BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers
No 532

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by Aytek Malkhozov, Philippe Mueller, Andrea Vedolin and Gyuri Venter

Monetary and Economic Department

December 2015

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Mortgage Risk and the Yield Curve[*]

Aytek Malkhozov[†]   Philippe Mueller[‡]   Andrea Vedolin[§]

Gyuri Venter[¶]

Abstract

We study the feedback from the risk of outstanding mortgage-backed securities (MBS) on the level and volatility of interest rates. We incorporate the supply shocks resulting from changes in MBS duration into a parsimonious equilibrium dynamic term structure model and derive three predictions that are strongly supported in the data: (i) MBS duration positively predicts nominal and real excess bond returns, especially for longer maturities; (ii) the predictive power of MBS duration is transitory in nature; and (iii) MBS convexity increases interest rate volatility, and this effect has a hump-shaped term structure.

Mortgage-backed securities (MBS) and, more generally, mortgage loans constitute a major segment of US fixed income markets, comparable in size to that of Treasuries. As such, they account for a considerable share of financial intermediaries' and institutional investors' exposure to interest rate risk.[1] The contribution of MBS to the fluctuations in the aggregate risk of fixed income portfolios over short to medium horizons is even more important. Indeed, because most fixed-rate mortgages can be prepaid and refinanced as interest rates move, the variation of MBS duration can be very large, even over short periods of time.[2]

In this paper we study the feedback from fluctuations in the aggregate risk of MBS onto the yield curve. To this end, we build a parsimonious dynamic equilibrium term structure model in which bond risk premia result from the interaction of the bond supply driven by mortgage debt and the risk-bearing capacity of specialized fixed income investors.

The equilibrium takes the form of a standard Vasicek (1977) short rate model augmented by an affine factor, aggregate MBS dollar duration, which captures the additional interest rate risk that investors have to absorb. Intuitively, a fall in mortgage duration is similar to a negative shock to the supply of long-term bonds that has an effect on their prices. In addition to duration itself, its sensitivity to changes in interest rates, measured by aggregate MBS dollar convexity, also plays a role. Because MBS duration falls when interest rates drop, mortgage investors who aim to keep the duration of their portfolios constant for hedging or portfolio rebalancing reasons will induce additional buying pressure on Treasuries and thereby amplify the effect of an interest rate shock. As a result, the MBS channel can simultaneously affect bond prices and yield volatility.

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[1]Between 1990 and 2014 the average value of outstanding mortgage-related and Treasury debt was USD 5tr each. Financial intermediaries and institutional investors hold approximately 25% of the total amount outstanding in Treasuries, and approximately 30% of the total amount outstanding in MBS. Government-sponsored enterprises (GSEs) hold on average around 13% of all outstanding MBS (see Securities Industry and Financial Markets Association (2013) and the Flow of Funds Tables of the Federal Reserve).

[2]Aggregate MBS duration can drop by more than two years within a six-month period (see also Figure 1). The duration of Treasuries does not experience changes of such magnitude over short horizons. Taking into account the value of outstanding mortgage debt, we calculate that a one standard deviation shock to MBS duration is a dollar duration equivalent of a USD 368bn shock to the supply of 10-year Treasuries.

Our model makes a range of predictions for which we find strong empirical evidence. First, MBS duration predicts both nominal and real bond excess returns. This effect is stronger for longer maturity bonds that are more exposed to interest rate risk. At the same time, the effect is weaker for real bonds if real rates are imperfectly correlated with nominal rates and are less volatile. Accordingly, we find an economically significant relationship between duration and bond risk premia, in particular at longer maturities: A one standard deviation change in MBS duration implies a 381 basis point change in the expected one-year excess return on a 10-year nominal bond and a 199 basis point change in the expected one-year excess return on a 10-year real bond. These effects imply an approximately $38 \approx 381/10$ (20) basis point increase in nominal (real) 10-year yields, assuming that most of the effect on returns happens within a year.

Second, while large in size, shocks to MBS duration and their effect on bond excess returns are transient. Our model captures the fast mean-reversion in aggregate MBS duration by linking it to both interest rate mean-reversion and the renewal of the mortgage pool through refinancing. For example, running predictive regressions for different return horizons, we find little additional effect of MBS duration on bond excess returns beyond one year.

Finally, in our model the feedback between changes in long-term yields and MBS duration translates into higher yield volatility: Lower interest rates decrease duration, which in turn decreases the term premium and further lowers long-term rates. Different from noncallable bonds, callable bonds such as MBS typically feature a concave relationship between prices and yields, the so-called negative convexity. More negative convexity implies that the duration and, therefore, the market price of risk are more sensitive to changes in interest rates. Empirically, we find that the effect is hump-shaped and most pronounced for maturities between two and three years. In terms of magnitude, any one standard deviation change in MBS dollar convexity changes 2-year bond yield volatility by approximately 37 basis points. A calibrated version of our model reproduces the above predictions with similar economic magnitudes.

The statistical significance and the magnitude of our estimates remains stable when we control for a range of standard predictors of bond risk premia and yield volatility,

including yield factors, macroeconomic variables, and bond market liquidity measures. Overall, we find little overlap between the predictive power of MBS duration and convexity and that of other factors, which justifies the narrow focus of the paper on the MBS channel.

Our paper builds on the premise that fluctuations in MBS duration prompt fixed income investors to adjust their hedging positions, rebalance their portfolios, or more generally revise the required risk premia at which they are willing to hold bonds. To support this view, we show that the increase in the share of outstanding MBS held by Fannie Mae and Freddie Mac (government-sponsored enterprises or GSEs) can be associated with a strengthening of the MBS duration channel, whereas the subsequent decrease in that share and the increasingly important role of the Federal Reserve made it weaker. GSEs actively manage their interest rate risk exposure, while the Federal Reserve has no such objective. These findings are in line with our interpretation of the main results of the paper.

The MBS channel analyzed in our paper has attracted the attention of practitioners, policy makers, and empirical researchers alike. Perli and Sack (2003), Chang, McManus, and Ramagopal (2005), and Duarte (2008) test the presence of a linkage between various proxies for MBS hedging activity and interest rate volatility. Unlike those papers, we look at the effect that MBS convexity has on the entire term structure of yield volatilities, and find that it is strongest for intermediate (and not long, as previously assumed) maturities. Our model provides an explanation for this finding. In contemporaneous work, Hanson (2014) reports results similar to ours regarding the predictability of nominal bond returns by MBS duration. In contrast to the theoretical framework that guides the author's analysis, our dynamic term structure model allows us to jointly explain the effect of mortgage risk on real and nominal bond risk premia, and bond yield volatilities across different maturities.

Our work is also related to the literature on government bond supply and bond risk premia. We make use of the framework developed by Vayanos and Vila (2009). In their model, the term structure of interest rates is determined by the interaction of preferred habitat investors and risk-averse arbitrageurs, who demand higher risk premia

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as their exposure to long-term bonds increases. Thus, the net supply of bonds matters. Greenwood and Vayanos (2014) use this theoretical framework to study the implications of a change in the maturity structure of government debt supply, similar to the one undertaken in 2011 by the Federal Reserve during "Operation Twist". Our paper is different in at least three respects. First, in our model the variation in the net supply of bonds is driven endogenously by changing MBS duration, and not exogenously by the government. Second, the supply factor in Greenwood and Vayanos (2014) explains low frequency variation in risk premia, because movements in maturity-weighted government debt to GDP occur at a lower frequency than movements in the short rate. Our duration factor, on the other hand, explains variations in risk premia at a higher frequency than movements in the level of interest rates. Finally, Greenwood and Vayanos (2014) posit that the government adjusts the maturity structure of its debt in a way that stabilizes bond markets. For instance, when interest rates are high, the government will finance itself with shorter maturity debt and thereby reduce the quantity of interest rate risk held by agents. Our mechanism goes exactly in the opposite direction: Because of the negative convexity in MBS, the supply effect amplifies interest rate shocks.[3,4]

Our paper is related to Gabaix, Krishnamurthy, and Vigneron (2007), who study the effect of limits to arbitrage in the MBS market. The authors show that, while mortgage prepayment risk resembles a wash on an aggregate level, it nevertheless carries a positive risk premium because it is the risk exposure of financial intermediaries that matters. Our paper is based on a similar premise. Different from these authors, however, we do not study prepayment risk, but changes in interest rate risk of MBS that are driven by the prepayment probability, and their effect on the term structure of interest rates.

The remainder of the paper is organized as follows. Section 1 sets up a model of the term structure of interest rates and MBS duration, and solves for its equilibrium.

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[3]Corporate debt constitutes another important class of fixed income instruments and its supply has been shown to be negatively correlated with the supply of Government debt. For example, Greenwood, Hanson, and Stein (2010) show that firms choose their debt maturity in a way that tends to offset the variations in the supply and maturity of Government debt. However, the authors find no relationship between corporate debt and MBS supply, which provides us with an additional motivation to focus on the latter.

[4]Domanski, Shin, and Sushko (2015) argue that a negative convexity gap between German insurance sector assets and liabilities gives rise to a similar amplification effect.

Section 2 discusses the qualitative predictions of the model. Section 3 describes the data used and Section 4 empirically tests our hypotheses. Section 5 calibrates the model to the data and quantitatively confirms our empirical findings. Finally, Section 6 concludes. Proofs and additional robustness checks are deferred to the Online Appendix.

1 Model

In this section we propose a parsimonious dynamic equilibrium term structure model in which changes in MBS duration are equivalent to long-term bond supply shocks: When the probability of future mortgage refinancing increases but before refinancing happens and investors have access to new mortgage pools, the interest rate risk profile of mortgage-related securities available to investors resembles that of relatively short maturity bonds.[5]

1.1 Bond market

Time is continuous and goes from zero to infinity. We denote the time t price of a zero-coupon bond paying one dollar at maturity $t + \tau$ by Λ_t^τ, and its yield by $y_t^\tau = -\frac{1}{\tau} \log \Lambda_t^\tau$. The short rate r_t is the limit of y_t^τ when $\tau \to 0$. We take r_t as exogenous and assume that its dynamics under the physical probability measure are given by

$$dr_t = \kappa \left(\theta - r_t \right) dt + \sigma dB_t, \tag{1}$$

where θ is the long run mean of r_t, κ is the speed of mean reversion, and σ is the volatility of the short rate.

At each date t, there exists a continuum of zero-coupon bonds with time to maturity $\tau \in (0, T]$ in total net supply of s_t^τ, to be specified below. Bonds are held by financial institutions who are competitive and have mean-variance preferences over the instanta-

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[5]Market participants can invest in new mortgage loans by buying corresponding MBS. Up to 90 days before those MBS are issued, investors have access to them through the "to-be-announced" (TBA) market (see e.g., Vickery and Wright (2010)).

neous change in the value of their bond portfolio. If x_t^τ denotes the quantity they hold in maturity-τ bonds at time t, the investors' budget constraint becomes

$$dW_t = \left(W_t - \int_0^T x_t^\tau \Lambda_t^\tau d\tau \right) r_t dt + \int_0^T x_t^\tau \Lambda_t^\tau \frac{d\Lambda_t^\tau}{\Lambda_t^\tau} d\tau, \qquad (2)$$

and their optimization problem is given by

$$\max_{\{x_t^\tau\}_{\tau \in (0,T]}} \mathrm{E}_t \left[dW_t \right] - \frac{\alpha}{2} \mathrm{Var}_t \left[dW_t \right], \qquad (3)$$

where α is their absolute risk aversion. Since financial institutions have to take the other side of the trade in the bond market, the market clearing condition is given by

$$x_t^\tau = s_t^\tau, \quad \forall t \text{ and } \tau. \qquad (4)$$

The ability of financial institutions to trade across different bond maturities simplifies the characterization of the equilibrium market price of interest rate risk. From the financial institutions' first order condition and the absence of arbitrage, we obtain the following result:

Lemma 1. *Given (1)-(4), the unique market price of interest rate risk is proportional to the dollar duration of the total supply of bonds:*

$$\lambda_t = \alpha \sigma \frac{d \left(\int_0^T s_t^\tau \Lambda_t^\tau d\tau \right)}{dr_t}. \qquad (5)$$

Lemma 1 implies that in order to derive the equilibrium term structure it is not necessary to explicitly model the maturity structure of the bond supply, but it is sufficient to capture its duration.

1.2 MBS duration

The supply of bonds is determined by households' mortgage liabilities. Without explicitly modeling them, we think about a continuum of households who do not themselves

invest in bonds but take fixed-rate mortgage loans that are then sold on the market as MBS. The aggregate duration of outstanding MBS is driven by two forces: (i) changes in the level of interest rates that affect the prepayment probability of each outstanding mortgage, and (ii) actual prepayment that changes the composition of the aggregate mortgage pool. The earlier literature has adopted two main ways of describing prepayment behavior: Prepayment can be modelled as an optimal decision by borrowers who minimize the value of their loans; see, e.g., Longstaff (2005). Alternatively, since micro-level evidence suggests that individual household prepayment is often non-optimal relative to a contingent-claim approach, Stanton and Wallace (1998) add an exogenous delay to refinancing (see also Schwartz and Torous (1989) and Stanton (1995)). In the following, we posit a reduced-form model of aggregate prepayment in the spirit of Gabaix, Krishnamurthy, and Vigneron (2007). Our motivation for using a reduced-form approach is twofold. First, we avoid making strong assumptions regarding the optimal prepayment. Second, the incentive to prepay on aggregate is well explained by interest rates themselves.

