BIS Working Papers
No 124

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January 2003

BANK FOR INTERNATIONAL SETTLEMENTS

BIS Working Papers

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Abstract

This paper proposes a continuous-time framework that explains some stylised facts in recent “twin crises” episodes. I show that access to the world capital market enables the domestic economy to achieve a more efficient allocation of resources. However, the banking sector becomes more fragile when this international borrowing is wealth-constrained. A temporary shock is amplified and becomes persistent due to the interaction between the value of bank assets and the borrowing constraint. Depending on the exchange rate regime arrangement and the policy of the central bank, this financial fragility can evolve into a banking crisis, a currency crisis, or the simultaneous occurrence of both.

JEL Classification Numbers: F3, G21
Keywords: borrowing constraint; banking crises; currency crises

Copies of publications are available from:

Bank for International Settlements
Press & Communications
CH-4002 Basel, Switzerland

E-mail: publications@bis.org

Fax: +41 61 280 9100 and +41 61 280 8100

This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)

ISSN 1682-7678 (web)

1 Introduction[1]

The past two decades have witnessed a surge in financial crises and currency crises. Kaminsky and Reinhart (1999) and Shen and Wang (2000) analyse a large number of crisis episodes, and find that the two types of crisis often come together (the so-called "twin crises" phenomenon). They propose that the financial crises help predict the currency crises but not vice versa. The empirical study by Glick and Hutchison (2000) confirms this regularity and, in addition, highlights the importance of financial liberalisation in these episodes. Their study suggests that openness to international capital flows makes the countries concerned particularly vulnerable to twin crises. In particular, twin crises have been characterised by the following stylised facts:

- Sharp reversal of capital flows. As documented in Calvo and Mendoza (1996, 2000), Goldfajn and Valdés (1997) and Gourinchas et al (1999), most crises were preceded by a surge in portfolio capital inflows and featured sharp reversals in the downward phase.

- Financial liberalisation. Glick and Hutchison (2000) and Weller (1999) find that the twin crises phenomenon is not a general characteristic in all countries. Instead, it is most common in financially liberalised emerging markets. Demirgüc-Kunt and Detragiache (1998) confirm the important role of financial liberalisation, and suggest that banking crises are more likely to occur in liberalised financial systems, especially when the institutional environment is weak.

- Financial fragility. Currency crises and banking crises became intertwined after the financial liberalisation that occurred during the 1980s. The twin crises often start with problems in the financial sector. The banking crisis is then followed by an external crisis, which in turn exacerbates the banking crisis.

- Market overreaction. Even though most crises, such as the collapse of the East Asian economies in 1997,[2] are related to fundamental problems, the punishment is often too se-

[1]E-mail: haibin.zhu@bis.org. I thank Gaetano Antinolfi, Hui Guo, Enrique Mendoza, Kostas Tsatsaronis, and seminar participants at the Bank for International Settlements, Duke University and the Federal Reserve Bank of St Louis for helpful comments and suggestions. Some of the work on this paper was done while I was visiting the Federal Reserve Bank of St Louis, whose hospitality I gratefully acknowledge. The views in this paper are mine and should not be interpreted as reflecting those of the BIS or the Federal Reserve System.

[2]See Corsetti et al (1998).

vere to be justifiable. A trivial sin is often amplified and can trigger a sudden reversal of economic activity and the collapse of the whole system.

Despite the connections between banking crises and currency crises, most of the existing literature has developed models to explain the two types of event separately. On the one hand, most currency crisis literature explains the crises as the result of flawed government policies. Some economists, following Krugman (1979) and Flood and Garber (1984), propose the balance of payments crises as the anticipated demise of an inconsistent policy regime. In these models, the central bank holds international reserves not only to maintain a fixed exchange rate regime, but also to cover the country's fiscal deficit. Investors observe this policy conflict and understand that it is not sustainable in the long run. As a result, a speculative attack occurs when the reserve falls below a critical value. More recently, a number of papers have focused instead on the possibility of multiple equilibria. Obstfeld (1986, 1994) proposes that in a fixed exchange rate regime, if investors begin to suspect that the government will choose to abandon the parity, the resulting pressure on the interest rates can force the government to devalue the currency. In other words, arbitrary expectational shifts could turn a fairly credible exchange rate peg into collapse.

On the other hand, the bulk of the existing literature on banking crises has been based on the seminal work by Diamond and Dybvig (1983), which focuses on banks' role as liquidity transformers. In a typical D-D framework, individual agents face uncertainty in their liquidity needs and the banking sector provides a risk-sharing mechanism to diversify this preference risk. However, there are two different explanations as to why bank runs happen in such an economy. Traditional D-D models explain bank runs as the result of the liquidity problems that exist in the banking sector. As banks borrow short but lend long, they are structurally vulnerable to bank runs. A common belief that a bank run will occur will induce agents to run on the bank immediately, forcing a solvent bank to run into liquidity crises.[3] More recently, there have been a few variants of the model[4] that try to explain the formation of the agents' belief, which underpins the switch from the good equilibrium to the bank run equilibrium. They propose that the occurrence of bank runs is related to economic fundamentals. Therefore, a bank run situation is more of an insolvency problem than

[3]There are a number of papers advocating this line, including Bhattacharya and Padilla (1996), Chang and Velasco (2000a) and Cooper and Ross (1998).

[4]See Allen and Gale (1998), Goldstein and Pauzner (2000) and Zhu (2001).

a pure liquidity issue.

There are a few important exceptions that look into the connections between banking crises and currency crises. These include Chang and Valesco (2000b) and Allen and Gale (2000).[5] Chang and Velasco (2000b) extend the Diamond-Dybvig framework and explain the currency crises as the by-product of self-fulfilling liquidity crises. They show that self-fulfilling bank runs make banking crises and speculative attacks possible. However, whether this possibility will develop into reality depends on the exchange rate regime and the lending policy of the central bank. Allen and Gale (2000) develop a model where crises are "fundamental" and result from low asset returns. They show that, under certain circumstances, large movements in exchange rates can be desirable in that they allow better risk-sharing between domestic bank depositors and international investors. Furthermore, the welfare effects of central bank intervention are ambiguous.

This paper is similar to the above two articles in highlighting the role of the banking sector in twin crises. However, there are two major differences. First, in contrast to Chang and Velasco (2000b), my analysis does not depend on the existence of self-fulfilling bank runs but considers only "fundamental" crises.[6] Second, my analysis emphasises the role of financial liberalisation, which is absent in other papers. By introducing an international credit imperfection, my model provides a transmission mechanism through which a small, temporary shock would generate large, persistent domestic balance sheet effects.[7] This amplification effect can explain why a small fundamental problem can evolve into a large-scale deterioration of economic performance.

I model emerging markets as a small country that is faced with a shortage in capital goods, therefore has a higher rate of return on investment. Domestic investors are uncertain about their liquidity needs, and the bank deposits provide a risk-sharing mechanism to diversify this preference risk. Capital account liberalisation allows the domestic economy to borrow capital from the world market on relatively cheaper terms but subject to a wealth constraint. This external funding allows

[5]The so-called "third-generation" currency crises models, represented by Krugman (1998, 1999), have also highlighted the importance of the financial sector in recent East Asian crises. Nevertheless, the banking sector is not explicitly modelled.

[6]If multiple equilibria are allowed, the banking sector is also vulnerable to self-fulfilling bank runs. The increased financial fragility will make the results in this paper even stronger.

[7]Existence of credit market imperfections has been widely used to model the "financial accelerator" mechanisms. These include the borrowing constraint in Kiyotaki and Moore (1997) and costly state verification in Bernanke and Gertler (1989).

banks to invest more in more productive long-term projects, and therefore improves the welfare of domestic investors. However, the credit constraint interacts with aggregate economic activity over the business cycle and generates *asymmetric* effects in response to unexpected productivity shocks. While a positive shock has only a small effect, a negative shock can cause large losses for banks. In particular, a temporary reduction in productivity reduces the value of bank assets and forces the banks to call in the collateral at a discounted liquidation value. It deteriorates the balance sheet of bank assets and further cuts down banks' borrowing capacity. Since the liquidated assets cannot be restored when the shock is over, the amplification effect becomes persistent. A small productivity shock can evolve into a full-blown banking crisis.

This financial fragility exerts heavy pressure on the external sector. When the exchange rate is flexible, banks can transfer the balance sheet losses to depositors through domestic inflation. There will be no banking crisis, yet the exchange rate will fluctuate wildly.

If the exchange rate is fixed and banks cannot change the terms of deposit contracts, banks will be unable to fulfil the promised interest payments in the long run. Knowing this, depositors will run on the banks immediately and a banking crisis happens. As a result of the run, banks are forced to liquidate earlier and aggregate welfare is unduly reduced. Moreover, if the central bank uses international reserves in order to protect the banking sector and to maintain the fixed exchange rate regime, the banking crisis can have a big impact on the external sector and under certain circumstances may lead to a collapse of the currency regime.

The paper is organised as follows. Section 2 describes the economic environment. Section 3 studies the optimal demand deposit contracts before and after financial liberalisation and shows that limited access to the international capital market improves the welfare of domestic investors. Section 4 lays out the equilibrium outcomes after a productivity shock under various scenarios. Section 5 introduces the exchange rate arrangement and discusses how it affects the equilibrium outcomes. Section 6 discusses the policy implications and Section 7 concludes.

2 The economic environment

The model is based on the continuous-time version of the Diamond-Dybvig framework developed by Von Thadden (1998). The details are as follows.

2.1 Investors

The economy consists of a single good and a continuum of identical agents of measure 1. Time is measured continuously, with $t \in [0, 1]$.

At time 0, each agent is endowed with one unit of the good, which can be either deposited in the banks or stored without depreciation. Following Diamond and Dybvig (1983), the economy is subject to a preference shock. That is, for each agent, there is a certain time T at which he needs to consume all his wealth. The agent's utility depends solely on what he can consume at time T. However, this time point is not known to him in advance. Liquidity needs occur entirely unexpectedly and are assumed to be distributed continuously over (0,1] with a probability distribution function $f(t)$ ($F(t)$ is the cdf function). As these shocks are identically and independently distributed across consumers, the function $f(t)$ also represents the distribution of liquidity needs in the whole economy across time. In other words, there is no aggregate uncertainty regarding the aggregate liquidity needs in the economy.

A representative agent has a utility function $u(c) = \frac{c^{1-\beta}}{1-\beta}$, where $\beta > 1$. It satisfies the usual Inada conditions: $u' > 0$, $u'' < 0$, $u'(0) = \infty$ and $u'(\infty) = 0$.

2.2 Investment technology

The economy has access to an investment technology at time 0, which yields a gross return of $R_t = R_0 e^{rt}$ at time t. The return function $R(\cdot)$ is differentiable and satisfies $R_0 \leq 1$, $\frac{\dot{R}_t}{R_t} = r > 0$ and $E(R) > 1$. That is, the investment is more productive in the long run, and it yields a lower return if it is liquidated prematurely.

I also assume that investment can be made only at time 0. In other words, once an investment is liquidated, the proceeds cannot be reinvested using the above investment technology. This is an easy way to capture the same idea as in the Diamond-Dybvig (1983) model that the investment technology is illiquid.

