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**IS THERE EXCESS COMOVEMENT OF BOND YIELDS  
BETWEEN COUNTRIES?**

by  
**Gregory D. Sutton**

**July 1997**

**BANK FOR INTERNATIONAL SETTLEMENTS  
Monetary and Economic Department  
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# IS THERE EXCESS COMOVEMENT OF BOND YIELDS BETWEEN COUNTRIES? \*

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## **Abstract**

This paper examines the issues of excess volatility and excess comovement of interest rates among global bond markets. The base model of interest rate behaviour is the expectations theory of the term structure. The empirical evidence presented in the paper indicates that ten-year government bond yields in five major markets – the United States, Japan, Germany, the United Kingdom and Canada – have in the past displayed both excess volatility and excess comovement relative to the base model. This suggests that term premia at the long end of the term structure are both time-varying and positively correlated across markets.

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## Introduction

Many empirical studies have shown that the term structure of nominal interest rates contains information potentially useful for the conduct of monetary policy. For example, the studies by Mishkin (1990) and Jorion and Mishkin (1991) show that, in a number of countries, the spread between long and short-term nominal interest rates is a useful predictor of domestic price inflation. Also embedded in a country's term structure of interest rates is information concerning future economic activity. The studies by Estrella and Mishkin (1996) and Bernard and Gerlach (1996) show that, for many countries, the long/short spread is a useful indicator of the likelihood of a future recession.

Long/short interest rate spreads help to predict inflation and economic activity because they reflect in part expectations of future short rates. However, much empirical work on the term structure of interest rates indicates that, while expectations of future short rates are important determinants of long/short spreads, other factors also play a role. This suggests that the use of interest rate spreads as indicators for the conduct of monetary policy may be complicated by the need to distinguish between changes in spreads that are due to shifting views about future short rates and changes attributable to other factors.

For instance, consider the case of the widening of spreads in Japan and Germany that coincided with the global back-up in yields which began in early 1994 and continued throughout the year (see Graph 1). At that time considerable differences existed between the cyclical position of the United States, where monetary policy was tightened in February 1994, and those of Japan and Germany. However, long-term interest rates in the G-3 countries displayed a striking tendency to move together before, during and after 1994. The relatively high degree of comovement of long rates between these countries, coupled with the global nature of the 1994 bond market reversal, has led to speculation that international factors may at times override domestic considerations in the determination of long rates.

One question that naturally arises is whether there has been a tendency in the past for long-term interest rates to covary excessively between domestic markets. This paper addresses this question by examining the historical behaviour of bond yields in five major markets - the United States, Japan, Germany, the United Kingdom and Canada. The base model of interest rate behaviour is the expectations theory of the term structure. Most empirical investigations of interest rate behaviour which have taken the expectations theory of the term structure as the base model have focused on the US market and have come to the conclusion that long-term bond yields deviate from the predictions of the model.<sup>1</sup> A much smaller, but rapidly growing, literature tests the expectations

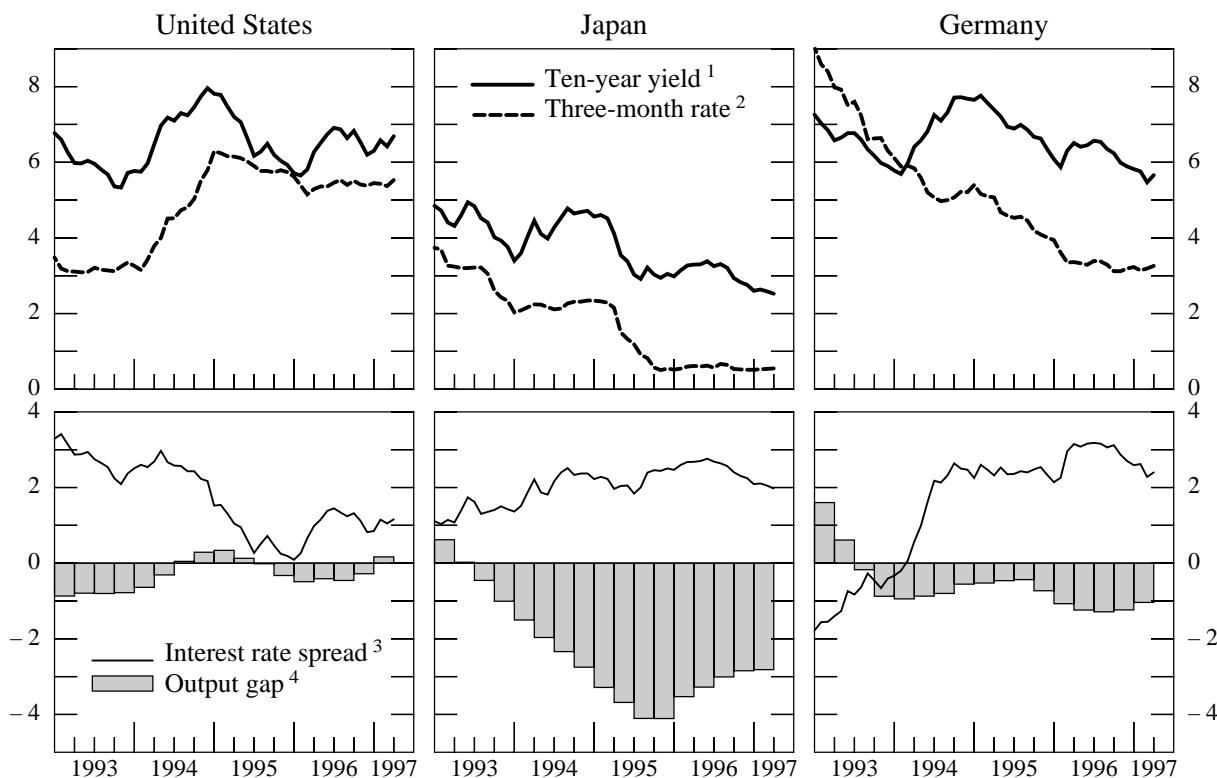
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<sup>1</sup> See, for example, the studies by Shiller (1979), Shiller et al. (1983) and Campbell and Shiller (1984).

