INTERNATIONAL CURRENCY SUBSTITUTION AND THE APPARENT
INSTABILITY OF VELOCITY IN SOME WESTERN EUROPEAN
ECONOMIES AND IN THE UNITED STATES

by Bruce Brittain

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1. Introduction

The purpose of this paper is to describe and interpret some
empirical regularities in the movements of money's income velocity of
circulation in major industrial countries. These movements have tended
to be closely coincident, particularly since the beginning of the period
of floating exchange rates. These two facts suggest that anomalous
velocity movements in any one country may be related to the anomalous
velocity movements in the others. Furthermore, all of these movements
considered together may have something to do with the shift to flexible
exchange rates.

A country of principal concern in this discussion is the
United States - where unexpectedly low levels of velocity in the early
1970s and unexpectedly high values after 1975 have been one of the main
reasons for the monetary authorities' scepticism regarding the advisabil-
ity of monetary targeting. Attention has focused on the period after
mid-1974. Since that time, money-demand forecasts based on equations
such as those estimated by Enzler, Johnson and Paulus (1976) signifi-
cantly overpredict the growth of money demand or underpredict money's
income velocity of circulation. (Money in those studies was narrowly
defined US money, or M_1.) Hamburger (1977) has clarified the issue by
showing that the prediction errors can, to a large degree, be eliminated
by adopting a more general form of the money-demand function. The more
general form includes rates of return on long rather than short-dated
government bonds and the price-earnings ratio for equities.

While Hamburger's explanation is compelling - movement in
rates of return on US dollar-denominated substitutes for US currency
seems to be a reasonable source of change in the behaviour of US money -
it is interesting to note a coincidence of monetary policy problems and

velocity movements in the United States and other countries. Unexpected velocity movements in Switzerland, Germany, Italy and the United Kingdom have been problematic from the point of view of monetary targeting. Furthermore, velocity movements in these countries have been coincident with movements in the United States. In Switzerland and Germany, for example, money's income velocity of circulation has been unexpectedly low since early 1977, while the US M_1 velocity measure has been high. Conversely, the Swiss and German measures were high prior to 1977, when the US measure was low. In addition, anomalous velocity movements can be observed in the United Kingdom and Italy and these, too, follow the pattern of the US movements closely.

As I implied in the first paragraph, these empirical regularities would seem to imply certain limits on any potential explanation of velocity's behaviour. Firstly, the factors invoked to explain movements in the United States ought to be admissible as an explanation of movements in Germany, Switzerland, Italy and the United Kingdom. Secondly, as coincidental movements of velocity in these five countries have been observed principally since the breakdown of the Bretton Woods fixed exchange rate agreements, they ought to be in some way connected with floating exchange rates. In Part II of this paper, I shall describe the empirical regularities that justify this line of argument.

In the subsequent section, I shall discuss potential explanations as to why these regularities may have occurred. One possible explanation is that demand for any single national currency may reflect a demand for money to be held in a diversified currency portfolio. Rules for optimal currency diversification could be imposed on this portfolio, so that one might expect to see the proportions of various currencies in the portfolio - as well, therefore, as conventional measures of velocity - varying with the risk and expected rates of return of the specific currencies in the portfolio.

Work by Kouri and Braga de Macedo (1978) would seem potentially fruitful in this regard. Based on their model of currency portfolio behaviour, I would expect the relevant measures of risk and rate of return to involve the levels and variability of the spot and forward exchange rates as well as the variability of the difference between national inflation rates and interest rates on financial instruments

denominated in the domestic currency. Such arguments would be of interest because, on the one hand, they could imply that the proportions of various currencies in the portfolio, and therefore measures of velocity's circulation, might be stabilised by fixing the exchange rate. On the other hand, they are interesting because they contrast sharply with others that rely on purely domestic variables to explain velocity's recent behaviour.

In Part IV of the paper, I shall present estimates of money-demand functions for the pound sterling, the Deutsche Mark and the US dollar. In each case the money-demand formulations reflect the influence of portfolio variables. The estimates are an attempt to test specific forms of the more general portfolio model suggested by the empirical regularities described in the second section of this paper.

The statistical results offer support for the portfolio view. The German equations are most convincing: the portfolio variables are not only significant and of the expected sign, but they are also important in eliminating the bulk of the unexplained increase in German money demand since the end of 1976.

The US equations perform well, with the portfolio variables contributing significantly to the behaviour of money demand. The variables offer little insight into the reasons for the major unexplained shifts in US money demand, however. Either the portfolio approach is of considerable statistical and little practical significance for revealing why US money demand shifted or the portfolio variables were poorly chosen.

The UK equations perform poorly - both from the point of view of domestic variables and from that of the portfolio variables.

II. The problems of explaining recent velocity movements in the United States and Germany

There are a number of reasons for being concerned about the behaviour of money's income velocity of circulation. The reason that interests me is that velocity's instability has been used to justify a lack of urgency in attempts to target the money stock in a number of different countries. Successful targeting would lower the average

money-stock growth rate, reduce its variability and inevitably, it is to
be hoped, bring down the inflation rate.

This tendency to discount the desirability of stable monetary
aggregates appears to have a certain justification. As regards the
United States, Graph 1 shows that velocity often strays from trend and
may do so because the demand for money rather than conditions involving
its supply are changing. Indeed, between the middle of 1969 and the
middle of 1975 velocity moved from a level near 5 per cent. above its
trend value to a level more than 4 per cent. below. That movement was
unexpectedly reversed between end-1975 and late 1978, as the velocity
of M_1 rose from 4 per cent. below trend to almost 7 per cent. above.
Is this behaviour of US M_1 velocity unexplainable?

Hamburger (1979) has told us that it is not. By including
rates of return on equity and on long-term bonds in simple money-demand
functions, he succeeds statistically in explaining a good part of US
velocity's unexplained variability. While the argument is attractive
as far as it goes, it seems to me that the issue is not closed. Firstly,
velocity instability has been an excuse - possibly quite adequate - for
exceeding monetary targets in Germany and dropping them altogether in
Switzerland. Secondly, the German and Swiss explanations as to why
velocity altered have implications for the movement of velocity in
the United States, and the Hamburger explanation has neither been made
explicitly consistent with these arguments nor shown them to be incorrect.

The German and Swiss argument was in its roughest outline that
during 1977 and 1978 the demand for Deutsche Mark and for Swiss francs
increased as the result of a shift by holders of internationally
diversified currency portfolios out of US dollars and into Swiss francs
and the Deutsche Mark. The Swiss and German authorities were explicit
in saying that this unexpectedly lowered velocity in Germany and Switzer-
land. Implicitly, their argument intimates an increase in US velocity.
In the Hamburger framework, the US shift would be explained by the
increasing relative attractiveness of investments in non-money US assets,
and not by a shift in international currency portfolios. The Hamburger
explanation predicts no coincident movement of foreign and US velocity
movements.

