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# A DEFENCE OF THE EXPECTATIONS THEORY AS A MODEL OF US LONG-TERM INTEREST RATES

by

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#### Abstract

This paper re-examines the empirical content of the expectations theory of the term structure by employing the Campbell-Shiller (1987) methodology to study the behaviour of 10-year/three-month US government yield spreads. The methodology is implemented in two ways. First, theoretical spreads satisfying the expectations theory are constructed from in-sample forecasts of future changes in short rates generated from a small-scale VAR. Second, theoretical spreads are computed from out-of-sample forecasts of changes in short rates with the parameters of each VAR equation updated with a Bayesian procedure. When the procedure is restricted to give less weight to new data than would be the case with OLS estimation over an expanding sample, theoretical spreads computed from out-of-sample forecasts track actual spreads closely in pre-1979 data. This is also the case as from the start of 1984 if data from the 1979Q4–1982Q4 period of non-borrowed reserve targeting are given zero weight when estimating the parameters of the VAR.

\* I thank Craig Furfine for helpful discussions. E-mail address: greg.sutton@bis.org.

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## **1.** Introduction

In an influential article, Campbell and Shiller (1987) propose a methodology for testing present value models of asset price determination. The key advantage of the approach is that it allows an informal evaluation of the "fit" of the model under investigation. This is accomplished by constructing a theoretical asset price from vector autoregressive (VAR) based forecasts of relevant fundamentals and by comparing the behaviour of actual and theoretical prices. If there is a high degree of similarity between the movements of the two series over time, then one might be inclined to view the theory as a success, even if it is possible to reject the model with formal statistical tests.

Campbell and Shiller (1987) use their methodology to assess the empirical merit of the expectations theory of the term structure, an example of a present value model. In particular, they compare the evolutions of actual and theoretical long/short yield spreads. Under the expectations theory the long/short spread is a function of expected future one-period changes in the short rate, and Campbell and Shiller (1987) construct theoretical long/short yield spreads from VAR-based forecasts of future changes in short rates. Because actual and theoretical spreads move closely together over time for a sample that ends in 1978, Campbell and Shiller (1987) conclude that there is an important element of truth to the expectations theory as a model of US long-term interest rates.

More recent research, however, has reported results less favourable to the expectations theory of the term structure. For instance, Hardouvelis (1994) employs the Campbell-Shiller (1987) methodology to study the relationship between short- and long-term US government yields from the mid-1950s to mid-1992. He concludes that there are large deviations of long rates from levels predicted by the expectations theory.

This paper employs the Campbell-Shiller (1987) methodology to re-examine the empirical evidence on the expectations theory at the long end of the term structure. The methodology is implemented in two ways. First, following standard practice, theoretical spreads satisfying the expectations theory are constructed from in-sample forecasts of future changes in short rates generated from a small-scale VAR. Second, theoretical spreads are computed from out-of-sample forecasts of changes in short rates with the parameters of each VAR equation updated with a Bayesian procedure.

The empirical results can be briefly described as follows. First, the expectations theory of the term structure receives considerable support from pre-1979 data on the basis of in-sample forecasts of future changes in short rates. This is also the case if theoretical spreads are computed from out-of-sample forecasts, provided that the Bayesian procedure is restricted to give less weight to new data than would be the case with OLS estimation over an expanding sample. But starting in the 1980s, there is a noticeable divergence between actual spreads and theoretical spreads computed from

out-of-sample forecasts. However, if the period of non-borrowed reserve targeting (1979Q4–1982Q4) is given zero weight when estimating the parameters of the VAR, theoretical spreads track actual spreads closely from the start of 1984 to the end of the 1990s.

The remainder of the paper is organised as follows. Section 2 is a brief review of selected literature. Section 3 discusses in greater detail the Campbell-Shiller (1987) approach to assessing the empirical content of the expectations theory. The section applies the method in its traditional manner (estimating the parameters of the VAR over the full sample) to a study of US long/short government yield spreads. It is shown that the expectations theory performs well over a period studied by Campbell and Shiller (1987) but that the theory appears to perform poorly over Hardouvelis' sample.

