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Risk premia across markets: information from option prices[1]

A measure of risk premium is derived from the comparison of spot and option prices across the US equity and eurodollar markets. Risk premia in both markets co-move with volatility risk. Option prices, however, seem to underreact to changes in return volatility forecasts.

JEL classification: G120, G130, G140.

Financial market commentary often focuses on the identification and analysis of shifts in risk premia embedded in asset prices. Risk premia relate to the compensation that investors expect to receive for bearing risks. The analysis, however, is complicated by the fact that neither the premia nor their main drivers are directly observable. Inferences are typically made on the basis of comparisons between the prices of different securities with slightly different risk characteristics.

The compensation for risk naturally depends on investors' perception of the underlying risks and on the price they require per unit of risk, which relates directly to their attitude towards risk. Disentangling the two is key in deriving correct inferences from asset prices. The price of a security will decline if investors become more uncertain about the associated risk, even if they do not revise downwards their expectations of future cash flows. Alternatively, lower prices might signal investors' increased uneasiness with the uncertain nature of cash flows. In the first case, the price decline suggests a change in expectations about economic fundamentals that might be specific to the particular asset class. In the second case, it could be symptomatic of a more general shift in investor preferences that is likely to have implications for the pricing of risk across a spectrum of asset classes and might also affect other market functioning attributes such as liquidity.

In this article, we calculate risk premia on the basis of information regarding investors' risk attitudes that are extracted from option prices using techniques that have been developed recently in the academic literature. The

[1] The views expressed in this article are those of the authors and do not necessarily reflect those of the BIS. Dimitrios Karampatos provided excellent help with the data, graphs and table.

main innovation of our methodology is that it combines information from two different sources: the equity market and the money market. In doing so, it casts a broader net than other methods which also use information from option prices but typically focus on a single asset class. This results in reduced sensitivity of the estimates to technical aspects specific to any one market while making it easier to distinguish between diverging fundamentals across the two markets. Moreover, by deriving measures of market-specific risk premia in the context of a common specification of investor risk preferences, the methodology is consistent with the notion of an integrated financial system.

The article is organised in four sections. The first section discusses the definition of risk premia and gives a general description of the empirical methodology. The second focuses on the particular application to the two asset classes we examine. It discusses our findings and how investor perceptions about underlying risks relate to risk premia. The third section characterises the relationship between the estimated risk premia and the behaviour of asset price returns and volatilities. The final section focuses on the implications of assumptions regarding perceptions of risk for the estimates of risk premia and of investors' appetite for risk-taking.

Methodology[2]

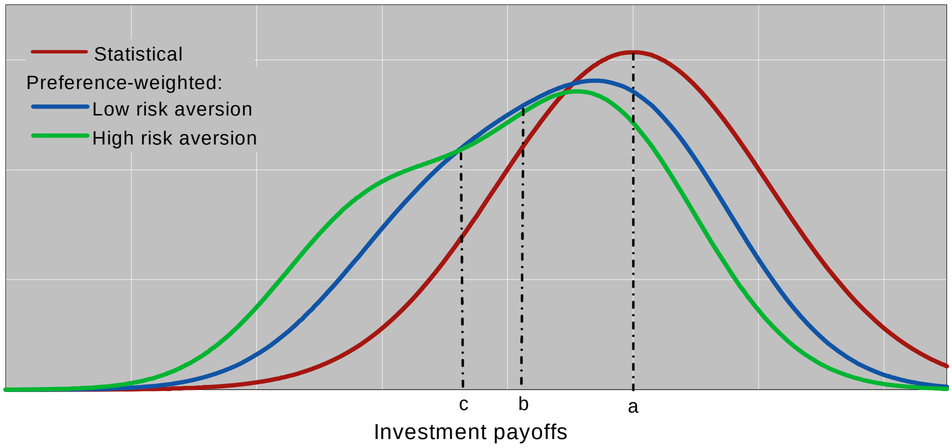
Investors' aversion to risk ...

