

BIS Working Papers

No 173

Measuring default risk premia from default swap rates and EDFs

by Antje Berndt, Rohan Douglas, Darrell Duffie, Mark Ferguson and David Schranz

Monetary and Economic Department

March 2005

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Copies of publications are available from:

Bank for International Settlements Press & Communications CH-4002 Basel, Switzerland

E-mail: publications@bis.org

Fax: +41 61 280 9100 and +41 61 280 8100

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ISSN 1020-0959 (print) ISSN 1682-7678 (online)

Foreword

On 9-10 September 2004, the BIS held a workshop on "The pricing of credit risk". This event brought together central bankers, academics and market practitioners to exchange views on this issue (see the conference programme in this document). This paper was presented at the workshop. The views expressed are those of the author(s) and not those of the BIS.

BIS workshop on "The pricing of credit risk" 9-10 September 2004, Basel, Switzerland

Conference programme

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William White, Bank for International Settlements

Paper 1: The credit risk component in corporate spreads and swap spreads

- Authors: David Lando, Copenhagen Business School (joint with Peter Feldhütter)
- Discussants: Tony Rodrigues, Federal Reserve Bank of New York Etienne Varloot, Citigroup

Paper 2: Default and recovery implicit in the term structure of sovereign CDS spreads

| Authors: | Ken Singleton, Stanford University (joint with Jun Pan) |
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| Discussants: | Naohiko Baba, Bank of Japan Richard Cantor, Moody's |

Paper 3: Valuation of a CDO and a n-th to default CDS without Monte Carlo simulation

| ohn Hull, University of Toronto (joint with Alan White) |
|---|
| lichael Gibson, Federal Reserve Board ominic O'Kane, Lehman Brothers |
| |

Paper 4: The pricing of unexpected credit losses

Authors: Jeffery Amato and Eli Remolona, Bank for International Settlements

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Paper 5: Measuring default risk premia from default swap rates and EDFs

Authors: Darrell Duffie, Stanford University (joint with Antje Berndt, Rohan Douglas, Mark Ferguson and David Schranz)

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Paper 6: On the relation between credit spread puzzles and the equity premium puzzle

| Authors: | Pierre Collin-Dufresne, UC Berkeley (joint with Long Chen and Bob Goldstein) |
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|----------|--|

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Paper 7: Explaining the level of credit spreads: option-implied jump risk premia in a firm value model

| Authors: | Joost Driessen, University of Amsterdam (joint with Martijn Pascal Maenhout and David Weinbaum) | Cremers, |
|--------------|---|----------|
| Discussants: | /arqa Khadem, Lehman Brothers Garry Young, Bank of England | |

Measuring Default Risk Premia from Default Swap Rates and EDFs^{*}

Antje Berndt,[†] Rohan Douglas,[‡] Darrell Duffie,[§] Mark Ferguson,[¶] and David Schranz^{\parallel}

First Version: April 16, 2003. Current Version: March 29, 2004

Abstract

This paper estimates recent default risk premia for U.S. corporate debt, based on a close relationship between default probabilities, as estimated by Moody's KMV EDFs, and default swap (CDS) market rates. The default-swap data, obtained through CIBC from 22 banks and specialty dealers, allow us to establish a strong link between actual and risk-neutral default probabilities for the 69 firms in the three sectors that we analyze: broadcasting and entertainment, healthcare, and oil and gas. We find dramatic variation over time in risk premia, from peaks in the third quarter of 2002, dropping by roughly 50% to late 2003.

^{*}We thank CIBC for default swap data; Moody's KMV, Ashish Das, Jim Herrity, Roger Stein, and Jeff Bohn for access to Moody's KMV EDF data; and Moody's Investor Services for a research grant to Antje Berndt that partially supported her work. We are also grateful to Gustavo Manso and Leandro Saita for research assistance, and to Linda Bethel and Sandra Berg for technical assistance.

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1 Introduction

This paper estimates recent default risk premia for U.S. corporate debt, based on a close relationship between default probabilities, as estimated by the Moody's KMV EDF measure, and default swap (CDS) market rates. The default-swap data, obtained by CIBC from 22 banks and specialty dealers, allow us to establish a strong link between actual and risk-neutral default probabilities for the 69 firms in the three sectors that we analyzed: broadcasting and entertainment, healthcare, and oil and gas.

Based on over 49,000 CDS rate quotes, we find that 5-year EDFs explain over 70% of the cross-sectional variation in 5-year CDS rates, after controlling for sectoral and temporal effects. We find that the marginal impact of default probability on credit spreads is significantly greater for high-creditquality firms than for low-credit-quality firms. We also find that, for a given default probability, there is substantial variation over time in credit spreads. For example, after peaking in the third quarter of 2002, credit risk premia declined steadily and dramatically through late 2003, when, for a given default probability, credit spreads were on average roughly 50% lower than at their peak, after controlling for sectoral effects.

If a firm's risk-neutral default intensity λ^* and risk-neutral expected fraction L^* of notional lost at default are assumed to be relatively stable over time, the firm's CDS rate and its par-coupon credit spread would be approximately equal to the risk-neutral mean loss rate, $\lambda^* L^*$, ignoring illiquidity effects.¹

The lowest annual cross-sectional sample mean of loss given default during our sample period was reported by Altman, Brady, Resti, and Sironi (2003) to be approximately 75%. Using 75% as a rough estimate for L^* , our measured relationship between CDS and EDF implies that risk-neutral default intensities are roughly double actual default intensities (proxied by EDFs), on average, although this premium is much higher for high quality

¹This close relationship between risk-neutral mean loss rate and par credit spreads is from Duffie and Singleton (1999). The close relationship between par credit spreads and CDS rates is explained by Duffie (1999). With illiquidty, however, Duffie and Singleton (1999) show a divergence between spreads and risk-neutral mean loss rates. Longstaff, Mithal, and Neis (2003) claim that there are indeed illiquidity effects, measuring yield spreads relative to Treasury yields. Blanco, Brennan, and Marsh (2003) suggest that the illiquidity differences causing bond spreads and default swap rates to diverge are small if using interest-rate swap yields as a benchmark for bond spreads.

firms, and lower for low quality firms. This ratio of risk-neutral to actual default intensities is a default-timing risk premium whose size and behavior over time is a primary objective of our analysis.

This simple estimate of the default risk premium does not consider: (i) the effect of random fluctuations in actual and risk-neutral default intensities, (ii) the potential impact of illiquidity on CDS rates, (iii) variation between actual and risk-neutral mean fractional losses given default, (iv) correlation between fluctuations over time in risk-neutral mean losses given default and risk-neutral default intensities, (v) the effect of cheapest-to-deliver settlement options on default swap rates, and (vi) sample noise. We shall address the impact of each of these later in the paper.

Fons (1987) gave the earliest empirical analysis, to our knowledge, of the relationship between actual and risk-neutral default probabilities. By using *ex-post* default rates and excess returns on corporate debt, he was able to show that, for his data set treating the early 1980s, that market implied risk-neutral default rates were about 5% larger than actual default rates.

Driessen (2002) recently estimated the relationship between actual and risk-neutral default probabilities, using U.S. corporate bond price data (rather than CDS data), and using average long-horizon default frequencies by credit rating (rather than contemporaneous firm-by-firm EDFs). Driessen reported an average risk premium across his data of 1.89, after accounting for tax and liquidity effects, that is roughly in line with the estimates that we provide here. While the conceptual foundations of Driessen's study are similar to ours, there are substantial differences in our respective data sources and methodology. First, the time periods covered are different. Second, the corporate bonds underlying Driessen's study are less homogeneous with respect to their sectors, and have significant heterogeneity with respect to maturity, coupon, and time period. Each of our CDS rate observations, on the other hand, is effectively a new 5-year par-coupon credit spread on the underlying firm that is not as corrupted, we believe, by tax and liquidity effects, as are corporate bond spreads. Most importantly, we do not rely on historical average default rate by credit rating as a proxy for current conditional default intensity.

Because the corporate bonds in Driessen's study involve taxable coupon income, extracting credit spreads required an estimation by him of the portion of the bond yield spread that is associated with taxes. As for the estimated actual default probabilities, Driessen's reliance on average frequency of default for bonds of the same rating rules out conditioning on current market conditions, which Kavvathas (2001) and others have shown to be significant. Reliance on default frequency by rating also rules out consideration of distinctions in default risk among bonds of the same rating. Moody's KMV EDF measures of default probability provide significantly more power to discriminate among the default probabilities of firms (Kealhofer (2003), Kurbat and Korbalev (2002)). Finally, while Longstaff, Mithal, and Neis (2003) and Blanco, Brennan, and Marsh (2003) show that bond yield spreads and CDS rates provide roughly contemporaneous information, our enquiries of market participants have led us to the view that default swaps, because they are "un-funded exposures," in the language of dealers, have rates that are less sensitive to liquidity effects than are bond yield spreads.

Fisher (1959) took a simple regression approach to explaining yield spreads on corporate debt in terms of various credit-quality and liquidity related variables.

Bohn (2000), Delianedis and Geske (1998), G. Delianedis Geske and Corzo (1998), and Huang and Huang (2000) use structural approaches to estimating the relationship between actual and risk-neutral default probabilities, generally assuming that the Black-Scholes-Merton model applies to the asset value process, and assuming constant volatility.

The potential applications of our study are numerous, and include: (i) the relationship between risk and expected return for the credit component of corporate debt, and (ii) analysis of the extent to which the default-risk premia of different firms have common factors, as well as the dynamics and macroeconomics of these common factors. These applications can, in turn, be further applied to a range of pricing and portfolio investment decisions involving corporate credit risk.

An example of our results is illustrated in Figure 1, which shows estimated actual and risk-neutral 1-year default probabilities for Vintage Petroleum, based on EDF and CDS rate data from Moody's KMV and CIBC, respectively. Figure 1 shows the typical pattern in our sample of high default risk premium in the third quarter of 2002.

The remainder of the paper is structured as follows. Section 2 describes our data, including a brief introduction to default swaps and to the construction of the Moody's KMV EDF measure of default probability. Section 3 presents simple descriptive statistical evidence of a strong relationship between CDS rates and EDFs across several sectors. Section 4 introduces a simple approximate time-series model of actual default intensities, and a maximum-likelihood approach to parameter estimation. Section 4 also con-

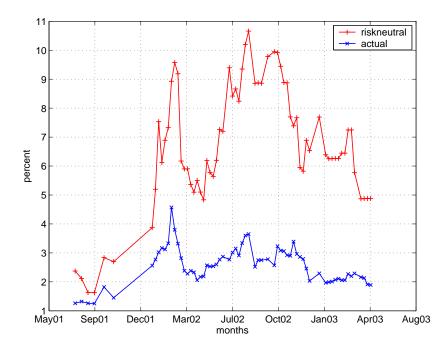


Figure 1: Estimated 1-year default probabilities for Vintage Petroleum. Sources: Moody's KMV and CIBC.

tains parameter estimates for each firm, based on 10 years of monthly observations of 1-year EDFs for each firm. Section 5 provides a reduced-form pricing model for default swaps, based on time-series models of actual and risk-neutral default intensities. Section 5.2 introduces our parameterization of the time-series model for risk-neutral default intensities, using both EDFs and CDS rates. Section 5.3 provides estimates of the parameters for each of the three sectors. Section 6 discusses the results, and then concludes.

2 The EDF and CDS Data

This section discusses our data sources for conditional default probabilities and for default swap rates.

2.1 The EDF Data

Moody's KMV provides its customers with, among other data, current firmby-firm estimates of conditional probabilities of default over time horizons that include the benchmark horizons of 1 and 5 years. For a given firm and time horizon, this "EDF" estimate of default probability is fitted nonparametrically from the historical default frequency of other firms that had the same estimated "distance to default" as the target firm. The distance to default of a given firm is, roughly speaking, the number of standard deviations of annual asset growth by which its current assets exceed a measure of book liabilities. The liability measure is, in the current implementation of the EDF model, equal to the firm's short-term book liabilities plus one half of its long-term book liabilities. Estimates of current assets and the current standard deviation of asset growth ("volatility") are calibrated from historical observations of the firm's equity-market capitalization and of the liability measure. The calibration is based on the model of Black and Scholes (1973) and Merton (1974), by which the price of a firm's equity may be viewed as the price of an option on assets struck at the level of liabilities. Crosbie and Bohn (2002) and Kealhofer (2003) provide more details on the KMV model and the fitting procedures for distance to default and EDF. While one could criticize the EDF measure as an estimator of the "true" conditional default probability, it has a number of important merits for business practice and for our study, relative to other available approaches to estimating conditional default probabilities. First, it is readily available for essentially all public U.S. companies, and for a large fraction of foreign public firms. (There is a private-firm EDF model, which we do not rely on, since our CDS data are for public firms.) Second, while the EDF model is based on a single covariate, distance-to-default, for default prediction, and one might wish to exploit additional covariates (Duffie and Wang (2003), Shumway (2001)), the distance-to-default (DD) covariate has a strong underlying theoretical basis in the Black-Scholes-Merton model, within which DD is a sufficient statistic for conditional default probabilities.