Households refinance their mortgages when the incentive to do so is sufficiently high. Prepaying a mortgage is equivalent to exercising an American option. As shown in Richard and Roll (1989), the difference between the fixed rate paid on a mortgage and the current mortgage rate is a good measure of the moneyness of this prepayment option. Because households can have mortgages with different characteristics, we focus on the average mortgage coupon (interest payment) on outstanding mortgages, c_t. Following Schwartz and Torous (1989), we approximate the current mortgage rate by the long-term interest rate $y_t^{\bar{\tau}}$ with reference maturity $\bar{\tau}$. According to Hancock and Passmore (2010), it is common industry practice to use either the 5- or 10-year swap rate as a proxy for MBS duration. To match the average MBS duration, which is 4.5 years in our data sample, we set $\bar{\tau} = 5$. In sum, we define the refinancing incentive as $c_t - y_t^{\bar{\tau}}$.

On aggregate, refinancing activity does not change the size of the mortgage pool: when a mortgage is prepaid, another mortgage is issued. However, the average coupon c_t is affected by prepayment, because the coupon of the newly issued mortgages depends

on the current level of mortgage rates. We assume that the evolution of the average coupon is a function of the refinancing incentive:

$$dc_t = -\kappa_c \left(c_t - y_t^{\bar{\tau}} \right) dt, \tag{6}$$

with $\kappa_c > 0$. This means that a lower interest rate $y_t^{\bar{\tau}}$, i.e., a higher refinancing incentive, leads to more prepayments, and new mortgages issued at this low rate decrease the average coupon more. Because our focus is the feedback between the MBS market and interest rates, we also assume that on aggregate there is no additional uncertainty about refinancing. The upper left panel of Figure 1 provides empirical motivation for (6). We plot the difference between the 5-year yield and the average MBS coupon, together with the subsequent change in the average coupon. The two series are closely aligned with the coupon reacting with a slight delay to a change in the refinancing incentive.

The distinctive feature of mortgage-related securities is that their duration depends primarily on the likelihood that they will be refinanced in the future. The MBS coupon and the level of interest rates proxy for the expected level of prepayments and the moneyness of the option (see Boudoukh, Whitelaw, Richardson, and Stanton (1997)). We thus assume that the aggregate dollar duration of outstanding mortgages is a function of the refinancing incentive:

$$D_t = \theta_D - \eta_y \left(c_t - y_t^{\bar{\tau}} \right), \tag{7}$$

where duration, $D_t \equiv -dMBS_t / dy_t^{\bar{\tau}}$, is the observable sensitivity of the aggregate mortgage portfolio value (MBS_t) to the changes in the reference long-maturity rate $y_t^{\bar{\tau}}$, and θ_D, $\eta_y > 0$ are constants. The upper right panel of Figure 1 provides empirical motivation for (7). We plot the difference between the 5-year yield and the average MBS coupon, together with aggregate MBS duration. The two series are again very closely aligned. In addition, we consider in Figure 1 (lower left panel) a simple scatter diagram of the two series. In general, the relationship between interest rates and prepayment is found to be "S-shaped" (see, e.g., Boyarchenko, Fuster, and Lucca (2014)). We note that the link between MBS duration and the refinancing incentive is approximately lin-

ear, although the relationship becomes more dispersed when interest rates rise and the option becomes more out-of-the-money. Overall, we conclude that our model captures well the key stylized properties of aggregate refinancing activity.

[Insert Figure 1 here.]

Combining (6) and (7) gives us the dynamics of D_t:

$$dD_t = \kappa_D \left(\theta_D - D_t \right) dt + \eta_y dy_t^{\bar{\tau}}, \tag{8}$$

where $\kappa_D = \kappa_c$. Note that dollar duration is driven both by changes in long-term interest rates and refinancing activity. The parameter $\eta_y = dD_t/dy_t^{\bar{\tau}}$ is the negative of the dollar convexity: When $\eta_y > 0$, lower interest rates increase the probability of borrowers prepaying their mortgages in the future, leading to a lower duration. The lower right panel of Figure 1 plots the MBS convexity series showing that in our sample it always stays negative. Comparative statics with respect to η_y allow us to derive predictions regarding the effect of negative convexity on interest rate volatility.[6]

1.3 Discussion

We now discuss bond supply and the identity and behavior of investors within the context of our model. We understand the former as the net supply of bonds coming from the rebalancing of fixed income portfolios in response to fluctuations in MBS duration. For instance, the hedging positions of the GSEs analyzed in Section 4.4 would be one of its components.

On the other hand, rather than modeling all bond market investors, we abstract from buy-and-hold investors and focus directly on those who end up absorbing this additional net supply. In particular, we have in mind financial institutions such as investment

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[6]A model where η_y itself follows a stochastic process would not fall into one of the standard tractable classes of models. The Online Appendix presents a version of the model that accommodates time-varying convexity. While this model implies a quadratic instead of an affine term structure, it leads to identical qualitative predictions.

banks, hedge funds, and fund managers, who specialize in fixed income investments, trade actively in the bond market, and act as marginal investors there in the short to medium run.[7]

The risk-bearing capacity of financial institutions is key to why shocks to MBS duration matter. The mortgage choice of households determines the supply of fixed income securities, s_t^τ, through the duration of mortgages, D_t, but in addition to this channel, households are not present on either side of the market-clearing condition (4). In other words, except for having a constant amount of mortgage debt, in the model households do not take part in fixed income markets.[8] As a result, the variation in the supply of bonds induced by changes in MBS duration is not washed out and matters for bond prices.[9]

To summarize the mechanism, while lower interest rates trigger a certain amount of refinancing of the most in-the-money mortgages, they also increase the probability of future prepayment and, thus, decrease the duration of all outstanding mortgages. In fact, empirical evidence shows that households' refinancing is gradual (see Campbell (2006)). The progressive nature of refinancing ($\kappa_c < \infty$) leaves financial institutions who invest in MBS on aggregate short of duration exposure after a negative shock to

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[7]The role of financial institutions in our model is similar to Greenwood and Vayanos (2014). Fleming and Rosenberg (2008) find that Treasury dealers are compensated by high excess returns when holding large inventories of newly issued Treasury securities. More generally, financial intermediaries and institutional investors hold approximately 25% of the total amount outstanding in Treasuries, and daily trading volume is almost 10% of the total amount outstanding. In addition, these financial intermediaries hold around 30% of the total amount outstanding in MBS, and daily trading is almost 25% of the total amount outstanding. GSEs hold on average around 13% of all outstanding MBS. Data is for the period 1997 to 2014; see Securities Industry and Financial Markets Association (2013) and the Flow of Funds Tables of the Federal Reserve.

[8]Home mortgages represent approximately 70% of household liabilities. While households invest in Treasuries (their holdings account for approximately 6% of the total amount outstanding in 2014), to the best of our knowledge, there is no evidence suggesting that they actively manage the duration of their mortgage liabilities by trading fixed income instruments. Looking at the Flow of Funds Tables of the Federal Reserve we find no relationship between the duration of MBS and the value of households' bond portfolio either in absolute level or relative to the outstanding amount of Treasuries. Consistent with this pattern, Rampini and Viswanathan (2015) argue that households' primary concern is financing, not risk management.

[9]Gabaix, Krishnamurthy, and Vigneron (2007) make a related point that from the perspective of financial intermediaries who are the marginal investors in MBS, mortgage prepayment risk cannot be hedged and therefore is priced. Note that the prepayment risk of MBS is different from their interest rate risk.

interest rates. The opposite happens when interest rates increase and MBS duration lengthens.

1.4 Equilibrium term structure

Because in the model mortgages underlie the supply of bonds, we replace the dollar value of bond net supply in (5) with the aggregate mortgage portfolio value MBS_t to obtain

$$\lambda_t = \alpha\sigma\frac{dMBS_t}{dr_t}. \tag{9}$$

Using a simple chain rule, $\frac{dMBS_t}{dr_t} = \frac{dMBS_t}{dy_t^{\bar{\tau}}}\frac{dy_t^{\bar{\tau}}}{dr_t}$, we rewrite (9) in terms of the sensitivity to the reference long-maturity rate $y_t^{\bar{\tau}}$:

$$\lambda_t = -\alpha\sigma_y^{\bar{\tau}}D_t, \tag{10}$$

where $\sigma_y^{\bar{\tau}} \equiv \frac{dy_t^{\bar{\tau}}}{dr_t}\sigma$, the volatility of $y_t^{\bar{\tau}}$, is a constant to be determined in equilibrium.

We look for an equilibrium in which yields are affine in the short rate and the duration factor. Under the conjectured affine term structure, the physical dynamics of MBS duration (8) can be written as

$$dD_t = (\delta_0 - \delta_r r_t - \delta_D D_t)\, dt + \eta_y \sigma_y^{\bar{\tau}} dB_t, \tag{11}$$

where δ_0, δ_r and δ_D are constants to be determined in equilibrium. In turn, equations (1), (10) and (11) together imply that the dynamics of the short rate and the MBS duration factor under the risk-neutral measure are

$$dr_t = \left(\kappa\theta - \kappa r_t + \alpha\sigma\sigma_y^{\bar{\tau}}D_t\right) dt + \sigma dB_t^{\mathbb{Q}} \text{ and} \tag{12}$$

$$dD_t = \left(\delta_0 - \delta_r r_t - \delta_D^{\mathbb{Q}} D_t\right) dt + \eta_y \sigma_y^{\bar{\tau}} dB_t^{\mathbb{Q}}, \tag{13}$$

where $\delta_D^{\mathbb{Q}} \equiv \delta_D - \alpha\eta_y\left(\sigma_y^{\bar{\tau}}\right)^2$.

We now have all the ingredients to solve for the equilibrium term structure.

Theorem 1. *In the term structure model described by (12) and (13), equilibrium yields are affine and given by*

$$y_t^\tau = \mathcal{A}(\tau) + \mathcal{B}(\tau) r_t + \mathcal{C}(\tau) D_t, \tag{14}$$

where the functional forms of $\mathcal{A}(\tau)$, $\mathcal{B}(\tau)$, and $\mathcal{C}(\tau)$ are given in the Online Appendix, and the parameters $\sigma_y^{\bar\tau}$, δ_r, δ_D, and δ_0 satisfy

$$\sigma_y^{\bar\tau} = \frac{\sigma \mathcal{B}(\bar\tau)}{1 - \eta_y \mathcal{C}(\bar\tau)}, \ \delta_r = \frac{\kappa \eta_y \mathcal{B}(\bar\tau)}{1 - \eta_y \mathcal{C}(\bar\tau)}, \ \delta_D = \frac{\kappa_D}{1 - \eta_y \mathcal{C}(\bar\tau)}, \ and \ \delta_0 = \delta_r \theta + \delta_D \theta_D. \tag{15}$$

Equations (15) have a solution whenever α is below a threshold $\bar\alpha > 0$.

2 Model Predictions

Our model has a series of implications that characterize the effect of MBS risk on the term structure of bond risk premia and bond yield volatilities. We summarize them in five propositions that will guide our empirical analysis.

