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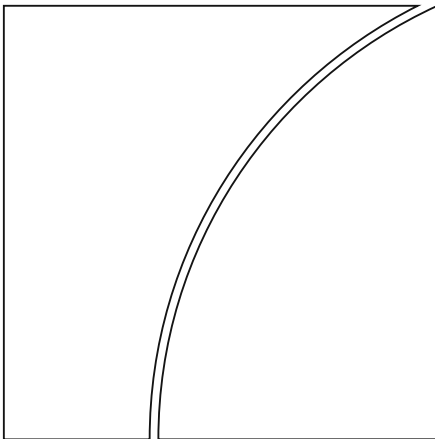
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Liquidity Squeeze, Abundant Funding and Macroeconomic Volatility*

Enisse Kharroubi[†]

Abstract

This paper studies the choice between building liquidity buffers and raising funding ex post, to deal with liquidity shocks. We uncover the possibility of an inefficient liquidity squeeze equilibrium. Agents typically choose to build smaller liquidity buffers when they expect cheap funding. However, when agents hold smaller liquidity buffers, they can raise less funding because of limited pledgeability, which in the aggregate depresses the funding cost. This incentive structure yields multiple equilibria, one being an inefficient liquidity squeeze equilibrium where agents do not build any liquidity buffer. Comparative statics show that this inefficient equilibrium is more likely when the supply of funding is large, and/or when aggregate shocks display low volatility. Last, the effectiveness of policy options to restore efficiency is limited because the net gain to intervention decreases with the availability of funding. In other words, policy becomes ineffective when the equilibrium becomes inefficient.

Key-words: Liquidity, Monetary Policy, Pledgeable Income, Reinvestment, Self-Insurance.

JEL : D53, D82, D86

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1 Introduction

Financial crises usually find their roots in boom periods that tend to precede them. The 2008-2009 financial crisis is no exception in this respect: significant vulnerabilities developed in the run-up to the crisis. For example, liquidity buffers, e.g. cash, claims on the central bank and claims on the government, which still accounted for around 10% of US banks total assets in the late 1990s dropped down to around 5% in 2007, at the onset of the financial crisis.^{1,2} This undoubtedly made US banks more vulnerable to financial distress, given that holding liquidity buffers is key for funding during adverse conditions.³ Yet, a key question is why did banks decide to reduce their liquidity buffers so much. One answer is that US banks decided to reduce their liquidity buffers in the pre-crisis period because of very easy funding conditions: even institutions which had no collateral or guarantees were able to raise funding. Yet, when funding became suddenly more expensive and scarce, US banks lacked the relevant assets as financial distress rose.⁴

Notwithstanding the role that unexpected changes in funding conditions can play in triggering financial crises, this paper focuses on an alternative answer which highlights the externalities at play in liquidity buffer holdings and how they relate to funding conditions. More specifically, we provide an analytical model to show that agents rationally hold too few liquidity buffers compared with the social optimum when the funding supply is sufficiently large. In this model, the economy paradoxically runs short of liquidity because funding is abundant, not because it suddenly becomes very scarce.

We build on the seminal paper by Holmström and Tirole (1998) in which entrepreneurs build liquidity buffers to self-insure against shocks affecting illiquid projects. Illiquid projects typically display a higher yield but are subject to liquidity shocks and then require reinvestment.⁵ To carry out reinvestment, entrepreneurs

¹Source: IMF-IFS. Liquid assets are the sum of reserves at the central bank (line 2:20), other claims on monetary authorities (line 2:20:N), claims on central government (line 2:22:A) and claims on state and local governments (line 2:22:B). Total Assets are the sum of liquid assets (defined as above), foreign assets (line 2:21), claims on the private sector (line 2:22:D) and claims on other financial corporations (line 2:22:G).

²To be sure, many assets that banks held on their balance sheet before the crisis -such as asset or mortgage backed securities- were considered at that time as liquid assets. However, when the crisis happened, these assets quickly became very illiquid. This is why we focus in the definition of liquid assets on cash and claims on the government whose liquidity is not state dependent, or put differently is similar ex ante and ex post.

³For example, claims on the government can be sold against cash when need be, a reason why they are also called "safe haven" assets.

⁴Financial institutions inability to evaluate current funding conditions at that time as abnormal was due either to an inherent myopia of banks and/or to the perception that public authorities would support banks in case such conditions were to evaporate.

⁵Reinvestment risk works here as a roll-over risk since getting the final pay-off once a reinvestment shock happened, requires

can either use liquidity buffers, or they can raise funds ex post. Yet, raising funds faces limits because future income streams are not fully pledgeable. Hence, besides providing self-insurance, holding liquid buffers also helps entrepreneurs raise more funds on the market. To this framework we add an exogenous supply of funding and ask how it affects entrepreneurs' decision to build liquidity buffers.

The main result of the model is that there can be multiple equilibria. The economy can coordinate on a "liquidity squeeze" equilibrium where agents do not build liquidity buffers and are unable to meet reinvestment needs when a liquidity shock occurs. This results from a positive externality of aggregate liquidity buffers on the funding cost. Let us detail the mechanism. First, agents typically choose to hold liquidity buffers on the funding cost. A high cost of funding therefore leads agents to build large liquidity buffers. Second, when future income streams are not fully pledgeable, agents holding large liquidity buffers can raise a large amount of funding. Large liquidity buffers in the aggregate hence lead to a high demand for funding which drives up the funding cost. And with a large funding cost, agents are willing to build large liquidity buffers. Larger *aggregate* liquidity buffers in the economy therefore raise *individual* incentives to hold liquidity buffers through the positive effect on funding cost. This externality yields two possible equilibria, one where agents build large liquidity buffers and another one where agents do not build any liquidity buffer. Importantly, the first equilibrium where agents prefer to build liquidity buffers always dominates. This is because illiquid projects achieve a high return even if they face a liquidity shock owing to large reinvestment while liquidity buffers provide a high return owing to the high cost of raising funds.⁶

Next, we investigate when this externality is more likely to hold: The positive feedback loop between individual and aggregate liquidity buffer holdings is more likely when the exogenous supply of funding is large. Yet, it is less likely when aggregate shocks display high volatility.⁷ Indeed, suppose agents hold large liquidity buffers, then those facing the liquidity shock can raise a large amount of funding -because of imperfect pledgeability- but those not facing the liquidity shock can also supply a large amount of funding. Now if the exogenous supply of funding is large, the total supply of funding is barely affected. As a result,

fresh funds to be raised.

⁶This mechanism illustrates the dangers of abundant and easy funding as it leads to over-investment in illiquid projects.

⁷We restrict ourselves to the analysis of aggregate supply shocks here.

the increase in the demand for funding dominates and the cost of raising funds needs to go up to balance the market.⁸ The positive externality of aggregate liquidity buffers hence requires a large exogenous supply of funding. Conversely the externality is less likely if aggregate shocks display high volatility. The reason is that with aggregate shocks, agents have an incentive to hold large liquidity buffers, even if the cost of funding is low. As a result, the externality which works through the cost of funding is less likely and so is indeterminacy. An important implication of this last result is that a reduction in the volatility of aggregate shocks, as was the case during the Great Moderation period, can be detrimental as the inefficient equilibrium with no liquidity holdings becomes more likely.

Last, we focus on policy options to avoid the inefficient equilibrium. The inefficiency comes in the model, from funding being too cheap. A natural policy to combat this inefficiency therefore consists in making a commitment to raise the funding cost.⁹ To do so, the central bank can, for example, commit itself to issuing bonds. This will raise the demand for funding and thereby the cost of funds. There are, however, two issues with such a policy. First, the larger the exogenous supply of funding, the larger the amount of bonds the central bank needs to commit to issue to induce the "good" equilibrium -where agents prefer to build liquidity buffers. Yet if issuing bonds entails costs which increase with the issuance size, then the central bank intervention will be more costly with a larger exogenous supply of funding. Second, the welfare loss in the "bad" equilibrium -where agents prefer not build liquidity buffers- decreases with the exogenous supply of funding, which reduces the gains to policy intervention. Intervention can hence bring more costs than benefits when the exogenous supply of funding is large, which is precisely when the equilibrium can be inefficient. This illustrates the difficulty to address the inefficiency stemming from abundant funding as this inefficiency tends to materialize precisely when policy becomes not worth being carried out.

This paper contributes to our understanding of the mechanics of liquidity crises. As noted above, we build on the Holmström and Tirole (1998) approach in which entrepreneurs use liquidity buffers to meet refinancing needs stemming from shocks affecting illiquid projects. We also closely follow Caballero and

⁸On the contrary, when the exogenous supply for capital is low then the total supply for capital increases significantly when agents hold more liquidity buffers. The cost of raising funds then needs to go down to balance the market.