To show why the portfolio shift arguments may be important,
I shall consider for a moment the simple correlation coefficients relating

Graph 1
- 5 -

The income velocity of circulation of narrowly defined money in the United States

(log values)
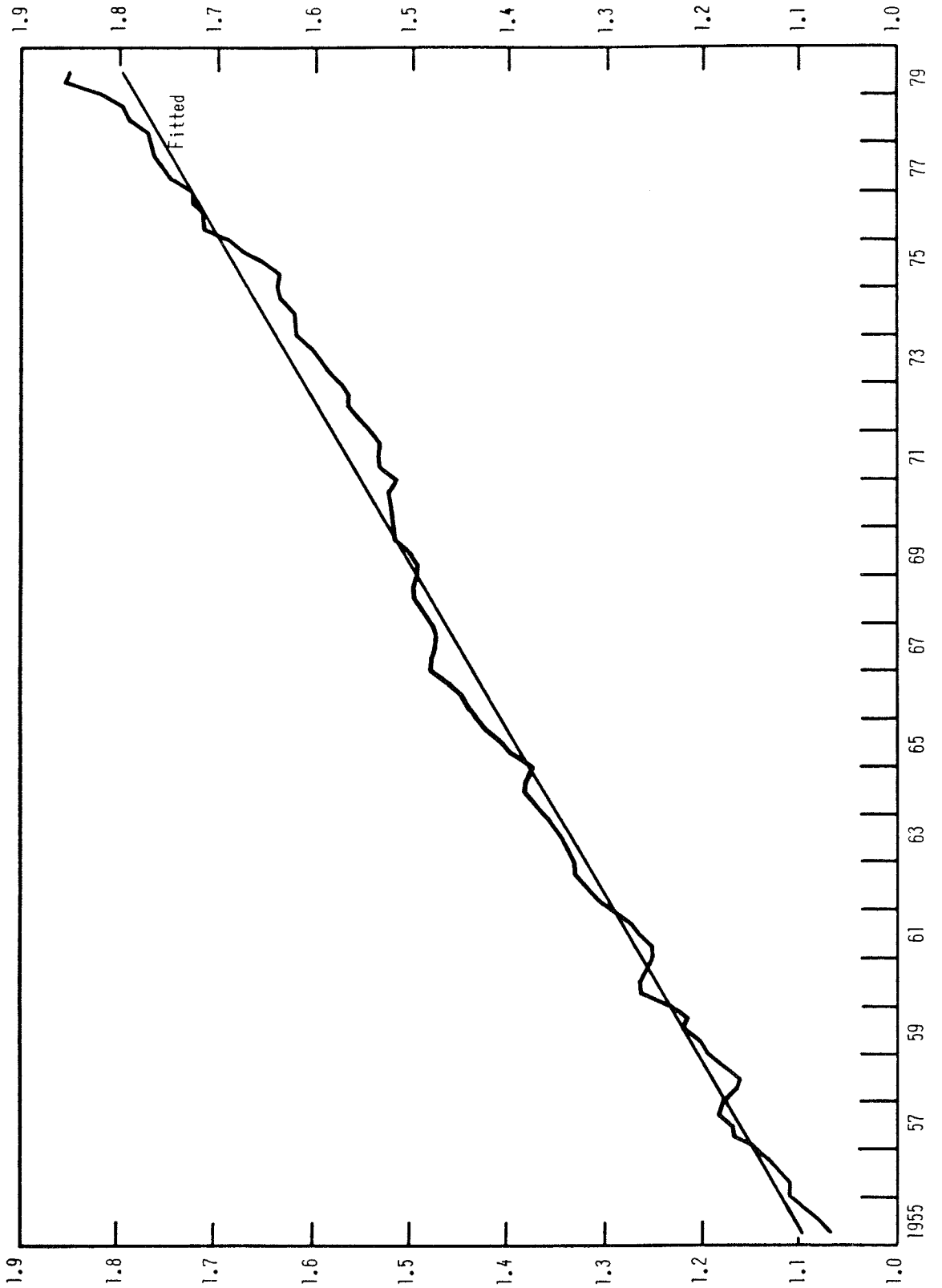
the deviations of the log of velocity from trend in the United States, Germany, Italy, Switzerland and the United Kingdom. The construction of the trends is described in Table 1. The correlation coefficients are reported in Table 2.

There is a strong negative correlation between deviations in Switzerland and Germany, on the one hand, and the United States, Italy and the United Kingdom, on the other. There are also strong positive correlations within each group. The simple correlation coefficients seem consistent with the idea that individuals favour the Deutsche Mark and the Swiss franc whenever they disfavour the Italian lira, the pound sterling and the US dollar, and vice versa.

As another partial illustration of these points, I have included Graph 2, which is a scattergram relating the deviations of US velocity from trend to the deviation of German velocity from trend. A least-squares line is plotted through the scatter.

These crude statistical measures seem to imply systematic interdependence among deviations of velocity from trend in Italy, the United Kingdom, Germany, Switzerland and the United States. A worthwhile question is whether this interdependence is useful in explaining any of the particularly sharp, anomalous movements in velocity in the countries concerned, or whether the interdependence reflects coincident movements of a less interesting nature. Is this dependence useful in explaining the tendency for US velocity, for example, to be below trend between 1971 and 1975 and above trend thereafter?

To give a rough answer to this question, I have regressed the log of US M_1's income velocity of circulation against time and against various combinations of deviation of other countries' velocity from trend. The results are presented in Table 3. Because of the high degree of collinearity between the Italian and the UK variables and between those for Germany and Switzerland, I have excluded the UK and Swiss variables from among the list of potential explanatory variables. German and Italian variables are highly correlated, so when the two would have been included together in any regression equation I have subtracted the Italian from the German deviation. The regression results for both annual and quarterly data are presented in Table 3.

Table 1

Calculation of velocity trends

("t" statistics in brackets)

	Constant	Time	$\bar{R}^2$	D.W.	S.E.E.	Frequency	Period
LN(VUK)	7.807 (174.8)	-6.38[a] (-.141)	-.0156	.165	.068	Q	1959:1 - 79:1
LN(VUS)	.857 (123.1)	.0072 (89.71)	.988	.168	.023	Q	1955:1 - 79:2
LN(VIT)	.718 (27.94)	-.0057 (-20.67)	.850	.444	.053	Q	1960:1 - 78:4
LN(VCH)	-8.682 (-205.3)	.0021 (4.637)	.212	.473	.088	Q	1960:1 - 79:1
LN(VDE)	2.300 (174.9)	-2.9[b] (-2.090)	.043	.317	.027	Q	1960:1 - 79:1
LN(VUS)	.851 (65.52)	.029 (47.7)	.990	.517	.020	A	1955 - 78
LN(VIT)	.723 (14.89)	-.023 (-11.05)	.871	.413	.049	A	1960 - 78
LN(VDE)	2.294 (92.95)	-9.6[b] (-.919)	-.009	.806	.025	A	1960 - 78

[a] $x\ 10^{-5}$

[b] $x\ 10^{-4}$

The deviation of foreign velocities from trend, as the earlier correlation coefficients implied they would, significantly improve the explanation of US velocity's behaviour, particularly in the turbulent period after 1971. Perhaps the best way of demonstrating the nature of the improvement is to examine the deviations of US velocity from trend and the error terms from the regression equations reported in Table 3 and which include velocity as a dependent variable. Graph 3 includes plots of residuals from the equation with both German and Italian variables included, estimated using quarterly data.

