Global contagion of volatilities and volatility risk premiums

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March 25, 2010

Draft for BoJ-BIS 3rd Annual Workshop of the Asian Research Networks

Abstract

This paper proposes a method for measuring volatility risk premiums using option prices and high-frequency intra-day price data, which then apply to stock indices in Japan, Europe, and the US. The paper also investigates how volatilities and volatility risk premiums propagate among the markets and how the interdependency through the propagation changes during the course of the global financial market turmoil after the summer of 2007. Our studies reveal that the return shocks and the successive increases in volatilities and the volatility risk premiums evolved through global equity markets. Specifically, we identify i) our estimate of the volatility risk premiums show stronger correlation with market risk indicators than those reported in earlier studies, ii) the positive spillover effects among equity returns remain positive with additional counter feedback to the US market during the turmoil, iii) the volatility shows strong reciprocal dependency among the three markets after the Lehman Brothers bankruptcy. As for the contagion of volatility risk premiums, iv) while the interdependency is weakened after the summer of 2007, it grows stronger after the Lehman Brothers bankruptcy in most of the directions, particularly from Europe to US.

Keywords: volatility risk premium; model-free implied volatility; realized volatility; VIX; volatility spillover; time-varying parameter VAR

JEL Classification: G12, G13, G14

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*I would like to thank Peter Carr, Sacha Stoikov, Toshiaki Watanabe, and seminar participants at Diawa International Workshop on Financial Engineering 2008, Conference on High-frequency Data Analysis in Financial Markets at Hitotsubashi University for useful comments and suggestions. I also thank Jochi Nakajima for giving me many advices and providing his MCMC program for the estimation of a stochastic volatility VAR model. Views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan.

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1 Introduction

The turmoil of global financial markets since 2007 exhibited the existence of various contagion channels through which a plunge of an asset price in a region triggered subsequent falls in asset prices in the other regions across borders. Propagations of collapse in financial markets were accelerated by de-leverage or risk reduction at globally active financial institutions. There are many arguments on the propagation mechanisms. Some argue that large financial institutions play important rolls in various financial markets including mortgages, credit derivatives, corporate loans, commodities and money markets all over the world, and that large losses from an asset might lead to fire sales of other assets independent of their fundamental values. Others point out that globally active financial institutions unintentionally made up tightly connected network of debts and credits through risk transfer businesses, namely "originate and distribute" model.

While these arguments describe credit or liquidity channels of contagion, another channel of contagion exists, that is contagion of volatilities and risk premiums across financial markets. The volatility addresses the magnitude of shocks. The risk premium includes market participants' view for uncertainty in asset prices, more accurately, their expectation for future volatility, along with risk aversion defined in an investor's utility function. In addition to uncertainty in future asset price levels, market participants face another uncertainty in future volatilities. The fluctuating secondary moment requires the volatility risk premium (VRP) in the same way as the first moment requires the general (first order) risk premium.

This paper examines how volatilities and VRPs in three major equity markets are affected each other in the period of the global market turmoil including the highly volatile period of post Lehman Brothers bankruptcy. VRP is simply defined by the difference between the squared implied volatility under the risk neutral measure and the squared volatility under the real measure given a period of observation. The VIX index traded in the Chicago Board Options Exchange is such an implied volatility measure on the S&P500 stock index. The realized volatility (RV) is such a volatility measure under the real measure.

The concept of VRP has been well known and earlier studies have already examined VRPs in different manners from our approach. Scott [1987], Wiggins [1987] and Hull and White [1987] introduced the stochastic volatility (SV) model with a VRP, while Wiggins [1990] derived a VRP from the partial differential equation for a derivative price on the assumption that an underlying asset price followed the SV model. Bakshi and Kapadia [2003] applied this method and showed that the VRP was negative for options buyers and that the absolute value of the VRP was likely to remain quite large in periods of marked volatility. Uchida and Miyazaki [2008] have applied the same method as Bakshi and Kapadia [2003] to the Japanese stock market with similar findings.

On the other hand, some recent works such as Carr and Wu [2009] and Bollerslev, Tauchen, and Zhou [2009] have examined non-parametric estimates for a VRP defined by the difference between the model-free integrated implied variance, thereafter IIV, and integrated realized variance, thereafter IRV. IIV is a variance under the risk neutral measure, which is estimated from European option prices with different strike prices without any specific models for underlying asset price processes. IIV is called "integrated" due to the integration of instantaneous implied volatilities from current date to the option maturity. IRV is a non-parametric estimator of the variance under the real measure, which is the sum of daily RVs up to the maturity. RV is calculated from high-frequency intra-day price data. Matching integration periods for IIV and IRV makes it possible to estimate the VRP defined by the difference between the risk neutral measure and the real measure.

This paper analyses the non-parametric estimate of VRP. While the non-parametric estimation is easier to implement than the parametric approach using a SV model, a filtration inconsistency problem stems from the application of IRV in the non-parametric approach. Market expected IRV cannot be obtained from market data on date t while IIV can be estimated on the date t from option prices with expiration date T $(> t)$. If we employ an ex post IRV obtained at the date T, the filtration for IRV does not match the filtration for IIV on the date t. To determine the expected IRV under the information set with the same time span for IIV, we assume the market participants' expectation for future realized volatility is described by time series models for RV on daily basis. We choose the Heston [1993]'s model and the ARFIMAX (autoregressive fractionally integrated moving average plus X) model proposed by Giot and Laurent [2004] for such models. The expected VRP is identified as the gap between the model-base estimate for IRV and the current IIV. For comparison, we examine a VRP on an alternative assumption that future volatilities, in average, keep the same level as the previous period with the same integration length.

This paper applies the methods to major stock indices and their options in Japan, Germany, and the US from July 2003 to October 2008 and compares our proposed VRP measure with the VRP measures based on earlier studies. The paper also examines how RVs and VRPs, along with the equity return of the underlying asset prices, propagated among the three major equity markets in the period of the global financial market turmoil. The evaluation of the spillover is conducted by the time-varying-parameter structural vector autoregressive model with stochastic volatility in the residuals, which seems to be a new approach for the spillover evaluation as far as the author knows.

The analyses reveal that the synchronicity of equity returns, RVs, and VRPs among the three markets. The positive spillover effects among the equity returns remain positive with reinforcement of the counter feedback to the US market during the turmoil. The volatility shows reciprocal dependency among the three markets after the Lehman shock. As for the risk premium contagion, while the interdependency is weakened during the period from the summer of 2007 to the Lehman shock, it grows stronger after the Lehman shock in most of the directions, particularly from Europe to US. The strong dependency of volatilities reinforces the magnitude of returns propagated from overseas markets, along with the up-welling market sentiments evaluated by VRPs. This cyclical propagation seemed to maintain high volatility in the global equity markets and lead to consecutive escalation of risk premium the market participants required. Our studies shed light on the two aspects of contagion among financial markets; namely contagion of shock's magnitude measured by RV and contagion of the magnitude's uncertainty measured by VRP. Our empirical studies supports that our proposed method of volatility risk premium are informative to identify contagion in volatility uncertainty which plays a complement roll in spreading market turmoil.

The remainder of this paper is organised as follows: Section 2 defines the IIV and the IRV. Section 3 describes the four measures of VRP and discusses the filtration inconsistency problem arising from the mismatch of integration period. We also discuss how these VRPs relate to the VRP under the SV model which is often used in the parametric approach. Section 4.1 explains the data and basic statistics of IIV, RV, and VRP, and Section 5 discusses propagation among the global equity markets in the financial turmoil after the sub-prime mortgage loan crisis with respect to RV contagion and VRP contagion. Lastly, Section 6 offers a conclusion regarding these discussions.

2 Integrated Implied and Realized Variances

Suppose an asset price at time t, S_t, generates i.i.d. instantaneous returns. The integrated quadratic variation for S_t at time t until time T ($T > t$) is defined as:

$$\langle S \rangle_{t,T} = \frac{1}{T-t} \int_t^T \left(\frac{dS_u}{S_{u^-}} \right)^2 = \frac{1}{T-t} \int_t^T \frac{1}{S_{u^-}^2} d[S,S]_u, \tag{1}$$

where $1/(T-t)$ is an annualizing conversion coefficient and S_{t^-} is the stock price just before the time t. The expected value of the future quadratic variation is estimated by option prices, and the realized value of the past quadratic variation is estimated by the underlying asset processes. The former estimator is the IIV, defined as the expected variance under the risk neutral measure $\mathcal{Q}$ by

$$\sigma_{IIV}^2(t,T) = E^{\mathcal{Q}}[\langle S \rangle_{t,T} | \mathcal{F}_t], \tag{2}$$

where $\mathcal{F}_t$ is the filtration generated by the asset prices process until time t. The latter estimator is the IRV, defined as a consistent estimator of the realized variance under the real measure $\mathcal{P}$ by

$$\sigma_{IRV}^2(t,T) = E^{\mathcal{P}}[\langle S \rangle_{t,T} | \mathcal{F}_T] = \langle S \rangle_{t,T}. \tag{3}$$

Note that the IRV is a time-T value, hence not observable at time t.

The IIV is estimated from out-of-the-money European put and call options prices across all strikes. Demeterfi *et al.* [1999] and Britten-Jones and Neuberger [2000] derived the IIV by option prices as

$$\sigma^2_{IIV}(t,T) = \frac{2}{(T-t)B(t,T)} \left(\int_0^{F(t,T)} \frac{P(t,T,K)}{K^2} dK + \int_{F(t,T)}^{\infty} \frac{C(t,T,K)}{K^2} dK \right), \quad (4)$$

where $B(t,T)$, $F(t,T)$, $P(t,T,K)$ and $C(t,T,K)$ denote a risk-free discounted bond price, a forward price, and a put and a call option price at time t with a maturity time T, and a strike price K, respectively. See Appendix A for the derivation of Eq.(4). The IIV represents the aggregated expectation of the future volatility in the options market. The volatility index on the S&P500, the VIX index traded on the Chicago Board Options Exchange, is based on the discretised formula of Eq.(4).

The IRV is estimated from high-frequency ex post intra-day stock returns by discretising the integral in Eq.(1). Assume we have M days from time t to time T and let $\{t_i\}_{i=1}^{M}$ denote a set of days from t to T where $t_0 = t$ and $t_M = T$. The IRV is estimated as

$$\sigma^2_{IRV}(t,T) = \frac{1}{T-t} \sum_{i=1}^{M} \sigma^2_{RV}(t_i), \quad (5)$$

where $\sigma^2_{RV}(t_i)$ is the RV on day i. Further assuming there are N i.i.d. intra-day returns on each day i, $\{r_{j,i}\}_{j=1}^{N}$, we can estimate $\sigma^2_{RV}(t_i)$ as

$$\sigma^2_{RV}(t_i) = \sum_{j=1}^{N} r^2_{j,i}. \quad (6)$$

Theoretically, when N is large enough, the right hand side of Eq.(5) converges in probability to Eq.(1), i.e.,

$$\operatorname*{plim}_{N\to\infty} \sigma^2_{IRV}(t,T) = \langle S \rangle_{t,T}. \quad (7)$$

However, many earlier studies showed that, when the data frequency, or N, is too high, Eq.(6) is likely to overestimate RV because of an increase in market microstructure noises such as a price reciprocal development between the bid and offer prices within a short time period, so-called “bid ask bounce.”

3 Volatility Risk Premiums

3.1 The definitions and filtration inconsistency problem

Using IIV and IRV defined in Section 2, this paper defines the VRP as the gap between them, i.e.,

$$\mathrm{VRP} = \mathrm{IRV} - \mathrm{IIV}. \tag{8}$$

VRP is equivalent with the covariance between the pricing kernel and the quadratic variation defined in Eq.(1). Let ξ_t denote the Radon-Nikodým derivative process as

$$\xi_t = E\left[\left.\frac{d\mathcal{Q}}{d\mathcal{P}}\right| \mathcal{F}_t\right], \tag{9}$$

and let $\{M_{t,u}\}_{u\geq t}$ the pricing kernel process as

$$M_{t,u} = \xi_u/\xi_t \quad (u \geq t). \tag{10}$$

Note that ξ_t is a martingale under the real measure $\mathcal{P}$, i.e., $E^{\mathcal{P}}[M_{t,u}|\mathcal{F}_t] = 1$ for any $u > t$. Then IIV and IRV have the following relationship.

$$\begin{aligned}
\sigma^2_{IIV}(t,T) &= E^{\mathcal{Q}}[\langle S\rangle_{t,T}|\mathcal{F}_t] \\
&= E^{\mathcal{P}}\left[\left.\int_t^T M_{t,u}\langle S\rangle_{t,u}du\right| \mathcal{F}_t\right] \\
&= E^{\mathcal{P}}[\langle S\rangle_{t,T}|\mathcal{F}_t] + \int_t^T \mathrm{Cov}^{\mathcal{P}}\left[M_{t,u}, \langle S\rangle_{t,u}|\mathcal{F}_t\right]du, \\
&= \tilde{\sigma}^2_{IRV}(t,T) - \tilde{\lambda}(t,T),
\end{aligned} \tag{11}$$

where $\tilde{\sigma}^2_{IRV}(t,T) = E^{\mathcal{P}}[\langle S\rangle_{t,T}|\mathcal{F}_t]$, and $\tilde{\lambda}(t,T) = -\int_t^T \mathrm{Cov}^{\mathcal{P}}(M_{t,u}, \langle S\rangle_{t,u}|\mathcal{F}_t)du$ denotes the true VRP at time t integrated from time t to time T. In Eq.(11), we add a tilde $\tilde{\ }$ on the IRV to emphasise the difference in filtration from our previously defined IRV; $\sigma^2_{IRV}(t,T)$ defined in Eq.(3) is $\mathcal{F}_T$-measurable, whereas $\tilde{\sigma}^2_{IRV}(t,T)$ in Eq.(11) is $\mathcal{F}_t$-measurable. $\tilde{\sigma}^2_{IRV}(t,T)$, and hence $\tilde{\lambda}(t,T)$, cannot be computed from historical market data.