2.1 Predictability of nominal bond excess returns

The predictability of bond excess returns by the dollar duration of MBS is a natural outcome of our model. The market price of interest rate risk depends on the quantity of the risk that financial institutions hold to clear the supply. In turn, bonds with higher exposure to interest rate risk are more affected. As a result, MBS duration predicts excess bond returns and the effect is stronger for longer maturity bonds.[10]

We define the excess return of a τ-year bond over an h-year bond for the holding period $(t, t + h)$ as $rx_{t,t+h}^\tau \equiv \log\Lambda_{t+h}^{\tau-h} - \log\Lambda_t^\tau + \log\Lambda_t^h = h\left(y_t^\tau - y_t^h\right) - (\tau - h)\left(y_{t+h}^{\tau-h} - y_t^\tau\right)$. Then, running a univariate regression of these excess returns on the MBS duration factor,

$$rx_{t,t+h}^\tau = \beta_0^{\tau,h} + \beta^{\tau,h} D_t + \epsilon_{t+h}, \tag{16}$$

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[10]Note that the effect of MBS dollar duration on the level of yields is not necessarily monotonic in maturity. A yield depends on the average of risk premia over the life of the bond. Higher risk premia increase yields. However, because of mean reversion in interest rates and duration, we expect risk premia at longer horizons to be lower. We are not testing this implication empirically, because duration itself depends on yields, thus causing an endogeneity problem for identification.

leads to the following result on the theoretical slope coefficient:

Proposition 1. *Holding h fixed, we have $\lim_{\tau \to h} \beta^{\tau,h} = 0$ and $d\beta^{\tau,h}/d\tau > 0$ for all $\tau > 0$. Hence, $\beta^{\tau,h}$ is positive and increasing across maturities.*

An additional prediction of the model allows us to disentangle the role played by the MBS duration factor from that of the level of interest rates. Even though the model has only one shock, long-term yields are a function of two separate factors: the short rate and the aggregate dollar duration of MBS. This is the case because duration does not only depend on the current mortgage rate, but also on the entire history of past mortgage rates that determine the coupon of currently outstanding mortgages.[11]

Formally, running a bivariate regression of excess returns over horizon h of bonds with maturity τ on the MBS duration factor while controlling for the short rate,

$$rx^\tau_{t,t+h} = \beta^{\tau,h}_0 + \beta^{\tau,h}_1 D_t + \beta^{\tau,h}_2 r_t + \epsilon_{t+h}, \tag{17}$$

we obtain the following result on the theoretical slope coefficients:

Proposition 2. *Holding h fixed, we have $\lim_{\tau \to h} \beta^{\tau,h}_1 = \lim_{\tau \to h} \beta^{\tau,h}_2 = 0$ and $d\beta^{\tau,h}_1/d\tau > 0 > d\beta^{\tau,h}_2/d\tau$ for all $\tau > 0$. Hence, the slope coefficient on duration, $\beta^{\tau,h}_1$, is positive and increasing in maturity, while the slope coefficient on the short rate, $\beta^{\tau,h}_2$, is negative and decreasing (i.e., becoming more negative) in maturity.*

The model implies that slope coefficients on the two factors should have opposite signs. The level of interest rates does not contain any information about the current market price of risk beyond that already encoded in duration. However, including the short rate (or more generally the level) allows to control for the mean reversion in interest rates, and, therefore, to better predict the mean reversion in duration over the return horizon h; hence the negative sign on the level of interest rates.

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[11]Formally, when $\kappa_D \neq 0$, interest rates in our model are non-Markovian with respect to the short rate r_t alone. However, their history dependence can be summarized by an additional Markovian factor, namely the duration D_t.

We also study model implications regarding return predictability over different horizons while keeping bond maturity fixed. Revisiting regression (16), we obtain the following result on the slope coefficient:

Proposition 3. *Holding τ fixed, we have $\lim_{h\to 0} \beta^{\tau,h} = \lim_{h\to\tau} \beta^{\tau,h} = 0$. Moreover, $\beta^{\tau,h}$ is hump-shaped across horizons: $d\beta^{\tau,h}/dh > 0$ for short horizons and $d\beta^{\tau,h}/dh < 0$ after that.*

The excess return over an investment horizon h depends on the difference between the return earned on a maturity-τ long-term bond and that on a maturity-h bond, given by hy_t^h. A longer investment horizon increases both components, but the relative impact is different. Due to the transitory nature of duration, long bond return predictability is an increasing and concave function of h; it is concentrated in the short run and its increments deteriorate for longer horizons. A bond yield, on the other hand, depends on the average risk premium over a time interval. Thus, the second component is akin to a slow-moving average of the first. As a result, at short to medium horizons, the impact of MBS duration on the former dominates the latter and $\beta^{\tau,h}$ increases, but for longer horizons the difference disappears. We conclude that the effect of the dollar duration of MBS on excess returns is hump-shaped across investment horizons.

2.2 Bond yield volatility

Our model predicts a positive and hump-shaped effect of negative convexity η_y on the term structure of bond yield volatilities σ_y^τ. Formally, we have the following comparative statics result:

Proposition 4. *We have $d\sigma_y^\tau/d\eta_y > 0$ for all $\tau > 0$. In addition, $\lim_{\tau\to 0} \sigma_y^\tau = \sigma$ and $\lim_{\tau\to\infty} \sigma_y^\tau = 0$, where neither limit depends on η_y. Hence, $d\sigma_y^\tau/d\eta_y$ is hump-shaped across maturities.*

An intuitive way to understand the effect of negative convexity on volatility within the model is to consider an approximation of the results in Theorem 1 where we replace $\mathcal{B}(\bar{\tau})$ and $\frac{\mathcal{C}(\bar{\tau})}{\alpha}$ that are non-trivial functions of yield volatility with constants

$b = \mathcal{B}(\bar{\tau})|_{\alpha=0}$ and $c = \frac{\mathcal{C}(\bar{\tau})}{\alpha}|_{\alpha=0}$. When $c\alpha\eta_y < 1$, i.e., the risk aversion is below the threshold $\bar{\alpha} = \frac{1}{c\eta_y}$, we have an affine equilibrium where the volatility of the reference maturity yield solves $\sigma_y^{\bar{\tau}} = b\sigma + c\alpha\eta_y\sigma_y^{\bar{\tau}}$. This fixed-point problem is the result of a feedback mechanism between long rates and duration: lower interest rates decrease duration, which in turn decreases the term premium and further lowers long rates. More negative convexity implies that MBS duration and, therefore, the market price of risk are more sensitive to changes in interest rates. Through this mechanism the volatility increases by a factor $\frac{1}{1-c\alpha\eta_y} = 1 + c\alpha\eta_y + (c\alpha\eta_y)^2 + ... > 1$ that captures the combined effect of the successive iterations of the feedback loop. The feedback explains why negative convexity can cause potentially significant interest rate volatility even for moderate levels of risk aversion.

Moreover, the link between convexity and volatility has a term structure dimension. Short-maturity yields are close to the short rate and therefore are not significantly affected by variations in the market price of risk. For long maturities, we expect the duration of MBS to revert to its long-term mean. At the limit, yields at the infinite horizon should not be affected by current changes in the short rate and MBS duration at all. As a result, the effect of MBS convexity on yield volatilities has a hump-shaped term structure.

2.3   Predictability of real bond excess returns

A distinctive feature of MBS duration and, more generally, supply factors is that they affect the pricing of both nominal and real bonds.[12] To the extent that real and nominal interest rates are correlated, fixed income investors would demand an additional premium on real bonds when they have to absorb more aggregate duration risk. We extend our baseline model to study the joint impact of mortgage risk on nominal and real bonds, with additional details provided in the Online Appendix.

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[12]For instance, Hanson and Stein (2015) argue that the mechanism described in our paper can provide one possible explanation for the sensitivity of long real rates to changes in short nominal rates.

We keep our assumptions that the nominal short rate process follows (1) and that at each date t there exist zero-coupon nominal bonds in time-varying net supply s_t^τ for all $\tau \in (0, T]$. Further, we assume that a real short rate process under $\mathbb{P}$ is given by

$$dr_t^* = \kappa^* \left(\theta^* - r_t^*\right) dt + \sigma^* dB_t^* \tag{18}$$

whose instantaneous correlation with the nominal short rate is $dB_t dB_t^* = \rho dt$, and that an inflation index exists and follows

$$\frac{dI_t}{I_t} = \mu_t^\pi dt - \sigma^\pi dB_t^\pi, \tag{19}$$

where dB_t^π can be correlated with both dB_t and dB_t^*. In (19), we set the diffusion of I_t, σ^π, exogenously, but allow the drift μ_t^π to be any adapted process for now; later we derive it in equilibrium to satisfy no arbitrage between nominal and real bonds.[13] Finally, we assume that at each date t, there exist a continuum of real zero-coupon bonds with time to maturity $\tau \in (0, T]$ in zero net supply.[14]

Our assumptions imply that the nominal interest rate sensitivity of all fixed income securities in the economy is still driven by MBS duration, and so is the risk premium on nominal bonds; i.e., (5) holds. Furthermore, the equilibrium risk premium on real bonds depends on how their returns co-move with those on nominal bonds. Hence, equilibrium nominal yields are given by (14), as before, and we show that real yields are affine in MBS duration and the nominal and real short rates.

In this generalized setting, running a univariate regression of the horizon-h excess return of a real bond with maturity τ over that of the maturity-h real bond on the MBS duration factor,

$$rx_{t,t+h}^{\tau*} = \beta_0^{\tau,h*} + \beta^{\tau,h*} D_t + \epsilon_{t+h}, \tag{20}$$

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[13]In particular, we obtain that the ex-ante Fisher relation holds in equilibrium: the risk-neutral drift, $\mu_t^{\pi\mathbb{Q}}$, must equal the difference between the nominal and real short rates; see the Online Appendix.

[14]For our sample period the size of the TIPS market does not exceed 5% of the outstanding nominal Treasuries and MBS, suggesting that fixed income investors' portfolios are primarily exposed to the nominal interest rate risk.

and contrasting the loading with the theoretical slope coefficient $\beta^{\tau,h}$ obtained for the nominal bond, we get the following result:

Proposition 5. *When $\kappa^* \approx \kappa$, we have $\beta^{\tau,h*} \approx \frac{\rho\sigma^*}{\sigma}\beta^{\tau,h}$.*

Propositions 1 and 5 together imply that the duration coefficients in regression (20) are positive and increasing with maturity, mirroring the predictions for nominal bonds. In particular, for any shock in MBS duration, risk premia on real bonds move $\rho\sigma^*/\sigma$ for one with risk premia on nominal bonds, where the ratio $\rho\sigma^*/\sigma$ represents the coefficient from a regression of real short rate innovations on nominal short rate innovations. Thus, estimated coefficients for real bonds are smaller than for nominal bonds if real rates are imperfectly correlated with nominal rates and less volatile.

3 Data

Data is monthly and spans the time period from December 1989 through December 2012.

We use estimates of MBS duration, convexity, index, and average coupon from Barclays available through Datastream.[15] The Barclays US MBS index covers mortgage-backed pass-through securities guaranteed by Ginnie Mae, Fannie Mae, and Freddie Mac. The index is comprised of pass-throughs backed by conventional fixed rate mortgages and is formed by grouping the universe of over one million agency MBS pools into generic pools based on agency, program (i.e., 30-year, 15-year, etc.), coupon (e.g., 6.0%, 6.5%, etc.), and vintage year (e.g., 2011, 2012, etc.). A generic pool is included in the index if it has a weighted-average contractual maturity greater than one year and more than USD 250m outstanding. We construct measures of dollar duration and dollar convexity by multiplying the duration and convexity time series with the index level.[16]

The upper right panel of Figure 1 depicts MBS dollar duration and the lower right panel plots MBS dollar convexity. Dollar duration and dollar convexity are calculated as

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[15]Datastream tickers for MBS duration and convexity are LHMNBCK(DU) and LHMNBCK(CK), respectively.

[16]In the following, units are expressed in US dollars assuming that the portfolio value is equal to the index level in dollars.

the product of the Barclays US MBS index level and duration and convexity, respectively. Overall, the average MBS dollar duration is 457.43 with a standard deviation of 59.85 and the average dollar convexity is -163.73 with a standard deviation of 57.23.[17] Table 1 presents a summary statistic of all main variables used.

[Insert Table 1 and Figure 1 here.]

We use the Gürkaynak, Sack, and Wright (2007, GSW henceforth) zero-coupon nominal yield data available from the Federal Reserve Board. We use the raw data to calculate annual Treasury bond excess returns for 2- to 10-year bonds. We also download interest rate swap data from Bloomberg from which we bootstrap a zero-coupon yield curve. To calculate real bond excess returns, we use liquidity-adjusted real bond yields (see D'Amico, Kim, and Wei (2014)).[18]

We denote the annual return between time t and one year later on a τ-year bond with price Λ_t^τ by $r_{t,t+1y}^\tau = \log \Lambda_{t+1y}^{\tau-1y} - \log \Lambda_t^\tau$. The annual excess bond return is then defined as $rx_{t,t+1y}^\tau = r_{t,t+1y}^\tau - y_t^{1y}$, where $y_t^{1y} = -\log \Lambda_t^{1y}$ is the one-year yield. From the same data, we also construct a tent-shaped factor from forward rates, the Cochrane and Piazzesi (2005) factor (CP factor, labeled cp_t). Real annual excess bond returns are denoted by $rx_{t,t+1y}^{\tau*}$.

Using the GSW yields ranging from one to ten years, we estimate a time-varying term structure of yield volatility. We sample the data at the monthly frequency and take monthly log yield changes. We then construct rolling window measures of realized volatility using a 12-month window which represent the conditional bond yield volatility. The resulting term structure of unconditional volatility exhibits a hump shape consistent with the stylized facts reported in Dai and Singleton (2010), with the volatility peak being at the two-year maturity (see Table 1, Panel C).

Choi, Mueller, and Vedolin (2014) calculate measures of model-free implied bond market volatilities for a one-month horizon using Treasury futures and options data from

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[17]Units are expressed in dollars assuming that the portfolio value is equal to the index level in dollars.

[18]We find similar results when using liquidity-adjusted real bond yields from Pflueger and Viceira (2015). We thank Min Wei and Carolin Pflueger for sharing the data with us.

the Chicago Mercantile Exchange (CME). We use their data for the 30-year Treasury bond and henceforth label this measure tiv_t.

From Bloomberg, we also get implied volatility for at-the-money swaptions for different maturities ranging from one to ten years and we fix the tenor to ten years. We label these volatilities $\text{iv}^{\tau 10y}$. Further, we collect implied volatilities on 3-month-maturity swaptions with tenors ranging between 1 and 10 years, denoted by $\text{iv}^{3m\tau}$.