Section 4 reports the results of an application of the Campbell-Shiller (1987) methodology based on out-of-sample forecasts of changes in short rates. A Bayesian procedure to estimate the VAR is adopted which allows various degrees of influence of new data on the estimated parameter values. As a special case, the procedure reduces to OLS estimation over an expanding sample. Up to 1978, theoretical spreads under the expectations theory track actual spreads very closely when the influence of new data is restricted to be less than that implicit in OLS estimation, but there appears to be a deterioration in the performance of the expectations theory afterwards. The section investigates the possibility that the deterioration in the performance of the expectations theory is a consequence of the behaviour of interest rates during the period of non-borrowed reserve targeting. Taking the OLS estimates of the parameters of each VAR equation over the 1954Q3–1979Q3 period as prior parameter values, theoretical spreads are computed as from 1984 on the basis of out-of-sample forecasts of changes in short rates. In this way, the period of non-borrowed reserve targeting is given zero weight when estimating the parameters of the VAR. This approach leads to an improvement in the performance of the expectations theory over the 1984Q1–1999Q3 period. The final section offers some conclusions and interpretations of these results.

# 2. Review of selected literature

A large number of articles have tested the implications of the expectations theory of the term structure with long-term interest rate data from the United States.<sup>1</sup> Many of these studies report statistical rejections of the model. However, a more meaningful assessment of the merit of the theory might be based on an informal evaluation of the "fit" of the model. This is the approach taken by Campbell and Shiller (1987) in their study of monthly government bond yields. They find that, although they are able to reject the implications of the expectations theory at a high level of statistical significance, the theory

<sup>&</sup>lt;sup>1</sup> See for example the studies of Shiller et al. (1983), Campbell and Shiller (1984, 1991) and Fama and Bliss (1987), among others.

nevertheless explains a very large proportion of the variance of 20-year/one-month interest rate spreads.

In a more recent study, however, Hardouvelis (1994) employs the Campbell-Shiller (1987) methodology and finds the data less supportive of the expectations theory. He examines quarterly interest rate series and concludes that there are large deviations of 10-year/three-month yield spreads from their theoretical counterparts under the expectations theory.

A potential explanation of the different conclusions reached by Campbell and Shiller (1987) and Hardouvelis (1994) is that the studies examine different time periods. Campbell and Shiller (1987) examine the period 1959–78 while Hardouvelis studies the longer sample 1954Q3–1992Q2. Thus, anomalous behaviour of interest rates outside the Campbell-Shiller sample might be responsible for the discrepancies between the two studies.

An obvious candidate for a time of anomalous behaviour of interest rates outside the Campbell-Shiller sample is the period of non-borrowed reserve targeting by the US Federal Reserve which began in October 1979 and ended in late 1982. Evidence supporting the view that this period is associated with changes in the behaviour of interest rates is given in Huizinga and Mishkin's (1986) study. They find that the stochastic process governing the evolution of the US real rate of interest changed around 1979 and again around 1982. Interestingly, they identify October 1979 as the first break in the process for the real rate and October 1982 as the second. The first date coincides exactly with the shift to non-borrowed reserve targeting and the second date coincides exactly with the time when the Fed began to pay less attention to the growth of monetary aggregates.<sup>2</sup> These findings indicate that shifts in the process for US interest rates were associated with the changes in Fed operating procedures.

Previous researchers have noted the potential for shifts in the process for interest rates to influence standard tests of the expectations theory. For example, Hamilton (1988) develops a model for the evolution of short rates that captures in part the type of interest rate behaviour associated with the Fed's shift to non-borrowed reserve targeting. In his model, the mean level of the short rate and innovation variance undergo discrete shifts associated with changes in regime. A Markov-switching process determines the exact times of changes in regime. When interest rates evolve according to Hamilton's specification, non-linear forecasts of short rates will in principle dominate forecasts based on linear time series models, and this has implications for standard tests of the expectations theory of the term structure.

Fuhrer (1996) also proposes a model intended to capture the impact of changes in Fed operating procedures on the term structure of interest rates. He assumes that market participants' expectations are consistent with a small VAR model that includes measures of output and inflation, the long-term

<sup>&</sup>lt;sup>2</sup> See Meulendyke (1989) for a history of US monetary policy.

interest rate and the federal funds rate. The parameters of the federal funds rate equation, the monetary policy reaction function, are allowed to undergo discrete shifts associated with changes in regime. The possibility of discrete changes in the parameters of the policy reaction function helps to reconcile the behaviour of the long rate with the predictions of the expectations theory of the term structure over the 1966–1994 sample.