Third, the EDF is fitted non-parametrically to the distance-to-default, and is therefore not especially sensitive, at least on average, to model misspecification. While the measured distance-to-default is itself based on a theoretical option-pricing model, the function that maps DD to EDF is consistently estimated in a stationary setting. That is, conditional on only the distance to default, the measured EDF is equal to the "true" DD-conditional default probability as the number of observations goes to infinity, under typical mixing and other technical conditions for non-parametric qualitativeresponse estimation.

An alternative industry measure of default likelihood is the average historical default frequency of firms with the same credit rating as the target firm. This measure is often used, for example, in implementations of the Credit Metrics approach (www.creditmetrics.com), and is convenient given the usual practice by financial-services firms of tracking credit quality by internal credit ratings based on the approach of the major recognized rating agencies such as Moody's and Standard and Poors. The ratings agencies, however, do not claim that their ratings are intended to be a measure of default probability, and they acknowledge a tendency to adjust ratings only gradually to new information, a tendency strongly apparent in the empirical analysis of Behar and Nagpal (1999), Lando and Skødeberg (2000), Kavvathas (2001), Nickell, Perraudin, and Varotto (2000), among others.

The Moody's KMV EDF measure is also extensively used in the financial services industry. For example, from information provided to us by Moody's KMV, 40 of the world's 50 largest financial institutions are subscribers. Indeed, it is the only widely used name-specific major source of conditional default probability estimates of which we are aware, covering over 26,000 publicly traded firms.

Our basic analysis in Section 3 directly relates daily observations of 5year CDS rates to the associated daily 5-year EDF observations. In order to develop a time-series model of default intensities, however, we turn in Section 4 to monthly observations of 1-year EDFs. By sampling monthly rather than daily, we mitigate equity market microstructure noise, including intra-week seasonality in equity prices, and we also avoid the intra-month seasonality in EDFs caused by monthly uploads of firm-level accounting liability data. By using 1-year EDFs rather than 5-year EDFs, our intensity estimates are less sensitive to model mis-specification, as the 1-year EDF is theoretically much closer to the intensity than is the 5-year EDF.

2.2 Default Swaps and the CDS Database

A default swap, often called, with inexplicable redundancy, a "credit default swap" (CDS), is an over-the-counter derivative security designed to transfer credit risk. With minor exceptions, a default swap is economically equivalent to a bond insurance contract. The buyer of protection pays periodic (usually quarterly) insurance premiums, until the expiration of the contract or until a contractually defined credit event, whichever is earlier. For our data, the stipulated credit event is default by the named firm. If the credit event occurs before the expiration of the default swap, the buyer of protection receives from the seller of protection the difference between the face value and the market value of the underlying debt, less the default-swap premium that has accrued since the last default-swap payment date. The buyer of protection normally has the option to substitute other types of debt of the underlying named obligor. The most popular settlement mechanism at default is for the buyer of protection to submit to the seller of protection debt instruments of the named firm, of the total notional amount specified in the default-swap contract, and to receive in return a cash payment equal to that notional amount, less the fraction of the default-swap premium that has accrued (on a time-proportional basis) since the last regular premium payment date.

The CDS rate is the annualized premium rate, as a fraction of notional. Using an actual-360 day-count convention, the CDS rate is thus four times the quarterly premium. Our observations are at-market, meaning that they are bids or offers of the default-swap rates at which a buyer or seller of protection is proposing to enter into new default swap contracts, without an up-front payment. Because there is no initial exchange of cash flows on a standard default swap, the at-market CDS rate is, in theory, that for which the net market value of the contract is zero. In practice, there are implicit dealer margins that we treat by assuming that the average of the bid and ask CDS rates is the rate at which the market value of the default swap is indeed zero.

For the purpose of settlement of default swaps, the contractual definition of default normally allows for bankruptcy, a material failure by the obligor to make payments on its debt, or a restructuring of its debt that is materially adverse to the interests of creditors. The inclusion, or not, of restructuring as a covered default event has been a question of debate among the community of buyers and sellers of protection. ISDA, the industry coordinator of standardized OTC contracts (www.isda.org), has arranged a consensus for a standardized contractual definition of default that, we believe, is likely to be reflected in most of our data. This consensus definition of default has been adjusted over time, and to the extent that these adjustments during our observation period are material, or to the degree of heterogeneity in our data over the definition of default that is applied, our results could be affected. The contractual definition of default can affect the estimated risk-neutral implied default probabilities, since of course a wider definition of default implies a higher risk-neutral default probability.

If restructuring is included as a contractually covered credit event, then there is the potential for significant heterogeneity at default in the market values of the various debt instruments of the obligor, as fractions of their respective principals, especially when there is significant heterogeneity with respect to maturity. The resulting cheapest-to-deliver option can therefore increase the loss to the seller of protection in the event of default. Without, at this stage, data bearing on the heterogeneity of market value of the pool of deliverable obligations for each default swap, we are in effect treating the cheapest-to-deliver option value as a constant that is absorbed into the estimated risk-neutral fractional loss L^* to the seller of protection in the event of default. While we vary L^* as a parameter, we generally assume that L^* is constant across the sample. To the extent that L^* varies over time or across issuers, our implied risk-neutral default probabilities would be corrupted. This is not crucial, as we shall show, when modeling the CDS rates implied by a given EDF. This robustness also applies to the mark-to-market pricing of old default swaps, which is an increasingly important activity, given that the notional amount of debt covered by default swaps is almost doubling each year, and is expected to reach 4 trillion U.S. dollars in 2004, according to the British Bankers Association (www.bba.org).

For a given level of seniority (our data are based on senior unsecured debt instruments), there is less recovery-value heterogeneity if the event of default is bankruptcy or failure to pay, for these events normally trigger cross-acceleration covenants that cause debt of equal seniority to convert to immediate obligations that are *pari passu*, that is, of equal priority. In any case, the option held by the buyer of protection to deliver from a list of debt instruments will cause the effective fractional loss given default to the seller of protection to be the maximum fractional loss given default of the underlying list of debt instruments. If restructuring is included as a covered default event, the impact of this cheapest-to-deliver option is, within the current "modified-modified" ISDA standard contract, mitigated by a contractual restriction on the types of deliverable debt instruments, especially with respect to maturity.

Ignoring the cheapest-to-deliver effect, the CDS rate is, in frictionless markets, extremely close to the par-coupon credit spread of the same maturity as the default swap, as shown by Duffie (1999). Our results thus speak to the relationship between EDFs and corporate credit spreads. We are told

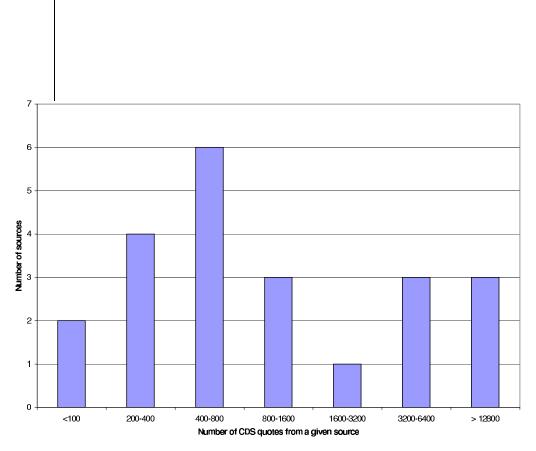


Figure 2: Distribution of CDS quote providers by number of quotes provided

by market participants that asset swaps, synthetic approximations of parcoupon bonds, as explained in Duffie (1999), trade at "par spreads" that are, on average, becoming closer to CDS rates as the CDS market matures and grows in volume, liquidity, and transparency. This is confirmed to some extent in empirical studies by Longstaff, Mithal, and Neis (2003) and Blanco, Brennan, and Marsh (2003), provided one measures bond spreads relative to interest-rate swap yields.

Our CIBC data set consists of over 49,000 intra-day CDS rate quotes on 69 firms from three Moody's industry groups. The sources of these quotes include 12 investment banks and 10 default-swap brokers. The cross-sectional concentration of the number of quotes by source is shown in Figure 2. A breakdown of the number of quotes by banks and by default-swap brokers is given in Table 1.

The three representative Moody's industry groups that we selected for analysis are North American Broadcasting and Entertainment, North Amer-

| | Total | Median | Average | Sources | Min | Max |
|---------|-------|--------|---------|---------|-----|------|
| Banks | 34462 | 871 | 2872 | 12 | 116 | 9523 |
| Brokers | 15030 | 620 | 1503 | 10 | 147 | 5431 |
| All | 49492 | 702 | 2250 | 22 | 116 | 9523 |

Table 1: Breakdown of number of CDS quotes by type of source

ican Oil and Gas, and North American Healthcare. The quotes are all for 5-year, quarterly premium, senior unsecured, US-Dollar-denominated, atthe-money default swaps. A company from any of these 3 sectors is included in our study if and only if at least 50 historical CDS bid/ask quotes for that firm were available during the sample period. The range of credit qualities of the included firms may be judged from Figure 3, which shows, for each credit rating, the number of firms in our study of that median Moody's rating during the sample period. Figure 3 indicates a concentration of Baa-rated firms.

Daily CDS mid-point rate quotes were generated from intra-day bid and ask quotes using the following algorithm.

- 1. If a bid and an ask were present, we record the bid-ask spread.
- 2. If the bid is missing, we subtract the average bid-ask spread to estimate the ask.
- 3. If the ask is missing, we add the average bid-ask spread to estimate the bid.
- 4. From the resulting bid and ask, we calculate the mid-quote as the average of the bid and ask quotes.

The firms that we studied from the broadcasting-and-entertainment industry are listed in Table 2, along with their median 1-year EDF, median Moody's credit rating during the sample period, and the number of CDS quotes available for each. The same information covering firms from the healthcare and oil-and-gas industries is provided in Tables 8 and 9 of Appendix C, respectively.

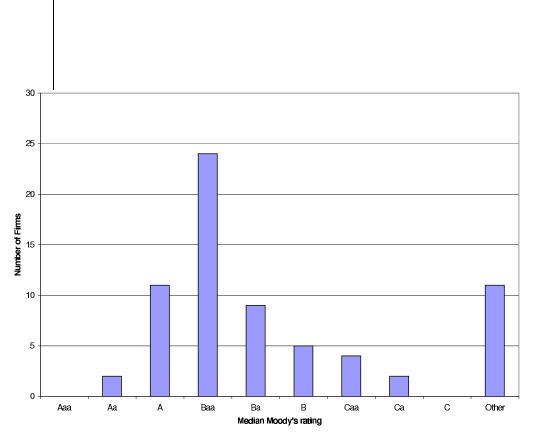


Figure 3: Distribution of firms by median credit rating during the sample period. Source: CIBC.

3 Descriptive EDF-CDS Model

A simple preliminary linear model of the relationship between a firm's 5-year CDS (Y_i) and the 5-year EDF (X_i) measured in basis points on the same day is

$$Y_i = 52.26 + 1.627X_i + e_i,$$
(1)
(1.58) (0.007)

where X_i is the observed 5-year EDF of a given firm on a given day, Y_i is an observed CDS rate of the same firm on the same day, e_i is a random disturbance. Standard errors are shown parenthetically. The ordinary-leastsquares coefficient estimates are based on 18,259 paired EDF-CDS observations from September 2000 to August 2003, with most observations during 2002.

The associated coefficient of determination, R^2 , is 0.718. Figure 4 illustrates the fit of (1), for all firms in our study, and all time periods. The 5-year

| Name of Firm | Median EDF | Median Rating | No. Quotes |
|------------------------------|----------------|---------------|------------|
| | (basis points) | | |
| Adelphia Communications | 349 | B2 | 279 |
| AOL Time Warner | 11 | N/A | 3447 |
| Charter Communications | 281 | B3 | 444 |
| Clear Channel Communications | 29 | Baa3 | 1698 |
| Comcast | 39 | Baa3 | 1043 |
| COX Communications | 36 | Baa1 | 2153 |
| Insight Communications | 173 | B3 | 303 |
| Liberty Media | 21 | Baa3 | 515 |
| Mediacom Communications | 286 | Caa1 | 168 |
| Primedia | 65 | Ba3 | 325 |
| Royal Caribbean Cruises | 134 | Baa2 | 462 |
| Viacom | 13 | Baa1 | 2458 |
| Walt Disney | 7 | N/A | 2745 |

Table 2: Broadcasting and Entertainment Firms

CDS rate is thus estimated to increase by approximately 16 basis points for each 10 basis point increase in the 5-year EDF. If one were to take the riskneutral expected loss given default to be, say, 75% and the default intensities (actual and risk-neutral) to be constant, this would imply an average ratio of risk-neutral to actual default intensity φ of approximately (16/0.75)/10, or 2.0.