Table 2

Correlation coefficients between deviations of money's
income velocity of circulation for five countries

|	United States	Germany	Italy	Switzer-land	United Kingdom
United States ..	1.0				
Germany	-.695	1.0			
Italy	.504	-.252	1.0		
Switzerland	-.443	.447	-.417	1.0	
United Kingdom..	.516	-.351	.532	-.506	1.0

In Graph 3 the residuals from equation 1 are compared with the
simple deviation of velocity from trend - the trend being derived from
the results in Table 1. The first set of residuals appears similar in
both the pre and post-1971 periods. The deviations from trend, by
contrast, appear more marked after 1971. In addition, the substantial
"unexplained" undershooting of velocity between 1971 and 1975 disappears,
as does the current overshooting.*

I have argued that US velocity behaviour is consistent with,
and can be partly explained by, shifting composition of the international
money portfolio. Nonetheless, a better argument based on these
results is that the behaviour of velocity in Germany, Switzerland,
Italy, the United Kingdom and the United States is jointly consistent
with this shifting composition. To see the implications of this point,
consider Graph 4, which includes plots of the residuals of a regression

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* These results were based on an examination of velocity measures cal-
culated using the monetary aggregate which is most closely targeted by
the monetary authority in question and thus on M_1 for the United States.
I have recalculated regressions (1), (2) and (3) reported in Table 3
using US M_2. The results are reported as equations (1)', (2)' and (3)'
in Table 3.

Graph 2

Deviation of US M₁ velocity of circulation from trend versus deviation of German central-bank money from trend
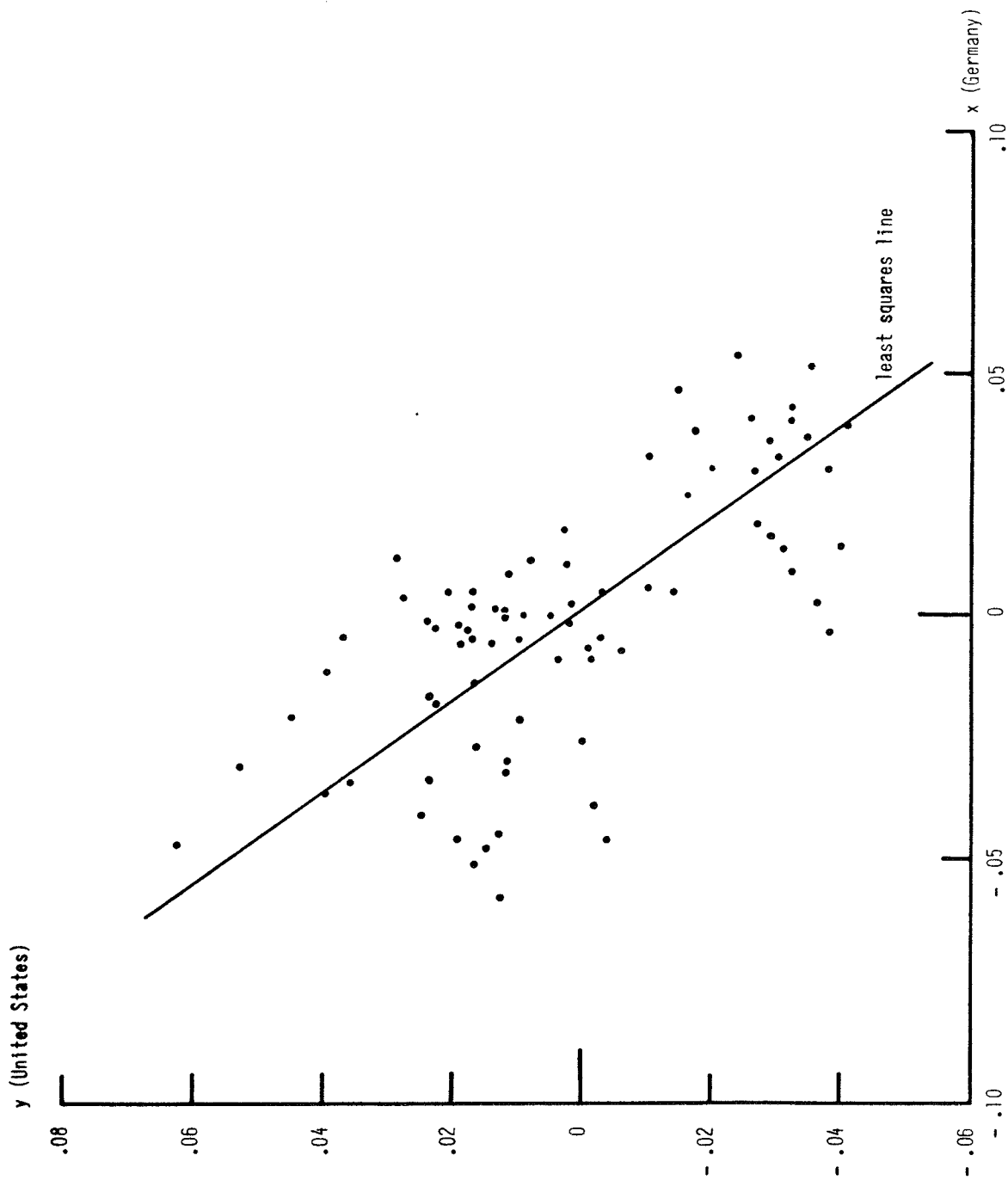
Graph 3

Deviation of M_1 velocity of circulation from trend and error term from equation 3
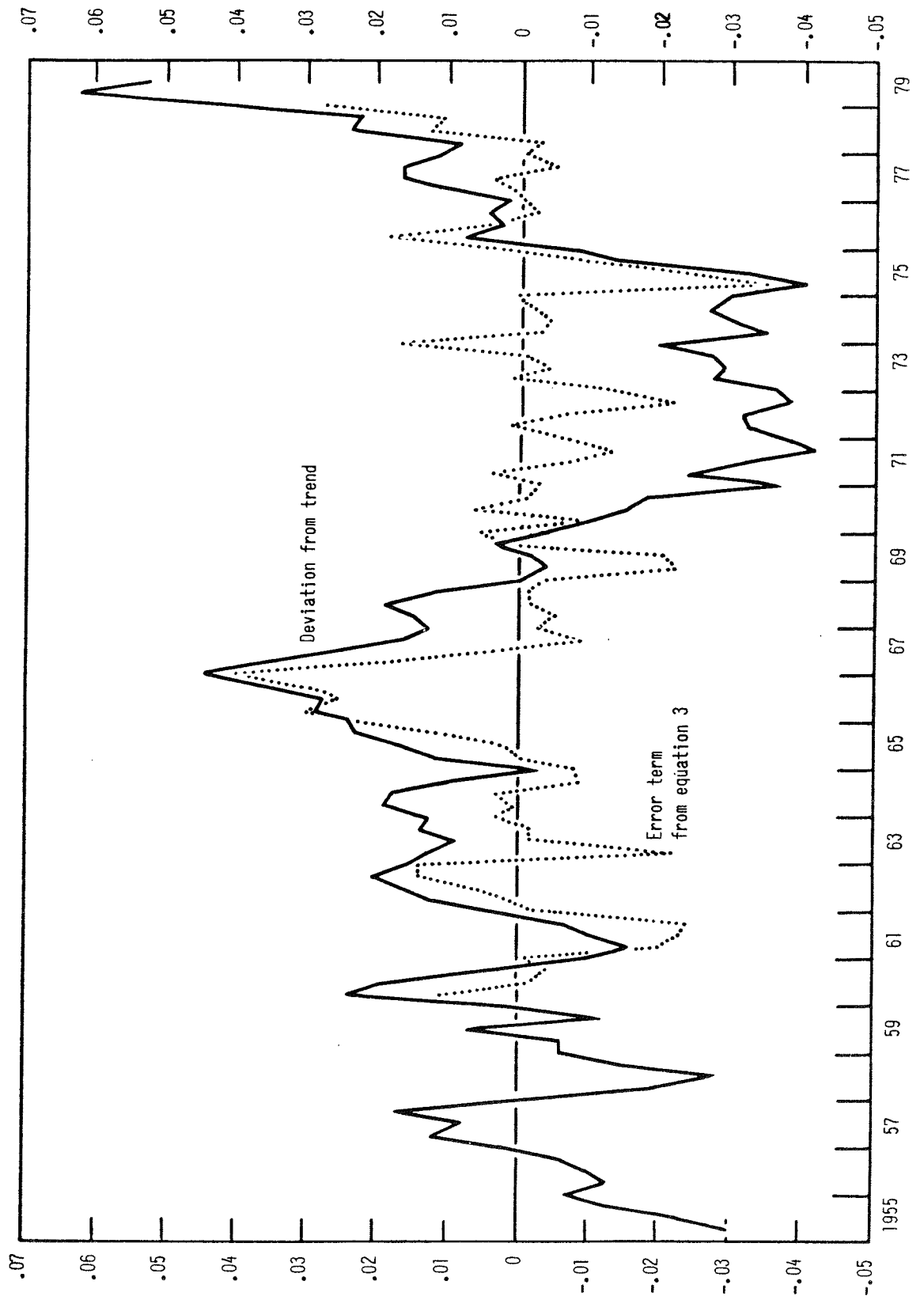
Table 3

