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# RESERVE CURRENCY ALLOCATION: AN ALTERNATIVE METHODOLOGY

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**Abstract**

This paper provides a quantitative framework for choosing the composition of reserve currencies. Assuming that the central bank's performance objectives are defined in terms of ex post returns in different currency numeraires, the currency allocation problem is formulated as a multi-objective optimisation problem. The advantage of the proposed methodology is that it does not require any explicit assumptions about the risk preferences of the central bank or knowledge of the currency numeraire. Using some proxy values for the possible range of ex post returns measured in different currency numeraires, the study shows how the currency allocation problem can be solved. In particular, the proposed method borrows the concept of the degree of satisfaction from fuzzy decision theory and maximises such a function defined on the least favourable return outcome. In this sense, the proposed method differs from standard utility-based approaches which look for solutions that are best on average. The results of the study indicate that central banks on average are dollar-based investors on the basis of current allocations. Further, the study also indicates that if central banks consider an ex post return profile that safeguards the purchasing power of the reserves, then the currency distribution of reserves should more closely resemble the SDR basket.

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# Contents

# 1. Introduction

Currency reserves, in general, are held by central banks for a variety of reasons including among others transaction needs, intervention needs and wealth diversification needs. Transaction needs are of minor importance to developed economies that have good access to international capital markets. However, for countries that have strict exchange rate controls, transactions needs may play a larger role. Intervention needs arise when countries desire to have convertible currencies while at the same time wishing to retain the power to influence exchange rates. This type of demand for reserves is by far the most important one for those countries that have well-developed capital markets. Lastly, wealth effects may play some role in the final choice of the currency composition of the reserves. During recent years, this aspect has been receiving greater attention as the need for central banks to be more transparent about the role and use of currency reserves, which are considered to be part of the national savings, has grown. Moreover, the size and growth of currency reserves may provide signals to global financial markets on the credibility of the country's monetary policy and creditworthiness. In such a case, the return on the reserves held may not be inconsequential. We refer readers interested in additional information on the purpose and role of currency reserves to the articles by Roger (1993), Hann (1995) and Nordman (1987) among others. In this paper we will focus primarily on developing a methodology for choosing the currency composition of foreign exchange reserves.

One common approach among reserve managers to choosing the composition of reserve currencies is to match the currency composition of reserves to the currency composition of imports. The motivation for choosing this currency distribution is that it hedges the exchange rate risk that will otherwise be present when buying a given volume of imports. However, the allocation of currency reserves along these lines assumes that the primary motivation for holding reserves is to finance imports. Such an assumption may be questionable, to say the least, for many central banks.

Seen from a quantitative perspective, a standard tool used in finance for asset allocation purpose is to apply the mean-variance optimisation technique that is based on expected utility maximisation. However, for some central banks an obstacle to the application of this technique to the currency allocation problem has been the lack of agreement on the choice of the currency numeraire. Given the depth and liquidity of the foreign exchange markets, at least in the major currencies, it is difficult to argue that one particular currency should be preferred to another. Since mean-variance optimisation seeks to find an optimal trade-off between risk and return in a given numeraire, it is not clear how a mean-variance efficient portfolio can be chosen without specifying the numeraire.

In general, the problem with the theory of rational choice for portfolio selection is that it is necessary to identify investor risk preferences to choose an optimal portfolio. In an uncertain world such risk preferences are subject to change. Given this, it is not obvious what constitutes an optimal risk aversion level for central banks. Moreover, the composition of the portfolios themselves is very sensitive to the assumption about future expected return. Most central bankers will be reluctant to provide forecasts on the expected returns for risk of losing reputation. Further, most central banks would prefer to see a stable composition for the reserves that would help to explain a consistent investment policy. In formulating such an investment policy, the central bank might indicate the minimum objectives that have to be met for the reserve portfolio. The identification of such objectives can also help explain to the public why a particular currency allocation was chosen and to this extent will improve the transparency of reserve management policy.

The approach taken in this paper to solving the reserve currency allocation problem avoids explicit references to either the currency numeraire or the risk aversion level for the central bank. Instead, the objective of the central bank is posed in terms of minimising the worst possible return outcome in different currency numeraires. The advantage of such an approach is that it leads to the definition of a performance criterion in terms of an observable variable that is constantly monitored by the central bank. In solving the currency allocation problem, the following assumptions are made. First, reserve managers tend to care more about possible worst case ex post returns than ex ante expected returns. Secondly, there is no obvious numeraire to measure the ex post performance of the reserve currency portfolio. Lastly, there are only loosely defined acceptable ranges for the proportion of various currencies in the reserve portfolio that are based on liquidity and supply considerations. Under these assumptions, the currency allocation decision problem is solved within the framework of a multi-objective optimisation problem using tools from fuzzy decision theory (see Bellman and Zadeh (1970)).

The results of the study show that the proposed currency allocation technique, although conceptually simple, can uncover some facts that explain the composition of world currency reserves. As a first approximation, it appears from the results of this study that central banks tend to favour the US dollar as their numeraire currency. The study also shows that if central banks would favour the euro as a potential reserve currency in addition to the US dollar, the composition of world currency reserves would have to be reallocated more in line with the composition of the SDR basket.

The paper is organised as follows. Section 2 provides the motivation for the formulation of the currency allocation problem given here that is different from mean-variance theory. Section 3 discusses the formulation of the currency allocation problem that takes into account the objectives and constraints of many central banks. Section 4 presents the numerical results from the studies carried out and compares them with the actual distribution of the world currency reserves and the SDR currency basket. The final section concludes.

