Ingo Fender
+41 61 280 8415
ingo.fender@bis.org

Nikola Tarashev
+41 61 280 9213
nikola.tarashev@bis.org

Haibin Zhu
+41 61 280 9164
haibin.zhu@bis.org

Credit fundamentals, ratings and value-at-risk: CDOs versus corporate exposures[1]

This article compares the linkages between credit fundamentals, ratings and value-at-risk measures for CDO tranches with those for corporate bond exposures. A sensitivity analysis incorporating market information and rating migrations data reveals that the behaviour of CDO tranche ratings can differ markedly from that of corporate ratings. In addition, tranching is found to have an important impact on the probability of large losses. This highlights how investors who narrowly focus on ratings and draw direct parallels with corporate exposures can seriously misjudge the value-at-risk of CDOs.

JEL classification: G24, G32.

Owing to weakening house prices and declining underwriting standards in 2006 and 2007, mortgage markets in the United States have seen a significant deterioration. Large numbers of rating downgrades on securitised mortgage products, in turn, have revived questions about the nature of structured finance ratings, their sensitivity to changes in credit fundamentals, the degree to which rating transitions for products such as collateralised debt obligations (CDOs) should be expected to differ from those for corporate bonds, and the extent to which ratings can serve as universal measures of credit risk.[2]

In an attempt to address these questions, this article analyses the risk profile of CDOs, mainly through comparison with that of corporate exposures. The analysis is based on a hypothetical CDO that reflects key features of the market for structured products backed by mortgage collateral. A number of stylised but realistic scenarios, motivated by market reports and observed rating migrations, are applied to a set of baseline ratings for different CDO tranches. The results shed some new light on the recent downgrade activity experienced by these products and the extent to which these downgrades could have been anticipated by market participants. In addition, the results extend the existing literature by lending new quantitative support to earlier

[1] The views expressed in this article are those of the authors and should not be taken to reflect the views of the BIS; any errors and omissions remain those of the authors. The authors would like to thank Marjorie Santos for her help with graphs and tables.

[2] See Kiff and Mills (2007) for details on the US mortgage market, and Fender and Mitchell (2005) for an overview of the key issues related to structured finance ratings.

findings on the characteristics of tranche ratings (eg CGFS (2005)) and by adding comparisons across like-rated exposures in different asset classes to existing analyses of CDO risk (eg Gibson (2004)).

This article is organised as follows. The first section briefly introduces CDOs and how they are rated, using so-called structured finance CDOs as an example. This is followed by a second section focusing on the impact of credit fundamentals on CDO ratings. A key finding of this exercise, namely that expected losses and, hence, ratings of CDO tranches can be substantially more sensitive to changes in credit fundamentals than ratings of like-rated corporate bonds, is taken further in the third section. That section argues that dimensions of credit risk not captured by ratings can drive substantial differences between credit value-at-risk (VaR) measures of like-rated instruments. These differences surface both in VaR levels and in their sensitivity to changes in credit fundamentals. The last section concludes.

Overview: CDOs and how they are rated

Market structure and recent developments

CDOs are structured finance products in which a distinct legal entity, a so-called special purpose vehicle, issues claims against an underlying pool of assets (CGFS (2005)). These claims, in turn, are prioritised by creating classes of securities with different levels of seniority, including senior and mezzanine tranches and an equity (first loss) piece. Senior tranches are insulated from default risk up to the point where credit losses deplete the more junior ones.

CDOs are structured finance products ...

While CDO collateral pools can consist of various forms of debt (such as loans, bonds or synthetic exposures), recent vintages have increasingly been based on other structured products (such as tranches of mortgage-backed securities or of other CDOs). Issuance data for these so-called *structured finance* CDOs suggest that they accounted for some 49% of the $560 billion worth of CDOs issued during 2006. This was up from 45% in 2005 and 40% in 2004. In 2007, despite the turmoil in credit markets during the second half of the year, the share remained at around 46%, with some $182 billion issued up to year-end (Graph 1, left-hand panel).

Whereas early structured finance CDOs had relatively diversified pools, more recent vintages have increasingly been based on mortgage collateral. As a result, by 2006, mezzanine structured finance CDOs (ie those backed primarily by BBB-rated mezzanine tranches of other securitisations) had almost 90% of their assets invested in home equity loan and residential mortgage-backed securities (RMBSs; Graph 1, right-hand panel). According to Moody's (2007b), 45% of these pools were on average devoted to subprime exposures, with variation around that level ranging from close to zero to as high as 88%.