Economic theory links the value of a security to the present discounted value of the associated stream of financial benefits. Investors' views of the likelihood of those benefits and their disposition towards uncertainty are the factors that determine the value of the security. Investors are assumed to assign a declining incremental value to additional benefits as the level of their wealth increases. This implies that, everything else constant, securities that offer higher payoffs when the level of wealth is lower are valued more highly. Another implication is that investors are risk-averse. The economic value of the uncertain payoff of a lottery ticket would be smaller than the statistical expectation of this payoff calculated on the basis of the lottery's odds. The difference between this statistical expectation and the economic (or preference-weighted) value of the uncertain payoff is often referred to as the risk premium.

Graph 1 illustrates this point. The red curve depicts the hypothesised statistical likelihood of future returns on a particular security. The value of the security to a risk neutral investor who shares this outlook about future returns should be equal to the statistical average of these payoffs, depicted by point **a**. By contrast, the preferences of a risk-averse investor can be summarised by the preference-weighted likelihood, shown as the blue curve, which puts greater weight on lower than on higher payoffs. The economic value of the security to this investor would be the average payoff calculated under this preference-weighted likelihood, depicted by point **b**.

[2] The discussion and graphical exposition in this section draw on Tarashev et al (2003).

Statistical versus risk preference-weighted probability densities



Graph 1

... gives rise to risk premia ...

The distance between **a** and **b** reflects the risk premium, or the expected excess return over the statistical expectation of payoffs that a risk-averse investor requires as compensation for risk. The size of the premium is closely related to the shape of the statistical likelihood curve (the nature of uncertainty surrounding the payoff) and to the difference between it and the curve that incorporates the risk preferences of the investor. In fact, the green curve in the graph corresponds to the subjective likelihood for an investor who is less inclined to bear risk. The preferences of such an investor imply a larger gap between the statistical expectation of the payoff and the average payoff under the preference-weighted likelihood (point **c**) and, hence, a larger risk premium.

Our methodology for calculating risk premia is based on this framework and broadly follows Rosenberg and Engle (2002). It consists in (i) estimating the statistical likelihood of future payoffs on the basis of historical patterns in the price dynamics of a security and (ii) deriving a mapping between this likelihood and the preference-weighted likelihood by reference to a cross section of observed prices on option contracts on the same security. Even though the mapping is derived on the basis of option prices, it can be used to calculate the risk premium associated with the underlying security since it is assumed to represent the same set of fundamentals and investor preferences. (The box on page 96 provides further details.)

... in equity and money markets

In recent years, there has been a growing literature that discusses the extraction of measures of investors' risk attitudes and risk premia on the basis of information contained in asset prices. The works of Rosenberg and Engle (2002), Bliss and Panigirtzoglou (2004), Aït-Sahalia and Lo (2000), Tarashev et al (2003), Misina (2005) and Gai and Vause (2005) present different methodologies aimed at isolating the effect of investors' aversion to risk on the pricing of financial securities. Another strand of this literature focuses on the impact of risk preferences on risk premia in different markets. Bollerslev et al (2005) compare the realised volatility in S&P 500 returns to the implied volatility in the prices of options on the same equity index to derive a measure

Deriving risk premia

We define risk premia as the difference between actual (or statistical) expectations and preference-weighted expectations of asset returns. In the most general terms, we first quantify two types of uncertainty: one regarding the return on the representative investors' overall wealth portfolio, and one regarding the payoffs from individual option positions. By parameterising the preferences of the representative investor, we can relate the two types of uncertainty to observed option prices. The values for the preference parameters are calibrated in order to match most closely the option prices observed in the data. Having quantified the relevant types of uncertainty and traders' preferences, we can calculate statistical and preference-weighted expectations and, thus, risk premia.