Linearity of the CDS-EDF relationship, however, is placed in doubt by the sizable intercept estimate of roughly 50 basis points, more than 30 times its standard error. Absent an unexpectedly large liquidity impact on CDS rates, the fitted default swap rate should be closer to zero at low levels of EDF. While there may be mis-specification due to the assumed homogeneity of the relationship over time and across firms, we have verified with sector and quarterly regressions that the associated intercept estimates are unreasonably large in magnitude. We also noted that scatter plots of the CDS-EDF relationship indicated a pronounced concavity at low levels of EDF. That is, the sensitivity of credit spreads to a firm's estimated default probability seems to decline at larger levels of default risk. There is also apparent heteroskedasticity; with greater variance for higher EDFs. The slope of the fit illustrated in Figure 4 is thus heavily influenced by the CDS-to-EDF relationship for

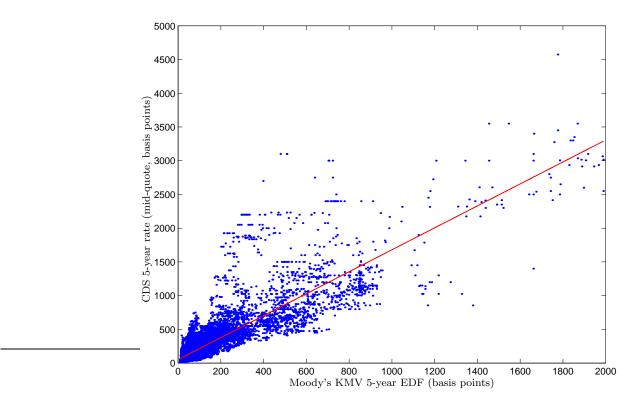


Figure 4: Scatter plot of EDF and CDS observations and OLS fitted relationship. Source: CIBC (CDS) and Moody's KMV (EDF).

lower-quality firms. Accordingly, we next turned to the specification²

$$\log Y_i = \alpha + \beta \log X_i + z_i,\tag{2}$$

for coefficients α and β , and a residual z_i . The fit, illustrated in Figure 5, shows much less heteroskedasticity, although some potential corruption from the granularity of EDFs of extremely high-quality firms.

We have taken CDS rate observations (Y_i) by two approaches: *(i)* the daily median CDS for each given name, and *(ii)* all CDS observations for that day. The second approach, which has substantially more CDS observations

²We also examined the fit, by non-linear least squares, of the model, $Y_i = \alpha X_i^{\beta} + u_i$, which differs from (2) by having a residual that is additive in levels, rather than additive in logs. An informal comparison shows that the non-linear least-squares model is somewhat preferred for lower-quality firms.

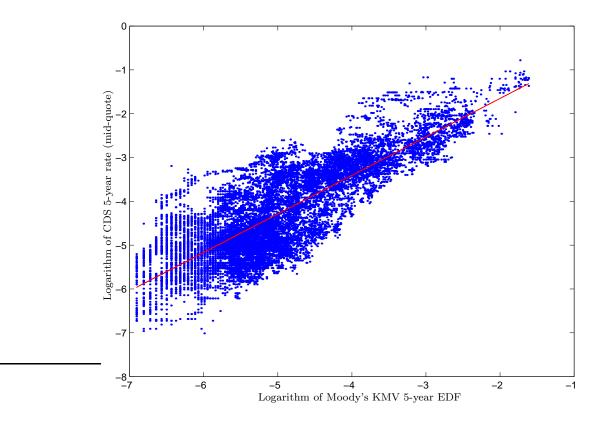


Figure 5: Scatter plot of EDF and CDS observations, logarithmic, and OLS fitted relationship. Source: CIBC (CDS) and Moody's KMV (EDF).

per EDF observation, has by construction a lower coefficient of determination (R^2) , and is likely to have more precise estimates of the intercept and slope coefficients, a and b. (This is necessarily so if the model is correctly specified.) It is from this model with more observations that we would thus anticipate getting a more precise notion of how CDS rates are related to EDFs.³ All of our regressions were fit by ordinary least squares.

We anticipate that practitioners will fit CDS rates to EDFs along these lines a tool for marking-to-market positions for which liquid CDS quotes are

³Technically, the two cases (daily median CDS observations, and all CDS observations) would not both be consistent with equation (2), since the median is an order statistic that depends on sampling noise in a non-linear fashion. We prefer, in any case, the median to the average daily CDS observation as we believe it to be more robust to outliers induced by observation noise.

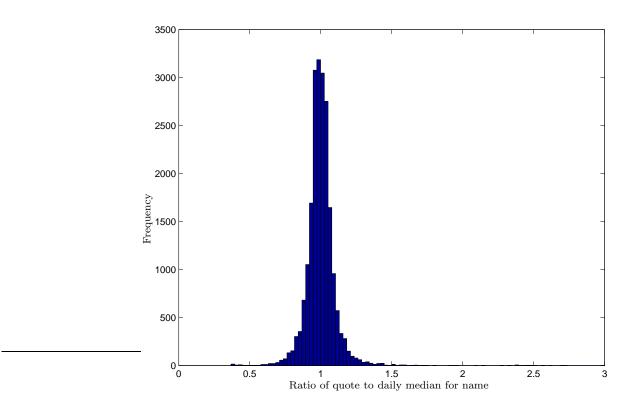


Figure 6: Intraday distribution of ratio of five-year CDS bids to median bid, after removing the median bids. Source: CIBC.

not always available.

Figure 6 shows a histogram of the ratio of quotes to the daily median quote for the same name, after removing the points associated with the median quote itself (of which there are approximately 19,500). The plot shows substantial intraday variation in CDS quotes of a given name.

One might have considered a model in which the CDS rate is fit to both 5-year and 1-year EDF observations, given the potential for additional influences of near-term default risk on CDS rates. We have found, however, that the 1-year and 5-year EDFs are highly correlated, presumably because they are both images of the same covariate, distance to default. As might be expected, adding 1-year EDFs to the regression has no major impact on the quality of fitted CDS rates, and involves substantial noise in the slope coefficients, presumably from multi-collinearity. We do not report the results for

the multiple regressions. In any case, the 5-year EDF captures the average effect of default risk over the 5-year period, as does the CDS. This is not to suggest, however, that default risk premia implicit in the CDS rates necessarily have the same term structure. We have little information about this term structure to report at this time. (We plan to later analyze short-maturity CDS data.)

Moving from a pooled regression, we control for changes in the CDSto-EDF relationship across time and across sectors. Table 10, found in the appendix, presents the results of a regression of the logarithm of the daily median CDS rate on the logarithm of the associated daily 5-year EDF observation (18, 259 observations in all), including dummy variables for sectors and months. For example, extracting from Table 10 the fit implied for the oil-and-gas sector, we have

$$\log \text{CDS}_{i} = 0.912 + 0.828 \log \text{EDF}_{i} + \sum \hat{\beta}_{j} D_{\text{month } j}(i) + z_{i}, \quad (3)$$
(0.029) (0.004)

where $\hat{\beta}_j$ denotes the estimate for the dummy multiplier for month j, with j running from December 2000 through August 2003, and z_i denotes the residual. We obtain an R^2 of about 74%. From the one-standard-deviation confidence band implied by normality of the residuals for the logarithmic fit, the associated confidence band for a given CDS rate places it between 58% and 171% of the fitted rate.

From the dummy coefficient estimate for the healthcare sector, the CDS rate for a healthcare firm is estimated to be 20% higher than that of an oiland-gas firm with the same EDF. A broadcasting-and-entertainment firm is estimated to have a 39% higher CDS than an oil-and-gas firm with the same reported EDF. As one can see from Figure 7, showing selected Moody's average sectoral default recoveries for 1982 to 2003, some of these sectoral spread-to-EDF differences are due to sectoral differences in default recovery. For example, assuming that the ratio of the risk-neutral mean loss given default in the oil-and-gas sector to another sector is the same as the ratio of the empirical average loss given default, then broadcasting-entertainment spreads would be approximately 62%/52% - 1 = 19% higher than oil-andgas sector, for equal risk-neutral default probabilities. Similarly, healthcare spreads would be approximately 67%/52% - 1 = 29% higher than oil-and-gas sector, for equal risk-neutral default probabilities.⁴

⁴From the Moody's sectoral data, the average recovery for the oil and gas sector is

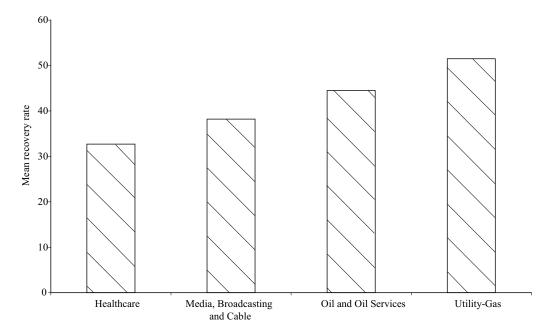


Figure 7: Sectoral differences in average default recovery, 1982-2003. Source: Moody's Investor Services.

The fitted model also shows highly significant variation in risk premia across the months of 2002 and 2003, with the highest risk premia during the third quarter of 2002, when, for a given EDF, spreads are estimated to have been roughly 50% higher than they were in August, 2003. Figure 8 illustrates this variation over time with a plot of the dummy variables of the regression model 3, indicating the percentage increase in CDS rates at a given EDF assocaited with each month. Figure 9 tells a similar story with a plot of the weekly average of the daily median, by sector, of the ratio of the 5-year CDS rate to the 5-year EDF. This index of default risk premium peaks, for every sector, during July and August of 2002.

estimated from the simple average of the of the Moody's "Oil and Oil Services" and the "Utility-Gas" sectors, at 48%. Broadcasting and Entertainment recoveries are estimated at the 'Media Broadcasting and Cable' average of 38%, and Healthcare at 32.7%.

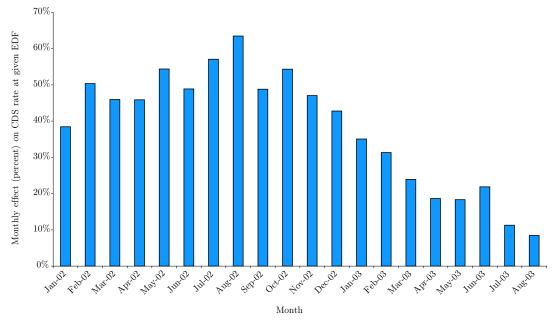


Figure 8: Monthly dummy multipliers in CDS-to-EDF fit.

4 Actual Default Intensity from EDF

The default intensity of an obligor is the instantaneous mean arrival rate of default, conditional on all current information. To be slightly more precise, we suppose that default for a given firm occurs at the first event time of a (non-explosive) counting process N with intensity process λ , relative to a given probability space (Ω, \mathcal{F}, P) and information filtration $\{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions. In this case, so long as the obligor survives, we say that its default intensity at time t is λ_t . Under mild technical conditions, this means that, conditional on survival to time t and all information available at time t, the probability of default between times t and t + h is approximately $\lambda_t h$ for small h. We also adopt the relatively standard simplifying doubly-stochastic, or Cox-process, assumption, under which the conditional probability at time t, for a currently surviving obligor, that the obligor survives to some later time T, is

$$p(t,T) = E\left(e^{-\int_t^T \lambda(s) \, ds} \mid \mathcal{F}_t\right). \tag{4}$$

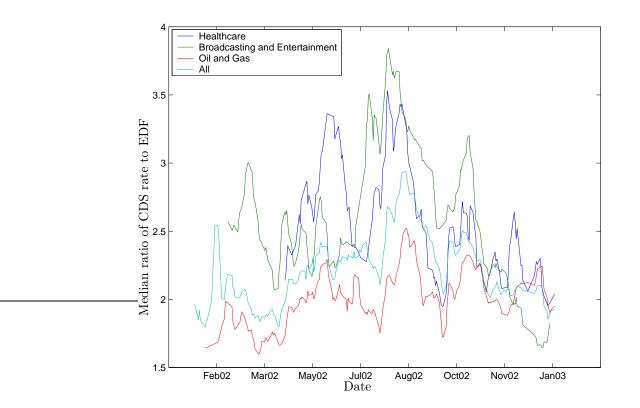


Figure 9: Weekly average of sector-median 5-year CDS-to-EDF ratio.

For our analysis, we ignore mis-specification of the EDF model itself, by assuming that 1 - p(t, t + 1) is indeed the current 1-year EDF. From the Moody's KMV data, then, we observe p(t, t + 1) at successive dates t, t + h, $t + 2h, \ldots$, where h is one month. From these observations, we estimate a time-series model of the underlying intensity process λ , for each firm. In total, we analyzed 69 firms.