⁹In the second best, the amount of liquid asset holdings is not contractible. The simple policy consisting in imposing a minimal liquidity ratio is therefore not possible as that would boil down to assuming that the policy maker can contract on the amount of liquid asset holdings.

Krishnamurthy (2001), who look extensively at the problem of underinsurance against refinancing shocks in an open economy context. A key difference from their approach is that we do not get into the problem of entrepreneurs facing a need for refinancing non-tradable assets with limited tradable resources. Besides, in our framework, inefficiencies stem from the abundance and not the shortage of interim refinancing. We also build on the seminal Diamond and Dybvig (1983) paper in which banks provide liquidity to depositors while investing in long term assets, thereby facing a risk of bank run.¹⁰ Bhattacharya and Gale (1987) extends their framework and looks at how liquidity provided by the interbank market affects banks willingness to hold liquidity. Bolton, Santos and Scheinkman (2010) provides a model where agents' reliance on inside liquidity as opposed to outside liquidity can affect the timing of trades on the market for liquidity. This model also features a multiple equilibria mechanism. An important difference however is that outside liquidity is efficient in their framework while our model points to abundant funding as a potential source of problems. It should also be clear that there are many different and important aspects relating to the notion of liquidity -see for instance Gorton and Pennachi (1990) for an information-based approach to liquidity- that we simply do not consider in this paper. Finally the paper by Acharya, Shin and Yorulmazer (2007) is also related: this paper looks at how foreign bank entry reduces domestic banks' incentives to hold liquid assets. The focus there however is on fire sales.

The paper is organized as follows. The following section sets out the main assumptions of the model. Section 3 describes the decentralized equilibrium. Section 4 examines the externality at the source of the multiple equilibria property and discusses efficiency considerations. Section 5 introduces aggregate shocks into the original model. Policy options to improve social welfare are investigated in Section 6. Conclusions are drawn in Section 7.

¹⁰Note however that in the standard Diamond-Dybvig framework (1983), multiple equilibria relate to depositors' behavior, for a given allocation between liquid assets and illiquid investments. In our framework, it is this allocation decision which can be indeterminate.

2 Timing and technology

We consider a single good economy populated with a unit mass continuum of entrepreneurs and a unit mass continuum of investors. The economy lasts for three dates; 0, 1 and 2. Agents are risk neutral and derive utility from profits at date 2. They can freely store capital at any date t with a unit return at date $t + 1$. An entrepreneur storing capital will be said to hold liquidity buffers and L will denote an entrepreneur's liquidity buffer holdings.

Entrepreneurs. Each entrepreneur starts with a unit endowment at date 0 and can invest an amount $I > 0$ in an illiquid project. At date 1, entrepreneurs experience an idiosyncratic liquidity shock with a probability α . If no liquidity shock hits, the project returns $\rho_g I$ at date 2. But if the liquidity shock hits, the project returns only $\rho_b I$ at date 2 with $\rho_b < 1 < \rho_g$. Yet, entrepreneurs hit with a liquidity shock have a reinvestment option: reinvesting an amount J of fresh resources at date 1 in the project returns $\rho_1 \min\{J, I\}$ at date 2, with $\rho_1 \geq (1 - \alpha)\rho_g + \alpha\rho_b \geq 1$. Total output for an entrepreneur hit with a liquidity shock is hence $\rho_b I + \rho_1 \min\{J, I\}$. Last, entrepreneurs can only pledge a fraction ϕ of illiquid projects' output and $\phi\rho_1 < 1$. Imperfect pledgeability will introduce a positive relationship between liquidity buffer holdings L at date 0 and reinvestment J at date 1.¹¹

Investors. Investors start with an endowment at date 1 denoted λ . They are essentially fund providers: they can trade with entrepreneurs once liquidity shocks are realized.¹²

Frictions. This economy is subject to two frictions. First, liquidity shocks are *not contractible* and hence cannot be diversified. Entrepreneurs therefore need to hold liquidity buffers as a self-insurance device. Second, entrepreneurs' allocation at date 0 between building liquidity buffers and investing in illiquid projects

¹¹The pledgeability constraint for distressed entrepreneurs will hence be binding since distressed entrepreneurs' pledgeable return $\phi\rho_1$ will be lower than the opportunity cost of capital (equal to one here).

¹²The assumption that investors' endowment is available at date 1 is a matter of simplification. If investors had on top of that a capital endowment at date 0, there would be two further issues that would either reinforce or complement the mechanism described in the paper: first entrepreneurs could issue claims at date 0. This possibility -introducing leverage for entrepreneurs at date 0- would actually reinforce the indeterminacy property. Second investors could decide strategically to provide their funds at date 0 or at date 1 depending on the return attached to each of these investment strategies. Under some conditions, this possibility to choose strategically the timing of capital supply can itself be a source of indeterminacy, independently of the mechanism highlighted in the paper.

is *observable* but *not verifiable*. This assumption precludes investors from charging funding costs which would depend on entrepreneurs' individual liquidity buffer holdings. The cost of raising funds at date 1 will actually depend on the aggregate amount of liquidity buffers in the economy, which will be the source of the pecuniary externality.

Timing. At date 0, entrepreneurs choose how much to invest in illiquid projects and how much liquidity to build. At date 1, a proportion α of entrepreneurs are hit with a liquidity shock. These entrepreneurs can then use their liquidity buffers and/or raise funds to carry out reinvestment. Investors and entrepreneurs who have not been hit with the liquidity shock can either lend to entrepreneurs who are reinvesting or stick to their liquidity buffers if this is more profitable. Finally at date 2, entrepreneurs pay back their liabilities if any and consume.

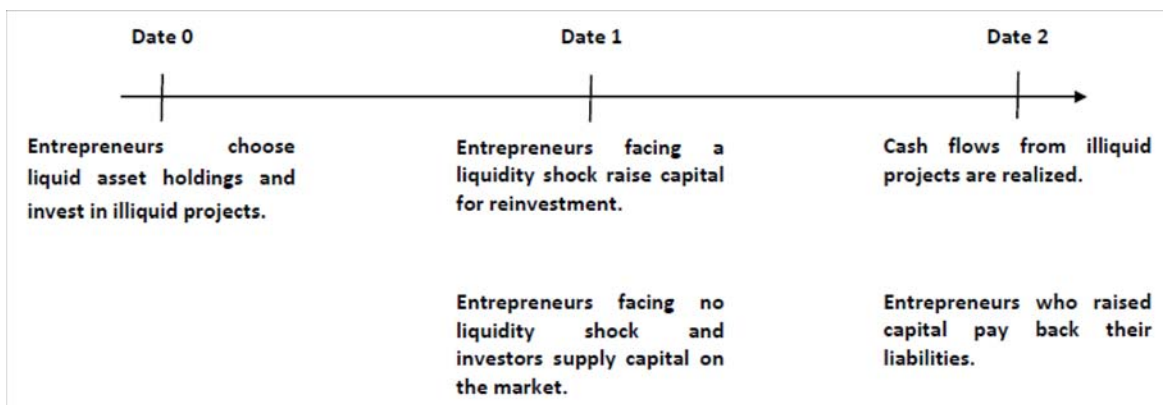


Fig. 1: Timing of the model.

At the heart of the model is a trade-off for entrepreneurs who have to compromise between the initial investment I on the one hand and the reinvestment J in the event of a liquidity shock on the other hand. Maximizing initial investment I requires liquidity buffer holdings L to be minimized and thereby pledgeable income. This in turn cuts reinvestment J in the event of a liquidity shock. Conversely, maximizing liquidity holdings L to mitigate liquidity shocks requires sacrificing initial investment I .

3 The decentralized equilibrium

The decentralized equilibrium is based on two building blocks: first entrepreneurs' choice at date 0 between holding liquidity and investing in illiquid projects and second the equilibrium of the market at date 1 and where entrepreneurs hit with a liquidity shock raise funding from investors and entrepreneurs facing no liquidity shock. We start by looking at the entrepreneurs' optimal allocation of liquidity buffer holdings and illiquid investments. To do so, we use backward induction. Since no decision is taken at date 2 - apart from executing previously agreed contracts-, we first look at date 1 where entrepreneurs hit with a liquidity shock choose the amount of funding to raise for reinvestment. Then we will move back to date 0 where entrepreneurs make a choice between building liquidity buffers and investing in illiquid projects.

