Approximation of changes in option values and hedge ratios: how large are the errors?

by

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Abstract

When changes in portfolio values must be calculated over asset price shocks of different sizes such as in stress tests over a variety of scenarios, constraints imposed by computation speed as well as database structure sometimes lead analysts to use approximations to portfolio values instead of exact portfolio revaluations. This paper examines the effect of time to maturity and moneyness of an option on the magnitude of the approximation error. In addition, the approximation error is also examined in a portfolio consisting of the outstanding stock of options on Eurodollar interest rate futures on the Chicago Mercantile Exchange.

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* The views expressed in this paper are the authors' and do not necessarily reflect the positions of the Federal Reserve Bank of New York, the Federal Reserve System, the Bank for International Settlements, or the Eurocurrency Standing Committee.

Approximation of changes in options values and hedge ratios: how large are the errors?

When changes in portfolio values must be calculated over asset price shocks of different sizes such as in stress tests over a variety of scenarios, constraints imposed by computation speed as well as database structure sometimes lead analysts to use approximations to portfolio values instead of exact portfolio revaluations (see Gibson 1997). Given this tradeoff between computation and database costs and accuracy, this paper examines the magnitude of the approximation error in the most common approximation methods. Other papers that provide insight on this issue would include papers by Estrella (1996), Pritsker (1996), Robinson (1996), and Rouvinez (1997).

The paper examines first and second order Taylor series approximations of option values and hedge ratios. For option values, the second order approximation is the option's delta plus gamma, and for the hedge ratio, the second order approximation is the gamma plus the third derivative of the option value. The approximations in the paper are estimates around a single price (the initial price). Better approximations can be obtained by means of piecewise approximations over a number of different "initial" prices. However, such approaches would be more demanding in computation time and might undercut one principal motivation for approximations: computation speed.

In Section 1, the effect of time to maturity and moneyness on the approximation error are examined. To consider the practical significance of the maturity and moneyness effects, the magnitude of the approximation error in a market portfolio is addressed in Section 2. Using the open interest of options on Eurodollar interest rate futures on the Chicago Mercantile Exchange, some insight to the size of the approximation error in practice can be obtained. In both sections, the option parameters of interest are the value of the option and the hedge ratio. The hedge ratio and its change may be of interest to analysts or risk managers who need to anticipate the volume of hedge transactions required after a sharp change in the price of the underlying asset.

The options examined are interest rate options, and the approximation error is examined for interest rate changes equal to a largest one day change and a 99th percentile two-week change. In both cases, movements in 3-month Eurodollar rates over the period 1/91 through 12/95 were expressed in percentage changes, and the largest and 99th percentile changes were applied to the initial interest rate. The option valuation function used was Black's forward interest rate option model (Hull 1993).

1. Effects of maturity and moneyness

Change in option value

As expected, a first order approximation (delta only) leads to large approximation errors across a wide range of maturities and strike prices. Errors relative to the true change are above 35% across a wide range (Figure 1, left panel).

Second order approximations (delta and gamma) produce relatively small approximation errors across a wide range of strikes and maturities. The error relative to the true change, however, are large in the case of deep out-of-the money options with short maturities (Figure 1, right panel). For a three month option with a strike 80 bp out-of-the money, the error is 10% of the true change, while for a six month option the error is less than 5%.

Figure 2 shows that in absolute terms, without adjusting for relative size, the largest approximation errors occur in options that are close-to-the-money.

Change in hedge

Relative to the true change in the hedge ratio, the approximation error is large across a wide range of maturity and strike price combinations -- for both the first order (gamma only) and second order (gamma and third derivative) approximations (Figure 3). In the case of the gamma only approximation, for a three month option with a strike 80 bp in the money (strike=0.052), the error is 100% of the true change in hedge, and for a six month option the error is 50%.

Relative to the initial hedge, the approximation error is large for deep out-of-the money options with short maturities (Figure 4). In the case of the gamma only approximation, for a two month option with a strike 80 bp out of the money (strike=0.068), the approximation error is 160% of the initial hedge position, while for a six month option the error falls to 15%.

Figure 5 shows that in absolute terms, without adjusting for relative size effects, the largest approximation errors occur in options that are close-to-the-money.