## 2. Motivation

The choice of the currency composition of foreign exchange reserves is strongly influenced by the purpose for which reserves are held. If it is assumed that the primary purpose for holding reserves is to meet the foreign liabilities on the balance sheet of the central bank, then the choice is rather obvious, namely one that matches the foreign liabilities. On the other hand, if it is assumed that there is no netting of government or other foreign debt against the reserves, then the composition of currency reserves cannot be selected on the basis of matching assets against liabilities. Another purpose for which currency reserves are used is to pay for imports. In this case what matters is not the value of the reserves but the volume of commodities that they can be used to purchase. To meet this objective the composition of currency reserves should ensure that their purchasing power does not decline. Again, if the central bank focuses on reserves as a store of wealth, then the currency composition could be chosen such that it is mean-variance efficient in a preferred currency numeraire. However, documented evidence on the application of mean-variance optimisation suggests that the optimal portfolios are extremely sensitive to the assumption about expected returns.

In an attempt to overcome the difficulty mentioned above, Black and Litterman (1992) suggest that incorporation of the global CAPM equilibrium risk premium can significantly improve the usefulness of mean-variance optimal portfolios. Their argument is that the extreme sensitivity of optimal portfolios to small changes in expected return arises because mean-variance optimisation completely ignores the assets side of the market. One can avoid this problem by incorporating an additional information that equates the supply and demand side for the assets by assuming the existence of an equilibrium risk premium. Black and Litterman show that investors can compute balanced optimal portfolios by tilting the equilibrium risk premiums to incorporate their market views.

An immediate question of interest in the context of choosing the currency composition of reserves is whether a similar framework can be used to choose an optimal reserve currency portfolio. Unfortunately, several difficulties arise in trying to extend such an approach to the reserve currency allocation problem. For instance, reserve managers may be reluctant to make a forecast that deviates from the equilibrium risk premium either because there is a lack of incentive or so as not to put their reputation at risk. Even otherwise one can argue that since central banks aim to keep their reserve currency composition stable, they may prefer to stick to the equilibrium framework by having neutral views. One difficulty with such an approach is that a market neutral portfolio on the basis of a CAPM framework may lead to a currency composition that does not meet the central bank's internal constraints.

The above discussion leads us to the following question: how could a central bank formulate the reserve currency allocation problem from a quantitative perspective? The availability of a quantitative methodology may provide the central bank with an alternative tool to choose the composition of currency reserves. In this paper we will try to formulate the currency allocation problem from such a perspective that takes into account a variety of objectives and constraints that a representative central bank could face. Before doing this we will first examine some of the shortcomings of the mean-variance optimisation theory from a central bank's point of view. Identifying these would help us seek alternative formulations that take into account the concerns of a central bank.

In general, classical approaches to portfolio selection require an appropriate utility function that quantifies the risk preferences of the investor. Given the probability distribution of asset returns, one can use the expected utility theorem to select portfolios that are optimal for a given level of risk aversion. In trying to apply portfolio selection schemes that are based on the expected utility theorem one can identify several difficulties from a central bank's perspective. First, choosing the appropriate level of risk aversion is not easy. This is because the purpose and role of currency reserves change dramatically depending on global economic conditions and any potential shocks that the economy may be subject to. This implies that the risk preferences of central banks can change dramatically within a short period of time in the event of a crisis that is most often unpredictable. Secondly, expected utility theory only ensures that the portfolio return will be good "on average" for the chosen risk level. However, there is no guarantee that this return will be achieved over the investment period. Invariably, a risk-averse central bank might prefer to have a guarantee that the realised return during the investment period will be above some minimum level. Stated differently, central banks tend to prefer setting target values for variables that are directly observable because it is much easier from the point of view of monitoring and reporting. One such variable is the realised or ex post return of the portfolio over the investment or accounting period. Assuming that ex post return is an important variable of interest to the central bank a related question that arises is in what currency this return should be measured. Having argued earlier that maintaining the purchasing power of currency reserves could be an important objective for the central bank, one can postulate that ex post returns will be measured in several currency numeraires. Imposing the constraint that the ex post return in the currency numeraire that appreciated the most over the investment period lies above a minimum value, the central bank could indirectly ensure that the purchasing power of reserves is safeguarded. Moreover, the representative central bank could also express a relative preference for some currencies over others to meet internal constraints.

The discussion so far on the attitude of central banks towards choosing an appropriate currency composition for reserves leads us to the following conclusion: although central banks may in general have certain "preference functions" ex-ante that motivate them to hold a particular currency composition, ex post their "degree of satisfaction" will depend on the realised return of the currency portfolio they hold. One can think of the realised return of the reserve portfolio as a random outcome generated from the underlying joint distribution of currency assets. Postulating that central banks are primarily concerned with the possible worst case returns ex post, a useful quantitative tool for currency allocation would be one that operates on the central bank's “degree of satisfaction” that depends only on the terminal wealth measured probably in several currency numeraires.