Entrepreneurs' optimal liquidity holdings. Consider an entrepreneur who built up L liquidity buffer and invested $I = 1 - L$ in an illiquid project at date 0. Then if no liquidity shock hits at date 1, the project returns $(1 - L)\rho_g$ at date 2. In addition, the entrepreneur has L available funds at date 1 which can either be stored with a unit return or lent to distressed entrepreneurs with a return denoted r . The entrepreneur therefore reaps $L \max\{1, r\}$ at date 2, depending on whether storing or lending is more profitable. When there is no liquidity shock, the entrepreneur's total profit π_g at date 2 can be written as

$$\pi_g(L) = (1 - L)\rho_g + L \max\{1, r\} \quad (1)$$

When $r < \rho_g$, the entrepreneur's profit π_g decreases with liquidity buffers L . Holding liquidity buffers is therefore costly for an entrepreneur who does not face the liquidity shock.

If now a liquidity shock hits at date 1, the illiquid project returns only $(1 - L)\rho_b$ at date 2. But the entrepreneur can reinvest. To do so, she can rely on liquidity buffers L ; she can also raise an amount D of funds -from investors and entrepreneurs not facing the liquidity shock- against the promise to pay back rD at date 2. Reinvestment hence yields a cash flow $(L + D)\rho_1$ so that the entrepreneur's total profit π_b at date 2 writes as

$$\pi_b(D, L) = (1 - L)\rho_b + (L + D)\rho_1 - rD \quad (2)$$

Moreover entrepreneurs confronted with a liquidity shock face the pledgeability constraint: repayments for funds raised at date 1 should not exceed pledgeable output:

$$rD \leq \phi [(1 - L) \rho_b + (L + D) \rho_1] \quad (3)$$

Then assuming the cost r to raise funds at date 1 satisfies $\phi \rho_1 \leq r \leq \rho_1$, i.e. raising capital for reinvestment is profitable but constrained by imperfect pledgeability, entrepreneurs choose to raise the maximum amount of capital D^* that is compatible with the pledgeability limit,

$$D^*(L) = \frac{\phi [(1 - L) \rho_b + L \rho_1]}{r - \phi \rho_1} \quad (4)$$

Given that $\rho_1 > \rho_b$, an entrepreneur with more liquidity L chooses to raise more capital when hit with a liquidity shock, since $D^*(L)$ increases with L . Holding more liquidity alleviates the constraint on the amount of funds that can be raised. The entrepreneur's profits π_b can therefore be written as:

$$\pi_b(L) = \frac{(1 - \phi) r}{r - \phi \rho_1} [(1 - L) \rho_b + L \rho_1] \quad (5)$$

Entrepreneurs hit with a liquidity shock benefit from carrying on larger liquidity buffers L : profits π_b increase with L since the return to reinvestment ρ_1 is larger than the return to an illiquid project hit with a liquidity shock ρ_b . This is the self-insurance aspect of holding liquidity buffers. With larger liquidity holdings, entrepreneurs reap larger profits when hit with a liquidity shock.

Now that we have determined entrepreneurs' profits in the cases with and without a liquidity shock, we can turn to the date 0 problem. The probability of a liquidity shock being α , the problem for an entrepreneur

consists in choosing the amount of liquidity L which maximizes expected profits:

$$\begin{aligned} \max_L \pi &= (1 - \alpha) \pi_g(L) + \alpha \pi_b(L) \\ \text{s.t. } L + D^*(L) &\leq 1 - L \end{aligned} \quad (6)$$

this problem being valid under the assumption that $1 \leq r \leq \rho_1$. Under this assumption, (i) raising funds is profitable for entrepreneurs facing a liquidity shock but constrained by imperfect pledgeability and (ii) lending funds is profitable for entrepreneurs not facing the liquidity shock. Last, reinvestment $L + D^*(L)$ is limited by initial investment $1 - L$. This translates into an upper bound on the amount of liquidity L entrepreneurs can hold at date 0, which will be denoted $L(r)$. The following proposition then describes the entrepreneurs' optimal choice for liquidity buffer holdings at date 0.

Proposition 1 *Assuming $1 \leq r \leq \rho_1$, entrepreneurs choose to hold at date 0 an amount L^* of liquidity which satisfies*

$$L^* = \begin{cases} L(r) & \text{if } r \geq \frac{(1-\alpha)\rho_g}{1-\alpha+\alpha(1-\phi)\frac{\rho_1-\rho_b}{r-\phi\rho_1}} \\ 0 & \text{if } r \leq \frac{(1-\alpha)\rho_g}{1-\alpha+\alpha(1-\phi)\frac{\rho_1-\rho_b}{r-\phi\rho_1}} \end{cases} \quad (7)$$

where $L(r)$ satisfies $L(r) + D^*(L(r)) = 1 - L(r)$.

Proof. When the return r satisfies $1 \leq r \leq \rho_1$, deriving the expression for entrepreneurs' expected profits yields

$$\frac{\partial \pi}{\partial L} = \left[1 - \alpha + \alpha(1 - \phi) \frac{\rho_1 - \rho_b}{r - \phi\rho_1} \right] r - (1 - \alpha) \rho_g \quad (8)$$

Entrepreneurs do not build liquidity buffers when this expression is negative. Conversely, when positive, they choose to build an amount $L(r)$ of liquidity. They are then able to reinvest at date 1 as much as they invested at date 0 in the illiquid project if the liquidity shock hits, i.e. $L(r) + D^*(L(r)) = 1 - L(r)$. ■

An entrepreneur allocating her endowment between liquidity buffers and an illiquid project faces a simple trade-off: building liquidity implies foregoing profits if no liquidity shock hits but contributes to higher profits if a liquidity shock hits. The cost r of raising funds at date 1 affects this trade-off in two ways. First, a larger funding cost reduces profits for entrepreneurs facing the liquidity shock and hence incentives to build

liquidity buffers. Second, a larger funding cost raises the return on liquidity buffers for entrepreneurs who do not face a liquidity shock; it hence provides incentives to build larger liquidity buffers. When this second effect dominates, entrepreneurs choose to build larger liquidity buffers at date 0 if they expect a larger funding cost r at date 1. In what follows, we will assume this is indeed the case -the second effect does dominate- and denote r^* the cost of raising funds at date 1 such that entrepreneurs are indifferent at date 0 between building liquidity buffers and investing in illiquid projects: $\partial\pi/\partial L|_{r=r^*} = 0$.¹³

The market for reinvestment. Let us now turn to the market for reinvestment which opens at date 1. In this market, entrepreneurs confronted with a liquidity shock can raise funding from investors and entrepreneurs facing no liquidity shock, to finance reinvestment. On the demand side of the market, there is a fraction α of entrepreneurs facing a liquidity shock and the demand for funding from each of them is D^* . Conversely, on the supply side of the market, there is a fraction $1 - \alpha$ of entrepreneurs not facing the liquidity shock and the supply of funding from each of them is L^* . Moreover, there is measure one of investors who can supply λ units of funding. The equilibrium of the market for reinvestment at date 1 therefore writes as

$$\mathbf{1}_{[r \leq \rho_1]} \alpha D^* = \mathbf{1}_{[r \geq 1]} [\lambda + (1 - \alpha) L^*] \quad (9)$$

where $\mathbf{1}_{[x]}$ is one if x is true and zero otherwise. The cost r of raising funds needs to be larger than one for investors and entrepreneurs not facing the liquidity shock to provide incentives to supply their capital on the market. Similarly, the cost r of raising funds needs to be lower than the return to reinvestment ρ_1 . Otherwise entrepreneurs facing a liquidity shock are better off not raising any funding.¹⁴ We can then derive the following result.

¹³For instance, the second effect dominates when the pledgeable share ϕ of output from illiquid projects is low, which is consistent with our assumption that $\phi\rho_1 < 1$. More generally, we focus on the case where $\partial\pi/\partial L$ is increasing in the return r with $\partial\pi/\partial L|_{r=\rho_1} > 0 > \partial\pi/\partial L|_{r=1}$, i.e. entrepreneurs prefer to build liquidity buffers when the cost of funding is ρ_1 while they prefer to invest in illiquid projects when the cost of funding is 1. The first condition $\partial\pi/\partial L|_{r=\rho_1} > 0$ simplifies as $\rho_1 > \alpha\rho_b + (1 - \alpha)\rho_g$ which we assumed does hold while the second condition $\partial\pi/\partial L|_{r=1} < 0$ holds when ϕ is sufficiently low.