2. Portfolio effects

This section examines the approximation error for a portfolio consisting of the outstanding stock of options on Eurodollar interest rate futures traded on the Chicago Mercantile Exchange. The data for each option consists of: the strike price, the option maturity, the amount outstanding (open interest), and the market value of the option. Table 1 lists some features of this option portfolio. This portfolio does not include contracts whose open interest was very small relative to other contracts.

Estimation

For each option, the implied volatility that returns the observed market price was estimated using Black's forward interest rate option model (Hull 1993). Given this estimated implied volatility, the approximations to the change in each option's value and the change in its hedge ratio were calculated. Finally, the value of each option was weighted by its amount outstanding, and summed to produce a portfolio value. For the option value, this weighted sum is the value of the option portfolio, for the hedge ratio, the weighted sum is the weighted average of the hedge ratios of each option.

Table 1

Portfolio of interest Rate Caps
(Puts on Eurodollar Futures)

Maturity	Number of Strikes	Range of Strikes	At-the-money Rate
2 months	10	4% - 6.25%	5.65%
5 months	14	4% - 7.5%	5.72%
8 months	19	3.5% - 8.25%	5.87%
11 months	18	4.5% - 9%	6%
14 months	18	4.5% - 9.25%	6.17%
17 months	12	4.25% - 9%	6.24%

Data as of October 14, 1996

Change in portfolio value

In the case of second order approximations (delta and gamma), approximation errors relative to the true values of interest rate caps are large for decreases in interest rates (Table 2). A similar, but opposite, relationship holds for interest rate floors because of the reversal of the moneyness relationship between puts and calls. While approximation errors would be the same for puts and calls with the same strike because of put call parity, the cap and floor portfolios have different strike distributions.

Table 2

Relative Approximation Errors of Portfolio Value
Interest Rate Cap Portfolio
99th percentile two-week change in rates

	Fall in rates	Rise in rates
Relative to change in value	22%	2%
Relative to initial portfolio value	16%	4%

In comparison to the figure in Table 2, the largest one-day change in rates produces an error of 6% relative to both the change and level of portfolio value, in the case of a fall in rates.

Change in portfolio hedge

For first order approximations (gamma only) to the change in the portfolio hedge ratio, approximation errors relative to true values are large for interest rate decreases (Table 3).

Table 3

Relative Approximation Errors of the Portfolio Hedge
Interest Rate Cap Portfolio
99th percentile two-week change in rates

	Fall in rates	**Rise in rates**
Relative to hedge adjustment	58.1%	2.0%
Relative to initial hedge position	39.1%	2.1%

For comparison with the figures in Table 3, the largest one-day change in rates produces an approximation error of 30% relative to the hedge adjustment and 14% relative to the initial hedge in the case of a fall in rates. The approximation error in hedge adjustments is not monotonically increasing in the size of the shock, because the sign of gamma changes at the at-the-money strike, and the delta function is bounded from above.

FIGURE 1

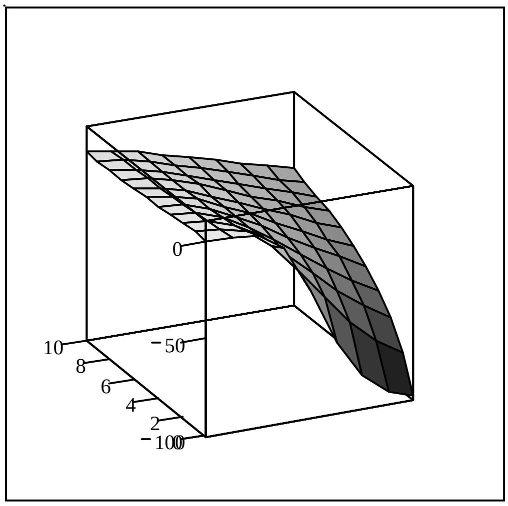
Change in option value: Approximation error relative to true change (%)

Maturity and strike variations

IR Caps

Delta only

Delta plus Gamma



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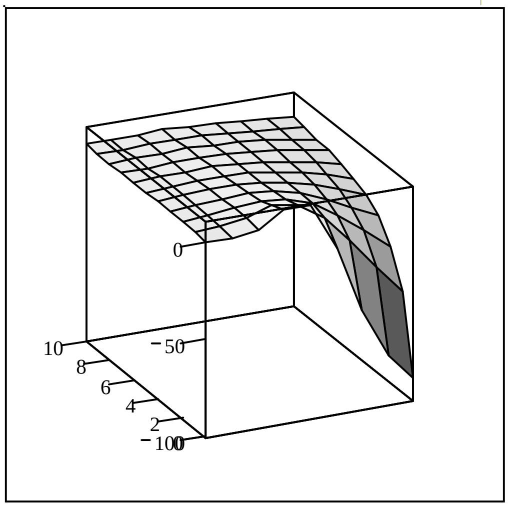