More concretely, our derivation starts from the idea that an option price is a preference-weighted expectation of the option's payoff. The latter is denoted by $g(R)$ and is fully specified contingent on the return of the underlying security, R_t. If we denote the return on investors' overall portfolio by W_t, the price, P_t, of a European-style option contract with an expiry date $t+T$ can be written as:

$$P_t\,(\theta_t) = e^{-r_tT}\sum g(R_{t+T})M(W_{t+T};\theta_t)\Pr(R_{t+T},W_{t+T})$$

where r is the risk-free discount rate, $\Pr(R,W)$ denotes a statistical likelihood of the joint realisation or R and W as perceived by the representative investor, and the summation is taken over all the possible realisations of the pair (R_{t+T},W_{t+T}). The function M, commonly known as the pricing kernel, transforms the statistical probabilities into preference-weighted probabilities, $M(W,\theta)\Pr(W,R)$, when preferences depend on the parameters θ and the aggregate investment return.

To estimate the preference parameters (θ), we need to be able to calculate the implied option price, $P(\theta)$, for any values of these parameters. For the main part of the analysis, we apply a three-parameter orthogonal-polynomial specification to the pricing kernel (see Rosenberg and Engle (2002)). This specification is flexible enough to be applied to data from two different asset markets but, at the same time, is relatively robust to the risk of corruption from noise in the price data. In the last section of the paper, we use a two-parameter specification of the kernel, which directly delivers an indicator of risk aversion, but it is less robust to noise, and thus requires a further filtering of the option data.

The remaining task is to estimate the statistical probabilities of asset returns $\Pr(R,W)$, as perceived by the representative investor. We assume that these probabilities are based on statistical models that fit as closely as possible the observed return series. In addition, we pay particular attention to two aspects of the distributions of W and R: their volatility and correlation. To allow for time-varying asset volatility, we estimate an asymmetric GARCH model, first suggested by Glosten et al (1993), for each of the two returns separately. The model incorporates two established characteristics of asset returns: the persistence of volatility and the tendency of volatility to change with the level of returns. At each desired date t, we simulate the estimated GARCH models T days (roughly one month) into the future. In simulating the models, we draw pairs of shocks whose correlation coefficient equals the sample correlation between W and R over a two-year period prior to date t. For each date t, we repeat these simulations 5,000 times, thus deriving an empirical joint statistical likelihood $\Pr(W_{t+T},R_{t+T})$.

The above procedure leads to an implied option price $P_t\,(\theta_t)$, which is a function of the values of the preference parameters. On each date t, these implied prices are then matched to the observed option prices for a cross section of contracts. The parameters (θ) are chosen to minimise

$$\sum_k \left(\left[P_t^k - P_t^k(\theta_t)\right] / P_t^k \right)^2$$

where k indexes the option contracts in the cross section and P^k denotes an observed option price.

Once we have estimated the preference parameters, we can calculate a risk premium for any asset. The risk premium is defined as the difference between the statistical expectation of an asset's return and the preference-weighted expectation of the same return. Taking an option's underlying asset as an example, its implied risk premium equals:

$$\sum r_{t+T}\Pr(R_{t+T}) - \sum r_{t+T}\,M(W_{t+T};\theta_t)\Pr(W_{t+T},R_{t+T})$$

of investors' pricing of equity market risk. In articles that have appeared in this *Quarterly Review*, Fornari (2005) analyses risk premia in fixed income markets using swaption prices, while Amato (2005) uses the preference-filtered likelihood of corporate bond payoffs embedded in CDS prices to derive measures of time-varying risk premia in the corporate bond market.

One important innovation in this article is that the estimation of the premia and the mapping between the statistical and preference-filtered likelihoods are based on information derived from two different markets. In an integrated financial system, there are strong a priori reasons for assuming that the preferences of the representative investor should have a similar impact on the pricing of different securities. Chief among these is the existence of arbitrageurs, who would take positions to exploit pricing discrepancies across markets.

The second innovation in this article is that the pricing filter is specified on the basis of an aggregate financial portfolio which includes equity and fixed income securities issued by both the public and the private sectors.[3] The composition of the portfolio corresponds closely to the composition of aggregate financial wealth and is thus better suited for the calibration of the representative investor's preferences. By contrast, existing methodologies focus on a single market (typically equities) and assume that the returns on that asset class are sufficient for characterising changes in overall investor wealth.