However, these attempts to reconcile the behaviour of long rates with the predictions of the expectations theory of the term structure for samples that include the period of non-borrowed reserve targeting have been less than completely successful. Driffill (1992) shows that over the 1963Q1–1987Q3 period there are relatively large discrepancies between actual long rates and those predicted by the expectations theory, conditional on Hamilton's model for the evolution of short rates. Fuhrer's (1996) model also delivers theoretical long rates that differ substantially from the values predicted by the expectations theory. Thus, whether the behaviour of US long rates can be reconciled with the expectations theory of the term structure for samples that include the period of non-borrowed reserve targeting remains an open question.

The next section explains in greater detail the Campbell-Shiller (1987) methodology for testing the expectations theory of the term structure. The approach is applied in its traditional manner to US interest rate data. The expectations theory receives considerable support for a sample that ends in 1978, but the theory appears to perform poorly for a longer sample that includes the period of non-borrowed reserve targeting.

## 3. The expectations theory and VAR-based specification tests

#### Theory and testing methodology

According to the expectations theory of the term structure, a long-term interest rate equals the sum of a constant term premium and a weighted average of current and expected future short rates. More formally, let  $R_t^e$  denote the *n*-period bond yield at time *t* implied by the expectations theory and let  $r_t$  represent the one-period rate of interest. Following Shiller (1979), the expectations theory of the term structure is the statement that these two rates satisfy the relation

(1) 
$$R_t^e = \theta + \sum_{j=0}^{n-1} w_j E_t r_{t+j}$$

where  $\theta$  is a constant term premium,  $E_t$  is the expectations operator given time *t* information and  $\{w_i\}$  are weights which depend upon the duration of the *n*-period bond. The weights satisfy

(2) 
$$w_i = g^j (1-g)/(1-g^n)$$

where  $g \equiv 1/(1 + \overline{R})$  and  $\overline{R}$  is the average *n*-period bond yield over the sample period.

The theoretical long/short interest rate spread satisfying the expectations theory is  $S_t^e \equiv R_t^e - r_t$ . It is easily shown that equation (1) is equivalent to

(3) 
$$S_t^e = \theta + \sum_{j=1}^{n-1} \omega_j E_t \Delta r_{t+j}$$

where  $\omega_j = w_j + w_{j+1} + ... + w_{n-1}$  and  $\Delta r_t = r_t - r_{t-1}$ . Thus, the expectations theory implies that the long/short spread is equal to a constant plus a linear combination of expected changes of the one-period (short) rate. Let

(4) 
$$\varepsilon_t \equiv S_t - S_t^e$$

be the difference between the actual spread,  $S_t \equiv R_t - r_t$ , and its theoretical counterpart. The expectations theory implies that  $\varepsilon_t \equiv 0$ ; however, if the term premium  $\theta$  is in fact not constant over time, a violation of the theory, then this would be reflected in time variation in  $\varepsilon_t$ .

The Campbell-Shiller approach to testing the expectations theory of the term structure relies on a comparison of the evolutions of actual and theoretical spreads, where a time series of theoretical spreads satisfying equation (3) is computed from forecasts of future short rate changes generated from a bivariate VAR.<sup>3</sup> The VAR includes the actual spread,  $S_t$ , and the change in the short rate,  $\Delta r_t$ .<sup>4</sup> The main advantage of the Campbell-Shiller methodology is that it permits an assessment of the economic significance of deviations of interest rates from levels predicted by the expectations theory. For example, the methodology allows one to compute an estimate of the standard deviation of the discrepancies ( $\varepsilon_t$  s). From this estimate, one can form a view on the relative importance of time variation in term premia for interest rate spread volatility.

An implicit assumption underlying the Campbell-Shiller method is that forecasts of short rate changes generated from a bivariate VAR adequately represent market participants' expectations. The small size of the VAR is not necessarily a drawback of the technique. The VAR includes the long/short spread which is, under the expectations theory, the optimal forecast of a weighted average of future short rate changes. Thus, the spread is a reasonable proxy for a vast array of information variables that agents have at their disposal.

<sup>&</sup>lt;sup>3</sup> Actually, it is the time-varying component of the right-hand side of equation (3) which is identified as the theoretical spread. All test statistics reported below are invariant to the addition of a constant term premium to the theoretical spread.