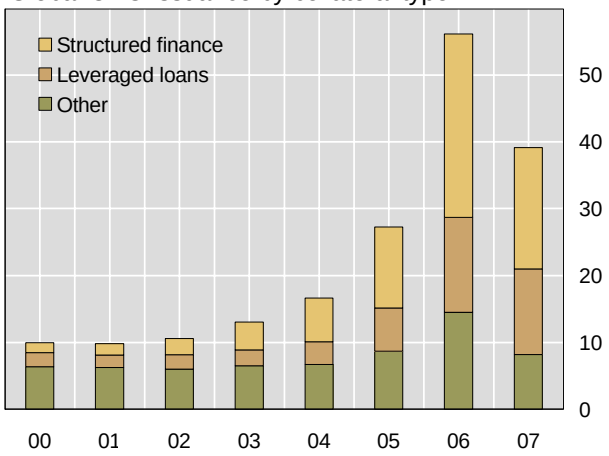
... increasingly based on tranches of mortgage-backed securities

In 2007, given the high exposures of these pools to US mortgage collateral, deterioration in credit quality became increasingly evident at the end of the securitisation chain. As rising mortgage delinquencies triggered large numbers of negative rating actions on RMBSs referencing subprime collateral, these downgrades subsequently fed into CDOs as well. Specifically, between

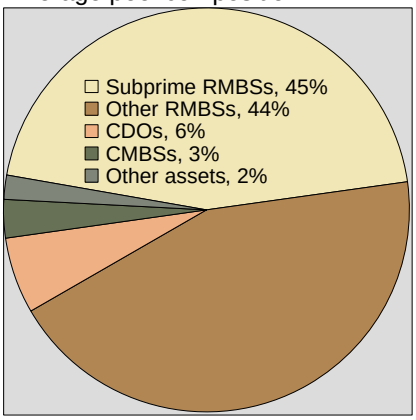
CDOs suffered large-scale rating downgrades in 2007 ...

CDO market structure

Global CDO issuance by collateral type[1]



Average pool composition[2]



[1] In billions of US dollars; combines funded deals as well as the funded portions of synthetic deals. [2] Mezzanine structured finance CDOs issued in 2006.

Sources: Citigroup; Lehman Brothers; Moody's; authors' calculations. Graph 1

January and December 2007, Moody's alone downgraded almost 1,400 CDO tranches from 462 transactions originally valued at about $76 billion. These included numerous cases of rapid rating transitions by six notches or more and affected tranches with original ratings as high as Aaa.[3]

There are several channels through which this happened.[4] One of these, which is the focus of the remainder of this article, works through the effect of credit quality deterioration on the protection provided through the tranching process. That is, as credit quality deterioration leads to collateral downgrades, it becomes increasingly likely that, given an otherwise unchanged CDO structure, at least some of the tranches are also downgraded.

CDO rating methodology

... reflecting higher expected loss estimates

Ratings of CDOs, just as those of more traditional debt instruments, are indicators of default risk based on expected loss (EL) or probabilities of default (PDs).[5] In assigning these ratings, the rating agencies rely on an iterative, two-stage process that combines estimated loss distributions (the result of *credit risk modelling*) with expert judgment based on deal-specific contractual information (the result of *cash flow analysis*). This process delivers estimates of tranche EL (or PD), which are translated into alphanumeric ratings via

[3] See Moody's (2008). The observed average downgrade of about 7.5 notches for the 2006 and 2007 vintages compares to an average downgrade of about 3.8 notches for earlier (1997 to 2006) vintages of US CDO tranches (Moody's (2007a)).

[4] Collateral downgrades below investment grade, in particular, can trigger structural provisions within CDOs that, in turn, may cause quasi-automatic downgrades of the affected CDO tranches. One example are "event-of-default" tests linked to the ratings composition of the CDO collateral pool. Of the 700 or so CDOs that saw collateral downgrades in 2007, at least 50 are reported to have experienced such events.

[5] Moody's ratings are based on EL whereas those of Standard & Poor's and Fitch Ratings are based on PDs. See Fender and Kiff (2005) for more detail on CDO rating methodology.

historical "mappings" benchmarked to the performance of *corporate bonds*. By implication, like-rated instruments are deemed by the rating agencies to have broadly similar ELs (or PDs).[6]

At the credit risk modelling stage, the major rating agencies rely heavily on Monte Carlo simulations, especially when assigning ratings to CDOs backed by synthetic or structured finance instruments (eg Fitch Ratings (2006), S&P (2005)). In operationalising these simulations, it is standard practice to approximate the complex structure of losses on a CDO pool by assuming that these losses are the *direct* consequence of *hypothetical* asset values falling below a prespecified threshold. Such an assumption allows recourse to so-called *structural* credit risk models (designed for corporate bonds), which require estimates of instrument-specific EL and pairwise asset return correlations as inputs (eg Moody's (2005)). Rating agencies obtain such estimates from observed ratings dynamics and feed them into the models in order to simulate the risk properties of the entire CDO collateral pool through repeated draws of random credit losses.

This delivers an estimate of the probability distribution of pool losses, the exposure to which is then parcelled out across tranches, taking the results of agencies' cash flow analysis into account. Specifically, the *attachment point* (ie the minimum loss on the underlying pool that affects the tranche) and the *detachment point* (ie the minimum pool loss that wipes out the entire tranche) are chosen so that the resulting EL of the tranche matches the level required for a desired rating. On this basis, a typical CDO comprises tranches with different levels of seniority, rated as high as Aaa/AAA at the senior end. Taken together, these tranches will amount to some 95% of the pool, with the remainder issued or retained by the CDO originator as (typically unrated) equity (Fender and Mitchell (2005)).

