The approach proposed by the Committee to generate correlation parameters for the three scenarios (para 54) fails to ensure that the resulting correlation matrices are positive-semidefinite – a fundamental property of correlation matrices which guarantees that for any such matrix $\boldsymbol{P}$, the scalar number given by $\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x}$ is non-negative, i.e. $\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x} \geq 0$, for any column vector of real numbers $\boldsymbol{x}$. The use of matrices which do not satisfy this property during the aggregation of risk-factor-level charges may thus result in a negative quantity appearing under the square root function, with in turn necessitates the use of ad-hoc flooring procedures being applied in the Standard (for example, the requirement that "*the quantity within the square root function is floored at zero*" found at the end of para 51(c) of the Standard).

In order to rectify this methodological flaw, the Committee should make use of the fact that the set of positive-semidefinite symmetric matrices is convex, which implies that for any two positive-semidefinite matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, and for any $\alpha \in (0,1)$, the matrix $\boldsymbol{C} = \alpha\boldsymbol{A} + (1-\alpha)\boldsymbol{B}$ is also positive-semidefinite. Thus, assuming that the correlation matrices used in the base "medium" correlation scenario, $\boldsymbol{P}_{base}$, are positive semi-definite by construction, then the two correlation matrices to be used in the "low" and "high" correlation scenarios may be constructed via

$$\boldsymbol{P}_{low} = \alpha_{low}\boldsymbol{P}_{base} + (1-\alpha_{low})\boldsymbol{L}$$

and

$$\boldsymbol{P}_{high} = \alpha_{high}\boldsymbol{P}_{base} + \left(1-\alpha_{high}\right)\boldsymbol{H}$$

where $\boldsymbol{L}$ and $\boldsymbol{H}$ are some positive-semidefinite matrices of "target" low and high correlations respectively. In particular, for the low correlations one natural choice for the target matrix is $\boldsymbol{I}$, the identity matrix with 1 on the diagonal and 0 everywhere else. With this choice for $\boldsymbol{L}$, it can be seen that the off-diagonal element of $\boldsymbol{P}_{low}$ are given by

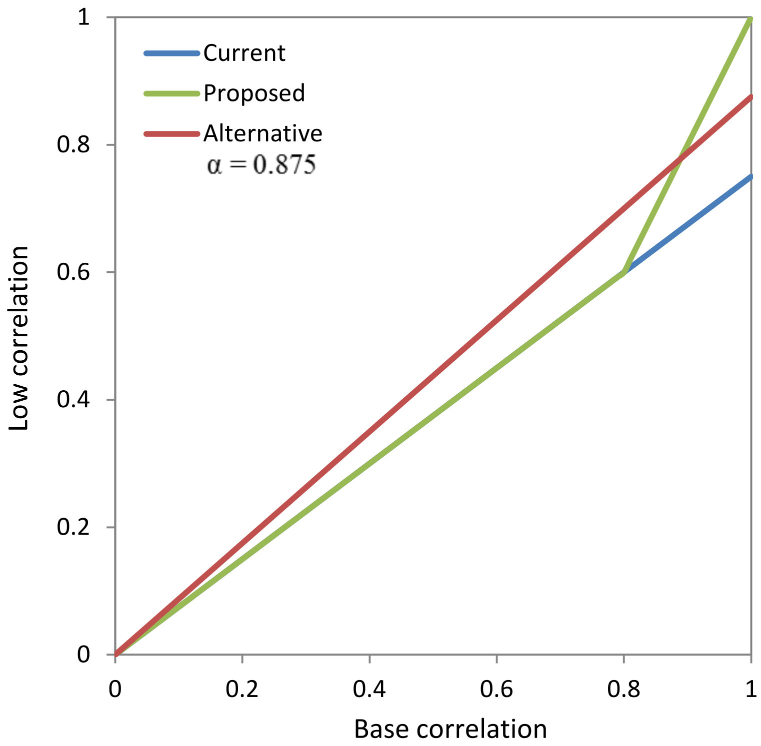
$$(\boldsymbol{P}_{low})_{i,j} = \alpha_{low}(\boldsymbol{P}_{base})_{i,j} + (1-\alpha_{low})(\boldsymbol{I})_{i,j} = \alpha_{low}(\boldsymbol{P}_{base})_{i,j} = 0.75(\boldsymbol{P}_{\boldsymbol{0}})_{i,j}$$

where in the last step we set $\alpha_{low} = 0.75$ to achieve the same result as is envisaged in January 2016 version of the Standard. For, the target high correlation matrix $\boldsymbol{H}$, a matrix with 1 on the diagonal and with, for example, 0.999 everywhere else might be chosen, in which case the off diagonal elements of $\boldsymbol{P}_{high}$ become

$$\left(\boldsymbol{P}_{high}\right)_{i,j} = \alpha_{high}(\boldsymbol{P}_{base})_{i,j} + \left(1-\alpha_{high}\right)(\boldsymbol{H})_{i,j} = 0.75(\boldsymbol{P}_{base})_{i,j} + 0.24975$$

where in the last step we set $\alpha_{high} = 0.75$ to achieve consistency with the "low" scenario settings.

In addition to the formulas contained in the January 2016 version of the Standard for generation of correlation parameters in the three scenarios, the consultative document "Revisions to the minimum capital requirements for market risk" issued on March 2018 also contains a proposal in para 54(c) of Annex A1 to modify the way in which "low" correlations are calculated. That proposal, unfortunately, also does not guarantee that the resulting correlation matrix is positive-semidefinite. Therefore, in order to achieve the stated goal of the revision to limit the impact of the reduction in correlations in cases where the observed base correlations are high, while at the same time ensuring that the resulting correlation matrices are positive-semidefinite, a different choice of α_{low} could be made. Figure 1 below compares correlations produced under the current and proposed methods, as well as the one described here with of $\alpha_{low} = 0.875$.



To summarise, in order to ensure that the correlation matrices used in the three scenarios are positive-semidefinite and thus to avoid introducing ad-hoc flooring in the aggregation formulas, the convex property of the set of positive-semidefinite symmetric matrices could be used and, subject to calibration of α_{low} and α_{high}, the following wording for para 54(a) and 54(c) is proposed:

(a) In the first scenario, "high correlations", the correlation parameters ρ_{kl} and γ_{bc} that are specified in Sections 4 to 6 are replaced by $\rho_{kl}^{high} = \alpha_{high} \cdot \rho_{kl} + 0.999 \cdot (1 - \alpha_{high})$ and $\gamma_{bc}^{high} = \alpha_{high} \cdot \gamma_{bc} + 0.999 \cdot (1 - \alpha_{high})$

(c) In the third scenario, "low correlations", the correlation parameters ρ_{kl} and γ_{bc} that are specified in Sections 4 to 6 are $\rho_{kl}^{low} = \alpha_{low} \cdot \rho_{kl}$ and $\gamma_{bc}^{low} = \alpha_{low} \cdot \gamma_{bc}$.