As a proxy of illiquidity in bond markets, we use the noise proxy from Hu, Pan, and Wang (2013), which measures an average yield pricing error from a fitted yield curve. As a proxy for economic growth we use the three-month moving average of the Chicago Fed National Activity Index. Negative (positive) values indicate a below (above) average growth. We also use a measure of inflation proxied by the consensus estimate of professional forecasts available from Blue Chip Economic Forecasts.

4 Empirical analysis

In this section we study the predictive power of MBS dollar duration and convexity for bond excess returns (nominal and real) and bond yield volatility. We start with univariate regressions to document the role of our main explanatory variables. Then, for robustness and to address a potential omitted variable bias, we also control for other well-known predictors of bond risk premia and interest rate volatility. We find that not only MBS duration and convexity remain statistically significant, but also the economic size of the coefficients stays stable across different specifications.

The start date for volatility regressions is dictated by the availability of the MBS convexity time series that starts in January 1997. Daily data for TIPS is available from the Federal Reserve Board website starting in January 1999, which is the start date for the real bond return regressions. For all other regressions we start in December 1989. With each estimated coefficient, we report t-statistics adjusted for Newey and West (1987) or Hansen and Hodrick (1980) standard errors. The lag length is set to 18.

4.1 Nominal bond risk premia

Hypothesis 1. *A regression of bond excess returns on the duration of MBS yields a positive slope coefficient for all maturities. Moreover, the coefficients are increasing in bond maturity and remain significant when we control for the level of interest rates.*

This hypothesis is derived from Propositions 1 and 2. To test it, we run linear regressions of annual excess returns on the duration factor. The regression is as follows:

$$rx_{t,t+1y}^{\tau} = \beta_0^{\tau} + \beta_1^{\tau}\text{duration}_t + \beta_2^{\tau}\text{level}_t + \epsilon_{t+1y}^{\tau},$$

where duration_t is MBS dollar duration and level_t is the one-year yield. The univariate results are depicted in the upper two panels of Figure 2, which plot the estimated slope coefficients of duration, $\hat{\beta}_1^{\tau}$ (upper left panel), and the associated adjusted R^2 (upper right panel). Both univariate and multivariate results are presented in Table 2.

[Insert Figure 2 and Table 2 here.]

The univariate regression results indicate that MBS duration is a significant predictor of bond excess returns across all maturities. In line with the theoretical prediction, the coefficient has a positive sign and is increasing with maturity.[19] The estimated coefficients are also economically significant, especially for longer maturities. For example, for any one standard deviation increase in MBS dollar duration, there is a $0.0636 \times 59.85 = 381$ (slope coefficient times standard deviation of MBS dollar duration)

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[19]We can also test whether the estimated slope coefficients are monotonically increasing using the monotonicity test developed by Patton and Timmermann (2010), i.e., we can test whether

$$\mathcal{H}_0 : \hat{\beta}_1^{10y} \leq \hat{\beta}_1^{9y} \leq \cdots \leq \hat{\beta}_1^{2y}$$

versus

$$\mathcal{H}_1 : \hat{\beta}_1^{10y} > \hat{\beta}_1^{9y} > \cdots > \hat{\beta}_1^{2y},$$

where $\hat{\beta}_1^{\tau}, \tau = 2, \ldots, 10$ years are estimated slope coefficients from an univariate regression from bond excess returns with maturity τ onto MBS dollar duration. First, note that the spread between the coefficients on the 2-year and 10-year bond excess returns is 0.0577. The associated t-statistic is 4.26 and is therefore statistically highly significant. Turning to the tests for monotonicity, using 10,000 bootstrap iterations, we find that the p-value is almost zero and we hence strongly reject the null hypothesis of no monotonic relationship between estimated coefficients.

basis point increase in the expected 10-year bond excess returns. Adjusted R^2s range from 7% for the shortest maturity to 23% for the longest maturity.[20] To put these effects into perspective, we can translate the above numbers into the yield space: for any one standard deviation change in MBS duration, there is a $381/10 \approx 38$ basis point increase in the 10-year yield, assuming that most of the effect on returns happens within the first year (which we verify in the data below).[21]

One might suspect that the predictive power of MBS duration could result from its close relationship to the level of interest rates. Proposition 2, however, allows us to disentangle the contribution of the two factors. To this end, we include the latter as a control in our multivariate test. The results presented in Table 2 indicate that the slope coefficient on duration remains positive and increasing with maturity, while the slope coefficient on the level of interest rates is negative and decreasing for longer maturities. This is in line with our theory, where the level of interest rates is not directly related to bond risk premia, but helps to predict the mean reversion in duration at longer horizons, and, thus, its coefficients have the opposite sign.

Finally, we study the persistence of the MBS dollar duration effect on bond risk premia by varying the horizon of excess bond returns in our predictive regression:

$$rx_{t,t+h}^{10y} = \beta_0 + \beta_1 \text{duration}_t + \epsilon_{t+h},$$

where h is 3, 6, 12, 24, and 36 months, respectively. We formulate the following hypothesis in line with Proposition 3:

Hypothesis 2. *A regression of bond excess returns on the duration of MBS yields coefficients that are hump-shaped across horizons, i.e., they are largest for intermediate horizons.*

[Insert Figure 3 here.]

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[20]Our conclusions remain the same when we use interest rate swaps instead of Treasury data, and the duration of the Bank of America US Mortgage Master index (Bloomberg ticker M0A0) instead of Barclays data (see the Online Appendix).

[21]One can also relate this to the 91 basis point estimated effect on the 10-year yield of the QE1 program (see Gagnon, Raskin, Remache, and Sack (2010)) whose dollar duration impact is approximately twice that of a one standard deviation MBS dollar duration shock.

The results are presented in Figure 3. We find that coefficients increase up to approximately a one-year horizon, but then plateau and decrease, suggesting that the effect of MBS duration on bond returns is transitory. Our model provides one possible explanation for this: both the mean reversion in interest rates and refinancing activity contribute to the fast mean reversion in aggregate MBS duration. The short-lived effect of MBS duration on bond returns could also be explained by the dynamics of arbitrage capital that, while slow-moving, ultimately flows into fixed income markets to absorb additional duration risk.[22] While both factors are likely to play a role, Section 5 presents a calibration of our model that can quantitatively account for the pattern of multi-horizon regression coefficients.

4.2 Real bond risk premia

According to our model, the variation in MBS duration should affect the pricing of both nominal and real bonds. This prediction allows us to differentiate between duration and factors that are related exclusively to inflation risk. To this end, we test the following hypothesis based on Propositions 1 and 5:

Hypothesis 3. *A regression of real bond excess returns on the duration of MBS yields a positive slope coefficient that is increasing with maturities. Moreover, real slope coefficients are approximately equal to nominal slope coefficients adjusted by the correlation between real and nominal rates times the ratio of their volatilities.*

Univariate results are presented in Figure 2 (middle two panels) and Table 3. We find that MBS dollar duration significantly predicts real bond excess returns at longer maturities. For instance, for the 10-year real bond, the estimated coefficient is positive, highly significant, and implies that any one standard deviation change in MBS dollar duration leads to a $0.0293 \times 68.25 = 199$ (regression coefficient times the standard deviation of MBS dollar duration between 1999 and 2013) basis point change in real excess returns.

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[22]See, for example, Greenwood, Hanson, and Liao (2015), who study slow-moving capital in partially segmented markets.

The relative magnitude of real and nominal coefficients supports a duration risk explanation of return predictability. Over our sample period from 1999 to 2013, real rates are less volatile than nominal rates ($\frac{\sigma^\star}{\sigma} \approx 0.72$) and the two series exhibit less than perfect correlation ($\rho \approx 0.87$). Accordingly, the effect of duration on real bond returns is lower. For the 10-year maturity, the ratio of real to nominal coefficients is $199/341 = 0.58$, in line with the $0.87 \times 0.72 = 0.62$ predicted by our theory.[23]

[Insert Table 3 here.]

4.3 Bond yield volatility

Proposition 4 allows us to formulate the following hypothesis regarding the effect of MBS convexity on bond yield volatility across maturities:

Hypothesis 4. *A regression of conditional yield volatility on the negative convexity of MBS results in a positive slope coefficient for all maturities. Moreover, the coefficients are the largest for intermediate maturities, i.e., they have a hump-shaped term structure.*

Due to the amplification channel described earlier, we expect a more negative convexity of MBS to result in larger bond yield volatility. To this end, we run the following univariate regression from conditional bond yield volatility onto MBS convexity:

$$\text{vol}_t^\tau = \beta_0^\tau + \beta_1^\tau \text{convexity}_t + \epsilon_t^\tau,$$

where vol_t^τ is the conditional bond yield volatility at time t of a bond with maturity $\tau = 1, \ldots, 10$ years.

The univariate results are presented in the lower two panels of Figure 2 and in Table 4. In line with our intuition, we find a significant effect from convexity onto bond

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[23]Following the model, these numbers are based on changes in short real and nominal yields. 10-year yields are more highly correlated, but imply a similar overall ratio of coefficients $0.97 \times 0.65 = 0.63$. The 341 basis point effect from MBS duration onto the 10-year nominal bond risk premium is for the 1999 to 2013 period.

yield volatility, and the effect is most pronounced for intermediate maturities.[24] The estimated slope coefficients produce the hump-shaped feature similar to the one observed in the unconditional averages of yield volatility. Adjusted R^2s range from 19% for the shortest maturities, increase to 22% for the two- and three-year maturities, and decrease again to 14% for longer maturities. Estimated coefficients are not only statistically but also economically significant: For the two-year maturity, any one standard deviation change in MBS dollar convexity is associated with a 37 (= 0.0764 (slope coefficient) $\times$ 57.23 (standard deviation of MBS dollar convexity) $\times$ 2.43 (level of 2-year yield) $\times \sqrt{12}$) basis point increase in annual bond yield volatility.

[Insert Table 4 here.]

An obvious concern with our regression results is that negative convexity could itself depend on volatility. Note, that it is a priori unclear in which direction volatility affects convexity as this depends on whether a particular MBS is in-, out-, or at-the-money.[25] For an at-the-money MBS, an increase in volatility will lead to an increase in negative convexity. Discount (i.e., small negative to positive convexity) and premium (negative convexity) mortgages will in general have a much lower sensitivity to changes in volatility, and the effect could go in the opposite direction.[26]

To address causality, we run Granger tests between MBS dollar convexity and volatility and present the results in Figure 4. In the left panel, we plot F-statistics of Granger causality tests that assess the null hypothesis whether negative convexity does not Granger cause volatility. On the right panel, we plot the corresponding F-statistics of the reversed Granger regression, i.e., we test the null hypothesis whether volatility does not Granger cause negative convexity. We also plot the 10% critical values. We

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[24]While there are no formal procedures which specifically test for a hump-shape, we can test whether the estimated coefficient on the 2-year bond yield volatility is statistically different from the 3-year volatility. Indeed, the difference, which is 0.0192, has a t-statistic of 2.75 and is hence different from zero. We can then again test for monotonicity between the 3-year and 10-year coefficients. Using the procedure from Patton and Timmermann (2010), we strongly reject the null of no relationship as the p-value is basically zero.

[25]This is analogous to the Zomma (sensitivity of an option's Gamma with respect to changes in the implied volatility) for equity options.

[26]We thank Bruce Phelps at Barclays Capital for insightful discussions on this.

note that for standard confidence levels, we can reject the null of no Granger causality from convexity to volatilities for any maturity. On the other hand, longer-maturity yield volatility does seem to Granger cause convexity as indicated by the F-statistics.

[Insert Figure 4 here.]

4.4 Two MBS investor types: the GSEs and the Federal Reserve

Our paper builds on the premise that fluctuations in MBS duration prompts investors to adjust their hedging positions, rebalance their portfolios, or, more generally, revise the required risk premia at which they are willing to hold bonds. While we do not observe the behavior of all mortgage investors, we can gauge the validity of the duration channel by looking at market participants with well-defined institutional mandates, and test whether the change in the composition of MBS ownership over time has an effect on expected bond returns.

The two GSEs, Fannie Mae and Freddie Mac, play a central role in the US housing finance system. In addition to their business of issuing and providing credit guarantee for a large fraction of pass-through MBS, these institutions also retain a significant portfolio of mortgage loans and MBS. Unlike that of the guaranteed portfolio, all of the interest rate risk of the retained portfolio lies with the GSEs. Moreover, Fannie Mae and Freddie Mac see the hedging of this exposure as part of their mandate, including the part driven by mortgage prepayment.[27] Hedging is done through interest rate swaps under which they trade the fixed-rate interest payments of mortgage loans for floating-rate interest rate payments that correspond more closely to their short-term borrowing costs. To hedge prepayment risk, the GSEs issue callable debt and buy swaptions. If interest rates fall, the GSEs can redeem their callable debt at lower rates, or similarly, exercise their swaptions. Historically, the GSEs have started hedging during the 1990s (see Howard (2013)).