2.3 Banks

The banks function as liquidity transformers as in the Diamond-Dybvig framework. The banking sector is perfectly competitive. At time 0, banks offer a demand deposit contract that specifies the term structure of deposit rates. Each investor chooses whether to deposit his endowment in the banks or not. Throughout this paper, I assume that the banking industry is competitive and, therefore, a representative bank makes zero profit in equilibrium.

As time passes, the preference shock for each agent is realised and it is observed only by himself. Each agent can withdraw his deposits at any time. However, he cannot redeposit his money in the banks. In other words, once an agent withdraws his deposits, he will have to consume them immediately or store them for future consumption.

To avoid the controversial multiple equilibria property, I follow Allen and Gale (1998, 2002) and rule out all inessential equilibria (ie Pareto-dominated equilibria) when more than one equilibrium exists. Consequently, a bank run occurs only if there is no other equilibrium in the economy.

2.4 Financial liberalisation

I label financial liberalisation as the situation where the domestic banking sector is allowed to borrow from the international capital market.[8] The risk-free international interest rate is assumed to be 0, which captures the fact that most emerging markets are capital-constrained and therefore the marginal return of capital is higher than the world average. However, this international borrowing is subject to a wealth constraint. To be more specific, the maximum borrowing from the international capital market cannot exceed a certain proportion (η) of the market value of bank assets.

The credit constraint can be justified in two ways. First, the limit (η) can be understood as the proportion of assets that are qualified to be international collateral (see Kiyotaki and Moore (1997)). Because the domestic investors borrow international loans at a risk-free rate, a certain amount of collateral is required to guarantee that foreign investors do not face default risk. The above assumption actually implies that the proportion of assets that can be used as international collateral is constant.[9] Second, it is similar to the margin requirement proposed by Aiyagari and Gertler (1999) and Mendoza and Smith (2002). In the small open economy examined here, domestic investors are the agents facing a margin requirement. They must finance at least a fraction κ of their assets out of equities. Such a margin requirement is equivalent to the above assumption when

[8] "Financial liberalisation" is thus equivalent to capital account liberalisation, or (limited) access to the international capital market.

[9] In practice, this collateral-based international borrowing may turn out to be procyclical in that the borrowing capacity (η) is higher during economic booms. This, however, will only make the amplification effect in this paper even stronger.

$\eta = \frac{1-\kappa}{\kappa}$.

3 Optimal demand deposit contracts

Under autarky, each private agent invests his endowment in the investment technology, and liquidates it whenever his liquidity need is realised. The expected utility of a representative agent is

$$U^{au} = \int_0^1 u(R_t)f(t)dt$$

3.1 Optimal arrangement before financial liberalisation

Now suppose that agents act collectively through a banking sector. Given that there is no uncertainty regarding aggregate liquidity needs, a bank is able to diversify the idiosyncratic preference risk and agents will obtain a better consumption path than in autarky. A deposit contract can be specified as a function x_t which represents the fraction of the original deposits that will be liquidated for each consumer at time t. Therefore, the deposit interest rate at time t is the yield from this asset liquidation ($x_t R_t$). In a competitive world, a representative bank will choose the optimal portfolio and optimal consumption allocation to maximise a representative agent's expected utility.[10]

$$\max \quad \int_0^1 u(c_t)f(t)dt$$

$$\text{s.t.} \quad c_t \leq x_t R_t \tag{3.1}$$

$$x_t \geq 0 \text{ for all } t \in [0,1] \tag{3.2}$$

$$\int_0^1 x_t f(t)dt = 1 \tag{3.3}$$

Equation (3.1) is the budget constraint at time t, specifying that the only resource for interest payments at time t is through asset liquidation. Obviously, in equilibrium the equality holds because the long-term project is more productive and it is inefficient to liquidate more assets than the bank's obligations. Equations (3.2) and (3.3) are the resource constraints in the economy.

[10] Competition among banks leads them to maximise the expected utility of a typical depositor subject to a zero profit constraint. As a result, a representative bank acts like a "social planner" and chooses the best demand deposit contract for agents.

The first-order condition is:

$$u^{'}(x_t R_t) \cdot R_t = constant \tag{3.4}$$

The familiar Euler equation requires that the marginal utility of each unit of bank assets be equalised across the time. Given the CRRA utility function and the aggregate resource constraint (equation (3.3)), the optimal demand deposit contract in a closed economy[11] is characterised by:

$$x_t^c \equiv \frac{R_t^{\frac{1}{\beta}-1}}{\int_0^1 R_t^{\frac{1}{\beta}-1} f(t)dt} \tag{3.5}$$

$$c_t^c = x_t^c R_t = \frac{R_t^{\frac{1}{\beta}}}{\int_0^1 R_t^{\frac{1}{\beta}-1} f(t)dt} \tag{3.6}$$

The optimal contract has the property that $\frac{\dot{x}_t^c}{x_t^c} = \frac{1-\beta}{\beta} \cdot \frac{\dot{R}_t}{R_t} = \frac{1-\beta}{\beta} r < 0$ and $\frac{\dot{c}_t^c}{c_t^c} = \frac{r}{\beta} \in (0, r)$. In other words, when the relative risk aversion coefficient is greater than 1, the optimal consumption profile is always flatter than the return profile of the investment opportunity. An agent hit by an early consumption shock actually consumes more than his endowment has physically produced up to that time. The counterpart of this is that late consumption is actually reduced compared with autarky. However, as interest payments increase over time, late consumers have no incentive to withdraw prematurely.[12]

These properties are typical in conventional banking literature. The demand deposit contract allows the domestic agents to pool their endowments together and invest in a more efficient way. It acts like an insurance contract among these agents who are faced with uncertainties in liquidity needs. Not surprisingly, a representative agent is better off in this banking equilibrium than under autarky.[13]

[11] Throughout this paper, a "closed" economy is equivalent to the economy before capital account liberalisation, and an "open" economy is equivalent to the economy after capital account liberalisation.

[12] There exists a second equilibrium in which a bank run is self-fulfilling (see Diamond and Dybvig (1983)). This paper restricts attention to the "essential run" which has been proposed in a series of papers by Allen and Gale (1998, 2002). In addition, there are some other channels through which the inessential run equilibrium can be removed. See Zhu (2001) and Goldstein and Pauzner (2000).

[13] Mathematically, the autarky economy is a special case of problems (3.1) to (3.3) in which x_t equals 1 at any time. A deposit contract actually expands the choice set of agents' consumption path.

3.2 Optimal arrangement with international borrowing

Now consider the situation after financial liberalisation. The domestic bank can borrow from the international capital market at a risk-free rate of zero.[14] Compared with domestic investors, international lenders obtain a "safer" return over time but receive a lower interest payment on average. In addition, this international borrowing is subject to a wealth constraint. In particular, the maximum borrowing from the international market cannot exceed a certain proportion (η) of the market value of bank assets at any time.

A representative bank's optimisation problem becomes:

$$\begin{aligned} \max \quad & \int_0^1 u(c_t) f(t) dt \\ \text{s.t.} \quad & c_t f(t) = x_t R_t f(t) + \dot{b}_t \qquad (3.7) \\ & x_t \geq 0 \\ & \int_0^1 x_t f(t) dt = 1 \\ & b_t \leq \eta \cdot \int_t^1 x_\tau f(\tau) d\tau \cdot R_t \qquad (3.8) \\ & b_0 = b_1 = 0 \qquad (3.9) \end{aligned}$$

Equation (3.7) specifies that the bank can pay depositors either by liquidating the assets or by borrowing from the international capital market. The resource constraints are the same as with the closed economy. Equation (3.8) is the borrowing constraint at time t. The maximum amount that a bank can borrow from the international market is a certain proportion of the value of the bank's remaining assets. Equation (3.9) states two boundary conditions. First, the initial level of international loans is zero. This assumption allows us to focus on the case in which the domestic economy uses international borrowing as a refinancing tool.[15] Second, the bank must repay all international loans by time 1.

Comparing this optimisation problem with the one in a closed economy, the latter problem turns out to be a special case in which international borrowing is restricted to be zero at any time.

[14] The international interest rate can be greater than zero. All that is important is that the domestic return increases over time at a faster rate than the international return.

[15] If we remove this restriction, the bank can use some international loans to invest in domestic projects. In this case, the borrowing constraint becomes binding earlier and the balance sheet effects discussed below are stronger.

Not surprisingly, the new contract after financial liberalisation is superior to the one in the closed economy.

Under the new setting, the key issue is how the bank takes advantage of its borrowing capacity. International capital flows have two advantages. First, the opportunity cost of internal funding, r, is higher than the cost of the international loans (0). In other words, capital account liberalisation allows the bank to borrow on more advantageous terms from the international capital market in order to increase investment in the more productive long-term project. Second, by using international loans, the bank can maintain a higher level of assets, and its borrowing capacity increases accordingly. As a result, the bank would prefer to finance through the international capital market so long as the borrowing capacity has not been reached. Define $X_t = \int_t^1 x_\tau f(\tau) d\tau$ as the bank's remaining assets at time t, it must be:

Lemma 1 $X_t = 1$ *(or* $x_t = 0$*) so long as equation (3.8) is not binding.*

Once the bank's borrowing constraint becomes binding, the bank has to liquidate assets to pay the depositors. At that time, its borrowing capacity decreases as the value of collateral assets drops. Therefore, more assets need to be liquidated to keep the international borrowing within limit. However, the bank will always choose to borrow up to its capacity for the same reasons mentioned above.

Lemma 2 *Once the borrowing constraint is binding, it will be binding always.*

Combining the above arguments, the bank's borrowing behaviour has the following properties. After gaining access to the international capital market, the bank will initially finance the domestic interest payments entirely through international loans. The level of international debt increases until the bank's borrowing limit is reached. Afterwards, the bank will have to liquidate bank assets to pay domestic investors and repay its international debt. From that point on, the borrowing constraint will be always binding.

Suppose that the borrowing constraint is not binding until a certain time t_0, the above maximisation problem has an appealing property in that it can be expressed in a separable fashion. Lemma 1 and Lemma 2 suggest that it can be divided into two problems before and after t_0:

$$E(U^o) = \max_{t_0} \; E[U(t_0)] = E[U_1(t_0)] + E[U_2(t_0)]$$

where

$$\begin{aligned} E[U_1(t_0)] &= \max \int_0^{t_0} u(c_t)f(t)dt \quad (3.10) \\ \text{s.t.} \quad c_t f(t) &= \dot{b}_t \\ b_0 &= 0, \quad b_{t_0} = \eta R_{t_0} \end{aligned}$$

and

$$\begin{aligned} E[U_2(t_0)] &= \max \int_{t_0}^{1} u(c_t)f(t)dt \quad (3.11) \\ \text{s.t.} \quad c_t f(t) &= -\dot{X}_t R_t + \dot{b}_t \quad (3.12) \\ b_t &= \eta X_t R_t \quad (3.13) \\ X_{t_0} &= 1, \quad X_1 = 0 \quad (3.14) \end{aligned}$$

That is, the initial optimisation problem can be solved in two stages. First, conditional on a given time t_0, the bank chooses the optimal consumption allocations in each subperiod. Second, the bank chooses the time t_0 to maximise the expected utility of a typical depositor.