¹⁴Note that the equilibrium is trivial if $\lambda \geq \alpha$, since, the aggregate supply for funding $[\lambda + (1 - \alpha) L^*]$ would always be larger than the aggregate demand for funding αD^* given that $D^* \leq 1$. Given this excess supply for funding, the cost of funding r would therefore always be one and entrepreneurs would not build any liquidity buffers. We will therefore restrict our attention in what follows to the case where $\lambda < \alpha$.

Proposition 2 *Assuming $1 \leq r \leq \rho_1$, the equilibrium cost of capital at date 1 satisfies*

$$r = \phi \left[\rho_1 + \alpha \frac{\rho_1 L^* + \rho_b (1 - L^*)}{(1 - \alpha) L^* + \lambda} \right] \quad (10)$$

Proof. First the equilibrium requires $1 \leq r \leq \rho_1$. If $r < 1$, there would be an excess demand for funding since investors and entrepreneurs not facing the liquidity shock would prefer holding liquidity buffers to lending while entrepreneurs facing the liquidity shock would be willing to raise funding. Conversely if $r > \rho_1$, there would be an excess supply of funding since entrepreneurs facing the liquidity shock would not be willing to raise funding while investors and entrepreneurs not facing the liquidity shock would be willing to lend. The equilibrium therefore necessarily satisfies $1 \leq r \leq \rho_1$, and based on expressions (4) and (9), it can be written as

$$\alpha \frac{\phi [(1 - L^*) \rho_b + L^* \rho_1]}{r - \phi \rho_1} = \lambda + (1 - \alpha) L^* \quad (11)$$

which finally yields expression (10) for the equilibrium cost of raising funds at date 1. ■

Expression (10) shows that the cost r of raising funds can be either a positive or a negative function of the amount of liquidity buffers L^* entrepreneurs build at date 0. It is typically positive (negative) when the investors' supply of funding satisfies $\lambda \geq \frac{(1-\alpha)\rho_b}{\rho_1-\rho_b}$ ($\lambda \leq \frac{(1-\alpha)\rho_b}{\rho_1-\rho_b}$). When entrepreneurs increase their liquidity buffer holdings at date 0, the supply of funding at date 1 goes up because entrepreneurs not facing the liquidity shock have more funds available. However entrepreneurs facing the liquidity shock can also demand more funds because the pledgeability constraint limiting the amount of funds that can be raised is relaxed. To determine which of these two effects dominates, take the case where the investors' supply of funding λ is large. Then a change in entrepreneurs' liquidity buffers L^* has a minor impact on the aggregate supply $\lambda + (1 - \alpha) L^*$. The relative increase in the supply of funding will therefore be small compared with the relative increase in the demand for funding. The increase in the demand will therefore dominate and lead to an increase in the cost r of raising funds.

Conversely when the investors' supply of funding λ is low, a change in entrepreneurs' liquidity holdings L^* has a large relative impact on the aggregate supply of funding $\lambda + (1 - \alpha) L^*$ which typically dominates

the relative increase in the demand for funding and hence leads to a reduction in the cost r of raising funds.

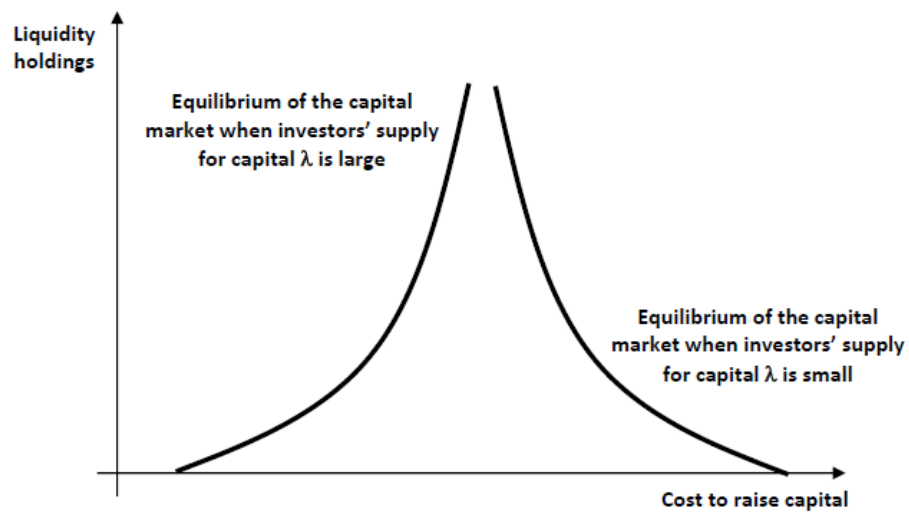


Fig. 2: Equilibrium of the market for funding.

To wrap up, the supply of funding λ from investors has two types of effects on the relationship between entrepreneurs' liquidity holdings L^* and the cost r of raising funds; a level effect and a slope effect. First, a larger supply of funding λ reduces the *level* of the cost r of raising funds. Second, a larger supply of funding λ raises the *slope* of the cost r of raising funds -w.r.t. aggregate liquidity holdings L^* - (which can turn from negative to positive as in Fig. 2).

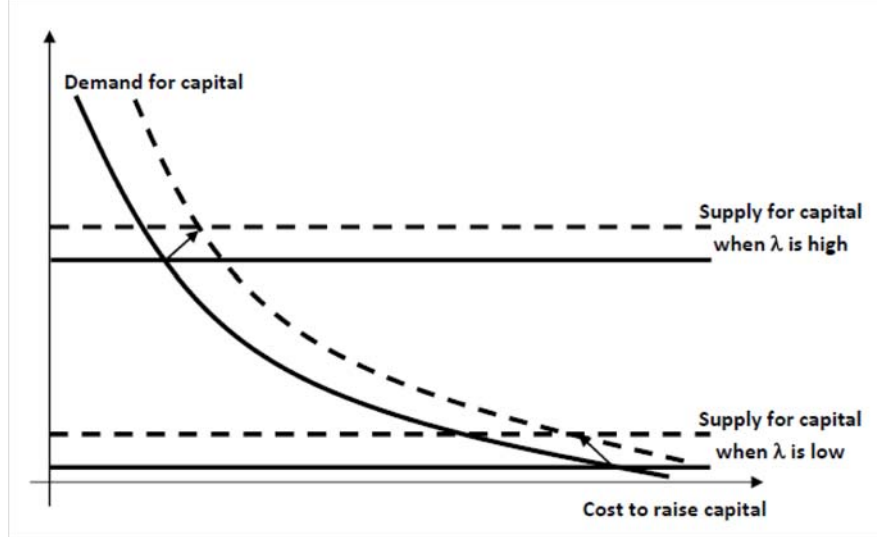


Fig 3: The effect of an increase in entrepreneurs' liquidity buffer holdings.

We can now determine the decentralized equilibrium using the two relations described above: the optimality condition (7) determines how large a liquidity buffer L^* each entrepreneur chooses to build depending on the cost of funding r and on the other hand the equilibrium relationship (10) which determines the cost of funding r as a function of the aggregate amount of liquidity buffers L^* that entrepreneurs hold. Given that entrepreneurs choose either to hold no liquidity buffer or to hold as large a liquidity buffer as possible, there are two corresponding possible equilibria, which we investigate below.

The equilibrium with liquidity buffers. Consider first the case where entrepreneurs prefer to build liquidity buffers at date 0. This is an equilibrium when the cost r of raising funds that comes out of the equilibrium at date 1 is such that entrepreneurs are effectively better off building liquidity buffers at date 0. The following proposition derives the condition under which this situation is an equilibrium.

Proposition 3 *A decentralized equilibrium in which entrepreneurs prefer to build liquidity buffers at date 0 exists if and only if investors' supply of funding λ satisfies*

$$\lambda \leq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} + \left[\alpha \frac{\phi(\rho_1 - \rho_b)}{r^* - \phi\rho_1} - (1 - \alpha) \right] \frac{\alpha - \lambda}{1 + \alpha} \quad (12)$$

Proof. When entrepreneurs prefer to build liquidity buffers at date 0, the amount of liquidity buffers L satisfies $L + D = 1 - L$ while the equilibrium at date 1 writes as $\alpha D = \lambda + (1 - \alpha)L$. Based on these two equalities, entrepreneurs' liquidity buffers at date 0 write as

$$L = \frac{\alpha - \lambda}{\alpha + 1} \quad (13)$$

Plugging (13) into expression (10) for the cost r of raising funds, the condition $r \geq r^*$ under which entrepreneurs are better off building liquidity buffers writes as

$$\phi \left[\rho_1 + \alpha \frac{\rho_b + (\rho_1 - \rho_b) \frac{\alpha - \lambda}{\alpha + 1}}{\lambda + (1 - \alpha) \frac{\alpha - \lambda}{\alpha + 1}} \right] \geq r^* \quad (14)$$

which simplifies as (12). Hence there exists a decentralized equilibrium in which entrepreneurs prefer to build liquidity buffers at date 0 if and only if (12) holds. ■

Condition (12) shows that the equilibrium in which entrepreneurs are better off building liquidity buffers is more likely when investors' supply of funding λ is sufficiently low. This is because it ensures a large cost r of raising funds so that entrepreneurs are willing to build liquidity buffers.