<sup>&</sup>lt;sup>4</sup> It is appropriate for the VAR to include these variables if the short rate is stationary in first differences rather than levels and the long/short spread is stationary. See Campbell and Shiller (1987) for a discussion of these and related issues. Both Campbell and Shiller (1987) and Hardouvelis (1994) present evidence supporting the view that the US short rate is integrated of order one while the long/short spread is stationary.

Perhaps a greater concern is that a constant-coefficient VAR estimated over the full sample may not adequately capture the evolution of market participants' expectations. Assuming that the parameters of the VAR remain unchanged throughout the sample is especially troubling if the period under study includes post-1978 data, which appear to include shifts in the process for interest rates. This section reports results from a traditional application of the Campbell-Shiller methodology which relies on the assumption that a constant-coefficient VAR can adequately represent market participants' expectations. The next section relaxes this assumption.

There are two standard VAR-based specification tests of the expectations theory associated with the Campbell-Shiller approach. The first is the sample correlation between actual and theoretical spreads,  $\rho \equiv corr(S, S^e)$ . The second is the ratio of standard deviations  $\sigma(S^e)/\sigma(S)$ , where  $\sigma$  denotes sample standard deviation. In large samples, these statistics should be unity if the expectations theory is true. When the expectations theory fails, the distance from unity of the values of these statistics sheds light on the empirical content of the expectations theory as a model of interest rate behaviour.

In order to supplement these standard tests of the expectations theory, this study reports two additional specification tests of the model. The first is the slope coefficient of a regression of the theoretical spread on the actual spread:

(5) 
$$S_t^e = \beta_0 + \beta_1 S_t$$

Under the expectations theory, the OLS slope coefficient  $\beta_1$  should equal one.<sup>5</sup> The second additional specification test is the sample standard deviation of the discrepancies between actual and theoretical long/short spreads,  $\sigma(\epsilon)$ . These measures may also provide information about the empirical content of the expectations theory.

#### Data and empirical results

This subsection employs the Campbell-Shiller (1987) methodology to study the relation between a short- and long-term interest rate over selected time periods. The short-term (one-period) rate is the three-month Treasury bill discount rate converted to a bond equivalent yield. The long rate is the 10-year government bond yield. Thus, in the context of equation (1), one period equals three months and n = 40. The data are quarterly and all interest rates are end-of-quarter values expressed as

<sup>&</sup>lt;sup>5</sup> As noted above,  $S_t^e$  is only identified up to an additive constant; therefore, the restriction  $\beta_0 = 0$  is not implied by the expectations theory of the term structure.







\* The vertical lines indicate the beginning and end of the period of non-borrowed reserve targeting.

percentage points at annual rates.<sup>6</sup> These interest rate series are plotted in Figure 1. The bivariate VAR model underlying the Campbell-Shiller approach includes the quarterly change in the short rate and the long/short spread. Following Hardouvelis (1994), a fourth-order VAR was specified.

Figure 2 on the next page plots actual and theoretical spreads over the 1959Q1–1978Q2 period, a sample studied by Campbell and Shiller (1987).<sup>7</sup> Theoretical spreads are computed, via equation (3), from VAR-based forecasts of the one-period change in the short rate. The VAR is estimated over the 1959Q1–1978Q2 period. As shown in the figure, movements in actual spreads can be explained to a large extent by shifts in VAR-based forecasts of future short rates changes. Table 1 reports numeric values of the specification tests discussed above. For the 1959Q1–1978Q2 period the correlation coefficient  $\rho$  is 0.94, the ratio of standard deviations is 0.90 and the slope coefficient  $\beta_1$  is 0.85. The measure of the size of the discrepancies,  $\sigma(\epsilon)$ , is about 30 basis points.

<sup>&</sup>lt;sup>6</sup> Up to 1977, the three-month T-bill rate is the average for the last month of the quarter and afterwards it is the value as of the last day of the quarter. Up to 1961, the 10-year bond yield is the average for the last month of the quarter and afterwards it is the value as of the last day of the quarter.

<sup>&</sup>lt;sup>7</sup> Campbell and Shiller (1987) study monthly data from the start of 1959 through August 1978.





Actual and theoretical spreads\*

\* Theoretical spreads are computed from VAR-based forecasts of future short rate changes. The VAR is estimated over the 1959Q1-1978Q2 period. Both spreads have been demeaned.

Table 1 also reports numeric values for the specification tests associated with the spreads shown in Figure 2 for the 1964Q3–1978Q2 period. The values of the statistics are about the same as those reported in the first column of the table and are useful for evaluating results reported in the next section.