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[27]Fannie Mae specifically stress this fact in the 10K filings. They write "Risk management derivative instruments are an integral part of our management of interest rate risk. We supplement our issuance of debt securities with derivative instruments to further reduce duration risk, which includes prepayment risk. We purchase option-based risk management derivatives to economically hedge prepayment risk."

The top panel of Figure 5 illustrates the relationship between the notional value of the GSEs' derivative contracts and MBS duration.[28] We note that the value of the hedging position on average exceeds one trillion USD, and its peaks coincide with the large drops in MBS duration around 2003 and 2008.

The middle panel of Figure 5 shows the growth of Fannie Mae and Freddie Mac's retained portfolio from approximately USD 200bn in the 1990s to almost USD 1.6tr in 2003. The increase in the GSEs' retained portfolio occurred in parallel with the overall growth of the MBS and Treasury markets. The bottom panel of Figure 5 presents the value of the retained portfolio as a share of total outstanding MBS. We note that the fraction of MBS held by the GSEs is positively associated with the predictive power of MBS duration on bond excess returns, with both increasing until the mid-2000s and falling subsequently. While we do not observe the portfolio of other MBS investors (with an important exception of the Federal Reserve considered below), this evidence supporting the role of actively hedging GSEs is in line with the MBS duration channel.[29]

[Insert Figure 5 here.]

Whereas the GSEs represent a class of investors who actively manage the interest rate exposure of their MBS portfolio, the Federal Reserve does not aim to hedge the duration risk of its MBS holdings. On 7 September 2008 the Federal Housing Finance Agency (FHFA) together with the Treasury outlined a plan to (i) place both GSEs into conservatorship and (ii) having the Treasury enter into senior preferred stock purchase agreements with both firms. The latter require both Fannie Mae and Freddie Mac to wind down their retained investment portfolio at a rate of at least 10% per year until they each fall below USD 250bn. This large reduction in the actively hedged GSE portfolios is partly offset by the increase in Federal Reserve holdings (see Malz, Schaumburg, Shimonov, and Strzodka (2014)). As of end 2014, the Federal Reserve holds USD 1.7tr of agency MBS.

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[28] According to the Financial Accounting Standard (FAS) 133, any firm is required to publish the fair value of derivatives designated as hedging instruments.

[29] At the same time, the GSEs holdings data is available to us only at an annual frequency, making more formal statistical inference difficult.

To study the effect of this shift in MBS ownership from the GSEs to the Federal Reserve on bond risk premia, we run the following regression:

$$rx_{t,t+1y}^{\tau} = \beta_0^{\tau} + \beta_1^{\tau}\text{duration}_t + \beta_2^{\tau}\text{fed share}_t + \beta_3^{\tau}\text{duration}_t \times \text{fed share}_t + \epsilon_{t,t+1y}^{\tau},$$

where fed share$_t$ is the Federal Reserve's share of total MBS holdings. If the effect of MBS duration is dampened as the Federal Reserve's share goes up, we would expect the loading on the interaction term, β_3^{τ}, to be negative.

The results in Table 5 reveal that the coefficient on MBS duration is still highly significant and increasing with maturity. In line with our intuition that the increased MBS holdings of the Federal Reserve have weakened the duration channel, we find a significant and negative coefficient on the interaction term that is increasing with maturity.

[Insert Table 5 here.]

4.5   MBS duration and other predictors of bond returns

In this section we study MBS duration in relation to other predictors of bond returns proposed by the literature but not included in our model. First, mortgage refinancing decisions and hence MBS duration could be a function of the information already contained in the yield curve. We run the following regression:

$$rx_{t,t+1y}^{10y} = \beta_0 + \beta_1\text{duration}_t + \beta_2\text{level}_t + \beta_3\text{slope}_t + \beta_4\text{curve}_t + \epsilon_{t+1y},$$

where level$_t$, slope$_t$ and curve$_t$ are the first three yield PCs. Table 6 (column 1) reveals that the economic and statistical significance of the duration factor remains very close to the results reported in Table 2. In the second column, we control for the Cochrane and Piazzesi (2005) factor, which is a linear combination of forward rates. Again, we note that MBS dollar duration is highly significant.

[Insert Table 6 here.]

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MBS duration could also be related to the business cycle as empirical evidence shows that the refinancing incentive of mortgage holders depends on the economic state (see, e.g., Chen, Michaux, and Roussanov (2013)). Therefore we control for business cycle measures that have also been shown to have a significant bearing on bond returns, namely economic growth and inflation.[30] We run the following regression:

$$rx_{t,t+1y}^{10y} = \beta_0 + \beta_1 \text{duration}_t + \beta_2 \text{inflation}_t + \beta_3 \text{growth}_t + \epsilon_{t+1y}.$$

The results are presented in Table 6 (third column). Again, we find that estimated coefficients remain very similar to the baseline regression results presented in Table 2. Finally, the last column presents regression results when including both yield and macro factors; we find MBS duration to remain highly statistically significant.

We conclude that the predictive power of MBS duration is not subsumed by either yield or macroeconomic factors and constitutes a separate channel. We also note that shocks to MBS duration are much more transient than shocks to either first two yield PCs or macro variables. For example, we find that MBS dollar duration has a half-life of around 4 months, whereas the level, slope, inflation, and growth factors have a half-life of 83, 23, 15, and 14 months, respectively.

4.6 MBS convexity, other determinants of yield volatility, and swaption implied volatility

We now control for additional determinants of yield volatility that have been documented in the literature. For example, it is well-known that volatility tends to increase in periods of high illiquidity (see, e.g., Hu, Pan, and Wang (2013)). In our multivariate specification, we therefore add a proxy for illiquidity and a proxy of fixed-income implied volatility, similar to the VIX in equity markets. We run the following regression from conditional bond yield volatility onto MBS dollar convexity and a set of other predictors:

$$\text{vol}_t^\tau = \beta_0^\tau + \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{illiq}_t + \beta_3^\tau \text{tiv}_t + \epsilon_t^\tau,$$

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[30]See e.g., Joslin, Priebsch, and Singleton (2014). We also use the eight principal components from macro variables as in Ludvigson and Ng (2009) and find that our results remain unchanged.

where vol_t^τ is the conditional bond yield volatility at time t of a bond with maturity $\tau = 1, \ldots, 10$ years, illiq_t is the illiquidity factor at time t, and tiv_t is the Treasury-implied volatility at time t. Results are reported in Table 7 (panel A). We find that when we add illiquidity and tiv to the regression, convexity still remains highly statistically significant. The estimated coefficients in the bond yield volatility regressions reveal that the effect is largest for the intermediate maturity of two years as indicated by the size of the coefficient. All three factors together explain between 27% and 43% of the time variation in bond yield volatility across different maturities.

[Insert Tables 7 here.]

As hedging can potentially take place both in the bond and in the fixed-income derivatives market, we also test the impact of MBS dollar convexity on measures of implied volatility from swaptions. For example, Wooldridge (2001) notes that non-government securities were routinely hedged in the Treasury market until the financial crisis of 1998 when investors started hedging their interest rate exposure in the swaptions market.

Table 7, panels B and C present estimated coefficients when regressing either implied volatility of τ-maturity swaptions written on the 10-year swap rate (panel B) or implied volatility of 3-month swaptions written on τ-maturity swap rates (panel C). We find that the effect is stronger for shorter maturities and tenors, and all coefficients are positive and highly significant.

5 Calibration

In this section we calibrate our model to test its quantitative performance.

5.1 Estimated and calibrated parameters

We estimate the parameters of the short rate process (1), real short rate (18), and the dollar duration process (8). We note that equation (8) provides a very good description

of the monthly series of MBS dollar duration as the associated R^2 is 83%. In order to fully characterize the theoretical effect of MBS duration and convexity on bond returns and yield volatility, we set the risk aversion of financial institutions to $\alpha = 88$. This value allows the model to match the R^2 of the predictive regression of 10-year nominal bond excess returns on duration reported in Table 2. Note that α is the coefficient of absolute risk aversion. In order to interpret this value, we multiply it by financial institutions' wealth to obtain a coefficient of relative risk aversion. In a setting similar to ours, Greenwood and Vayanos (2014) use financial institutions' capital to GDP ratio of 13.3%. Because we use the dollar duration of the MBS index to calibrate the model and the average value of the index itself is standardized to one dollar, we also need to adjust for the size of the MBS market relative to GDP. Between 1997 and 2012 the average value of outstanding mortgage-related debt was equal to 53% of the GDP. This implies a coefficient of relative risk aversion of approximately $22 \approx 88 \times 13.3\%/53\%$.

We summarize all calibrated parameter values in Table 8.

[Insert Table 8 here.]

5.2 Return predictability and volatility

The calibrated model provides a benchmark for the empirical results in Section 4. The two top panels of Figure 6 report the term structure of theoretical $\beta^{\tau,h}$ and $\beta^{\tau,h*}$ together with our empirical estimates. The coefficients implied by the model are within the 95% confidence intervals for maturities up to 8 years, but it underpredicts them at the long end.

The lower left panel of Figure 6 reports the theoretical slope coefficients $\beta^{10y,h}$ for different return horizons. In line with our empirical estimates, the coefficients increase steeply up to approximately one year before they plateau and then decrease. This suggests that the mean reversion in aggregate MBS dollar duration is enough to account for the transitory nature of its effect on bond returns.

The lower right panel of Figure 6 reports the total change in yield volatility across maturities that can be attributed to negative convexity. For instance, the calibrated

30

model implies a 48 basis point increase in the two-year yield volatility relative to the case where the negative convexity channel is shut down. This can be compared to the estimated 131 basis point change in the two-year yield volatility that would result from a 2.9-standard-deviation shock bringing convexity from its average value to zero.[31] In line with our empirical findings, the calibrated model implies that the effect of negative convexity is hump-shaped and strongest for maturities around two and three years.

[Insert Figure 6 here.]

5.3 MBS duration and the cross-section of yields

We also look at the ability of the calibrated model to match additional stylized facts regarding the information in MBS duration and its relation to the information encoded in yields. Table 6 reveals that the predictive power of our duration factor remains largely unaffected when we control for the first three principal components of yields. Moreover, regressing duration on these yield factors results in an R^2 of a mere 22%.

While our stylized model is not designed to address the possibility that MBS duration is unspanned, the calibration exercise nevertheless speaks to the empirical facts. In the calibrated model the short rate factor explains over 96% of the variation in yields across maturities, but only around 7% of the variation in MBS duration and only around 1% of the one-year excess return on the 10-year bond. At the same time, duration accounts for all the return predictability and explains the same proportion of 10-year bond returns $(R^2 = 24\%)$ in the model as in the data. In other words, the factor that accounts for all the predictive power is not strongly related to the factor that accounts for a dominant fraction of the cross sectional variation in yields.

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[31]One reason the model underpredicts the basis point effect of convexity is that it produces a lower level of interest rate volatility compared to the data, and hence the volatility amplification mechanism is applied to a lower base level of volatility.

6 Conclusion

In this paper we study both theoretically and empirically the feedback from the fluctuations of aggregate MBS risk onto the yield curve. Our model makes the following predictions. First, MBS duration increases both nominal and real bond excess returns and the effect is strongest for longer maturities. Second, the predictive power of MBS duration for bond excess returns is transient. Third, MBS convexity positively affects bond yield volatilities and the relationship is hump-shaped across maturities.

We test these predictions in the data and find strong support. In particular, any one standard deviation change in MBS duration increases expected annual 10-year bond returns by 381 basis points, while real bond returns increase by 199 basis points. Since the effect of MBS duration on expected returns is transient in nature and becomes insignificant for a horizon beyond one year, this translates to a rise in nominal (real) 10-year yields of 38 (20) basis points today. Our results are comparable in magnitude to the impact of the recent Quantitative Easing programs implemented by the Federal Reserve: the cumulative effect of all large-scale asset purchases taken together is estimated to be between 80 and 120 basis points (see Stein (2012)). For volatility, we find that a one standard deviation change in MBS convexity, changes 2-year bond yield volatility by 37 basis points. Finally, we calibrate our model to the data and find that our model successfully produces effects which are quantitatively in line with their empirical counterparts.

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Appendix A Tables

<div align="center">

Table 1

Summary statistics

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This table reports the summary statistics for the duration and convexity variables, the one year bond excess returns, and bond yield volatilities. Duration is available for the full sample period from December 1989 to December 2012, while the convexity time series starts in January 1997. Nominal bond excess returns are also available for the full sample period, and real bond excess returns start in January 1999. Bond yield volatilities are calculated using monthly yield changes and a 12-month rolling window for the period from January 1997 to December 2012. One year excess returns and volatilities are annualized and expressed in percent.