Regressions with the log of US narrowly defined money's income

velocity of circulation as the dependent variable

("t" statistics in brackets)

| Equa-tion | Explanatory variables | | | | | $\bar{R}^2$ | D.W. | S.E.E. | Fre-quency | Time period |
	Constant	Time	(X) ERRORDE	(Y) ERRORIT	(X-Y)					
1	.884 (131.3)	.0069 (96.12)	-.324 (-4.30)		-.155 (-4.95)	.992	.730	.014	Q	1960:1 - 78:4
2	.886 (97.89)	.0069 (71.23)		.211 (5.15)		.986	.326	.019	Q	1960:1 - 78:4
3	.880 (111.8)	.0070 (83.02)	-.592 (-8.55)			.989	.513	.016	Q	1960:1 - 79:1
4	.875 (74.16)	.028 (55.49)	-.359 (-2.33)		-.174 (-2.79)	.994	1.981	.0119	A	1960 - 78
5	.875 (50.88)	.028 (38.07)		.261 (3.034)		.987	.871	.0174	A	1960 - 78
6	.875 (62.14)	.028 (46.50)	-.643 (-4.65)			.992	1.533	.0142	A	1960 - 78
1'	.835 (121.2)	.0004 (5.30)	-.010 (-.13)		-.132 (-4.16)	.404	.510	.014	Q	1960:1 - 78:4
2'	.836 (118.3)	.0004 (5.02)		.147 (4.60)		.372	.470	.015	Q	1960:1 - 78:4
3'	.825 (95.6)	.0005 (5.69)	-.287 (-3.69)			.364	.302	.018	Q	1960:1 - 79:2

of the log of German velocity on time and the residuals from a regression
using time and the US velocity residual as explanatory values.* The
portfolio argument seems particularly useful in explaining the anomalous
movement in German velocity after 1977. Thus, the standard error of
the regression of the German velocity equation is reduced from .027 to
.020 by inclusion of the deviation of US velocity from trend. The
US velocity equation standard error is reduced from .023 to .016 by
the inclusion of the German error term.

On the basis of these rough comparisons, it would seem worth-
while undertaking further empirical study of international shifts in
currency portfolios. On the surface of it, unexplained velocity move-
ments in the United States seem related to similar movements in Italy
and the United Kingdom, and to opposite movements in Switzerland and
Germany. All seem related to exchange rate flexibility. Whether
these movements reflect systematic shifts into and out of the Swiss
franc and the Deutsche Mark at the expense of the US dollar, the Italian
lira and the pound sterling or whether they are due to factors unrelated
to portfolio shifts is the question taken up in the remainder of the
paper.

III. Implications of multicurrency portfolio models for the estimation of money-demand functions

The argument of the preceding section established only the
coincidence of movements in the velocity of circulation of several major
currencies. Although that coincidence suggests that similar factors may
have simultaneously influenced velocity in different countries, it does
not establish that the movements are behaviourally related and, if they
are, what form the relationship might have. This section is an initial
attempt at explaining how the coincident movements might have occurred.

An immediately plausible explanation for the transnational
similarities in velocity movements is that currencies should be con-
sidered elements of internationally diversified portfolios. The
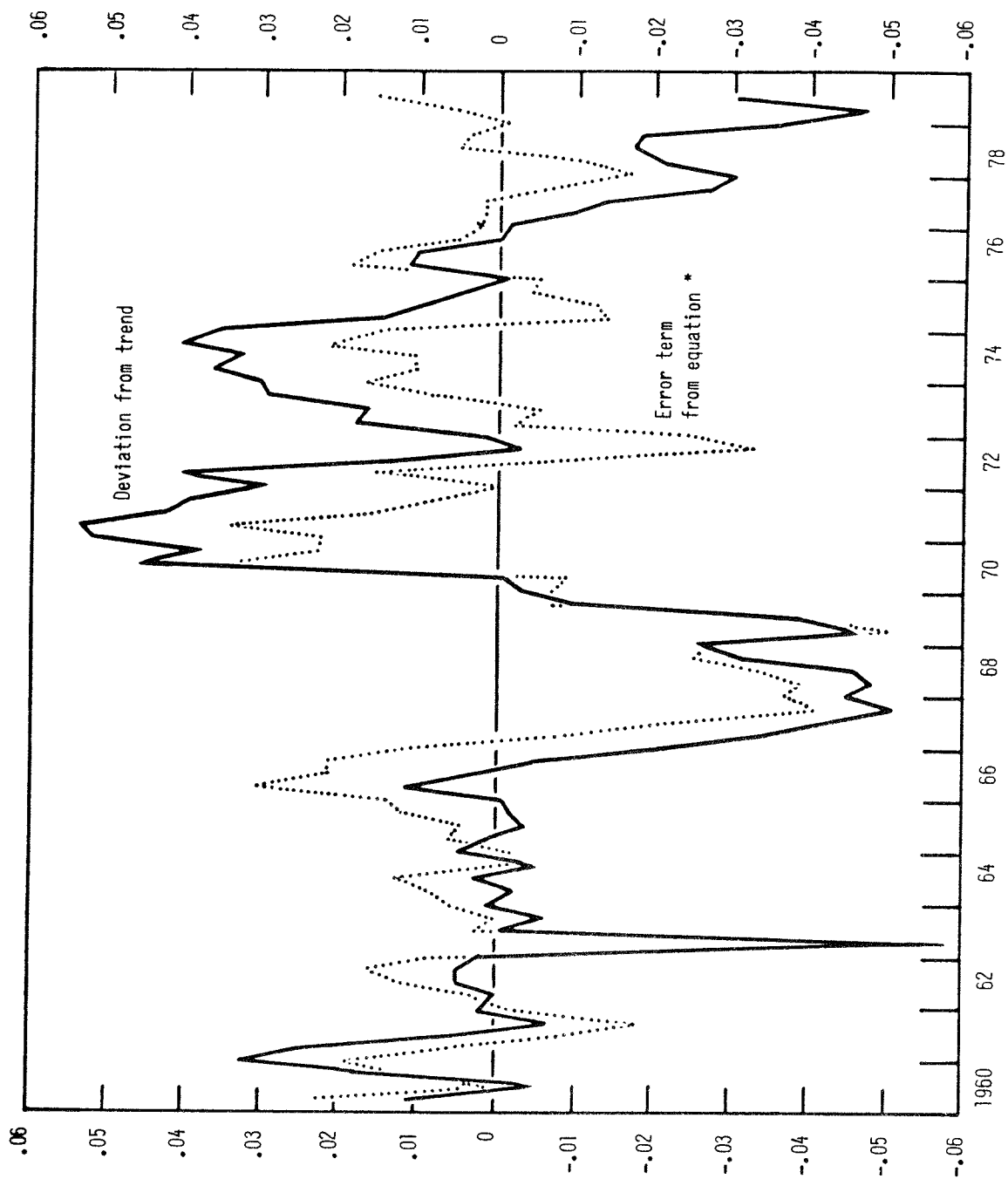
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* The actual regression results were:

