Generating market risk scenarios using principal components analysis: methodological and practical considerations

by

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Abstract

In this paper, I study a number of statistical issues that arise in the formulation of stress scenarios for market risk in financial instruments. The possibility of reducing the number of scenarios through the use of data-based, statistical dimension reduction methods is explored. Using data on returns to spot exchange, stock market and interest rate products for a number of countries, I show that principal components analysis may be used to reduce the effective dimensionality of the scenario specification problem in several cases. Given the data dimensionality uncovered by PCA for the datasets considered, various methods for specifying stress scenarios are discussed.

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* Views expressed in this paper need not reflect the views of the Board of Governors of the Federal Reserve System or of other members of its staff or of the Eurocurrency Standing Committee. Any errors are my own.

1. Introduction and general issues in market risk scenario specification

Market risk is commonly defined as the susceptibility of portfolio values to changes in asset prices, volatilities of prices, and related functions of asset prices. Measuring market risk may seem to require specifying a very large number of perturbations of prices and volatilities. However, in empirical practice, many asset price and volatility movements are highly correlated contemporaneously. The "effective dimensionality" of market risk is therefore often considerably less than the number of assets held in a typical portfolio. "Risk factors" are often defined and used to summarise observed changes in market prices and volatilities. This paper discusses some of the statistical issues that arise in the search for market risk factors and scenarios that describe stressful market risk events.

The remainder of this section discusses some general methodological considerations. The need for applying statistical methods for scenario specification is reviewed. Principal Components Analysis (PCA) is proposed as a tractable and easy-to-implement method for extracting market risk factors from observed data. Section 2 presents the returns series analysed in this paper, and tests whether the data are in fact amenable to PCA methods. Section 3 performs PCA on several groupings of these series. I find that the stock market and the exchange rate returns series are more highly correlated than, say, short term interest rates. This suggests that dimensionality reduction may apply for certain groups of series, but not for others. On the basis of the PCA results, I provide suggestions for stress scenarios for stock market and spot exchange rate shocks. Section 4 concludes and mentions several shortcomings of PCA not dealt with elsewhere in the paper. An appendix discusses some of the mathematical aspects of nonparametric density estimation and of PCA.

It is important to note that the dimensionality of the market risk scenario problem is, to a considerable extent, a choice variable for the researcher. Increasing the number of market risk factors tends to enhance descriptive accuracy or the amount of data variability captured by the scenarios, but also risks increasing the methodological complexity and unwieldiness of the study. An optimal cut-off for specifying additional risk factors will depend, in general, on the purposes for which the risk factors are being constructed.

When the number of series is small, say one or two, it is usually possible to simply "eyeball" scatter plots of the data and to decide heuristically what a relevant stress scenario might be. Unfortunately, "eyeball methods" become infeasible when the data are high-dimensional. To specify stress scenarios in such cases, it is necessary to resort to formal statistical methods. The statistical methods should provide answers to issues such as the effective dimensionality of the data and nature of data-coherent stress scenarios. Formulating market risk factors and extracting their distributions from the data is an intermediate step between assembling the data and specifying scenarios.

One may distinguish between model-driven and data-driven statistical methods for generating risk factors.[1] Model-driven methods rely heavily on hypothesised relationships between asset prices, returns, and volatilities (which are then estimated from the data). Examples of model-driven methods are the capital asset pricing model (CAPM) for returns and "GARCH" models for volatilities. Data-driven methods, on the other hand, impose less structure on the data. When a researcher is unwilling to impose a lot of structure on the data and would rather extract risk factors directly, data-driven methods are preferable. One method which is in widespread use among statistical practitioners is "Principal Components Analysis" (PCA). This method, whose technical details are described in the appendix to this paper, is frequently employed when one needs to reduce the data dimensionality to a tractable threshold without being willing to commit to strong hypotheses about the nature of the data generating process.

2. Data and preliminary data analysis

The data series I study in this paper are daily-frequency observations on spot exchange rates, stock market indexes, and long-term and short-term interest rates, and were obtained from the Federal Reserve Board's internal economic database. I consider data for nine countries: Belgium, Canada, France, Germany, Japan, the Netherlands, Switzerland, the United Kingdom, and the United States. The exchange rate series consist of the bilateral spot exchange rates of the first eight countries vis-a-vis the United States.[2] For each of the nine countries, a leading stock market index was chosen to represent movements in equity prices. Both short-term (3-month) and long-term (10-year) interest rates were collected for each of the nine countries. In addition, a nine-point term structure series for the US Treasury returns and four separate stock market indexes for the United States (S&P 500, Dow Jones Industrials-30 Average, Nasdaq Composite, and Wilshire 5000) were studied. The observations run from 2 January 1990 to 8 October 1996, or slightly less than 1,700 observations. Cross-sectional missing values, caused chiefly by differing national market holiday conventions, were deleted prior to further analysis. I first took natural logarithms of the exchange rate and stock market index series, and then first-differenced all series to induce stationarity.

Prior to applying PCA to these returns series, it is important to determine whether PCA is in fact a meaningful procedure given the distributional properties of the data. The main distributional requirement is "axis symmetry," i.e., that the joint distribution of the data be symmetric about its

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1 In practice, of course, one finds that successful model-driven methods are congruent with the data, and successful data-driven methods can be interpreted to conform to certain statistical models.

2 The spot exchange rates are measured in units of foreign currency per US$ except for Sterling, where the inverse convention was applied.

axes.[3] Unfortunately, formal statistical techniques for testing axis symmetry are not well developed. To test the proposition of axis symmetry, I chose the following informal "eyeball method:" I computed the joint density of various pairs of series and graphed their contour plots. Significant deviations from axis symmetry are then readily apparent to the eye. Appendix B contains a brief discussion of the nonparametric density estimation methods that I employed to obtain the contour plots.