The modelling approach of the rating agencies has at least two important limitations if CDOs are not backed by corporate bonds (or loans) – that is, if the pool underlying a CDO comprises tranches of mortgage-backed securities. First, this approach will introduce approximation errors as long as default-related losses on *individual* structured finance exposures in the CDO pool are not captured appropriately by structural models designed to account for corporate defaults.[7] Second, by approximating the default behaviour of the *overall* collateral pool via pairwise correlations of hypothetical asset returns, rating agencies may not fully account for the default clustering within and across the different instruments in this pool. That said, the impact of such approximation errors on credit ratings is difficult to assess – not least because the rating agencies themselves attempt to correct for these errors by making

[6] The remainder of this article will focus only on the first part of the rating process (pool credit risk modelling), and will assume that ratings are assigned on the basis of EL (not PD).

[7] Mortgage-backed securities do not default in the sense of a singular corporate default event. Instead, reflecting delinquencies and prepayments on the underlying collateral, such securities will experience cash flow shortfalls and writedowns over the lifetime of the underlying collateral.

adjustments at the cash flow analysis stage or via the specific calibration of their credit risk models.[8]

The impact of credit fundamentals on CDO ratings

When exposed to changes in credit fundamentals ...

This section employs sensitivity analysis to gauge the impact of changes in credit fundamentals on tranche ratings, based on a hypothetical CDO structure. The exercise is implemented by "shocking" two key credit risk fundamentals, PDs and asset return correlations, using various scenarios inspired by recent market developments. The analysis then proceeds to illustrate how rating migrations of CDO tranches can differ from those of corporate exposures.

Setting up a hypothetical CDO pool

For tractability, the following analysis abstracts from the heterogeneity and complexity of actual CDO deals and focuses on a hypothetical pool that incorporates realistic, albeit stylised, credit risk features. The hypothetical CDO pool is composed of 100 equally sized individual assets with the same "Baa3" rating on the Moody's scale (ie "BBB–" on the Fitch/S&P scale). Furthermore, each underlying asset is assumed to have the same degree of exposure to a single common factor or, equivalently, all pairwise asset return correlations are fixed at 15%.[9] Finally, loss-given-default (LGD) is assumed to be independent of default events and to follow a symmetric triangular distribution in the range of [0.1, 1]. The resulting mean value of 55% corresponds to the assumption employed by Moody's in deriving PDs from their "idealised" expected loss data. These same data are used here to infer PD estimates from the assumed ratings of a CDO's underlying assets, and to map EL estimates into indicative ratings for various CDO tranches.

Under these assumptions, baseline tranche ratings for the hypothetical CDO pool can be derived through Monte Carlo simulations of its loss distribution, calculating the expected loss for each tranche and assigning ratings accordingly. The results of such an exercise are reported in Table 1. Two sets of tranche specifications are included, one corresponding to a typical tranche structure (as used, for example, in the CDS index market) and the other comprising two alternative tranches that are tailored to have the same EL corresponding to a Baa3 rating.

The chosen approach to assigning CDO ratings warrants some remarks. First, it follows market practice by essentially treating the assets in the CDO's

[8] Another way to mitigate approximation errors is the use of so-called "look-through" approaches that attempt to capture overlapping credit risks among underlying tranches in CDOs backed mainly by tranches of other CDOs.

[9] The heterogeneous asset pools contained in actual CDOs would typically necessitate a more complex correlation structure in which default clustering depends on asset sector and asset type composition. The correlation assumption adopted here was chosen for simplicity, but is in line with estimates reported in related studies. For instance, Lopez (2004) documents an average asset return correlation of 12.5% for a large number of US firms. A similar average asset return correlation arises for typical structured finance CDOs, as depicted in Graph 1, under standard correlation assumptions (eg Moody's (2005)).

Tranche ratings: hypothetical CDO pool

Baseline scenario

Attachment (%)	Detachment (%)	Tranche EL (%)	Rating
0.0	3.0	7.5748	B3
3.0	7.0	0.0916	Baa2
7.0	10.0	0.0028	A1
10.0	15.0	0.0002	Aa1
15.0	30.0	0.0000	Aaa
30.0	100.0	0.0000	Aaa
Tailor-made tranches			
2.6	5.0	0.231	Baa3
0.9	25.0	0.231	Baa3

In the hypothetical CDO pool, there are 100 homogeneous assets with the same Baa3 rating and the same pairwise asset return correlation of 15%. LGD is assumed to follow a symmetric triangular distribution between 10 and 100%. The ratings are assigned on the basis of Moody's idealised EL data. Tranche attachment and detachment points are defined as percentages of pool value; tranche EL is as a percentage of tranche notional.

Source: Authors' calculations. Table 1

underlying pool as corporate bonds. This "shortcut" approach, as mentioned above, simplifies the analysis of pool credit risk at the cost of introducing a source of approximation error.[10] Second, the analysis focuses on credit losses that are realised over a single one-year period and abstracts from cash flow analysis. As a result, factors such as default timing assumptions, amortisation/prepayment effects, cash flow redistributions resulting from structural features, and servicer or asset manager quality are ignored in deriving the results reported below.

Sensitivity analysis

In what follows, asset-level PD and correlation assumptions are "shocked" to gauge the sensitivity of tranche ratings to deteriorations in credit conditions, with various scenarios inspired by actions taken by rating agencies in the unfolding subprime crisis. Shocks are assumed to affect a maximum of 45% of the pool's assets, the average share of subprime RMBSs in a typical structured finance CDO (Graph 1).[11] The first set of scenarios introduces PD stresses in which the affected pool assets are downgraded by either one or six notches (on Moody's rating scale).[12] In the second set of scenarios, it is assumed that 45%

[10] This special feature does not address these issues directly. Nevertheless, the results of the sensitivity analysis suggest that miscalibration of the credit fundamentals of underlying assets could have significant implications for the ratings of CDO tranches.