$$\ln(VDE) = 2.31 \quad -.0004* \text{ TIME} \quad -.775* \text{ ERRORUS}$$
$$(244.6) \quad\quad (-4.17) \quad\quad (-8.23)$$

$$\bar{R}^2 = .489$$
$$\text{D.W.} = .671$$
$$\text{S.E.E.} = .0196$$

Graph 4

Deviation of German central-bank money velocity from trend and
error term from German velocity equation*

* Footnote on page 12.

theoretical groundwork for this suggestion has been laid by Kouri and
Braga de Macedo (1978). There is, of course, ample indirect support
for the view among "realists" who explain recent swings in the US dollar
in terms of changes in confidence and the rearrangement of currency
portfolios.

A potentially useful way of formulating the money-demand
function to take account of these portfolio considerations is equation (1):

$$(1) \quad M_i = a + B.R' + C.V.C' + D.Y'$$

where: M_i is the nominal quantity demanded of currency i,
  one of n currencies,

a is a constant term,

B, C and D are (1 x n) row vectors of coefficients,

R is a (1 x n) row vector of expected opportunity costs of
  holding the various currencies,

V is an (n x n) variance-covariance matrix of expected
  opportunity costs,

Y is a (1 x n) row vector of income in the various countries
  under consideration.

The derivation of a money-demand function incorporating cur-
rency portfolio assumptions is presented in the technical appendix here-
to. Equation (1) differs somewhat from the form developed there. Rather
than expressing the money-demand variable as the proportion of a given
currency in the total portfolio, equation (1) expresses the amount in
absolute terms. Furthermore, for the purposes of equation (1) I have
normalised the money variable with a measure of nominal income. These
differences require explaining.

I did not use particular Ms as proportions of the total port-
folio because of the high degree of correlation evident between dif-
ferences in interest rate levels in the United States and Germany,
and the value of the dollar in terms of the Deutsche Mark. The possi-
bility of spurious correlation because of the choice of variables with
which to deflate M led me to choose a measure of nominal income. I
chose income, rather than prices, for example, to make my money-demand
estimating forms more directly comparable with studies incorporating
closed economy assumptions.

When there is only one currency, equation (1) very closely approximates the estimating form of standard money-demand functions (cf. Hamburger). The only difference is that standard forms exclude the variance of the expected return on money - an exclusion which seems unexceptionable in practice if not in theory.

When there are several currencies, the exclusions implied by this simple formulation are somewhat less acceptable. When there are two currencies, this point is easily made. Let us look at the demand, M_C, for country C's currency.

$$(2) \quad M_C = a + b'_C \cdot r_C + b_C^2 \, r_{US} + \sum_{i=1}^{2} \sum_{j=1}^{2} c_{ij} \, \sigma_i \, \sigma_j$$

$$+ d'_C \, Y_C + d_C^2 \, Y_{US}; \quad i,j = US, C$$

With regard to the effect that changes in US income might have on M_C, let us suppose that C is a trading partner of US, that trade between C and US (partly denominated in C's currency) accounts for a significant proportion of US trade, and that the proportion of US trade to income in C is quite large. An increase in US income could, therefore, produce a measurable increase in the demand for C's currency. The increased demand would be a demand to finance a greater volume of transactions and would be related to the growth of trade. To exclude US income from C's money-demand equation would be to misspecify the function.

Consider now the effects that changes in variances and covariances of expected returns might have on the demand for C's currency. Should σ_C^2 rise relative to σ_{US}^2 - perhaps because US inflation is expected to be less variable than C's - then holders of C's currency might be expected to substitute US currency for their own. This effect might be measurable if, for example, residents in C already found it convenient to hold relatively large quantities of US currency for transaction purposes or to finance trade. Similar arguments could be advanced concerning $\sigma_C \, \sigma_{US}$, the covariance of expected returns.

As a final point regarding the importance of the variance and covariance terms in the multicurrency model, σ_C^2 would, for all practical purposes, equal σ_{US}^2, and $\sigma_C \, \sigma_{US}$ would equal unity as long as the exchange rate between the two currencies was expected to remain fixed. Thus, a move to flexible exchange rates would free σ_C^2 and σ_{US}^2 from

one another, and σ_C σ_{US} from unity, producing shifts in the demand for C's currency.

Lastly, let us examine the effect that expected rates of return might have on the demand for C's currency. Since increases in r_C represent increases in the opportunity cost of holding C's currency, they should cause a decline in M_C. By contrast, an increase in r_{US} represents an increase in the opportunity cost of holding US currency and should induce an increase in the demand for that of C.

To this argument the following objection can be made. Because differntials in the interest rates on assets with similar risk character- istics denominated in different currencies equal the forward exchange agio – by arbitrage – the expected rates of return on investments in national currencies are equal regardless of the currency of denomination. I have, however, used the uncovered interest differential in my calcula- tions. I did so for two reasons. The first has to do with the relative bias of the forward exchange rate as a predictor of the future spot rate. The second reflects the way in which the level of interest rates roughly approximates the variability of inflation rates.

As is widely argued, the forward exchange rate tends to under- estimate the increase in the spot exchange rate when the exchange rate is rising, and to underestimate the decline in the spot exchange rate when it is falling. This possible systematic forecasting error raises the possibility that the forward exchange rate need not be the expected future spot rate. In this case the uncovered differential could be taken as an indicator of expected opportunity costs.

The second argument for including the uncovered differential is as an approximation of the difference in expected variances in the rate of return on two different assets. If the nominal interest rate reflects inflationary expectations and if inflation rates are more variable when they are high, the level of interest rates may also serve to represent the expected variability of the opportunity cost of holding the relevant money. The interest differential may then reflect diver- gence between the variance of two expected rates of return.