Contours of six (randomly chosen) bivariate densities are plotted in Figures 1 through 6. The returns data are standardised for this exercise. Density estimates are provided for a -5 to +5 standard deviations range from the joint mean of the data. The heights of the displayed contours are 0.40, 0.30, 0.20, 0.10, 0.04, 0.01, and 0.001.[4] The height of the outermost lines is only 1/400th of the height of the innermost "ring." For a practical assessment of axis symmetry, though, it is more practical to consider the shapes of the lines with heights between 0.01 and 0.40. The first three figures depict bivariate data sets that are not highly correlated; the second group of three figures depicts series that are highly correlated. In no case does failure of axis symmetry appear to be a prominent problem. Since axis symmetry cannot be rejected, at least not on the basis of the informal tests conducted, I conclude that we may indeed apply PCA methods to the data at hand.

3. Principal components analysis and effective dimensionality of the data

3.1 Fraction of variance explained by principal components

For a collection of returns series, the number of principal components (PCs) to be retained for further analysis is determined by the correlation structure of the data. If the data are all highly mutually correlated, one or two PCs will suffice to explain a large fraction of total data variation. On the other hand, if the data are either uncorrelated or only correlated across subgroups, more PCs need to be retained. By studying the fraction of the variance that is explained by successive PCs, one may obtain an estimate of the effective dimensionality of the data.

Since PCA is sensitive to the units of measurement of the data, we report our results both for the "raw" and for "standardised" (zero mean and unit variance) series. Standardisation is found to have little qualitative effect except when groups of series with differing group variances, such as exchange rates and interest rates, are analysed.

In Table 1, I list the fractions of the total variance explained by successive principal components. Numbers that exceed $1/N$ (where N is the number of series under consideration) are

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[3] Intuitively, axis symmetry can be thought of as an absence of non-linear dependence among the series. Multivariate normality is sufficient but not necessary for axis symmetry. Other well-known distributions, such as the multivariate Student-*t*, are also elliptic and hence axis-symmetric.

[4] The height of 0.001 was chosen deliberately so that even a single data point would "show up" in the contour plots.

italicised, and numbers that exceed $2/N$ are underlined. I start with several more narrowly defined groups of series, and then go on to study larger data groups. For the eight groupings considered, I find:

(A) *Short Term Interest Rates, 9 Countries.* In the sample period, correlations among the nine short term interest rates were quite low. This is reflected in Panel (A) of Table 1: Whether standardised or raw interest rate changes are considered, the first two PCs barely explain 50% of total variance.

(B) *Long Term Interest Rates, 9 Countries.* In this case, the first PC alone explains ca. 50% of total data variability, and first three jointly explain about 75% of the variance.

(C) *9-Point US Term Structure Series.* Here, the first PC explains more than 80% of total variation, and the second explains about 10%. None of the other seven PCs explains more than about 3% of total variation.

(D) *Spot Exchange Rates, 8 Countries.* All of the series are very highly correlated, and the first PC explains more than 70% of the total variance. No other PC explains more than 15% of the variance.

(E) *Stock Market Indexes, 9 Countries.* The first PC explains about 40% of the variance, and the next two each contribute more than 10%.

(F) *4 US Stock Market Indexes.* All series are well known to be very highly correlated at daily frequencies; this is borne out in the PCA, where the first (of four) PCs explains close to 90% of total variance.

(G) *Combination of Stock Market Indexes and Exchange Rates, 17 Series.* For the raw data, only the first four PCs each explain more than a $1/N$ fraction of total variance, but none of these four is particularly dominant. A similar results applies for the standardised returns series.

(H) *Combination of Stock Market Indexes, Exchange Rates, and Long Term Interest Rates, 26 Series.* For the unstandardised series, the first PC explains 50% of the variance, and two more PCs explain more then a $2/N$ fraction of the variance. However, upon standardisation the influence of the first PC is diminished to 26%, and the second PC has roughly equal weight (21%).

From these numbers, it would appear that there is considerable scope for dimension reduction among the equity returns series and exchange rate series, as well as within the US term structure of interest rates. However, the two broad asset classes (G) and (H) are less mutually correlated, leading to a lower contribution to the total variance provided by the first few leading principal components.

3.2 Correlations of estimated principal components with observed time series

In the preceding subsection we found that, in several cases, one or two PCs suffice to explain most of the variability present in the data. This suggests that the effective dimensionality of the data groups is smaller than the number of series in the groups. However, this finding alone does not let us attribute an economic interpretation to the PCs, since it does not tell us whether the PCs are correlated with *all* of the series in the respective group, or only with a subset of the series

Since PCs are linear functions of the data, it is useful to study their correlations with the observed returns series to uncover their economic interpretation (if one exists). In Table 2, we list correlations for the first four PCs (computed from the raw as well as the standardised returns series) with the corresponding observed series. The discussion below focuses, to the most part, on the correlations between the observed series and the PCs obtained after first standardising the data. We find:

(A) *Short Term Interest Rates, 9 countries.* In keeping with the finding reported above that none of the PCs explains a large fraction of the total variance in the data, we find that each of the first four PCs is highly correlated with only one or at most two of the individual 3-month interest rate series. This finding precludes the use of PCA to reduce the dimensionality of the multivariate short-rate process.

(B) *Long Term Government Bond Interest Rates, 9 countries.* In contrast to the short rate case, the long rates (especially the six European series) are highly correlated with each other and with the first PC. The Canadian and US series are highly correlated with P2, and the Japanese long rate is highly correlated with P3. This suggests that for purposes of scenario specification, the nine series can be reduced to three "meta series:" one "European" dimension, one "North American" dimension, and one "East Asia" dimension.

(C) *9-Point US Term Structure Series.* For this group of time series, the first PC is highly correlated with all nine series, and the correlations are of the same sign. The second PC is negatively correlated with the short-maturity series and positively correlated with the long-maturity series. The third PC is positively correlated with the short- and long-maturity series, and negatively correlated with the intermediate-maturity series. This finding lets us interpret the first principal component as a factor that shifts the whole term structure, the second PC as a factor that tilts or rotates the yield curve, and the third as a factor that affects curvature. In many cases, it will be quite satisfactory to concentrate

(D) *Spot Exchange Rates, 8 Countries.* Here, all series except the Can$/US$ are highly correlated with the first PC. The Can$/US$ series is highly correlated with P2, and Yen/US$ series is highly correlated with P3 (as well as with P1). This means that these data show one dominant risk factor at work, *viz.* the joint comovements of all exchange rates (except the Canadian series) against the US$; the fluctuations of the Canadian currency vis-a-vis the US$ are governed by a separate risk factor, given by P2.