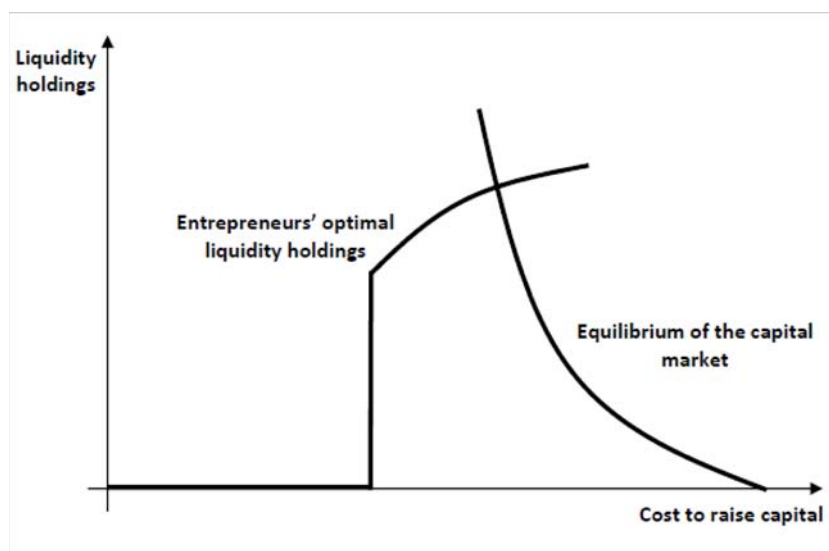


Fig. 4: The equilibrium with liquidity holdings.

The comparative statics show that the equilibrium with liquidity buffers is more likely to hold if the probability α of a liquidity shock is larger. The need for entrepreneurs to build liquidity buffers naturally increases if liquidity shocks are more likely.

The equilibrium without liquidity buffers. Consider now the case where entrepreneurs prefer to invest in illiquid projects. This arises if the cost r of raising funds that comes out of the equilibrium at date 1 leaves entrepreneurs better off investing in illiquid projects at date 0.

Proposition 4 *A decentralized equilibrium in which entrepreneurs invest in illiquid projects and hold no liquidity buffer exists if and only if investors' supply of funding λ satisfies*

$$\lambda \geq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} \quad (15)$$

Proof. When entrepreneurs hold no liquidity buffers, the equilibrium funding cost is $r = \phi\left(\frac{\rho_b\alpha}{\lambda} + \rho_1\right)$ (based on expression (10)). As a result, there exists a decentralized equilibrium in which entrepreneurs are better off holding no liquidity buffers if and only if $\phi\left(\frac{\rho_b\alpha}{\lambda} + \rho_1\right) \leq r^*$ which simplifies as (15). ■

We identify this equilibrium as a "liquidity squeeze" equilibrium since entrepreneurs facing the liquidity shock have a profitable reinvestment opportunity which they are unable to exploit due to the lack of pledgeable income. Note that this situation is not related to investors suddenly reducing their funding supply λ as is the case in sudden stops or capital flow reversals. On the contrary, this situation emerges because the large funding supply from investors λ reduces the funding cost r and thereby the profits entrepreneurs can reap from building liquidity buffers at date 0. Entrepreneurs are therefore better off investing in illiquid projects. Note finally that the low funding cost r hides a large shadow cost of capital since entrepreneurs are not able to exploit their reinvestment option fully. The low funding cost r rather reflects here entrepreneurs' limited pledgeable income.

Last, this equilibrium is more likely if the probability α of a liquidity shock is lower. If liquidity shocks are less likely, the need for entrepreneurs to build liquidity buffers is reduced.

4 Multiple equilibria and efficiency

In this section, we derive two properties. The first has to do with indeterminacy in entrepreneurs' decision between building liquidity buffers and investing in illiquid projects. The second property relates to efficiency.

Proposition 5 *Entrepreneurs' choice between building liquidity buffers and investing in illiquid projects is indeterminate when investors' funding supply λ satisfies*

$$\frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} \leq \lambda \leq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} + \left[\alpha \frac{\phi(\rho_1 - \rho_b)}{r^* - \phi\rho_1} - (1 - \alpha) \right] \frac{\alpha - \lambda}{1 + \alpha} \quad (16)$$

Proof. The two different equilibria described above coexist when condition (12) and (15) hold altogether which is equivalent to condition (16). Moreover for this condition to hold, it is necessary that $\alpha \frac{\phi(\rho_1 - \rho_b)}{r^* - \phi\rho_1} > 1 - \alpha$ which simplifies as $\rho_1 > (1 - \alpha)\rho_g + \alpha\rho_b$. ■

The presence of multiple equilibria is related to the positive externality of aggregate liquidity buffers on the cost of raising funds at date 1, which determines entrepreneurs' individual decision to build liquidity buffers. Entrepreneurs prefer to build liquidity buffers if they expect a high funding cost. Yet, when investors' funding supply is sufficiently large, the cost of raising funds increases with entrepreneurs' liquidity buffer holdings. As a result, there can be two possible equilibrium outcomes: if entrepreneurs prefer to build liquidity buffers, the funding cost will be high and expecting this, entrepreneurs prefer to build liquidity buffers at date 0. And conversely, if entrepreneurs prefer to invest in illiquid projects, the funding cost will be low; expecting a low funding cost, entrepreneurs will indeed prefer to invest in illiquid projects. The indeterminacy property hence arises from investors' funding supply λ being sufficiently large.

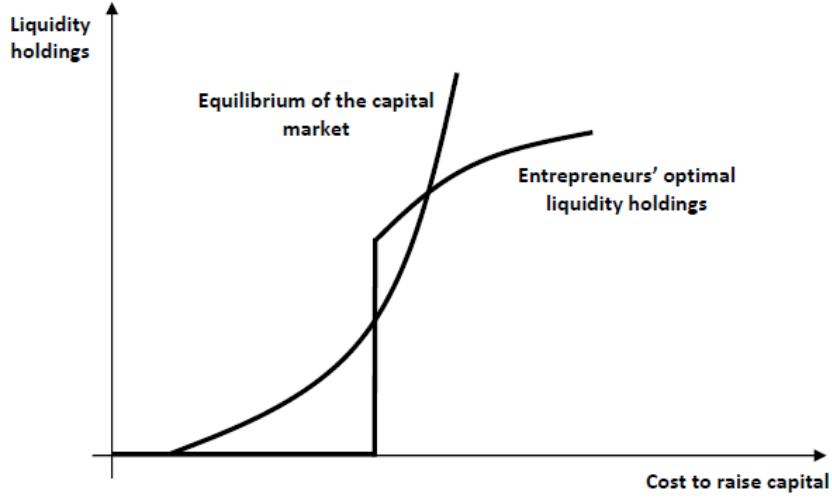


Fig. 5: Multiple equilibria.

Multiple equilibria also require that the condition $\alpha \frac{\phi(\rho_1 - \rho_b)}{r^* - \phi\rho_1} > 1 - \alpha$ holds. To understand the logic of this condition, assume the funding cost is r^* so that entrepreneurs are indifferent between holding liquidity and investing in illiquid projects. Then the equilibrium condition (11) shows that if entrepreneurs hold larger liquidity buffers, the demand for funding from entrepreneurs facing the liquidity shock increases by $\alpha \frac{\phi(\rho_1 - \rho_b)}{r^* - \phi\rho_1}$ while the funding supply from investors and entrepreneurs not facing the liquidity shock increases by $1 - \alpha$. If $\alpha \frac{\phi(\rho_1 - \rho_b)}{r^* - \phi\rho_1} > 1 - \alpha$, the demand increases more than the supply and the funding cost hence needs to go up in order to balance the market. Moreover a higher funding cost further helps to raise entrepreneurs' liquidity buffers. The funding cost therefore increases up to the point where an equilibrium is reached. In this equilibrium, entrepreneurs hold a large amount of liquidity and the cost of raising capital is high. A similar but opposite argument can be made to derive the other equilibrium where entrepreneurs invest in illiquid projects, build no liquidity buffer and the funding cost is low.