theory with interest rate data for other countries. A recent example is the study by Hardouvelis (1994), which examines the behaviour of ten-year government bond yields for G-7 countries. Hardouvelis concludes that bond yields in the majority of G-7 countries deviate from the predictions of the expectations theory.<sup>2</sup> The main contribution of the present paper is an investigation of the comovements of bond yields between countries with the aim of determining whether the deviations of bond yields from the levels suggested by the expectations theory are correlated across markets.

Graph 1

**Interest rates and the output gap: United States, Japan and Germany**



<sup>1</sup> For the United States, Treasury notes and bonds; for Japan, government and for Germany, federal public bonds. <sup>2</sup> For the United States and Japan, CDs, and for Germany, FIBOR. <sup>3</sup> Long minus short rates. <sup>4</sup> As a percentage of potential GDP.

The empirical results can be briefly summarised as follows. First, the restrictions that the expectations theory of the term structure imposes on the behaviour of yields in groups of countries allow the rejection of the model at high levels of statistical significance in the case of every country examined. Thus, this paper provides additional international evidence against the expectations theory as a model of the behaviour of long-term government bond yields. Secondly, the empirical evidence suggests that term premia at the long end of the term structure are positively correlated across markets. This is consistent with the existence of an international component to global bond yield fluctuations beyond that attributable to common movements in short-term interest rates and inflation.

<sup>2</sup> It is important to note that the empirical evidence also indicates that there is an important element of truth to the expectations theory. This was observed in the context of the US market by Campbell and Shiller (1987). Hardouvelis (1994) reaches a similar conclusion for other G-7 countries.

The rest of the paper is organised as follows. Section 1 develops a simple empirical framework which nests the expectations theory of the term structure, as it applies to interest rate behaviour within a group of countries, in a more general model. The particular alternative hypothesis entertained regarding the joint behaviour of bond yields is Shiller's (1989) notion of excess comovement of asset prices and is related to the concept of excess volatility of an asset price. Loosely speaking, an asset price displays excess volatility, relative to a specific asset pricing model, if there are deviations of the asset price from the theoretical values predicted by the model. Excess comovement of two asset prices is the case where two asset prices display excess volatility and the deviations of prices from predicted values are positively correlated across assets.

Section 2 examines the historical behaviour of ten-year government bond yields in the United States, Japan, Germany, the United Kingdom and Canada within the context of the empirical framework presented in Section 1. Using this framework, measures of excess volatility and excess comovement of bond yields are estimated and their statistical significance judged on the basis of the small sample distributions of the relevant test statistics computed from Monte Carlo simulations. The results of the Monte Carlo simulations indicate that over the period examined the comovements of bond yields between all countries studied were excessive, given the inter-country correlations of changes in short-term interest rates.

## **1. The expectations theory and comovements of bond yields**

The expectations theory of the term structure is among the most popular models of interest rate behaviour. The theory starts from the observation that the investment strategy of rolling over a sequence of short-term bonds is an alternative to holding a long-term bond. According to the theory, the expected rates of return on these alternative investment strategies differ by a constant term premium. This implies that long-term bond yields are related to expectations of current and future short-term interest rates, and that comovements of bond yields between countries are related to comovements of expectations of future short rates between countries.

Of course, the comovements of interest rates between countries can be in agreement with the expectations theory even if the model fails to explain the behaviour of interest rates in each domestic market. This may occur, for example, if the deviations of interest rates from the predictions of the theory are uncorrelated across countries. On the other hand, if there is an international component to departures of interest rates from the expectations theory, then the comovements of interest rates between countries may violate the predictions of the model.

This section presents an empirical framework that nests the expectations theory of the term structure, as it applies to interest rate behaviour within a group of countries, in a more general model. In this context, restrictions that the expectations theory imposes on the comovements of bond yields between countries are derived. The particular alternative hypothesis entertained concerning the

joint behaviour of bond yields is Shiller's (1989) notion of excess comovement of asset prices, which is closely related to the concept of excess volatility of an asset price. In particular, excess comovement of bond yields is the special case where the deviations of yields from levels predicted by the expectations theory are positively correlated across countries.

In order to develop these ideas more formally, let  $R_{it}$  denote the yield to maturity on an  $n$ -period bond in country  $i$  at time  $t$  and let  $r_{it}$  denote the one-period rate of interest. In what follows it is convenient to write the term premium as

$$(1.1) \quad \varphi_{it} \equiv R_{it} - \sum_{j=0}^{n-1} w_{ij} E_t r_{i,t+j},$$

where  $E_t$  is the expectations operator given all publicly available information at time  $t$ .<sup>3</sup> The  $\{w_{ij}\}$  are weights which depend upon the duration of the  $n$ -period bond.<sup>4</sup> For a zero coupon bond,  $w_{ij} = 1/n$  for all  $j$  and the term premium  $\varphi_{it}$  is the difference between the  $n$ -period bond yield and an arithmetic average of expected short rates. For coupon bonds, which will be employed in the empirical analysis which follows, the weights decline monotonically and sum to one. In this case, equation (1.1) places more weight on expected short rates in the near term.