(E) *Stock Market Indexes, 9 Countries.* Concentrating on the standardised-PC correlations with the observed series, it is obvious that all but one of the series (the French stock market index returns) are highly correlated with the first PC. In addition, the Canadian and US series are also highly correlated with P2. Given these findings, one can easily conclude that there is one dominant global risk factor as well as a separate "North American" risk factor.

(F) *4 US Stock Market Indexes.* All four series are highly correlated with P1; in addition, the Nasdaq Composite returns series is also somewhat correlated with P2. It seems, though, that it would suffice for many purposes to specify a single risk factor that governs the daily-frequency returns of all four indexes.

(G) *Combination of Stock Market Indexes and Exchange Rates, 17 Series.* (Here, it is definitely preferable to concentrate on the second part of panel (G) of Table 2, since the two types of series have differing levels of variance.) From the correlation numbers, P1 may be interpreted as an "exchange rate shock" and P2 as a "stock market shock." However, these first two principal components explain only 56% of the total data variability (*cf.* Table 1). Hence, a simple two-factor model may not be satisfactory for capturing a sufficiently large fraction of the variance in the data.

(H) *Combination of Stock Market Indexes, Exchange Rates, and Long Term Interest Rates, 26 Series.* Attributing economic significance to the PCs computed from the joint behaviour of all 26 series is even more difficult than in the previous case. P1 is negatively correlated with most stock market returns series; the exchange rate returns are negatively correlated with P1 but positively with P2; finally, the long term interest rates are positively correlated with both P1 and P2. These findings strongly suggest that it is not fruitful to study all 26 series jointly if the objective is reducing the dimensionality of the data.

To sum up, PCA applied to the various groupings of the data reveals that it is feasible to reduce the dimensionality of the scenario specification problem for certain groups of assets, especially for exchange rates and stock market index fluctuations. On the other hand, we also found groups of series—most notably the set of short-term interest rates—where there appears to be little scope for dimension reduction. Both the "positive" and the "negative" results are useful since they point out the types of groupings of the data for which dimension reduction is appropriate, as well as the ones for which it is not.

3.3 Stress scenarios based on principal components analysis

The preceding analysis suggests that several groupings of the data are well characterised as possessing only one or at most two "meta-dimensions." How does one specify scenarios that make use of this information? Consider first the case where a single principal component suffices to capture most of the variance of the data. Since the first PC is a one-to-one transformation of the observed data, it is possible to "reverse" the calculations and to compute the values of each of the series that correspond to given values of the first PC. Next, since the PC is a random variable we may pick tail-event quantiles of the empirical distribution of the PC to generate corresponding tail events of the observable series.

When more than one PC is required to describe a sufficient amount of the total variance in the data, one may proceed by specifying separate "shocks" in each of the directions given by the retained PCs, in analogy to the case of a single relevant PC. Alternatively, one may choose to form arbitrary linear combinations of the estimated PCs to generate "combined" shocks. Or, if the PCs are highly correlated with one of the observable series, one could simply sort the data by that series, and associate stress scenarios with particularly large realisations of that series.

Frye (1996) and Jamshidian and Zhu (1996) explain in detail how trading firms may use PCA as a basis for their risk management process. Once the "relevant dimensions" of market risk are established via PCA, scenarios are generated by taking various linear combinations of the first two or three PCs of the data.

In the remainder of this section, we report the results of specifying shock scenarios for the following four groupings of the data: spot exchange rates (8 series), the US T-Bond term structure (9 series), long-term government bond returns (9 series), and stock market indexes (9 series). For each of these datasets, four separate types of scenarios were generated. The first three are based on fluctuations in the direction specified by each of the first three PCs of the data; the fourth scenario is created by taking the direct sum of the first three scenarios. To indicate how the potential computational burden might be reduced for firms that would calculate their exposure to each of these shocks, "fluctuations" that do not exceed at least 0.5% per day or 1 basis point per day are set to zero.

For each of these four types of scenarios, the following quantiles of the resulting distributions are reported: 0.5%, 1%, 5%, 10%, 90% 95%, 99%, and 99.5%. By measuring the exposure to shocks of increasing severity-from 10% to 0.5%, and from 90% to 99.5%-it may be possible to determine if there is "curvature" in the exposure, i.e., if there is gamma risk that could lead to systemic breakdowns if these exposures are hedged by dynamic trading strategies. Note that the quantiles of the shock distributions should not be interpreted as meaning that any of these particular scenarios will occur with the specified probabilities; "real world" shocks are combinations of the shocks in the directions of the various PC-shocks. The results are listed in Table 3.

Turning first to the scenarios for the eight exchange rates (Panel A), we note that the shocks generated by fluctuations along the first PC affect mainly the European series; the second shock affects mostly the Can$/US$ exchange rate, and the third induces fluctuations in the Yen/US$ rate. The fourth shock, which is a weighted sum of the first three shocks, leads to fluctuations in all series except the Can$/US$ series

The scenarios for shocks to the US term structure, tabulated in Panel B of Table 3, show that shocks in the direction of the first PC-which was identified above as a "shift" factor-indeed lead to a shift in all rates, with the changes being largest for the longer-term bonds. The second scenario is a "tilt" of the yield curve, and the third serves to increase or decrease curvature. The numerical magnitude of the shocks, measured in basis points, may seem somewhat small. However, it should be remembered that they are "pure factor shocks," and that "actual" shocks are combinations of the "pure" shocks. To wit, the fourth scenario, which is a simple combination of the first three, does lead to fluctuations that exceed 20 basis points at either end of the distribution.

In Panel C of Table 3, various scenarios for fluctuation in long term bond rates across nine countries are presented. The first PC-shock leads to sizeable changes in all long rates except for Japan and the US; these two series are affected by PCs 2 and 3, respectively. Interestingly, a simultaneous shock to all three PCs leads to a scenario in which the Canadian and US long rates fluctuate strongly while the other series do not show much action.