Next, we turn to determining which of the two equilibria described above dominates the other in the presence of multiplicity.

Proposition 6 *When there are multiple equilibria, the equilibrium in which entrepreneurs prefer to build liquidity buffers ensures higher welfare.*

Proof. When entrepreneurs prefer to build liquidity buffers, expected profits π_l can be written as

$$\pi_l = \left[(1 - \alpha) \rho_g + \alpha (\rho_1 + \rho_b) - (1 + \alpha) \lambda \frac{\phi (\rho_1 + \rho_b)}{1 + \lambda - (\alpha - \lambda)} \right] \frac{1 + \lambda}{1 + \alpha} \quad (17)$$

On the contrary, when entrepreneurs prefer to invest in illiquid projects, then expected profits π_i can be written as

$$\pi_i = (1 - \alpha) \rho_g + (1 - \phi) (\rho_b \alpha + \rho_1 \lambda) \quad (18)$$

The equilibrium in which entrepreneurs prefer to hold liquidity dominates if $\pi_l \geq \pi_i$. Given that $\lambda \leq \alpha$, the condition $\pi_l \geq \pi_i$ can be simplified as

$$\left[\rho_b - \frac{\rho_1 - \rho_b}{1 - \alpha} \lambda \right] (1 - \phi) \leq \frac{(1 + \lambda) (\rho_1 + \rho_b - \rho_g) + (\alpha - \lambda) \rho_g}{1 + \alpha} \quad (19)$$

Now in the equilibrium in which entrepreneurs prefer to build liquidity buffers, the funding cost is $r = \frac{1+L}{1+L-(\alpha-L)} \phi (\rho_1 + \rho_b)$ and the equilibrium holds if and only if $r \geq \frac{(1-\alpha)\rho_g}{(1-\alpha)+\alpha(1-\phi)\frac{\rho_1-\rho_b}{r-\phi\rho_1}}$. Given the expression for the funding cost, this last condition simplifies as

$$\left[\rho_b - \frac{\rho_1 - \rho_b}{1 - \alpha} \lambda \right] (1 - \phi) \leq \frac{(1 + \lambda) \rho_b + (\alpha - \lambda) \rho_1}{(1 + \lambda) (\rho_b + \rho_1)} \frac{(1 + \lambda) (\rho_1 + \rho_b - \rho_g) + (\alpha - \lambda) \rho_g}{1 + \alpha} \quad (20)$$

As is clear, given that the inequality

$$\frac{(1 + \lambda) \rho_b + (\alpha - \lambda) \rho_1}{(1 + \lambda) (\rho_b + \rho_1)} \leq 1 \quad (21)$$

always holds, (20) implies (19). In other words, if the equilibrium in which entrepreneurs prefer to build liquidity buffers exists, then it necessarily dominates the equilibrium where entrepreneurs prefer to invest in illiquid projects. ■

This result relates to the pecuniary externality of aggregate liquidity buffer on the cost of raising funds. In the equilibrium in which entrepreneurs prefer to build liquidity buffers, illiquid projects display high

productivity because large reinvestment compensates for the fall in productivity due to liquidity shocks. Moreover, the return on liquidity buffers is also large. Hence aggregate output and welfare are relatively high. By contrast, in the equilibrium in which entrepreneurs do not build liquidity buffers, illiquid projects hit with a liquidity shock are relatively unproductive because reinvestment is low. As a result, aggregate output and welfare are relatively low. The equilibrium in which entrepreneurs prefer not to build liquidity buffers hence provides lower welfare.

5 Introducing aggregate shocks

The model developed up to now includes only idiosyncratic shocks. Yet, liquidity crises are highly connected to the occurrence of aggregate shocks that can leave the economy short of pledgeable income. This section introduces aggregate shocks and uses this extended framework to show that the indeterminacy property developed above holds especially when aggregate shocks display low volatility.

We introduce aggregate shocks on the return, ρ_b , to distressed illiquid projects. A state of nature s at date 1 determines the return at date 2 of illiquid projects in the presence of a liquidity shock. Then using expressions (1) and (5) for entrepreneurs' profits, and assuming that the pledgeability constraint binds at date 1, entrepreneurs' date 0 expected profits conditional on state s at date 1 can be written as

$$\pi_s = (1 - \alpha) [(1 - L) \rho_g + Lr_s] + \alpha \frac{(1 - \phi) r_s}{r_s - \phi \rho_1} [(1 - L) \rho_s + L\rho_1] \quad (22)$$

where ρ_s denotes the return in state s to an illiquid project affected by a liquidity shock and r_s denotes to the cost of raising funds in state s . Denoting r as the average funding cost, $r = E_s r_s$, and ρ_b as the average return to distressed illiquid projects, $\rho_b = E_s \rho_s$, we can write expected profits' variations as

$$\frac{\partial E_s \pi_s}{\partial L} = \left[1 - \alpha + \alpha (1 - \phi) \frac{\rho_1 - \rho_b}{r - \phi \rho_1} \right] r - (1 - \alpha) \rho_g + \alpha (1 - \phi) (\rho_1 - \rho_b) E \left[\frac{r_s}{r_s - \phi \rho_1} - \frac{r}{r - \phi \rho_1} \right] \quad (23)$$

In the absence of aggregate shocks, the last part $\alpha (1 - \phi) (\rho_1 - \rho_b) E \left[\frac{r_s}{r_s - \phi \rho_1} - \frac{r}{r - \phi \rho_1} \right]$ of this expression

is zero and we are back to the case with idiosyncratic shocks only where a larger cost r to raise funding increases entrepreneurs' incentives to build liquidity buffers (cf. proposition 1).

Let us now turn to the market at date 1 where entrepreneurs facing the liquidity shock raise funding from investors and entrepreneurs not facing the liquidity shock. Assuming the cost of raising funds in state s satisfies $1 < r_s < \rho_1$, the equilibrium of the market at date 1 in state s writes as

$$\alpha \frac{\phi [(1 - L^*) \rho_s + L^* \rho_1]}{r_s - \phi \rho_1} = \lambda + (1 - \alpha) L^* \quad (24)$$

L^* denoting entrepreneurs' optimal liquidity buffer holdings. Using this equilibrium condition (24) to derive the funding cost r_s in state s , the expression (23) for $\frac{\partial E\pi_s}{\partial L}$ can be simplified as

$$\frac{\partial E\pi_s}{\partial L} = \left[1 - \alpha + \alpha (1 - \phi) \frac{\rho_1 - \rho_b}{r - \phi \rho_1} \right] r - (1 - \alpha) \rho_g + \varphi(L^*) \quad (25)$$

where

$$\varphi(L^*) = (1 - \phi) (\rho_1 - \rho_b) \rho_1 (1 - L^*) \frac{\lambda + (1 - \alpha) L^*}{(1 - L^*) \rho_b + L^* \rho_1} E \left[\frac{\rho_b - \rho_s}{(1 - L^*) \rho_s + L^* \rho_1} \right] \quad (26)$$

We can now detail the different possible equilibria using (24), (25) and (26).

First, there exists an equilibrium where entrepreneurs are better off investing in illiquid projects if and only if $\frac{\partial E\pi_s}{\partial L} \leq 0$ which simplifies as

$$\left[1 - \alpha + \alpha (1 - \phi) \frac{\rho_1 - \rho_b}{r - \phi \rho_1} \right] r \leq (1 - \alpha) \rho_g - \varphi(L^*=0) \quad (27)$$

Condition (27) defines a threshold funding cost r_0^* such that entrepreneurs prefer to invest in illiquid projects than to build liquidity buffers whenever the average funding cost r satisfies $r \leq r_0^*$. Note that the presence of aggregate shocks reduces entrepreneurs' incentives to invest in illiquid projects since $r_0^* \leq r^*$.

Second, there exists an equilibrium where entrepreneurs are better off building liquidity buffers if and

only if $\frac{\partial E\pi_s}{\partial L} \geq 0$. In this case entrepreneurs' liquidity buffer holdings are $L^* = \frac{\alpha - \lambda}{1 + \alpha}$.¹⁵ The equilibrium condition therefore writes as

$$\left[1 - \alpha + \alpha(1 - \phi) \frac{\rho_1 - \rho_b}{r - \phi\rho_1} \right] r > (1 - \alpha)\rho_g - \varphi_{(L^* = \frac{\alpha - \lambda}{1 + \alpha})} \quad (28)$$

Condition (27) defines a threshold funding cost r_1^* such that entrepreneurs prefer to build liquidity buffer than to invest in illiquid projects whenever the average funding cost satisfies $r \geq r_1^*$. The presence of aggregate shocks here raises entrepreneurs' incentives to build liquidity buffers since $r_1^* \leq r^*$. We can then derive the following proposition.