After some preliminary diagnostic analysis of the EDF data set, we opted to specify a model under which the logarithm $X_t = \log \lambda_t$ of the default intensity satisfies the Ornstein-Uhlenbeck equation

$$dX_t = (a - \kappa X_t) dt + \sigma \, dB_t,\tag{5}$$

where B is a standard Brownian motion, and a, κ , and σ are constants to be estimated. The behavior for $\lambda = e^X$ is sometimes called a Black-Karasinski model.⁵ This leaves us with a vector $\Theta = (a, \kappa, \sigma)$ of unknown parameters to estimate from the available monthly EDF observations of a given firm. We have 123 months of 1-year EDF observations for most of the firms in our sample, for the period January, 1993 to August, 2003.

In general, given the log-autoregressive form of the default intensity in (5), there is no closed-form solution available for the 1-year EDF, 1 - p(t, t + 1)from (4). We therefore rely on numerical lattice-based calculations of p(t, t + 1). We have implemented the two-stage procedure for constructing trinomial trees proposed by Hull and White (1994), as well as a more rapid algorithm, explained in the Appendix B, based on approximation of the solution in terms of a basis of Chebyshev polynomials. (Our current parameter estimates are for the trinomial-tree algorithm.)

The maximum likelihood estimator (MLE) Θ of the parameter vector Θ is then obtained, firm by firm, using a fitting algorithm described in the appendix. That is, for a given firm, $\hat{\Theta}$ solves

$$\sup_{\Theta} \quad \mathcal{L}\left(\left\{1 - p(t_i, t_i + 1) : 1 \le i \le N\right\}; \Theta\right),\tag{6}$$

where t_1, t_2, \ldots, t_N are the N observation times for the given firm, and \mathcal{L} denotes the likelihood score of observed EDFs given Θ . This is not a routine MLE for a discretely-observed Ornstein-Uhlenbeck model, for several reasons:

1. Evaluation of the likelihood score requires a numerical differentiation of the modeled EDF,

$$G(\lambda(t);\Theta) = 1 - E_{\Theta} \left(e^{-\int_{t}^{t+1} \lambda(s)} ds \mid \lambda(t) \right),$$
(7)

where E_{Θ} denotes expectation associated with the parameter vector Θ .

- 2. As indicated by Kurbat and Korbalev (2002), Moody's KMV caps its 1-year EDF estimate at 20%. Since this truncation, if untreated, would bias our estimator, we explicitly account for this censoring effect on the associated conditional likelihood, as explained in Appendix A.
- 3. Moody's KMV also truncates the EDF below at 2 basis points. Moreover, there is a significant amount of integer-based granularity in EDF data below approximately 10 basis, as indicated in Figure 5. We therefore remove from the sample any firm whose sample-mean EDF is below 10 basis points.

⁵See Black and Karasinski (1991).

- 4. There were occasional missing data points. These gaps were also treated exactly, assuming the event of censoring is independent of the underlying missing observation.
- 5. For a small number of firms, an exceptional 1-month fluctuation in the 1-year EDF generated an obviously unrealistic estimate of the meanreversion parameter κ for that company. We ignored Enron's data point for December 2002, the month it defaulted. Similarly, Magellan Health Services filed for protection under Chapter 11 in March 2003 (we used the EDFs through February 2003), and Adelphia Communications petitioned for reorganization under Chapter 11 in June 2002 (we used the EDFs through May 2002). For Forest Oil, we ignored the outlier months of January and February 1993. Finally, we removed Dynergy from our data set as its 1-year EDF is capped at 20% for most of 2002 and 2003.

We have not imposed a joint distribution of EDFs across firms. It could be feasible, for example, to impose joint normality of the Brownian motions driving each firm's EDFs, with a specified cross-firm correlation structure. (This is planned for subsequent research.)

Table 3 lists the firms for which we have EDF data, showing the number of monthly observations for each as well as the number of EDF observations that were truncated at 20%. Frequency plots of the estimated volatility and mean-reversion coefficients, σ and κ , are shown in Figures 11 and 10, respectively. The estimated parameter vector for each firm is provided in Table 12, found in Appendix C.

One notes significant dispersion across firms in the estimated parameters. Monte-Carlo analysis revealed substantial small-sample bias in the MLE estimators, especially for mean reversion. We therefore obtained sector-by-sector estimates for κ and σ , shown in Table 4, while allowing for a firm-specific drift parameter *a*, listed in Table 13 in Appendix C.⁶

5 Risk-Neutral Intensity from CDS and EDF

This section explains our methodology for extracting risk-neutral default intensities, and probabilities, from CDS and EDF data.

⁶The intensity λ is measured in basis points.

| Ticker | uncensored | capped at 20% | total | Ticker | uncensored | capped at 20% | total |
|---------------|------------|------------------|-------|----------------|------------|------------------|-------|
| ABC | 92 | 0 | 92 | HAL | 123 | 0 | 123 |
| ABT | 123 | 0 | 123 | HCA | 123 | 0 | 123 |
| ADELQ | 97 | 16 | 113 | HRC | 123 | 0 | 123 |
| AGN | 123 | 0 | 123 | HUM | 121 | 0 | 121 |
| AHC | 123 | 0 | 123 | ICCI | 44 | 0 | 44 |
| AMGN | 123 | 0 | 123 | JNJ | 123 | 0 | 123 |
| AOL | 123 | 0 | 123 | KMG | 123 | 0 | 123 |
| APA | 123 | 0 | 123 | KMI | 123 | 0 | 123 |
| APC | 123 | 0 | 123 | KMP | 121 | 0 | 121 |
| BAX | 123 | 0 | 123 | \mathbf{L} | 81 | 0 | 81 |
| BEV | 121 | 2 | 123 | LLY | 123 | 0 | 123 |
| BHI | 123 | 0 | 123 | MCCC | 35 | 0 | 35 |
| BJS | 123 | 0 | 123 | MDT | 123 | 0 | 123 |
| BMY | 123 | 0 | 123 | MGLH | 103 | 19 | 122 |
| \mathbf{BR} | 123 | 0 | 123 | MRO | 123 | 0 | 123 |
| BSX | 123 | 0 | 123 | NBR | 123 | 0 | 123 |
| CAH | 123 | 0 | 123 | NEV | 123 | 0 | 123 |
| CAM | 92 | 0 | 92 | OCR | 123 | 0 | 123 |
| CCU | 123 | 0 | 123 | OEI | 123 | 0 | 123 |
| CHIR | 123 | 0 | 123 | OXY | 123 | 0 | 123 |
| CHK | 109 | 13 | 122 | PDE | 123 | 0 | 123 |
| CHTR | 33 | 7 | 40 | PKD | 123 | 0 | 123 |
| CMCSA | 123 | 0 | 123 | \mathbf{PRM} | 86 | 3 | 89 |
| CNG | 85 | 0 | 85 | PXD | 123 | 0 | 123 |
| COC | 45 | 0 | 45 | RCL | 120 | 0 | 120 |
| COP | 123 | 0 | 123 | RIG | 119 | 0 | 119 |
| COX | 95 | 0 | 95 | SBGI | 92 | 0 | 92 |
| CVX | 123 | 0 | 123 | THC | 123 | 0 | 123 |
| CYH | 76 | 0 | 76 | TLM | 123 | 0 | 123 |
| DCX | 53 | 0 | 53 | TRI | 46 | 0 | 46 |
| DIS | 123 | 0 | 123 | TSO | 123 | 0 | 123 |
| DO | 86 | 0 | 86 | VIA | 123 | 0 | 123 |
| DVN | 123 | 0 | 123 | VLO | 123 | 0 | 123 |
| DYN | 0 | 0 | 0 | VPI | 123 | 0 | 123 |
| ENRNQ | 106 | 1 | 107 | WFT | 123 | 0 | 123 |
| EP | 122 | 1 | 123 | WLP | 123 | 0 | 123 |
| F | 123 | 0 | 123 | WMB | 115 | 8 | 123 |
| FST | 121 | 0 | 121 | WYE | 123 | 0 | 123 |
| GENZ | 123 | 0 | 123 | YBTVA | 99 | 0 | 99 |
| GM | 123 | 0 | 123 | | | - | |

Table 3: Number of observations of 1-year EDFs. Data: Moody's KMV.

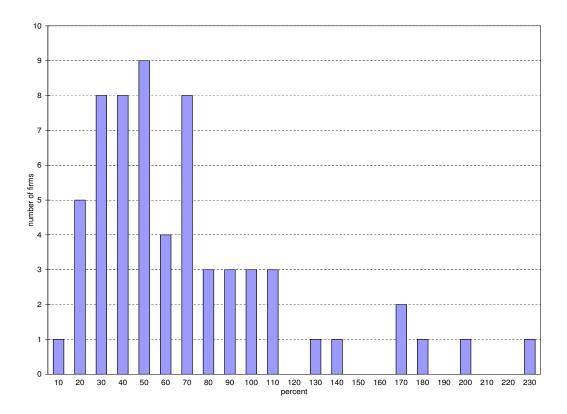


Figure 10: Distribution of estimated default intensity mean-reversion parameters (κ).

5.1 Default Swap Pricing

We begin with a simple reduced-form arbitrage-free pricing model for default swaps. Under the absence of arbitrage and market frictions, and under mild technical conditions, there exists a "risk-neutral" probability measure, also known as an "equivalent martingale" measure, as shown by Harrison and Kreps (1979) and Delbaen and Schachermayer (1999). In our setting, markets should not be assumed to be complete, so the martingale measure is not unique. This pricing approach nevertheless allows us, under its conditions, to express the price at time t of a security paying some amount, say W, at some stopping time $\tau > t$, of

$$S_t = E^Q \left(e^{-\int_t^\tau r(u) \, du} \, W \, \middle| \, \mathcal{F}_t \right), \tag{8}$$

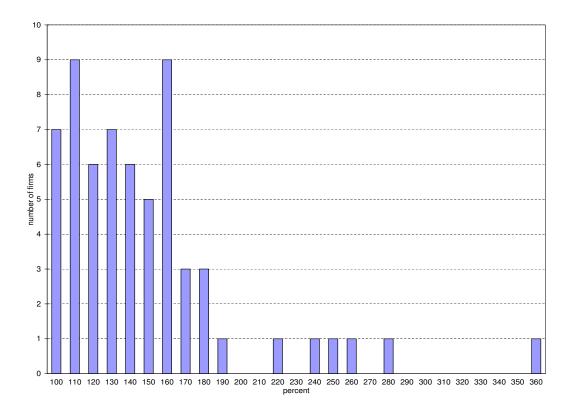


Figure 11: Distribution of estimated default intensity volatility parameters (σ) .

where r is the short-term interest-rate process⁷ and E^Q denotes expectation with respect to an equivalent martingale measure Q, that we fix. One may view (8) as the definition of such a measure Q. The idea is that the actual measure P and the risk-neutral measure Q differ by an adjustment for risk premia.

Under our earlier assumption of default timing according to a default intensity process λ (under the actual probability measure P that generates our

⁷Here, r is a progressively measurable process with $\int_0^t |r(s)| ds < \infty$ for all t, such that there exists a "money-market" trading strategy, allowing investment at any time t of one unit of account, with continual re-investment until any future time T with a final value of $e^{\int_t^T r(s) ds}$.

Table 4: Sector EDF-implied default intensity parameters.

| | $\operatorname{mean}(\hat{\theta})$ | $\hat{\kappa}$ | $\hat{\sigma}$ | no. firms |
|--------------------------------|-------------------------------------|----------------|----------------|-----------|
| Oil and Gas | 3.3309 | 0.4663 | 1.2910 | 32 |
| Healthcare | 3.3784 | 0.6559 | 1.5123 | 17 |
| Broadcasting and Entertainment | 4.2625 | 0.7082 | 1.6372 | 14 |

data), Artzner and Delbaen (1992) show that there also exists a default intensity process λ^* under Q. Even though we have assumed the double-stochastic property under P, this need not imply the same convenient double-stochastic property under Q as well. Indeed, Kusuoka (1999) gave a counterexample. We will nevertheless assume the double-stochastic property under Q. (Sufficient conditions are given in Duffie (2001), Appendix N.) Thus, we have

$$Q(\tau > T \mid \mathcal{F}_t) = p^*(t, T) = E^Q \left(e^{-\int_t^T \lambda^*(u) \, du} \mid \mathcal{F}_t \right), \tag{9}$$

provided the firm in question has survived to t.

For convenience, we assume independence, under Q, between interest rates on the one hand, and on the other the default time τ and loss given default. We have verified that, except for levels of volatility of r and λ^* far in excess of those for our sample, the role of risk-neutral correlation between interest rates and default risk is in any case negligible for our parameters. This is not to suggest that the magnitude of the correlation itself is negligible. (See, for example, Duffee (1998).) It follows from (8) and this independence assumption that the price of a zero-coupon defaultable bond with maturity T and zero recovery at default is given by

$$d(t,T) = \delta(t,T)p^*(t,T), \tag{10}$$

where $\delta(t,T) = E_t^Q \left(e^{-\int_t^T r(s) ds} \right)$ is the default-free market discount and $p^*(t,T)$ is the risk-neutral conditional survival probability of (9).