One way to devise such a portfolio selection rule is through the application of concepts from fuzzy decision theory that operates on the satisfaction or membership functions that quantify the investor's relative preferences to uncertain events (see Ramaswamy (1998)). Moreover, if we assume that the representative central bank will focus on ex post returns in various currency numeraires, there will be multiple objective functions to the optimisation problem. An advantage of using fuzzy decision theory concepts to solve the currency allocation problem is that they are well suited to handle multiple objective functions. In this paper we will assume that readers are familiar with the concepts on fuzzy decision theory presented in Zimmermann (1985) and Sakawa (1993). In the next section we formulate the currency allocation problem that will enable a central bank to explicitly set constraints on the worst case scenarios of ex post returns in several currency numeraires.

## 3. Problem formulation

Traditional approaches to portfolio selection revolve around the notion that investors focus on the mean and variance of the portfolio returns in domestic currency terms. In contrast, many central banks have no motivating reason for choosing one particular foreign currency and ignoring the others for measuring performance. Choosing a currency basket to measure performance, on the other hand, leads to solving a problem having similar complexity to the original currency allocation problem itself. In this case we are merely transforming the currency allocation problem into one of choosing an optimal currency basket with which to measure performance. Moreover, as pointed out earlier, the trade-off between risk and return for a central bank can change dramatically over time. This is because changing economic scenarios will have a considerable impact on the purpose for which reserves will be used. From an operational point of view, however, one can state what the soft constraints on the currency composition of reserves are and the minimum objectives that the reserve currency portfolio should fulfil. Given such requirements on the composition and objectives to be met by the reserve currency portfolio, the job of the reserve manager is to select a suitable currency composition for the reserves. In this section we will present a methodology for addressing the currency allocation problem faced by many central banks.

### 3.1 Currency composition

A broad objective to which many central banks will subscribe is the fact that diversification of reserves into different currencies helps to protect the value of reserves against event risks and international inflation. This is one important reason for central banks to diversify their reserve holdings into various currencies. Depending on the relative size and importance of the respective currencies, it is possible for central banks to define acceptable ranges for different currencies that make up the reserves. It will be assumed in the present study that a representative central bank would wish to invest only in the four major world currencies, namely the US dollar, euro, yen and pound sterling. Among these currencies, the central bank could indicate acceptable ranges for the percentage of a given currency in the reserve holdings. A basis for such a choice could be the depth and liquidity of the particular currency in question in the world foreign exchange market. We will now provide an example of how such a trade-off can be established using linear membership functions that model the constraints of a central bank on the proportion of a given currency in the reserves.

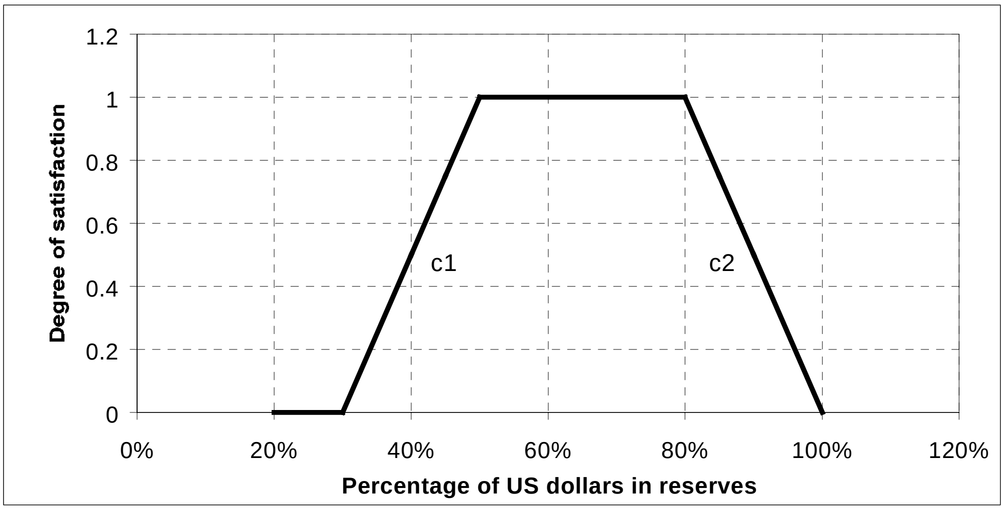
Let us consider the case of choosing the constraints on the proportion of US dollar in the reserves. Assume that a representative central bank is indifferent if the proportion of US dollars in the reserves is in the range 50-80%. Considering that there is no reason why the central bank should be worried if the percentage of US dollars deviates slightly outside this range, we will not impose hard constraints that prevent deviation from this range. Rather, we will assume that the degree of satisfaction declines steadily in proportion to the deviation from this desired range. One can capture this decline in satisfaction through linear membership functions. To illustrate this, let us denote the proportion of US dollars in the portfolio as $x_{usd}$ and let the preference of the central bank be expressed as "the proportion of US dollar in the reserves should be much greater than $x_{\min}$". Further, let us assume that the degree of satisfaction of the central bank improves up to a point where the US dollar proportion in the portfolio is $x_{\max}$ and then remains constant. In this case one can define a linear membership function that characterises the central bank's degree of satisfaction as

$$(1) \qquad \mu(x_{usd}) = \begin{cases} 0 & ; \quad x_{usd} \leq x_{\min} \\ \dfrac{x_{usd} - x_{\min}}{x_{\max} - x_{\min}} & ; \quad x_{\min} < x_{usd} \leq x_{\max} \\ 1 & ; \quad x_{usd} > x_{\max} \end{cases}$$

If, for instance, we consider that the minimum and maximum acceptable ranges for the proportion of US dollars in the reserves are 30% and 100%, respectively, then the linear membership functions that model the degree of satisfaction of the central bank to the proportion of US dollars in the reserves are as shown in Figure 1. In a similar manner, one can also construct suitable membership functions for the acceptable ranges of the other currencies in the reserves. We will now proceed to indicate how the dilemma over the choice of the currency numeraire can be overcome.