IV. The estimation of a minimal currency portfolio model

Table 4 presents the results of demand equations for US M_1, German central-bank money and UK M_3 based on quarterly data, and of

Table 4

Regression results: Money-demand equations

("t" statistics in brackets)

Equation number	Dependent variable	Constant	Lagged dependent variable	r_d	$r_f - r_d$	$\bar{R}^2$	D.W.	S.E.	Time period
1	MDDE	-2.235 (-282.7)		-.0102 (-5.151)		.249	.374	.024	1960:1 - 79:2
2		-.513 (-3.624)	.767 (12.17)	-.0044 (-3.581)		.747	2.209	.014	1960:2 - 79:2
3		-2.260 (-316.6)		-.0069 (-4.233)	.0014 (6.937)	.536	.651	.019	1960:1 - 79:2
4		-.795 (-4.754)	.645 (8.755)	-.0042 (-3.507)	.0034 (2.862)	.769	2.099	.013	1960:2 - 79:2
5	MDUK	-7.866 (-310.2)		.008 (2.603)		.082	.175	.066	1963:1 - 79:2
6		-.453 (-1.607)	.941 (17.48)	-.001 (-.737)		.844	1.687	.027	1963:2 - 79:2
7		-7.874 (-266.9)		.011 (2.057)	.004 (.558)	.072	.173	.066	1963:1 - 79:2
8		-.472 (-1.109)	.939 (17.41)	.0004 (.186)	.003 (.879)	.843	1.703	.027	1963:2 - 79:2
9	MDUS	-1.045 (-34.10)		-.089 (-14.23)		.675	.188	.118	1955:1 - 79:2
10		-.011 (-1.249)	.996 (123.6)	-.0007 (-.846)		.998	1.737	.009	1955:2 - 79:2
11		-1.141 (-32.4)		-.059 (-6.857)	.043 (4.508)	.730	.277	.108	1955:1 - 79:2
12		-.024 (-2.368)	.987 (114.6)	-.000 (-.034)	.002 (2.358)	.998	1.813	.009	1955:1 - 79:2
1'	MDDE	-2.237 (-150.9)		-.010 (-2.714)		.261	.722	.021	1960 - 78
2'		-.865 (-2.346)	.602 (3.724)	-.010 (-3.564)		.585	1.600	.016	1961 - 78
3'		-2.258 (-171.7)		-.007 (-2.476)	.010 (3.440)	.549	1.178	.016	1960 - 78
4'		-1.258 (-3.274)	.436 (2.609)	-.008 (-3.203)	.007 (2.069)	.659	1.903	.014	1961 - 78
9'	MDUS	-1.016 (-15.6)		-.098 (-6.958)		.673	.956	.116	1955 - 78
10'		-.027 (-.895)	1.004 (34.94)	.0004 (.115)		.994	1.988	.015	1956 - 78
11'		-1.133 (-16.24)		-.0594 (-3.42)	.058 (2.853)	.758	1.308	.101	1955 - 78
12'		-.039 (.979)	.995 (28.88)	.0007 (.197)	.0017 (.475)	.996	1.991	.015	1956 - 78

demand equations for US M_1 and German central-bank money based on annual data. The UK equation was not estimated using annual data because of the poor quarterly results. Demand functions for the Italian lira were not estimated because adequate short-term interest rate data were unavailable for Italy. The Swiss income approximation, an index of industrial production, was considered too crude to give meaningful results. Germany and the United States alone, however, provide an interesting sub-set of countries.

To preserve consistency with Hamburger's article, I estimated equation (1) in the form (1'):

$$(1') \quad M/Y = a + b \cdot (M/Y)_{t-1} + c \cdot r_d + d \cdot (r_f - r_d),$$

where $r_f - r_d$ is a foreign less the domestic interest rate.

I have explained my motivation for estimating the models using uncovered interest differentials, and for deflating nominal money stocks with nominal income. The results of equation estimates using the quarterly and annual data support the portfolio approach in the case of the United States and Germany. Estimation of the UK equation gives no satisfactory results. In both the dollar and the Deutsche Mark equations the interest differential is significant and of the predicted sign in all formulations of the equation. This is not the case with the level of US interest rates in the dollar demand equations. Inclusion of the lagged dependent variable reduces the significance of the US interest measure.

Graphs 5 and 6 show the marginal contribution of the portfolio variable to the explanatory power of the regression equations estimated without the lagged value of the dependent variable. The graph contains plots of residuals of the two relevant regression equations. In the case of the US dollar equation, the portfolio variable reduces the variability of the regression error terms, but does not explain the post-1971 anomalies in velocity's behaviour as the results of Part II had suggested it might. I am inclined, however, to believe that this failure is due to the specific form of the portfolio model that was estimated. The results of the Deutsche Mark equation are striking since not only is the standard error of the equation significantly smaller in a statistical sense, but the overshooting of money demand in the post-1976 period is largely explicable in terms of the portfolio variable.
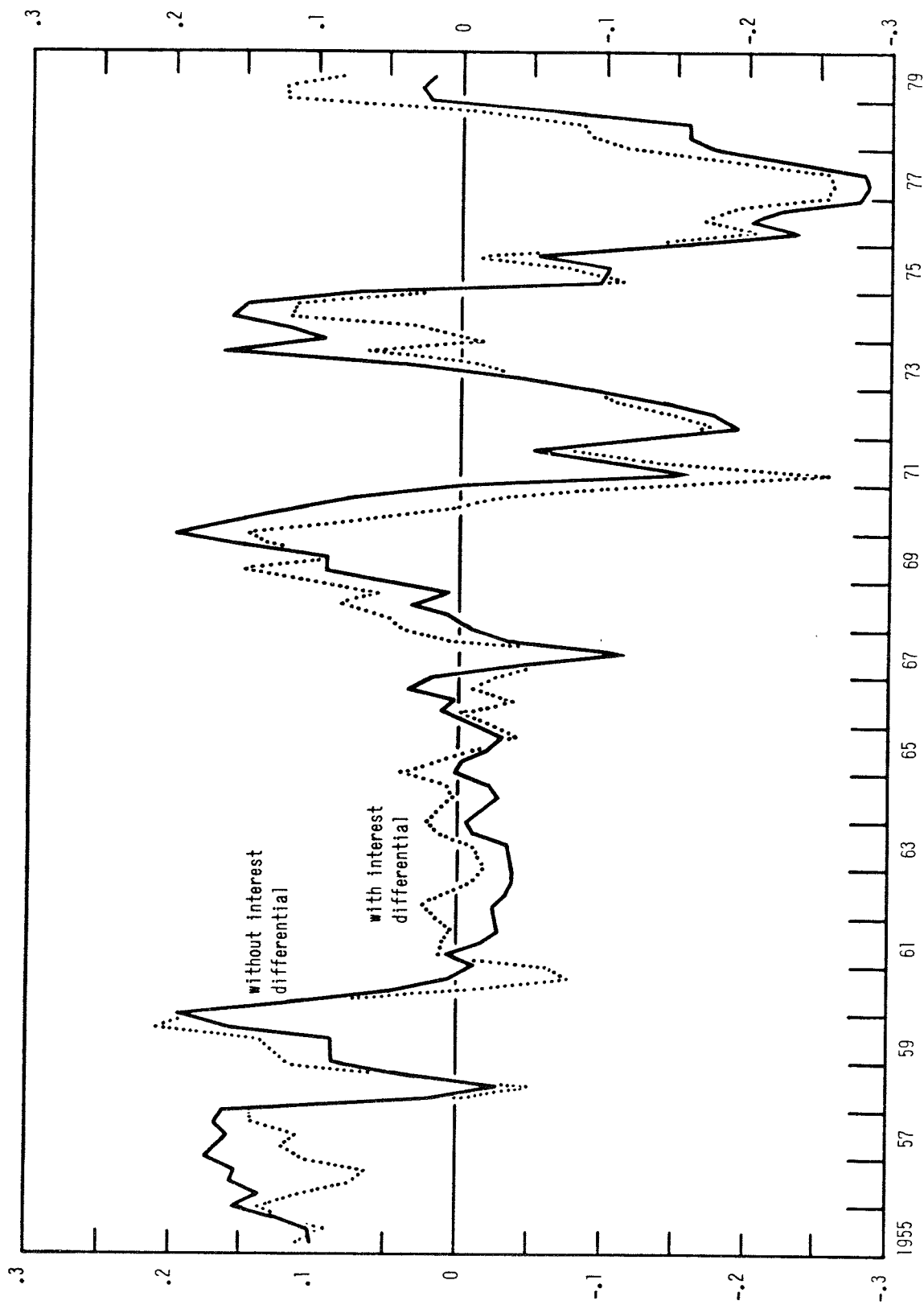
Graph 5