As this paper is concerned with the joint behaviour of bond yields in a group of countries, it is necessary to make assumptions concerning the joint evolution of term premia. In what follows it is assumed that term premia in the group of countries under investigation evolve according to a stationary (vector) stochastic process. Let

$$\theta_i \equiv E\varphi_{it},$$

where  $E$  is the unconditional expectations operator. With this notation, equation (1.1) can be expressed as

$$(1.2) \quad R_{it} = \theta_i + \sum_{j=0}^{n-1} w_{ij} E_t r_{i,t+j} + \varepsilon_{it},$$

where  $\varepsilon_{it} \equiv \varphi_{it} - E\varphi_{it}$  is a mean-zero random variable which represents the time  $t$  deviation of the term premium in country  $i$  from its average value.

Equation (1.2) nests the expectations theory of the term structure in a more general framework. Under the expectations theory term premia are constant over time, which is equivalent to  $\varepsilon_{it} \equiv 0$ . Let

$$R_{it}^e \equiv \theta_i + \sum_{j=0}^{n-1} w_{ij} E_t r_{i,t+j}$$

---

<sup>3</sup> In this section it is assumed that the required expectations exist. Issues related to non-stationarity of the interest rate series will be addressed in the next section.

<sup>4</sup> The weights  $\{w_{ij}\}$ , which will be discussed in more detail below, correspond to a linear representation of the expectations theory of the term structure. This linear representation is discussed in greater depth in Shiller et al. (1983).

denote the theoretical  $n$ -period bond yield implied by the expectations theory of the term structure. Note that  $R_{it}^e$  is known at time  $t$  and that

$$R_{it} = R_{it}^e + \varepsilon_{it}.$$

A rejection of the condition that  $\varepsilon_{it} = 0$  for all  $t$  is a rejection of the expectations theory of the term structure as a model of  $n$ -period bond yields in country  $i$ .

As mentioned above, empirical evidence indicates that long-term bond yields in many countries deviate from the predictions of the expectations theory. This suggests that non-zero  $\varepsilon_{it}$ s may be an important source of bond yield volatility. In this case, the degrees of comovement of bond yields between countries are determined in part by the inter-country correlations of the  $\varepsilon_{it}$ s.

It is useful in what follows to let

$$(1.3) \quad R_{it}^* \equiv \theta_i + \sum_{j=0}^{n-1} w_{ij} r_{i,t+j}.$$

$R_{it}^*$  will be referred to as a *perfect foresight* bond yield. With this notation, the expectations theory of the term structure can be expressed as

$$(1.4) \quad R_{it}^e = E_t R_{it}^*.$$

Let  $R_t^e \equiv (R_{1t}^e, R_{2t}^e, \dots, R_{kt}^e)'$  denote the  $k \times 1$  vector of time  $t$  bond yields implied by the expectations theory of the term structure. Then

$$(1.5) \quad R_t^e = E_t R_t^*,$$

where  $R_t^* \equiv (R_{1t}^*, R_{2t}^*, \dots, R_{kt}^*)'$ . Let

$$(1.6) \quad U_t \equiv R_t^* - R_t^e$$

denote the  $k \times 1$  vector of discrepancies between perfect foresight bond yields and  $R_t^e$ .<sup>5</sup> The expectations theory of the term structure imposes restrictions on the random vector  $U_t$ . For instance, it follows from equation (1.5) that  $U_t$  is a mean-zero random vector that is unforecastable given information publicly available at time  $t$ . In particular, as  $R_t^e$  is known at time  $t$  the expectations theory requires that the unconditional covariance,  $Cov(U_t, R_t^e)$ , equals zero.<sup>6</sup>

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<sup>5</sup> Note that  $U_{it}$  positive (negative) corresponds to  $R_{it}^e$  less (greater) than the perfect foresight long rate  $R_{it}^*$ . Of course, the realisation of the random vector  $U_t$  is not known at time  $t$ .

<sup>6</sup>  $Cov(A, B) \equiv E\{(A - EA)(B - EB)'\}$ .

Let  $\varepsilon_t \equiv (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{kt})'$ . It follows from previous assumptions that  $\varepsilon_t$  evolves according to a stationary vector stochastic process. It will prove convenient to make the additional assumption that the stochastic process  $\{\varepsilon_t\}$  is statistically independent of the vector stochastic process  $\{(r_{1t}, r_{2t}, \dots, r_{kt})\}$ .<sup>7</sup> With this assumption, it is straightforward to relate bond yield volatilities and comovements to the elements of the covariance matrix  $\Omega \equiv E(\varepsilon_t \varepsilon_t')$ , because in this case  $\varepsilon_t$  is uncorrelated with both  $R_t^*$  and  $R_t^e$ , which implies that  $\varepsilon_t$  and  $U_t$  are also uncorrelated.