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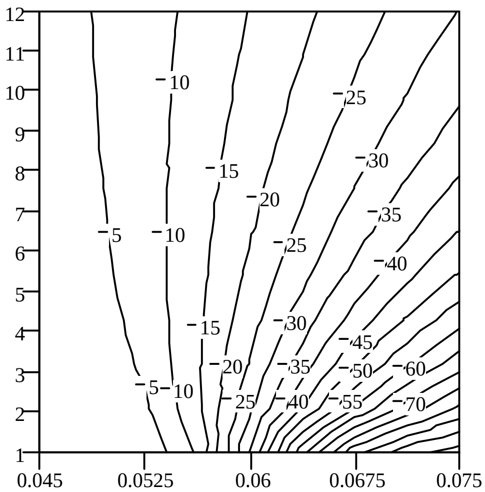

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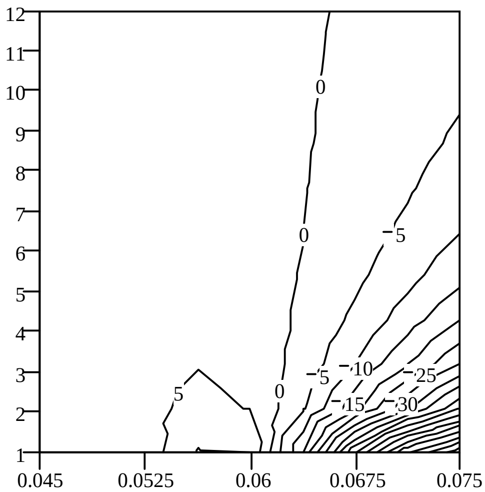


V2

Out-of-the money

Out-of-the money

Strike rates range from 0.045 to 0.075

at-the-money strike is 0.06

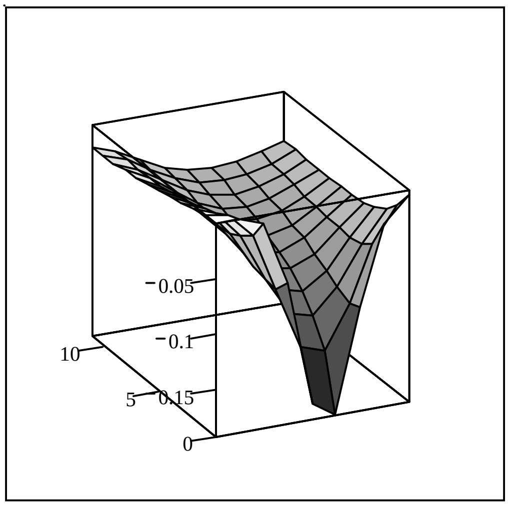
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 2
Option value: Approximation error
Maturity and strike variations
IR Caps

Delta only

Delta plus Gamma



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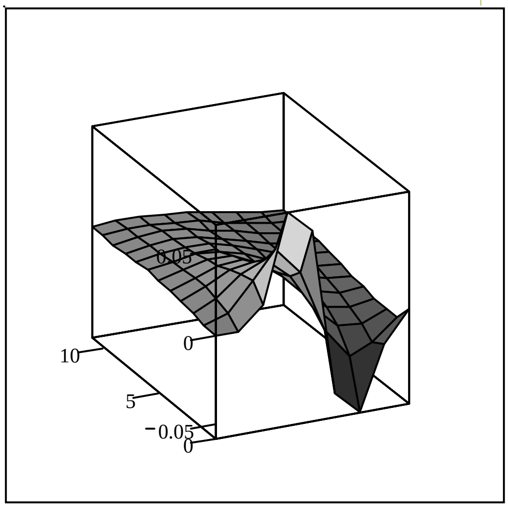