Alternatively, we define the realized VRP, $\overline{\lambda}(t,T)$, hereafter rVRP, using $\mathcal{F}_T$-measurable variance as:

$$\begin{aligned}
\overline{\lambda}(t,T) &= E^{\mathcal{P}}[\langle S\rangle_{t,T}|\mathcal{F}_T] - E^{\mathcal{Q}}[\langle S\rangle_{t,T}|\mathcal{F}_t] \\
&= \sigma^2_{IRV}(t,T) - \sigma^2_{IIV}(t,T),
\end{aligned} \tag{12}$$

which is computed from historical market data any time after time T. The rVRP are knows to be related to the delta-hedged gain of options. Though we can conduct an ex post calculation of Eq.(12), rVRP is still not a market expected VRP.

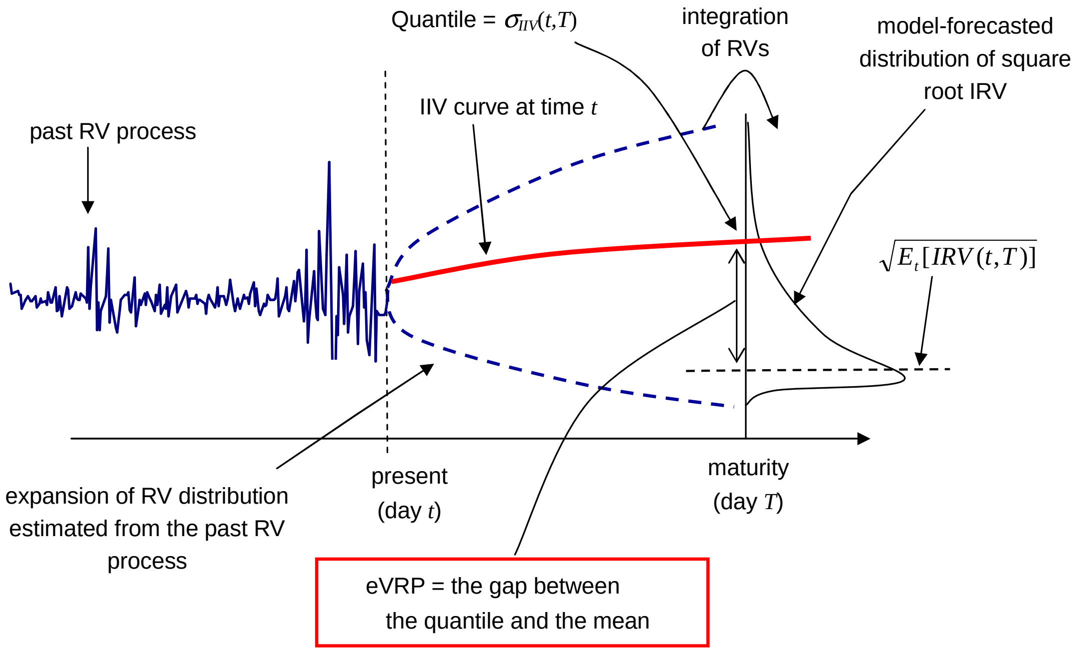

Figure 1: The concept of eVRP

In order to evaluate VRP which reflects market expectations, the filtration inconsistency should be resolved, which arises from the difference in the measurement periods of IIV and IRV. Because this problem stems from the prediction of the future real measure, we introduce a model to predict the future IRV based on the past time series of RV. This approach gives a new VRP, namely a model-base expected VRP, or eVRP.

In the estimation for eVRP, we model RV process rather than IRV process to avoid the overlapping observation problem discussed by Christensen, Hansen, and Prabhala [2002], that is the adjacent IRVs share the same RVs in integration. We simply assume the model that best fit to the past RV process is a natural estimate of the future real measure. Let $\tilde{S}_u$ denote the model-based estimation of the future prices at time $u(> t)$ under the real measure, and let $\mathcal{G}_T = \sigma\left(\{S_u\}_{u=0}^{t}, \{\tilde{S}_u\}_{u>t}^{T}\right)$, then eVRP, $\lambda(t,T)$, is defined under the filtration $\mathcal{G}_T$ as:

$$\lambda(t,T) = E^{\mathcal{P}}[\langle S\rangle_{t,T}|\mathcal{G}_T] - \sigma_{IIV}^2(t,T), \tag{13}$$

or

$$\lambda(t,T) = -\int_t^T \mathrm{Cov}^{\mathcal{P}}(M_{t,u}, \langle S\rangle_{t,u}|\mathcal{G}_u)du. \tag{14}$$

The concept of eVRP is displayed in Figure 1.

The simplest estimation of $\mathcal{G}_T$ is the linear estimation; simply set $\tilde{S}_u = S_{t-(T-t)+u-t}$ using lagged RVs in the period $[t-(T-t), t]$. We call this IRV the trailed IRV denoted as

Table 1: The definition, notation and difference of VRPs

term	notation	explanation
true VRP	$\tilde{\lambda}(t,T)$	True expected VRP, unobservable at any time.
realized VRP (rVRP)	$\overline{\lambda}(t,T)$	Ex-post VRP, related to delta-hedged gain of options, observable after time T.
trailed VRP (tVRP)	$\underline{\lambda}(t,T)$	Expected VRP under static expectation assumption on future IRV, observable at time t.
expected VRP (eVRP)	$\lambda(t,T)$	Expected VRP using model-base forcast of IRV, observable at time t.

$\underline{\sigma}^2_{IRV}(t,T)$ and defined as:

$$\underline{\sigma}^2_{IRV}(t,T) = \langle S \rangle_{t-(T-t),t}. \tag{15}$$

The trailed VRP, or tVRP hereafter, is defined as:

$$\underline{\lambda}(t,T) = \underline{\sigma}^2_{IRV}(t,T) - \sigma^2_{IIV}(t,T). \tag{16}$$

tVRP is the expected VRP under the assumption that the IRV in the adjacent one month is the best prediction of the IRV in the following one month.

So far, we have introduced four types of VRPs. The definitions and the differences of VRPs are summarised in Table 1.

3.2 Valuation method for expected VRP

Two models to forecast future RV process are introduced in this section, which is then applied to eVRP evaluation defined in Section 3.1. The first model is the Heston [1993]'s type model, where RV process is subject to χ^2 distribution. The second model is ARFIMAX model proposed by Giot and Laurent [2004], which provides a good fit to the Japanese RV time series according to Watanabe and Sasaki [2007] or Shibata [2008].

3.2.1 Two models for RV forecasting

The first model is the Heston type model defined as:

$$dS_t = \mu S_t dt + \sigma_{RV}(t) S_t dW^1_t,$$

$$d\sigma^2_{RV}(t) = \kappa(\theta - \sigma^2_{RV}(t))dt + \sigma_V \sigma_{RV}(t) \left(\rho dW^1_t + \sqrt{1-\rho^2} dW^2_t \right), \tag{17}$$

where dW^1_t, dW^2_t are independent Weiner processes and $\mu, \kappa, \theta, \sigma_V, \rho$ are parameters that satisfy $\sigma_V > 0, |\rho| \leq 1, \sigma^2_V/(\kappa\theta) \geq 2$. The original model of Eq.(17) was proposed by Heston [1993], which is the stochastic volatility model with the mean-reverting square root volatility process. The model in Eq.(17) is constructed just by substituting the volatility process in the original Heston model for RV. Since the Heston model is commonly used among practitioners, eVRP based on the model is expected to represent the market expectation.

The second model for RV forecast is the ARFIMAX model

$$R_t = \sigma_{RV}(t) z_t,$$

$$(1-L)^d \{\ln(\sigma_{RV}^2(t)) - \mu_0 - \mu_1 |R_{t-1}| - \mu_2 \mathbf{1}_{\{R_{t-1}<0\}} |R_{t-1}|\} = (1+\delta L) u_t, \tag{18}$$

$$z_t \sim N(0, \sigma_z^2), \quad u_t \sim N(0, \sigma_u^2),$$

where L is a lag operator[1], $R_t = \ln(S_t/S_{t-1})$ is the one-day return on day t, and $\mu_0, \mu_1, \mu_2, \delta, \sigma_z$ and σ_u $(\sigma_z, \sigma_u > 0)$ are parameters.

Though both the Heston type and ARFIMAX models have mean-reverting property, the levels of the mean differs; the Heston type model has a fixed and deterministic mean of RV, whereas the ARFIMAX model has a stochastic mean which varies based on the previous day's return. The ARFIMAX model takes into account the volatility's persistence and asymmetric features. The former is the feature that the present level of volatility depends on the past level of volatilities, which is expressed in the fractionally integration in Eq.(18). The latter is the feature that the volatility is normally higher when the previous day's return is negative than when it is positive. This feature is expressed by the μ_3 parameter and $\mathbf{1}_{\{R_{t-1}<0\}}$ in Eq.(18) which takes non-zero value only when the previous day's return was negative. The "X" in ARFIMAX addresses this asymmetry. We assume the mean-reverting property of RV in those models can better express the market expectation for the future RV than the simple static expectation assumption in tVRP.

3.2.2 The calculation procedures

eVRP is computed by the following procedures.

First, IIV is estimated by applying option prices data to Eq.(4). IIV with one-month-terms to maturity is calculated by interpolating market-traded terms. For the risk-free discount bond prices $B(t,T)$, the interpolated rate of LIBOR and swap rates, or government bond prices, are used. [2]

Second, RV is computed from intra-day stock price data by Eq.(6). The appropriate choice of N, or the choice of a time interval to measure returns, is an ongoing issue for RV estimation. Because five-minute returns are widely used in earlier studies, we follow this choice of N. Note that the return during lunch break[3] and the return from closing to the next morning's opening are included in the daily RV. The first sample, $j = 1$ in Eq.(6),

[1] $Lx_t = x_{t-1}$.
[2] See Section 4.1 for the computation procedures for IIV.
[3] Lunch break exist only for Japanese data.

corresponds to the nighttime return from the last day's closing to the next morning's opening.

Third, the parameters of the two models are estimated. The parameters are estimated at each day t_i by the quasi-maximum likelihood method using the previous one-year of ex post RV data including the day t_0 itself. [4] Based on the analyses in Sugihara [2010], market's volatility expectation relies on the recent level of RV, the parameters are updated daily for better tracking the level.

Forth, RV process is simulated from the day t_0 to day t_M using the estimated parameters, and based on the simulated results, IRV is computed under the information $\mathcal{F}_t \vee \mathcal{G}_{t,T}$, by ten thousand times of Monte Carlo simulation on each day.

Lastly, eVRP is computed from IIV on day t_0 and the simulated IRV. The Monte Carlo simulation is again used for the calculation of eVRP. We repeat the above process for each day to get the time series of eVRP.

4 Data and basic statistics

4.1 Data

Before discussing empirical analyses, we briefly summarise the data used in the analyses. We analyse volatility of major stock indices in Japan (JP), Europe (EU) and the US (US), selecting the Nikkei 225 Stock Index (Nikkei), the DAX Index (DAX) and the S&P 500 Stock Index (S&P500). The Nikkei is the arithmetic mean of prices for 225 stocks for major Japanese corporations. The DAX is the total return for 30 major stock prices listed on the Frankfurt Stock Exchange. The S&P500 is the market-value weighted average of prices for 500 major stocks traded in the US.[5]

IIV is calculated much the same way among the Nikkei, DAX and S&P500. IIV on Nikkei is computed directly from a tick-by-tick options prices database [6] based on the computation method proposed by Jiang and Tian [2007]. In brief, the method involves interpolation and extrapolation implemented by a cubic spline function after converting option prices to Black-Scholes implied volatilities. We interpolate in the Black-Scholes implied volatilities rather than directly in option prices because the implied volatility surface is smoother and more readily interpolated by a cubic spline function; additionally, we can thereby avoid negative interpolated option prices. For the Nikkei, the range of strikes traded or priced is mostly from 11-tick ups and downs from at the money with a tick size of 500 yen.[7] For risk-free discount bond price $B(t,T)$, we use the interpolated rate for Yen

[4] The reason for the inclusion of the day t_0 is that the options market for the Nikkei closes at 15:10 whereas the stock market closes at 15:00. By comparing the closing IIV with the IRV, which are computed with past data including the current day, an unbiased VRP can be computed.