Table 1
VAR-based tests of expectations theory: in-sample forecasts

	1959Q1-1978Q2	1964Q3-1978Q2	1954Q3-1992Q2
ρ	0.942	0.950	0.765
$\sigma(S^e)/\sigma(S)$	0.902	0.902	0.316
$\beta_1$	0.850	0.857	0.244
σ(ε)	0.315	0.338	0.934

Figure 3 on the next page plots actual and theoretical spreads over the 1954Q3–1992Q2 period computed from a VAR estimated over this period, which corresponds to the sample studied by Hardouvelis (1994). The expectations theory appears to receive less support from these data. As reported in Table 1, the correlation coefficient falls to 0.77, the ratio of standard deviations declines to 0.32 and the slope coefficient  $\beta_1$  falls to 0.24. The measure of the size of the discrepancies,  $\sigma(\varepsilon)$ ,



#### Actual and theoretical spreads\*



<sup>\*</sup> Theoretical spreads are computed from VAR-based forecasts of future short rate changes. The VAR is estimated over the 1954:Q3-1992:Q2 period. Both spreads have been demeaned.

rises to over 90 basis points. For comparison, the standard deviation of 10-year/three-month spreads over this period is about 120 basis points. Thus, a standard application of the Campbell-Shiller methodology to Hardouvelis' (1994) sample suggests that there are in fact large deviations of long rates from levels predicted by the expectations theory.

For comparison with the results shown in Table 1, the next section assesses the empirical merit of the expectations theory of the term structure on the basis of out-of-sample forecasts of changes in short rates.

# 4. VAR-based tests of the expectations theory: out-of-sample forecasts of changes in short rates

The previous section reported results of standard applications of the Campbell-Shiller (1987) methodology to the study of the term structure of interest rates. This entailed generating forecasts of future changes in short rates from a VAR estimated over the entire sample. Estimating the parameters of the VAR on data for the full sample is appropriate if market participants know the joint stochastic process for short rates and spreads that must be uncovered by an econometrician. However, in the case where market participants are learning about the process for interest rates, a VAR estimated over the

full sample may not accurately capture the evolution of expectations. This possibility is especially relevant if the sample period includes times of changes in the process for interest rates.

This section reports results of further applications of the Campbell-Shiller methodology to the same interest rate data studied in the previous section. However, in this section theoretical spreads satisfying the expectations theory are computed from out-of-sample forecasts of future changes in short rates. Therefore, only information actually available to market participants at the time when interest rates are determined is used to construct a theoretical spread under the expectations theory.

Perhaps the most straightforward way to generate out-of-sample forecasts of future changes in short rates would be to estimate the parameters of each VAR equation by OLS over an expanding sample. Under standard assumptions, the time t parameter estimates would be the maximum likelihood estimates for the system of VAR equations conditional on the information available as of time t. In what follows, however, a more general approach to estimating the VAR parameters is adopted. The parameters of each VAR equation are estimated by a Bayesian procedure which subsumes OLS estimation over an expanding sample but in general allows new data to have various degrees of influence on the estimated values of the parameters.

In order to derive the formulae for the Bayesian parameter estimates, prior distributions for the parameters of each VAR equation must be specified. It is assumed that prior information about the parameters of the spread equation is represented by a N( $\overline{\mu}_{sp}, \sigma_{sp}^2 \Sigma_{sp}$ ) distribution, where  $\sigma_{sp}^2 > 0$  is the variance of the innovation of the spread equation of the VAR. Likewise, it is assumed that prior information about the parameters of the equation for the change in the short rate is represented by a N( $\overline{\mu}_{sr}, \sigma_{sr}^2 \Sigma_{sr}$ ) distribution. Combining the prior distributions for the parameters of each equation of the VAR with the likelihood function for the normal linear statistical model gives posterior distributions for the parameters of each equation of the VAR. Given data from time *s* through time *t*, Hamilton (1994) shows that the means of the posterior distributions,  $\mu_{sp}$  and  $\mu_{sr}$ , are given by

(6a) 
$$\mu_{sp}(t) = (\Sigma_{sp}^{-1} + X(s,t)'X(s,t))^{-1}(\Sigma_{sp}^{-1} \cdot \overline{\mu}_{sp} + X(s,t)'Y_{sp}(s,t))$$