The solution to the first suboptimisation problem is relatively straightforward. Since the total amount of consumption in the first subperiod is constant ($\int_0^{t_0} c_t f(t)dt = b_{t_0} - b_0 = \eta R_{t_0}$), the best allocation is to have a flat consumption path. The preference risk is fully diversified to the international market. Hence:

$$\begin{aligned} c_t &= c = \frac{\eta R_{t_0}}{F(t_0)} \quad \forall t \leq t_0 \quad (3.15) \\ E[U_1(t_0)] &= u(c) \cdot F(t_0) \quad (3.16) \end{aligned}$$

The second suboptimisation problem is to choose the optimal consumption path when the borrowing constraint is always binding. Combining equations (3.12) and (3.13), we can write the Lagrangian equation as follows:

$$L = \int_{t_0}^{1} u(c_t)f(t)dt - \int_{t_0}^{1} \lambda_t[c_t f(t) + (1-\eta)\dot{X}_t R_t - \eta X_t \dot{R}_t]dt$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial L}{\partial c_t} &= 0 \rightarrow u^{'}(c_t) - \lambda_t = 0 \\ \frac{\partial L}{\partial X_t} &= 0 \rightarrow \dot{R}_t \lambda_t + R_t(1-\eta)\dot{\lambda}_t = 0 \end{aligned}$$

When the utility function is of CRRA form, it can be shown that:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\beta} \cdot \frac{1}{1-\eta} \cdot \frac{\dot{R}_t}{R_t} \tag{3.17}$$

Compared with the optimal consumption allocation in the closed economy environment, the consumption path in the second subperiod is steeper, implying a higher compensation for patience. This is because the marginal resource cost of early consumption is higher when limited international borrowing is allowed. A marginal increase in interest payments to early consumers reduces the resources available to late consumers not only through the direct impact of asset liquidation, but also indirectly through the reduction of the bank's borrowing capacity, which in turn forces the bank to liquidate more assets to pay international loans. This borrowing capacity consideration, which is absent in the closed economy, induces the bank to pay a smaller proportion of resources to early consumers.

Equation (3.17) can be used to derive the path of $\{X_t\}$ by using the fact that $c_t f(t) = -\dot{X}_t R_t + \dot{b}_t = (\eta-1)\dot{X}_t R_t + \eta X_t \dot{R}_t$.[16]

$$\frac{\dot{c}_t}{c_t} = \frac{(\eta-1)\ddot{X}_t R_t + (2\eta-1)\dot{X}_t \dot{R}_t + \eta X_t \ddot{R}_t}{(\eta-1)\dot{X}_t R_t + \eta X_t \dot{R}_t} = \frac{1}{\beta} \cdot \frac{1}{1-\eta} \cdot \frac{\dot{R}_t}{R_t}$$

After some rearrangement, the motion function for $\{X_t\}$ becomes:

$$\ddot{X}_t + A_1 \dot{X}_t + A_2 X_t = 0 \tag{3.18}$$

where $A_1 = -\frac{r[1+\beta(2\eta-1)]}{\beta(1-\eta)}$ and $A_2 = [\frac{1}{\beta(1-\eta)} - 1] \cdot \frac{\eta r^2}{1-\eta}$.

Equation (3.18), together with the two boundary conditions that $X_{t_0} = 1$ and $X_1 = 0$, consists of a second-order differential equation problem that determines the transition path of $\{X_t\}$.[17] After some tedious algebra, the solution to this problem is:

$$X_t = \frac{e^{z_1(t-1)} - e^{z_2(t-1)}}{e^{z_1(t_0-1)} - e^{z_2(t_0-1)}} \tag{3.19}$$

$$c_t = \frac{r(1-\beta)}{\beta} \cdot \frac{R_t e^{z_2(t-1)}}{e^{z_1(t_0-1)} - e^{z_2(t_0-1)}} \tag{3.20}$$

$$E[U_2(t_0)] = \frac{\beta^\beta (1-\eta)(R_0 e^r)^{1-\beta}}{r^\beta (1-\beta)^{\beta+1}} \cdot \frac{[e^{z_1(t_0-1)} - e^{z_2(t_0-1)}]^\beta}{e^{z_1(t_0-1)}} \tag{3.21}$$

[16] I am assuming that $f(t)$ is uniformly distributed across time in the remaining part in order to obtain more qualitative results.

[17] Boyce and DiPrima (2001) describe in detail how to solve this type of problem.

where $z_1 = \frac{r\eta}{1-\eta}$ and $z_2 = \frac{r[1+\beta(\eta-1)]}{\beta(1-\eta)}$.

The remaining part of the problem is to find an optimal divisible time (t^*) that maximises the expected utility of a representative agent, which is the sum of $E[U_1(t_0)]$ and $E[U_2(t_0)]$. Based on the solutions in the two suboptimisation problems, it can be shown that t^* has the following properties:

Lemma 3 *The properties of the optimal divisible time (t^*) include:*

1) $t^* \in [0, 1]$.

2) t^* is increasing in η. In particular, $t^* = 0$ when $\eta = 0$.

3) The consumption path is continuous at t^*.

Proof: See Appendix A.

These properties are quite intuitive. When no international borrowing is allowed ($\eta = 0$), the problem degenerates into the closed economy one and the consumption path is increasing over time. When the domestic bank has limited access to the international capital market, the bank can borrow international loans on relatively cheaper terms. When the bank's borrowing capacity is higher, the bank will increase its engagement in higher-yielding investment domestically, and rely more on international debt to repay customers (t^* increases). In the extreme case where the banking sector has unlimited access to the international capital market, the optimal consumption path is flat across time and the bank is able to invest all deposits in the more productive domestic long-term project.

3.3 A numerical example

I provide a numerical example to demonstrate the properties of optimal demand deposit contracts under the two economic environments, and will use it as a benchmark for later analysis. I assume that the return function, the distribution of liquidity needs, the utility function and the borrowing constraint coefficient are as follows: $R_t = 0.8e^t$, $f(t) = 1$, $\beta = 2$ and $\eta = 0.5$.

With this parameterisation, the optimal contract in a closed economy can be derived directly from equations (3.5) and (3.6). A consumer at time t receives an interest payment of $c_t^c = \frac{0.4e^{\frac{1}{2}t}}{1-e^{-\frac{1}{2}}}$, and the expected utility of a representative agent is $E(U^c) = -0.7741$.

The optimal contract in an open economy is chosen by following the two-stage algorithm described in Section 3.2. I first solve the two suboptimisation problems before and after a given t_0, and then maximise the aggregate expected utility by choosing the best divisible time t^*. Using equations (3.16) and (3.21), this optimisation problem is:

$$max_{\{t_0\}} \quad -\frac{t_0^2}{0.5 \cdot 0.8e^{t_0}} - \frac{2(e^{t_0-1}-1)^2}{0.8e^{t_0}}$$

After some rearrangement, the first-order condition satisfies $1-t^* = e^{t^*-1}$ and the best divisible time t^* is 0.43. Accordingly, the optimal contract under credit-constrained international borrowing is characterised by:

i) when $t < t^*$, $X_t^o = 1$, $x_t^o = 0$, $c_t^o = \frac{\eta R_{t^*}}{t^*}$ and $b_t = t \cdot \frac{\eta R_{t^*}}{t^*}$

ii) when $t \geq t^*$, $X_t^o = \frac{e^t - e}{e^{t^*} - e}$, $x_t^o = -\frac{e^t}{e^{t^*} - e}$, $c_t^o = -\frac{R_t e}{2(e^{t^*} - e)}$ and $b_t^o = \frac{R_t(e^t - e)}{2(e^{t^*} - e)}$

Under this contract, the expected utility for a representative agent is $E^o(U) = -0.6077$. Not surprisingly, the welfare for a representative agent is higher when the domestic economy has access to the international capital market.

Figure 1 shows the paths of contract variables under the two economic settings. In a closed economy, the banking sector provides a risk-sharing mechanism among investors, and the consumption path becomes smoother across time than under autarky. When the domestic economy has limited access to the international capital market, the bank can borrow on relatively cheaper terms and increase its investment in the more productive long-term technology, thereby improving the welfare of domestic investors.

4 Financial fragility

While the positive and limited international borrowing improves the welfare of domestic investors, it also makes equilibrium more fragile. The credit constraint introduces important indirect effects through reduction of borrowing capacity and brings about a shock amplification mechanism. A small, temporary reduction in productivity, which only has a small effect in the closed economy, may cause a large-scale disruption in the open economy environment.

In the remaining part of this paper, I define a temporary productivity shock as follows: for unexpected reasons, the return on bank assets suddenly changes to $R_{t,new} = (1 + \varepsilon)R_t$ during a short interval $[t_1, t_1 + \triangle t]$.[18] The asset returns are the same as before in any other time.

I will discuss the effects of this productivity shock in the two economies, each under two different scenarios: (1) the bank cannot renegotiate the contract and must stick to the prespecified deposit rate; and (2) the bank renegotiates the terms of the contract when the shock occurs. As we will see, the effects of productivity shocks differ under the two scenarios, and they also differ in the closed economy and the open economy environments.

4.1 Case 1: closed economy / no renegotiation of the contract

Before capital account liberalisation, such a temporary shock has only a modest effect. When a temporary adverse shock ($\varepsilon < 0$) occurs, the bank has to liquidate more assets to meet its contractual obligations. The extra amount of assets that are liquidated to pay each depositor at time t ($t \in [t_1, t_1 + \triangle t]$) is $\frac{-\varepsilon}{1+\varepsilon} x_t^c$. Therefore, during the shock period, the aggregate amount of extra assets that are liquidated is:

$$\triangle X_{t_1+\triangle t}^c = \int_{t_1}^{t_1+\triangle t} \frac{-\varepsilon}{1+\varepsilon} x_\tau^c f(\tau) d\tau \tag{4.1}$$

When the shock is over, the economy returns to the normal state and the bank pays back depositors as promised in the initial contract. However, as the bank has lost some assets during the shock period, it will face a shortage of assets eventually. At a certain time before all agents are repaid, the bank will lose all its assets and go into bankruptcy. The default time (t_2) is determined by:

$$X_{t_2}^c = \triangle X_{t_1+\triangle t}^c$$

At the time when the bank goes into bankruptcy, there are still $1 - F(t_2)$ agents who fail to receive interest payments. The liabilities of the bank to these agents are $c_{t_2}^c (1 - F(t_2))$.

The timing of default depends on two factors.[19] First, default occurs earlier if the size of the negative shock is larger. This is quite intuitive. A larger shock forces the bank to liquidate more assets in bad time and therefore it goes into bankruptcy more quickly. Second, an earlier shock can cause the bank to default earlier. The reason is a little tricky. Because x_t^c is decreasing over time

[18] $\varepsilon > 0$ represents a positive productivity shock and $\varepsilon < 0$ represents a negative one.