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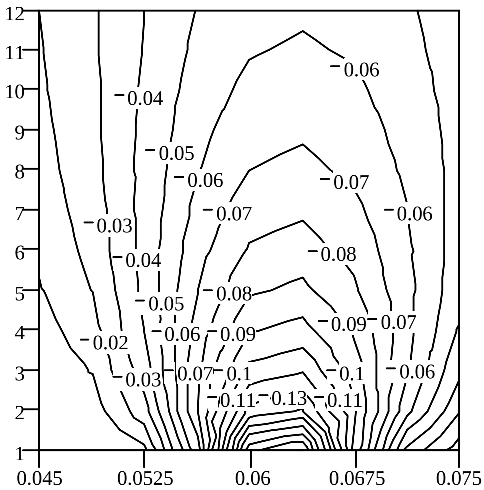

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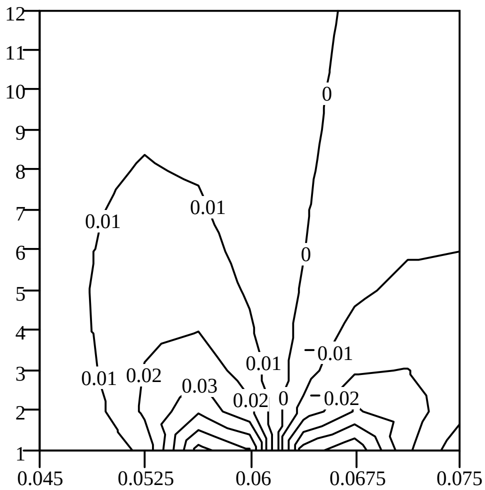


V2

Out-of-the money

Out-of-the money

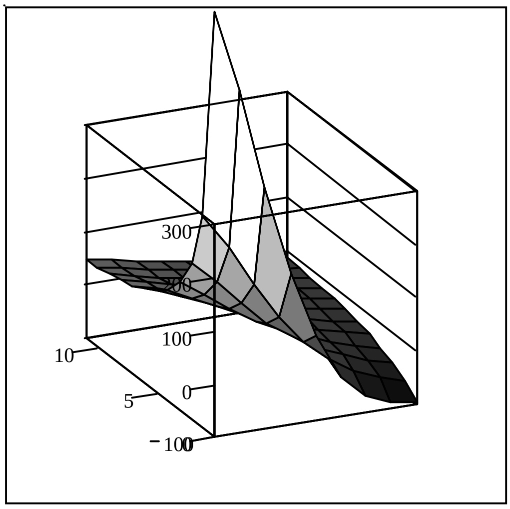
Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 3
Change in hedge: Approximation error relative to change in hedge (%)
Maturity and strike variations
IR Caps

Gamma only

Gamma and third derivative



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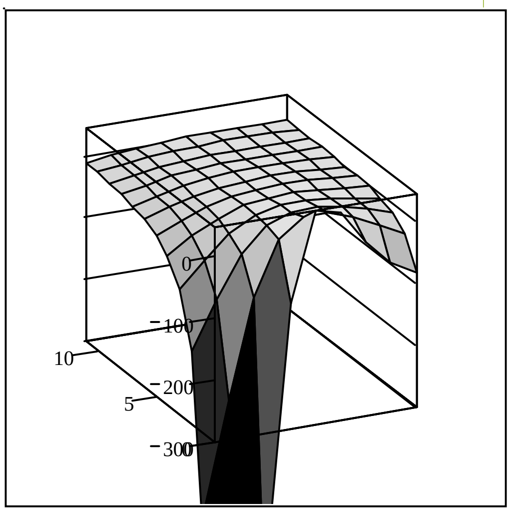

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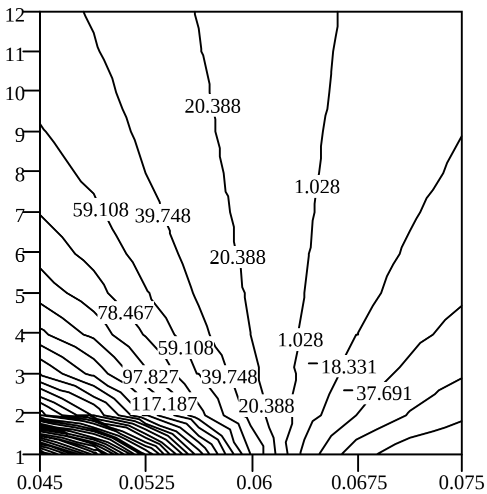