Proposition 7 *Entrepreneurs' choice between building liquidity buffers and investing in illiquid projects is indeterminate if and only if investors' funding supply λ satisfies*

$$\frac{\alpha\phi\rho_b}{r_0^* - \phi\rho_1} \leq \lambda \leq \frac{\alpha\phi\rho_b}{r_1^* - \phi\rho_1} + \left[\alpha\phi \frac{\rho_1 - \rho_b}{r_1^* - \phi\rho_1} - (1 - \alpha) \right] \frac{\alpha - \lambda}{1 + \alpha} \quad (29)$$

Proof. The situation where entrepreneurs do not build liquidity buffers is an equilibrium when the average cost of capital satisfies $r \leq r_0^*$. According to the equilibrium at date 1, the average funding cost is $r = \phi \left(\rho_1 + \alpha \frac{\rho_b}{L} \right)$ when entrepreneurs do not hold liquidity buffers, so that the equilibrium condition $r \leq r_0^*$ can be simplified as

$$\lambda \geq \frac{\alpha\phi\rho_b}{r_0^* - \phi\rho_1} \quad (30)$$

Similarly, the situation where entrepreneurs prefer to build liquidity buffers is an equilibrium when the average funding cost satisfies $r \geq r_1^*$. According to the equilibrium at date 1, the average funding cost is $r = \phi \left(\rho_1 + \alpha \frac{(1 - L^*)\rho_b + L^*\rho_1}{(1 - \alpha)L^* + \lambda} \right)$ with $L^* = \frac{\alpha - \lambda}{1 + \alpha}$. The equilibrium condition $r \leq r_1^*$ can hence be simplified as

$$\lambda \leq \frac{\alpha\phi\rho_b}{r_1^* - \phi\rho_1} + \left[\alpha\phi \frac{\rho_1 - \rho_b}{r_1^* - \phi\rho_1} - (1 - \alpha) \right] \frac{\alpha - \lambda}{1 + \alpha} \quad (31)$$

¹⁵ When entrepreneurs prefer to hold liquid assets, reinvestment from distressed entrepreneurs needs to equal initial investment $L + D = 1 - L$ while the equilibrium of the market for reinvestment states that $(1 - \alpha)L + \lambda = \alpha D$. Using these two expressions yields the amount of reserves $l = \frac{\alpha - \lambda}{1 + \alpha}$.

There are hence multiple equilibria if and only if the two conditions (30) and (31) are satisfied altogether. ■

Condition (29) shows that the introduction of aggregate shocks introduces a wedge between the funding cost r_0^* below which entrepreneurs prefer to invest in illiquid projects and the funding cost r_1^* above which entrepreneurs prefer to build liquidity buffers, the former being lower than the latter, i.e. $r_0^* \leq r_1^*$. When the funding cost falls in between, i.e. $r_0^* < r < r_1^*$, there is a continuum of mixed strategies equilibria where entrepreneurs are indifferent between building liquidity buffers and investing in illiquid projects. In the absence of aggregate shocks, this continuum of mixed strategies equilibria disappears since the funding costs r_0^* and r_1^* are both equal to the threshold r^* . The presence of aggregate uncertainty therefore reduces the scope for multiple equilibria since (29) may be impossible to satisfy if the threshold cost of capital r_0^* is sufficiently low compared to r_1^* .

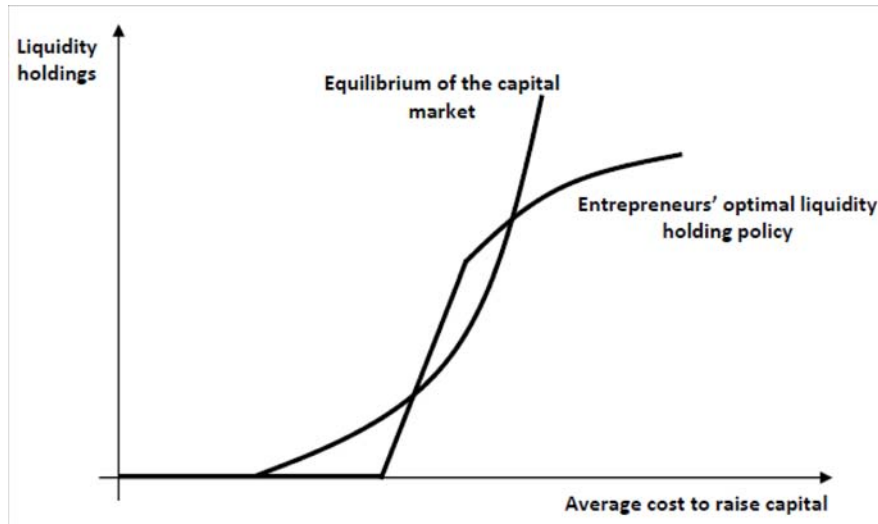


Fig. 6: Multiple equilibria in the presence of aggregate uncertainty.

The presence of aggregate shocks introduces a strategic substitutability which counteracts the strategic complementarity developed above. Profits from reinvestment are typically larger with aggregate shocks, so that entrepreneurs have an incentive to build larger liquidity buffers. However, the equilibrium funding cost is less volatile when entrepreneurs hold larger liquidity buffers. This reduces profits from reinvestment and thereby incentives to build liquidity buffers. Hence because of aggregate shocks, each individual entrepreneur has less of an incentive to build liquidity buffers when the aggregate amount of liquidity buffers in the economy is larger. This is the strategic substitutability introduced with aggregate shocks. A reduction in the volatility

of aggregate shocks will therefore reduce this strategic substitutability and raise the likelihood of multiple equilibria.

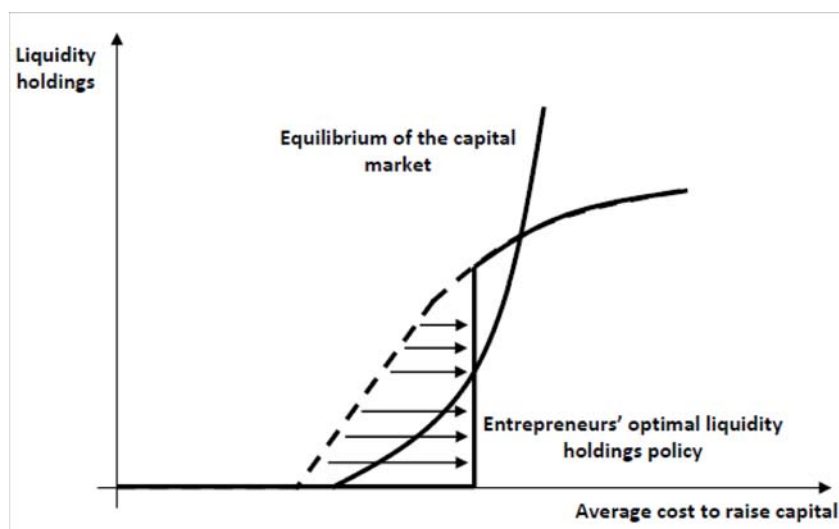


Fig. 7: The effect of a reduction in the volatility of aggregate shocks.

6 Policy options to restore efficiency

When the economy faces multiple equilibria, the equilibrium in which entrepreneurs do not hold liquidity buffers is dominated. As a result, if entrepreneurs coordinate on this equilibrium, there is room for policy to improve welfare. We investigate this question here in the context of the model with idiosyncratic shocks only.

The ineffectiveness of lender of last resort policy. Let us first examine the implications of a lender of last resort policy. Such a policy would typically consist in providing funding to entrepreneurs who need to reinvest in their illiquid projects but fail to do so because they lack pledgeable income. In this framework, this would consist in raising the exogenous supply of funding from λ to $\bar{\lambda}$, $\bar{\lambda}$ being the sum of investors' and the lender last of resort funding supply ($\bar{\lambda} > \lambda$).