[5] Those price data set of indices used for RV calculation is obtained from Nexa Technologies Inc.

[6] Data is obtained from Nikkei Media Marketing Inc.

[7] The option market system has changed in Japan since September 2008 in such a way that the options

LIBOR and swap rates. For IIV on S&P and DAX, we use the VIX Index and VDAX Index. The VIX Index and VDAX Index are calculated and disseminated each trading day by the Chicago Board Options Exchange and by the Deutsche Boerse Market Data and Analytics, respectively. [8] See CBOE [2003] for detailed information on the VIX Index.

The time frame examined in this paper begins from July 2003 for RV, IRV, rVRP and tVRP data, and from July 2004 for eVRP data, since we need one year RV data for the previous year to compute eVRP. We compute data up to the end of October 2009.

All integrated data (IRV, IIV and VRPs) are exactly one-month-term to maturity. We assume 21 trading days per month, setting $M = 21$ in Eq.(5). For IIV on Nikkei, we compute exactly one-month-term IIV by interpolating IIVs with market-traded terms IIVs with a cubic spline function. For IIVs on S&P500 and DAX, linear interpolation is used.

Note that all data are computed daily as the closed price in each region. In local time, this is 15:00 for Japanese RV data (GMT+9 hours), 15:10 for Japanese IIV data, 17:30 for European data (GMT+1 hour), and 15:00 for US data (GMT−5 hours).

4.2 IIV and IRV time series

Figure 2 shows the time series for IIV and IRV for the Nikkei, DAX and S&P500 for all the sample period, plotted in the dimension of volatility – i.e., the square root of the integrated variance in Eq.(4) and Eq.(5). Two types of IRV are shown: future one-month IRVs and past one-month IRVs. The former covers the same integration period as the IIV, whereas the latter uses lagged samples for the daily RV. Future IRV is $\sigma_{IRV}(t, t + 1\text{month})$, while past IRV is $\sigma_{IRV}(t - 1\text{month}, t)$.

On the one hand, we see two characteristics: i) both IIV and IRV levels go up after September 2008, and surge prominently after October 2008, ii) IIV levels exceed those of IRVs through much of the time until September 2008; iii) IIV generally moves more in parallel to past IRV than to future IRV; iv) future IRV sometimes exceeds IIV, prominently in October 2008 for DAX and S&P500; and v) the gap between IIV and IRVs is greater in Japan than in Europe or the US.

The gap between IIV and IRVs corresponds to the VRP. [9] According to the definitions from Section 3.1, we consider the gap between future IRV and IIV to be rVRP and that

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with maturity less than 3 month have strike prices with 250 yen tick while the options other than that have 500 yen tick strikes. In order to keep consistency, we only use 500 yen tick of strike prices for Japanese options.

[8] The procedures used to calculate VIX and VDAX differs slightly from the method used by Jiang and Tian [2007] to calculate IIV for the Nikkei. The primary difference lies in how market traded option prices are interpolated and extrapolated. VIX and VDAX simply discretise Eq.(4) with the market-traded tick size of strike prices and provide a summary without interpolating option prices. According to the analysis by Jiang and Tian [2007], this method, when applied to VIX, may leads to results that are skewed high. We disregard this potential bias on the assumption that the bias would prove negligible compared to the volatility risk premiums.

[9] These values are computed in the dimension of volatility and may be somewhat apart from the square root of the VRP. We disregard such differences for the purpose of this discussion.

between past IRV and IIV to be tVRP. Based on the definition of Eq.(8) and above characteristic i), the signs for the VRPs are generally negative. Note that based on Eq.(??), the negative VRP reduces the risk premium on an underlying asset price itself, since the vega of European options is positive.

The negative sign on the VRPs indicates that option sellers require a premium for uncertainties in future volatility, or volatility risk. The above characteristics ii) and iii) lead to the following hypotheses: a) option sellers require a premium determined by observing past IRV to compensate for future volatility risk, and option buyers are willing to pay the premium; and b) the premium covers the volatility risk for option sellers most of time, but is sometimes undervalued. Particularly in late 2008 (more precisely, after the Lehman shock), IRV surged so rapidly and dramatically that IIV levels failed to follow or cover realized levels of IRV (i.e., future IRV) in Europe or the US, resulting in large positive realised VRP.

These results show the risk aversion of the financial markets and how market participants form expectations. Hypothesis a) supports our assumptions regarding tVRP calulation and eVRP calculation as well since eVRP is evaluated by ex post RV samples; expectations for future RV appears to be based on past developments in RV. We would surmise that model-base eVRP is a better estimate of expected VRP than tVRP based on static expectations for future RV.

4.3 Basic statistics

Table 2 summarises basic statistics for RV, IRV, IIV and VRPs. Those for IRV, IIV, rVRP, tVRP, eVRP-Heston and eVRP-ARFIMAX in Japan, Europe and the US are displayed in top-down fashion. The samples are divided into the three periods: A) from July 2003 to July 2007 (or from July 2004 to July 2007 for VRPs); B) from August 2007 to September 2008; and C) from October 2008 to October 2009. During the first period, A), the gravity of the problem in the US mortgage market was not yet fully recognised. We call this the "ordinary" period. We set the boundary of the phases of the subprime mortgage problem at August 2007, when global equity prices dropped dramatically in response to the announcement that cash withdrawals from funds managed by BNP Paribas had been frozen due to illiquidity in the US mortgage-backed securities market, so-called the "Paribas shock". The second period, B), marks the period following the severe recognition of the problem. We refer to this period as "Post Paribas Shock" The third period, C), which we call "Post Lehman Shock," is period after the dramatic spike in volatility following the Lehman Brothers' filing for Chapter 11 bankruptcy on September 15, 2008, so-called the "Lehman shock", up to the end of our sample period.

First, we see how realized volatility changes before and after the Paribas shock by comparing mean values of RV and IRV in Japan, Europe and the US. The values are

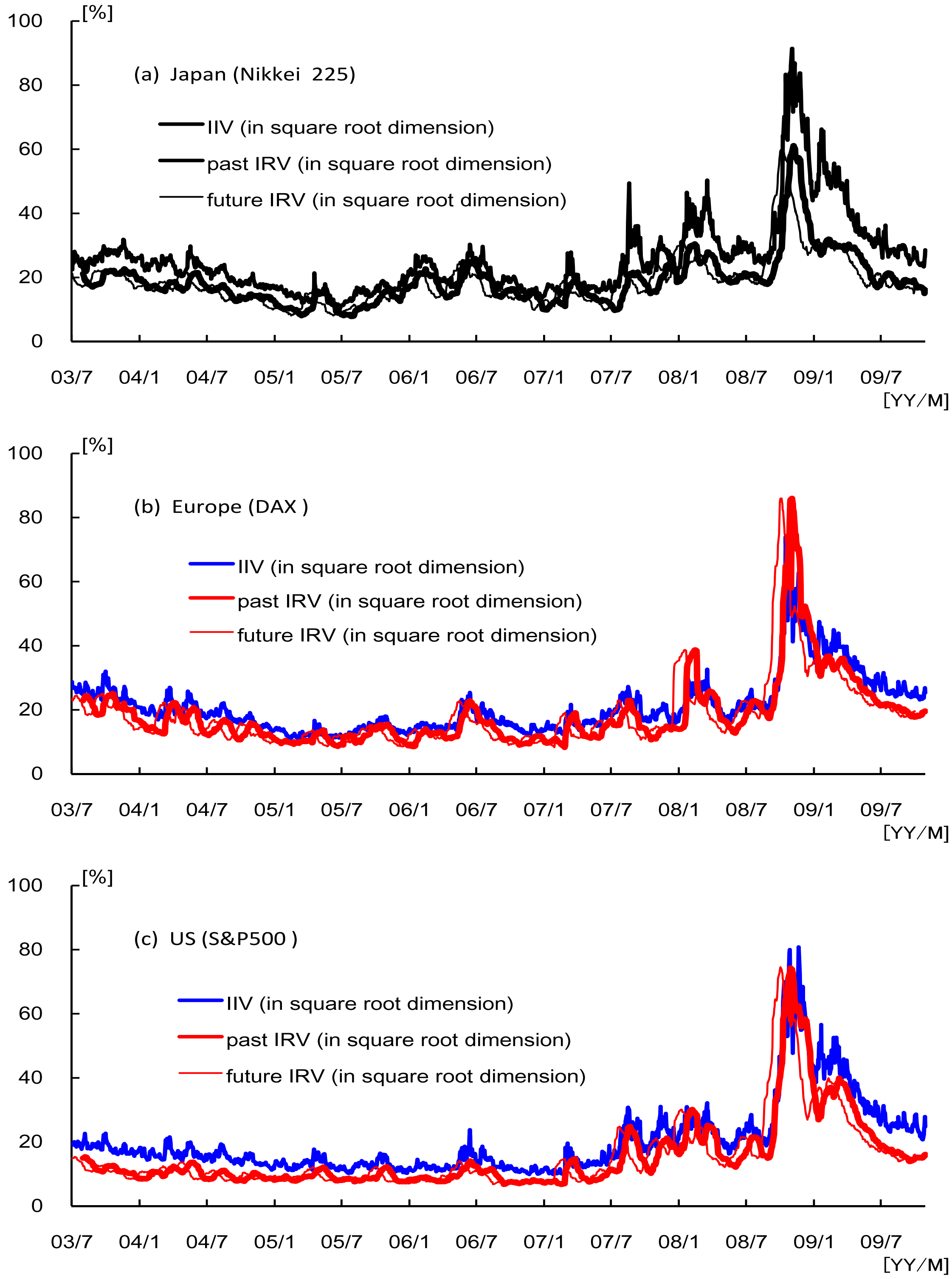


Figure 2: Time series of square root of IRV and IIV from July 2003 to October 2009

Notes: Thick dark line, thick light line, thin red line indicate square root of IIV, past IRV, and future IRV, respectively, with 1 month term plotted in percent scale. The future IRV is $\sigma_{IRV}(t, t + 1\text{month})$, and the past IRV is $\sigma_{IRV}(t - 1\text{month}, t)$.

highest in Japan before the Paribas shock. However, the mean values in Europe and the US increase after the Paribas shock, beating in the levels of Japan after Lehman shock. On the contrary, mean values of IIV are highest in Japan and lowest in Europe or the US. As a result, mean values of negative VRP are largest in Japan and smallest in Europe in any periods.

According to "standard deviation," the sixth column in each row of RV and IRV, we see the standard deviation for RV and IRV, the volatility's volatility, increases dramatically following the Paribas shock in every region. Particularly, it is prominent in Europe and in the US, which leads to positive means and positive skews for realised VRP. This indicates that VRP levels in Europe and in the US are sometimes turned out to be insufficient and that option sellers experience losses due to unexpected spikes in future volatility. On the contrary, the VRP in Japanese market is set high enough to cover unexpected increases in volatility, even after the Lehman shock.

4.4 Difference in VRPs

This section considers which VRP should be referred to in the four types of VRPs defined in Section 3. The discussion in this section is not based on a rigorous theory but from an intuitive approach since the true level of VRP is not rigorously known.

Figure 3 plots the time series of four types of VRPs. The magnitude of the change in VRP is quite large for all equity indices just after the Lehman shock. And at the same time, rVRP surge to the positive region in all indices, followed by tVRP's increase in Europe and US after a few months' interval. However, the absolute value of eVRP jumps up but stays negative in most of the period after the Lehman shock for any indices. This implies that negative eVRP is consistent with the intuitive movement of risk aversion which is considered to become larger in the turmoil.

We then compare VRPs to certain risk indicators regarded to reflect market risk appetite/aversion. The Citi Macro Risk Index (CMRI), [10] credit default swap indices (CDS indices), [11] and swap spreads [12] with five years to maturity are chosen as risk indicators. CMRI measures overall risk aversion of global investors, CDS indices measure credit

[10] The Citi Macro Risk Index is an equally weighted index of emerging market sovereign spreads, US credit spreads, US swap spreads and implied FX, equity and swap rate volatilities. The index is expressed in a rolling historical percentile and ranges between 0 (low risk aversion) to 1 (high risk aversion). Data source is Citibank Ltd.

[11] The iTraxx Japan, iTraxx Euro and CDX North America are chosen for the CDS indices. Those refers around 50 to 150 of most liquid investment grade credit default swap premiums on Japanese, European and North American entities, respectively. The indication is arithmetic average of index CDS premiums collected from broker dealers. It starts to be computed from July 2004 for iTraxx and November 2003 for CDX. All have five years term to maturity. Data source is Markit Group Ltd.