[19] From equation (4.1), it is obvious that the bank loses more assets if the shock is larger. In addition, $\triangle X_{t_1+\triangle t}^c$ is increasing in t_1 because x_t^c is decreasing in t in a closed economy.

in the initial contract and the amount of extra liquidation is proportional to x_t^c, an earlier shock will lead to more liquidation and an earlier bankruptcy.

On the other hand, if the temporary shock is positive, it will have a symmetric effect. During the time when the assets are more productive, the bank can liquidate fewer amounts of assets to pay depositors. The assets that are not liquidated during the shock period, which are in the amount of $\triangle X = \int_{t_1}^{t_1+\triangle t} \frac{\varepsilon}{1+\varepsilon} x_t^c f(t) dt$, will be retained as long-term investments and yield a net profit of $\triangle X \cdot R(1)$ for the bank at time 1.

Figure 2 shows the balance sheet effects of temporary shocks (the duration is always 0.01) in the benchmark example. As we can see, the magnitude of the effects is mainly affected by the size of the shocks. Timing matters, but only plays a trivial role. Moreover, the effects of positive and negative shocks are symmetric. But overall, in a closed economy, such temporary shocks only have small balance sheet effects.[20]

To summarise, in a closed economy, if the bank is unable to renegotiate the deposit contract, it will eventually become insolvent after a negative productivity shock occurs. Knowing this, the remaining agents no longer have the incentive to wait until their true liquidity needs are realised. Given that there is no third-party guarantee, the potential insolvency scenario will trigger a bank run immediately.

Proposition 1 *When the bank cannot renegotiate the contract, a negative productivity shock will trigger a bank run immediately.*

Appendix B provides a strict proof of the proposition. The intuition is quite straightforward using the backward induction method. When a shock occurs at time t_1, agents know that the bank will be insolvent at some future time, say t_2. Therefore all agents will withdraw before t_2. Suppose that they decide to wait until a time point t_3 ($t_1 < t_3 < t_2$), a representative agent at time t_1 receives the contractual rate if he needs to consume before t_3 and he obtains a lower rate if his liquidity need is realised after (including) t_3 because a bank run occurs at that time. However, this equilibrium is not stable because an individual agent can make himself better off by deviating from the above strategy. He can choose a similar strategy except that his "run" moment is a little earlier than

[20] The quantitative effects are in the order of $\epsilon \triangle t$.

others'. By switching to this new strategy, the agent always receives a contractual rate but never gets involved in the bank run situation. The fact that each agent has an incentive to run ahead of others causes an immediate bank run when the shock occurs. Therefore, a small "fundamental" problem tends to generate overreaction of the market and a sudden reversal of economic activity.

4.2 Case 2: closed economy / renegotiation of the contract

If the bank can respond to the negative shocks by renegotiating the deposit contract, the losses related to low productivity will be shared among all remaining investors and no bank runs will happen. Whenever a shock occurs, the bank is able to choose a new consumption path by changing its liquidation strategy. The new optimisation problem is to maximise the expected utility of a typical depositor at time t_1 in the new environment:

$$\begin{aligned} \max \quad & \int_{t_1}^{1} u(c^c_{t,new}) f(t)dt \qquad (4.2)\\ \text{s.t.} \quad & c^c_{t,new} = x^c_{t,new} R_{t,new} \\ & \int_{t_1}^{1} x^c_{t,new} f(t)dt = \int_{t_1}^{1} x^c_t f(t)dt = X^c_{t_1} \end{aligned}$$

This optimisation problem is quite similar to the initial one except that the new return profile has a temporary deviation during the shock period. Following Section 3.1, the new portfolio allocation is characterised by:

$$x^c_{t,new} = \frac{\int_{t_1}^{1} R_t^{\frac{1}{\beta}-1} f(t)dt}{\int_0^1 R_t^{\frac{1}{\beta}-1} f(t)dt} \cdot \frac{R_{t,new}^{\frac{1}{\beta}-1}}{\int_{t_1}^{1} R_{t,new}^{\frac{1}{\beta}-1} f(t)dt} = x^c_t \cdot \left(\frac{R_{t,new}}{R_t}\right)^{\frac{1}{\beta}-1} \cdot \frac{\int_{t_1}^{1} R_t^{\frac{1}{\beta}-1} f(t)dt}{\int_{t_1}^{1} R_{t,new}^{\frac{1}{\beta}-1} f(t)dt} \qquad (4.3)$$

$$c^c_{t,new} = \frac{\int_{t_1}^{1} R_t^{\frac{1}{\beta}-1} f(t)dt}{\int_0^1 R_t^{\frac{1}{\beta}-1} f(t)dt} \cdot \frac{R_{t,new}^{\frac{1}{\beta}}}{\int_{t_1}^{1} R_{t,new}^{\frac{1}{\beta}-1} f(t)dt} = c^c_t \cdot \left(\frac{R_{t,new}}{R_t}\right)^{\frac{1}{\beta}} \cdot \frac{\int_{t_1}^{1} R_t^{\frac{1}{\beta}-1} f(t)dt}{\int_{t_1}^{1} R_{t,new}^{\frac{1}{\beta}-1} f(t)dt} \qquad (4.4)$$

Equations (4.3) and (4.4) suggest that, when the reduction of productivity is only temporary ($\int_{t_1}^{1} R_t^{\frac{1}{\beta}-1} f(t)dt$ is slightly smaller than $\int_{t_1}^{1} R_{t,new}^{\frac{1}{\beta}-1} f(t)dt$), the new contract will spread the losses among all remaining investors. Compared with the initial contract, this new one offers more resources ($x^c_{t,new}$ is higher than x^c_t for $t \in (t_1, t_1 + \triangle t)$) to those consumers in bad time. However, the new interest payment is still lower than the initial contractual rate ($c^c_{t,new} < c^c_t$ for $t \in (t_1, t_1 + \triangle t)$). Meanwhile, the rest of the agents will share the costs by receiving slightly lower payments in the future.

An important feature of the new contract is that it constructs a stable equilibrium. Although everyone is worse off, the new contract is the best available outcome. Since the new consumption path is increasing over time, and the bank has no insolvency problem after switching to the new contract, all agents will choose to leave their deposits in the bank until their liquidity needs are realised.

4.3 Case 3: open economy / no renegotiation of the contract

When the domestic economy has access to the international capital market, the external financing allows the domestic economy to take advantage of the cheaper loans from the international capital market. However, this wealth-constrained borrowing tends to exert heavy pressure on the banking sector if the value of bank assets decreases. Indeed, it introduces a new channel through which a temporary shock becomes amplified and persistent and causes large balance sheet effects. More importantly, the balance sheet effects are asymmetric. While a positive shock has a modest effect on the bank's balance sheets, a negative shock can result in huge losses. Consequently, the banking sector becomes more fragile in the open economy environment.

We first consider the situation under which the bank is unable to renegotiate the terms of the deposit contract. Compared with the closed economy, the balance sheet effects after capital account liberalisation have two important new properties. First, the timing of the shock plays a key role. Second, the balance sheet effect is asymmetric in that an amplification mechanism is absent for positive shocks but it causes huge losses for negative shocks.

The timing of the shock matters in two ways. On the one hand, if the shock occurs at an early stage ($t_1 < t^*$), it has no real effect so long as the borrowing constraint is not binding. Since the bank depends exclusively on foreign debt to pay the early depositors, the productivity shock does not affect the bank's investment strategy at all. The financial sector functions as if nothing happened. However, if the shock happens when the borrowing constraint is binding ($t_1 \geq t^*$), or if the shock is big enough to make the original non-binding borrowing constraint binding, it will have a large balance sheet effect. In the remaining part, I will show how a small, temporary adverse shock can evolve into a disaster for the banking sector.

A temporary shock at time t_1 has two immediate effects. The first effect, which is the same as in a closed economy, is that the bank needs to liquidate more assets to pay depositors. The amount of extra asset liquidation is in the order of $\frac{-\varepsilon}{1+\varepsilon}x_t^o \cdot \triangle t$. In addition, there is a second effect related to the external borrowing capacity. Because the shock reduces the value of assets by $-\varepsilon X_{t_1}^o R_{t_1}$, the bank is forced to pay back some international loans to keep the debt level within the limit. The amount of loan payment is determined by:

$$\begin{aligned} b_{t_1,new}^o &= b_{t_1}^o - \triangle b_{t_1} = \eta[(X_{t_1}^o R_{t_1}(1+\varepsilon) - \triangle b_{t_1}] \\ &\rightarrow \triangle b_{t_1} = -\varepsilon X_{t_1}^o R_{t_1} \\ &\quad \triangle X_{t_1}^o = \frac{\triangle b_{t_1}}{(1+\varepsilon)R_{t_1}} = \frac{-\varepsilon}{1+\epsilon} X_{t_1}^o \end{aligned} \tag{4.5}$$

Notice that the duration of the shock ($\triangle t$) is absent in equation (4.5). Hence, the second effect is a much larger effect than the first one.

In addition to an amplified within-period effect, the temporary shock also has a persistent effect due to the interaction between the value of bank assets and the borrowing constraint. The dynamics of $\{X_t^o\}$ is as follows:

1. During the shock period $(t_1, t_1 + \triangle t)$, the bank reduces its international borrowing and keeps it within limit. To meet its contractual obligations and to abide by the limit rule, it has to liquidate some long-term productive projects. The budget constraint at a particular time point is:

$$\begin{aligned} c_t^o f(t) &= -\dot{X}_{t,new}^o R_{t,new} + \dot{b}_{t,new}^o \\ &= (\eta - 1)\dot{X}_{t,new}^o R_{t,new} + \eta X_{t,new}^o \dot{R}_{t,new} \end{aligned} \tag{4.6}$$

   in which c_t^o is the predetermined contractual deposit rate as specified in Section 3.2. This first-order differential equation, together with the initial condition of $X_{t_1,new}^o = X_{t_1}^o - \triangle X_{t_1}^o$ (equation (4.5)), determines the path of $\{X_t^o\}$ during the shock period.

2. When the shock is over $(t_1 + \triangle t)$, the productivity returns to the normal level. The value of bank assets increases, and the borrowing constraint is no longer binding. From Lemma 1, the bank will choose to use international loans to pay depositors until the new borrowing limit is reached (denote that time as t_{11}). That is, during a certain period $t \in [t_1 + \triangle t, t_{11}]$, the dynamics of the bank's portfolio holding is:

$$\dot{b}_{t,new}^o = c_t^o$$

$$X^o_{t,new} = X^o_{t_1+\triangle t,new}$$
$$t_{11} \text{ is determined by } : b^o_{t_{11},new} = \eta X^o_{t_1+\triangle t,new} R_{t_{11}}$$

3. After the bank reaches its borrowing limit, it needs to liquidate domestic assets to pay depositors and foreign debt. The borrowing constraint, again, is always binding. Following the analysis in Section 3.2, the dynamics of $\{X^o_{t,new}\}$ is determined by:

$$c^o_t f(t) = (\eta - 1)\dot{X}^o_{t,new} R_t + \eta X^o_{t,new} \dot{R}_t,$$

with an initial condition of $X^o_{t_{11},new} = X^o_{t_1+\triangle t,new}$. Whenever $X^o_{t,new}$ drops to zero, the bank becomes insolvent and claims bankruptcy.