In order to derive the implications of the structure of  $\Omega$  for bond yield volatilities and comovements, note that from (1.6) and the relation  $R_{it} = R_{it}^e + \varepsilon_{it}$  it follows that

$$(1.7) \quad R_t^* = R_t + U_t^o,$$

where  $U_t^o \equiv U_t - \varepsilon_t$ . Applying the unconditional variance operator to both sides of the relation (1.7) gives

$$(1.8) \quad \text{Var}(R_t^*) = \text{Var}(R_t) + \text{Var}(U_t^o) + \text{Cov}(R_t, U_t^o) + \text{Cov}(U_t^o, R_t),$$

where  $\text{Var}(\cdot)$  denotes unconditional variance. Restrictions that the expectations theory imposes on the joint behaviour of bond yields in the  $k$  countries under study are easily derived from the relation (1.8).

By substituting for  $U_t^o$  and  $R_t$ , it follows that

$$\text{Cov}(U_t^o, R_t) = \text{Cov}(U_t - \varepsilon_t, R_t^e + \varepsilon_t) = -\text{Var}(\varepsilon_t),$$

where the last equality follows from the fact that  $\text{Cov}(U_t, R_t^e) = \text{Cov}(\varepsilon_t, R_t^e) = \text{Cov}(U_t, \varepsilon_t) = 0$ . Thus, equation (1.8) is equivalent to

$$(1.8') \quad \text{Var}(R_t^*) = \text{Var}(R_t) + \text{Var}(U_t^o) - 2 \cdot \text{Var}(\varepsilon_t).$$

If  $n$ -period bond yields in the  $k$  countries under study are exactly determined by the expectations theory, then  $\varepsilon_t \equiv 0$  and equation (1.8') becomes

$$(1.9) \quad \text{Var}(R_t^*) = \text{Var}(R_t) + \text{Var}(U_t^o).$$

The diagonal elements of the matrix relation (1.9) take the form

$$(1.10) \quad \text{Var}(R_{it}^*) = \text{Var}(R_{it}) + \text{Var}(U_{it}^o),$$

a relation that pertains to bond yield volatility in a single country. A violation of equation (1.10) of the form

$$(1.11) \quad \text{Var}(R_{it}^*) < \text{Var}(R_{it}) + \text{Var}(U_{it}^o)$$

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<sup>7</sup> Hardouvelis (1994) concludes that the deviations of long rates from levels predicted by the expectations theory can, for the majority of G-7 countries, be attributed to discrepancies of the type modeled by  $\{\varepsilon_t\}$ .

will be referred to as *excess volatility* of bond yields in country  $i$ . From equation (1.8') it easily follows that the inequality (1.11) is equivalent to  $Var(\varepsilon_{it}) > 0$ , which, in turn, is equivalent to a negative correlation between  $U_{it}^o$  and  $R_{it}$ . In this case, when  $R_{it}$  is high, it is typically too high, relative to the fundamental value  $R_{it}^*$ .

The definition of excess volatility of bond yields given by the inequality (1.11) is different from that which has been employed in previous studies.<sup>8</sup> Because  $Var(U_{it}^o)$  is necessarily non-negative, it follows from equation (1.10) that the expectations theory of the term structure implies that  $Var(R_{it}^*) \geq Var(R_{it})$ . Thus, the condition  $Var(R_{it}^*) < Var(R_{it})$  is a violation of the model which may also be considered as a case of excess volatility of bond yields. Clearly, this condition implies that the inequality (1.11) is satisfied, but the converse is not true.

The expectations theory of the term structure also places restrictions on the comovements of bond yields between countries. The off-diagonal ( $i \neq j$ ) elements of the expression (1.8) are of the form

$$Cov(R_{it}^*, R_{jt}^*) = Cov(R_{it}, R_{jt}) + Cov(U_{it}^o, U_{jt}^o) + Cov(R_{it}, U_{jt}^o) + Cov(U_{it}^o, R_{jt}).$$

If the expectations theory holds, then  $R_t = R_t^e$ ,  $U_t^o = U_t$  and  $Cov(R_{it}, U_{jt}^o) = Cov(U_{it}^o, R_{jt}) = 0$ ; therefore,

$$(1.12) \quad Cov(R_{it}^*, R_{jt}^*) = Cov(R_{it}, R_{jt}) + Cov(U_{it}^o, U_{jt}^o).$$

Expression (1.12) pertains to the comovements of bond yields between countries. A violation of restriction (1.12) of the form

$$(1.13) \quad Cov(R_{it}^*, R_{jt}^*) < Cov(R_{it}, R_{jt}) + Cov(U_{it}^o, U_{jt}^o)$$

will be referred to as a case of *excess comovement* of bond yields between countries  $i$  and  $j$ . Clearly, the condition (1.13) is equivalent to

$$(1.14) \quad Cov(R_{it}, U_{jt}^o) + Cov(U_{it}^o, R_{jt}) < 0.$$

This represents a violation of the expectations theory of the term structure, because according to the theory both covariances in the inequality (1.14) are zero.

The same arguments used in the derivation of equation (1.8') give

$$(1.15) \quad Cov(R_{it}, U_{jt}^o) = Cov(U_{it}^o, R_{jt}) = -Cov(\varepsilon_{it}, \varepsilon_{jt}).$$

From the relation (1.15) it follows that excess comovement of bond yields between countries  $i$  and  $j$  is equivalent to  $Cov(\varepsilon_{it}, \varepsilon_{jt}) > 0$ . Therefore, a necessary condition for excess comovement of bond

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<sup>8</sup> See Shiller (1981) for a discussion of volatility measures and tests of present value models of asset price behaviour.

yields between two countries is excess volatility of bond yields in each country. It also follows from the relation (1.15) that  $Cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$  is a sufficient condition for equation (1.12) to be satisfied. Thus, the degree of comovement of bond yields between two countries can be consistent with the expectations theory of the term structure even if bond yields in both countries display excess volatility, provided that the discrepancies between yields and levels predicted by the expectations theory are uncorrelated across countries.