Extensions to the case of correlated interest rates and default times are treated, for example, in Lando (1998).

A default swap stipulates quarterly payments by the buyer of protection of premiums at an annual rate of c, as a fraction of notional, until the defaultswap maturity or default, whichever is first. From (10), the market value of the payments by the buyer of protection at the origination date of a default swap of unit notional size is thus cg(t), where

$$g(t) = \frac{1}{4} \sum_{i=1}^{n} \delta(t, t(i)) p^*(t, t(i)),$$
(11)

for premium payment dates $t(1), \ldots, t(n)$. The market value of the potential payment by the seller of protection on this default swap is

$$h(t,c) = E^{Q} \left(\delta(t,\tau) W_{\tau}^{c} \mathbf{1}_{\tau \le t(n)} \mid \mathcal{F}_{t} \right),$$
(12)

for the payment at default, if it occurs at time t, of

$$W_t^c = L_t^* - c\left(t - \frac{\lfloor 4t \rfloor}{4}\right),\tag{13}$$

where $\lfloor x \rfloor$ denotes the largest integer less than x, and where L_t^* denotes the risk-neutral expected fractional loss of notional at time t, assuming immediate default.⁸ The second term in (13) is a deduction for accrued premium.

The current CDS rate is that choice C(t) for the premium rate c at which the market values of the payments by the buyer and seller of protection are equal. That is, C(t) solves

$$C(t)g(t) = h(t, C(t)).$$
 (14)

Noting that h(t, c) is linear with respect to c, this is a linear equation to solve for C(t).

We turn to the calculation of h(t, c). By the doubly-stochastic property (see, for example, Duffie (2001), Chapter 11), we first condition on (λ^*, L^*) , and then use the conditional risk-neutral density $e^{-\int_t^s \lambda^*(u) du} \lambda^*(s)$ of τ at time s to get

$$h(t,c) = \int_{t}^{t(n)} \delta(t,s) E^{Q} \left(e^{-\int_{t}^{s} \lambda^{*}(u) \, du} \lambda^{*}(s) W_{s}^{c} \middle| \mathcal{F}_{t} \right) ds.$$
(15)

We take L^* to be constant and use, as a numerical approximation of the integral in (15),

$$h(t,c) \simeq \sum_{i=1}^{n} \delta\left(t, \frac{t(i) + t(i-1)}{2}\right) \left[p^{*}(t, t(i-1)) - p^{*}(t, t(i))\right] \left(L^{*} - \frac{c}{8}\right),$$
(16)

⁸A more precise definition of L_t^* is given on page 130 of Duffie and Singleton (2003).

which involves a time discretization of the integral in (15) that, in effect, approximates, between quarter ends, with a linear discount function and risk-neutral survival function. Then C(t) is calculated from (14) using this approximation.

5.2 Model Specification

For a parametric specification of the risk-neutral default intensity process λ^* , motivated by our regression results, we suppose that

$$\log \lambda_t^* = \alpha + \beta \log \lambda_t + u_t,\tag{17}$$

where α and β are constants, $X = \log \lambda$ is as specified earlier by (5), and u satisfies

$$du_t = -\kappa_u u_t \, dt + \sigma_u \, d\xi_t,\tag{18}$$

where, under the actual probability measure P, we take ξ to be a standard Brownian motion independent of the Brownian motion B of (5).

The risk-neutral distribution of (λ^*, λ) is specified by assuming that

$$dB_t = -(\gamma^0 + \gamma^1 X_t) dt + d\tilde{B}_t \tag{19}$$

and

$$d\xi_t = -(\gamma_u^0 + \gamma_u^1 u_t) dt + d\tilde{\xi}_t, \qquad (20)$$

where $(\tilde{B}, \tilde{\xi})$ is a two-dimensional standard Brownian motion under Q, and where $\gamma^0, \gamma^1, \gamma^0_u$, and γ^1_u are constants. In addition to the parameter vector Θ , the model for λ^* requires an estimator of the parameter vector

$$\Theta^* = (\alpha, \beta, \kappa_u, \sigma_u, \gamma^0, \gamma^1, \gamma_u^0, \gamma_u^1).$$

5.3 Estimation Strategy and Results

For any given firm, we estimate the parameters (Θ, Θ^*) for the joint model of actual and risk-neutral intensity processes in a two-step procedure. First, we estimate the parameter vector Θ of the actual intensity model λ following the procedure described in Section 4. In a second step, fixing the estimate of Θ , and treating this estimate as though in fact equal to the true parameter vector, we estimate the parameter vector Θ^* governing the risk-neutral intensity process λ^* on a sector-by-sector basis. For this second step, our data consists of weekly observations of 5-year default swap rates and 1-year EDFs, over a time period from 9/27/00 until 4/9/03. As with the actual default intensity model, this is not a routine MLE procedure since the evaluation of the likelihood function requires a numerical differentiation of the modeled CDS rate C(t) determined by (14), which we approximate using (16). In the current implementation, we only use pairs of CDS-EDF observations where neither the CDS or the EDF data is missing, and for which the EDF is not censored at 20%. In addition, we remove from the sample any firm whose sample-mean EDF is below 10 basis points.

Preliminary investigations have shown that, by restricting Θ^* so that the parameter-implied stationary mean of u is equal to its model-implied sample mean across all firms in a given sector, one obtains considerable improvement in the interpretability of the parameter estimates, facilitating the comparison of the implied values for λ and λ^* . On the same grounds, and because the data were unable to sharply pin down the market-price-of-credit-risk factors γ^0 and γ_u^0 exactly, we set $\gamma_0^u = 0$ and further restrict Θ^* so that the parameter-implied risk-neutral stationary mean of λ^* for a given firm is equal to the sample mean of CDS rates divided by the risk-neutral mean loss rate at default. In addition, we also impose, in this current implementation, that λ^* is a one-dimensional lognormal intensity process under Q, by taking $\tilde{\kappa} = \tilde{\kappa}_u$. Here, $\tilde{\kappa}$ and $\tilde{\kappa}_u$ denote the risk-neutral mean-reversion parameters of X and u, respectively.⁹ We anticipate relaxing the latter two restrictions in a subsequent version of the model.

Preliminary sector-by-sector parameter estimates for the broadcastingand-entertainment, healthcare, and oil-and-gas industries are summarized in Table 5.¹⁰ We allow for a firm-specific market-price-of-default-risk parameter γ^0 . Estimates by firm are listed in Table 14, Appendix C. Figure 12 shows the implied sample paths of λ^* and λ for Vintage Petroleum, and Figure 13 displays the time series of Vintage's estimated default risk premia, that is, the ratio of its risk-neutral to its actual default intensities.

For example, extracting from Table 5 the fit implied for the healthcare sector, we have

 $\log \lambda_t^* = 2.49 + 0.63 \log \lambda_t + u_t,$

⁹From Equations (5) and (18) through (20) we have $\tilde{\kappa} = \kappa + \gamma^1 \sigma$ and $\tilde{\kappa}_u = \kappa_u + \gamma_u^1 \sigma_u$. ¹⁰Both λ and λ^* are measured in basis points.

| | Oil and Gas | Healthcare | Broadcasting and Entertainment |
|--|-------------|------------|-----------------------------------|
| $\hat{\alpha}$ | 2.8987 | 2.4917 | 4.5272 |
| \hat{eta} | 0.4167 | 0.6290 | 0.2451 |
| $\hat{\kappa}_u$ | 1.6218 | 1.7509 | 0.8603 |
| $\hat{\sigma}_u$ | 1.8311 | 2.2719 | 1.6911 |
| $\operatorname{mean}(\hat{\gamma}^0)$ | 0.4025 | 0.4637 | 1.3007 |
| $\hat{\gamma}_1$ | -0.0790 | -0.0668 | -0.2880 |
| $\hat{\gamma}_1 \\ \hat{\gamma}_u^1$ | -0.6867 | -0.5264 | -0.3688 |
| $\operatorname{mean}(\lambda^*/\lambda)$ | 2.53 | 5.59 | 5.50 |
| no. firms | 29 | 14 | 12 |

Table 5: Preliminary sector CDS-implied risk-neutral default intensity parameter estimates

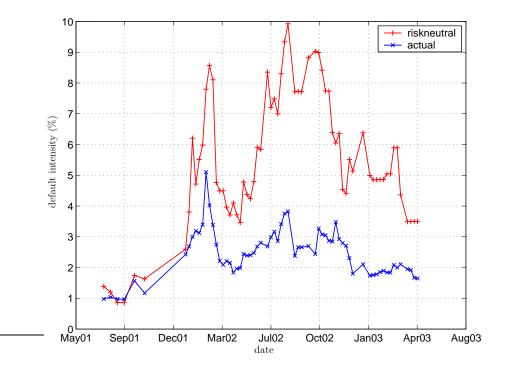


Figure 12: Implied default intensities for Vintage Petroleum. Sources: Moody's KMV and CIBC.

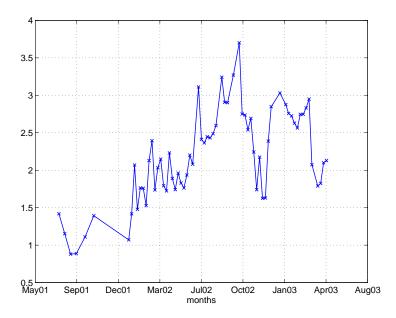


Figure 13: Estimated default risk premia, λ^*/λ , for Vintage Petroleum.

or equivalently,

$$\lambda_t^* = 12.06 \,\lambda_t^{0.63} \, e^{u_t},$$

where λ_t and λ_t^* are measured in basis points. So, for an actual default intensity of 100 basis points and $u_t = 0$, we get a risk-neutral default intensity of roughly 219 basis points. The risk-neutral distribution of λ^* is estimated as

$$d \log \lambda_t^* = (\hat{\tilde{a}} - 0.55 \log \lambda_t^*) dt + 2.46 dB_t^*,$$

where $\tilde{a} = \hat{a} - \hat{\gamma}^0 \hat{\sigma}$. The sample averages of the estimated risk premia are 2.53, 5.59, and 5.50 for the oil-and-gas, healthcare, and broadcasting-and-entertainment sector, respectively.

As a diagnostic check, we examine the behavior of the standardized innovations $\epsilon_{t+h}, \epsilon_{t+2h}, \ldots$ of u_t , defined by

$$u_{t+h} = e^{-\kappa_u h} u_t + \sigma_u \sqrt{\frac{1 - e^{-2\kappa_u h}}{2\kappa_u}} \epsilon_{t+h}.$$

Under the specified model, and under the actual probability measure P, these innovations are independent standard normals. Table 6 lists the sample mean and the sample standard deviation (SD) of the fitted versions of these standardized innovations, for each of the three sectors. Finally, Figure 14 shows the associated histogram of fitted ϵ_t , merging across all firms, plotted along with the standard normal density curve.

| | Mean | SD |
|--------------------------------|---------|--------|
| Healthcare | -0.0101 | 0.9962 |
| Oil and Gas | -0.0590 | 0.9760 |
| Broadcasting and Entertainment | -0.0006 | 1.0008 |
| All | -0.0347 | 0.9862 |

Table 6: Sample moments for standardized innovations

6 Discussion and Conclusion

We compare our results on default risk premia to those available in the literature. Using the structural model of Leland and Toft (1996), Huang and Huang (2000) calibrated parameters for the model determining actual and risk-neutral default probabilities, by credit rating, that are implied from equity market risk premia, recoveries, initial leverage ratios, and average default frequencies. All underlying parameters were obtained from averages reported by the credit rating agencies, Moody's and Standard and Poors, except for the equity-market risk premia, which were obtained by rating from estimates by Bhandari (1999). At the five-year maturity point, the estimated ratios of annualized risk-neutral to actual five-year default probabilities are reported in Table 7. In magnitude, the results are roughly consistent with those of Driessen (2002). One notes that the risk premium typically declines as default probability increases, as suggested by the results of our basic log-log regression model, and as captured by our time-series formulation.

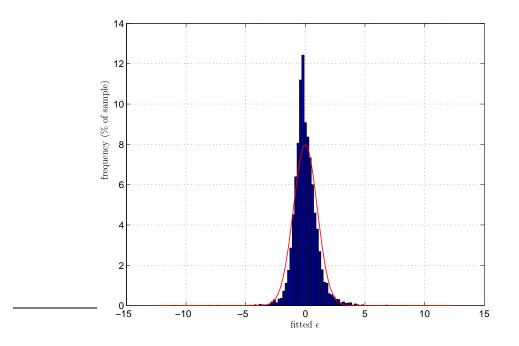


Figure 14: Estimated innovations ϵ across all sectors, and the standard normal density.