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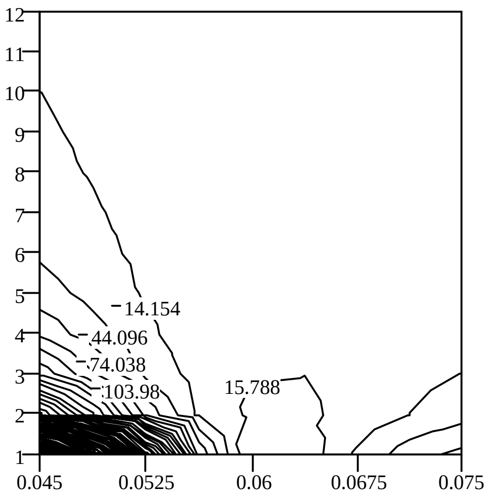
Out-of-the money



G3

Out-of-the money

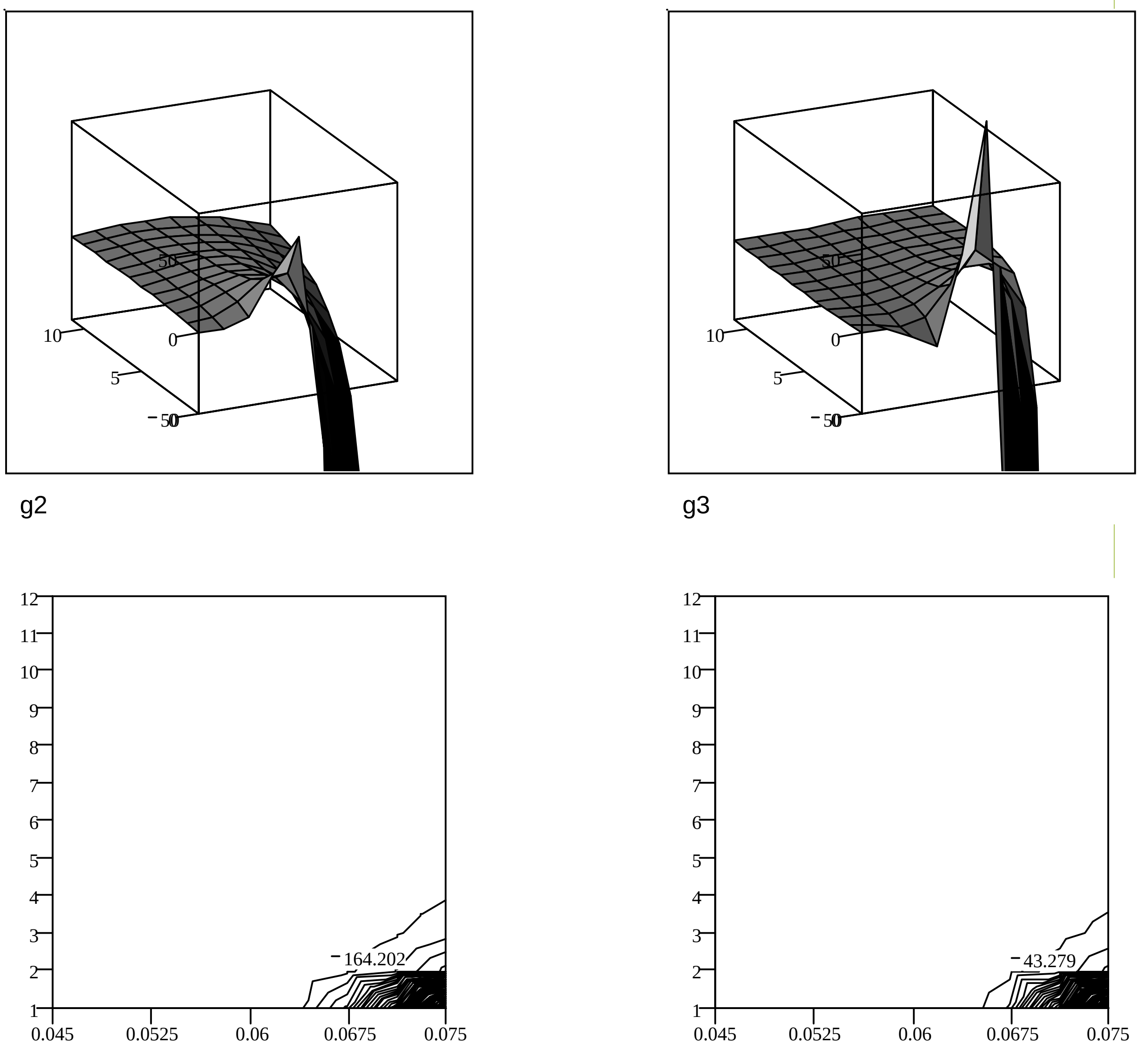
Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 4
Change in hedge: Error relative to (old) hedge position (%)
Maturity and strike variations
IR Caps