[11] Obviously, if the share of pool assets that are subject to credit deterioration increases, the impact on ratings of CDO tranches is greater.

[12] In response to the onset of the subprime crisis, all three major rating agencies decided to make adjustments to their rating methodologies for structured finance CDOs, mainly by stressing PD inputs in the credit risk assessment. In particular, Fitch Ratings increased all rating-implied PDs for subprime RMBSs issued since 2005 by 125%, while Moody's downgraded subprime RMBSs by between zero and six notches depending on vintage year and rating. Standard & Poor's, in turn, downgraded the ratings of subprime RMBSs issued between the first quarter of 2005 and mid-July 2007 by between zero and two notches.

of the CDO pool's assets are subject to increased exposure to the systemic risk factor, which raises the corresponding pairwise asset return correlations from 15% to 45% and 65%.[13] A third and final set of scenarios allows for joint shocks to both PDs and asset return correlations (Table 2).[14] The main findings are as follows.

... CDO tranche ratings can change by much more than ...

First, the impact of PD shocks on CDO tranche ratings depends on the magnitude and clustering of the shocks and tends to be non-linear. For instance, one-notch downgrades of 45% of the pool's underlying assets have only a small impact on tranche ratings (downgrades at most by one notch; see scenario 1). By contrast, six-notch downgrades on the same group of assets can cause mezzanine tranches to be downgraded by as much as 10 notches (scenario 3). Interestingly, multi-notch downgrades for a small set of pool assets have greater effects than single-notch downgrades for a large set of assets, even when the total number of notch downgrades is similar (scenario 2 vs scenario 1; Table 2). This finding, dubbed the *dispersion effect*, results from the non-linear relationship between rating grades and rating-implied PDs, which leads to greater changes in PD per notch for multi-notch relative to single-notch downgrades. As a result, a higher dispersion in ratings of the underlying assets implies a higher average PD and increases the risk across CDO tranches.

... corporate ratings

Second, correlation stresses can trigger significant downgrades for mezzanine and senior tranches, *even in the absence of downgrades in the underlying pool*. For instance, an increase in within-group correlation from 15% to 65% changes the rating of tranche [15, 30] from Aaa to A3, the same effect as if 45% of the underlying assets were downgraded by six notches (scenario 5 vs scenario 3; Table 2). The reason for this effect is that higher correlations do not affect expected loss but push probability mass into the tails of the loss distribution. Therefore, the equity tranche tends to benefit (because the probability of zero default increases) at the expense of senior tranches.

Third, the impact of credit fundamentals on CDO ratings depends on tranche specifications, including seniority and thickness (ie the difference between detachment and attachment points). The equity tranche is adversely affected by increases in PD, but benefits from increases in asset return correlations, as noted above. By contrast, mezzanine and senior tranches are vulnerable to increases in both PDs and correlations. The impact on ratings is usually most remarkable for mezzanine tranches, for which the loss distribution is most sensitive to changes in credit fundamentals. In addition, a comparison between the two like-rated, tailor-made, mezzanine tranches reveals that the thinner one depends more on credit fundamentals. This reflects the increased importance of the credit quality of any one collateral asset for tranches that can be wiped out by a small rise in pool losses.

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[13] These assumptions appear to be deemed conservative by the rating agencies. See, for example, Moody's (2005).

[14] Empirical studies suggest that default correlation increases when the credit quality of underlying assets deteriorates.

Tranche downgrades: sensitivity analysis

In notches relative to baseline tranche ratings

	Original rating	PD shocks (# assets x # notches)			Correlation shocks (# assets x correlation)		Joint shocks (# assets x # notches x correlation)	
		1 (45x1)	2 (7x6)	3 (45x6)	4 (45x45)	5 (45x65)	6 (7x6x45)	7 (45x6x45)
[0.0, 3.0]	B3	–1	–3	–4	0	0	–2	–3
[3.0, 7.0]	Baa2	–1	**–2**	**–9**	**–2**	**–3**	**–4**	**–9**
[7.0, 10.0]	A1	–1	–2	**–10**	–4	–5	–3	**–13**
[10.0, 15.0]	Aa1	–1	–1	**–9**	–5	–7	–2	**–14**
[15.0, 30.0]	Aaa	0	0	–6	–3	–6	0	**–12**
[30.0, 100.0]	Aaa	0	0	0	0	0	0	0
Tailor-made tranches								
[2.6, 5.0]	Baa3	**–1**	**–3**	**–9**	**–2**	**–2**	**–4**	**–9**
[0.9, 25.0]	Baa3	**–1**	**–2**	**–7**	**–1**	**–1**	**–3**	**–7**

"# assets" refers to the number of underlying assets that are exposed to PD or correlation shocks; "# notches" refers to the degree of PD shocks, ie the number of notch downgrades with which each of these assets will be faced; "correlation" refers to the new pairwise correlation within the group of underlying assets that experience shocks. The bold results represent tranche downgrades from investment grade to speculative grade.