Residuals from German money-demand equations

without interest
differential

with interest
differential
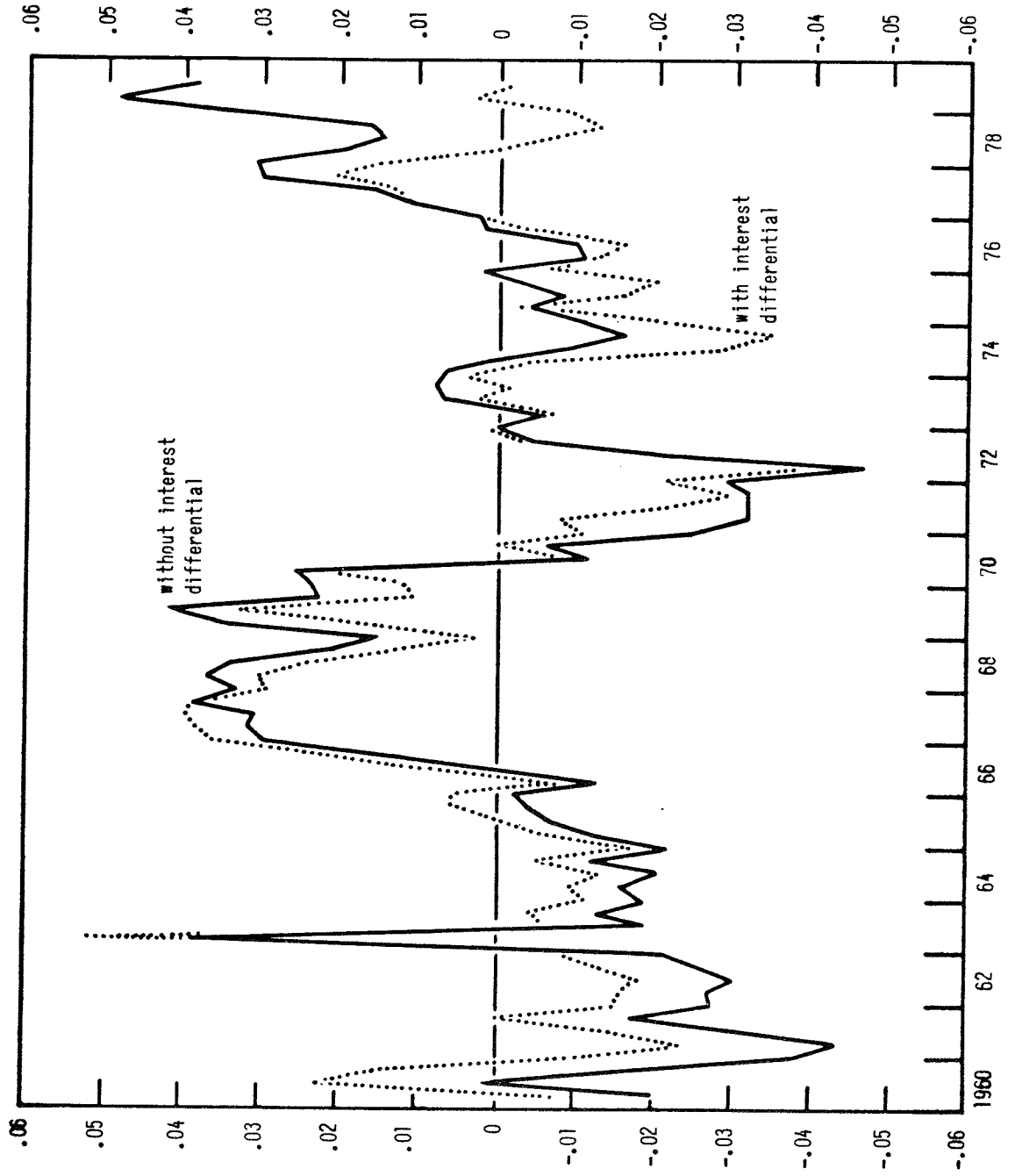
Graph 6

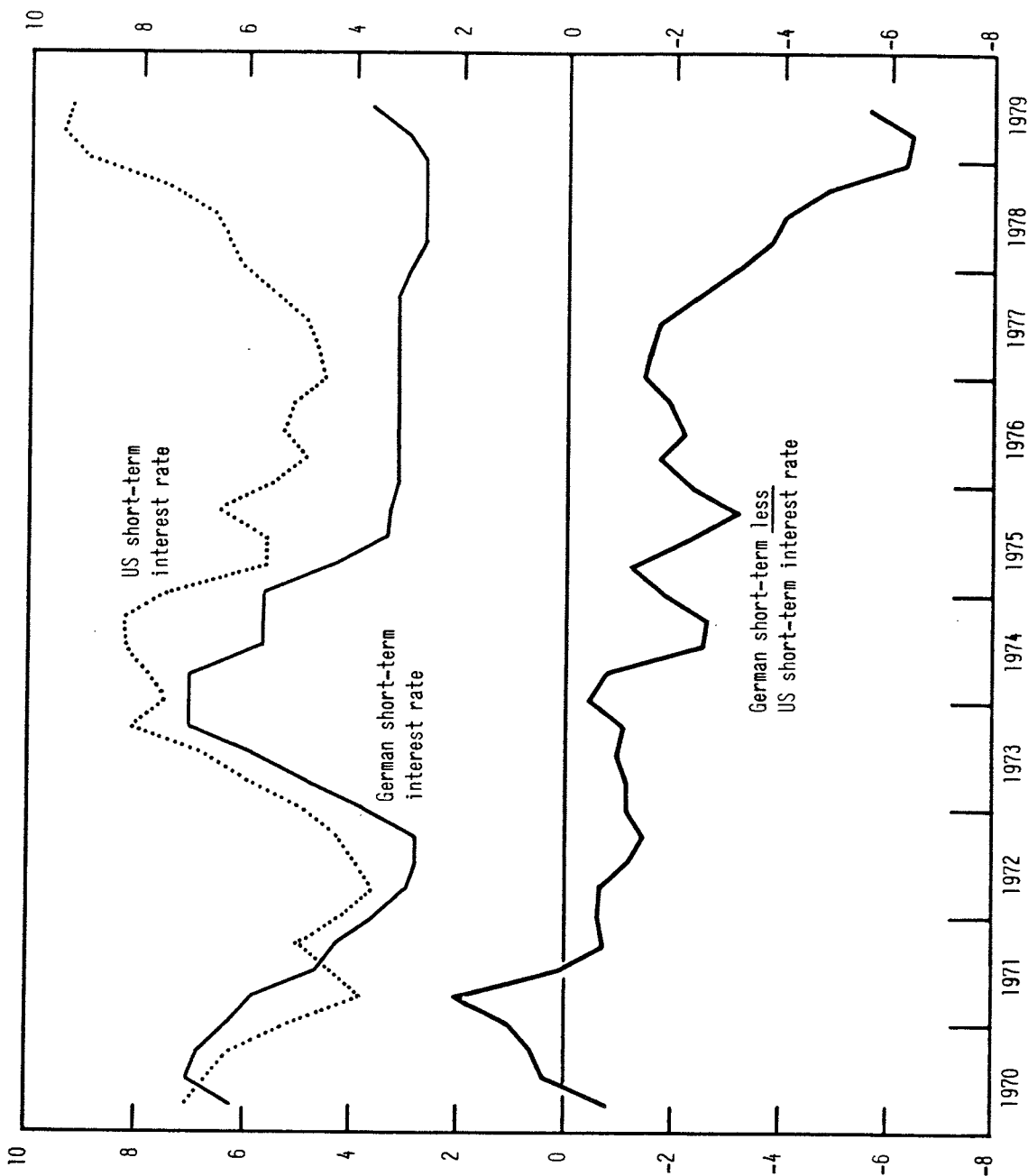Residuals from US money-demand equations

V. Conclusion

These results and Graph 7 - which plots the level of short-term interest rates in the United States and Germany, and the difference between the two - suggest that basic supply conditions expressed in terms of relative opportunity costs and variances of expected rates of return have since 1970 grown slowly to favour the Deutsche Mark. Since 1976 that shift has speeded up drastically. US M_1 velocity movements, while largely determined by factors suggested by Hamburger (1978), have nonetheless been significantly affected by shorter-term fluctuations in the variables reflecting these international portfolio shifts. The German central-bank money's velocity of circulation was equally significantly affected, but particularly so in the period since 1976. During that period, increases in the opportunity cost and in the risk associated with holding dollars have argued strongly in favour of a shift in currency demand towards the Deutsche Mark.

Graph 7

US and German interest rates

US short-term
interest rate

German short-term
interest rate

German short-term less
US short-term interest rate

<u>A note on the data</u>

All data are from the OECD's <u>Main Economic Indicators.</u>
"Money" is narrowly defined money (M_1) for the United States,
sterling M_3 for the United Kingdom, central-bank money for Germany,
M_2 for Italy and narrowly defined money (M_1) for Switzerland. As
measures of income, gross domestic product figures were used for the
United Kingdom and Italy; gross national product for the United States
and Germany; and for Switzerland an index of industrial production
multiplied by the consumer price index.

References

1. Enzler, J., Johnson, L. and Paulus, J., 1976. Some problems of money demand. <u>Brookings Papers on Economic Activity</u> I, 266-280.

2. Hamburger, Michael J., 1977. Behaviour of the money stock: Is there a puzzle? <u>Journal of Monetary Economics</u>, 265-288.

3. Kouri, Pentti J.K. and Braga de Macedo, 1978. Exchange rates and the international adjustment process. <u>Brookings Papers on Economic Activity</u> I, 111-150.

Appendix

The purpose of this appendix is explicitly to derive a variant of the demand-for-money function, estimates of which are discussed in the main body of the text. I have assumed that the individual portfolio holder's motive is simultaneously to minimise the expected opportunity cost on his currency portfolio, subject to his attitudes towards risk and to the cost of holding transaction balances denominated in various currencies.