Figure 3 demonstrates the quantitative effects of temporary shocks (with a duration of 0.01) in an open economy. In contrast to the closed economy case, a temporary negative shock has not only a larger within-period effect but also a dynamic effect. The bank has to liquidate more long-term assets and the impact on its balance sheet is larger. For example, a 10% decrease in productivity during time $[0.43, 0.44]$ will bring down the bank at time $t_2 = 0.9755$, and the balance sheet losses are $c^o_{t_2}(1 - t_2) = 0.0598$. In contrast, the same productivity shock in a closed economy leads to bankruptcy at time $t = 0.9985$ and the liability cost is 0.0025.

Moreover, the effects of temporary shocks are no longer symmetric in an open economy environment. While a negative shock has large-scale balance sheet effects, a positive shock brings a modest level of profits for the bank as in a closed economy. This asymmetry lies in the fact that the effects of negative shocks are persistent while those of positive shocks are only temporary. More specifically, an adverse shock forces the bank to liquidate more assets and these assets cannot be recovered when the shock is over. Therefore, the balance sheet effects become persistent. In contrast, when the productivity increases temporarily, the bank's borrowing capacity increases and it is able to rely on international borrowing to pay depositors. However, when the shock is over, the borrowing capacity returns to the initial level, forcing the bank to liquidate assets and pay back the new loans that have been incurred during the shock period. The borrowing capacity effect is temporary and is cancelled out as soon as the shock is over. In the given example, a temporary shock of $\varepsilon = 0.1$ at time $t \in [0.43, 0.44]$ yields a profit of 0.0005 at time 1, which is much smaller than the losses caused by a negative shock of the same magnitude.

Figure 3 also suggests that, if the borrowing constraint is binding during the shock period, the balance sheet effects are larger for early shocks. This is because, after financial liberalisation, a temporary shock also has a dynamic effect due to the interaction between the value of bank assets and the credit constraint. An earlier shock has a larger dynamic effect. Hence, the bank will default earlier, with larger balance sheet losses.

To summarise, although access to the international capital market allows the domestic banking industry to write a better contract, it also increases the vulnerability of the financial sector. A small, unexpected adverse productivity shock may evolve into a large hit to the banking industry. Even worse, this amplification effect works only on the "bad" side and is absent when the shock is positive.

The amplified balance sheet effect will induce domestic investors to run on the bank immediately whenever a negative shock occurs. The reason is exactly the same as in Section 4.1, except that investors know that the bank will have a more severe solvency problem in the open economy environment. The coordination failure among investors causes severe disruption to the economy and brings about huge losses to domestic investors.

4.4 Case 4: open economy / renegotiation of the contract

If the bank can renegotiate the terms of the contract, the equilibrium outcome is completely different. As in a closed economy, the bank can design a new contract that maximises the expected utility of remaining investors, and investors will accept the new contract and no bank run occurs.

Suppose that a negative shock occurs at time t_1. Following Section 4.3, the bank has to liquidate extra assets in the amount of $\frac{-\epsilon}{1+\epsilon} X^o_{t_1} R_{t_1}$ to pay foreign debt that exceeds its borrowing capacity. And the bank's new optimisation problems is:

$$\max \quad \int_{t_1}^{1} u(c_{t,new}) f(t) dt \tag{4.7}$$

$$\text{s.t.} \quad c_{t,new} f(t) = x_{t,new} R_{t,new} f(t) + \dot{b}_{t,new}$$

$$X_{t_1,new} = X^o_{t_1} - \triangle X^o_{t_1} = \frac{1+2\epsilon}{1+\epsilon} X^o_{t_1} \tag{4.8}$$

$$X_1 = 0 \tag{4.9}$$

$$X_{t,new} = X_{t_1+\triangle t,new} \quad \forall t \in (t_1 + \triangle t, t_2) \tag{4.10}$$

$$b_{t,new} = \eta X_{t,new} R_{t,new} \quad \forall t \in (t_1, t_1 + \triangle t) \; and \; t > t_2 \tag{4.11}$$

Equations (4.7) to (4.11) represent the expected utility of a typical depositor at time t_1. The bank repays the domestic investors and foreign creditors either by liquidating domestic assets or by rolling over international debt. Equations (4.8) and (4.9) are the two new boundary conditions for $\{X_{t,new}\}$, in which the former equation reflects the within-period amplification effect that is related to the reduction of borrowing capacity. Equation (4.9) also implies that the bank will not liquidate all assets until time 1. Hence, there is no insolvency problem in the new contract. Equations (4.10) and (4.11) are results from Lemma 1 and Lemma 2, suggesting that the new optimisation problem consists of three separable parts:

- During the shock period $(t_1, t_1 + \triangle t)$. As the borrowing constraint is always binding during the shock period, this suboptimisation problem is similar to the one specified in equations (3.11) to (3.14).
- When the shock is over and the borrowing constraint is not binding ($t \in [t_1 + \triangle t, t_2]$).[21] The portfolio choice is similar to that in problem (3.10), where the bank depends exclusively on foreign debt. The level of remaining assets is constant over this period (equation (4.10)), and the consumption path is flat.
- When the shock is over and the borrowing constraint is binding again ($t \in [t_2, 1]$). It is similar to that in problem (3.11), except that the new boundary conditions are $X_{t_2,new} = X_{t_1+\triangle t,new}$ and $X_1 = 0$.

The problems can be solved using the same method as in Section 3.2.[22] The terms of the new contract depend on two key parameters, $X_{t_1+\triangle t,new}$ and t_2, which are endogenously chosen by the bank. As we will see later, the new contract can spread part of the burden among all future consumers. Moreover, because the bank will not fall into bankruptcy after switching to the new contract, the new equilibrium is stable and no bank runs will happen.

To summarise, in an open economy, a negative productivity shock tends to have a larger balance sheet effect due to the interaction of the value of bank assets and the bank's borrowing capacity.

[21] t^2 is the first time that the bank's borrowing constraint becomes binding again. It can be endogenously determined as in Section 3.2.

[22] Appendix C provides the algorithm in detail.

Depending on whether the bank is able to change the terms of deposit contracts, the economy may evolve into a new "good" equilibrium or a "bad" (an immediate bank run) one.

5 Exchange rate arrangements

So far, all of my analysis has been in real terms. Now I start to introduce exchange rate regimes into the economy. I assume that all deposit contracts are denominated in local currency (Thai baht), and international loans are denominated in foreign currency (US dollar). The foreign price is assumed to be constant at 1, and the domestic price (ie the exchange rate) is p. In addition, the asset returns are in real terms (in units of goods or in units of foreign currency).

A demand deposit contract requires investors to deposit their endowments with the bank at time 0. In return, depositors are given the right to withdraw D_t bahts at time t. The bank's investment strategy is to choose the term structure of its investment portfolio (x_t).

A flexible exchange rate regime is defined as the arrangement in which domestic prices (or exchange rates) can move freely with economic fundamentals. The central bank's monetary policy is to provide enough domestic currency for the bank's interest payments (ie $M_t = D_t f(t)$ always).

In contrast, under a fixed exchange rate regime, the domestic price is fixed at $p = 1$. That is to say, the money supply is determined by the real resources available in the economy, rather than by the liquidity needs of deposit payments (ie $M_t = x_t R_t f(t)$).

5.1 Flexible exchange rate regime

When the exchange rate is flexible, the money supply is predetermined, and the domestic prices fluctuate in response to changes in productivity. This domestic inflation is an automatic absorber of the productivity shocks. The bank is able to change its investment strategy (without changing the terms of the contract) and offer a new contract that in real terms is the same as in the "good" new equilibrium (in Sections 4.2 and 4.4). That is, under a flexible exchange rate regime, a productivity shock tends to cause fluctuations in exchange rates and does not cause the failure of the domestic banking sector.

5.1.1 Before capital account liberalisation

Before financial liberalisation, the best deposit contract (in Section 3.1) can be implemented in the following way. At the beginning, the bank takes deposits from investors. It agrees to pay a deposit rate of $D_t = c_t^c$ to each consumer at time t. At the same time, it chooses to liquidate a faction of $x_t^c f(t)$ of the original deposits for interest payments at time t. On the money side, the domestic money supply is set to equal the amount of deposit payments at any time ($M_t = D_t f(t)$). Under this arrangement, the exchange rate is constant at $p = 1$. Everything works out perfectly and the economy achieves the optimal outcome.

When a temporary adverse shock ($\varepsilon < 0$) occurs at time $t \in [t_1, t_1 + \triangle t]$, the bank sees lower returns ($(1+\varepsilon)x_t R_t f(t)$) if it abides by the initial portfolio strategy. Given that the money supply is predetermined ($M_t = c_t f(t)$), domestic prices will increase to $p = \frac{1}{1+\varepsilon}$ during the shock period. In other words, domestic currency depreciates and consumers during that time interval bear all of the welfare losses if the bank does not change its investment strategy.

However, the bank can respond to the negative shock by adjusting its portfolio allocation. Given that the domestic money supply is predetermined and the prices fluctuate automatically, the bank is able to change the real terms of the contract by adopting a new investment strategy. In particular, the bank chooses a new path of $x_{t,new}$ ($t > t_1$) to maximise a typical investor's expected utility:

$$\max \quad \int_{t_1}^{1} u(c_{t,new}) f(t) dt$$

$$\text{s.t.} \quad D_t = c_{t,new} p_{t,new} = x_{t,new} R_{t,new} p_{t,new} \tag{5.1}$$

$$M_t = D_t f(t) = x_t R_t f(t) \tag{5.2}$$

$$\int_{t_1}^{1} x_{t,new} f(t) dt = \int_{t_1}^{1} x_t f_t dt = X_{t_1} \tag{5.3}$$

Notice that price does not matter here because agents care about the real consumption. Therefore, the new optimisation problem is exactly the same as the one in Section 4.2, meaning that the bank is able to achieve the "good" equilibrium under a flexible exchange rate regime. Accordingly, the exchange rate is determined by:

$$p_{t,new} = \frac{M_t}{c_{t,new} f(t)} = \frac{c_t}{c_{t,new}} = \left(\frac{R_t}{R_{t,new}}\right)^{\frac{1}{\beta}} \cdot \frac{\int_{t_1}^{1} R_{t,new}^{\frac{1}{\beta}-1} dt}{\int_{t_1}^{1} R_t^{\frac{1}{\beta}-1} dt}$$

The four left-hand panels in Figure 4 demonstrate the effects of a temporary negative shock ($\varepsilon = -10\%$) during $t = [0.43, 0.44]$. It is obvious that automatic adjustment in the domestic price level plays a major role. The domestic price increases during the shock period. This inflation "tax" forces the agents to accept a welfare loss. Meanwhile, the bank is able to liquidate more assets in bad time and spread the costs among all future consumers. However, the fact that future real consumption is only slightly lower than the original contractual level suggests that this cost-sharing is limited and consumers during the shock period still suffer the most. This is not surprising. Because the marginal rate of return is too low during the shock period, it is very costly to completely share the losses by liquidating more assets during that interval.

Another important conclusion is that there is no bank run when the exchange rate is flexible. The banking sector is resilient because domestic prices fluctuate automatically to absorb the effects of productivity shocks. After the portfolio reallocation, late consumers are still better off than early consumers. Therefore no agents have the incentive to misreport their preference types and no banking crisis happens.