Figure 1

**Membership function for percentage of US dollars in reserves**




### 3.2 Currency numeraire

It is reasonable to assume that currency reserves in general represent a store of value for central banks. However, the difficult question for central banks to address is how the value of the reserves is to be measured. In other words, should a central bank measure the value of reserves in one particular currency and, if so, which one? If the central bank is using a currency basket, how should the relevant one be chosen? There are no definitive answers to any of these questions. We will assume in the following analysis that when measuring the performance of the reserve currency portfolio the central bank may have a relative preference for one currency, but on the other hand will not be completely indifferent to performance in the other currencies. If, for instance, the size of reserves is being reported in US dollars for accounting purposes, then the return in US dollar terms may be considered more important in comparison to the other currencies. However, if it happens that the US dollar depreciates considerably against other currencies, then this would result in lost income if the reserves are not diversified in other currencies. Although it would appear in this case that the performance of the reserve currency portfolio is reasonably good in US dollar terms, the performance in other currency numeraires may indicate a negative return over the holding period. Put differently, during periods when the US dollar depreciates against other currencies the purchasing power of the currency reserves will decline if the reserve currency holding is primarily in US dollars.

One way to avoid this happening is to structure a portfolio so as to ensure that the returns measured in other currencies over the holding period of interest lie above a minimum value. Imposing such a constraint would imply that there is no one single absolute currency numeraire. We will now focus our attention on choosing a data generating process for the exchange rate process. This will be used to compute the ex post returns in various currencies, since we argued that such a performance measure would be of interest to the central bank when allocating the currency reserves.

### 3.3 Realised returns

Given the type of constraints faced by a central bank it was postulated that a key variable of interest is the realised return over some time horizon. The realised return over the horizon of interest for a given portfolio will be generated by one sample vector drawn from the underlying joint distribution of the assets. Let us assume that a central bank would like to ensure that these sample outputs satisfy certain minimum requirements on the portfolio return. In order to generate these sample outputs, we will have to assume a model for the data generating process for the evolution of exchange rates. We consider two such models in this paper. The first model is based on the assumption that the evolution of exchange rates satisfies the uncovered interest parity condition. Although a consistent finding in the international literature is that there are systematic deviations from uncovered interest rate parity (see Meese and Rogoff (1983), Lewis (1995) and Frankel and Rose (1995)), most of these studies test the model's predictability over a one to three-month horizon. However, over a one-year or longer time period, which is of interest in this paper, there is less empirical evidence supporting this point of view. An argument in favour of the uncovered interest parity condition is that it provides a long-run equilibrium model under the assumption that there is no steady state risk premium for holding a given currency. This is a reasonable assumption at least for the major world currencies. However, in order to explore the implications for currency allocation of choosing a different data generating process for exchange rates, we will use the random walk model as the second data generating model in the study.

Under the uncovered interest parity condition, the evolution of the exchange rate is given by

(2) $$dS(t) = S(t)[r_0 - r_f]dt + \sigma S(t)dw(t)$$

where $r_0$ and $r_f$ denote the risk-free rates in the chosen currency numeraire and the foreign currency, respectively, $S(t)$ denotes the exchange rate at time $t$ with respect to the numeraire currency (i.e. $S$ units of numeraire currency are required to purchase one unit of foreign currency), $w(t)$ is a Wiener process and $\sigma$ is the volatility of the exchange rate returns. For multiple currencies, the evolution with respect to the numeraire currency can be modelled by taking into account the correlation between the currency returns as follows:

(3) $$dS_i(t) = S_i(t)[r_0 - r_i]dt + \sigma_i S_i(t)\sum_{j=1}^{N}\alpha_{ij}dw_j(t), \qquad i = 1,2,\cdots,N$$

where $S_i(t)$ denotes the exchange rate of the $i$th currency at time $t$ and the matrix $[\alpha_{ij}]$ is the Cholesky decomposition of the correlation matrix $[\rho_{ij}]$ between the currency returns. To model the evolution of exchange rates as a random walk process we need to set all the risk-free rates to zero in equation (3). We will assume that this is done when we use the random walk model as the data generating process for exchange rates.

Using the discretised version of the stochastic differential equations given by (3), one can simulate the sample functions of the exchange rates against the numeraire currency. If we denote the exchange rate of the *i*th currency for the *k*th sample function at time *T* by $S_{ik}(T)$, then the total return for the *k*th sample function expressed in the numeraire currency is given by

(4) $$R_{ik}(T) = \left[(1+r_i)^T \, S_{ik}(T)/S_i(0)\right] - 1, \qquad i = 1,2,\cdots,N$$

where it is assumed that *T* is expressed in years and $r_i$ is the one-year risk-free interest rate for the *i*th currency. For the investment in the numeraire currency the total return is given by

(5) $$R_0(T) = (1+r_0)^T - 1$$

We will assume in the analysis that all returns are annualised and drop the dependency on time. If we denote the weight of the *i*th currency in the reserves as $x_i$, then the annualised return in the numeraire currency for the *k*th sample output is given by