Stock market shock scenarios are given in Panel D of Table 3. Here, the first PC induces shocks for all European series except France. The second shock affects US stock returns strongly, but has a smaller impact on the stock returns of Belgium, Canada, France and Germany as well. The third PC leads to large fluctuations in the series for France, and affects Japanese stock market returns as well. A combination of these three scenarios affects all stock markets except the ones for Canada and the United States.

We close this section by observing that the numerical values given in all of the scenarios confirm the qualitative interpretation of the nature of the PCs derived earlier in this paper. The numerical values presented here serve mainly to give a "flavour" of the severity of market risk scenarios that can be generated by PCA.

4. Conclusions

In this note, I have set out to discuss some of the technical issues that need to be addressed in the process of specifying scenarios that are based on data driven methods such as principal components analysis. The methodological points were illustrated empirically with a dataset that consists of daily-frequency observations on long- and short-term interest rates, stock market indexes, and exchange rates for nine industrialised countries. I find that the effective dimensionality of several subgroups of these time series is considerably smaller than the number of series included. These results would allow us to reduce the number of market risk scenarios to groups. Several methods for generating scenarios in terms of observables on the basis of the PCA-based results were discussed, and numerical values of several simple scenarios were presented.

We close by discussing some shortcomings of PCA that have not been mentioned up to this point. First, and most importantly, PCA is strongly affected by the choice of units of the series. An important consequence of this fact is that PCA will not detect risk factors that do not contribute significantly to the total variability of the data. This shortcoming could be remedied, at least in principle, by multiplying the series with appropriate portfolio weights. However, this requires knowledge of the actual asset holdings of participants in the reporting exercise.

A second shortcoming, less serious than the first, is that PCA is suitable for detecting risk factors that are linear functions of the data. Volatility factors, which are of interest for the valuation of options and of products with embedded-option characteristics, are more difficult to derive by PCA. To obtain volatility factors, it appears to be preferable to use a more model-driven approach to data analysis, say by specifying and estimating a multi-factor GARCH process. Third, by construction, the factors derived from PCA are mutually orthogonal. If the true market risk factors (assuming that there is such a thing as a "true" risk factor!) are not orthogonal, then the PCA-based factors will be linear combinations of the true factors, and it will be harder to give economic interpretations to the PCA-derived factors.

Appendix A

Technical exposition of principal components analysis

Consider a collection of T observations of N asset returns. Let X denote the resulting $T \times N$ data matrix, and assume without loss of generality that X has full column rank. (Otherwise, one or more of the returns series are redundant and may be omitted.) Our goal is to find a linear combination of the observed asset returns that "explains" as much as possible of the observed variability of the data. We will demonstrate that principal components analysis, PCA for short, achieves this objective.

The following discussion is based on Theil (1971, pp. 46–56). Let P denote the $T \times N$ matrix of the eigenvectors of XX' that correspond to the N non-zero eigenvalues (sorted in descending order) of XX'. (Since XX' is positive semi-definite, exactly N of its eigenvalues are positive and the remaining T-N are zero.) One can show that the first column of P, i.e., the first "principal component" (PC) of X, maximises the explained variance ("R^2") of the multivariate regression of X on any linear combination of the columns of X. Thus, the first PC solves the objective set out above. Similarly, the second column of P, i.e., the second PC, maximizes the explained variability in the data, given the explanation already provided by the first PC. Since the eigenvectors are mutually orthogonal, all of the principal components are uncorrelated with each other. Note that principal components are not unique up to sign, i.e., multiplying a PC by -1 has no effect on the explanatory power of the PC.

One may write $X = P\,A$, where A is the $N \times N$ matrix of "loadings" of the data on each of the principal components. This representation shows that PCA is a special form of the general statistical method of "factor analysis." In PCA, the "factors" are not directly observed, but are constructed by taking linear combinations of the data. Since each of the PCs is (in principle) a function of all N data vectors, PCA is a function of the *joint* distribution of all data points. This distinguishes it from regression analysis, which is concerned with the *conditional* distribution of the "dependent" variable(s) given observations on the "independent" variables. In PCA, one does not distinguish between dependent and independent variables.

The fraction of the data variance explained by each of the successive PCs is given by $\lambda_i / \left(\sum \lambda_i\right)$, where λ_i is the i'th (sorted) eigenvalue of XX', $i = 1,\ldots,N$. The cumulative fraction of the data variance explained by the first j PCs is given by $\left(\lambda_1 + \ldots + \lambda_j\right) / \left(\sum \lambda_i\right)$.

In empirical practice, when the data are correlated, the first few PCs tend to capture most of the variability. The leading PCs, then, can be used to represent the "meta-dimensions" in which the data fall. One could also say that the number of leading PCs, say, those that capture between 50% and 90% of the total variance, represents the effective dimensionality of the data, which will be well less than in general.

Appendix B

Nonparametric density estimation

Technical references to the field of nonparametric density estimation are Silverman (1986), Green and Silverman (1994) and Wand and Jones (1995) and the references contained in these works. The pieces cited explain both the intuition that underlies nonparametric density estimation methods as well as many of the mathematical subtleties and computational considerations that arise in this field in practice.

The key idea in nonparametric density estimation—as in other areas of nonparametric statistics—is to apply "local smoothing" techniques to obtain estimates of the probability density of the data. Local smoothing means that the estimate of the density at a point is influenced mostly by the number of observations close to that point, whereas it is little affected by the properties of the data far away from the point of interest. Generally, the local smoothing estimators are so-called "kernel methods." In all kernel methods, the crucial parameter is the "bandwidth." The bandwidth parameter determines the size of the region (around the point of interest) which is used to perform the smoothing operation.

The bivariate density estimates reported in the paper were computed using a two-dimensional Gaussian kernel and a (scalar) bandwidth chosen as $\sigma N^{-0.2}$, where σ is the average standard deviation of both series. The estimation routines were coded in the "Gauss" programming language by the author.

Table 1

Fractions of variance explained by successive principal components

Note: There are two lines for each group of series. Line 1 applies to the raw returns series, the second for the standardised returns series. Numbers greater than 1/N are italicised, numbers greater than 2/N are underlined, where N is the number of series included in the group.