[12] The swap spread is computed by subtracting government bond rates from swap rates with same term to maturity. For swap rate, LIBOR (London inter-bank offered rate) are chosen for Japan and German data and US money market fixing rate is used for the US data. All have five years term to maturity. Data source is Bloomberg.

Table 2: Basic Statistics

		mean	standard error	median	standard deviation	excess kurtosis	skewness	number of samples
A) Ordinary period (Jul-03 $\sim$ Jul-07)								
RV [$\%^2$]	JP	1.02	0.03	0.80	0.80	6.70	2.05	1,003
	EU	0.86	0.03	0.61	0.81	12.13	2.86	1,043
	US	0.39	0.01	0.32	0.30	21.05	3.41	1,028
$\sqrt{\text{IRV}}$ [%]	JP	15.44	0.12	15.14	3.90	−0.69	0.16	982
	EU	14.01	0.12	12.92	3.96	−0.06	0.88	1,021
	US	9.64	0.06	9.14	1.85	−0.13	0.77	1,006
$\sqrt{\text{IIV}}$ [%]	JP	19.87	0.15	18.78	4.62	−0.93	0.27	1,002
	EU	17.27	0.13	16.14	4.31	0.06	0.85	1,043
	US	14.29	0.09	13.75	2.74	−0.15	0.66	1,028
rVRP [$\%^2 p$]	JP	−160.39	4.19	−146.87	132.62	−0.18	−0.17	1,003
	EU	−72.39	2.91	−78.37	81.83	2.40	0.56	791
	US	−83.93	2.45	−84.21	68.24	9.46	1.35	776
tVRP [$\%^2 p$]	JP	−121.06	2.62	−113.06	72.16	3.48	−1.19	759
	EU	−76.15	2.05	−73.88	56.93	2.63	−0.14	769
	US	−91.79	1.73	−82.69	47.66	6.74	−1.71	756
eVRP [$\%^2 p$] (Heston)	JP	−75.30	5.14	−32.76	141.64	0.88	−1.18	758
	EU	−42.06	3.13	−33.56	87.93	2.39	−1.20	789
	US	−83.54	2.08	−68.75	57.81	8.14	−1.98	776
eVRP [$\%^2 p$] (ARFIMAX)	JP	−85.66	7.65	−43.73	210.75	0.46	−0.88	758
	EU	−46.88	4.34	−53.28	121.77	3.92	0.18	789
	US	−84.94	2.16	−69.56	60.11	8.50	−2.25	775
B) Post Paribas Shock (Aug-07 $\sim$ Sep-08)								
RV [$\%^2$]	JP	1.94	0.09	1.55	1.45	7.77	2.36	287
	EU	1.75	0.17	1.12	2.87	104.46	8.77	297
	US	1.80	0.14	1.14	2.39	21.84	4.25	295
$\sqrt{\text{IRV}}$ [%]	JP	21.26	0.22	20.59	3.70	−0.11	0.62	287
	EU	19.40	0.37	18.60	6.40	1.33	1.23	297
	US	19.21	0.31	18.68	5.37	0.98	0.92	295
$\sqrt{\text{IIV}}$ [%]	JP	29.52	0.35	28.12	5.96	0.72	0.94	287
	EU	21.81	0.21	21.40	3.68	−0.04	0.60	297
	US	23.49	0.24	23.12	4.13	3.73	1.25	295
rVRP [$\%^2 p$]	JP	−340.38	28.46	−335.32	482.15	3.19	0.47	287
	EU	116.49	46.82	−125.54	806.93	15.24	3.75	297
	US	33.43	43.64	−172.33	749.47	10.92	3.26	295
tVRP [$\%^2 p$]	JP	−441.65	17.69	−369.12	299.74	5.51	−1.72	287
	EU	−71.88	13.05	−115.89	224.86	6.34	2.32	297
	US	−170.94	8.88	−167.30	152.56	0.68	0.11	295
eVRP [$\%^2 p$] (Heston)	JP	−510.06	22.86	−388.94	387.24	2.08	−1.31	287
	EU	−129.89	9.97	−103.63	171.83	−0.61	−0.34	297
	US	−256.05	11.32	−239.52	194.43	1.75	−0.83	295
eVRP [$\%^2 p$] (ARFIMAX)	JP	−671.31	22.51	−596.28	381.41	1.82	−1.24	287
	EU	−292.39	10.22	−240.22	176.14	0.94	−1.01	297
	US	−436.53	14.39	−415.79	247.07	7.08	−1.40	295

		mean	standard error	median	standard deviation	excess kurtosis	skewness	number of samples
C) Post Lehman Shock (Oct-08 $\sim$ Oct-09)								
RV [%^{2}]	JP	3.69	0.28	2.31	4.57	13.54	3.32	264
	EU	5.59	0.59	2.62	9.83	30.24	5.06	275
	US	5.05	0.47	2.38	7.77	38.56	4.97	274
$\sqrt{\text{IRV}}$ [%]	JP	28.63	0.72	28.21	11.68	0.85	1.24	264
	EU	33.92	1.02	29.71	16.85	1.53	1.49	275
	US	32.41	1.02	28.74	16.82	-0.22	0.90	274
$\sqrt{\text{IIV}}$ [%]	JP	44.12	1.09	38.49	17.71	-0.06	0.92	264
	EU	35.71	0.63	34.45	10.46	0.52	0.98	275
	US	39.20	0.85	37.41	14.10	-0.18	0.78	274
rVRP [%^2p]	JP	-644.68	51.53	-394.04	905.76	17.67	-4.00	309
	EU	-185.24	68.76	-378.56	$1,140.18$	17.89	4.10	275
	US	-655.62	51.57	-576.13	853.61	8.94	1.07	274
tVRP [%^2p]	JP	$-1,303.87$	70.40	-742.96	$1,143.83$	3.06	-1.65	264
	EU	49.79	58.94	-231.37	977.42	11.51	3.16	275
	US	-402.39	36.87	-379.41	610.27	8.89	1.56	274
eVRP [%^2p] (Heston)	JP	$-1,746.59$	109.27	-883.12	$1,775.41$	1.51	-1.45	264
	EU	-257.03	54.39	-92.04	901.88	1.27	-0.48	275
	US	-31.88	64.58	309.30	1069.04	3.67	-1.57	274
eVRP [%^2p] (ARFIMAX)	JP	$-1,616.78$	120.14	-899.09	$1,952.11$	2.04	-1.55	264
	EU	-525.74	116.96	-778.08	$1,939.51$	10.21	2.43	275
	US	-911.32	125.48	$-1,013.56$	$2,076.99$	3.10	1.03	274

Notes: “JP” refers to Japanese data, “EU” for European data and “US” the US data. See Section 4.1 for details of data. Number of samples differs due to the difference in integration periods and national holidays. The pre-integrated data of all except for RV are overlapped and may be autocorrelated.

risk in the referenced major enterprises, and swap spreads measure counter-party risk for major inter-bank market players (more accurately, the differences in credit risk between government and those players). All indicators rise as market risk appetite or sentiment deteriorates. Since risk preference in the stock market generally parallels risk preference in the credit or inter-bank markets, volatilities or VRPs are expected to exhibit stronger correlation with those indicators.

Table 3 displays correlations of IIV, IRV, VRPs with the risk indicators. Comparing IIV and IRVs (the first eleven rows), we see that IIV shows a relatively stronger correlation with risk indicators. Specifically, IIV strongly correlates with CDS indices for every region, which suggests that implied volatilities are priced by accounting for the credit risk of major enterprises. Hence, IIV is good metrics for market risk aversion. Comparing correlations for each VRP, we see that tVRP and eVRPs are negatively correlated with risk indicators, particularly in Japan, but it does not hold for rVRP. The negative correlation indicates that risk aversion among option sellers matches market sentiments. Comparing trailed, realized and expected VRPs, we see that eVRPs show an even stronger correlation with risk indicators, attributable to the coherent cycle of eVRPs and the risk indicators, which indicates that eVRP is a better measure of market risk aversion than trailed or realized VRPs.

5 Global contagion

This section evaluates interdependencies of equity markets in Japan, Europe, and US. In particular, we determine the country which originates shocks (i.e., increase or decrease in volatilities or risk premiums), and the direction of contagion during the course of the financial turmoil.

For this purpose, we apply the time-varying-parameter structural vector autoregressive models (TVP-VAR) to evaluate the interdependencies. TVP-VAR has recently became popular in economic literatures such as Cogly and Sargent [2005], Primiceri [2005], or Nakajima, Kasuya, and Watanabe [2009]. While those literatures examine macroeconomics data, we apply the model to examine spillovers of financial markets. We consider two types of TVP-VAR: the constant volatility type where the volatility of the structural shock is a constant over time, and the stochastic volatility type where the volatility varies over time as well.

5.1 Model and parameter estimation

5.1.1 The model to evaluate contagion

Suppose we have $T+1$ samples of end-of-the-day market data vector set $\{\mathbf{y}_t\}_{t=0}^{T}$ from day 0 to day T with $\mathbf{y}_t = (y_t^J, y_t^E, y_t^U)^\top$ where y_t^J, y_t^E, and y_t^U denote market data of Japan,

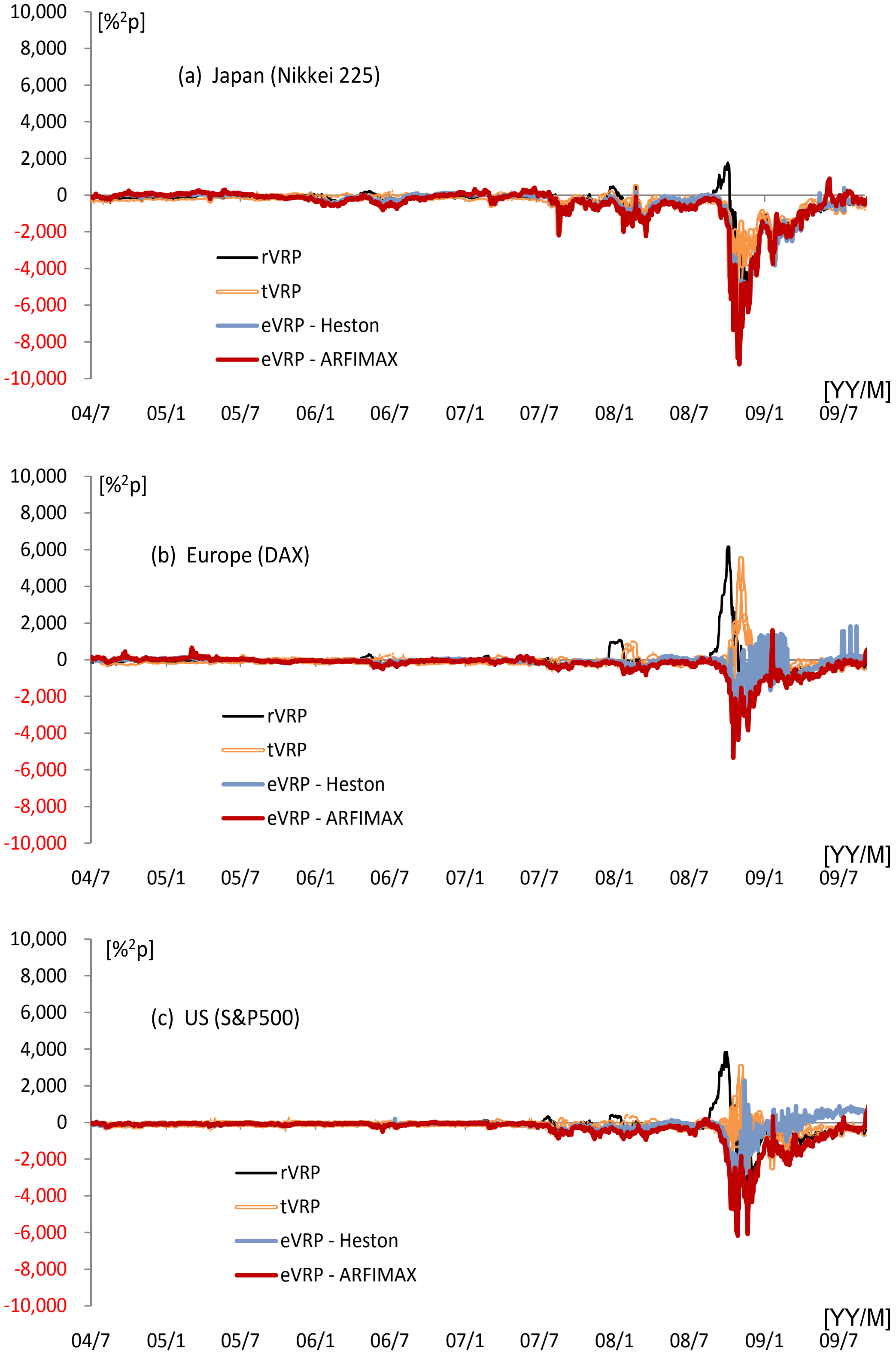


Figure 3: VRP time series

Table 3: Correlation with risk indicators

		CMRI	CDS Index	Swap Spread
IIV	JP	0.55	0.79	0.47
	EU	0.42	0.88	0.73
	US	0.52	0.91	0.57
past IRV	JP	0.54	0.70	0.47
	EU	0.45	0.79	0.73
	US	0.56	0.87	0.63
future IRV	JP	0.58	0.60	0.49
	EU	0.46	0.70	0.74
	US	0.00	0.79	0.68
rVRP	JP	−0.33**	−0.66**	−0.29**
	EU	0.19	−0.02	0.23
	US	0.00	−0.39	0.08
tVRP	JP	−0.42**	−0.70**	−0.34**
	EU	0.20	0.15	0.30
	US	−0.08**	−0.40**	−0.04
eVRP (Heston)	JP	−0.47**	−0.66**	−0.35**
	EU	−0.35**	−0.31**	−0.33**
	US	−0.49**	−0.15**	−0.52**
eVRP (ARFIMAX)	JP	−0.52**	−0.60**	−0.39**
	EU	−0.41**	−0.39**	−0.42**
	US	−0.47**	−0.52**	−0.52**

Notes: The correlations to CMRI and swap spreads are computed using data from July 2003 to October 2009 (for eVRPs from July 2004), whereas those to CDS indices use data from July 2004 to October 2009. "JP" stands for Japanese data (iTraxx Japan and Yen LIBOR swap spreads from Japanese government bonds), "EU" for German data (iTraxx Euro and Euribor spreads from German government bonds), and "US" stands for North American data (CDX North America and US dollar swap spreads from US government bonds). CDS indices and swap spreads are five years to maturity. Numbers with ** indicate statistically significant negative correlations with 1% significance level.