Source: Authors' calculations. Table 2

Lastly, the sensitivity analysis sheds some light on the severity of credit shocks necessary to push Aaa-rated senior tranches into sub-investment grade territory. While the most senior Aaa tranche appears to be quite safe, downgrades can be quite pronounced for more junior tranches with the same rating. For instance, it takes six notch downgrades on 45% of pool assets and a within-group correlation of 45% for the rating of the [15, 30] Aaa tranche to be lowered to Ba2. This partly explains the large magnitude of downgrades of CDO tranches in 2007 (see footnote 3), when more than 125 CDOs experienced collateral downgrades in excess of 45% of the underlying pool.

Simulating rating migrations

This subsection introduces an additional perspective to the preceding analysis of CDO tranche ratings by comparing the migration rates of these tranches with those of corporate bonds with the same original rating. Given that mezzanine tranches have received much attention recently, the baseline results reported below relate to the two tailor-made CDO tranches that share the same Baa3 rating (Table 1).

Rating migrations of CDO tranches differ substantially from …

The simulation of migration rates of CDO ratings is implemented as follows. Consistent with the above analysis, the hypothetical CDO pool is assumed to consist of 100 identical underlying assets. The credit quality of these assets is assumed to change over time, subject to the typical rating transition probabilities of Baa3-rated corporate issuers observed during 1983–2003 (as reported by Moody's (2004)) and the asset return correlation specified above. The realisation of credit quality of the underlying assets is simulated 1,000 times and, in each simulation, the credit rating of CDO tranches is

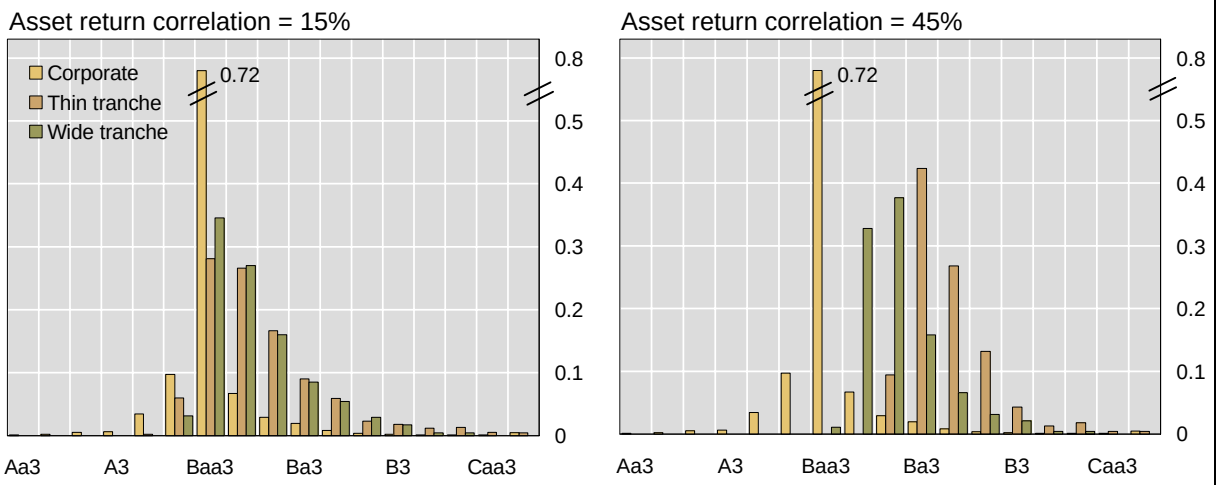
reassessed accordingly. The results are then aggregated to obtain simulated migration rates, and compared with observed migration rates of Baa3-rated corporate issuers (Graph 2).

... rating migrations of corporate bonds

The one-year rating transitions for CDO tranches can be strikingly different from those of corporate bonds, in terms of the likelihood, direction and size of rating changes. Specifically, the two mezzanine tranches examined here are more likely to face rating revisions than the like-rated corporate exposures. In addition, when revisions occur for CDO tranches, downgrades are more likely than upgrades and the probability of large-scale downgrades is not negligible. By contrast, rating revisions for Baa3 corporate bonds tend to be symmetric and of a limited scale.[15] Moreover, the downgrade risk of CDO tranches is more pronounced when the asset return correlation is higher, suggesting that tranches are particularly vulnerable when the credit deterioration of underlying assets is mainly driven by increased exposure to systematic risk (eg during a cyclical downturn). Lastly, the migration rates depend on tranche specification. Comparing the two like-rated tranches, the thinner one is more sensitive to changes in credit conditions, which is consistent with the sensitivity analysis conducted above.

Migration rates: hypothetical CDO tranches

CDO vs corporate exposures



The two panels plot the rating transition probabilities of corporate bonds and two tailor-made CDO tranches (defined in Table 1) with the original rating of Baa3. The hypothetical CDO consists of 100 identical underlying assets with the same Baa3 rating. At each time, the realisation of credit quality of the 100 assets is simulated on the basis of the transition probabilities of Baa3 corporate issuers and a prespecified asset return correlation. The realised credit quality is fed into the CDO pool to infer new ratings for the CDO tranches. The simulation is repeated 1,000 times to plot the rating migration of the two CDO tranches.