(A) Short Term Interest Rates (9 countries)

0.355	0.232	*0.154*	0.090	0.069	0.046	0.032	0.014	0.008
0.202	*0.179*	*0.112*	0.102	0.097	0.093	0.080	0.074	0.060

(B) Long Term Government Bond Interest Rates (9 countries)

0.494	*0.159*	*0.094*	*0.084*	0.069	0.033	0.029	0.026	0.011
0.480	*0.122*	*0.100*	*0.087*	0.063	0.061	0.037	0.034	0.016

(C) 9-Point US Term Structure

0.843	0.093	0.028	0.011	0.007	0.007	0.005	0.004	0.003
0.810	*0.121*	0.031	0.013	0.009	0.006	0.004	0.003	0.003

(D) Spot Exchange Rates (8 countries)

0.812	0.084	0.043	0.022	0.022	0.010	0.007	0.001
0.716	*0.130*	0.079	0.042	0.017	0.008	0.006	0.001

(E) Stock Market Indexes (9 countries)

0.395	*0.192*	*0.164*	0.076	0.052	0.043	0.037	0.023	0.019
0.409	*0.130*	*0.113*	0.090	0.071	0.060	0.050	0.043	0.033

(F) US Stock Market Indexes (4 series)

0.868	0.107	0.018	0.007
0.883	0.090	0.019	0.008

(G) 9 Stock Market Indexes & 8 Exchange Rates

0.298	0.233	0.129	*0.113*	0.053	0.035	0.030	0.026	0.024
0.358	0.202	*0.069*	*0.064*	0.057	0.047	0.038	0.035	0.031

(H) 9 Stock Market Indexes, 8 Exchange Rates, & 9 Long Term Rates

0.500	0.147	0.087	*0.076*	*0.064*	0.032	0.027	0.025	0.010	0.009
0.257	0.213	*0.075*	*0.056*	*0.044*	*0.039*	0.034	0.032	0.028	0.027

Table 2

Correlations of the data series with the first four principal components, for various data groupings

Note: Two sets of correlations are reported for each group of returns, (i) between the data and the "raw-data PCs" and (ii) between the data and "standardised-data PCs." Correlations greater than 0.45 in absolute value are underlined.

(A) Short Term Interest Rates (9 countries)

Correlation between data and raw-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	<u>-0.580</u>	<u>0.813</u>	-0.008	0.031
CA	-0.184	-0.081	<u>0.978</u>	-0.001
FR	<u>-0.899</u>	-0.426	-0.099	0.039
GE	-0.218	0.070	0.043	-0.193
JA	0.021	0.021	0.042	-0.047
NE	-0.174	0.109	0.102	-0.182
SZ	-0.079	0.039	0.033	-0.278
UK	-0.130	-0.007	-0.044	<u>-0.958</u>
US	0.005	0.047	0.036	-0.080

Correlation between data and standardised-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	0.433	0.403	-0.163	-0.077
CA	0.224	0.220	-0.274	<u>0.742</u>
FR	0.390	<u>0.624</u>	-0.143	0.024
GE	<u>0.728</u>	-0.077	0.227	-0.058
JA	0.114	<u>-0.488</u>	-0.332	0.319
NE	<u>0.709</u>	-0.193	0.164	0.077
SZ	<u>0.537</u>	-0.431	0.114	-0.065
UK	0.269	0.059	-0.394	<u>-0.576</u>
US	0.014	-0.228	<u>-0.766</u>	-0.107

Table 2 (cont.)

(B) Long Term Government Bond Interest Rates (9 Countries)

Correlation between data and raw-data PCs

Index	**P1**	**P2**	**P3**	**P4**
BE	0.777	-0.247	0.290	-0.015
CA	0.695	0.655	0.006	0.293
FR	0.814	-0.218	0.279	-0.053
GE	0.829	-0.215	0.285	-0.060
JA	0.315	-0.031	0.185	-0.309
NE	0.811	-0.227	0.289	-0.038
SZ	0.407	-0.100	0.184	0.015
UK	0.777	-0.325	-0.532	0.065
US	0.416	0.503	-0.190	-0.708

Correlation between data and standardised-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	0.830	-0.180	-0.035	0.097
CA	0.579	0.525	-0.158	-0.094
FR	0.849	-0.110	-0.009	0.125
GE	0.894	-0.135	-0.055	0.094
JA	0.349	0.156	0.923	-0.011
NE	0.879	-0.163	-0.055	0.080
SZ	0.503	-0.239	-0.017	-0.819
UK	0.704	-0.036	-0.067	0.235
US	0.365	0.804	-0.113	-0.099

Table 2 (cont.)

(C) 9-Point US Term Structure

Correlation between data and raw-data PCs

Maturity	**P1**	**P2**	**P3**	**P4**
m03	0.624	-0.676	0.334	0.164
m06	0.807	-0.518	0.117	-0.119
y01	0.911	-0.294	-0.146	-0.198
y02	0.956	-0.109	-0.208	0.046
y03	0.975	0.005	-0.143	0.074
y05	0.979	0.120	-0.042	0.067
y07	0.960	0.223	0.070	0.041
y10	0.942	0.276	0.128	-0.004
y30	0.875	0.359	0.243	-0.134

Correlation between data and standardised-data PCs

Maturity	**P1**	**P2**	**P3**	**P4**
m03	0.664	-0.677	0.269	0.159
m06	0.836	-0.482	0.025	-0.208
y01	0.922	-0.224	-0.212	-0.109
y02	0.954	-0.039	-0.225	0.075
y03	0.968	0.067	-0.154	0.086
y05	0.969	0.170	-0.048	0.075
y07	0.949	0.261	0.067	0.042
y10	0.931	0.308	0.129	0.003
y30	0.865	0.385	0.258	-0.117

Table 2 (cont.)

(D) Spot Exchange Rates (8 countries)

Correlation between data and raw-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	-0.956	0.074	-0.106	0.174
CA	0.009	0.188	0.191	-0.028
FR	-0.969	0.085	-0.054	0.100
GE	-0.985	0.062	-0.080	0.037
JA	-0.608	-0.790	0.070	0.017
NE	-0.981	0.065	-0.086	0.051
SZ	-0.947	0.038	-0.087	-0.301
UK	0.822	-0.148	-0.547	0.003

Correlation between data and standardised-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	-0.955	0.023	0.080	0.126
CA	0.011	0.971	-0.232	0.050
FR	-0.969	0.030	0.091	0.075
GE	-0.983	0.006	0.076	0.094
JA	-0.617	-0.278	-0.735	-0.047
NE	-0.979	0.009	0.077	0.101
SZ	-0.941	0.008	0.046	0.091
UK	0.829	-0.130	-0.111	0.532

Table 2 (cont.)