Europe, and US at the end of the day t. Considering the time difference among those regions, we examine the interdependency by the following model.

$$\begin{cases} y_t^J = c_0^J + c_t^{JJ1} y_{t-1}^J + c_t^{EJ1} y_{t-1}^E + c_t^{UJ1} y_{t-1}^U + \cdots + c_t^{JJs} y_{t-s}^J + c_t^{EJs} y_{t-s}^E + c_t^{UJs} y_{t-s}^U + \varepsilon_t^J, \\ y_t^E = c_0^E + c_t^{JE0} y_t^J + c_t^{EE1} y_{t-1}^E + c_t^{UE1} y_{t-1}^U + \cdots + c_t^{JEs-1} y_{t-s-1}^J + c_t^{EEs} y_{t-s}^E + c_t^{UEs} y_{t-s}^U + \varepsilon_t^E, \\ y_t^U = c_0^U + c_t^{JU0} y_t^J + c_t^{EU0} y_t^E + c_t^{UU1} y_{t-1}^U + \cdots + c_t^{JUs-1} y_{t-s-1}^J + c_t^{EUs-1} y_{t-s-1}^E + c_t^{UUs} y_{t-s}^U + \varepsilon_t^U, \end{cases} \tag{19}$$

$$(t = 1, \ldots, T)$$

where ε_t's are stationary and independent shocks, $\mathbf{c}_0 = (c_0^J, c_0^E, c_0^U)^\top$ is a constant vector, and c_t^{**i} ($* = J, E, U$; $i = 0, \ldots, s$; $t = 1, \ldots, T$) are time-varying parameters which indicate spillover effects with i days lag. The first and the second characters in the over-script of c_t's indicate an origin and a destination of spillover, respectively, and the third number i indicates the lag. The larger c_t^{**i} implies the higher effect of spillover from the origin to the destination on the day t with i days lag. We consider lags of dependency up to s ($s \geq 1$) days, assuming the spillover effect in the global financial markets vanishes in s days. $\boldsymbol{\varepsilon}_t = (\varepsilon_t^J, \varepsilon_t^E, \varepsilon_t^U)^\top$ denotes the structural shock.

In the constant volatility type, the volatility of the structural shocks are simply considered to be a constant vector, hence

$$\boldsymbol{\varepsilon}_t \sim N(0, \Sigma_\varepsilon), \quad \Sigma_\varepsilon = \mathrm{diag}((\sigma_\varepsilon^J)^2, (\sigma_\varepsilon^E)^2, (\sigma_\varepsilon^U)^2),$$

where $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with the diagonal elements of $\mathbf{x}$. Whereas in the stochastic volatility type, we assume the logarithm of variances follow random walk processes. By letting $\mathrm{Var}(\varepsilon_t^*) = (\sigma_t^*)^2$ ($* = J, E, U$) and $\mathbf{h}_t = (\ln(\sigma_t^J)^2, \ln(\sigma_t^E)^2, \ln(\sigma_t^U)^2)^\top$ denote the log variances of shocks and the logarithm vector, respectively, we assume $\mathbf{h}_t$ obeys

$$\mathbf{h}_t = \mathbf{h}_{t-1} + \mathbf{u}_t, \quad \mathbf{u}_t \sim N(0, \Sigma_h), \quad \Sigma_h = \mathrm{diag}((\sigma_h^J)^2, (\sigma_h^E)^2, (\sigma_h^U)^2), \tag{20}$$

where σ_h^*'s are constant parameters which indicate volatilities of shocks' volatilities.

Let $\mathbf{x}_t = (x_t^J, x_t^E, x_t^U)^\top$ denote the deviation from the sample mean, i.e., $\mathbf{x}_t = \mathbf{y}_t - \sum_{t=0}^T \mathbf{y}_t/(T+1)$ composed of Japanese, European and US data on day t. Suppose the data process is covariance-stationary, then, Eq.(19) is written in the matrix form as

$$A_t \mathbf{x}_t = B_{1t} \mathbf{x}_{t-1} + B_{2t} \mathbf{x}_{t-2} + \cdots + B_{st} \mathbf{x}_{t-s} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Sigma_{\varepsilon t}), \tag{21}$$

where $\Sigma_{\varepsilon t} = \mathrm{diag}((\sigma_t^J)^2, (\sigma_t^E)^2, (\sigma_t^U)^2)$,

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ -c_t^{JE0} & 1 & 0 \\ -c_t^{JU0} & -c_t^{EU0} & 1 \end{pmatrix}, \quad B_{it} = \begin{pmatrix} c_t^{JJi} & c_t^{EJi} & c_t^{UJi} \\ c_t^{JEi} & c_t^{EEi} & c_t^{UEi} \\ c_t^{JUi} & c_t^{EUi} & c_t^{UUi} \end{pmatrix} \ (i < s),$$

$$B_{st} = \begin{pmatrix} c_t^{JJs} & c_t^{EJs} & c_t^{UJs} \\ 0 & c_t^{EEs} & c_t^{UEs} \\ 0 & 0 & c_t^{UUs} \end{pmatrix},$$

and $\mathbf{c}_0 = (A_t - \sum_{i=1}^{s} B_{it}) \sum_{t=0}^{T} \mathbf{y}_t/(T+1)$ for any t.

The time-varying parameters should also be modelled in TVP-VAR. We simply assume that all the parameters obey independent random walk processes as

$$c_t^{**i} = c_{t-1}^{**i} + u_t^{**i}, \quad u_t^{**i} \sim N(0, (\sigma_u^{**i})^2), \quad (* = J, E, \text{or } U;\ i = 0, 1, \ldots, s) \tag{22}$$

which is suggested by Primiceri [2005] and is also used in Nakajima, Kasuya, and Watanabe [2009].

5.1.2 The estimation procedures

The time-varying parameters (c_t^{**}) and volatilities (σ_t^*) are estimated via Markov chain Monte Carlo (MCMC) method as explained in Nakajima, Kasuya, and Watanabe [2009]. In the stochastic volatility type, our estimation procedures are closer to theirs. While Nakajima, Kasuya, and Watanabe [2009] generate a sample of elements in A_t and B_t by transforming the structural VAR model to a reduced one, we generate a sample directly from the structural VAR form as explained in Appendix B. This is because the identification of structural parameters from a reduced form becomes problematic if we sample after transforming to the reduced form, since the matrix B_{st} is upper-triangular in our model. This is not the case for Nakajima, Kasuya, and Watanabe [2009] since they consider a non-triangular matrix for any B_{it}. We follow the method used in Nakajima, Kasuya, and Watanabe [2009] when sampling the stochastic volatility h_t.[13]

The prior distribution should be set carefully for σ_u's, and for Σ_ε in the constant volatility type, or for Σ_h in the stochastic volatility type. We assume priors of either $(\sigma_u^{**i})^2$, Σ_ε, or Σ_h obey inverse Gamma distributions with degree of freedom 2 and mean 20. Robustness of this prior is confirmed by Nakajima, Kasuya, and Watanabe [2009]. Initial value of c_1^{**i} and $\mathbf{h}_0$ are set to be the OLS estimators.

We generate $10,000$ samples after the initial $1,000$ samples are discarded for burn-in. The simulation is executed using Ox version 5.0 developed and distributed by Doornik [2006]. If one of the three markets does not open, the whole data of the day is excluded from inputs.

[13] Jochi Nakajima gave us many advices regarding the estimation procedures. We also referred to and used some part of the MCMC program developed by Nakajima, Kasuya, and Watanabe [2009].

5.1.3 Impulse responses

In order to examine impulse responses of pure contagion effects, we compute impulse responses in a different manner from the usual way. While the estimation of parameters is conducted on the full matrix base, all of the diagonal elements in B_{it} $(i = 1, \ldots, s)$, or autoregressive parameters, are replaced by 0 in the successive computation of the responses. This procedure excludes the propagation of a shock from a country to the same country on the next day. For instance, responses to the US shock on the next day's US data is excluded up to s days lag.

In the stochastic volatility type model, the size of the shocks are given as the mean of the time-varying volatility in each country. In the constant volatility model, the size is equivalent with the constant volatility in each country. Hence the size of the shock is not time-varying in both models.

5.2 Result and analysis

Analyses of global contagion are provided in this section using the TVP-VAR model. Particularly, we focus on the level shifts of time-varying parameters c_t^{**i} and the changes in time-varying impulse response functions before and after the financial turmoil. We first analyse the interdependency of returns, then volatilities, and lastly VRPs.

5.2.1 Contagion of returns

First, we analyse the contagion of returns. Since volatilities of equity returns are non-constants, and since the spillover effect of equity returns is likely to diminish on the next day, we conduct analysis using the stochastic volatility type TVP-VAR with one day lag.[14]

The panels in Figure 4 (i) display the time series of estimated parameters $\hat{c}_t^{**}$, and those in (ii) plot the impulse responses from July 2003 to October 2009.[15] The solid lines in the upper panel in (i) indicate parameters that show spillovers to Japan, those in the middle panel to Europe, and those in the bottom panel to US. The dotted lines indicate a band with the estimated two standard deviations $2\hat{\sigma}_u$ from each estimate. The impulse responses in Figure 4 (ii) are evaluated by the response to the standardised shock with a size of the mean of the estimated volatilities in each country.

From panels in (i), we detect the following. First, all the coefficients except for $\hat{c}_t^{JU}$ are positively estimated in most of the period, whereas $\hat{c}_t^{JU}$ are almost zero. This shows that the equity returns in Europe and the US are positively affected each other at any time whereas the contagion between Japan and the US is one way. Those effects do not change during the course of the financial turmoil after the summer of 2007. Second, $\hat{c}_t^{UE}$ drops

[14] Though we have tried three days lag and five days lag, those results are quite similar to the one day lag case.

[15] The third number i in the superscript of c_t is omitted in this section since we only consider one day lag.

while $\hat{c}_t^{EU}$ rises after the middle of 2007, showing that the spillover of returns from the European market to the US market is strengthened while that in the opposite direction weakened.

We examine those characteristics from the impulse response of returns to the structural shocks displayed in Figure 4 (ii). While the level of time-varying parameters indicate direct contagion on the next day, the impulse responses encompass not only the direct contagion but also indirect contagion which goes through the other region. First, almost all the responses are positively estimated between any regions, which confirms the positive spillover among global equity returns. Second, the level of US response to the European and Japanese markets increases after the middle of 2007, and the interdependency between the Japanese and the European markets is strengthened as well.[16] Third, the responses are weakened after the second day in any directions, indicating the return's spillover effect diminishes quickly. Putting the above analyses altogether reveals that the positive spillover effects of returns remain positive and are strengthened in some directions during the financial turmoil.

5.2.2 Contagion of volatilities

We then examine the contagion of volatilities. While examinations in the previous section revealed the positive spillover of shocks in equity returns, we investigate the interdependencies of the magnitude of those shocks evaluated by RV in this section. Since RV is not stationary covariance, we take logarithm of RV in the evaluation; i.e., $\mathbf{x}_t$ in Eq.(21) is set to be $\ln \sigma_{RV}^2(t)$. We apply TVP-VAR model with stochastic volatility type with three days lag,[17] since the levels of volatilities are known to persist for several days after it spikes up. We only analyse the impulse response for RV.