(6b) 
$$\mu_{sr}(t) = (\Sigma_{sr}^{-1} + X(s,t)'X(s,t))^{-1}(\Sigma_{sr}^{-1} \cdot \overline{\mu}_{sr} + X(s,t)'Y_{sr}(s,t))$$

where X(s,t) is the  $(t-s) \times 9$  matrix of explanatory variables from time *s* to time *t*,  $Y_{sp}(s,t)$  is the  $(t-s) \times 1$  vector of observations on the long/short spread from time *s* to time *t* and  $Y_{sr}(s,t)$  is the  $(t-s) \times 1$  vector of observations on the one-period change in the short rate.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup> Strictly speaking, equations (6) cannot be given their usual Bayesian interpretation. The Bayesian interpretation assumes that the regressors of each equation are exogenous, an assumption that is violated in the VAR context.

In what follows we take the prior means  $\overline{\mu}_{sp}$  and  $\overline{\mu}_{sr}$  to equal the values of OLS estimates of the parameters over a given initial sample period, say time *u* through time *v*. Furthermore, it is assumed that  $\Sigma_{sp} = \Sigma_{sr} = \gamma \cdot (X(u,v)'X(u,v))^{-1}$ , for various values of  $\gamma$ . Thus, the covariance matrices governing the prior uncertainty about the parameters are assumed to be proportional to the values implied by OLS estimation over the initial sample.

In the first application of the Bayesian procedure reported below, the initial sample period is taken to be the first 10 years of interest data, so that u = 1954Q3 and v = 1964Q2. Theoretical spreads are computed over the period 1964Q3–1999Q3 from out-of-sample forecasts generated from a VAR with parameter values as of time t given by equations (6a) and (6b) with s = 1964Q3. Three values of  $\gamma$  are considered. The first is  $\gamma = 1$ . In this case Bayesian estimation via equations (6a) and (6b) is identical to OLS estimation over an expanding sample. The second is  $\gamma = 10$ , in which case new data are given more weight than would be the case for OLS estimation over an expanding sample. The third is  $\gamma = 0.1$ , in which case new data are given less weight than would be the case with OLS estimation over an expanding sample.

The results from this exercise are reported in Table 2 on the next page. The upper panel of the table is concerned with pre-1979 behaviour of interest rate spreads. The first column reports results for  $\gamma = 1$ , so that estimation of the parameters of the VAR via equations (6a) and (6b) is equivalent to OLS estimation over an expanding sample. As revealed by the numeric values of the statistics, the expectations theory of the term structure receives some support on the basis of out-of-sample forecasts of changes in short rates over the 1964Q3–1978Q2 period. The correlation between actual and theoretical spreads is 0.90, the ratio of standard deviations is 0.98 and the slope coefficient  $\beta_1$  is 0.89. The measure of the size of the discrepancies,  $\sigma(\varepsilon)$ , is about 45 basis points.

The upper panel, second column of Table 2 reports statistics which pertain to the case  $\gamma = 10$ . Much less support for the expectations theory obtains in this case. Although the ratio of standard deviations remains at about unity, the correlation coefficient falls to 0.48 and the slope coefficient  $\beta_1$  falls to 0.56. The measure of the size of the discrepancies,  $\sigma(\epsilon)$ , rises to about 120 basis points.

Figure 4 plots actual and theoretical spreads for the case  $\gamma = 0.1$  over the 1964Q3–1999Q3 period. This is the case where new data are given less weight than would be the case with OLS estimation over an expanding sample. The upper panel, third column of Table 2 confirms what is apparent from the figure: there is remarkable support for the expectations theory in this case in pre-1979 data. The correlation between actual and theoretical spreads is 0.99, while the ratio of standard deviations and slope coefficient  $\beta_1$  are both 1.15. The measure of the size of the discrepancies,  $\sigma(\varepsilon)$ , is less than 19 basis points.