(E) Stock Market Indexes (9 countries)

Correlation between data and raw-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	0.587	0.229	-0.087	0.151
CA	0.436	0.129	-0.110	-0.569
FR	-0.012	-0.654	-0.758	0.012
GE	0.718	0.387	-0.316	0.338
JA	0.806	-0.479	0.346	0.017
NE	0.688	0.368	-0.228	-0.015
SZ	0.667	0.354	-0.241	0.053
UK	0.613	0.269	-0.232	-0.313
US	0.400	0.164	-0.155	-0.734

Correlation between data and standardised-data PCs

Country	**P1**	**P2**	**P3**	**P4**
BE	0.661	-0.303	0.017	0.189
CA	0.577	0.636	-0.139	0.070
FR	-0.060	0.241	0.930	-0.242
GE	0.766	-0.285	0.068	-0.138
JA	0.496	-0.020	0.313	0.763
NE	0.824	-0.201	-0.016	-0.188
SZ	0.759	-0.179	0.002	-0.162
UK	0.729	0.017	0.021	-0.224
US	0.538	0.680	-0.159	-0.023

Table 2 (cont.)

(F) US Stock Market Indexes (4 series)

Correlation between data and raw-data PCs

Index	**P1**	**P2**	**P3**	**P4**
djia30	0.925	0.329	0.191	0.017
nasdaqc	0.890	-0.454	0.033	0.025
sp500	0.954	0.225	-0.179	0.086
wilt5000	0.982	0.065	-0.080	-0.161

Correlation between data and standardised-data PCs

Index	**P1**	**P2**	**P3**	**P4**
djia30	0.939	0.280	0.198	0.018
nasdaqc	0.862	-0.502	0.053	0.041
sp500	0.966	0.170	-0.167	0.098
wilt5000	0.986	0.006	-0.072	-0.149

Table 2 (cont.)

(G) 9 Stock Market Indexes & 8 Exchange Rates

Correlations between data and raw-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.520	-0.260	-0.236	0.112
CA	-0.372	-0.219	-0.137	0.123
FR	0.068	-0.118	0.693	0.709
GE	-0.628	-0.327	-0.386	0.352
JA	-0.609	-0.574	0.404	-0.368
NE	-0.713	-0.133	-0.308	0.239
SZ	-0.632	-0.224	-0.330	0.262
UK	-0.598	-0.180	-0.233	0.239
US	-0.325	-0.232	-0.183	0.174
Exchange Rate				
BE	-0.594	0.741	0.144	-0.024
CA	0.054	0.013	-0.011	-0.071
FR	-0.598	0.744	0.157	-0.056
GE	-0.617	0.753	0.146	-0.048
JA	-0.353	0.503	0.130	-0.006
NE	-0.614	0.750	0.148	-0.051
SZ	-0.650	0.685	0.113	-0.021
UK	0.485	-0.660	-0.103	0.036

Table 2 (cont.)

Correlations between data and standardised-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.241	-0.616	0.326	0.047
CA	-0.168	-0.563	-0.604	-0.213
FR	0.080	0.026	-0.289	0.553
GE	-0.262	-0.718	0.285	0.139
JA	-0.123	-0.491	0.046	0.130
NE	-0.466	-0.690	0.176	0.052
SZ	-0.336	-0.679	0.176	0.030
UK	-0.348	-0.640	-0.018	-0.026
US	-0.117	-0.545	-0.626	-0.266
Exchange Rate				
BE	-0.931	0.230	-0.015	-0.013
CA	0.043	0.074	0.254	-0.737
FR	-0.938	0.242	-0.023	-0.025
GE	-0.956	0.233	-0.014	-0.004
JA	-0.601	0.205	-0.098	0.240
NE	-0.952	0.234	-0.017	-0.008
SZ	-0.935	0.146	-0.014	-0.006
UK	0.800	-0.233	-0.009	0.126

Table 2 (cont.)

(H) 9 Stock Market Indexes, 8 Exchange Rates, & 9 Long Term Rates

Correlations between data and raw-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.380	0.071	-0.130	-0.023
CA	-0.306	-0.149	0.016	0.129
FR	0.044	-0.012	-0.025	0.026
GE	<u>-0.477</u>	0.140	-0.159	-0.047
JA	-0.185	0.006	-0.022	0.016
NE	-0.436	0.066	-0.030	-0.023
SZ	0.345	0.011	-0.052	0.002
UK	-0.411	0.076	0.086	0.047
US	-0.264	-0.166	0.055	0.309
Exchange Rate				
BE	-0.031	0.105	0.035	-0.100
CA	0.132	0.140	-0.008	0.106
FR	0.002	0.102	0.047	-0.100
GE	-0.029	0.107	0.043	-0.100
JA	0.026	0.023	0.055	-0.122
NE	-0.027	0.106	0.048	-0.101
SZ	-0.048	0.090	0.049	-0.094
UK	-0.016	-0.101	0.036	0.044
Long Term Rate				
BE	<u>0.783</u>	-0.248	0.292	0.027
CA	<u>0.710</u>	<u>-0.642</u>	-0.000	0.276
FR	<u>0.828</u>	-0.242	0.244	-0.018
GE	<u>0.833</u>	-0.244	0.272	-0.000
JA	0.349	-0.016	0.182	-0.414
NE	<u>0.821</u>	-0.250	0.277	0.017
SZ	<u>0.450</u>	-0.116	0.161	0.047
UK	<u>0.787</u>	-0.283	<u>-0.546</u>	0.015
US	<u>0.464</u>	<u>0.492</u>	-0.090	-<u>0.669</u>

Table 2 (cont.)