5.1.2 After capital account liberalisation

Similarly, a good equilibrium outcome can be achieved under the flexible exchange rate after financial liberalisation. Domestic inflation and automatic adjustment in the bank's investment strategy enable the economy to achieve the new equilibrium outcome as specified in Section 4.4. However, due to the existence of the credit constraint, the welfare loss to a representative agent is larger, and the exchange rate fluctuates more wildly.

The larger welfare losses come from the fact that domestic inflation can transfer the losses to investors if the liabilities are denominated in domestic currency, but this transfer channel no longer works if the claims are denominated in foreign currency. In the given example, suppose that a negative shock occurs at time t_1. The value of bank assets decreases by $-\varepsilon X^o_{t_1} R_{t_1}$. The bank has to liquidate more assets in the amount of $\frac{-\varepsilon}{1+\varepsilon} X^o_{t_1} R_{t_1}$ (see Section 4.3) to pay foreign debt that exceeds its borrowing capacity. Because international loans are denominated in foreign currency, the bank is not able to mitigate this adverse effect through exchange rate depreciation. Nevertheless, the bank can adjust its investment strategy to maximise the expected utility of domestic investors. The new optimisation problem is the same as problems (4.7) to (4.11), except that the level of the domestic price is determined by $p_{t,new} = \frac{M_t}{c_{t,new} f(t)} = \frac{c_t}{c_{t,new}}$.

The four right-hand panels in Figure 4 demonstrate the quantitative effects of the same shock when the banking sector has access to the international capital market and the exchange rate is flexible. The dynamics is similar to that in the previous subsection in two important ways. First, the automatic inflation process is an important absorber of the productivity shocks. The bank is able to transfer the welfare losses to depositors, and no bank run will happen. Second, by adjusting its investment portfolio, the bank is able to spread part of the burden among future consumers. However, as the balance sheet effect has been amplified, the real effects are much larger and the exchange rate fluctuates more wildly than in the closed economy.

5.2 Fixed exchange rate regime

Under the fixed exchange rate regime, the domestic price is fixed at $p = 1$. Given the absence of the inflation adjustment mechanism, the bank is not able to transfer the losses to depositors and it has to liquidate more assets to meet its contractual obligations. As shown in Sections 4.1 and 4.3, if the bank cannot renegotiate the terms of the contract, the future insolvency problem will cause an immediate attack on the bank because each agent has an incentive to recover his deposit ahead of others. When a bank run occurs, each agent receives an average amount of the bank assets (c_{run}). The payment is $\frac{X^c_{t_1} R_{t_1}(1+\epsilon)}{1-F(t_1)}$ in a closed economy and $\frac{X^o_{t_1} R_{t_1}(1+\varepsilon)-b^o_{t_1}}{1-F(t_1)}$ in an open economy.

5.3 Twin crises

A common practice in emerging market economies is that the domestic banking sector is either explicitly or implicitly protected by the government. The central bank holds international reserves to provide emergency liquidity for domestic banks. Such a policy is useful in preventing disruptive bank runs. However, the protection cost becomes extremely high when domestic banks borrow from the international capital market. Moreover, if the international reserves are also used to maintain a fixed exchange rate regime, the financial fragility can easily spread to the external sector and cause twin crises.

To implement the protection policy successfully, the central bank needs an emergency loan that at least enables domestic banks to fulfil their initial contracts and portfolio strategies. In a closed economy, this minimum requirement is $\int_{t_1}^{t_1+\triangle t} x^c_t (R_t - R_{t,new}) f(t) dt$. That is, domestic banks stick to the initial arrangements, and the central bank bails them out by making up for the shortage in liquidity. If the level of international reserves is below this critical value, domestic investors know that the banks have to liquidate more assets during the shock period, and will finally become insolvent before all investors are repaid. Thus, a bank run occurs immediately (following the same logic as in Section 4), and the fixed exchange rate regime also breaks down.[23]

In an open economy, however, the minimum amount of emergency loans is much higher because of the amplification effect. To avoid a bank run on domestic banks, the central bank needs to provide two layers of emergency liquidity. The first line of credit, which is used to pay customers during the shock period, is similar to the loans in the closed economy. Moreover, there is another liquidity need in order to repay the international loans that exceed the borrowing limit. The second liquidity need, which is in the order of $\eta X^o_{t_1}(R_{t_1} - R_{t_1,new})$, is much higher than the former one. Therefore, it is more difficult for the central bank to bail out the banking sector successfully after financial liberalisation.

Suppose that productivity decreases by 10% between time 0.43 and 0.44 in our benchmark example. Our numerical simulations show that the central bank needs emergency liquidity of at least 0.0629 (among which the second type of loan is 0.0615) to protect domestic banks from runs in an open economy environment, while in a closed economy the minimum requirement of international reserves is 0.0013. Obviously, the larger balance sheet effect associated with capital market openness increases the fragility of the domestic banking sector. Given that the central bank holds the same level of international reserves, the twin crises are more likely to happen in the financially liberalised emerging market economies.

6 Policy implications

My analysis shows that the bank run equilibrium outcome is disruptive to the economy and brings about huge welfare losses. A small productivity shock causes an insolvency problem to the bank, generating a severe liquidity crisis as a result of coordination failure. Understanding such a mechanism provides some room for a central bank (or international lender of last resort) to implement a successful bailout.

[23] I simply assume that the fixed exchange rate regime collapses whenever the central bank is unable to fulfil its commitment, or the level of international reserves drops to zero.

My results have several important policy implications. First, the bailout cost is smaller if the central bank provides emergency liquidity during the shock period rather than "waiting until the last minute". A prompt liquidity injection can save the bank from costly early liquidation. Besides, this emergency loan can be repaid by the bank when the shock is over. For example, in a closed economy, an emergency loan of $\int_{t_1}^{t_1+\triangle t} x_t R_t dt$ allows the bank to repay its customers without liquidating any asset in bad time. When the shock is over, the bank is able to repay the central bank in an amount of $\int_{t_1}^{t_1+\triangle t} x_t dt R_{t_1+\triangle t}$. In contrast, if the central bank waits until the last moment (ie until the bank claims bankruptcy), the outcome turns out to be completely different. Because the balance sheet effect builds up across time and affects the solvency situation of the banks, the bailout cost $(c^c_{t_2}(1 - F(t_2)))$ is higher and it cannot be repaid by the bank. The difference between the two outcomes reflects the "waiting cost" during a crisis if the central bank does not act promptly.

Second, the request for liquidity provision may turn out to be high, especially after financial liberalisation. An important difference between the two economic environments is that the liquidity needs in an open economy are much higher than in a closed economy. Most of the emergency liquidity is used to pay back international loans instead of domestic consumers. This partly explains why the magnitude of the IMF emergency liquidity package has increased significantly in the past two decades, a period when most of the recipient countries were recently financially liberalised emerging market economies.

To summarise, my model suggests that, once a temporary productivity shock occurs, it is better for the central bank to move quickly and to act aggressively. However, policymakers might be cautious about this conclusion. One particular reason is that, in practice, it is very difficult to judge whether the fundamental problem is temporary (more of a liquidity problem) or permanent (more of a solvency problem). If it turns out to be the latter case, the liquidity provision policy does not work out well. In some circumstances it may even be worse, because it gives the bank an excuse not to get rid of the problem loans and not to carry out reconstruction schemes. The absence of market discipline may deteriorate the economic fundamentals and weaken market confidence, in turn increasing the bailout cost for the central bank.

7 Conclusions

This paper shows that limited access to the international capital market, although helping the domestic economy achieve a more efficient outcome, makes the domestic banking sector more fragile. A small, temporary adverse shock can have large balance sheet effects. When the central bank holds international reserves to protect the banking industry and also to maintain a fixed exchange rate regime, this financial fragility can be readily spread to the external sector and cause twin crises. In contrast, when the exchange rate is flexible, a productivity shock will lead to exchange rate fluctuation instead of a banking crisis.

My analysis also has two important implications. First, the curse of financial fragility suggests that emerging market economies should be more cautious during the process of financial liberalisation. Policymakers should not only see the benefits from integration into the international capital market, but also have a clear understanding of the potential risks such as financial vulnerability. In particular, those developing countries with a weak domestic financial sector and insufficient prudential regulation may want to strengthen the domestic financial sector before allowing access to the international capital market. Second, this paper also highlights the importance of exchange rate arrangements. It suggests that a fixed exchange rate regime in a financially liberalised emerging market might be particularly vulnerable to attacks, especially when the domestic banks are explicitly or implicitly protected by the government.

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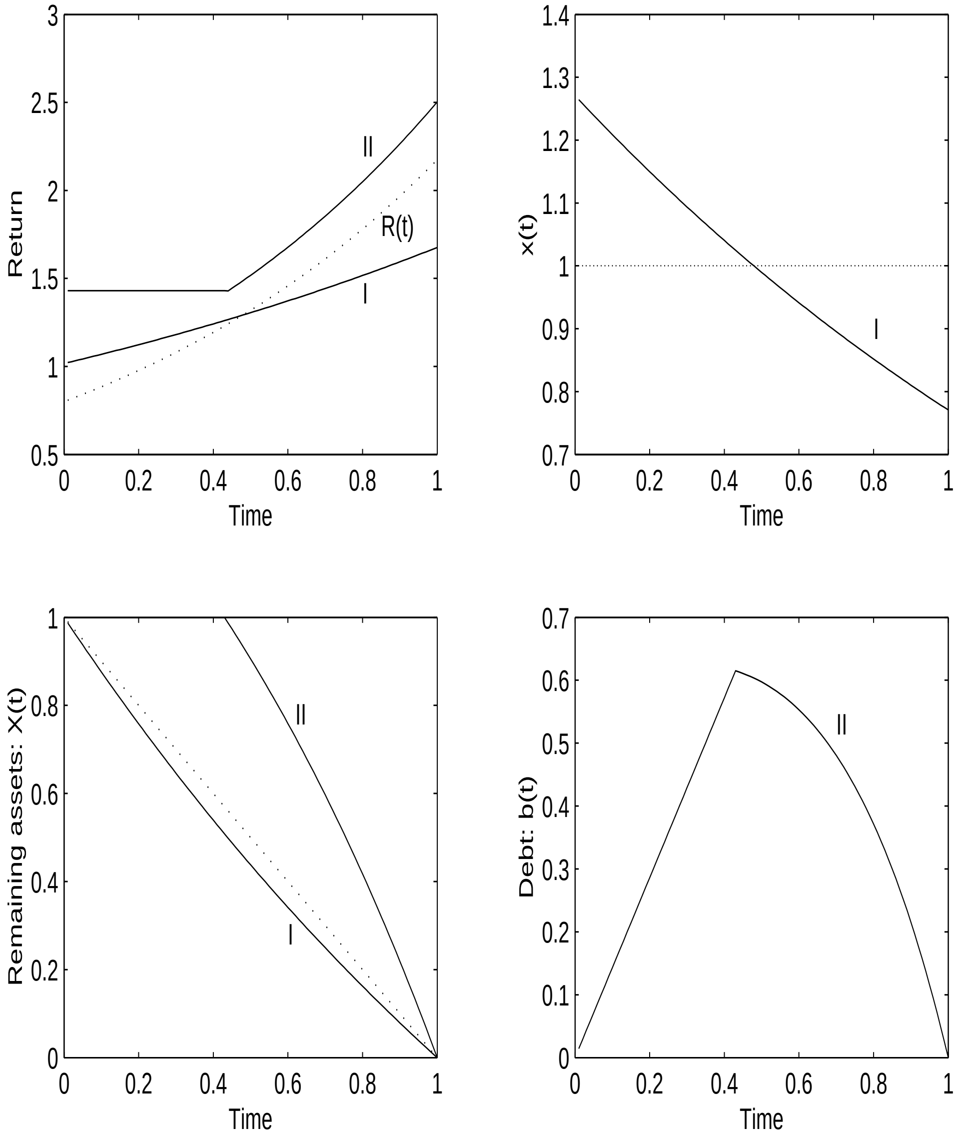
Figure 1