(6) $$\Re_k(\vec{x}) = x_0 R_0 + \sum_{i=1}^{N} x_i R_{ik}$$

Let us assume that the central bank is interested in ensuring that the realised return in the chosen numeraire currency lies above a given minimum value. There are a variety of ways in which such a constraint can be imposed. For instance, within the framework of fuzzy decision theory, one can construct a membership function that quantifies the degree of satisfaction to the central bank in question for a particular value of realised return. If we postulate that the central bank has several numeraire currencies of interest, then for a specific choice of the currency weights in the reserve portfolio the realised return for the *k*th sample output in the *j*th numeraire currency is given by

(7) $$\Re_k^j(\vec{x}) = x_0 R_0^j + \sum_{i=1}^{N} x_i R_{ik}^j$$

If the minimum and maximum ranges for the acceptable returns in the *j*th numeraire currency are given by $p_{\min}^j$ and $p_{\max}^j$, respectively, then the linear membership function that captures the degree of satisfaction for the *k*th sample output in the *j*th numeraire currency is given by

(8) $$\mu_{jk}\left(\Re_k^j(\vec{x})\right)=\begin{cases}0 & ; \quad \Re_k^j(\vec{x}) \le p_{\min}^j \\ \dfrac{\Re_k^j(\vec{x})-p_{\min}^j}{p_{\max}^j-p_{\min}^j} & ; \quad p_{\min}^j < \Re_k^j(\vec{x}) \le p_{\min}^j \\ 1 & ; \quad \Re_k^j(\vec{x}) > p_{\max}^j\end{cases}$$

The solution to the reserve currency allocation problem will be the one that ensures that under the given constraints the degree of satisfaction of the central bank is maximised for all possible sample outputs in every numeraire currency. We will now formulate an optimisation problem to find a suitable currency composition for the foreign exchange reserves of a central bank such that the degree of satisfaction is maximised.

### 3.4 Optimisation problem

Let us consider the case of a representative central bank interested in finding a suitable currency composition of reserves in US dollars, euros, yen and sterling. For sake of clarity we will use the currency code as the subscript to denote the weight of individual currencies in the reserves. In this notation, the currency weight vector will be given by $\vec{x}=[x_{usd}, x_{eur}, x_{jpy}, x_{gbp}]$. Let us assume that the currency numeraires of interest to the representative central bank are US dollar, euro and yen. Let the central bank's constraints on the proportion of individual currencies in the reserves be modelled as in Figure 1. In this case we need two membership functions for each currency to model the constraint functions. If we denote these two constraints as $c_1$ and $c_2$, respectively, then the membership functions that model the constraints on the proportion of the *i*th currency in the reserves are given by

(9) $$\mu_{i,c_1}(\vec{x})=\begin{cases}0 & ; \quad x_i \le x_{\min}^{i,c_1} \\ \dfrac{x_i - x_{\min}^{i,c_1}}{x_{\max}^{i,c_1}-x_{\min}^{i,c_1}} & ; \quad x_{\min}^{i,c_1} < x_i \le x_{\max}^{i,c_1} \\ 1 & ; \quad x_i > x_{\max}^{i,c_1}\end{cases}$$

(10) $$\mu_{i,c_2}(\vec{x})=\begin{cases}1 & ; \quad x_i \le x_{\min}^{i,c_2} \\ \dfrac{x_i - x_{\max}^{i,c_2}}{x_{\min}^{i,c_2}-x_{\max}^{i,c_2}} & ; \quad x_{\min}^{i,c_2} < x_i \le x_{\max}^{i,c_2} \\ 0 & ; \quad x_i > x_{\max}^{i,c_2}\end{cases}$$

Given the membership functions that model the goals and constraints of the central bank, the maximising decision can be computed by solving the following optimisation problem:

(11) $$\underset{\vec{x}}{Maximise} \quad Min \left\{ \mu_{jk}\left(\Re_k^j(\vec{x})\right), \ \mu_{i,c_1}(\vec{x}), \ \mu_{i,c_2}(\vec{x}) \right\}, \quad i \in I, \ j \in J, \ k \in K$$

subject to

(12) $$x_{usd} + x_{eur} + x_{jpy} + x_{gbp} = 1$$

(13) $$x_i \geq 0, \quad i \in I$$

In the above relations, the set $I = [usd, eur, jpy, gbp]$ is the set of all the currencies in the reserves, the set $J = [usd, eur, jpy]$ is the set of numeraire currencies and the set $K$ is the set of possible sample outputs in the $j$th currency numeraire.

By introducing the auxiliary variable $\lambda$, the above optimisation problem can be reduced to the following one:

(14) $$\underset{\vec{x}}{Maximise} \quad \lambda$$

subject to

(15) $$\mu_{jk}\left(\Re_k^j(\vec{x})\right) \geq \lambda, \quad j \in J, \ k \in K$$

(16) $$\mu_{i,c_1}(\vec{x}) \geq \lambda, \quad i \in I$$

(17) $$\mu_{i,c_2}(\vec{x}) \geq \lambda, \quad i \in I$$

(18) $$x_{usd} + x_{eur} + x_{jpy} + x_{gbp} = 1$$

(19) $$x_i \geq 0, \quad i \in I$$

It is useful to note here that the membership functions given by $\mu(\cdot)$ in the inequalities (15)-(17) are all linear in the variable $\vec{x}$. Hence, the above optimisation problem is a conventional linear programming problem. In the next section we will present the numerical results for particular choices of the constraint variables $p_{\min}$, $p_{\max}$, $x_{\min}$ and $x_{\max}$ that define the membership functions.