Sources: Moody's; authors' calculations. Graph 2

[15] These results are in partial accordance with studies of historical rating transitions (eg Moody's (2007a)). On the one hand, such studies reveal that rating changes are more seldom for CDO products than for corporate bonds. On the other, when historical CDO tranche ratings do change, the changes tend to be roughly twice as large as those of corporate bond ratings. Two factors may explain why this article derives a relatively higher probability of changes of tranche ratings. First, the analysis here assumes that ratings depend solely on estimated expected losses and ignores other potentially important factors. It also ignores any lags introduced by the rating surveillance process. Second, given that available rating histories are too short to embody a full credit cycle, observed migration rates might be biased away from long-run averages.

The high likelihood of CDO tranche downgrades, particularly significant downgrades, is attributable to two factors. First, the dispersion effect suggests that a higher dispersion in ratings translates into a deterioration of the average credit quality of the underlying pool. That is, a one-notch downgrade increases the implied average PD by more than a one-notch upgrade would decrease it. Second, as seen in the above sensitivity analysis, the structuring process redistributes losses across tranches. As a result, credit quality deterioration tends to have an amplified effect on particularly vulnerable CDO tranches, eg the thin mezzanine tranche.

From ratings to credit VaR

The preceding analysis shows that the sensitivity of ratings to changes in credit fundamentals (and, thus, to the business cycle) can be substantially stronger in the case of structured finance instruments than in the case of corporate bonds. The reason is that, being determined solely by estimates of average losses, ratings are only loosely related to other measures of credit risk. Credit VaR is one such measure, which, representing a high level of credit losses that can be exceeded only with a small probability, is of particular importance to market participants and supervisors.

It is thus useful to analyse how the tranching of structured finance instruments affects the relationship between ratings and VaR. In conducting such an analysis, this section focuses on the marginal contributions of two types of assets to the VaR of a hypothetical portfolio (which is kept in the background). The first asset is a corporate bond. The second asset is a CDO tranche whose collateral is based on a homogeneous pool of corporate bonds with the same PD and dependence on the common factor as the first asset.[16]

The rest of this section considers the marginal VaRs (henceforth MVaRs) of the two asset types from three different angles.[17] The first subsection compares MVaR *levels* across asset types, keeping the corresponding rating constant. The second subsection considers the *sensitivity* of tranche and corporate bond MVaRs to changes in PDs and default correlations. Lastly, the third subsection analyses how the difference between these MVaRs depends on the degree of *diversification* in the pool underlying the CDO.

Tranching and the level of MVaR

Tranching may increase VaR levels ...

Even when a CDO tranche and a corporate bond share the same rating, MVaR measures point to differences in the credit risk of each security. Referring to the example introduced in the previous section, a Baa3-rated corporate bond has an MVaR of 3.26%, which is more than *10 times* smaller than the 35% MVaR of a CDO tranche with the same rating (Table 3). This is a consequence

[16] The higher is the dependence on the common factor, the higher are asset-return correlations and the more correlated are default events.

[17] See the box on page 98 for theoretical underpinnings of the MVaR of a CDO tranche.

Credit VaR: sensitivity analysis

In per cent

	Original expected loss	Original credit VaR	PD shocks			Correlation shocks		Joint shocks	
			Scenario						
			1	2	3	4	5	6	7
Corporate	0.23	3.26	4.30	4.96	14.20	7.30	11.80	6.43	23.60
Tranches									
[2.6, 5.0]	0.23	35.36	59.64	74.86	100.00	96.51	100.00	95.47	100.00
[0.9, 25.0]	0.23	9.90	14.05	16.91	55.19	26.77	45.20	22.99	93.11

Original expected loss and VaR as well as shock scenarios 1 to 7 are based on the same credit risk parameters as those underlying Table 2. In addition, VaR numbers refer to perfectly granular portfolios comprising homogeneous corporate exposures or CDO tranches. The underlying pool of a CDO comprises 100 corporate exposures. Numbers in square brackets refer to the attachment and detachment points of the respective tranche. Table 3

of the tranching process, which concentrates the underlying credit risk in the more junior tranches.

That said, the difference between corporate and tranche MVaRs would be smaller if the tranche's detachment and attachment points were further apart (ie if the tranche were "thicker"). Table 3 illustrates this by considering two "nested" and like-rated CDO tranches. Since most of the extra collateral underpinning the thicker tranche is affected only after the entire collateral of the thinner tranche is wiped out,[18] the probability of a large loss on the thicker tranche is lower. In terms of this specific example, the "thicker" tranche features an MVaR that is less than one third of the MVaR of the "thinner" one, but is still much higher than that of the like-rated corporate bond.

Tranching, risk fundamentals and MVaR

... and the sensitivity of VaR to credit fundamentals ...

The different nature of the credit risk underlying corporate bonds and CDO tranches also affects the sensitivity of MVaR to changes in risk fundamentals (PD and default correlations). Quite naturally, deteriorating fundamentals would raise the MVaR of each member in a pool of corporate bonds and, thus, the *overall* risk of the CDO based on this pool. However, the sensitivity of MVaR to fundamentals changes substantially with tranche seniority, reflecting the high degree of non-linearity introduced by the structuring process.