Panel A: MBS duration and convexity

	Duration	Dollar duration	Convexity	Dollar convexity
Mean	4.510	457.43	-1.595	-163.73
Median	4.720	476.75	-1.590	-163.20
Min	1.840	189.37	-3.130	-337.70
Max	5.470	551.45	-0.440	-42.35
StDev	0.653	59.85	0.524	57.23

Panel B: Bond excess returns

		2y	3y	4y	5y	6y	7y	8y	9y	10y
Nominal	Mean	0.870	1.660	2.371	3.004	3.560	4.041	4.455	4.808	5.108
	Median	0.792	1.717	2.683	3.250	4.122	4.961	5.300	5.546	5.893
	Min	-2.522	-4.972	-6.917	-8.488	-10.091	-11.807	-13.419	-14.948	-16.410
	Max	3.581	7.027	10.008	12.455	14.472	16.234	17.969	19.475	20.959
	StDev	1.311	2.469	3.458	4.327	5.118	5.856	6.557	7.226	7.867
Real	Mean	0.607	1.247	1.819	2.315	2.745	3.124	3.461	3.764	4.040
	Median	0.525	1.087	1.709	2.465	2.764	3.190	3.630	4.195	4.715
	Min	-1.438	-2.695	-4.079	-5.497	-6.738	-7.838	-8.832	-9.743	-10.589
	Max	3.041	5.326	7.581	9.450	10.957	12.202	13.260	14.188	15.039
	StDev	1.083	1.934	2.557	3.024	3.414	3.777	4.140	4.512	4.892

Panel C: Bond yield volatilities

	1y	2y	3y	4y	5y	7y	10y
Mean	11.78	13.00	11.67	10.25	9.15	7.68	6.44
Median	9.72	11.80	10.39	9.03	7.72	6.60	5.36
Min	1.89	2.25	2.49	2.63	2.68	2.30	2.02
Max	32.81	32.95	26.96	21.67	18.41	16.49	15.30
StDev	9.53	9.41	7.58	6.05	5.03	3.95	3.32

Table 2
Nominal bond risk premia regressions

This table reports estimated coefficients from regressing annual bond excess returns constructed from Treasuries with maturity τ, $rx_{t,t+1y}^{\tau}$, onto a set of variables:

$$rx_{t,t+1y}^{\tau} = \beta_0^{\tau} + \beta_1^{\tau}\text{duration}_t + \beta_2^{\tau}\text{level}_t + \epsilon_{t+1y}^{\tau},$$

where duration_t is MBS dollar duration and level_t is the one-year yield. t-Statistics are calculated either using Newey and West (1987) (in parentheses) or Hansen and Hodrick (1980) (in brackets). Data is monthly and runs from December 1989 to December 2012.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
constant	-1.8494	-4.0917	-6.6335	-9.3851	-12.2709	-15.2213	-18.1790	-21.1034	-23.9711
	(-1.80)	(-2.08)	(-2.39)	(-2.69)	(-2.94)	(-3.15)	(-3.30)	(-3.43)	(-3.54)
	[-1.45]	[-1.68]	[-1.93]	[-2.17]	[-2.39]	[-2.57]	[-2.72]	[-2.84]	[-2.95]
duration	0.0059	0.0126	0.0197	0.0271	0.0346	0.0421	0.0495	0.0566	0.0636
	(2.46)	(2.87)	(3.31)	(3.71)	(4.03)	(4.25)	(4.41)	(4.51)	(4.59)
	[1.96]	[2.28]	[2.63]	[2.96]	[3.23]	[3.44]	[3.59]	[3.71]	[3.80]
Adj. R^2	7.40%	9.32%	11.65%	14.08%	16.44%	18.59%	20.47%	22.09%	23.47%
constant	-1.4361	-3.7956	-6.7248	-9.9961	-13.4558	-16.9873	-20.5043	-23.9497	-27.2914
	(-1.38)	(-1.83)	(-2.29)	(-2.71)	(-3.07)	(-3.36)	(-3.60)	(-3.78)	(-3.94)
	[-1.19]	[-1.56]	[-1.93]	[-2.26]	[-2.55]	[-2.78]	[-2.97]	[-3.12]	[-3.26]
duration	0.0041	0.0112	0.0201	0.0299	0.0400	0.0502	0.0601	0.0697	0.0788
	(1.54)	(2.18)	(2.80)	(3.37)	(3.87)	(4.28)	(4.60)	(4.85)	(5.04)
	[1.37]	[1.91]	[2.42]	[2.88]	[3.28]	[3.60]	[3.86]	[4.06]	[4.23]
level	0.1223	0.0876	-0.0270	-0.1808	-0.3506	-0.5225	-0.6880	-0.8421	-0.9824
	(1.43)	(0.55)	(-0.13)	(-0.71)	(-1.22)	(-1.68)	(-2.06)	(-2.39)	(-2.65)
	[1.39]	[0.54]	[-0.13]	[-0.72]	[-1.28]	[-1.81]	[-2.31]	[-2.79]	[-3.24]
Adj. R^2	11.18%	9.90%	11.72%	14.89%	18.52%	22.09%	25.30%	28.03%	30.29%

Table 3
Real bond risk premia regressions

This table reports estimated coefficients from regressing annual real bond excess returns, $rx_{t,t+1y}^{\tau*}$, onto MBS dollar duration:

$$rx_{t,t+1y}^{\tau*} = \beta_0^\tau + \beta_1^\tau \text{duration}_t + \epsilon_{t+1y}^\tau.$$

t-Statistics are calculated using Newey and West (1987) (in parentheses) or Hansen and Hodrick (1980) (in brackets). Data is monthly and runs from January 1999 through December 2012.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
constant	0.8248	1.0286	0.4685	-0.6596	-2.1407	-3.8123	-5.5661	-7.3341	-9.0765
	(1.28)	(0.89)	(0.31)	(-0.37)	(-1.08)	(-1.71)	(-2.24)	(-2.64)	(-2.94)
	[1.25]	[0.85]	[0.29]	[-0.33]	[-0.91]	[-1.41]	[-1.81]	[-2.12]	[-2.36]
duration	-0.0005	0.0005	0.0030	0.0066	0.0109	0.0155	0.0202	0.0248	0.0293
	(-0.25)	(0.14)	(0.68)	(1.31)	(1.96)	(2.57)	(3.09)	(3.50)	(3.81)
	[-0.22]	[0.12]	[0.59]	[1.10]	[1.61]	[2.08]	[2.47]	[2.79]	[3.04]
Adj. R^2	0.09%	0.03%	0.65%	2.25%	4.76%	7.83%	11.04%	14.06%	16.70%

Table 4
Bond volatility regressions

This table reports estimated coefficients from regressing bond yield volatility, vol_t^τ, onto MBS dollar convexity:

$$\mathrm{vol}_t^\tau = \beta_0^\tau + \beta_1^\tau \mathrm{convexity}_t + \epsilon_t^\tau.$$

t-Statistics are calculated using Newey and West (1987) (in parentheses) or Hansen and Hodrick (1980) (in brackets). Data is monthly and runs from January 1997 to December 2012.

	1y	2y	3y	4y	5y	7y	10y
constant	0.2650	0.9444	2.8631	3.7640	3.9861	3.6797	2.9920
	(0.10)	(0.31)	(1.01)	(1.54)	(1.91)	(2.35)	(2.60)
	[0.09]	[0.28]	[0.91]	[1.39]	[1.71]	[2.09]	[2.28]
convexity	0.0713	0.0764	0.0572	0.0429	0.0345	0.0267	0.0226
	(4.18)	(4.07)	(3.78)	(3.52)	(3.36)	(3.33)	(3.45)
	[3.81]	[3.67]	[3.40]	[3.16]	[3.01]	[2.94]	[3.01]
Adj. R^2	19.62%	22.30%	18.30%	15.59%	14.28%	13.95%	14.87%

Table 5
Regressions with Fed holdings

This table reports estimated coefficients from the following regression:

$$rx_{t,t+1y}^{\tau} = \beta_0^{\tau} + \beta_1^{\tau}\text{duration}_t + \beta_2^{\tau}\text{fed share}_t + \beta_3^{\tau}\text{duration}_t \times \text{fed share}_t + \epsilon_{t,t+1y}^{\tau},$$

where fed share$_t$ is the Federal Reserve's share of total MBS holdings. t-Statistics are calculated using Newey and West (1987) (in parentheses) or Hansen and Hodrick (1980) (in brackets). Data is monthly and runs from January 2009 to December 2014.

	2y	3y	4y	5y	7y	10y
constant	-0.7857	-2.5375	-5.3436	-9.0014	-17.6518	-30.5812
	(-2.36)	(-2.70)	(-2.85)	(-2.97)	(-3.26)	(-3.71)
	[-2.05]	[-2.36]	[-2.48]	[-2.57]	[-2.79]	[-3.17]
duration	0.0050	0.0141	0.0265	0.0409	0.0713	0.1116
	(4.30)	(4.38)	(4.27)	(4.17)	(4.15)	(4.33)
	[3.67]	[3.81]	[3.73]	[3.63]	[3.60]	[3.76]
interaction	-0.0294	-0.0770	-0.1368	-0.1982	-0.3044	-0.4040
	(-4.68)	(-4.11)	(-3.69)	(-3.41)	(-3.10)	(-2.88)
	[-3.92]	[-3.55]	[-3.20]	[-2.96]	[-2.70]	[-2.55]
Adj. R^2	43.84%	49.59%	50.45%	50.29%	51.72%	56.39%
constant	-0.3602	-3.0718	-7.4939	-12.6500	-22.6729	-33.8161
	(-0.90)	(-2.86)	(-3.61)	(-3.86)	(-3.99)	(-3.94)
	[-0.75]	[-2.67]	[-3.64]	[-4.00]	[-4.12]	[-3.96]
duration	0.0040	0.0153	0.0313	0.0491	0.0824	0.1188
	(3.01)	(4.47)	(4.98)	(5.07)	(4.96)	(4.72)
	[2.48]	[3.93]	[4.64]	[4.84]	[4.77]	[4.46]
fed share	-6.9698	8.7520	35.2216	59.7625	82.2438	52.9861
	(-1.24)	(0.50)	(0.98)	(1.04)	(0.81)	(0.35)
	[-1.03]	[0.45]	[0.91]	[0.97]	[0.76]	[0.33]
interaction	-0.0145	-0.0958	-0.2122	-0.3262	-0.4805	-0.5175
	(-1.08)	(-2.47)	(-2.77)	(-2.69)	(-2.28)	(-1.64)
	[-0.88]	[-2.18]	[-2.59]	[-2.57]	[-2.20]	[-1.57]
Adj. R^2	45.81%	50.74%	52.66%	52.76%	53.61%	57.54%

Table 6
Bond risk premia regressions with controls

This table reports estimated coefficients from regressing annual bond excess returns constructed from 10y Treasuries, $rx^{10y}_{t,t+1y}$, onto MBS dollar duration, the first three principal components from yields (level, slope and curvature), the Cochrane and Piazzesi (2005) factor, expected inflation and a growth index. Expected inflation is the consensus estimate from monthly forecasts on future inflation from Blue Chip Economic Indicators. Growth is a three-month moving average of the CFNAI. Data is monthly and runs from December 1989 to December 2012.

constant	-25.1188	-21.0410	-26.8405	-22.1520
	(-3.54)	(-3.19)	(-3.34)	(-2.87)
	[-3.67]	[-2.79]	[-3.21]	[-4.13]
duration	0.0663	0.0507	0.0757	0.0704
	(4.41)	(3.18)	(4.47)	(3.92)
	[4.34]	[2.85]	[4.71]	[4.19]
level	-0.1854			-0.0010
	(-1.37)			(0.00)
	[-1.66]			[0.00]
slope	-1.9324			-2.0780
	(-2.95)			(-3.31)
	[-4.48]			[-5.19]
curve	3.9107			4.2522
	(1.17)			(1.10)
	[1.13]			[1.10]
cp factor		1.4769		
		(1.64)		
		[1.78]		
inflation			-1.1202	-1.9282
			(-0.98)	(-0.78)
			[-1.28]	[-0.79]
growth			-2.1128	-0.7176
			(-2.45)	(-0.78)
			[-4.57]	[-1.09]
Adj. R^2	38.70%	27.03%	23.30%	34.37%

Table 7
Bond volatility regressions with controls

This table reports estimated coefficients from regressing bond yield volatility, vol_t^τ, or swaption implied volatility $\text{iv}_t^{\tau 10y}$ ($\text{iv}_t^{3m\tau}$) with maturity τ on a 10y swap rate (with maturity 3 months and a τ maturity swap rate) onto MBS dollar convexity, illiquidity, and Treasury implied volatility:

$$\text{vol}_t^\tau \text{ or } \text{iv}_t^{\tau 10y} \text{ or } \text{iv}_t^{3m\tau} = \beta_0^\tau + \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{illiq}_t + \beta_3^\tau \text{tiv}_t + \epsilon_t^\tau,$$

where illiq_t is the illiquidity factor and tiv_t is an implied volatility index from Treasury future options at time t. t-Statistics are calculated using Newey and West (1987) (in parentheses) or Hansen and Hodrick (1980) (in brackets). Data is monthly and runs from January 1997 through December 2012.