Correlations between data and standardised-data PCs

Stock Market Index	**P1**	**P2**	**P3**	**P4**
BE	-0.512	-0.275	0.230	0.336
CA	-0.385	-0.259	0.475	-0.386
FR	0.082	-0.027	0.027	-0.066
GE	-0.602	-0.351	0.223	0.363
JA	-0.287	-0.197	0.387	0.217
NE	-0.707	-0.164	0.344	0.240
SZ	-0.560	-0.206	0.420	0.252
UK	-0.591	-0.208	0.356	0.098
US	-0.326	-0.266	0.522	-0.464
Exchange Rate				
BE	-0.615	0.730	-0.070	-0.048
CA	0.110	0.059	0.036	0.245
FR	-0.602	0.756	-0.050	-0.050
GE	-0.630	0.750	-0.066	-0.051
JA	-0.357	0.524	-0.043	-0.101
NE	-0.627	0.749	-0.064	-0.053
SZ	-0.643	0.693	0.004	-0.030
UK	0.508	-0.654	0.075	0.029
Long Term Rate				
BE	0.606	0.473	0.341	-0.053
CA	0.407	0.412	0.091	0.378
FR	0.637	0.478	0.333	-0.043
GE	0.622	0.514	0.408	-0.020
JA	0.178	0.345	0.079	0.070
NE	0.611	0.508	0.418	-0.027
SZ	0.358	0.291	0.315	0.040
UK	0.539	0.391	0.254	0.079
US	0.265	0.349	-0.134	0.643

Table 3

Market risk scenarios generated by PC shocks

A. Exchange Rate "Shock Scenarios"
(measured in percent per day; values less than 0.5% are suppressed)

Shock in direction of first PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-2.25	-	-2.12	-2.28	-1.34	-2.28	-2.43	1.76
1%	-1.92	-	-1.80	-1.94	-1.15	-1.94	-2.07	1.50
5%	-1.09	-	-1.03	-1.11	-0.66	-1.10	-1.18	0.85
10%	-0.79	-	-0.74	-0.80	-	-0.80	-0.85	0.61
90%	0.79	-	0.74	0.80	-	0.80	0.84	-0.63
95%	1.10	-	1.03	1.11	0.64	1.11	1.18	-0.87
99%	1.87	-	1.76	1.90	1.10	1.89	2.01	-1.48
99.5%	2.17	-	2.04	2.20	1.27	2.19	2.32	-1.71

Shock in direction of second PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-	-0.76	-	-	-	-	-	-
1%	-	-0.70	-	-	-	-	-	-
5%	-	-	-	-	-	-	-	-
10%	-	-	-	-	-	-	-	-
90%	-	-	-	-	-	-	-	-
95%	-	-	-	-	-	-	-	-
99%	-	0.82	-	-	-0.55	-	-	-
99.5%	-	0.93	-	-	-0.63	-	-	-

Table 3 (cont.)

Shock in direction of third PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-	-	-	-	1.30	-	-	-
1%	-	-	-	-	1.17	-	-	-
5%	-	-	-	-	0.76	-	-	-
10%	-	-	-	-	0.54	-	-	-
90%	-	-	-	-	-0.58	-	-	-
95%	-	-	-	-	-0.79	-	-	-
99%	-	-	-	-	-1.44	-	-	-
99.5%	-	-	-	-	-1.67	-	-	-

Simultaneous positive shock to first three PCs:
(sorted by value of GE column)

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK
0.5%	-2.18	-	-2.05	-2.22	-1.65	-2.21	-2.37	1.68
1%	-1.90	0.57	-1.80	-1.93	-	-1.93	-1.99	1.48
5%	-1.12	-	-1.06	-1.13	-0.69	-1.13	-1.21	0.92
10%	-0.80	-	-0.76	-0.80	-	-0.80	-0.84	0.65
90%	0.79	-	0.75	0.82	-	0.81	0.84	-0.57
95%	1.09	-	1.03	1.11	-	1.10	1.13	-0.87
99%	1.88	-	1.76	1.90	1.58	1.89	2.04	-1.55
99.5%	2.18	-	2.05	2.22	1.67	2.21	2.36	-1.71

Table 3 (cont.)

B. US Term Structure "Shock Scenarios"
(measured in basis points; values less than 1 bp are suppressed)

Shock in direction of first PC:

Quantile	**m03**	**m06**	**y01**	**y02**	**y03**	**y05**	**y07**	**y10**	**y30**
0.5%	-11	-14	-18	-21	-22	-22	-21	-20	-16
1%	-8	-11	-14	-16	-17	-17	-16	-15	-12
5%	-5	-6	-8	-9	-9	-9	-9	-8	-7
10%	-4	-5	-6	-7	-7	-7	-7	-6	-5
90%	3	4	6	7	7	7	7	6	5
95%	5	6	8	9	10	10	9	9	7
99%	8	11	14	16	17	17	16	15	12
99.5%	10	13	17	20	21	21	20	18	15

Shock in direction of second PC:

Quantile	**m03**	**m06**	**y01**	**y02**	**y03**	**y05**	**y07**	**y10**	**y30**
0.5%	-11	-9	-5	-	1	4	6	7	7
1%	-9	-7	-4	-	1	3	4	5	6
5%	-5	-4	-2	-	-	2	2	3	3
10%	-4	-3	-1	-	-	1	2	2	2
90%	3	2	1	-	-	-1	-2	-2	-2
95%	5	3	2	-	-	-2	-3	-3	-3
99%	8	6	3	-	-1	-3	-4	-5	-5
99.5%	10	7	4	-	-1	-4	-5	-6	-6

Table 3 (cont.)