## 4. Numerical results

In Section 3, we outlined an approach for solving the reserve currency allocation problem through the use of concepts from fuzzy decision theory. It was shown that by using linear membership functions to define the goals and constraints of the central bank the reserve currency allocation problem reduces to solving a linear programming problem. In this section, we will indicate a methodology for generating the sample outputs from the underlying data generating process for exchange rates which is required for computing the membership functions. We will then provide numerical results for specific choices of the minimum and maximum ranges for the variables that determine the membership functions.

### 4.1 Quasi-random sequences

The number of constraints in the linear programming problem formulated above increases with the number of random samples simulated to generate the distribution for realised returns. In order to keep the sample size small in the simulations and yet be able to generate a reasonable distribution for the returns, we will use the quasi-random Sobol sequence (see Press et al (1992)). The advantage of using quasi-random sequences is that they generate sequences of *n*-tuples that fill *n*-dimensional space more uniformly than uncorrelated random points generated by pseudo-random number generators. In performing the numerical experiments we used 3-tuple Sobol sequences of 1024 points to generate the distribution of realised returns in each currency numeraire. Using the uncovered interest parity condition or a simple random walk to model the evolution of exchange rates, sample vectors of returns in the numeraire currency can be generated by drawing samples from a multi-normal distribution. For purpose of illustration, let us consider generating the returns in the numeraire currency US dollar. In this case, samples of random vectors $\vec{R}^{usd} = [R^{usd}_{eur}, R^{usd}_{jpy}, R^{usd}_{gbp}]$ can be drawn from the multi-normal distribution given by $N(\vec{\eta}^{usd}, \Sigma^{usd})$, where $\vec{\eta}^{usd}$ is the annualised mean return vector and $\Sigma^{usd}$ is the annualised covariance matrix of currency returns in US dollars. The mean return vector in the US dollar currency numeraire is given by

(20) $$\vec{\eta}^{usd} = [\eta^{usd}_{eur}, \eta^{usd}_{jpy}, \eta^{usd}_{gbp}]$$

where,

(21) $$\eta^{usd}_{eur} = (1 + (r_{usd} - r_{eur}))^T - 1$$

(22) $$\eta^{usd}_{jpy} = (1 + (r_{usd} - r_{jpy}))^T - 1$$

(23) $$\eta^{usd}_{gbp} = (1 + (r_{usd} - r_{gbp}))^T - 1$$

In equations (21)-(23), $r_i$ denotes the one-year risk-free interest rate in the *i*th currency and *T* is the investment horizon in years of interest to the central bank. Using the above equations in conjunction with equation (5) it is possible to generate the *k*th sample return in the *j*th numeraire, and hence compute the membership function $\mu_{jk}(\Re_k^j(\vec{x}))$. It is useful to note here that if the data generating process is a simple random walk, then the mean vector $\vec{\eta}^{usd}$ in equation (20) will be a zero vector.

### 4.2 Covariance matrix estimation

For generating the random return vectors in various currency numeraires, an estimate of the covariance matrix of currency returns is necessary. Although covariance matrices are relatively stable when estimated over longer historical time periods, they do change when estimated over shorter time periods such as one or two years. This raises the question as to what time period should be used for estimating the covariance matrix of currency returns. In the present study we addressed this problem by considering two sets of covariance matrices for generating the random vectors of returns in the numeraire currency. In the first case, we used quarterly returns over the historical time period December 1988 to December 1998 to estimate the covariance matrix in every currency numeraire of interest. These covariance matrices were then used to generate random sample vectors of returns using the Sobol sequence in order to compute the membership functions $\mu_{jk}(\Re_k^j(\vec{x}))$ in the *j*th numeraire currency for the *k*th sample return. In the other case, we used weekly samples of returns over two-year periods beginning December 1988 to estimate five different covariance matrices in each currency numeraire. This in turn leads to a total of 5120 linear constraints for the *j*th currency numeraire in the inequality given by (15).[1] For the sake of brevity, only the annualised covariance matrices in different currency numeraires estimated over the ten-year period from December 1988 to December 1998 using quarterly currency returns are shown in Tables 1-3. In computing the figures given in Tables 1-3, the Deutsche mark was used as a proxy for the euro.

Table 1

**Covariance matrix for returns in US dollars**

| Currency | EUR | JPY | GBP |
|---|---|---|---|
| **EUR** | 0.00803 | 0.00489 | 0.00512 |
| **JPY** | 0.00489 | 0.01208 | 0.00286 |
| **GBP** | 0.00512 | 0.00286 | 0.00686 |

1 In this particular case, the total number of linear constraints in the optimisation problem with three currency numeraires is equal to 15360.

Table 2

**Covariance matrix for returns in euros**

| Currency | USD | JPY | GBP |
|---|---|---|---|
| **USD** | 0.00803 | 0.00342 | 0.00243 |
| **JPY** | 0.00342 | 0.01069 | 0.00066 |
| **GBP** | 0.00243 | 0.00066 | 0.00473 |

Table 3

**Covariance matrix for returns in yen**

| Currency | USD | EUR | GBP |
|---|---|---|---|
| **USD** | 0.01115 | 0.00690 | 0.00837 |
| **EUR** | 0.00690 | 0.01061 | 0.00941 |
| **GBP** | 0.00837 | 0.00941 | 0.01297 |

### 4.3 Membership functions

The choice of the parameter values for $p_{\min}$, $p_{\max}$, $x_{\min}$ and $x_{\max}$ for the various currency numeraires that define the membership functions will have a significant influence on the composition of the foreign exchange reserves. We assumed in the numerical study that the annualised return measured in the US dollar numeraire should be greater than 0% whereas in the numeraires euro and yen the representative central bank would tolerate a worst case annualised return of –20% over the horizon of interest. Such an assumption will tend to bias the numeraire currency in favour of the US dollar. Table 4 shows the chosen values for the various parameters that define the membership functions used in the inequalities (15)-(17).