Table 7: Five-year default risk premium implied by structural-model results of Huang and Huang (2001)

| Initial | Premium | $Q(\tau < 5)$ | $P(\tau < 5)$ |
|---------|---------|---------------|---------------|
| Rating | (ratio) | (percent) | (percent) |
| Aaa | 1.7497 | 0.04 | 0.02 |
| Aa | 1.7947 | 0.09 | 0.05 |
| А | 1.7322 | 0.25 | 0.15 |
| Baa | 1.4418 | 1.22 | 0.84 |
| Ba | 1.1658 | 9.11 | 7.85 |
| В | 1.1058 | 25.61 | 23.41 |

A MLE for Intensity from EDFs

This appendix shows our methodology for MLE estimation of the parameters of the default intensity, including the effects of missing EDF data as well as censoring of EDFs by truncation above 20%. Our data is the monthly observed EDF level Y_i at month i, for each of N month-end times t_0, t_1, \ldots, t_N .

From (5), for any time t and time step h (which is 1/12 in our application), the discretely sampled log-intensity process X satisfies

$$X_{t+h} = b_0 + b_1 X_t + \epsilon_{t+h},$$
 (A.1)

where $b_1 = e^{-\kappa h}$, $b_0 = a/(\kappa(1-b_1))$, and $\epsilon_{t+h}, \epsilon_{t+2h}, \ldots$ are *iid* normal with mean zero and variance $\sigma_{\epsilon} = \sigma^2(1-e^{-2\kappa h})/(2\kappa)$.

For a given firm, we initialize the search for the parameter vector $\Theta = (a, \kappa, \sigma)$ as follows. First, we regress $\log(1 - Y_i)$ on $\log(1 - Y_{i-1})$, using only months at which both the current and the lagged EDF are observed and not truncated at 20%. The associated regression coefficient estimates, denoted by \hat{b}_0 and \hat{b}_1 , are considered to be starting estimates of b_0 and b_1 , respectively. The sample standard deviation of the fitted residuals, $\hat{\sigma}_{\epsilon}$, is our starting estimate for σ_{ϵ} . We then start the search for $\Theta = (a, \kappa, \sigma)$ at

$$\kappa_0 = -\frac{\log(\hat{b}_1)}{h},$$

$$a_0 = \frac{\hat{b}_0}{1-\hat{b}_1}\kappa_0,$$

$$\sigma_0 = \hat{\sigma}_{\epsilon}\sqrt{\frac{2\kappa_0}{1-\exp(-2\kappa_0 h)}}$$

If Θ is the true parameter vector, then $Y_i = G(\lambda(t_i); \Theta)$, where G is defined via (7).

Suppose, to pick an example of a censoring outcome from which the general case can easily be deduced, that for months k through $\bar{k} > k + 1$, inclusive, the EDFs are truncated at $\zeta = 20\%$, meaning that the censored and observed EDF is 20%, implying that the actual EDF was larger than or equal to 20%, and moreover that the EDF data from month l + 1 to month \bar{l} are missing. Let $\mathcal{I} = \{i : k + 1 \leq i \leq \bar{k}\} \cup \{i : l + 1 \leq i \leq \bar{l}\}$ denote the censored month numbers. Then the likelihood of the censored EDFs

 $Y = \{Y_i : i \notin \mathcal{I}\}$ evaluated at outcomes $y = \{y_i : i \notin \mathcal{I}\}$, using the usual abuse of notation for measures, is defined by

$$\begin{aligned} \mathcal{L}(Y,\mathcal{I};\Theta) \, dy &= \prod_{n=0}^{k-1} P(Y_{n+1} \in dy_{n+1}; Y_n = y_n, \Theta) \\ &\times P(Y_{k+1} \ge \zeta, \dots, Y_{\bar{k}} \ge \zeta, Y_{\bar{k}+1} \in dy_{\bar{k}+1}; Y_k = y_k, \Theta) \\ &\times \prod_{n=\bar{k}+1}^{l-1} P(Y_{n+1} \in dy_{n+1}; Y_n = y_n, \Theta) \\ &\times P(Y_{\bar{l}+1} \in dy_{\bar{l}+1}; Y_l = y_l, \Theta) \\ &\times \prod_{n=\bar{l}+1}^{N-1} P(Y_{n+1} \in dy_{n+1}; Y_n = y_n, \Theta), \end{aligned}$$

where $P(\cdot; Y_n = y_n; \Theta)$ denotes the distribution of $\{Y_{n+1}, Y_{n+2}, \ldots\}$ associated with initial condition y_n for Y_n , and associated with parameter vector Θ . A maximum likelihood estimator (MLE) $\hat{\Theta}$ for Θ solves

$$\sup_{\Theta} \mathcal{L}(Y, \mathcal{I}; \Theta).$$
(A.2)

For $z \in \mathbb{R}$, we let $g(z; \Theta) = G(e^z; \Theta)$, and let $Z_i^{\Theta} = g^{-1}(Y_i; \Theta)$ denote the logarithm of the default intensity at time t_i that would be implied by a non-censored EDF observation Y_i , assuming the true parameter vector is Θ . Letting $Dg(\cdot; \Theta)$ denote the partial derivative of $g(\cdot; \Theta)$ with respect to its first argument, and using standard change-of-measure arguments, we can rewrite the likelihood as

$$\mathcal{L}(Y,\mathcal{I};\Theta) = \prod_{n=0}^{k-1} P(Z_{n+1}^{\Theta}; Z_n^{\Theta}, \Theta) \left(Dg(Z_{n+1}^{\Theta}; \Theta) \right)^{-1} \\ \times P(Y_{k+1} \ge \zeta, \dots, Y_{\bar{k}} \ge \zeta; Y_k = y_k, Y_{\bar{k}+1} = y_{\bar{k}+1}, \Theta) \\ \times P(Z_{\bar{k}+1}^{\Theta}; Z_k^{\Theta}, \Theta) \left(Dg(Z_{\bar{k}+1}^{\Theta}; \Theta) \right)^{-1} \\ \times \prod_{n=\bar{k}+1}^{l-1} P(Z_{n+1}^{\Theta}; Z_n^{\Theta}, \Theta) \left(Dg(Z_{n+1}^{\Theta}; \Theta) \right)^{-1} \\ \times P(Z_{\bar{l}+1}^{\Theta}; Z_l^{\Theta}, \Theta) \left(Dg(Z_{\bar{l}+1}^{\Theta}; \Theta) \right)^{-1} \\ \times \prod_{n=\bar{l}+1}^{N-1} P(Z_{n+1}^{\Theta}; Z_n^{\Theta}, \Theta) \left(Dg(Z_{n+1}^{\Theta}; \Theta) \right)^{-1}.$$
(A.3)

The second term on the right-hand side of (A.3) is equal to

$$q(Y;\Theta) = P(Z_{k+1}^{\Theta} \ge g^{-1}(\zeta;\Theta), \dots, Z_{\bar{k}}^{\Theta} \ge g^{-1}(\zeta;\Theta);$$

$$Z_{k}^{\Theta} = g^{-1}(y_{k};\Theta), Z_{\bar{k}+1}^{\Theta} = g^{-1}(y_{\bar{k}+1};\Theta),\Theta).$$

In the remainder of this appendix, we describe how to compute $q(Y; \Theta)$ by Monte Carlo integration, and hence $P(Y_{k+1} \ge \zeta, \ldots, Y_{\bar{k}} \ge \zeta; Y_k = y_k, Y_{\bar{k}+1} = y_{\bar{k}+1}, \Theta)$. In order to simplify notation we suppress Θ in what follows. We observe that for any time t between times s and u, the conditional distribution of X(t) given X(s) and X(u) is a normal distribution with mean $M(t \mid s, u)$ and variance $V(t \mid s, u)$ given by

$$\begin{split} M(t \mid s, u) &= \frac{1 - e^{-2\kappa(u-t)}}{1 - e^{-2\kappa(u-s)}} M(t \mid s) + \frac{e^{-2\kappa(u-t)} - e^{-2\kappa(u-s)}}{1 - e^{-2\kappa(u-s)}} M(t \mid u), \\ V(t \mid s, u) &= \frac{V(t \mid s)V(u \mid t)}{V(u \mid s)}, \end{split}$$

where, for times t before u, we let

$$\begin{split} M(u \,|\, t) &= \theta + e^{-\kappa(u-t)} (X(t) - \theta) \\ V(u \,|\, t) &= \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(u-t)}) \\ M(t \,|\, u) &= e^{\kappa(u-t)} (X(u) - \theta(1 - e^{-\kappa(u-t)})) \end{split}$$

denote the conditional expectation and variance, respectively, of X(u) given X(t), and the conditional expectation of X(t) given X(u). As a consequence, letting $Z_k = X(t_k)$, we can easily simulate from the joint conditional distribution of $(Z_{k+1}, \ldots, Z_{\bar{k}})$ given Z_k and $Z_{\bar{k}+1}$ which is given by

$$P(Z_{k+1}, \dots, Z_{\bar{k}} | Z_k, Z_{\bar{k}+1}) = P(Z_{k+1} | Z_k, Z_{\bar{k}+1})$$
$$\prod_{j=1}^{\bar{k}-(k+1)} P(Z_{k+j+1} | Z_{k+j}, Z_{\bar{k}+1}).$$

We are now in a position to estimate the quantity in (A.4) by generating some "large" integer number J of independent sample paths $\{(Z_{k+1}^j, \ldots, Z_{\bar{k}}^j); 1 \leq j \leq J\}$ from the joint conditional distribution of $(Z_{k+1}, \ldots, Z_{\bar{k}})$ given Z_k and $Z_{\bar{k}+1}$, and by computing the fraction of those paths for which $Z_i^j \geq g^{-1}(\zeta)$ for all i in $\{k+1, \ldots, \bar{k}\}$.

B Solution of Log-Normal Intensity Model

This appendix provides an algorithm, prepared for this project by Gustavo Manso, for computing the survival probability of (4), and related expectations of the form $E(e^{-\int_0^t \lambda(s) \, ds} F(\lambda_t))$, for a well-behaved function $F : [0, \infty) \to \mathbb{R}$. The algorithm allows for a generalization of the log-normal intensity model to a model that is, in logarithms, autoregressive with a mixtureof-normals innovation, allowing for fat tails and skewness. Matlab code is downloadable at the web site www.stanford.edu/~/manso/numerical/.

INPUTS: Parameters $(k, m_1, v_1, p, m_2, v_2, m)$ and initial log-intensity $x \in [a, b]$.

OUTPUT: Let $y(j) = \lambda(t_j)$, for equally spaced times t_0, t_1, \ldots, t_m . The output is

$$S(0,x) = E\left[\exp\left(-\sum_{j=1}^{m} y(j)\right) F(y(m))\right],$$

where

$$\log y(j) = -k \log y(j-1) + W(j) + Z(j), \log y(0) = x,$$

and W(j) is normal, mean m_1 , variance v_1 , Z(j) is, with probability p, equal to 0 (no jump) and with probability 1 - p, normal with mean m_2 , variance v_2 . All W(j) and Z(j) are independent.

Step 1 Compute $K \ge N + 1$ Chebyshev interpolation nodes on [-1, 1]:

$$z_k = -\cos\left(\frac{2k-1}{2K}\pi\right), \ k = 1, \dots, K.$$

Step 2 Adjust the nodes to the [a, b] interval:

$$x_k = (z_k + 1) \left(\frac{b-a}{2}\right) + a, \ k = 1, \dots, K.$$

Step 3 Evaluate Chebyshev polynomials:

$$T_n(z_k) = \cos(n\cos^{-1} z_k), \ k = 1, \dots, K \text{ and } n = 1, \dots, N.$$

Step 4 Recursive Integration:

- Boundary condition: $S(m, x) = F(\exp(x))$, for $x \in [a, b]$.
- For j = m : -1 : 0,
 - 1. Numerical Integration:

$$S(j, x_k) = \pi^{-\frac{1}{2}} \sum_{i=1}^{I} \omega_i \left[pq(j+1, u_a(i, x_k)) + (1-p)q(j+1, u_b(i, x_k)) \right],$$

where

$$q(j, u) = S(j + 1, u) \exp(-\exp(u)),$$

$$u_a(i, x) = \sqrt{2v_1}\phi_i + (m_1 - kx),$$

$$u_b(i, x) = \sqrt{2(v_1 + v_2)}\phi_i + (m_1 + m_2 - kx),$$

and (ω_i, ϕ_i) , $i = 1, \ldots, I$, are *I*-point Gauss-Hermite quadrature weights and nodes.¹¹

2. Compute the Chebyshev coefficients:

$$c_n = \frac{\sum_{k=1}^{K} S(j, x_k) T_n(z_k)}{\sum_{k=1}^{K} T_n(z_k)^2} \text{ for } n = 0, \dots, N,$$

to arrive at the approximation for $S(j, x), x \in [a, b]$:

$$\widehat{S}(j,x) = \sum_{n=0}^{N} c_n T_n \left(2\frac{x-a}{b-a} - 1 \right).$$

¹¹See Judd (1998), page 262, for a table with (ω_i, ϕ_i) .