Risk premia in equity and money markets

We calculate time series of risk premia on the basis of option and futures prices and cash returns in the S&P 500 and eurodollar markets.[4] The data cover the period from February 1992 to February 2004. To avoid technical problems with option contracts too far away from or too close to expiration, we consider prices for contracts with one month to expiry date. Owing to a change in the frequency of eurodollar option expiry dates, we obtain quarterly risk premia estimates up to November 1995 and monthly estimates thereafter.[5]

Throughout the estimation, we conform to typical practice in the related literature. In particular, we closely follow Rosenberg and Engle (2002) in filtering out option contracts of suspect quality that could corrupt the estimation results. After suspect data have been eliminated, an average month features 34 strikes for the S&P 500 options and 14 strikes for the eurodollar options. In

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[3] The aggregate financial portfolio is proxied by the stocks in the S&P 500 Index, the government bonds in the Lehman Brothers US Treasury Index and the corporate bonds in the Lehman Brothers US Corporate Investment Grade Index.

[4] The available data relate to American-style options on S&P 500 and eurodollar futures. In order to be able to apply the methodology outlined in the box, we adjust the option prices to their European-style analogues by following Barone-Adesi and Whaley (1987). We are grateful to William Melick for providing us with the eurodollar option data.

[5] In addition, poor data availability prevents us from calculating risk premia for the following months: January and December 1996, and April, June, July and September 1997.

addition and in line with the literature, we do not estimate a time-varying statistical expectation of S&P 500 returns but use instead the unconditional mean of these returns over the entire sample.[6]

Risk premia react to …

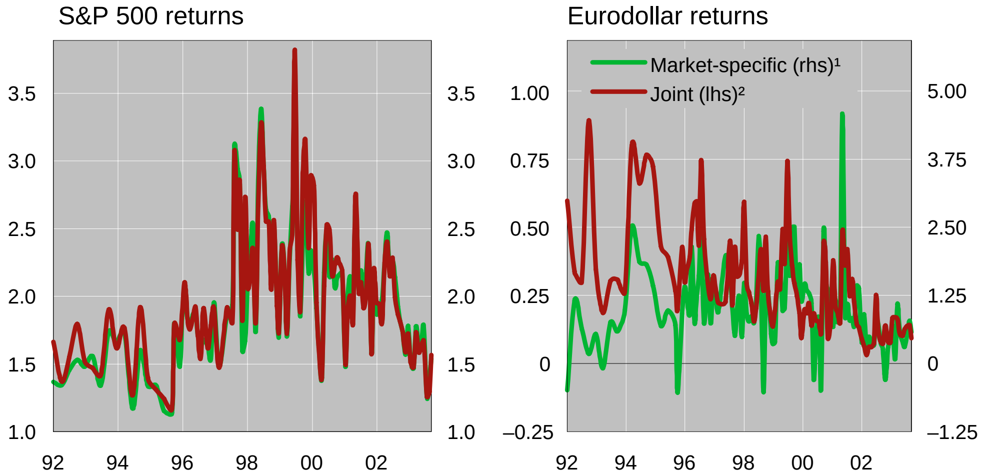
Graph 2 plots the estimated measures of risk premia for the two markets. Each panel compares the estimation based on information from each market separately to that performed jointly using information from both markets. A number of observations are worth highlighting. First, for both markets, separately and jointly estimated premia exhibit similar patterns. Second, looking more closely at the eurodollar market premia, short-term movements in the market-specific indicators are dampened when the estimation is performed jointly across the two markets.[7] Finally, premia follow different trends across markets over the sample period. Equity market premia were on an upward trend between 1996 and 2000 but have been on a declining one since then.[8] By contrast, money market premia were on a general downward trend over most of the sample period, albeit at times matching short-lived swings in the equity market premium.

… perceived risk …

Return volatility is arguably the most commonly used measure of risk in financial markets. Graph 3 compares the behaviour of risk premia to

Risk premia

In per cent



[1] Based on S&P 500 option data (left-hand panel) and eurodollar option data (right-hand panel). [2] Based on both S&P 500 and eurodollar option data.