## Table 2

	1964Q3-1978Q2	1964Q3-1978Q2	1964Q3-1978Q2
	γ <b>=1</b>	γ <b>=10</b>	γ <b>=0.1</b>
ρ	0.902	0.483	0.996
$\sigma(S^e)/\sigma(S)$	0.983	1.153	1.150
$\beta_1$	0.887	0.557	1.145
σ(ε)	0.468	1.177	0.185
	1978Q3-1999Q3	1984Q1-1999Q3	1984Q1-1999Q3
	γ= <b>0.1</b>	γ= <b>0.1</b>	γ= <b>0.1</b>
ρ	0.952	0.935	0.952
$\sigma(S^e)/\sigma(S)$	0.838	0.760	1.018
$\beta_1$	0.798	0.711	0.970
$\sigma(\epsilon)$	0.465	0.435	0.344

Note: In all cases, u = 1954Q3. For the lower panel, third column results, v = 1979Q3; otherwise, v = 1964Q2.

It is of interest to compare the values of the statistics reported in the upper panel, third column of Table 2 with those reported in the second column of Table 1. Based on the statistic  $\sigma(\epsilon)$ , theoretical spreads computed from out-of-sample forecasts of changes in short rates (with  $\gamma = 0.1$ ) track actual spreads more closely than do theoretical spreads computed from a traditional application of the

#### Figure 4





\* Theoretical spreads are computed from out-of-sample forecasts with  $\gamma$ =0.1. Both spreads have been demeaned.

Campbell-Shiller methodology in which the parameters of the VAR are estimated on data for the full sample.

The interest rate spreads plotted in Figure 4 suggest that there is a deterioration in the performance of the expectations theory beginning some time during second half of the 1980s. This is confirmed by the first two columns, lower panel of Table 2, which report statistics associated with the spreads shown in the figure. Over the 1978Q3–1999Q3 period the correlation coefficient, ratio of standard deviations and regression slope coefficient all decline noticeably and the measure of the size of the discrepancies,  $\sigma(\varepsilon)$ , rises to about 45 basis points. This is also the case over the shorter 1984Q1–1993Q3 period.

After 1983, the theoretical spreads shown in Figure 4 are computed from a VAR estimated on a sample that includes the period of non-borrowed reserve targeting. As mentioned in Section 2, previous studies have had difficulty explaining the behaviour of interest rates for samples that include this period, so that the deterioration in the performance of the expectations theory after 1983 might not be too surprising.

In order to evaluate the empirical merit of the expectations theory of the term structure for samples that include the period of non-borrowed reserve targeting, it is of course necessary to model the response of market participants' expectations to successive changes in Fed operating procedures. This is a potentially challenging task which to date has yet to be successfully accomplished. However, it need not be necessary to model the evolution of market participants' expectations over the period of non-borrowed reserve targeting if one is interested in the performance of the expectations theory outside the 1979Q4–1982Q4 period. This would be the case, for example, if after the abandonment of non-borrowed reserve targeting in late 1982 market participants did not expect the episode to be repeated and therefore looked to the pre-1979 behaviour of interest rates when forming expectations about the future course of the term structure. In this case, the behaviour of interest rates during the 1979Q4–1982Q4 period may not be relevant for testing the expectations theory of the term structure for other times. Given the limited success of previous attempts to reconcile the behaviour of short- and long-term interest rates with the expectations theory for samples that include the period of non-borrowed reserve targeting, this would appear to be a simple hypothesis worth investigating.

One way to evaluate the empirical merit of the expectations theory under this simple hypothesis concerning expectations formation is to construct theoretical spreads after 1983 on the basis of out-of-sample forecasts of changes in short rates with the period 1979Q4–1982Q4 given zero weight when estimating the parameters of the VAR. This may be accomplished by taking the OLS estimates of the parameters of each VAR equation over the 1954Q3–1979Q3 period as prior parameter values and estimating the VAR parameters as from 1984Q1 by equations (6). The theoretical spreads computed in this way, with  $\gamma = 0.1$ , are plotted in Figure 5, along with actual spreads. This exercise provides considerable support for the expectations theory in post-1983 data. The lower panel, third column of

Table 2 reports the numeric values of the specification tests associated with these spreads. The correlation between actual and theoretical spreads is 0.95, the ratio of standard deviations is 1.02 and the slope coefficient  $\beta_1$  is 0.98. The measure of the size of the discrepancies,  $\sigma(\epsilon)$ , is about 35 basis points.

#### Figure 5



Actual and theoretical spreads\*

\* Theoretical spreads are computed from out-of-sample forecasts with  $\gamma$ =0.1. Both spreads have been demeaned.