	1y	2y	3y	4y	5y	7y	10y
Panel A: Bond Yield Volatility							
constant	-16.9600	-15.6441	-11.0142	-7.6251	-5.4714	-3.2627	-2.0297
	(-3.99)	(-3.37)	(-2.52)	(-1.99)	(-1.64)	(-1.27)	(-1.04)
	[-3.88]	[-3.08]	[-2.26]	[-1.77]	[-1.47]	[-1.14]	[-0.96]
convexity	0.0830	0.0841	0.0629	0.0473	0.0381	0.0295	0.0251
	(4.79)	(4.35)	(4.11)	(3.88)	(3.69)	(3.51)	(3.46)
	[4.30]	[3.88]	[3.70]	[3.52]	[3.34]	[3.15]	[3.06]
tiv	1.8600	2.1738	1.8984	1.5908	1.3280	0.9495	0.6362
	(3.02)	(3.12)	(3.21)	(3.25)	(3.19)	(2.89)	(2.43)
	[2.79]	[2.81]	[2.86]	[2.90]	[2.85]	[2.63]	[2.28]
illiq	0.5466	-0.1593	-0.2763	-0.2856	-0.2497	-0.1379	-0.0091
	(1.43)	(-0.30)	(-0.59)	(-0.74)	(-0.77)	(-0.54)	(-0.04)
	[1.30]	[-0.27]	[-0.54]	[-0.69]	[-0.72]	[-0.51]	[-0.04]
Adj. R^2	43.63%	39.49%	35.25%	32.65%	30.92%	28.80%	27.27%
Panel B: Swaption Implied Volatility $\text{iv}_t^{\tau 10y}$							
constant	-19.9060	-15.5770	1.8475	3.4406	4.3976	6.6216	6.0767
	(-3.18)	(-2.80)	(0.54)	(1.08)	(1.42)	(2.55)	(2.25)
	[-2.87]	[-2.55]	[0.49]	[0.99]	[1.30]	[2.42]	[2.14]
convexity	0.1178	0.1113	0.0699	0.0633	0.0594	0.0595	0.0579
	(4.03)	(4.01)	(3.65)	(3.61)	(3.52)	(4.07)	(3.93)
	[3.56]	[3.55]	[3.23]	[3.19]	[3.10]	[3.63]	[3.53]
tiv	3.0654	2.7569	0.8754	0.6394	0.4606	-0.0444	-0.1894
	(3.85)	(3.70)	(1.68)	(1.35)	(1.03)	(-0.13)	(-0.56)
	[3.58]	[3.44]	[1.51]	[1.22]	[0.93]	[-0.12]	[-0.52]
illiq	1.1998	0.7860	0.3091	0.3373	0.3591	0.4729	0.4744
	(1.78)	(1.32)	(0.78)	(0.92)	(1.02)	(1.45)	(1.39)
	[1.72]	[1.26]	[0.73]	[0.86]	[0.95]	[1.37]	[1.31]
Adj. R^2	59.34%	56.16%	42.92%	42.16%	41.21%	45.04%	44.11%
Panel C: Swaption Implied Volatility $\text{iv}_t^{3m\tau}$							
constant	-49.1577	-40.4331	-31.5223	-26.0015	-23.0277	-17.5016	-13.6486
	(-4.61)	(-5.20)	(-4.76)	(-4.62)	(-4.56)	(-3.94)	(-3.31)
	[-4.45]	[-4.94]	[-4.53]	[-4.45]	[-4.44]	[-3.80]	[-3.19]
convexity	0.1895	0.1533	0.1433	0.1337	0.1252	0.1089	0.0956
	(3.89)	(3.80)	(3.82)	(3.73)	(3.76)	(3.70)	(3.76)
	[3.52]	[3.47]	[3.49]	[3.38]	[3.39]	[3.31]	[3.36]
tiv	7.8629	7.5338	6.0521	5.0691	4.5169	3.5586	2.8360
	(4.01)	(5.52)	(4.89)	(4.67)	(4.67)	(4.42)	(3.97)
	[3.55]	[4.90]	[4.33]	[4.16]	[4.18]	[4.00]	[3.64]
illiq	-0.0004	-0.8205	-0.6835	-0.3853	-0.0978	0.2604	0.6355
	(0.00)	(-0.86)	(-0.81)	(-0.52)	(-0.15)	(0.47)	(1.27)
	[0.00]	[-0.78]	[-0.73]	[-0.47]	[-0.13]	[0.42]	[1.14]
Adj. R^2	39.15%	44.85%	43.40%	44.27%	45.63%	46.95%	47.14%

Table 8
Calibrated parameters

This table reports parameters used for the calibration exercise. The mean reversion and the volatility of the nominal and real short rate processes are estimated directly from the short rate series. The sensitivity of mortgage refinancing to the incentive to refinance and the negative dollar convexity are set to match the aggregate MBS duration dynamics. The absolute risk aversion of financial institutions is chosen to match the predictive R^2 of the duration factor on the 10-year nominal bond excess returns. We use monthly data from 1990 to 2013.

κ	Nominal short rate mean reversion	0.13
σ	Nominal short rate volatility	1.33%
κ^*	Real short rate mean reversion	0.16
σ^*	Real short rate volatility	1.05%
ρ	Nominal-real rate correlation	0.87
κ_D	Sensitivity of refinancing to the incentive	1.05
η_y	Negative dollar convexity	1.05
α	Absolute risk aversion	88
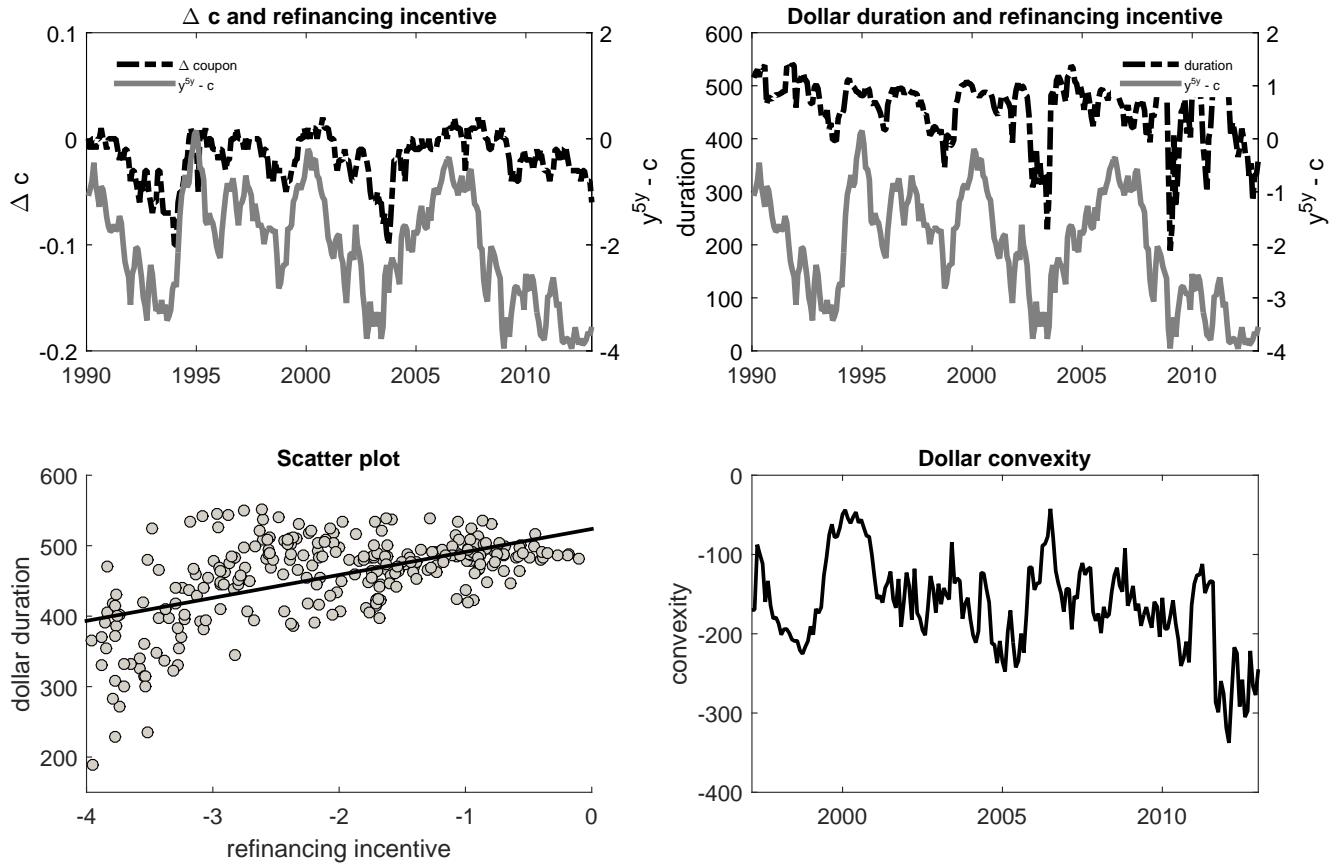
Figure 1. Average coupon, MBS dollar duration, and dollar convexity

The upper left panel plots the difference between the 5-year yield and the average MBS coupon (=refinancing incentive) together with the subsequent change in the average MBS coupon. The upper right panel plots the difference between the 5-year yield and the average MBS coupon together with MBS dollar duration. The lower left panel depicts a scatter plot between MBS dollar duration together with the refinancing incentive and a least-square line. The lower right panel plots MBS dollar convexity. Data is monthly and runs from December 1989 to December 2012 (duration) and from January 1997 to December 2012 (convexity), respectively.
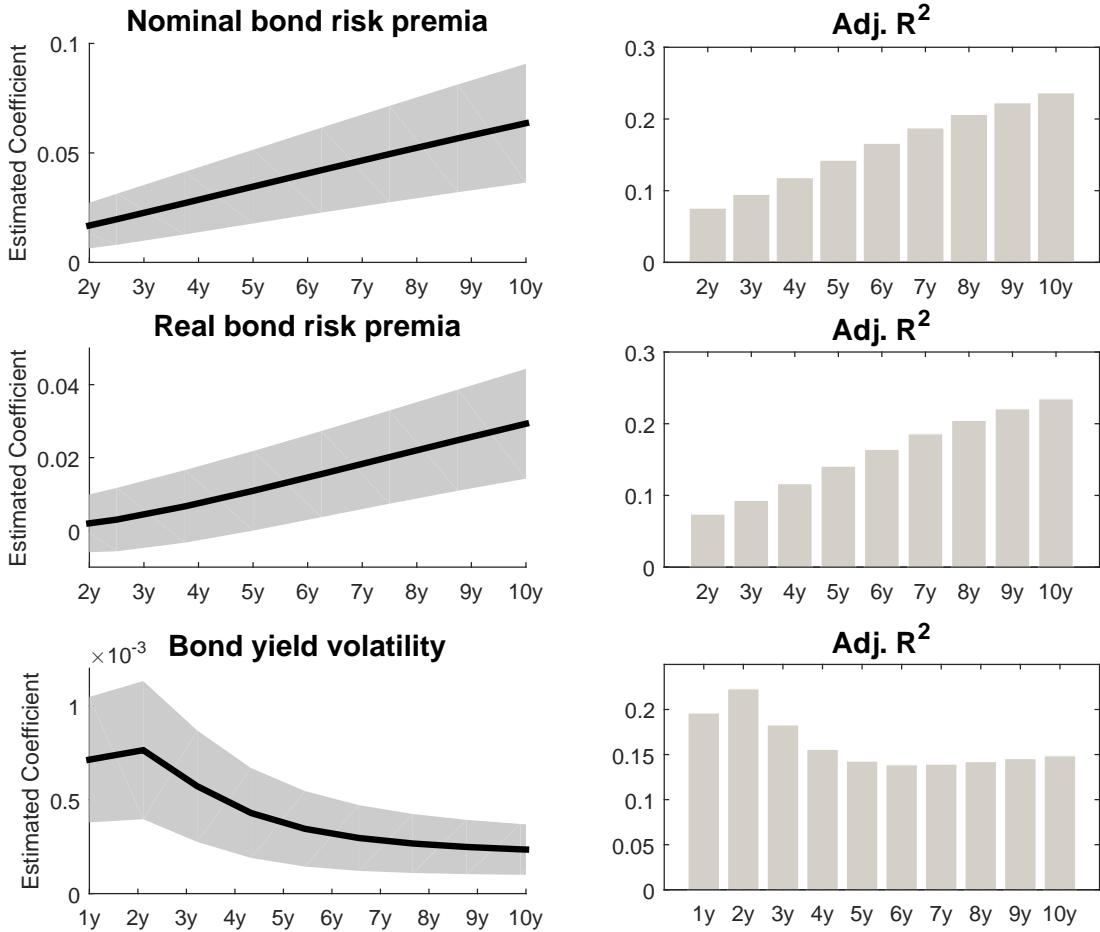
Figure 2. Univariate regression coefficients