The panels in Figure 5 displays the impulse responses. Those formats are the same as Figure 4 (ii). First, the Japanese responses to shocks in Europe or US are positive in most of the period, and the level of the response to the US grows stronger after the summer of 2007. In other words, the Japanese volatility is linked to the European and the US volatility regardless of examined periods, and the linkage to the US grows stronger during the financial turmoil. Second, the European RV positively responds to the US shock while the opposite direction is not significant. Those summarise that the shock generated in the US propagates to Japan and Europe while the Japanese volatility is also sensitive to the European volatility. Third, responses in any direction grow stronger after the Lehman shock in 2008 which gradually goes back to neutral in one year. The responses

[16] This indicates that the Japanese and European market become gradually volatile due to concerns for the US problem after the Paribas shock, which may sometimes generate the counter feedback to the US market.

[17] We also analyse the constant volatility type, but does not find significant differences. We set three days lag here though technically the lag dimension should be determined by the difference in marginal likelihood values.

of two or three days later are also strengthened after the Lehman shock. Those confirm that the positive feedback loop exists among RVs in those countries and that the positive interdependency strengthened after the Lehman shock. The surging volatility goes through the global equity market.

5.2.3 Contagion of VRPs

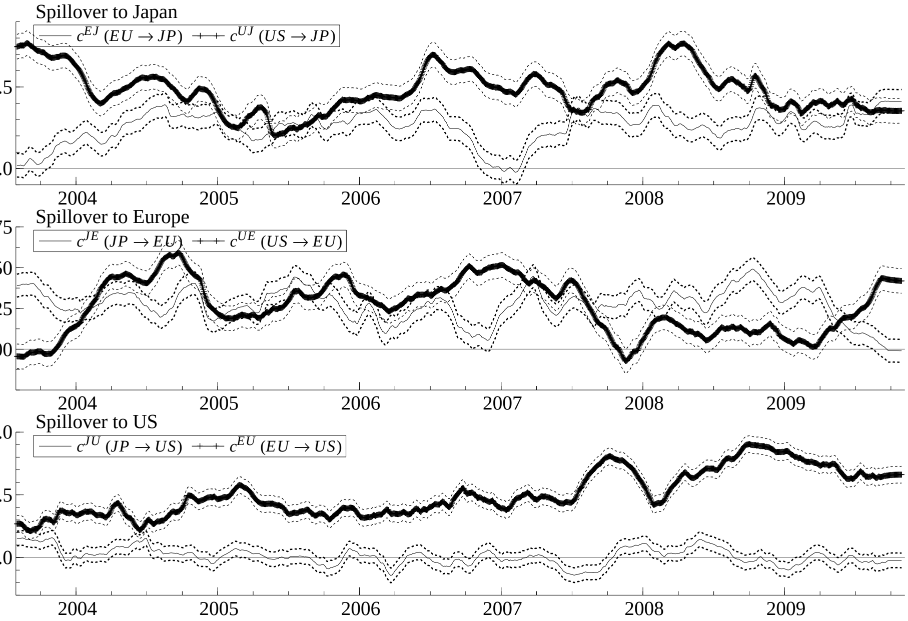
Lastly, we analyse the contagion of VRPs. We examine the interdependencies of tVRP, and eVRP with ARFIMAX model using the stochastic volatility type TVP-VAR.[18] Since VRPs are one-month integrated values, we input monthly data (end of month data) in TVP-VAR with one month lag for the evaluation.

The panels in Figure 6 (i) and (ii) plot the impulse responses for tVRP and eVRP, respectively. Though figures in (i) and (ii) show different features, we detect the following three trails common to those. First, the Japanese VRP has stronger effect on the European or the US VRPs, while the US VRP has relatively weaker effect on the Japanese or European VRPs. Secondly, the interdependency is weakened during the period from the summer of 2007 to the Lehman shock, though after the Lehman shock it grows stronger in most of the directions, particularly from Europe to US.

Examining in detail, we detect several differences between panels in (i) and those in (ii). The impulse responses evaluated by tVRP shows a small but negative link between Japan and Europe before the turmoil while US and Japan have a positive link, though we do not see such a relationship in eVRPs. And the impulse responses evaluated by eVRPs are positively estimated in any directions, indicating the positive feedback loop resides among eVRPs all through the examined period. Taking the above analyses together, we permit the following assessment. Firstly, the weakening VRP contagion during the Post Paribas Shock period indicates that the speed of market risk aversion to the future volatile market varies country by country. The concerns for the US subprime mortgage loan problem are gradually prevailing in the global market while each option market reacts differently to various policy responses adopted in each country. After the Lehman shock, however, the interdependency among VRPs grew stronger. The market participants become sensitive to the volatility risk which may result in the increase in strength of the positive feedback loop among global VRPs during the period. Additionally, contrary to the direction of RV contagion, the shock in VRP generated in Japan mostly affects the European and US VRPs all through the examined period. This may be explained by the fact that the Japanese VRP is largest than those in Germany or the US, hence the shock given in the impulse response is large.

[18] Though we investigate the constant volatility type TVP-VAR and obtain different results, the stochastic volatility type is examined in this section as that is a more comprehensive model.

(i) Time series of estimated TVP coefficients $\hat{c}_t^{**}$



(ii) Impulse responses

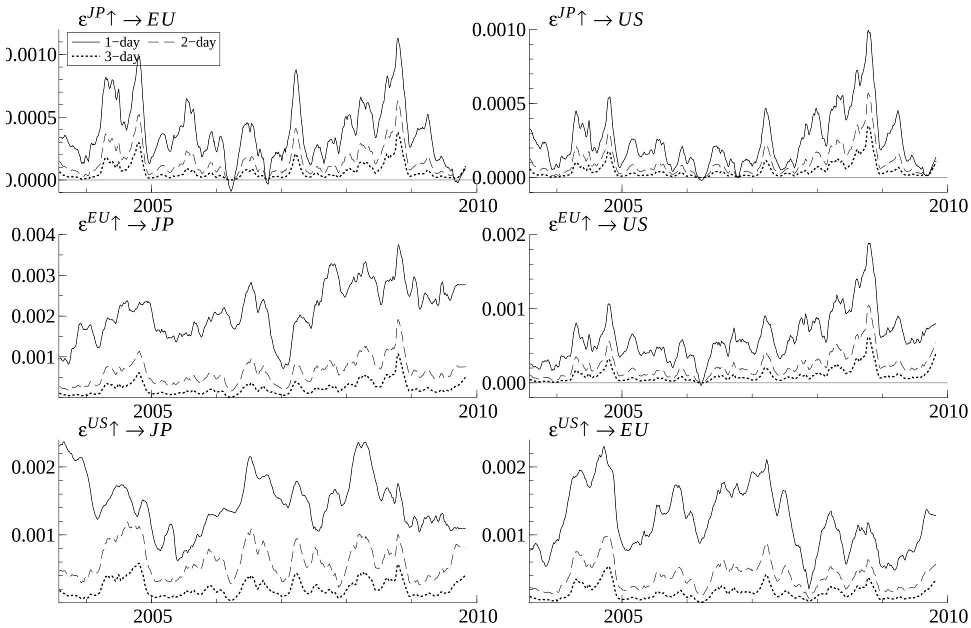


Figure 4: Time-varying coefficient estimates and impulse responses of equity return contagion

Notes: Dotted lines in panels in (i) indicate two standard deviation bands calculated by estimator of $\hat{\sigma}_u$. Solid lines, dashed lines, and dotted lines in panels (ii) indicate impulse responses with 1 day, 2 days, and 3 days lag, respectively. The shocks for the response calculation are given in percent scale. The stochastic volatility type model with one day lag is applied.

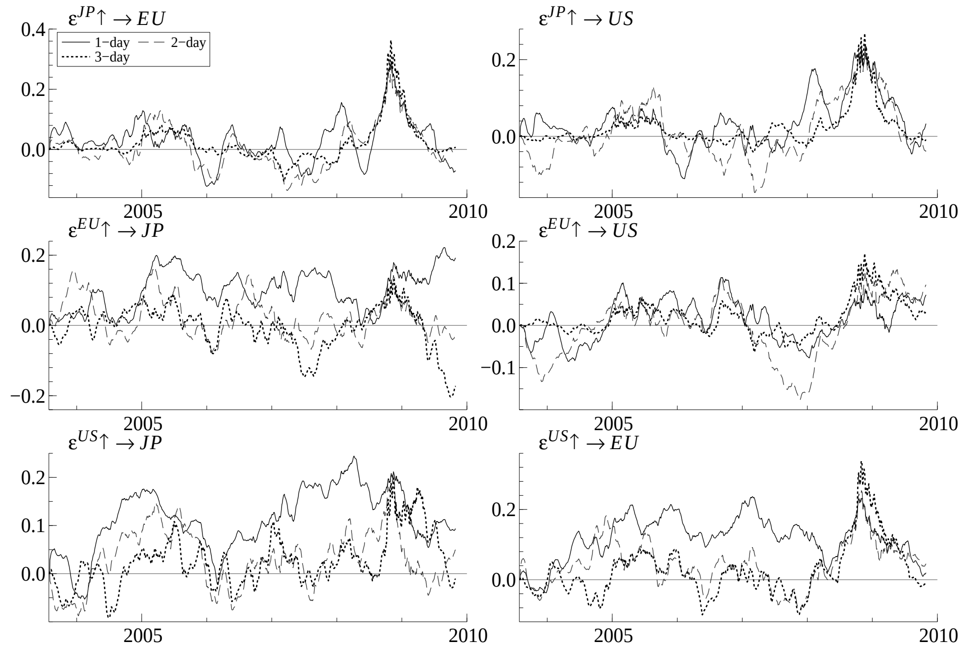


Figure 5: Time-varying impulse responses of log RV contagion

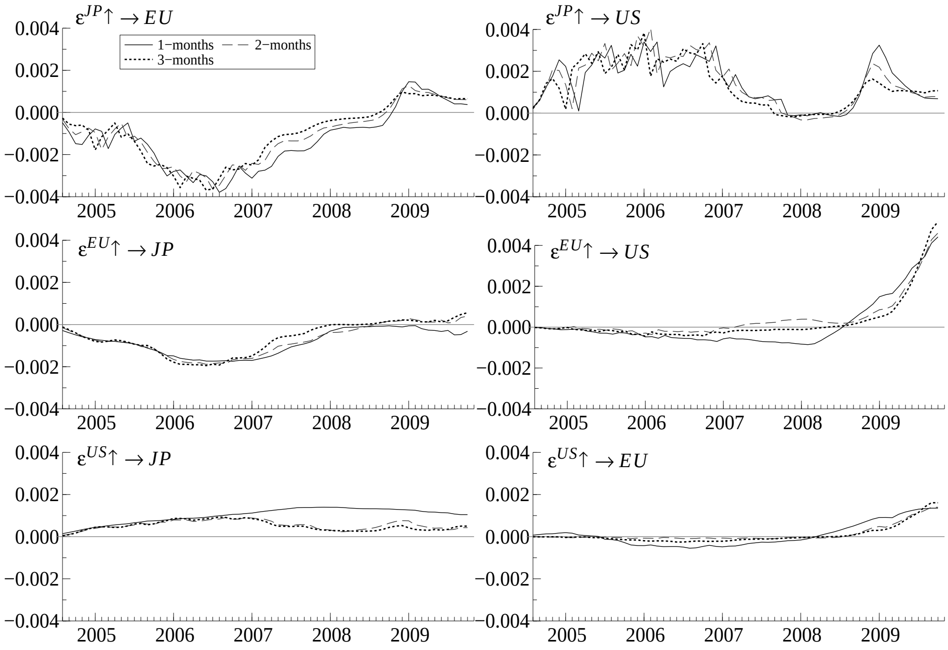
Notes: Solid lines, dashed lines, and dotted lines indicate impulse responses with 1 day, 2 days, and 3 days lag, respectively. The stochastic volatility type TVP-VAR with three days lag is applied. The shocks for the response calculation are given in the scale of the log of squared percent.

6 Summary

This paper has proposed the evaluation method of volatility risk premiums, seeking for improvement on methods applied in earlier studies. It analyses those volatility risk premiums on the global stock indices, along with volatility developments. The analyses focus on how those volatilities and the risk premiums propagated among the global equity markets during the global financial market turmoil after the summer of 2007. In the analyses, this paper has tried to apply the time-varying-parameter structural VAR model to evaluate the change of spillover effects of equity returns, volatilities, and risk premiums.