Table 4

**Parameters chosen to define membership functions**

| | $p_{\min}$ | $p_{\max}$ | $x_{\min}^{c_1}$ | $x_{\max}^{c_1}$ | $x_{\min}^{c_2}$ | $x_{\max}^{c_2}$ |
|---|---|---|---|---|---|---|
| **USD** | 0.00 | 0.20 | 0.30 | 0.50 | 0.80 | 1.00 |
| **EUR** | –0.20 | 0.20 | 0.00 | 0.10 | 0.40 | 0.60 |
| **JPY** | –0.20 | 0.20 | _ | _ | 0.20 | 0.40 |
| **GBP** | _ | _ | _ | _ | 0.05 | 0.10 |

It may be worth mentioning here that the worst case return of –20% in euro and yen terms may at first sight appear to be rather low. However, given that the historical volatilities of currency returns average 8-10% a constraint of –20% on returns only implies that the downside risk should not exceed a little more than two standard deviations. The advantage of formulating the constraints directly in terms of returns helps to draw the investor's attention to the magnitude of potential losses. The classical interpretation of the term "risk" as the standard deviation of returns fails to provide this intuition to investors who mostly focus on return.

### 4.4 Reserve currency composition

This section presents the results of the numerical study carried out using the parameter values given in Table 4 to define the membership functions. The linear programming problem formulated in Section 3.4 was solved using the NAG C-library software. It was assumed in the study that the time horizon of interest to the central bank is two years.[2] The simulations were carried out under both uncovered interest parity and random walk as potential models for the evolution of exchange rates. For easy reference we will refer to the currency allocation on the basis of the uncovered interest parity condition and time-invariant covariance matrix assumption for currency returns as UIP_TI. For the case where weekly data was used to estimate the covariance matrices over two-year time periods, we will refer to the model as UIP_TV to denote the time-varying feature of covariance matrices being incorporated. The corresponding abbreviations for the random walk model will be denoted as RWA_TI and RWA_TV, respectively. Table 5 shows the currency allocations for reserves using each of these models which differ only in their assumptions concerning the stability of covariance matrices or the mean of the return distribution. Also shown in this table is the actual currency distribution of world foreign exchange reserves held by central banks as of beginning of January 1999.

Table 5

**Composition of currency reserves**

| **Model** | **USD** | **EUR** | **JPY** | **GBP** |
|---|---|---|---|---|
| **UIP_TI** | 63.4% | 20.6% | 6.1% | 9.9% |
| **UIP_TV** | 62.4% | 14.6% | 14.0% | 9.0% |
| **RWA_TI** | 76.9% | 12.6% | 2.2% | 8.3% |
| **RWA_TV** | 75.8% | 9.5% | 8.7% | 6.0% |
| **World reserves [3]** | 70.0% | 19.7% | 6.1% | 4.2% |

[2] The currency allocation changed insignificantly for any choice of the time horizon between 1.25 years and 2.25 years.

[3] The share of the euro in world reserves includes that of the Swiss franc (0.9%).

Some interesting conclusions can be drawn from examining Table 5. We note that the composition of reserve currencies selected on the basis of model UIP_TI is fairly similar to the actual composition of world foreign exchange reserves. In particular, the percentage of the euro and yen in the currency allocation using this model is almost identical to the actual composition of world currency reserves. Another observation of interest here is that incorporating the feature that the covariance matrices of currency returns are time-varying through the use of model UIP_TV has very little impact on the proportion of US dollars in the reserves. This suggests that the proposed currency allocation technique is quite robust at least to changes in the higher order moments of the return distribution.

The choice of the random walk model, on the other hand, significantly changes the expected returns for the various assets compared to the model using the uncovered interest parity condition. For instance, holding US dollar assets leads to an expected excess return of 2.6% over euro assets and 5.2% over yen assets (on the basis of interest rate differentials as of May 1999, when the study was carried out). This tends to favour greater weights for the US dollar in the currency allocation chosen by the models RWA_TI and RWA_TV. However, it is interesting to note that all the portfolio allocations shown in Table 5 are still well balanced. One conclusion that arises from this study is that estimation errors that arise when computing the moments of the return distribution are less important, suggesting that the proposed currency allocation methodology is very robust. This is quite in contrast to the mean-variance portfolio allocation methodology, where estimation errors can have a big impact on the composition of the optimal portfolio.

Again examining Table 5 it can be said that all models recommend an increase in the proportion of the pound sterling in world currency reserves from existing levels. However, the lower weight for sterling in world currency reserves may be partly due to fact that sterling assets are considered to be less liquid. A final comment worth making here is that the currency allocations given in Table 5 were computed subject to the constraint that the returns in US dollar terms should not be negative over the investment horizon. Such a constraint can be said to favour a US dollar-based investor. Seen from this perspective it can be said that central banks holding at least 60% dollar reserves favour the US dollar as numeraire currency. This conclusion follows from the fact that the currency allocation using the uncovered interest rate model, which is a long-run equilibrium model, recommends around 63% weight for US dollars to ensure that returns on the reserves are not negative in dollar terms. Seen from the same perspective, the currency composition of world reserves suggests that central banks on average tend to favour the US dollar as their numeraire currency.