Sources: Chicago Mercantile Exchange; authors' calculations. Graph 2

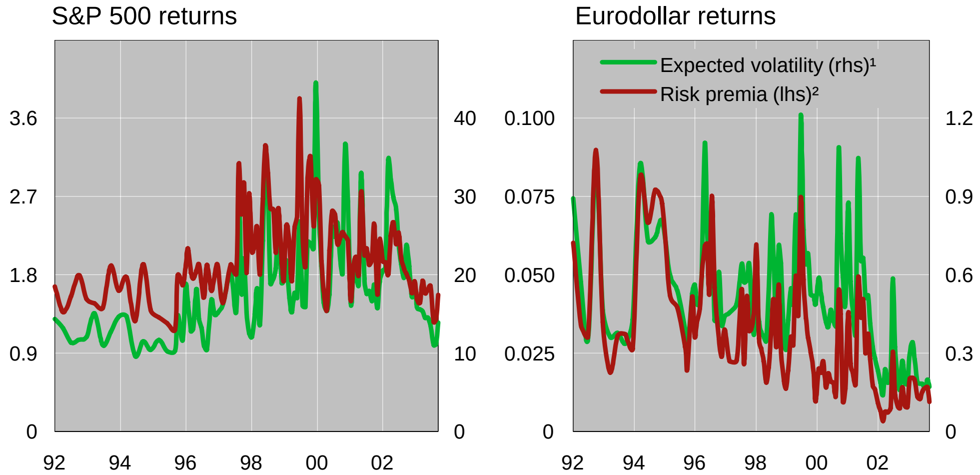
[6] The reason for not estimating a time-varying expected return is that the high volatility of stock returns introduces much uncertainty about their mean over short time horizons (one month, in our case). Allowing for time variation of the one-month statistical expectation of returns would tend to commingle the estimates of risk with those of expected return.

[7] The correlation between the risk premia estimated jointly and separately is 91% and 45% for the equity and the eurodollar markets, respectively. In addition, the standard deviation of market-specific risk premia on the eurodollar market equals 75% of the associated mean, while the same statistic drops to 63% for the jointly estimated risk premia.

[8] The upward movement of equity market premia is somewhat surprising as it coincides with a sustained bull market. The finding, however, is consistent with a higher return volatility over this period (see also the discussion in the next section).

Risk premia and expected volatility

In per cent



[1] Based on GARCH estimates; annualised. [2] Based on both S&P 500 and eurodollar option data.

Sources: Chicago Mercantile Exchange; authors' calculations. Graph 3

perceptions of risk, which are proxied by the expected short-term volatility of returns in the respective market. The volatility measure is based on the model of returns that underlies our estimation procedure. We observe that there is a fairly close co-movement between the two series. Premia seem to rise in anticipation of higher risk and to decline in more tranquil market conditions. This pattern is common across markets and throughout the sample period. This indicates that asset prices (in this case, options) do react to changes in perceived risk in the expected way.

Stylised patterns of risk, return and risk premia

This section examines further our estimates of risk premia by relating their dynamics to the dynamics of asset returns. A series of adverse investment returns is likely to induce investors to require greater compensation for bearing the risk of additional losses in subsequent periods. In addition, risk-averse traders would bid down the price of an asset if they perceived an increase in its volatility. With this in mind, we evaluate the relationship between risk premia and the level and volatility of asset returns and report the results in Table 1.