Our proposed measure of the volatility risk premium addressed the filtration inconsistency problem ignored in earlier studies to better reflect market expectations for future volatility risk. By our method, the premium is identified as the gap between the current integrated implied volatility and the integrated future volatility estimated by a time series model applied to preceding realized volatilities. We estimated two types of premiums on the assumption that one-day realized volatility behaves according to the Heston model or the ARFIMAX model. Analyses indicate that the premium obtained by our method

(i) Impulse responses of tVRP contagion



(ii) Impulse responses of eVRP (ARFIMAX) contagion

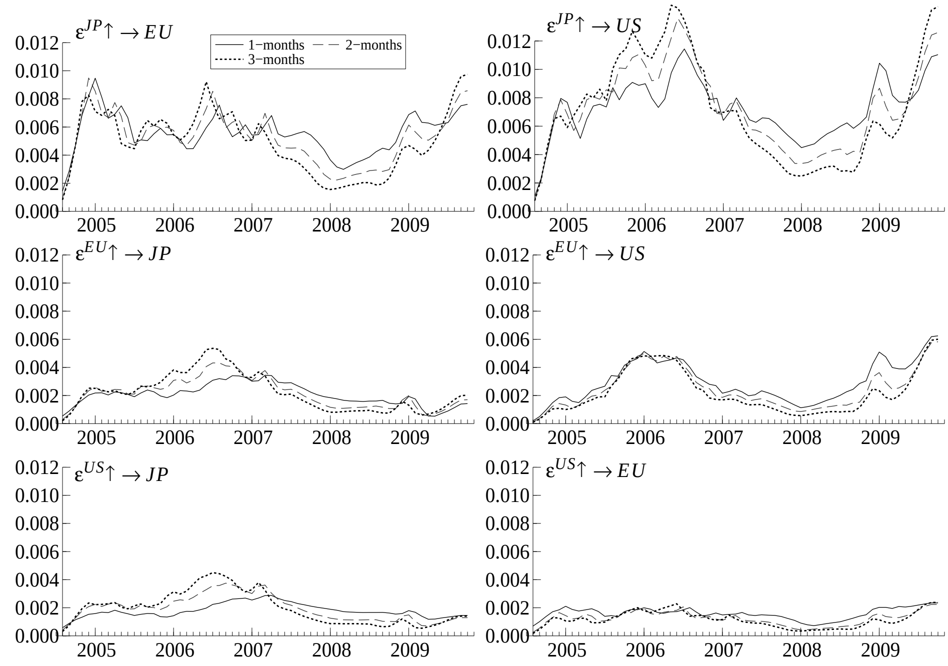


Figure 6: Time-varying impulse responses of tVRP and eVRP (ARFIMAX) contagion

Notes: Solid lines, dashed lines, and dotted lines in each panel indicate impulse responses with 1 month, 2 months, and 3 months lag, respectively. The stochastic volatility type TVP-VAR with three days lag is applied. The shocks for the impulse response calculation are given in in the normal scale though figures in the other tables/figures are shown in squared percent scale.

more strongly correlates with market risk indicators which generally used to measure risk aversion/appetite than those based on earlier methods.

We also applied the time-varying-parameter structural VAR model with stochastic volatility of residuals to analyse the spillover effects of equity returns, volatilities, and the volatility risk premiums among Japan, Europe, and the US. Using this model for the evaluation of market contagion is new as far as the author knows. Examination revealed that the positive spillover among the equity returns remained positive with reinforcement of the counter feedback to the US market during the turmoil. The contagion of shocks in volatilities, which had mainly originated in the US market before the turmoil, became reciprocal after the Lehman shock. As for the risk premium contagion, while the interdependency is weakened after the Paribas shock, it grows stronger after the Lehman shock in most of the directions, particularly from Europe to US. The enhanced propagation of returns, volatilities, and volatility risk premiums made markets more volatile, and let option market participants highly risk averse. These contagion effect may result in the dramatic increase in the level of volatility and volatility risk premiums during the turmoil.

Our proposed measure of volatility risk premium, along with volatility itself, seems to be useful to monitor investors risk aversion and the market sentiments. It does matter especially in disordered markets and attains to the knowledge of its propagation. Additionally, our new approach of using the time-varying-parameter structural VAR model brings useful scheme of evaluating such propagation dynamics.

A The derivation of IIV (Eq.(4))

A.1 Natural assumptions on underlying price processes

In this section, we derive Eq.(4), the representation of expected quadratic variation under risk neutral measure $\mathcal{Q}$, using European option prices. Suppose the forward price process at time t, $F(t,T) = F_t$, obeys the general jump diffusion process:

$$dF_t = F_{t^-}\sigma(t^-,\cdot)dW_t + F_{t^-}\int_{-\infty}^{\infty}(e^z - 1)[\mu(dz,dt) - \nu(dz,t)dt], \tag{B-1}$$

where t^-, σ, dW_t and z denote the time just before t, a diffusion coefficient, Weiner process and a jump size, respectively. The coefficient σ is assumed to be a function of time and any other variables. z is the jump size when $\mu(z,t) = 1$ and satisfies $z = \ln\frac{F_t}{F_{t^-}}$. By integrating in terms of z, the price can include every possible size of the jumps. $\mu(z,t)$ is an arbitrary process and $\nu(z,t)$ denotes a compensation process of the jump process:

$$J_t = \int_0^t\int_{-\infty}^{\infty} F_{s^-}(e^z - 1)\mu(dz,ds), \tag{B-2}$$

which satisfies

$$E_s^{\tilde{\mathcal{Q}}}\mu(dz,dt) = E_s^{\tilde{\mathcal{Q}}}\nu(dz,t)dt \quad (\forall s < t).$$

Here, we define the T-forward measure $\tilde{\mathcal{Q}}$, such that F_t is martingale under $\tilde{\mathcal{Q}}$. And for simplicity, we write $E_t^{\tilde{\mathcal{Q}}}[\cdot] = E^{\tilde{\mathcal{Q}}}[\cdot|\mathcal{F}_t]$ and $\mathcal{F}_t = \sigma(F_s; s \le t)$.

Furthermore, in order to restrict the variance of F_t not to diverge, we assume $\nu(z,t)$ should satisfy

$$\nu(0,t) = 0, \quad \int_{-\infty}^{\infty}(e^{2z}\wedge 1)\nu(z,t)dz < \infty, \quad \forall t \in [0,\infty). \tag{B-3}$$

A.2 Expansion of European type payoff

First we show any type of C^2 European payoff $f(y)$ $(y > 0)$ can be expanded as the following using the strictly positive value x.

$$f(y) = f(x) + f'(x)(y-x) + \int_0^x f''(K)(K-y)^+dK + \int_x^{\infty} f''(K)(y-K)^+dK \tag{B-4}$$

(proof)

By simple mathematics,

$$f(y) - f(x) = \mathbf{1}\{y \ge x\}\int_x^y f'(u)du - \mathbf{1}\{x > y\}\int_y^x f'(u)du$$

can be derived for any C^2 function $f(\cdot)$ and strictly positive variables x, y. Transforming the above equation yields

$$
\begin{aligned}
f(y) = f(x) + \mathbf{1}\{y \geq x\} \int_x^y \left[f'(x) + \int_x^u f''(v)dv \right] du \\
\qquad - \mathbf{1}\{x > y\} \int_y^x \left[f'(x) - \int_u^x f''(v)dv \right] du \\
= f(x) + f'(x)(y-x) + \mathbf{1}\{y \geq x\} \int_x^y f''(v) \int_v^y du dv + \mathbf{1}\{x > y\} \int_y^x f''(v) \int_y^v du dv \\
= f(x) + f'(x)(y-x) + \mathbf{1}\{y \geq x\} \int_x^y (y-v) f''(v)dv + \mathbf{1}\{y < x\} \int_y^x (v-y) f''(v)dv \\
= f(x) + f'(x)(y-x) + \int_x^\infty (y-v)^+ f''(v)dv + \int_0^x (v-y)^+ f''(v)dv.
\end{aligned}
$$

When $v = K$, we get Eq.(B-4). ■

A.3 Synthesis of the present value of any European-type payoff

The present value of any European type payoff $f(F_T)$, $V_t = B(t,T)E_t^{\tilde{Q}} f(F_T)$, can be synthesised by the riskless bond price $B(t,T)$, the forward price F_t, and put and call option values $C(t,T,K), P(t,T,K)$ as follows.

$$
V_t = f(F_t)B(t,T) + \int_0^{F_t} f''(K)P(t,T,K)dK + \int_{F_t}^\infty f''(K)C(t,T,K)dK. \tag{B-5}
$$

Specifically, the present value of the log type payoff can be decomposed as

$$
B(t,T)E_t^{\tilde{Q}} \left[-\ln \frac{F_T}{F_t} \right] = \int_0^{F_t} \frac{P(t,T,K)}{K^2} dK + \int_{F_t}^\infty \frac{C(t,T,K)}{K^2} dK. \tag{B-6}
$$

(proof)

Let $x = F_t$ and taking expectation at time t under the T-forward measure in Eq.(B-4) as

$$
\begin{aligned}
E_t^{\tilde{Q}} f(F_T) &= f(F_t) + f'(F_t)(E_t^{\tilde{Q}} F_T - F_t) \\
&\quad + E_t^{\tilde{Q}} \int_0^{F_t} f''(K)(K - F_T)^+ dK + E_t^{\tilde{Q}} \int_{F_t}^\infty f''(K)(F_T - K)^+ dK.
\end{aligned}
$$

Here, the second term is zero since F_t is martingale under T-forward measure. Therefore, the present value become

$$
V_t = f(F_t)B(t,T) + \int_0^{F_t} f''(K)P(t,T,K)dK + \int_{F_t}^\infty f''(K)C(t,T,K)dK,
$$

which results in Eq.(B-5). Additionally, when the payoff function has the form $f(x) =$

$\ln x$, Eq.(B-5) can be rewritten as

$$E_t^{\tilde{Q}} \ln F_T = \ln F_t - \int_0^{F_t} \frac{\tilde{P}(t,T,K)}{K^2} dK - \int_{F_t}^{\infty} \frac{\tilde{C}(t,T,K)}{K^2} dK,$$

which is Eq.(B-6). ■

A.4 Synthesis of IIV

We lastly derive Eq.(4), or the following equation, from Eq.(B-6).

$$\sigma_{IIV}^2(t,T) = \frac{2}{(T-t)B(t,T)} \left(\int_0^{F_t} \frac{P(t,T,K)}{K^2} dK + \int_{F_t}^{\infty} \frac{C(t,T,K)}{K^2} dK \right) + \varepsilon,$$

$$\varepsilon \sim o\left(\left(\frac{dF_s}{F_{s^-}} \right)^3 \right). \tag{B-7}$$

(proof)

By the assumption in Eq.(B-3),

$$\begin{aligned} \sigma_{IIV}^2(t,T) &= \frac{1}{T-t} E_t^{\tilde{Q}} \int_t^T \left(\frac{dF_s}{F_s^-} \right)^2 \\ &= \frac{1}{T-t} E_t^{\tilde{Q}} \int_t^T \left(\frac{1}{F_{s^-}} \right)^2 d[F,F]_s \\ &= \frac{1}{T-t} E_t^{\tilde{Q}} \int_t^T \left[\sigma^2(s^-,\cdot)ds + \int_{-\infty}^{\infty} (e^z-1)^2 \mu(dz,ds) \right]. \end{aligned}$$

Applying Ito's formulae to $\ln F_s$, we get

$$d \ln F_s = \frac{dF_s}{F_{s^-}} - \frac{1}{2}\sigma^2(s^-,\cdot)ds + \int_{-\infty}^{\infty} \left[\ln(F_{s^-} e^z) - \ln F_{s^-} - \frac{F_{s^-}e^z - F_{s^-}}{F_{s^-}} \right] \mu(dz,ds).$$

Substituting $\sigma^2(s^-,\cdot)$ in the above two equations yields

$$\begin{aligned} \sigma_{IIV}^2(t,T) &= \frac{2}{T-t} E_t^{\tilde{Q}} \left[-\ln \frac{F_T}{F_t} \right] + \frac{2}{T-t} E_t^{\tilde{Q}} \int_t^T \frac{dF_s}{F_{s^-}} \\ &\quad + \frac{1}{T-t} E_t^{\tilde{Q}} \int_t^T \int_{-\infty}^{\infty} \left(e^{2z} - 4e^z + 2z + 3 \right) \mu(dz,ds). \end{aligned}$$

The second term is zero as F_t is martingale under the T-forward measure. Applying

Eq.(B-6) to the first term yields

$$\begin{aligned}
\sigma_{IIV}^2 &= \frac{2}{(T-t)B(t,T)}\left[\int_0^{F_t}\frac{P(t,T,K)}{K^2}dK + \int_{F_t}^{\infty}\frac{C(t,T,K)}{K^2}dK\right] \\
&\quad + \frac{1}{T-t}E_t^{\tilde{Q}}\int_t^T\int_{-\infty}^{\infty}\left(e^{2z}-4e^z+2z+3\right)\mu(dz,ds) \\
&= \frac{2}{(T-t)B(t,T)}\left[\int_0^{F_t}\frac{P(t,T,K)}{K^2}dK + \int_{F_t}^{\infty}\frac{C(t,T,K)}{K^2}dK\right] + \varepsilon.
\end{aligned}$$

As shown, Eq.(4) is satisfied.