Relation (1.15) motivates a regression-based test for excess comovement of bond yields between countries  $i$  and  $j$ . The test is based on the system of regression equations

$$(1.16a) \quad U_{it}^o = \alpha_{ij} + \beta_{ij}R_{jt} + \zeta_t$$

$$(1.16b) \quad U_{jt}^o = \alpha_{ji} + \beta_{ji}R_{it} + \xi_t,$$

where the  $\alpha$ s and  $\beta$ s are parameters to be estimated by ordinary least squares (OLS) and  $\zeta$  and  $\xi$  are error terms. According to the expectations theory, both  $U_{it}^o$  and  $U_{jt}^o$  are unforecastable given information available at time  $t$ . Therefore, the expectations theory implies that  $\beta_{ij} = \beta_{ji} = 0$ . In contrast, from the relation (1.15) it follows that the case of excess comovement of bond yields between countries  $i$  and  $j$  is equivalent to the condition  $\beta_{ij} < 0$  **and**  $\beta_{ji} < 0$ . The case of excess volatility of bond yields in country  $i$  is subsumed by the system of equations (1.16) and is equivalent to  $\beta_{ii} < 0$ .

## 2. Historical comovements of bond yields

This section examines the joint behaviour of interest rates in two groups of countries within the context of the empirical model developed in Section 1. The first group, for which the interest rate series are available for the longest time span, consists of the United States, Canada and the United Kingdom. The second group is made up of the United Kingdom, Germany and Japan.

### 2.1 Data and data transformations

For each country, quarterly time series of three-month and ten-year yields were obtained.<sup>9</sup> The starting dates (year and quarter) of these series are reported in Table 1. Both interest rate series are available for all of the countries in the first group (the United States, Canada and the United Kingdom) from 1961Q1 and for Germany and Japan only from 1967Q1. For both groups of countries only data up to 1992Q2 are examined, so that the time of the 1992 European currency crisis is excluded from the analysis.

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<sup>9</sup> With the exception of the US data, these are the same interest rate series recently studied by Hardouvelis (1994).

Table 1

	<i>Starting dates</i>	
	Three-month yield	Ten-year bond yield
United States	1947Q1	1953Q1
Canada	1934Q1	1936Q1
United Kingdom	1961Q1	1961Q1
Germany	1963Q2	1967Q1
Japan	1961Q1	1961Q4

An essential component of the model developed in the preceding section is the variable  $R_t^*$ , the vector of perfect foresight bond yields. In order to construct  $R_t^*$ , the weights  $\{w_{ij}\}$  in equation (1.3) must be specified. Following Shiller et al. (1983), we set

$$w_{ij} = g_i^j(1 - g_i)/(1 - g_i^n),$$

where  $g_i \equiv 1/(1 + \bar{R}_i)$  and  $\bar{R}_i$  is the average ten-year bond yield over the sample period. Sample averages of ten-year bond yields are reported in Table 2, together with the average spread between the ten-year bond yield and the three-month rate over the sample period.

Table 2

	Sample period	$\bar{R}$ (in %)	Average spread (in %)
United States	1961Q1-1992Q2	7.74	1.06
Canada	1961Q1-1992Q2	8.87	0.63
United Kingdom	1961Q1-1992Q2	10.04	0.71
Germany	1967Q1-1992Q2	7.84	1.52
Japan	1967Q1-1992Q2	7.64	0.51

The construction of perfect foresight bond yields also requires a numerical value for the term premium  $\theta_i$  in equation (1.3). In what follows, it is assumed that for each country the term premium equals the average spread reported in Table 2. Because  $R_t^*$  is a function of the three-month rate of interest up to quarter  $t+39$ , perfect foresight bond yields are constructed over the period 1961Q1-1982Q3 for the United States, Canada and the United Kingdom. For Germany and Japan, perfect foresight bond yields are constructed over the period 1967Q1-1982Q3.

The tests for excess volatility and excess comovement of bond yields presented in Section 1 rely on an examination of the unconditional moments of the vector time series under review. The existence of unconditional moments requires the vector time series to be stationary. If short rates

are non-stationary in levels, then the expectations theory of the term structure implies that long rates will also be non-stationary in levels. In this case, it is necessary to transform the processes  $\{R_{it}:i=1,\dots,k\}$  and  $\{R_{it}^*:i=1,\dots,k\}$  before interpreting the estimates of the system (1.16) within the context of the empirical model presented in Section 1.

In what follows, potentially non-stationary variables are deflated by a moving average of lagged long rates. In particular, let  $\tilde{R}_{it}$  be the bond yield in country  $i$  measured relative to its previous five-year moving average. Let  $\tilde{R}_{it}^*$  be the perfect foresight bond yield measured relative to the previous five-year moving average of bond yields in country  $i$ .<sup>10</sup> For the United States, Canada and the United Kingdom these series are constructed over the period 1966Q1-1982Q3, while for Germany and Japan they are constructed over the period 1972Q1-1982Q3. Tests for excess volatility and excess comovement are applied to the transformed time series  $\{\tilde{R}_t^*\}$ ,  $\{\tilde{R}_t\}$  and  $\{\tilde{U}_t^o\}$ , where  $\tilde{U}_t^o \equiv \tilde{R}_t^* - \tilde{R}_t$  is the vector of transformed excess returns.