Table 1 presents the correlation coefficients of the estimated risk premia on the S&P 500 and eurodollar markets with three statistical characteristics of asset returns. The first characteristic is past realised returns and it is measured as the average return over the month ending on the date for which we calculate the risk premium. The second characteristic is the expected volatility of returns over the remaining life of the option. It is the volatility measure derived from the statistical likelihood that underpins our calculations, and represents an *ex ante* measure of perceived risk consistent with our statistical model of returns. The third characteristic, "realised volatility", is the actual realisation of asset price volatility around the expiry date of the associated option contract. In contrast to the second characteristic, this provides an *ex post* measure of volatility but

Risk premia and asset returns

Correlation coefficients[1]

Risk premium on:	Characteristics of corresponding asset		
	Past returns	Expected volatility	Realised volatility
S&P 500[2]	–0.32**	0.69**	0.39**
Eurodollar[3]	–0.20*	0.82**	0.33**
	Characteristics of aggregate portfolio[4]		
	Past returns	Expected volatility	Realised volatility
S&P 500	–0.30**	0.70**	0.39**
Eurodollar	0.12	–0.38**	–0.34**

* and ** denote statistical significance at the 5% and 1% levels, respectively.

[1] For the calculation of the correlation coefficients, a risk premium estimated for date *t* is aligned with, respectively: the average return between dates *t*–30 and *t* (past returns), the expected standard deviation of the return until the option's expiry date, as implied by GARCH estimates (expected volatility), and the standard deviation of returns between dates *t*+15 and *t*+45 (realised volatility). [2] Correlation between the risk premium and summary statistics of S&P 500 Index returns. [3] Correlation between the risk premium and summary statistics of eurodollar returns. [4] The market portfolio is proxied by the S&P 500 Index and the Lehman Brothers Government Bond Index and Corporate Investment Grade Index.

Source: Authors' calculations. Table 1

could be thought of as an alternative measure of expected risk under the assumption that investors' expectations are correct on average. We calculate the three characteristics for returns on the S&P 500 Index, the eurodollar market and the aggregate portfolio.

… past returns and …

Risk premia appear to react significantly to past returns. The first column of Table 1 indicates that abnormally low returns on the S&P 500 Index and the aggregate wealth portfolio tend to be followed by increases in the risk premium for equities. The same is true for eurodollar market returns and the corresponding premium. One explanation for the finding is that a series of low returns may put pressure on a trading operation's risk budget, which would drive up the required compensation for bearing risk in subsequent periods. Alternatively, the result might simply be a reflection of the frequently observed fact that price volatility increases when prices decline, coupled with persistence in bear market conditions. A period of low returns could be seen as foreshadowing high risk in the immediate future, hence raising the risk premium.

… realised return volatility

The second and third columns of the table provide evidence that the risk premium indeed compensates investors for expected and realised risk. The second column in the upper panel shows the positive link between risk premia and perceived asset return volatility, which was previously illustrated in Graph 3. For its part, the third column in the upper panel indicates that higher compensation for risk-taking is associated with higher realised risk. This also provides an indirect and partial validation of the statistical model of returns we use in our estimation method. The results reported in the lower panel of the table show that the risk premium on the S&P 500 Index is positively correlated with the perceived and realised volatility of returns on the aggregate wealth portfolio. Finally, we find a negative relationship between eurodollar risk premia and aggregate portfolio risk, mainly as a result of the low correlation between the volatility of eurodollar returns and the volatility of returns on the aggregate wealth portfolio.

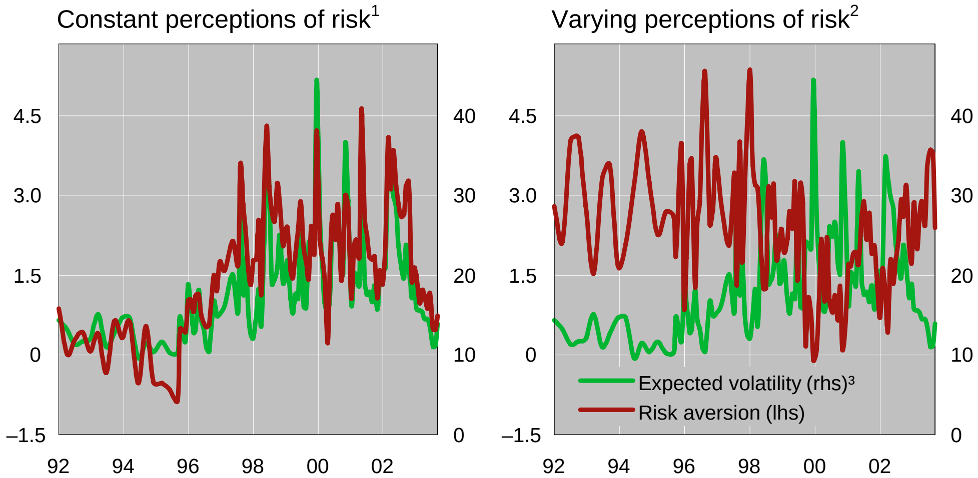
Perceptions of risk and measures of risk aversion