The figure plots estimated coefficients and adjusted R^2 from univariate regressions of nominal and real bond excess returns (upper and middle panels) onto MBS dollar duration, and bond yield volatilities (lower panels) onto MBS dollar convexity, respectively. Data is monthly and runs from December 1989 through December 2012 (nominal bond excess returns), January 1999 through December 2012 (real bond excess returns), and January 1997 to December 2012 (bond yield volatilities). Shaded areas represent confidence levels on the 95% level using Newey and West (1987) adjusted standard errors.
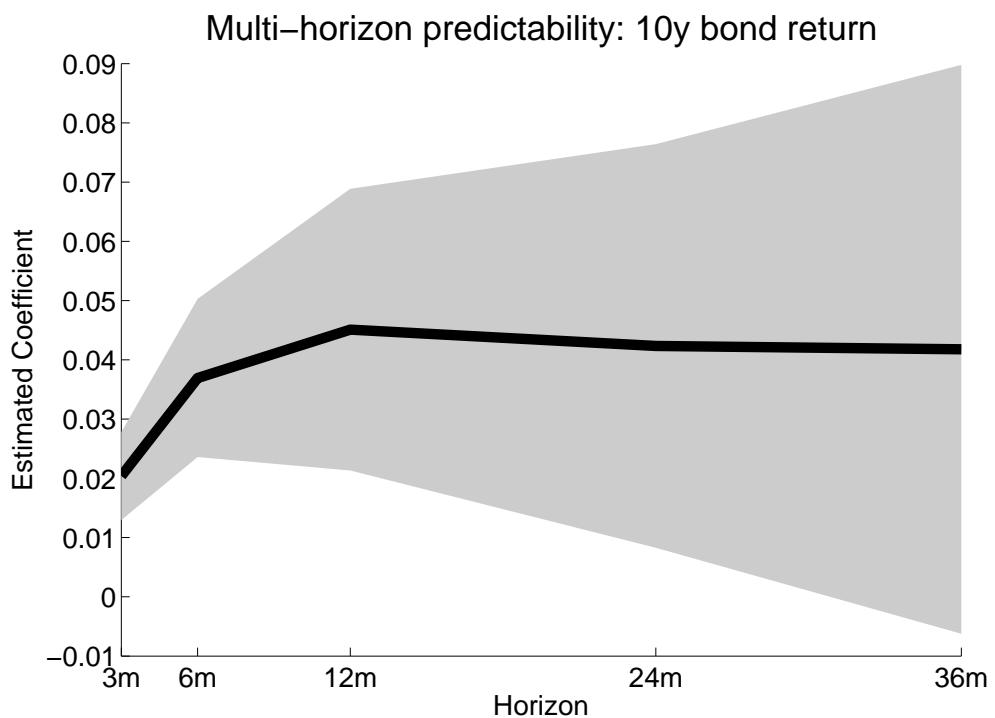
Figure 3. Multi-horizon predictability

This figure plots estimated coefficients from the following univariate regression:

$$rx_{t+h}^{10y} = \beta_0^h + \beta_1^h \text{duration}_t + \epsilon_{t+h},$$

where h is 3m, 6m, 12m, 24m, 36m. Shaded areas represent confidence levels on the 95% level using Newey and West (1987) adjusted standard errors.
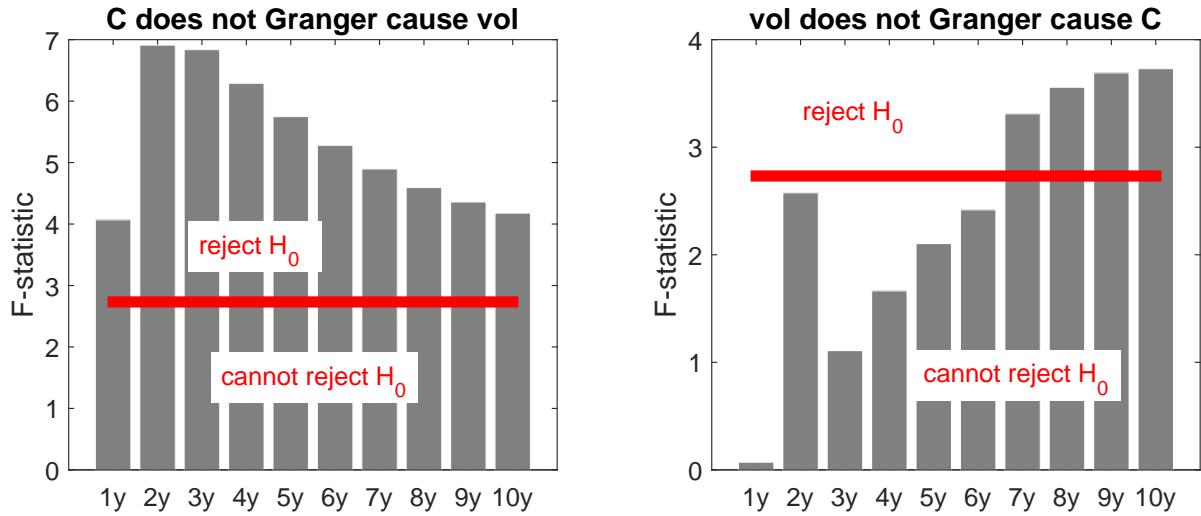
Figure 4. F-statistics of Granger causality tests

These figures present F-statistics for Granger causality tests. The line presents the critical value from the F-distribution on the 10% confidence level. The null hypothesis for the left (right) panel is that negative convexity (volatility) does not Granger cause bond yield volatility (convexity). The regressions are estimated on monthly data from January 1997 to December 2012.

Derivatives & MBS duration

Retained Portfolio

Rolling R^2
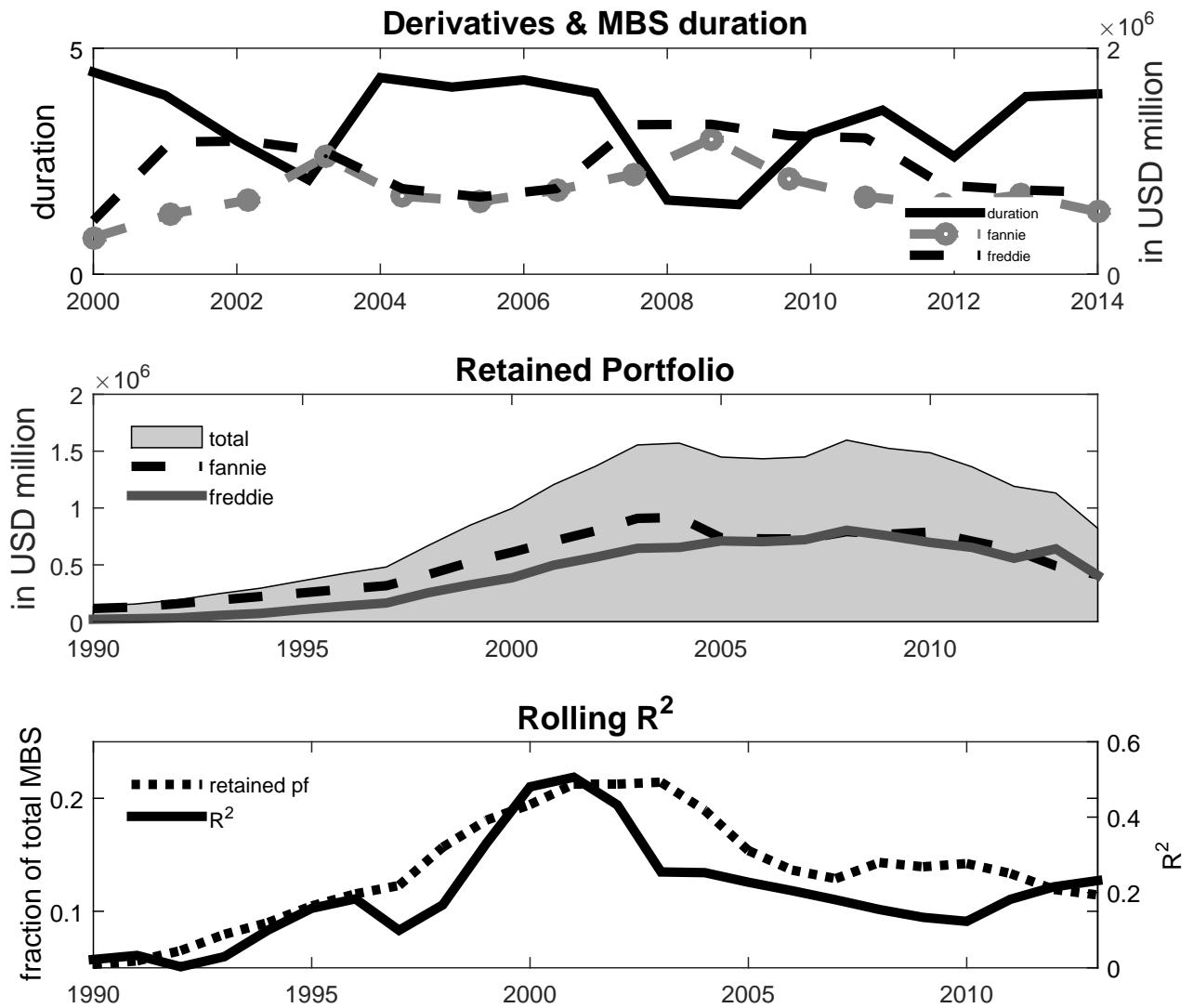
Figure 5. Retained portfolio and derivative holdings of GSEs

The upper panel plots the notional value of derivatives held by Fannie Mae and Freddie Mac in USD millions together with MBS duration. The middle panel plots the size of the retained portfolio of Fannie Mae and Freddie Mac in USD millions. The lower panel plots the size of the retained portfolio as a fraction of total MBS together with a rolling R^2 from regressing the 10-year bond excess return onto MBS dollar duration. Data is annual, source is the Federal Housing Finance Agency Annual Report to Congress.
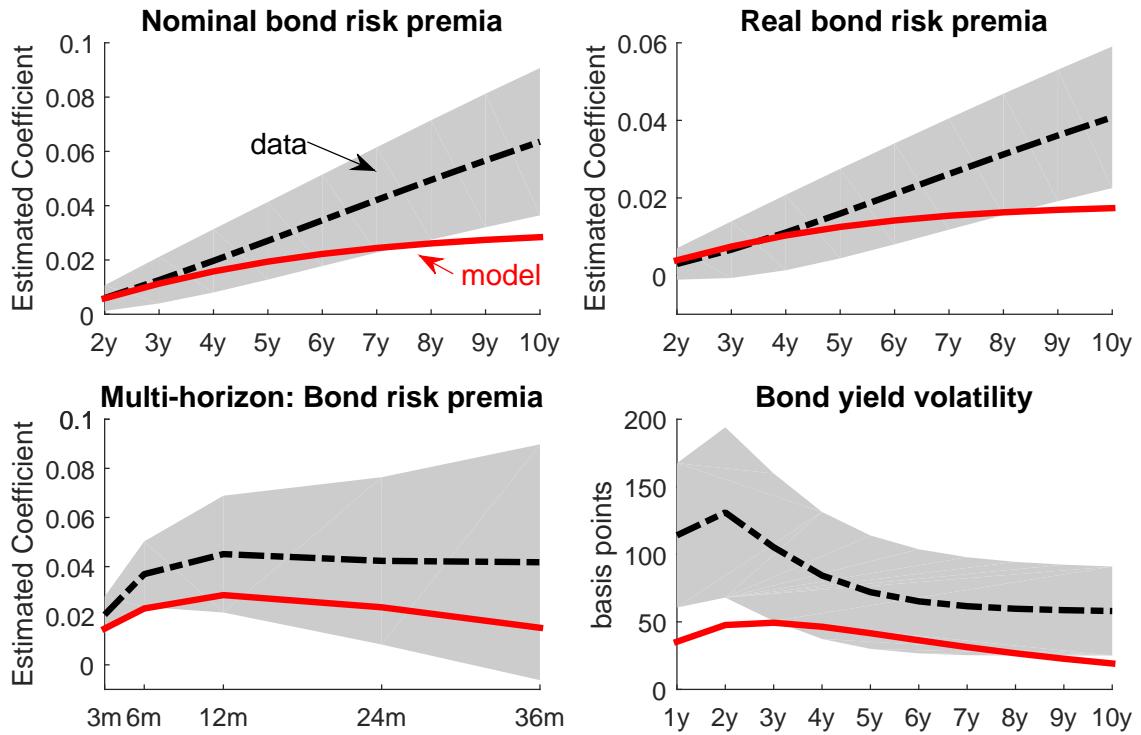
Figure 6. Calibration results

The top panels plot the theoretical slope coefficient of the regression of nominal (left) and real (right) bond excess returns on the duration factor together with the calibrated values from the model (see Table 8). The lower left panel plots the theoretical slope coefficient of the regression of 10-year nominal bond excess returns at different horizons on the duration factor together with the estimated values. The lower right panel plots the increase in yield volatility that can be attributed to negative convexity. In the model this effect is calculated as the difference between yield volatility in the benchmark calibration and an otherwise identical calibration where α is set to 0 and thus the negative convexity channel is shut down. Its empirical counterpart is based on our linear regression results. Shaded areas present 95% confidence intervals based on Newey and West (1987) standard errors.

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