C Additional Background Statistics

This appendix contains additional background statistics regarding the firms studied. Section 2 contains the data regarding firms from the broadcastingand-entertainment industry. This appendix includes information regarding the firms studied from the healthcare and the oil-and-gas industries.

| Firm Name | Median EDF | Median Rating | No. Quotes |
|----------------------------|------------|---------------|------------|
| Abbott Laboratories | 3 | Aa1 | 235 |
| AmerisourceBergen | 200 | N/A | 311 |
| Amgen | 2 | A2 | 776 |
| Baxter International | 13 | A3 | 741 |
| Beverly Enterprises | 432 | B1 | 256 |
| Boston Scientific | 84 | Baa3 | 443 |
| Bristol-Myers Squibb | 3 | Aaa | 504 |
| Cardinal Health | 25 | A2 | 323 |
| Chiron | 16 | Baa1 | 429 |
| Community Health Systems | 173 | B2 | 328 |
| Eli Lilly | 5 | Aa3 | 403 |
| Genzyme Corp-Genl Division | 14 | N/A | 242 |
| НСА | 46 | Ba2 | 540 |
| Healthsouth | 257 | Ba1 | 349 |
| Humana | 165 | Baa3 | 393 |
| Medtronic | 2 | N/A | 610 |
| Tenet Healthcare | 41 | Ba1 | 1129 |
| Triad Hospitals | 183 | B1 | 349 |
| Wellpoint Health Networks | 47 | Baa2 | 294 |
| Wyeth | 17 | A2 | 698 |

Table 8: Healthcare firms

| Firm Name | Median EDF | Median Rating | No. Quotes |
|-------------------------------|------------|---------------|------------|
| Amerada Hess | 26 | Baa1 | 866 |
| Anadarko Petroleum | 19 | Baa1 | 1215 |
| Apache | 38 | Baa1 | 688 |
| Baker Hughes | 45 | A2 | 738 |
| BJ Services | 32 | Baa2 | 245 |
| Burlington Resources | 35 | A3 | 590 |
| Chesapeake Energy | 279 | B3 | 778 |
| ChevronTexaco | 3 | N/A | 491 |
| Conoco | 32 | A3 | 532 |
| ConocoPhillips | 15 | N/A | 1334 |
| Consolidated Natural Gas | 9 | A2 | 443 |
| Devon Energy | 40 | Baa1 | 1463 |
| Diamond Offshore | 46 | A3 | 830 |
| EL Paso | 42 | Baa2 | 1307 |
| Enron | 12 | Baa1 | 678 |
| Forest Oil | 196 | B1 | 397 |
| Halliburton | 36 | Aa3 | 430 |
| Kerr-McGee | 38 | Baa1 | 596 |
| Kinder Morgan Energy Partners | 18 | Baa1 | 632 |
| Kinder Morgan | 11 | Baa2 | 505 |
| Marathon Oil | 62 | Baa2 | 447 |
| Nabors Industries | 27 | A3 | 1224 |
| Occidental Petroleum | 67 | Baa3 | 1027 |
| Parker Drilling | 323 | B1 | 307 |
| Pride International | 145 | Ba3 | 870 |
| Talisman Energy | 55 | Baa1 | 226 |
| Tesoro Petroleum | 171 | Ba3 | 243 |
| Transocean | 32 | N/A | 1038 |
| Valero Energy | 85 | Baa3 | 1131 |
| Vintage Petroleum | 118 | Ba1 | 449 |
| Weatherford International | 140 | Baa1 | 879 |
| Williams Cos | 55 | Baa2 | 749 |

Table 9: Oil and gas firms

| Results | for Daily Med | ian CDS Da | ta | |
|--------------------------|---------------|------------|---------|----------|
| | Levels | Standard | Log-Log | Standard |
| | Model | Error | Model | Error |
| Number of CDS Samples | 18259 | | 18259 | |
| Intercept | -28.877 | 7.846 | 0.912 | 0.029 |
| Slope | 1.541 | 0.007 | 0.828 | 0.004 |
| Broadcasting Dummy | 104.985 | 3.187 | 0.389 | 0.010 |
| Healthcare Dummy | 21.439 | 3.053 | 0.196 | 0.010 |
| Dec-00 Dummy | 2.605 | 29.525 | 0.545 | 0.094 |
| Jan-01 Dummy | 31.516 | 22.927 | 0.554 | 0.073 |
| Feb-01 Dummy | 42.600 | 21.568 | 0.594 | 0.069 |
| Mar-01 Dummy | 63.467 | 21.050 | 0.704 | 0.067 |
| Apr-01 Dummy | 45.407 | 18.248 | 0.556 | 0.058 |
| May-01 Dummy | 35.145 | 20.701 | 0.416 | 0.066 |
| Jun-01 Dummy | 22.945 | 21.569 | 0.215 | 0.069 |
| Jul-01 Dummy | 18.772 | 19.937 | 0.117 | 0.064 |
| Aug-01 Dummy | -8.447 | 15.164 | 0.049 | 0.048 |
| Sep-01 Dummy | 2.341 | 23.433 | 0.180 | 0.075 |
| Oct-01 Dummy | 20.670 | 17.013 | 0.264 | 0.054 |
| Nov-01 Dummy | 20.341 | 22.436 | 0.322 | 0.072 |
| Dec-01 Dummy | -8.964 | 69.162 | 0.194 | 0.221 |
| Jan-02 Dummy | 33.728 | 12.343 | 0.385 | 0.039 |
| Feb-02 Dummy | 51.169 | 11.568 | 0.504 | 0.037 |
| Mar-02 Dummy | 48.130 | 9.778 | 0.460 | 0.031 |
| Apr-02 Dummy | 43.675 | 9.485 | 0.459 | 0.030 |
| May-02 Dummy | 63.273 | 9.417 | 0.544 | 0.030 |
| Jun-02 Dummy | 55.498 | 9.631 | 0.489 | 0.031 |
| Jul-02 Dummy | 130.020 | 9.265 | 0.571 | 0.030 |
| Aug-02 Dummy | 152.160 | 9.375 | 0.635 | 0.030 |
| Sep-02 Dummy | 101.648 | 9.617 | 0.488 | 0.031 |
| Oct-02 Dummy | 124.762 | 9.230 | 0.544 | 0.030 |
| Nov-02 Dummy | 105.308 | 9.413 | 0.471 | 0.030 |
| Dec-02 Dummy | 74.419 | 9.884 | 0.428 | 0.032 |
| Jan-03 Dummy | 52.690 | 9.594 | 0.351 | 0.031 |
| Feb-03 Dummy | 43.064 | 9.892 | 0.313 | 0.032 |
| Mar-03 Dummy | 19.895 | 9.626 | 0.239 | 0.031 |
| Apr-03 Dummy | 15.600 | 9.618 | 0.187 | 0.031 |
| May-03 Dummy | 23.330 | 9.671 | 0.183 | 0.031 |
| Jun-03 Dummy | 27.770 | 9.605 | 0.219 | 0.031 |
| Jul-03 Dummy | 11.421 | 9.646 | 0.113 | 0.031 |
| Aug-03 Dummy | 4.165 | 10.808 | 0.085 | 0.034 |
| Sum of Squared Residuals | 517328076 | | 5270 | |
| Total Sum of Squares | 2064101382 | | 20300 | |
| R^2 | 0.749 | | 0.740 | |

Table 10: CDS-EDF regression results

| Resu | lts for Intraday | 7 CDS Data | | |
|--------------------------|------------------|------------|---------|----------|
| | Levels | Standard | Log-Log | Standard |
| | Model | Error | Model | Error |
| Number of CDS Samples | 40844 | | 40844 | |
| Intercept | -35.010 | 5.167 | 1.026 | 0.023 |
| Slope | 1.583 | 0.005 | 0.785 | 0.003 |
| Broadcasting Dummy | 58.204 | 1.552 | 0.322 | 0.006 |
| Healthcare Dummy | 30.340 | 1.928 | 0.229 | 0.007 |
| Dec-00 Dummy | 38.421 | 23.172 | 0.618 | 0.088 |
| Jan-01 Dummy | 62.357 | 17.417 | 0.655 | 0.066 |
| Feb-01 Dummy | 57.496 | 16.120 | 0.633 | 0.061 |
| Mar-01 Dummy | 79.335 | 15.178 | 0.789 | 0.058 |
| Apr-01 Dummy | 68.074 | 12.643 | 0.631 | 0.048 |
| May-01 Dummy | 51.590 | 14.461 | 0.443 | 0.055 |
| Jun-01 Dummy | 37.881 | 15.176 | 0.263 | 0.058 |
| Jul-01 Dummy | 25.792 | 13.116 | 0.194 | 0.050 |
| Aug-01 Dummy | 5.740 | 10.373 | 0.125 | 0.040 |
| Sep-01 Dummy | 5.060 | 16.124 | 0.207 | 0.061 |
| Oct-01 Dummy | 35.723 | 11.398 | 0.313 | 0.043 |
| Nov-01 Dummy | 36.374 | 15.471 | 0.391 | 0.059 |
| Dec-01 Dummy | 18.565 | 57.107 | 0.278 | 0.218 |
| Jan-02 Dummy | 45.780 | 7.755 | 0.484 | 0.030 |
| Feb-02 Dummy | 59.139 | 7.147 | 0.592 | 0.027 |
| Mar-02 Dummy | 49.396 | 6.082 | 0.531 | 0.023 |
| Apr-02 Dummy | 33.833 | 5.833 | 0.456 | 0.022 |
| May-02 Dummy | 61.333 | 5.873 | 0.575 | 0.022 |
| Jun-02 Dummy | 43.272 | 5.883 | 0.509 | 0.022 |
| Jul-02 Dummy | 112.068 | 5.708 | 0.622 | 0.022 |
| Aug-02 Dummy | 128.001 | 5.781 | 0.674 | 0.022 |
| Sep-02 Dummy | 91.658 | 5.854 | 0.534 | 0.022 |
| Oct-02 Dummy | 101.796 | 5.674 | 0.558 | 0.022 |
| Nov-02 Dummy | 89.819 | 5.775 | 0.499 | 0.022 |
| Dec-02 Dummy | 63.924 | 6.096 | 0.442 | 0.023 |
| Jan-03 Dummy | 42.351 | 6.127 | 0.355 | 0.023 |
| Feb-03 Dummy | 36.320 | 6.168 | 0.341 | 0.024 |
| Mar-03 Dummy | 21.240 | 6.078 | 0.276 | 0.023 |
| Apr-03 Dummy | 20.482 | 6.109 | 0.250 | 0.023 |
| May-03 Dummy | 21.402 | 6.093 | 0.221 | 0.023 |
| Jun-03 Dummy | 25.479 | 5.989 | 0.239 | 0.023 |
| Jul-03 Dummy | 9.514 | 6.196 | 0.117 | 0.024 |
| Aug-03 Dummy | 8.619 | 6.823 | 0.103 | 0.026 |
| Sum of Squared Residuals | 792797345 | | 11500 | |
| Total Sum of Squares | 2952660064 | | 35702 | |
| R^2 | 0.731 | | 0.678 | |