## 2.2 Comovements of US, Canadian and UK bond yields

This subsection examines the historical behaviour of ten-year government bond yields in the United States, Canada and the United Kingdom. First, estimates of the system (1.16), extended to the case of transformed bond yields, are reported which suggest the presence of excess comovement of bond yields between all three countries. Secondly, the results of a simple Monte Carlo study are reported which indicate that the evidence in favour of excess comovement of bond yields is statistically significant.

The extension of the system (1.16) to the case of transformed bond yields is:

$$(2.1a) \quad \tilde{U}_{it}^o = \alpha_{ij} + \beta_{ij}\tilde{R}_{jt} + \zeta_t$$

$$(2.1b) \quad \tilde{U}_{jt}^o = \alpha_{ji} + \beta_{ji}\tilde{R}_{it} + \xi_t,$$

where, as in (1.16), the  $\alpha$ s and  $\beta$ s are parameters to be estimated by ordinary least squares (OLS) and  $\zeta$  and  $\xi$  are error terms. Scatter plots of transformed excess returns against own country transformed bond yields are shown in Graph 2 for the first group of countries, and OLS estimates of the  $\beta$ s in the system (2.1) are reported in Table 3.<sup>11</sup> As shown in the table, all of the estimated  $\beta$ s are negative. This is consistent with excess volatility of bond yields in all three domestic markets and with excess comovement of bond yields between all three pairs of countries.

<sup>10</sup> This method of detrending interest rates is similar to the method employed by Campbell (1991) and Hodrick (1992).

<sup>11</sup> The sample period is 1966Q1-1982Q3.

Graph 2

**Transformed excess returns and transformed bond yields:  
the United States, Canada and the United Kingdom**

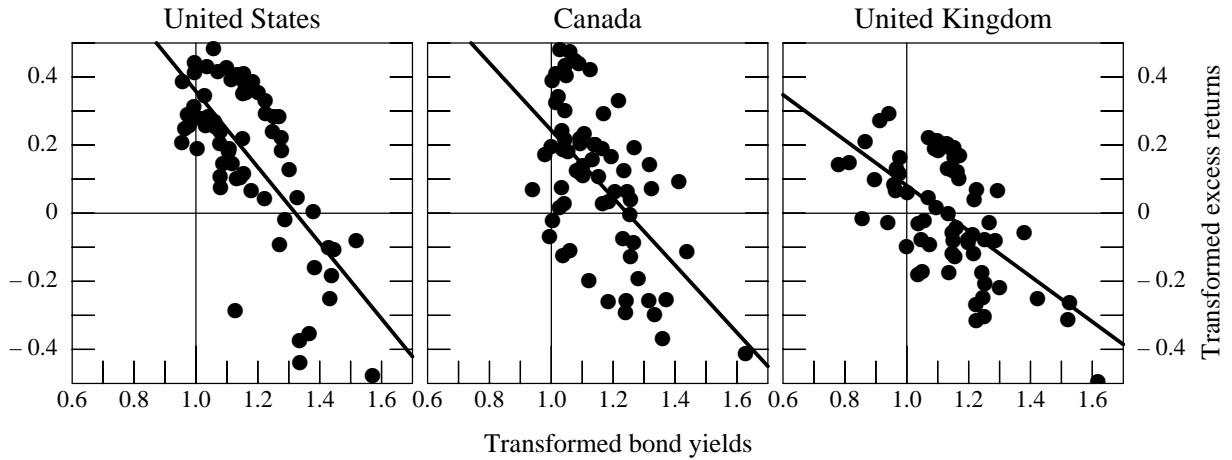


Table 3

OLS estimates of  $\beta_{ij}$  associated with the regression  $\tilde{U}_{it}^o = \alpha_{ij} + \beta_{ij}\tilde{R}_{jt} + \zeta_t$

		<i>j</i>		
		US	CA	UK
<i>i</i>	US	-1.1	-1.4	-0.3
	CA	-0.8	-1.0	-0.4
	UK	-0.4	-0.7	-0.7

Concerning the system (2.1), the null hypothesis motivated by the expectations theory of the term structure is  $\beta_{ij} = \beta_{ji} = 0$ . A test of this null hypothesis is complicated by two factors. First, the hypothesis imposes restrictions on coefficients in two separate regressions. Secondly, the error terms  $\zeta$  and  $\xi$  exhibit a high degree of serial correlation. It is, of course, in principle possible to derive valid test statistics of the null hypothesis  $\beta_{ij} = \beta_{ji} = 0$  in the presence of serially correlated error terms by invoking the relevant asymptotic theory. However, given the relatively short sample period under study, it is doubtful that critical values computed on the basis of large sample theory will be good approximations in the present case. Therefore, significance probabilities associated with the parameter estimates reported in Table 3 are calculated by Monte Carlo simulation.<sup>12</sup>

The Monte Carlo study is based on the following model, which is described in more detail in the Appendix. For each country, the natural logarithm of the three-month rate of interest is

<sup>12</sup> The importance of basing statistical inference on small sample distributions of test statistics in related contexts has been pointed out by Mishkin (1990) and Jorion and Mishkin (1991), among others.

assumed to evolve as a random walk without drift.<sup>13</sup> The vector comprising the innovations in the random walks is assumed to be normally distributed and independently and identically distributed over time. The inter-country correlations of the innovations in the random walks are set equal to the historical correlations over the period 1961Q2-1992Q2. Given this model of the evolution of short rates, long-term interest rates are constructed under the assumption that the expectations theory of the term structure holds. 1,000 trials were conducted for the experiment, with equations (2.1a) and (2.1b) estimated by OLS for each trial. Significance probabilities were determined by the frequency with which the estimated  $\beta$ s fell into particular regions of the parameter space.