The derivation of money-demand functions relying on cost minimisation in a situation of uncertainty is equivalent to the problem of maximising the value of a portfolio of risky securities over time, so I shall not go into detail on that aspect of the problem. I would, however, like to discuss the rôle of the income variables in the estimating equations. According to equation (1) in the text, the demand function for any particular currency should, in principle, include the levels of income in all countries. I would like to discuss one simple reason as to why that may be so.

Assuming the portfolio owner to be engaged in international trade, it may be reasonable to suppose that, in addition to the cost-minimising-subject-to-risk aspect of his demand for cash, he will also distinguish among currencies for transaction purposes. He will find it convenient to hold individual currencies as some function of his transactions denominated in that currency. If this is the case, then the costs associated with the various transactions technologies in the areas concerned might be expected to determine optimal - or cost-minimising - shares of various currencies in his portfolio. These shares will have certain values when the variances and covariances of expected returns are all zero, and I shall define these proportions as being the row vector $\bar{A}$ in the following discussion. Costs may be thought of as rising whenever one of the individual components of A, the actual shares, diverge from $\bar{A}$. I assume that these costs rise by amounts that are independent of the sign of the deviation of a_i from $\bar{a}_i$ and of the currency under consideration.

I assume that the cost function is of the form (A1):

(A1) $C = A.R' - .5BAVA' - .5G\bar{A}\bar{A}'$

where: A, R , V and $\bar{A}$ have already been defined,

B is a measure of risk aversion,

G is a measure of the costs associated with deviations
   of the portfolio shares from their zero risk values.

The problem is to derive an expression for A, which I do by maximising
(A1) subject to the construction that Ae', the sum of the portfolio shares,
equals unity (e is a (1 x n) vector of ones). Thus, I differentiate
the Lagrangean function (A2) with respect to A and to L, and set the
partial derivatives to zero.

$$(A2) \quad M = AR' - .5BAVA' - .5GA'A - L(Ae' - 1)$$

$$(A3) \quad \partial M/\partial A = R' - BVA' - G\bar{A} = Le'$$

$$(A4) \quad \partial M/\partial L = Ae' - 1 = 0$$

Equations (A3) and (A4) imply (A5).

$$(A5) \quad \begin{pmatrix} A' \\ -L \end{pmatrix} = \begin{pmatrix} -BV & -e' \\ e & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} R' - G\bar{A} \\ 1 \end{pmatrix}$$

It is easily verified that (A5) has the properties of the
model described by Kouri and Braga de Macedo (1978), for the last
column vector on the right-hand side of the equation.

(A5) implies that a currency's share in the portfolio should
rise as the result of a decline in its opportunity cost, a decline in
the variance of its own expected returns, an increase in the variance
of the other country's opportunity cost or a decrease in its expected
opportunity cost. Similarly, an increase in income in Y_i will increase
the share of M_i depending on how intensively M_i is used in transactions
denominated in terms of the currency of country i.