Table 11: CDS-EDF regression results

| Ticker | \hat{a} | $\hat{\kappa}$ | $\hat{\sigma}$ | Ticker | \hat{a} | $\hat{\kappa}$ | $\hat{\sigma}$ |
|---------------|-----------|----------------|----------------|----------------------|-----------|----------------|----------------|
| ABT | Ť | † | † | MGLH | 1.09 | 0.16 | 1.09 |
| ADELQ | 1.17 | 0.18 | 1.40 | MRO | 1.20 | 0.45 | 0.95 |
| AGN | 0.83 | 0.48 | 1.01 | NBR | 3.18 | 1.06 | 1.62 |
| AHC | 1.28 | 0.58 | 1.14 | NEV | 1.34 | 0.26 | 0.97 |
| AMGN | † | † | † | OCR | 0.69 | 0.24 | 1.32 |
| AOL | 0.90 | 0.20 | 1.10 | OEI | 1.38 | 0.33 | 1.23 |
| APA | 4.13 | 1.38 | 1.53 | OXY | 1.94 | 0.71 | 0.92 |
| APC | 0.93 | 0.32 | 0.89 | PDE | 3.98 | 0.89 | 1.70 |
| BAX | 1.56 | 0.64 | 1.19 | PKD | 0.79 | 0.11 | 1.22 |
| BEV | 1.11 | 0.17 | 1.14 | PXD | 1.87 | 0.46 | 1.38 |
| BHI | 1.24 | 0.44 | 0.88 | THC | 1.43 | 0.45 | 1.01 |
| BJS | 2.14 | 0.70 | 1.35 | TLM | 0.94 | 0.38 | 1.28 |
| BMY | † | † | † | TSO | 2.26 | 0.53 | 1.33 |
| \mathbf{BR} | 0.89 | 0.42 | 1.04 | VIA | 1.41 | 0.64 | 1.52 |
| BSX | 1.12 | 0.57 | 1.75 | VLO | 0.89 | 0.28 | 1.04 |
| CAH | 1.04 | 0.49 | 1.19 | VPI | 3.20 | 0.76 | 1.67 |
| CCU | 0.97 | 0.38 | 1.57 | WFT | 0.77 | 0.24 | 1.20 |
| CHIR | 2.31 | 0.85 | 1.24 | WLP | 1.93 | 0.68 | 1.47 |
| CMCSA | 2.04 | 0.56 | 1.03 | WMB | ‡ | ‡ | ‡ |
| CNG | † | † | † | WYE | † | † | † |
| COP | † | † | † | CHK | 3.42 | 0.67 | 1.75 |
| CVX | † | † | † | HUM | 1.73 | 0.37 | 1.45 |
| CYH | 3.89 | 0.90 | 1.61 | RCL | 0.95 | 0.26 | 1.14 |
| DIS | † | † | † | RIG | 0.81 | 0.28 | 1.43 |
| DVN | 0.92 | 0.34 | 1.46 | DYN | | | |
| ENRNQ | ‡ | ‡ | ‡ | YBTVA | 3.66 | 0.67 | 1.44 |
| \mathbf{EP} | ‡ | ‡ | ‡ | COX | 1.79 | 0.67 | 1.51 |
| F | 0.79 | 0.23 | 1.01 | ABC | 8.17 | 1.99 | 2.75 |
| FST | 4.79 | 1.05 | 1.53 | CAM | 3.22 | 0.94 | 1.32 |
| GENZ | 1.87 | 0.79 | 1.52 | SBGI | 3.27 | 0.61 | 1.54 |
| GM | 3.56 | 1.09 | 1.29 | \mathbf{PRM} | 4.69 | 1.29 | 2.47 |
| HAL | 1.23 | 0.41 | 1.53 | DO | 0.75 | 0.31 | 1.54 |
| HCA | 2.07 | 0.87 | 2.56 | \mathbf{L} | 0.54 | 0.05 | 0.95 |
| HRC | 0.70 | 0.14 | 1.26 | DCX | 2.22 | 0.49 | 1.32 |
| JNJ | † | † | † | COC | 4.81 | 1.78 | 1.79 |
| KMG | 0.46 | 0.13 | 0.86 | TRI | 8.20 | 1.66 | 1.22 |
| KMI | 3.28 | 2.20 | 3.57 | ICCI | 5.36 | 0.96 | 2.14 |
| KMP | 1.01 | 0.37 | 1.09 | CHTR | ‡ | ‡ | ‡ |
| LLY | † | † | † | MCCC | 9.02 | 1.62 | 2.30 |
| MDT | † | † | † | | | | |
| | | | | | | | |

Table 12: Fitted parameters of default intensity models

 \dagger $\,$ No estimates provided; the sample mean of the 1-year EDF is less than 10 basis points.

 \ddagger No estimates within admissible parameter region; the estimate for the mean-reversion parameter κ is negative.

 $\parallel~$ Firm removed from data set.

| Ticker | \hat{a} | Ticker | \hat{a} | Ticker | \hat{a} | Ticker | \hat{a} |
|-------------|-----------|----------------|-----------|--------------|-----------|----------------|-----------|
| ABC | 2.8120 | CHK | 2.2229 | HRC | 2.3440 | PKD | 2.0882 |
| ABT | † | CHTR | 5.0986 | HUM | 2.6410 | \mathbf{PRM} | 2.9253 |
| ADELQ | 3.4884 | CMCSA | 2.7299 | ICCI | 4.1459 | PXD | 1.8506 |
| AGN | 1.3468 | CNG | † | JNJ | † | RCL | 2.4718 |
| AHC | 1.1826 | COC | 1.4114 | KMG | 1.4174 | RIG | 1.1622 |
| AMGN | † | COP | † | KMI | 0.8738 | SBGI | 3.7737 |
| AOL | 2.4591 | COX | 2.0626 | KMP | 1.2905 | THC | 2.2631 |
| APA | 1.4619 | CVX | † | \mathbf{L} | 1.4229 | TLM | 1.2036 |
| APC | 1.3539 | CYH | 2.9018 | LLY | † | TRI | 3.3708 |
| BAX | 1.5443 | DIS | † | MCCC | 4.4058 | TSO | 2.0046 |
| BEV | 3.2271 | DO | 1.0159 | MDT | † | VIA | 1.7943 |
| BHI | 1.4137 | DVN | 1.2629 | MGLH | 3.7885 | VLO | 1.5457 |
| BJS | 1.4458 | DYN | ‡ | MRO | 1.3825 | VPI | 1.9995 |
| BMY | † | ENRNQ | 1.2787 | NBR | 1.4524 | WFT | 1.6014 |
| $_{\rm BR}$ | 1.2160 | \mathbf{EP} | 2.0104 | NEV | 2.3467 | WLP | 1.9974 |
| BSX | 1.2483 | \mathbf{FST} | 2.1402 | OCR | 1.8883 | WMB | 1.4543 |
| CAH | 1.6181 | GENZ | 1.5829 | OEI | 1.8829 | WYE | † |
| CCU | 1.5381 | HAL | 1.2546 | OXY | 1.3733 | YBTVA | 3.9456 |
| CHIR | 2.0109 | HCA | 1.0847 | PDE | 2.1021 | | |

Table 13: Fitted parameters of default intensity models

 \dagger $\,$ No estimates provided, mean EDF is less than 10 basis points.

‡ Firm removed from data set.

| $\begin{array}{c c c c c c c c c c c c c c c c c c c $ | | Oil and Gas | | Oil and Gas Healthcare | | | Broadcasting and Entertainment | | |
|---|---|-------------------------|---------|------------------------|------------------|----------------|-----------------------------------|--|--|
| APA 0.6787 BAX 0.3906 AOL 1.1280 APC 0.5199 BEV 0.6026 CCU 0.5715 BHI 0.6298 BSX 0.1852 CHTR 2.4714 BJS 0.4423 CAH 0.5710 CMCSA 1.2592 BR 0.3957 CHIR 0.7550 COX 0.8936 CHK 0.6863 CYH 0.6022 ICCI 1.9274 COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 HAL -0.0905 WLP 0.6793 KMG 0.44686 KMI -0.0102 KMP 0.5145 ICH ICH PL 0.5777 OXY 0.4744 | | Ticker $\hat{\gamma}^0$ | | Ticker | $\hat{\gamma}^0$ | | | | |
| APC 0.5199 BEV 0.6026 CCU 0.5715 BHI 0.6298 BSX 0.1852 CHTR 2.4714 BJS 0.4423 CAH 0.5710 CMCSA 1.2592 BR 0.3957 CHIR 0.7550 COX 0.8936 CHK 0.6863 CYH 0.6022 ICCI 1.9274 COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 HAL -0.0905 WLP 0.6793 KMG 0.4486 MRO 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 ICH ICH ICH ICH PKD 0.5145 ICH< | - | AHC | 0.2853 | ABC | 0.7332 | ADELQ | 1.6181 | | |
| BHI 0.6298 BSX 0.1852 CHTR 2.4714 BJS 0.4423 CAH 0.5710 CMCSA 1.2592 BR 0.3957 CHIR 0.7550 COX 0.8936 CHK 0.6863 CYH 0.6022 ICCI 1.9274 COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6608 TRI 1.0802 HAL -0.0905 WLP 0.6793 KMG 0.4686 KMI -0.0102 KMP 0.3446 IMRO 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 IMA 0.3290 TSO 0.4562 VLO 0.3695 VIA 0.3290 TSO 0.4562 | | APA | 0.6787 | BAX | 0.3906 | AOL | 1.1280 | | |
| BJS 0.4423 CAH 0.5710 CMCSA 1.2592 BR 0.3957 CHIR 0.7550 COX 0.8936 CHK 0.6863 CYH 0.6022 ICCI 1.9274 COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6608 TRI 1.0802 HAL -0.0905 WLP 0.6793 KMG 0.4486 KMI -0.0102 KMP 0.3446 | | APC | 0.5199 | BEV | 0.6026 | CCU | 0.5715 | | |
| BR 0.3957 CHIR 0.7550 COX 0.8936 CHK 0.6863 CYH 0.6022 ICCI 1.9274 COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 HAL -0.0905 WLP 0.6793 KMG 0.4486 KMI -0.0102 KMP 0.3446 NBR 0.5577 OXY 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 VFT 0.6459 VIA 0.593 | | BHI | 0.6298 | BSX | 0.1852 | CHTR | 2.4714 | | |
| CHK 0.8863 CYH 0.6022 ICCI 1.9274 COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 1444 0.8981 KMG 0.44866 KMI -0.0102 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1444 1 | | BJS | 0.4423 | CAH | 0.5710 | CMCSA | 1.2592 | | |
| COP 0.5617 GENZ -0.2647 L 0.4696 DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 1.4443 0.8981 FST 0.6808 TRI 1.0802 1.444 0.8981 FST 0.6808 TRI 1.0802 1.444 0.8981 KMG 0.4486 KMI -0.0102 1.444 0.8981 KMP 0.3446 MRO 0.4744 1.444 | | BR | 0.3957 | CHIR | 0.7550 | COX | 0.8936 | | |
| DO 0.2172 HCA -0.3030 MCCC 2.1241 DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 1.0038 HAL -0.0905 WLP 0.6793 1.0038 KMG 0.44686 1.0032 1.0032 1.0032 KMP 0.3446 1.0802 1.0032 1.0032 KMP 0.3446 1.00102 1.0032 1.0032 KMP 0.3446 1.00346 1.0032 1.0032 PDE 0.5742 1.00346 1.0032 1.0032 PKD 0.5145 1.0032 1.0032 1.0032 RIG 0.2383 1.0032 1.0032 1.0032 TLM 0.3290 1.0032 1.0032 1.0032 VLO 0.3695 1 | | CHK | 0.6863 | CYH | 0.6022 | ICCI | 1.9274 | | |
| DVN 0.2591 HRC 0.1678 PRM 1.2443 ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 0.8981 HAL -0.0905 WLP 0.6793 0.6793 KMG 0.44686 KMI -0.0102 0.6793 KMP 0.3446 MRO 0.4744 NBR 0.5577 0XY 0.4792 PDE 0.5742 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | COP | 0.5617 | GENZ | -0.2647 | \mathbf{L} | 0.4696 | | |
| ENRNQ 0.1907 HUM 0.8434 RCL 1.0038 EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 0.8981 HAL -0.0905 WLP 0.6793 0.6793 KMG 0.4686 0.4446 0.8981 MRO 0.4744 0.8981 0.8777 OXY 0.4792 0.4792 0.4792 PDE 0.5742 0.5145 RIG 0.2383 0.2383 TLM 0.3290 0.4562 VLO 0.3695 VPI VPI 0.4796 WFT 0.6459 | | DO | 0.2172 | HCA | -0.3030 | MCCC | 2.1241 | | |
| EP 0.3221 THC 0.4496 VIA 0.8981 FST 0.6808 TRI 1.0802 HAL -0.0905 WLP 0.6793 KMG 0.4686 KMI -0.0102 KMP 0.3446 MRO 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | DVN | 0.2591 | HRC | 0.1678 | \mathbf{PRM} | 1.2443 | | |
| $\begin{array}{llllllllllllllllllllllllllllllllllll$ | | ENRNQ | 0.1907 | HUM | 0.8434 | RCL | 1.0038 | | |
| HAL -0.0905 WLP 0.6793 KMG 0.4686 | | \mathbf{EP} | 0.3221 | THC | 0.4496 | VIA | 0.8981 | | |
| KMG 0.4686 KMI -0.0102 KMP 0.3446 MRO 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | FST | 0.6808 | | 1.0802 | | | | |
| $\begin{array}{llllllllllllllllllllllllllllllllllll$ | | | -0.0905 | WLP | 0.6793 | | | | |
| KMP 0.3446 MRO 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | - | | | | | | | |
| MRO 0.4744 NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | | | | | | | | |
| NBR 0.5577 OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | $_{\rm KMP}$ | 0.3446 | | | | | | |
| OXY 0.4792 PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | MRO | 0.4744 | | | | | | |
| PDE 0.5742 PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | - | 0.5577 | | | | | | |
| PKD 0.5145 RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | - | | | | | | | |
| RIG 0.2383 TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | | | | | | | | |
| TLM 0.3290 TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | | | | | | | | |
| TSO 0.4562 VLO 0.3695 VPI 0.4796 WFT 0.6459 | | | | | | | | | |
| VLO 0.3695 VPI 0.4796 WFT 0.6459 | | | | | | | | | |
| VPI 0.4796 WFT 0.6459 | | | | | | | | | |
| WFT 0.6459 | | | | | | | | | |
| | | | | | | | | | |
| WMB -0.0289 | | | | | | | | | |
| | - | WMB | -0.0289 | | | | | | |

Table 14: Fitted parameters of risk-neutral default intensity models

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