Proposition 8 *There is no lender of last resort policy that can restore efficiency.*

Proof. Suppose entrepreneurs coordinate on the equilibrium where they do not build liquidity buffers. Investors' funding supply therefore satisfies $\lambda \geq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1}$. As noted above, a lender of last resort policy would result in raising the exogenous funding supply from λ to $\bar{\lambda}$ ($\bar{\lambda} > \lambda$). To preclude the equilibrium in which entrepreneurs do not build liquidity buffers, the lender of last resort policy would need to satisfy $\bar{\lambda} < \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1}$ which is not possible given that $\bar{\lambda} \geq \lambda \geq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1}$. There is hence no lender of last resort policy which can preclude the equilibrium where entrepreneurs do not build liquidity buffers. ■

In our framework, the inefficiency of the decentralized equilibrium relates to the exogenous funding supply λ being too large. A lender of last resort policy, by further increasing this exogenous funding supply will simply aggravate the problem.

Alternative to lender of last resort policy. Because the inefficiency in the decentralized equilibrium relates to the low funding cost or to the large funding supply, we investigate in this section a policy which commits to a high funding cost. We start by assuming that entrepreneurs coordinate on the equilibrium where they do not build liquidity buffers. Investors' funding supply λ and entrepreneurs' welfare π_0 hence satisfy

$$\lambda \geq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} \text{ and } \pi_0(\lambda) = (1 - \alpha)\rho_g + (1 - \phi)[\rho_1\lambda + \alpha\rho_b] \quad (32)$$

Now in order to raise the funding cost at date 1, the central bank commits at date 0 to issue an amount λ_{cb} of bonds on the market at date 1. The central bank bond issuance will raise the total demand for funding, which will drive up the funding cost. To the extent that the central bank's announcement is credible and anticipated, entrepreneurs should build larger liquidity buffers, given the higher funding cost at date 1.¹⁶ Denoting r_{cb} the funding cost when the central bank issues an amount λ_{cb} of bonds, we assume that it costs the central bank $cr_{cb}\lambda_{cb}$ to issue an amount λ_{cb} of claims, c being a positive constant which scales the cost of central bank intervention ($c > 1$). Net social welfare of the central bank intervention is

$$\pi_{cb}(\lambda, \lambda_{cb}) = (1 - \alpha)[(1 - L)\rho_g + Lr_{cb}] + \alpha \frac{(1 - \phi)r_{cb}}{r_{cb} - \phi\rho_1} [(1 - L)\rho_b + L\rho_1] - cr_{cb}\lambda_{cb} \quad (33)$$

¹⁶We will indeed assume for simplicity that the central bank is perfectly credible and does not suffer any commitment problem.

with

$$r_{cb} = \phi(\rho_1 + \rho_b) \frac{1-L}{1-2L} \text{ and } L = \frac{\alpha - (\lambda - \lambda_{cb})}{1 + \alpha} \quad (34)$$

To be welfare improving, the central bank issuance λ_{cb} should satisfy

$$\lambda - \lambda_{cb} \leq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} \text{ and } \pi_{cb}(\lambda, \lambda_{cb}) \geq \pi_0(\lambda) \quad (35)$$

The first condition makes sure that entrepreneurs coordinate on the equilibrium where they prefer to build liquidity buffers while the second condition ensures that central bank intervention actually improves on social welfare.

Proposition 9 *When entrepreneurs coordinate on the equilibrium where they do not build liquidity buffers, the central bank cannot improve social welfare by committing to raise funds at date 1 if investors' funding supply λ satisfies*

$$\lambda \geq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} + \frac{\alpha}{1 + \alpha} \frac{r^* - \phi(\rho_1 + \rho_b)}{r^* - \phi\rho_1} \frac{\mu + (1 - \alpha)r^*}{(1 - \phi)\rho_1 + cr^*} \quad (36)$$

Proof. When entrepreneurs do not build liquidity buffer, the central bank can raise social welfare by raising an amount λ_{cb} of funding at date 1 if and only if λ_{cb} satisfies $\pi_{cb}(\lambda, \lambda_{cb}) \geq \pi_0(\lambda)$. Using above expressions, this inequality can be simplified to be

$$\frac{\lambda_{cb}}{\alpha - \lambda + \lambda_{cb}} \leq \frac{1}{1 + \alpha} \frac{(1 - \alpha)r_{cb} + \mu}{cr_{cb} + (1 - \phi)\rho_1} \quad (37)$$

with $\mu = (1 - \phi)(\rho_1 - \alpha\rho_b) - (1 - \alpha)\rho_g$. Moreover, entrepreneurs are better off building liquidity buffers if and only if the central bank demand for funding λ_{cb} satisfies

$$\lambda - \lambda_{cb} \leq \frac{\alpha\phi\rho_b}{r^* - \phi\rho_1} \quad (38)$$

This means in particular that, condition (37) must be met when (38) holds with equality. Otherwise, there is no central bank intervention which can induce entrepreneurs to switch from the no liquidity buffer equilibrium

to the equilibrium with strictly positive liquidity buffers. This simplifies as (36). ■

The central bank cannot improve on social welfare when investors' funding supply λ is too large. On one side, the amount of funding λ_{cb} the central bank needs to raise to get entrepreneurs to build liquidity buffers actually increases with investors' funding supply λ . A large funding supply from investors λ hence raises the cost of central bank intervention. On the other side, the benefits of central bank intervention decrease with investor's funding supply λ . There is hence an upper limit on investor's funding supply λ such that above this limit, the central bank intervention is simply not worth being carried out.

7 Conclusion

The model derived in this paper provides a framework for analyzing how the decision to build liquidity buffer as a insurance device against liquidity shocks is affected by the ability to raise funding after liquidity shocks are realized. In particular, the model illustrates that a positive externality from aggregate liquidity buffers on individual decisions to build liquidity buffers can emerge. As a result, the economy can coordinate on an inefficient equilibrium in which (i) agents do not build liquidity buffers, and (ii) are unable to cope with liquidity shocks. This externality is more likely to hold when there is large exogenous funding supply and/or aggregate shocks display low volatility.

Last, the paper investigates how policy can prevent the inefficient outcome. Lender of last resort policies are clearly not the right tool given that they help to reduce funding costs, while low funding costs are precisely at the root of the inefficiency. However, a policy that aims at raising funding costs can also fall short of its objective because it yields net benefits only if the exogenous supply of funding is not too large while the inefficient outcome occurs when the supply of ex post funding is sufficiently large. The model hence stresses the difficulty for policy to improve social welfare since the conditions for an inefficient outcome are precisely those under which policy becomes powerless.

8 Appendix

The first best allocation. To derive the first best allocation, we remove the two information frictions: entrepreneurs' decision at date 0 to build liquidity buffer as well as liquidity shocks affecting illiquid projects at date 1 are now contractible. In such a framework, there can be an insurance contract at date 0 whereby an entrepreneur who holds L liquidity buffers is paid $\frac{1}{\alpha}L$ at date 1 if distressed and nothing if intact. Given that an entrepreneur is distressed with probability α , this insurance yields zero profits. The first best allocation therefore solves

$$\begin{aligned} \max_L \left\{ \max_D \pi_{fb} = (1 - \alpha) (1 - L) \rho_g + \alpha [(1 - L) [\rho_b + (\frac{1}{\alpha}L + D) \rho_1] - rD] \right\} \\ \text{s.t.} \quad \begin{cases} \frac{1}{\alpha}L + D \leq 1 - L \\ rD \leq \phi [\rho_b (1 - L) + (\frac{1}{\alpha}L + D) \rho_1] \end{cases} \end{aligned} \quad (39)$$

When the return r satisfies $1 \leq r \leq \rho_1$, the pledgeability constraint for entrepreneurs facing a liquidity shock, which can be written as

$$D \leq \frac{\phi}{r - \phi \rho_1} \left[\rho_b (1 - L) + \frac{\rho_1}{\alpha} L \right]$$

is binding. The problem in the first best framework then simplifies as

$$\begin{aligned} \max_L \pi_{fb} = (1 - \alpha) (1 - L) \rho_b + \frac{(1 - \phi)r}{r - \phi \rho_1} [(1 - L) \rho_b \alpha + \rho_1 L] \\ \text{s.t.} \quad 0 \leq L \leq \frac{\alpha}{\alpha + \frac{r}{r - \phi(\rho_1 + \rho_b)}} \end{aligned} \quad (40)$$

We then have

$$\frac{\partial \pi_{fb}}{\partial L} = \frac{(1 - \phi)r}{r - \phi \rho_1} [\rho_1 - \alpha \rho_b] - (1 - \alpha) \rho_g \quad (41)$$

Entrepreneurs then prefer to build liquidity buffers when $\frac{\partial \pi_{fb}}{\partial L} \geq 0$. Given that the return r must satisfy $r \leq \rho_1$, entrepreneurs are always better off building liquidity buffers since by assumption $\rho_1 \geq (1 - \alpha) \rho_g + \alpha \rho_b$.

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