Note that the error ε is the accumulation of the third moment of price jumps from time t to time T. This is because, since we know $e^z = 1 + z + z^2/2 + o(z^3)$ by Taylor expansion, ε can be computed as

$$\begin{aligned}
\varepsilon &= \frac{1}{T-t}E_t^{\tilde{Q}}\int_t^T\int_{-\infty}^{\infty}\left(e^{2z}-4e^z+2z+3\right)\mu(dz,ds) \\
&= \frac{1}{T-t}E_t^{\tilde{Q}}\int_t^T\int_{-\infty}^{\infty}\left\{(1+2z+2z^2+o(z^3))-4(1+z+z^2/2+o(z^3))+2z+3\right\}\mu(dz,ds) \\
&\sim \frac{1}{T-t}E_t^{\tilde{Q}}\int_t^T\int_{-\infty}^{\infty}o(z^3)\mu(dz,ds). \quad \blacksquare
\end{aligned}$$

B MCMC sampling scheme for estimating parameters

We explain the MCMC sampling scheme for the parameter estimation in Eq.(19) or Eq.(21). The model is

$$A_t\mathbf{x}_t = \sum_{i=1}^{s} B_{it}\mathbf{x}_{t-i} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{x}_t = (x_t^J, x_t^E, x_t^U)^\top$ is a 3-by-1 data vactor A_t is a lower triangular matrix whose diagonal elements are 1, B_{it} $(i = 1,\dots,s-1)$ are 3-by-3 square matrices, and B_{st} is 3-by-3 upper triangular matrix. Let $\mathbf{b}_t$ denotes row stacked vector of the 3-by-3s matrix $(B_{1t}, B_{2t},\dots,B_{st})$ excluding 0 elements in the lower triangular of B_{st}, defined by,

$$\mathbf{b}_t = (\mathbf{c}_t^{J1}, \mathbf{c}_t^{J2},\dots,\mathbf{c}_t^{Js}, \mathbf{c}_t^{E1},\dots,\mathbf{c}_t^{Es-1}, c_t^{EEs}, c_t^{UEs}, \mathbf{c}_t^{U1},\dots,\mathbf{c}_t^{Us-1}, c_t^{UUs},)^\top,$$

where $\mathbf{c}_t^{*i}$ $(* = J, E, U;\ i = 1,\dots,s)$ denotes the coefficient vector of i-th lag to the country $*$, i.e., $\mathbf{c}_t^{*i} = (c_t^{J*i}, c_t^{E*i}, c_t^{U*i})$. And let $\mathbf{a}_t$ denote row stacked vector of A_t, i.e.,

$$\mathbf{a}_t = (-c_t^{JE0}, -c^{JU0}, -c_t^{EU0})^\top.$$

We apply the following algorithm of Gibbs sampler recursively.

1. Initialise $\{\mathbf{a}_t\}_{t=1}^T$, $\{\mathbf{b}_t\}_{t=1}^T$, $\{\mathbf{h}_t\}_{t=1}^T$ and set priors as $\boldsymbol{\sigma}_u^2 \sim IG(n_0/2, m_0/2)$, and also

set $\boldsymbol{\sigma}_h^2 \sim IG(n_2/2, m_2/2)$ for stochastic volatility case, or $\boldsymbol{\sigma}_\varepsilon^2 \sim IG(n_4/2, m_4/2)$ for constant volatility case.

2. Generate sample $\{\mathbf{b}_t\}_{t=1}^T \mid \{\mathbf{x}_t\}_{t=0}^T,\ \{\mathbf{a}_t\}_{t=1}^T,\ \{\mathbf{h}_t\}_{t=1}^T,\ \boldsymbol{\sigma}_u$ by applying the method explained in B.1.

3. Generate sample $\{\mathbf{a}_t\}_{t=1}^T \mid \{\mathbf{x}_t\}_{t=0}^T,\ \{\mathbf{b}_t\}_{t=1}^T,\ \{\mathbf{h}_t\}_{t=1}^T,\ \boldsymbol{\sigma}_u$ by applying the method explained in B.2.

4. Generate sample $\boldsymbol{\sigma}_u^2 \mid \{\mathbf{a}_t\}_{t=1}^T,\ \{\mathbf{b}_t\}_{t=1}^T \sim IG(n_1/2, m_1/2)$, where $n_1 = n_0 + T - 1$ and $m_1 = m_0 + \sum_t \mathbf{c}_t^\top \mathbf{c}_t$.

5. Generate sample $\{\mathbf{h}_t\}_{t=1}^T \mid \{\mathbf{x}_t\}_{t=0}^T,\ \{\mathbf{a}_t\}_{t=1}^T,\ \{\mathbf{b}_t\}_{t=1}^T, \{\sigma_h\}_{t=1}^T$ by applying the method explained in B.3 for the case of stochastic volatility. Generate sample $\boldsymbol{\sigma}_\varepsilon^2 \mid \{\mathbf{a}_t\}_{t=1}^T,\ \{\mathbf{b}_t\}_{t=1}^T \sim IG(n_5/2, m_5/2)$, where $n_5 = n_4 + T - 1$ and $m_5 = m_4 + \sum_t \boldsymbol{\varepsilon}_t^\top \boldsymbol{\varepsilon}_t$ for the case of constant volatility.

6. Generate sample $\boldsymbol{\sigma}_h^2 \mid \{\mathbf{h}_t\}_{t=1}^T \sim IG(n_3/2, m_3/2)$ where $n_3 = n_2 + T - 1$ and $m_3 = m_2 + \sum_t \mathbf{h}_t^\top \mathbf{h}_t$ for the case of stochastic volatility.

7. Go to 2.

B.1 Sampling elements in B_t

In order to sample elements in B_t, we rewrite Eq.(21) in a state space form as

$$\begin{cases} \tilde{\mathbf{x}}_t = X_b \mathbf{b}_t + \boldsymbol{\varepsilon}_t, \\ \mathbf{b}_t = \mathbf{b}_{t-1} + \boldsymbol{u}_t^b, \end{cases}$$

where $\tilde{\mathbf{x}}_t = A_t \mathbf{x}_t$, and

$$X_b = \begin{pmatrix} \langle \mathbf{x}_{t-1,t-s} \rangle & \mathbf{x}_{t-s}^\top & 0 & & \cdots & & 0 \\ 0 & 0 & \langle \mathbf{x}_{t-1,t-s-1} \rangle & x_{t-s}^E & x_{t-s}^U & 0 & 0 \\ 0 & & \cdots & & 0 & \langle \mathbf{x}_{t-1,t-s-1} \rangle & x_{t-s}^U \end{pmatrix},$$

where $\langle \mathbf{x}_{t-1,t-i} \rangle = (\mathbf{x}_{t-1}^\top, \ldots, \mathbf{x}_{t-i}^\top)$ $(i > 1)$. Since $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{u}_t^b$ are normally distributed independently, samples of $\mathbf{b}_t$ are generated using Kalman filter and the simulation smoother. Those algorithms are explained in Chapter 13 in Hamilton [1994], or Appendix A.1 in Nakajima, Kasuya, and Watanabe [2009].

B.2 Sampling elements in A_t

To sample elements in A_t from a posterior distribuiton, we rewrite Eq.(21) in another state space form. Since $A_t = I + \bar{A}_t$ where

$$\bar{A}_t = \begin{pmatrix} 0 & 0 & 0 \\ -c_t^{JE} & 0 & 0 \\ -c_t^{JU} & -c_t^{EU} & 0 \end{pmatrix}, \quad \text{and} \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

we transform Eq.(21) to $\bar{\mathbf{x}}_t = -\bar{A}_t \mathbf{x}_t + \boldsymbol{\varepsilon}_t$ where $\bar{\mathbf{x}}_t = \mathbf{x}_t - X_b \mathbf{b}_t$. It can be further rewritten as

$$\begin{cases} \bar{\mathbf{x}}_t = X_a \mathbf{a}_t + \boldsymbol{\varepsilon}_t, \\ \mathbf{a}_t = \mathbf{a}_{t-1} - \boldsymbol{\epsilon}_t^a, \end{cases}$$

where $\mathbf{a}_t = (-c_t^{JE}, -c_t^{JU}, -c_t^{EU})^\top$, and

$$X_a = \begin{bmatrix} 0 & \dots & 0 \\ -x_t^J & 0 & 0 \\ 0 & -x_t^E & -x_t^U \end{bmatrix}.$$

We apply Kalman filter and the simulation smoother as in Section B.1 to generate samples of A_t.

B.3 Sampling h_t

For the stochastic volatility type, we generate the sample of the volatility process $\mathbf{h}_t$. The algorithm of multi-move sampler for non-linear Gaussian state space model is applied. We set block size of 100 which is roughly equivalent to 15 times division of sample set in the estimation of return and RV contagion. In the estimation of monthly VRP contagion, we set block size 3 which is roughly equivalent to 20 times division of samples. See Appendix A.3 in Nakajima, Kasuya, and Watanabe [2009] for the procedure. We apply the program of generating h_t developed by them.

References

Bakshi, G. and N. Kapadia, “Delta-Hedged Gains and the Negative Market Volatility Risk Premium,” *The Review of Financial Studies*, 16(2), 2003, pp. 527–566.

Bollerslev, T., G. Tauchen, and H. Zhou, “Expected Stock Returns and Variance Risk Premia,” *The Review of Financial studies*, 22(11), 2009, pp. 4463–4492.

Britten-Jones, M. and A. Neuberger, “Option Prices, Implied Price Processes and Stochastic Volatility,” *Journal of Finance*, 55, 2000, pp. 839–866.

Carr, P. and L. Wu, "Variance Risk Premiums," *Review of Financial Studies*, 22(3), 2009, pp. 1311–1341.

CBOE, "VIX white paper," Chicago Board Options Exchange, 2003.

Christensen, B.J., C.S. Hansen, and N.R. Prabhala, "The Telescoping Overlapping Problem in Options Data," Working Paper, University of Maryland, 2002.

Cogly, T. and T. J. Sargent, "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.," *Review of Economic Dynamics*, 8, 2005, pp. 262–302.

Demeterfi, K., E. Derman, M. Kamal, and J. Zou, "More Than You Ever Wanted To Know About Volatility Swaps," Quantitative Strategies Research Notes, Goldman Sachs, 1999.

Doornik, J., *Ox: Object Oriented Matrix Programming*, London Timberlake Consultants Press, 2006.

Giot, P. and S. Laurent, "Modelling Daily Value-at-Risk Using Realized Volatility and ARCH Type Models," *Journal of Empirical Finance*, 11(3), 2004, pp. 379–398.

Hamilton, J. D., *Time Series Analysis*, Princeton University Press, 1994.

Heston, S., "A Closed Form Solutiob for Options with Stochastic Volatility with Applications to Bond and Currency Options," *The Review of Financial Studies*, 6, 1993, pp. 327–343.

Hull, J. and A. White, "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance*, 42(2), 1987, pp. 281–300.

Jiang, G.J. and Y.S. Tian, "Extracting Model-Free Volatility from Option Prices: An Examination of the VIX Index," *Journal of Derivatives*, Spring, 2007, pp. 35–60.

Nakajima, J., K. Kasuya, and T. Watanabe, "Bayesian Analysis of Time-Varying Parameter Vector Autoregressive Model for the Japanese Economy and Monetary Policy," IMES Discussion Paper Series 2009-E-13, Bank of Japan, 2009.

Primiceri, G. E., "Time Varying Structural Vector Autoregressions and Monetary Policy," *Review of Economic Studies*, 72, 2005, pp. 821–852.

Scott, L.O., "Option Pricing when the Variance Changes Randomly: Theory, Estimation, and an Application," *Journal of Financial and Quantitative Analysis*, 22(4), 1987, pp. 419–438.

Shibata, M., "Estimation for Volatility using Intraday Market Data: Survey for Realized Volatility and Application for Japanese Stock Index and Stock Index Futures," *Kin'yu Kenkyu (Monetary and Economic Studies)*, 27(1), 2008, pp. 1–54. in Japanese.

Sugihara, Y., "The Model Free Implied Volatility in Nikkei 225 Stock Index Options," *Kin'yu Kenkyu (Monetary and Economic Studies)*, 29(2), 2010, pp. 73–120. in Japanese.

Uchida, Y. and K. Miyazaki, "Volatility Risk Premium in Nikkei 225 Index Options Market," *Gendai Finance*, 23, 2008, pp. 35–59. in Japanese.

Watanabe, T. and K. Sasaki, "The Volatility Estimation by an ARCH Model, the Realized Volatility, and the Value at Risk," *Kin'yu Kenkyu (Monetary and Economic Studies)*, 29, 2007, pp. 39–74. in Japanese.

Wiggins, J.B., "Option Values under Stochastic Volatilities," *Journal of Financial Economics*, 19, 1987, pp. 351–372.

——— "The Pricing of Foreign Currency Options with Stochastic Volatility," *Journal of Econometrics*, 45, 1990, pp. 239–265.