Table 4 reports significance probabilities computed by Monte Carlo simulation. The  $i = j$  elements of Table 4 give the probability of obtaining an OLS estimate of  $\beta_{ii}$  less than or equal to the corresponding value reported in Table 3 when the expectations theory is true. For example, the upper left-hand element of Table 4 indicates that the probability of obtaining an estimate of  $\beta_{US,US}$  which is less than or equal to -1.1 if the expectations theory is true is 0.2%. Thus, the null hypothesis that  $\beta_{US,US} = 0$  is rejected in favour of the alternative  $\beta_{US,US} < 0$  at standard levels of significance. The same conclusion is reached for the case of Canada. The null hypothesis  $\beta_{UK,UK} = 0$  is rejected in favour of the alternative  $\beta_{UK,UK} < 0$  at the 6.2% level. Thus, when measured by the metric of the 5% confidence level, the evidence for excess volatility of bond yields in the United States and Canada is statistically significant while the evidence for excess volatility of UK bond yields is not.

Table 4  
Significance probabilities

		<i>j</i>		
		US	CA	UK
<i>i</i>	US	.002		
	CA	.001	.001	
	UK	.017	.005	.062

The  $i \neq j$  elements of Table 4 are concerned with the evidence of excess comovement of bond yields between countries. In particular, the  $i \neq j$  elements give the joint probability of obtaining OLS estimates of  $\beta_{ij}$  and  $\beta_{ji}$  which are less than or equal to the maximum of the two corresponding parameter estimates reported in Table 3. For example, the row 2, column 1 element of Table 4 indicates that the joint probability of obtaining OLS estimates of  $\beta_{US,CA}$  and  $\beta_{CA,US}$  which are both less

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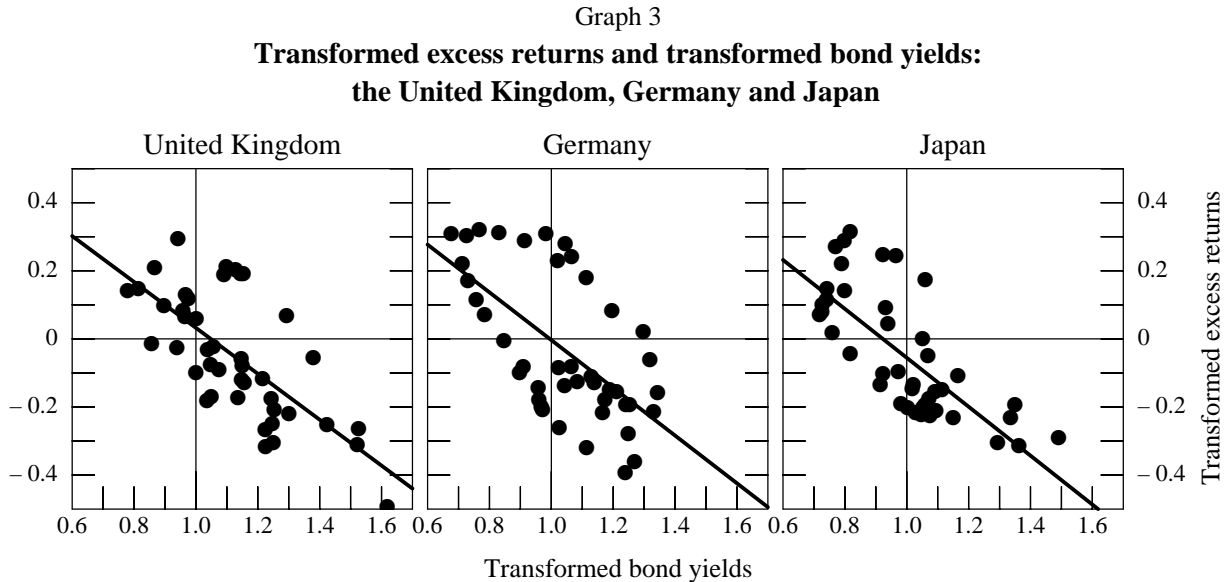
<sup>13</sup> Modelling the natural log of the short rate as a random walk can be defended as an approximation to the case in which the short rate is borderline stationary.

than -0.8 (if the expectations theory is true) is 0.1%. Thus, the hypothesis  $\beta_{US,CA} = \beta_{CA,US} = 0$  is rejected in favour of the alternative hypothesis  $\beta_{US,CA} < 0$  and  $\beta_{CA,US} < 0$  at a high level of significance. The same conclusion is reached for the other two pairs of countries.

According to the results of the Monte Carlo simulations, the evidence of excess comovement of bond yields is statistically significant at a high level of confidence for all country pairs. Because a necessary condition for excess comovement of bond yields between two countries is excess volatility of bond yields in each domestic market, the results indicate that yields in all three countries have exhibited excess volatility over the period studied.