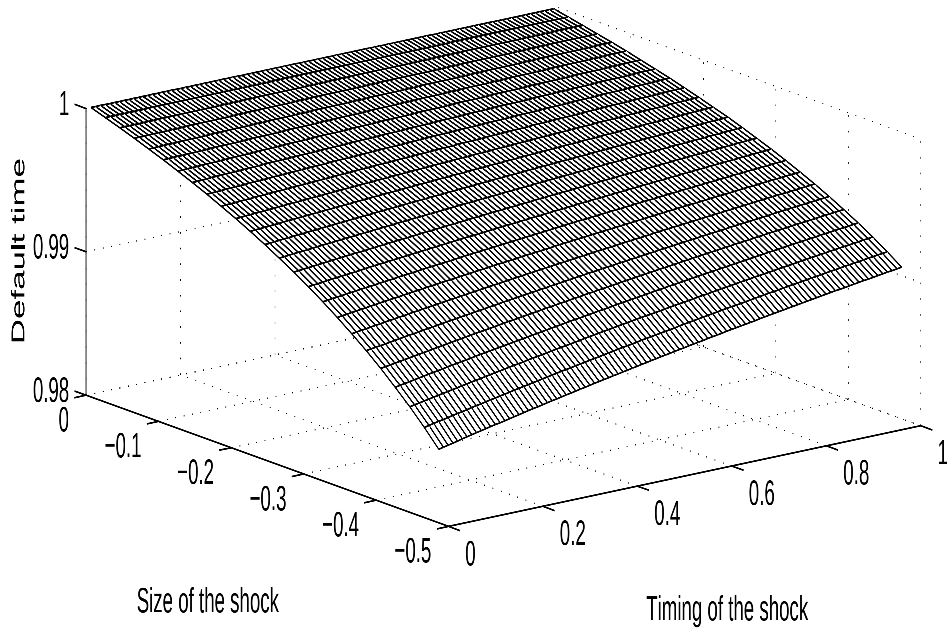
Optimal contracts in two economic environments

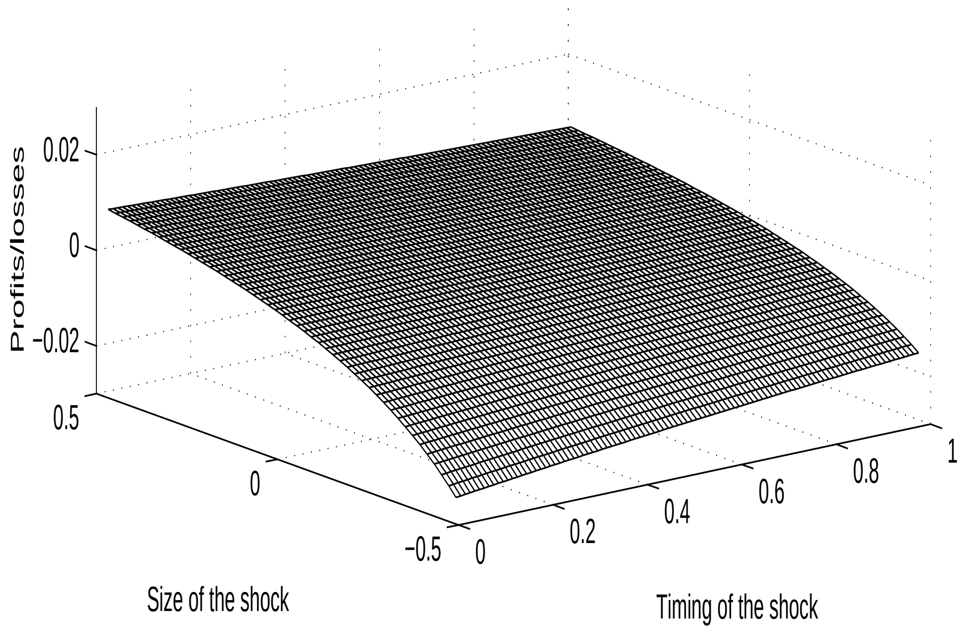


The above figure shows the optimal demand deposit contracts in two economic environments. Curve *I* characterises the optimal contract in a closed economy and *II* represents the optimal contract in an open economy with limited borrowing from the international capital market. The four panels refer to the path of interest payments specified in the contracts, the bank's asset liquidation and portfolio holdings, and foreign debt, respectively.

Figure 2

Effects of temporary shocks in a closed economy

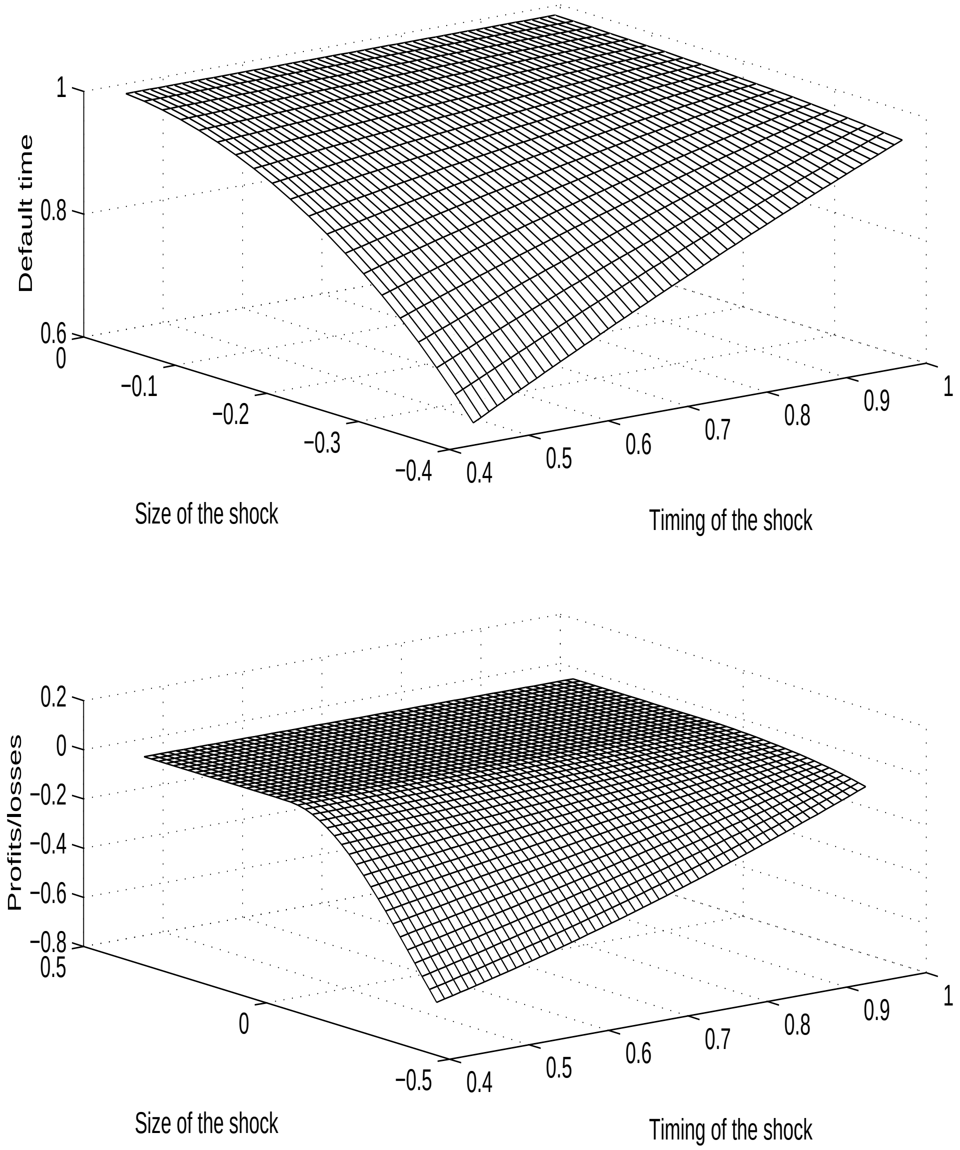




The effects vary with the timing and the size of the shocks. In general, the balance sheet effects are small in a closed economy. The upper graph shows the time when the bank goes into bankruptcy given that no bank run has occurred. The lower graph shows the balance sheet effects (profits/losses) measured at the default time.