Thus, the performance of the expectations theory after 1983 improves if the interest rate data for the period of non-borrowed reserve targeting are given zero weight when estimating the parameters of the VAR. However, while considerable, the evidence in favour of the expectations theory is not as strong as that documented for the pre-1979 sample, for which the standard deviation of the discrepancies was less than 19 basis points. However, a close inspection of Figure 5 suggests that the discrepancies between actual and theoretical spreads have not had a uniform size over the post-1983 sample period.

Figure 6 plots a rolling three-year standard deviation of the discrepancies between actual and theoretical spreads shown in Figure 5. This standard deviation trends down from the beginning of 1987 to around the middle of 1997 and then declines sharply. From the beginning of 1998, the standard deviation is less than 19 basis points, which implies that for the second half of the 1990s the performance of the expectations theory, when judged on the basis of the statistic  $\sigma(\varepsilon)$ , exceeds that observed in pre-1979 data.





#### Rolling standard deviation of discrepancies (ε)\*

\* Standard deviation computed over the previous three years. A zero mean for the discrepancies has been imposed.

#### 5. Conclusions

The expectations theory of the term structure is perhaps the most popular model of fluctuations of the yield curve. Some may find this surprising, because numerous studies have rejected the model with formal statistical tests. However, as shown by Campbell and Shiller (1987), it is possible for the expectations theory to explain a large proportion of the variance of interest rate spreads, even if the model is rejected at standard levels of statistical significance. Indeed, Campbell and Shiller (1987) find that, at least up to 1978, fluctuations of US long/short interest rate spreads are in strong agreement with the predictions of the expectations theory, which suggests that reported rejections of the theory may not have much economic significance.

More recent studies, however, which have examined the behaviour of US interest rates for samples that include the period of non-borrowed reserve targeting (1979Q4-1982Q4) have had greater difficulty reconciling the behaviour of US long rates with the predictions of the expectations theory of the term structure. These studies find relatively large discrepancies between actual long rates and the levels predicted by the expectations theory. This obtains even when the possibility of shifts in the process for interest rates, associated with changes in Fed operating procedures, are allowed for.

This paper re-examines the empirical content of the expectations theory of the term structure by employing the Campbell-Shiller (1987) methodology to study the behaviour of 10-year/three-month US government yield spreads. Perhaps the most interesting results are those associated with theoretical spreads computed from out-of-sample forecasts of future changes in short rates. When the parameters of the VAR model underlying the forecasts are estimated with a Bayesian procedure that gives less weight to new data than would be the case with OLS estimation over an expanding sample, theoretical spreads track actual spreads very closely in pre-1979 data. In fact, theoretical spreads computed from out-of-sample forecasts more closely than do theoretical spreads computed from in-sample forecasts. However, starting in the second half of the 1980s, there is a noticeable divergence between actual spreads and the theoretical spreads computed from out-of-sample forecasts.

The paper investigates the hypothesis that the deterioration in the performance of the expectations theory is a consequence of the behaviour of interest rates during the period of non-borrowed reserve targeting. This is accomplished by giving zero weight to data for the 1979Q4–1982Q4 period when constructing theoretical spreads after 1983. A simple hypothesis about market participants' expectations that would justify this approach to testing the expectations theory is that the shift to non-borrowed reserve targeting was not anticipated and after its abandonment in late 1982 was not expected to recur. This is clearly a simple assumption about the evolution of market participants' expectations, but it leads to discrepancies between actual and theoretical spreads with a standard deviation of less than 19 basis points in pre-1979 data and a standard deviation of about 35 basis points in post-1983 data. These values might be taken as benchmarks from which to judge the success of more sophisticated theories of investor expectations.

Moreover, the discrepancies between actual and theoretical spreads have not had a uniform size over the post-1983 sample period. They are relatively large in the early part of the sample, but are much smaller towards the end of the sample. This suggests that while the behaviour of interest rates during the period of non-borrowed reserve targeting may indeed have influenced market participants' expectations after 1982, the influence of this period has been declining over time and has essentially disappeared as of the mid-1990s.

Of course, the apparent success of the expectations theory of the term structure reported in this paper is conditional on the assumed model of expectation formation. The assumption that the expectations of market participants are consistent with a VAR model with parameter estimates that give relatively little weight to the behaviour of interest rates during the period of non-borrowed reserve targeting may not be too controversial. What is perhaps more surprising is that expectations appear to be consistent with relatively slow updating of the parameters of the VAR model. An interesting topic for future research is to identify the conditions under which expectations of this type are optimal.

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