### 2.3 Comovements of UK, German and Japanese bond yields

This subsection extends the analysis of the joint behaviour of bond yields to a second group of countries made up of the United Kingdom, Germany and Japan.



Scatter plots of transformed excess returns against own country transformed bond yields are shown in Graph 3 for the second group of countries, and Table 5 presents the estimates of the  $\beta$ s in the system (2.1) for all country pairs.<sup>14</sup> Every entry in the table is negative, which is consistent with excess volatility of bond yields in each domestic market and with excess comovement of bond yields between all three pairs of countries.

Table 6 reports significance probabilities associated with the parameter estimates shown in Table 5. These significance probabilities are computed in the same manner as those given in Table 4. As indicated by Table 6, the null hypothesis  $\beta_{ii} = 0$  cannot be rejected for any of the countries at

<sup>14</sup> The sample period is 1972Q1-1982Q3.

standard levels of confidence. However, the  $i \neq j$  elements of Table 6 indicate that the evidence for excess comovement of bond yields is statistically significant at better than the 5% level for each country pair. Because a necessary condition for excess comovement of bond yields between two countries is excess volatility of bond yields in each domestic market, the  $i \neq j$  elements of Table 6 indicate that bond yields in each domestic market have exhibited excess volatility over the period studied.

Table 5

**OLS estimates of  $\beta_{ij}$  associated with the regression  $\tilde{U}_{it}^o = \alpha_{ij} + \beta_{ij}\tilde{R}_{jt} + \zeta_t$**

		<i>j</i>		
		UK	DE	JP
<i>i</i>	UK	-0.7	-0.3	-0.7
	DE	-0.8	-0.7	-0.7
	JP	-0.5	-0.3	-0.7

Table 6

**Significance probabilities**

		<i>j</i>		
		UK	DE	JP
<i>i</i>	UK	.205		
	DE	.011	.632	
	JP	.014	.041	.216

## Conclusions

This paper examines the issues of excess volatility and excess comovement of interest rates among global bond markets. The base model of interest rate behaviour is the expectations theory of the term structure. The restrictions that the base model imposes on the behaviour of yields within groups of countries allow the rejection of the model at high levels of statistical significance in the case of every country examined. In particular, the empirical results indicate that, over the period studied, bond yields in the major markets displayed excess volatility relative to the base model. Thus, the present paper provides additional international evidence against the expectations theory as a model of the behaviour of long-term government bond yields. The empirical results also indicate that bond yields in the major markets displayed excess comovement relative to the base model over the period studied. Taken together, these findings suggest that term premia at the long end of the term structure are both time-varying and positively correlated across markets.

These findings have implications for the interpretation of movements in long/short interest rate spreads in the major markets. First, movements in long/short spreads in the major markets may at times reflect changes in term premia and not revisions of rational forecasts of future short rates. Secondly, the finding that long bond term premia are positively correlated across markets suggests that the factors responsible for time variation in term premia have an international component.

There remain interesting questions to be addressed by future research. One such question is the cause of time variation in term premia at the long end of the term structure. To what extent does it reflect changes in risk premia? Or does it reflect the failure of market participants to form expectations of future short-term interest rates in an optimal manner? Understanding the cause of time variation in term premia is the first step to answering the question of why these term premia move together across the major markets.

## Appendix: Monte Carlo experiments

This appendix describes in detail the assumptions underlying the Monte Carlo simulations and how they are conducted.

Let  $q_{it} \equiv \ln r_{it}$ , where  $\ln(\cdot)$  denotes natural logarithm. Let  $q_t \equiv (q_{1t}, q_{2t}, \dots, q_{kt})'$  denote the  $k \times 1$  vector of the logarithms of short rates in the  $k$  countries under study. It is assumed that the evolution of  $q_t$  obeys

$$(A.1) \quad q_t = q_{t-1} + e_t,$$

where  $e_t \equiv (e_{1t}, e_{2t}, \dots, e_{kt})'$  is an independent and identically distributed normal random vector with zero mean and covariance matrix  $\Omega_e$ . It follows from these assumptions that

$$(A.2) \quad E_t r_{i,t+j} = r_{it} \exp(j\sigma_i^2 / 2),$$

where  $\sigma_i^2$  is the  $i$ th main-diagonal element of the covariance matrix  $\Omega_e$ . The Monte Carlo simulations reported in the paper assume that short-rate dynamics are governed by (A.1) and that expectations of future short rates are formed according to (A.2).

In the Monte Carlo simulations involving the first group of countries,  $\Omega_e$  is set equal to the corresponding sample covariance matrix over the period 1961Q2-1992Q2 and realisations of the series  $\{q_t\}$  are generated over this period taking the value of  $q$  in 1961Q1 as an initial condition. In the Monte Carlo experiments involving the second group of countries,  $\Omega_e$  is set equal to the corresponding sample covariance matrix over the period 1967Q2-1992Q2.

1,000 (vector) series of short rates were generated for each group of countries according to (A.1). For each series of short rates, a time series of theoretical bond yields was computed under the assumption that the expectations theory of the term structure holds with the term premium for each country equal to the average spread reported in Table 2. From these generated interest rate series, time series of  $\{\tilde{U}_t^o\}$  and  $\{\tilde{R}_t\}$  were constructed and the system (2.1) was estimated by OLS for all country pairs. From the parameter estimates, the significance probabilities reported in Tables 4 and 6 were computed.

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