Our derivation of risk premia is based on a parameterisation of the risk preference filter of the representative investor. A direct by-product of the methodology is an indicator of investors' risk attitudes in the form of a time series of a key parameter in the estimated filter (see the box on page 96 for more details). In this section we discuss the behaviour of this indicator, which, in sharp contrast to the estimated risk premia, appears to be quite sensitive to how we model investors' perceptions of risk. We focus exclusively on the equity market.

Graph 4 plots measures of investor risk aversion on the basis of two alternative assumptions about investors' perceptions of risk.[9] The left-hand panel equates investors' expected volatility to the average estimated volatility over the entire sample, while the right-hand panel assumes that investors change their expectations of return risk consistently with the time series model we estimated. This time-varying measure of expected volatility is also depicted in both panels. The two assumptions about investors' perception of risk could be thought of as outlining the contours of a range of plausible alternatives. The fact that the estimates of risk premia under the two alternatives are virtually

Alternative indicators of risk aversion

S&P 500 futures market



[1] Risk aversion estimates are based on the unconditional volatility of S&P 500 futures and put options on these assets. [2] Risk aversion estimates are based on the conditional volatility of S&P 500 futures, as implied by GARCH estimates, and put options on these assets. [3] Based on GARCH estimates; annualised; in per cent.

Sources: Chicago Mercantile Exchange; authors' calculations. Graph 4

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[9] The risk aversion indicator plotted in the graph is based on a simpler version of the model that allows us to summarise risk aversion as a single parameter of the pricing kernel (see box for details) but restricts the estimation to using only put option prices. These simplifications have no material impact on our overall conclusions.

identical provides comfort as to the robustness of the conclusions in the previous section.[10]

Option prices seem to underreact to risk forecasts

By contrast, the estimated indicator of risk aversion is very sensitive to alternative assumptions about investors' risk perceptions. Under the assumption of constant risk perceptions, the co-movement between the risk aversion indicator implied by the estimated model and our estimates of risk is very close. Risk aversion seems to increase when risk is elevated and to reach pronounced peaks in periods when contemporary market commentary indicated that investors were particularly shy of risk-taking. When one allows for time-varying risk perceptions, however, the derived indicator has a counterintuitive behaviour. It is negatively correlated with expected risk and seems to decline in periods when one would a priori have expected it to peak.

The latter finding is puzzling but it is not unique to our methodology, as Bliss and Panigirtzoglou (2004) find similar patterns for risk aversion when allowing for time-varying perceptions of risk. An explanation consistent with the underlying model is that while option prices do react to changes in forecasts of future return volatility, this reaction is subdued and excessively influenced by the historical average volatility of returns. This topic requires further analysis that lies beyond the scope of the present article.

Conclusions

In this article, we combined information from equity and money market option prices to derive measures of risk premia in these markets that are consistent with a single price of risk required by the representative investor. The process yields a robust measure of premia that co-move with measures of risk, in the form of both expectations and realisations of return volatility. Premia are also negatively correlated with past market returns, suggesting that investor behaviour might generate feedback from past to future asset price performance. The results suggest that consistent estimates of risk premia across asset classes could be a useful tool in interpreting financial market conditions, as well as in assessing near-term prospects in securities markets.

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[10] The correlation between the two alternative premia over the sample period is equal to 94%. Given that the risk premium embedded in option prices increases with investors' perception of and aversion to risk, a change in the estimated risk aversion will counteract changes in estimated risk perceptions for a given set of observed option prices.

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