In order to examine the impact of a shift by central banks in favour of the euro as a reserve currency, another optimisation exercise was carried out with a slightly different choice for the membership functions. In particular, it was assumed that the central bank would be willing to tolerate an annualised return of –5% measured in US dollars over the time horizon of interest. However, the central bank would like to ensure that the annualised return of the reserve currency portfolio measured in euros is not lower than –10%. All other parameters in the membership functions were kept unchanged. It is useful to note here that by giving additional importance to the euro, the central bank will be able to hedge the risk of loss in purchasing power for the reserves if the US dollar depreciates. The resulting currency distribution for the reserves from the optimisation exercise is shown in Table 6. For purposes of comparison, we have also shown the composition of the SDR basket in this table as of May 1999, when this study was carried out.

Table 6

**Composition of currency reserves**

| Model | USD | EUR | JPY | GBP |
|---|---|---|---|---|
| **UIP_TI** | 43.0% | 32.3% | 15.0% | 9.7% |
| **UIP_TV** | 37.7% | 30.8% | 21.7% | 9.8% |
| **RWA_TI** | 52.1% | 29.6% | 8.7% | 9.6% |
| **RWA_TV** | 45.0% | 25.1% | 20.0% | 9.9% |
| **SDR basket** | 42.9% | 27.6% | 17.0% | 12.5% |

Examining the figures in Table 6, it is interesting to note that the currency composition resulting from the model UIP_TI is very similar to the composition of the SDR basket. Clearly, a currency composition such as the SDR basket tends to favour the criterion of maintaining the purchasing power of the reserves. A significant reallocation of reserves along these lines is only possible if central banks increase their tolerance to taking losses in US dollar terms.

## 5. Conclusions

This paper is concerned with the issue of finding a suitable currency composition for the foreign exchange reserves held by central banks. Taking the view that central banks have difficulty quantifying their risk preferences or identifying a currency numeraire, the approach adopted is to examine ex post returns on the portfolio. Making the assumption that the central bank's performance objectives are defined on ex post returns in different currency numeraires, the currency allocation problem was formulated as a multi-objective optimisation problem. From a practical perspective, this approach is easy to use and leads to balanced reserve currency composition. Moreover, the currency allocations are relatively insensitive to changes in the distribution of asset returns, and hence the proposed technique is quite robust as an optimisation tool for portfolio selections.

Several interesting conclusions emerge from this study. First, it appears that central banks on average tend to favour the use of the US dollar as their numeraire currency. This is quite evident from the choice of the parameters used to define the membership functions for the allocation given in Table 5. Secondly, if central banks tend to give importance to returns in other currencies that would favour an approach to maintain the purchasing power of the reserves, then a significant reallocation of the reserve currency composition would be necessary. Such a move could have a profound impact on foreign exchange markets. Thirdly, even if central banks tend to favour the US dollar as the numeraire currency, the proportion of the pound sterling in the currency portfolio could be increased at the expense of a reduction in US dollar holdings. This could in turn provide additional diversification for the currency reserves which would serve as a hedge against a potential weakening of the US dollar.

# References


Bellman, R and L A Zadeh (1970): "Decision-Making in a Fuzzy Environment". *Management Science*, Vol. 17, pp. 141-64.

Black, F and R Litterman (1992): "Global Portfolio Optimization". *Financial Analysts Journal*, Sept.-Oct., pp. 28-43.

Frankel, J A and A K Rose (1995): "Empirical Research on Nominal Exchange Rates", in G M Grossman and K Rogoff, *Handbook of International Economics*, Vol. 3, Elsevier Science, Amsterdam.

Hann, P (1995): "The Management of Canada's Foreign Exchange Reserves". *M.S Thesis*, Faculty of Commerce and Business Administration, The University of British Columbia.

Lewis, K (1995): "Puzzles in International Financial Markets", in G M Grossman and K Rogoff, *Handbook of International Economics*, Vol. 3, Elsevier Science, Amsterdam.

Meese, R and K Rogoff (1983): "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample? " *Journal of International Economics*, Vol. 14, pp. 3-24.

Nordman, T (1987): "Management of Foreign Exchange Reserves at the Bank of Finland". Presented at the World Bank Portfolio Management Seminar for Central Banks and Official Institutions, 4 December, Washington.

Press, H P, S A Teukolsky, W T Vetterling and B P Flannery (1992): "Numerical Recipes in C". Cambridge University Press.

Ramaswamy, S (1998): "Portfolio Selection Using Fuzzy Decision Theory". *BIS Working Papers*, No. 59.

Roger, S (1993): "The Management of Foreign Exchange Reserves". *BIS Economic Papers*, No. 38.

Sakawa, M (1993): *Fuzzy Sets and Interactive Multiobjective Optimisation.* Plenum Press, London.

Zadeh, L A (1965): "Fuzzy Sets". *Information and Control*, Vol. 8, pp. 338-53.

Zimmermann, H J (1985): *Fuzzy Set Theory and Its Applications.* Kluwer Academic Publishers, Dordrecht.

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