Figure 3

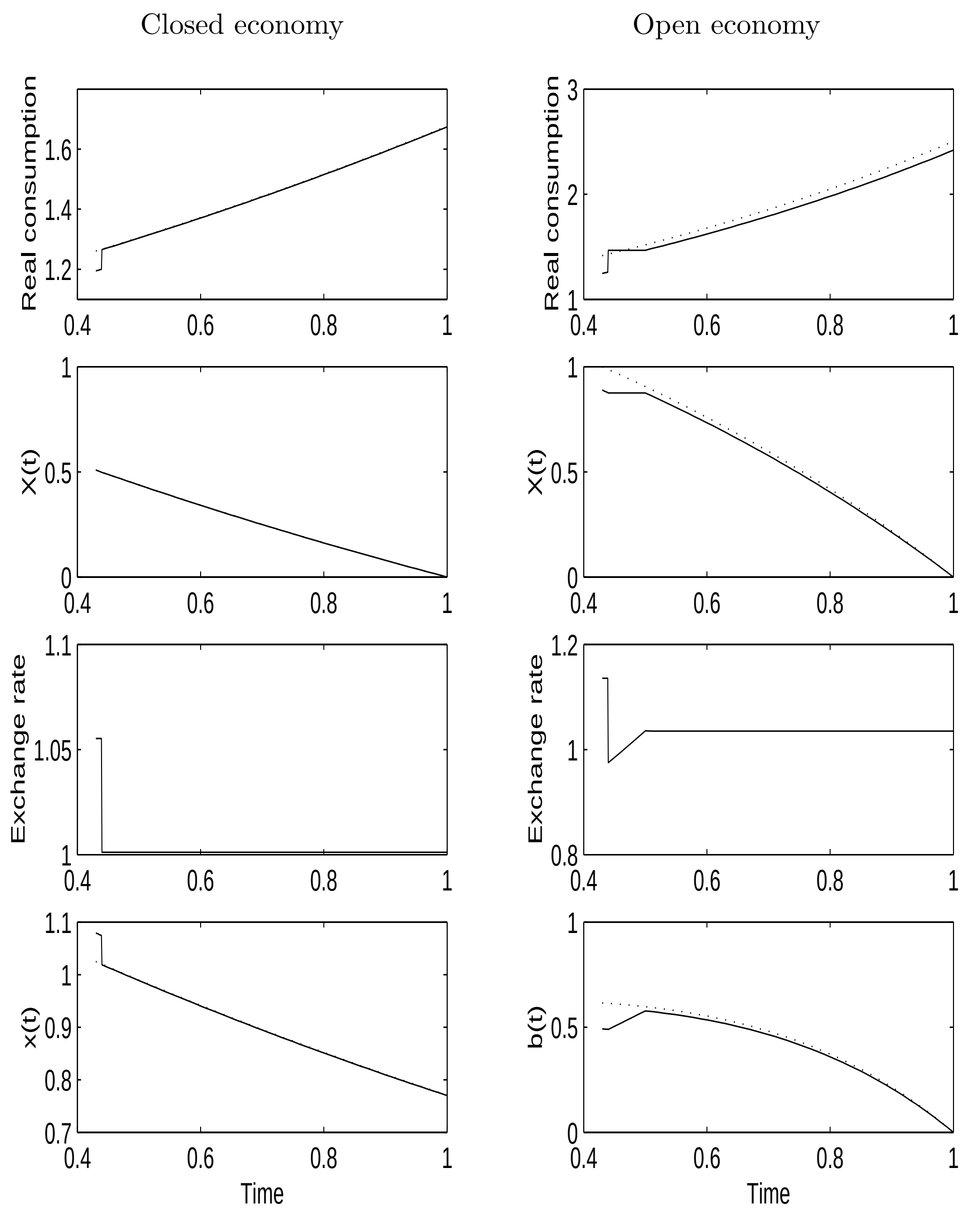
Effects of temporary shocks in an open economy with borrowing constraints



The two graphs demonstrate the balance sheet effects of temporary shocks (duration is 0.01 always) in an open economy. The balance sheet effects vary with the timing and the size of the shocks, and are much larger than in the closed economy.

Figure 4

Effects of a temporary shock under a flexible exchange rate regime



Under a flexible exchange rate regime, the exchange rate fluctuates to absorb the fundamental shocks. No bank run occurs. The same shock is used in both closed and open economy environments: productivity decreases by 10% between time 0.43 and 0.44. The left-hand four panels show the effects before capital account liberalisation and the right-hand four panels show the effects after liberalisation. The dotted lines represent the initial contractual arrangement, and the solid lines show the dynamics of contract variables after the shock occurs. The changes in the bank's portfolio are negligible in the closed economy.

Appendix

A Proof of Lemma 3

Proof: the problem is to choose t^* that maximises the objective function of $E[U_1(t_0)] + E[U_2(t_0)]$, where $E[U_1(t_0)]$ and $E[U_2(t_0)]$ are specified in equations (3.16) and (3.21), respectively. Starting from the first-order condition that $\frac{d(E[U_1(t_0)]+E[U_2(t_0)])}{d(t_0)}|_{t^*} = 0$, we are able to show that t^* is determined by

$$t^* = \frac{\beta\eta}{r(1-\beta)} \cdot [e^{(z_1-z_2)(t^*-1)} - 1] \tag{A.1}$$

Define the right-hand side as a new function $h(t_0)$. The function $h(\cdot)$ is decreasing in t_0.[24]

1) For any $\eta \in (0, 1)$, $h(0) > 0$ and $h(1) = 0$. Because the left-hand side is increasing in t_0 and $h(t_0)$ is decreasing in t_0, they must cross with each other at a unique point $t^* \in (0, 1)$.

2) When $\eta = 0$, $h(t) = 0$ always. Hence $t^* = 0$.

When $\eta \in (0, 1)$, equation (A.1) suggests that t^* is a function of η. Differentiate both sides with respect to t_0 and η, and use $z_1 = \frac{r\eta}{1-\eta}$ and $z_2 = \frac{r[1+\beta(\eta-1)]}{\beta(1-\eta)}$, we have

$$\frac{d(t^*)}{d(\eta)} = \frac{\frac{\beta}{r(1-\beta)} \cdot [e^{(z_1-z_2)(t^*-1)} - 1] - \frac{\eta(t^*-1)}{(1-\eta)^2} \cdot e^{(z_1-z_2)(t^*-1)}}{1 - h'(t^*)} > 0$$

3) Combine equation (A.1) with equations (3.15) and (3.20), it is straightforward to show that the consumption path is continuous at t^*.

B Proof of Proposition 1

Proof: when a negative shock occurs at time t_1, the bank will become insolvent at some future time t_2 (see Section 4.1). Knowing this, investors at time t_1 will change their withdrawal decisions. Because investors are identical, we only consider the symmetric equilibrium in the economy.

First, all agents will choose to withdraw their deposits before time t_2 because they will receive nothing after the bank defaults.

Now consider the case in which all investors decide to wait until a time t_3 ($t_1 < t_3 < t_2$). If this is an equilibrium, a representative agent receives the contractual rate c_t^c if his liquidity need is realised before t_3. Otherwise, he will share the remaining assets with other agents at time t_3, with each of them receiving a same payment of $\frac{(X_{t_3}^c - \triangle X)R_{t_3}}{1-t_3}$. Therefore, the expected utility of a representative agent is

$$\int_{t_1}^{t_3} u(c_t^c)f(t)dt + (1 - F(t_3)) \cdot u[\frac{(X_{t_3}^c - \triangle X)R_{t_3}}{1 - F(t_3)}]$$

[24] $h'(t_0) = \frac{\beta\eta(z_1-z_2)}{r(1-\beta)} \cdot e^{(z_1-z_2)(t^*-1)} < 0$ because $\beta > 1$ and $z_1 - z_2 > 0$.

However, this cannot be an equilibrium. A representative agent can deviate from this equilibrium and make himself better off. In particular, consider a new strategy under which an agent chooses to wait until a time $t_3 - \tau$, where τ is a very small interval. By adopting this new strategy, this agent is able to avoid the bank run outcome and his expected utility is $\int_{t_1}^{t_3-\tau} u(c_t^c) f(t)dt + (1 - F(t_3 - \tau))u(c_{t_3-\tau}^c)$. He will gain from this new strategy so long as

$$\int_{t_3-\tau}^{t_3} [u(c_t^c) - u(c_{t_3-\tau}^c)] f(t)dt + (1 - F(t_3)) \cdot [u(\frac{(X_{t_3}^c - \triangle X)R_{t_3}}{1 - F(t_3)}) - u(c_{t_3-\tau}^c)] < 0$$

Because x_t^c is decreasing over time ($\dot{x_t^c} < 0$), the liquidation payment upon a bank run at time t is much smaller than the contractual payment (c_t^c). Therefore there must be some $\tau > 0$ that satisfies the above condition. Since each investor wants to move ahead of others to take advantage of this benefit, the initial equilibrium breaks down and a bank run occurs earlier. In other words, existence of this externality effect leads to the coordination failure among investors.

Since this argument is valid for any t_3 that is before the default time (t_2) but after the shock (t_1), there is only one equilibrium left, that is, a bank run happens immediately at time t_1. Indeed, it is easy to show that this is truly a Nash Equilibrium. I will leave the proof to the readers.

C Terms of the new contract in an open economy

We rewrite problems (4.7) to (4.11) as the sum of three separable problems: $E[U(t_1, 1)] = E[U(t_1, t_1 + \triangle t)] + E[U(t_1 + \triangle t, t_2)] + E[U(t_2, 1)]$, where $E[U(\tau_1, \tau_2)]$ represents the highest expected utility of a representative agent between time τ_1 and τ_2. The algorithm is implemented in two steps. First, for any given $X_{t_1+\triangle t,new}$ and t_2, we calculate the dynamics and the expected utility in each subperiod. Second, we choose the optimal $X_{t_1+\triangle t,new}^*$ and t_2^* to maximise $E[U(t_1, 1)]$.

The expected utility in three subperiods, given the boundary conditions, can be calculated in the same way as in Section 3.2. In particular, $E[U(t_1 + \triangle t, t_2)]$ is similar to problem (3.10), and $E[U(t_1, t_1 + \triangle t)]$ and $E[U(t_2, 1)]$ are similar to problems (3.11) to (3.14) except that the boundary conditions are different. After some tedious algebra, we obtain the following results:

$$X_{t,new} = \begin{cases} \frac{e^{z_1(t-t_1)} \cdot (X_{t_1+\triangle t,new} - X_{t1,new} e^{z_2 \triangle t}) - e^{z_2(t-t_1)} \cdot (X_{t_1+\triangle t,new} - X_{t1,new} e^{z_1 \triangle t})}{e^{z_1 \triangle t} - e^{z_2 \triangle t}} & if\ t \in (t_1, t_1 + \triangle t) \\ X_{t_1+\triangle t,new} & if\ t \in (t_1 + \triangle t, t_2) \\ \frac{e^{z_1(t-1)} - e^{z_2(t-1)}}{e^{z_1(t_2-1)} - e^{z_2(t_2-1)}} \cdot X_{t_1+\triangle t,new} & if t \geq t_2 \end{cases}$$

$$c_{t,new} = \begin{cases} [(\eta - 1)\dot{X}_{t,new} + \eta X_{t,new} r] R_t (1 + \epsilon) & if\ t \in (t_1, t_1 + \triangle t) \\ \frac{\eta[R_{t_2} - R_{t_1+\triangle t}(1+\epsilon)]}{t_2 - t_1 - \triangle t} \cdot X_{t_1+\triangle t,new} & if\ t \in (t_1 + \triangle t, t_2) \\ \frac{r(1-\beta)}{\beta} \cdot \frac{R_t e^{z_2(t-1)}}{e^{z_1(t_2-1)} - e^{z_2(t_2-1)}} \cdot X_{t_1+\triangle t,new} & if\ t \geq t_2 \end{cases}$$

where $z_1 = \frac{r\eta}{1-\eta}$ and $z_2 = \frac{r[1+\beta(\eta-1)]}{\beta(1-\eta)}$ (the same as in Section 3.2).

The above results have an appealing property. The consumption level in the second and third subperiods is proportional to $X_{t_1+\triangle t,new}$. As a result, $E[U(t_1+\triangle t, t_2)]+E[U(t_2,1)]$ can be written in the form of $X^{1-\beta}_{t_1+\triangle t,new}G(t_2)$, where $G(t_2)$ is a function of t_2 only. That is, the choice of optimal t_2 is independent of $X_{t_1+\triangle t,new}$. Therefore, the maximisation problem can be solved as follows. (1) First, find t_2^* that maximises the function $G(t_2)$. (2) Second, find $X^*_{t_1+\triangle t,new}$ that maximises $E[U(t_1, t_1+\triangle t)] + X^{\frac{1}{\beta}-1}_{t_1+\triangle t,new}G(t_2^*)$. Once these two parameters are chosen, the terms of the new contract can be spelled out using the above results.

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