Tranche seniority that implies a moderate value of the MVaR would also imply high sensitivity of this value to changes in fundamentals (Table 3, third to last columns). If 7% of the corporate bonds underlying a CDO are downgraded by six notches as a result of a positive PD shock, the average MVaR of these securities increases by half to almost 5% (scenario 2). In parallel, the MVaR of a mezzanine tranche of this CDO more than doubles, from the initially moderate 35% to 75%. The downside risk of this tranche has, however, little

[18] This is because the difference between the attachment points of the two nested tranches is smaller than the difference between their detachment points. Had the two differences been the same, non-linearity of the loss distribution would have led to a higher PD (and, thus, a lower rating) for the thicker tranche. However, by introducing more senior collateral into the thicker tranche, the larger difference between the two detachment points lowers this tranche's PD to that of the thinner tranche.

room to increase further if fundamentals were to continue to deteriorate. Thus, a transition from scenario 2 to scenario 3, in which 45% of the underlying pool is downgraded by six notches, increases the MVaR of the tranche by (only) one third, to 100%. In this scenario, the average MVaR of the underlying corporate securities nearly triples to 14%.

Diversification and the MVaR of a CDO tranche

... in ways that depend on pool granularity

Smaller diversification of the CDO's underlying pool, also known as coarser granularity, increases the MVaR of low-risk tranches, but decreases the MVaR of high-risk tranches (see box). Coarser granularity increases both the

Derivation and features of the marginal VaR of a CDO tranche[①]

This box analyses the marginal contribution of a CDO tranche to portfolio VaR. For the calculation of its marginal VaR (henceforth MVaR), the tranche is treated as one of many credit-risky assets in an investment portfolio. It is assumed that the risk of this portfolio is governed by a single common factor and that the impact of idiosyncratic risk factors is diversified away owing to the large number of constituent assets (ie the portfolio is "perfectly granular"). Given these assumptions, the credit VaR of the portfolio equals the sum of the MVaRs of the individual assets included in the portfolio.

Furthermore, such an MVaR depends only on features specific to the particular asset, which allows the rest of the portfolio to be kept in the background. Concretely, the MVaR equals the expected loss on the asset over some horizon, conditional on a sufficiently adverse realisation of the common risk factor. This MVaR increases as credit fundamentals deteriorate, eg as the asset's PD or dependence on the common factor increases.

In order to build intuition about the MVaR of a CDO tranche, it is useful to consider a special case, in which the pool underlying the CDO is comprised of a very large number of homogeneous corporate bonds that are affected by a single common risk factor. When this factor is at the value used for calculating MVaR, the loss (per unit of exposure) on the perfectly granular pool would (by construction) be exactly equal to the MVaR of a constituent corporate bond. This loss wipes out the entire collateral of any CDO tranche with a detachment point lower than the corporate bond MVaR. Hence, the MVaR of such a tranche is 100% of the tranche's principal. However, since the same loss does not affect the collateral of any tranche with an attachment point higher than the corporate bond MVaR, the MVaR of such a tranche is 0%. Finally, a tranche with attachment and detachment points that straddle the corporate bond MVaR has an MVaR that falls between these two extremes.

This analysis is visualised by the red line in Graph 3 (left-hand panel). This line shows that conditional losses of a tranche increase one for one with its detachment point as long as this point is lower than the corporate bond MVaR. In addition, conditional losses on a tranche do not change if the detachment point changes above the corporate bond MVaR (which marks the kink of the line). Thus, the MVaR of a CDO tranche (as a share of the tranche's principal) equals the difference between the heights of the line at the detachment and attachment points divided by the difference between the detachment and attachment points. This ratio is the "slope" of the particular line segment.

Relaxing the assumption that the CDO's underlying pool is perfectly granular reveals additional insights. Coarse granularity introduces idiosyncratic risk, which affects the MVaR of a tranche in a way that depends strongly on the seniority of this tranche (Graph 3, left-hand panel, green and blue lines). For example, the MVaR of a junior tranche, with detachment/attachment points A^j/D^j, decreases as a result of coarser granularity. In terms of the plot, coarser granularity depresses the slope of the line segment associated with this tranche. However, the opposite is true for a senior tranche, with detachment/attachment points A^s/D^s.