Gamma only

Gamma and third derivative



Out-of-the money

Out-of-the money

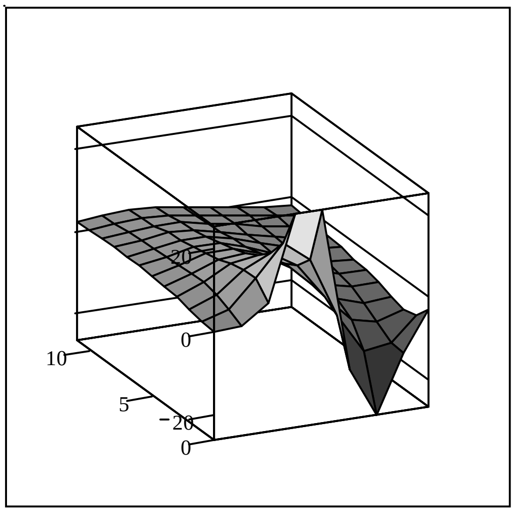
Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 5
Change in hedge: Approximation error
Maturity and strike variations
IR Caps

Gamma only

Gamma and third derivative



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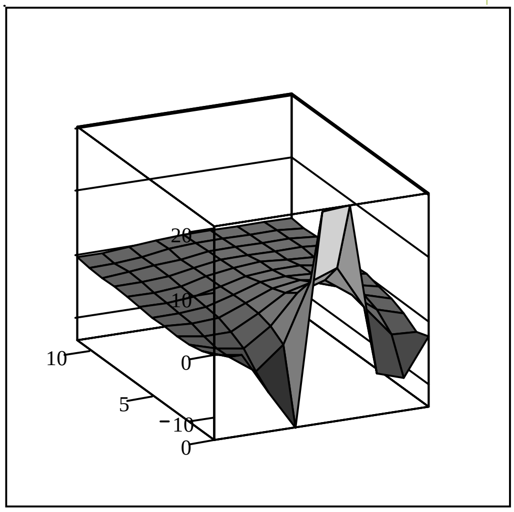

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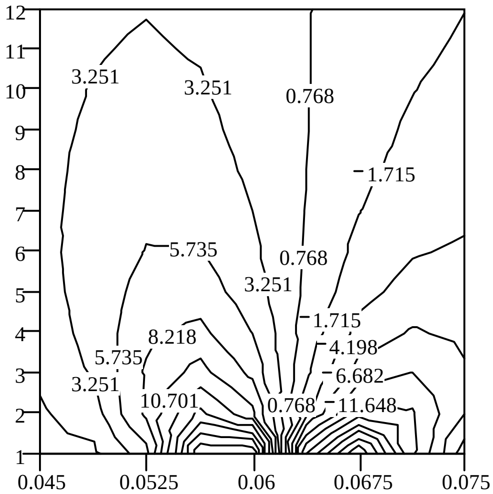

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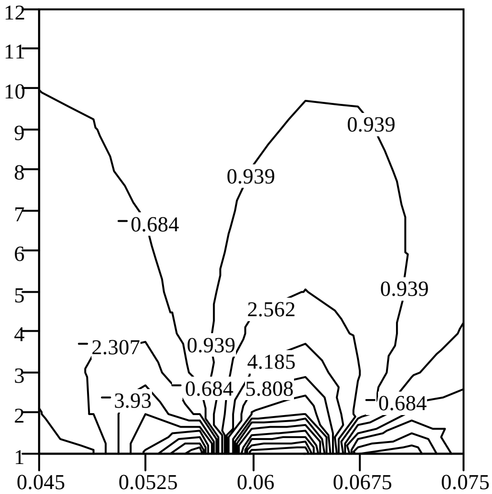

G3

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Maturity range from 1 month to 12 months

99th percentile 2 week IR change

References

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