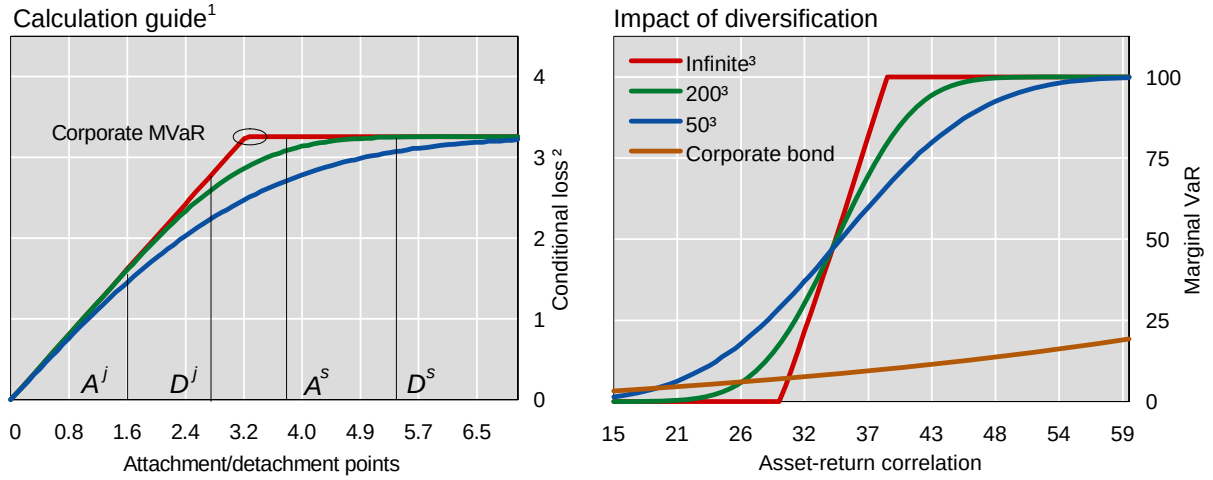
① For further detail on the main analytical results reported in this box, see Gordy (2003), who analyses portfolios of corporate exposures, and Gordy and Jones (2003), who conduct a similar analysis in the structured finance universe.

beneficial (ie loss-mitigating) and adverse (loss-enhancing) components of idiosyncratic (or asset-specific) risk in the underlying pool. The MVaR of a low-risk tranche, initially close to 0%, could be lowered only slightly by the beneficial component but would be raised substantially by the adverse component of idiosyncratic risk. Conversely, a high-risk tranche could only benefit from extra idiosyncratic risk and, thus, its MVaR decreases when granularity becomes coarser.

An important, albeit seemingly counterintuitive, implication of these results is that finer granularity in the underlying pool may render a CDO tranche more vulnerable to a change in credit fundamentals. This is illustrated by Graph 3 (right-hand panel), which focuses on a particular tranche and captures shocks to fundamentals via the level of asset-return correlations. When this level is relatively low (ie up to 30%), the tranche is a low-risk asset whose MVaR is lower than that of an underlying corporate bond if pool granularity is sufficiently fine. However, a small rise in correlations transforms the tranche into a high-risk security, whose MVaR is inflated by finer granularity. For example, if the underlying pool is perfectly granular (ie the number of underlying assets is infinite) and there is a 2 percentage point increase in correlations (from 29% to 31%), the MVaR of the tranche jumps from 0% to 15%, much above the MVaR of an underlying corporate bond.[19] Importantly, the increase in MVaR would have been much more muted, from 17% to 26%, under the coarser granularity implied by 200 assets in the underlying pool.

Marginal value-at-risk of a CDO tranche

In per cent



[1] The MVaR associated with a CDO tranche that has an attachment point *A* and a detachment point *D* equals the average slope of a line section delineated by points *A* and *D* on the horizontal axis. [2] Conditional loss is non-random only in the case of an infinite number of bonds in the CDO's underlying pool. In the other cases, the graph plots the expected value of this loss. [3] The number of homogeneous corporate bonds underlying a CDO contract. These bonds' credit risk parameters, which are shared by the corporate bond (the brown line), are reported in Table 1.

Source: Authors' calculations. Graph 3

[19] Greater asset-return correlation can be the result of stronger dependence of these returns on the common factor or higher volatility of this factor. Alternatively, greater asset-return correlation can surface when estimation errors are corrected for. Tarashev and Zhu (2008)

Conclusion

Recent, large-scale downgrades on structured finance CDOs are a reminder of the fact that rating transitions for structured finance products can be much more pronounced than what has historically been observed for more traditional credit instruments.

The preceding analysis suggests that at least two reasons can be put forward to explain such a pattern. First, the tranching process results in a non-linear relationship between the credit quality of underlying assets and that of tranched products. This can lead to a higher probability of rating downgrades as well as to more pronounced downgrades of CDO tranches than of corporate bonds. Second, ratings of tranched products are more sensitive to changes in the systematic risk factor than are ratings for corporate bonds. This implies that tranching will tend to leverage the cyclical deterioration of CDO credit quality relative to what is observed for underlying assets.

The same effect applies to other tranched instruments and is likely to be more pronounced for products that are themselves based on other tranched exposures (such as the structured finance CDOs reviewed above). In the current context, if ongoing adjustments in credit quality and related downgrades of collateral assets continue, further rapid rating migrations of CDO tranches (and, indeed, tranches of other securitisations) are to be expected.

A related observation is that measures of credit VaR can differ substantially across like-rated instruments, both with regard to levels and in terms of sensitivities to changes in credit fundamentals. As has been pointed out elsewhere, this implies that ratings are not an appropriate metric to fully capture and summarise the risks embodied in structured instruments. While this may be obvious for risk factors that are not covered by ratings (such as liquidity), investors need to appreciate that this also applies to default risk in that EL and PD do not give an indication of the higher moments of the loss distribution. These higher moments have important implications for rating transition behaviour and valuation, particularly for tranched instruments. Undue reliance on ratings, therefore, can lead to mispriced and mismanaged risk exposures as well as unfavourable market dynamics if these exposures have to be unwound.

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show that, given the limitation of data available in real-word situations, plausible small-sample errors in correlation estimates can be as high as 4 percentage points.

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