Shock in direction of third PC:

Quantile	m03	m06	y01	y02	y03	y05	y07	y10	y30
0.5%	-4	-	3	4	3	-	-1	-2	-4
1%	-3	-	3	3	2	-	-1	-2	-3
5%	-2	-	2	2	2	-	-	-1	-2
10%	-2	-	1	2	1	-	-	-1	-2
90%	1	-	-2	-2	-1	-	-	-	2
95%	2	-	-2	-2	-2	-	-	1	2
99%	3	-	-3	-4	-3	-	-	2	3
99.5%	4	-	-4	-4	-3	-1	1	2	4

Simultaneous positive shock to first three PCs:
(sorted by value of 30yr column)

Quantile	m03	m06	y01	y02	y03	y05	y07	y10	y30
0.5%	-15	-17	-19	-21	-21	-21	-20	-19	-15
1%	-8	-11	-14	-17	-18	-18	-18	-17	-14
5%	-4	-5	-6	-8	-9	-9	-10	-9	-8
10%	-3	-4	-5	-6	-6	-7	-7	-7	-6
90%	1	4	6	8	9	9	8	8	6
95%	2	3	3	5	6	8	8	9	8
99%	8	12	17	20	20	20	18	17	14
99.5%	11	17	25	29	29	27	25	22	17

Table 3 (cont.)

C. Long Term Interest Rate "Shock Scenarios"
(measured in basis points; values less than 1 bp are suppressed)

Shock in direction of first PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-13	-14	-15	-13	-5	-13	-5	-17	-6
1%	-12	-13	-14	-12	-5	-12	-4	-15	-6
5%	-7	-8	-8	-7	-3	-7	-3	-9	-4
10%	-5	-5	-6	-5	-2	-5	-2	-6	-3
90%	5	5	5	5	2	5	2	6	2
95%	7	7	8	7	3	7	2	9	3
99%	13	14	15	13	5	13	5	16	6
99.5%	20	21	22	19	7	19	7	25	10

Shock in direction of second PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-4	15	-3	-3	3	-3	-3	-1	17
1%	-3	12	-2	-2	2	-2	-2	-	13
5%	-2	7	-1	-1	1	-2	-1	-	8
10%	-1	5	-1	-	-	-1	-1	-	6
90%	-	-5	-	-	-1	-	-	-	-6
95%	1	-7	-	1	-1	1	1	-	-8
99%	2	-11	1	2	-2	2	2	-	-13
99.5%	3	-12	2	2	-2	2	2	-	-14

Table 3 (cont.)

Shock in direction of third PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-	-5	-	-1	16	-1	-	-2	-2
1%	-	-4	-	-	13	-	-	-2	-2
5%	-	-2	-	-	7	-	-	-1	-1
10%	-	-2	-	-	5	-	-	-	-
90%	-	1	-	-	-6	-	-	-	-
95%	-	2	-	-	-8	-	-	-	1
99%	-	3	-	-	-13	-	-	1	2
99.5%	-	4	-	-	-15	-	-	1	2

Simultaneous positive shock to first three PCs:
(sorted by value of US column)

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	3	-16	2	2	-3	3	3	-	-17
1%	-6	-17	-7	-6	-3	-6	-1	-9	-14
5%	-	-10	-1	-1	5	-	-	-3	-9
10%	-1	-8	-2	-2	3	-2	-	-3	-6
90%	4	8	5	4	7	4	1	6	6
95%	-	8	-	-	7	-	-	1	9
99%	3	16	5	4	13	4	-	6	15
99.5%	17	27	21	17	20	17	5	23	18

Table 3 (cont.)

D. Stock Market Shocks
(measured in percent per day; values less than 0.5% are suppressed)

Shock in direction of first PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-1.82	-1.33	-	-3.37	-3.10	-2.32	-2.64	-2.43	-1.59
1%	-1.34	-0.98	-	-2.50	-2.31	-1.71	-1.94	-1.79	-1.17
5%	-0.75	-0.55	-	-1.41	-1.33	-0.95	-1.08	-1.01	-0.64
10%	-0.52	-	-	-0.97	-0.94	-0.65	-0.73	-0.69	-
90%	0.56	-	-	1.00	0.83	0.73	0.83	0.74	0.52
95%	0.75	0.55	-	1.35	1.15	0.97	1.11	1.00	0.69
99%	1.32	0.97	-	2.41	2.10	1.71	1.95	1.76	1.20
99.5%	1.53	1.13	-	2.81	2.45	1.98	2.26	2.05	1.39

Shock in direction of second PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-0.66	1.23	0.99	-0.99	-	-	-	-	1.72
1%	-0.58	1.09	0.88	-0.87	-	-	-	-	1.52
5%	-	0.67	0.55	-0.52	-	-	-	-	0.94
10%	-	0.51	-	-	-	-	-	-	0.72
90%	-	-	-	-	-	-	-	-	-0.62
95%	-	-0.62	-	0.57	-	-	-	-	-0.85
99%	0.57	-0.94	-0.71	0.84	-	-	-	-	-1.29
99.5%	0.63	-1.05	-0.80	0.93	-	-	0.51	-	-1.43

Table 3 (cont.)

Shock in direction of third PC:

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-	-	-4.11	-	-1.81	-	-	-	-
1%	-	-	-3.01	-	-1.34	-	-	-	-
5%	-	-	-1.78	-	-0.82	-	-	-	-
10%	-	-	-1.35	-	-0.64	-	-	-	-
90%	-	-	1.45	-	0.56	-	-	-	-
95%	-	-	1.95	-	0.77	-	-	-	-
99%	-	-	3.16	-	1.28	-	-	-	-
99.5%	-	-	3.61	-	1.47	-	-	-	-

Simultaneous positive shock to first three PCs:
(sorted by value of GE column)

Quantile	BE	CA	FR	GE	JA	NE	SZ	UK	US
0.5%	-1.83	-1.19	-2.00	-3.51	-4.08	-2.28	-2.63	-2.46	-1.37
1%	-1.62	-	1.27	-2.82	-1.87	-1.77	-2.00	-1.46	-
5%	-0.87	-	0.88	-1.52	-1.00	-0.96	-1.08	-0.81	-
10%	-0.53	-	-	-1.01	-1.12	-0.65	-0.75	-0.71	-
90%	0.59	-	0.66	1.05	1.01	0.65	0.75	0.56	-
95%	0.93	-0.51	-2.61	1.42	-	0.93	1.03	-	-0.72
99%	1.28	1.16	2.03	2.48	3.07	1.70	1.97	1.92	1.44
99.5%	1.85	-	-2.28	3.13	1.57	2.08	2.34	1.66	-

Figure 1
Nonparametrically Estimated Bivariate Density
Switzerland/US Exchange Rate vs. German Stockmarket

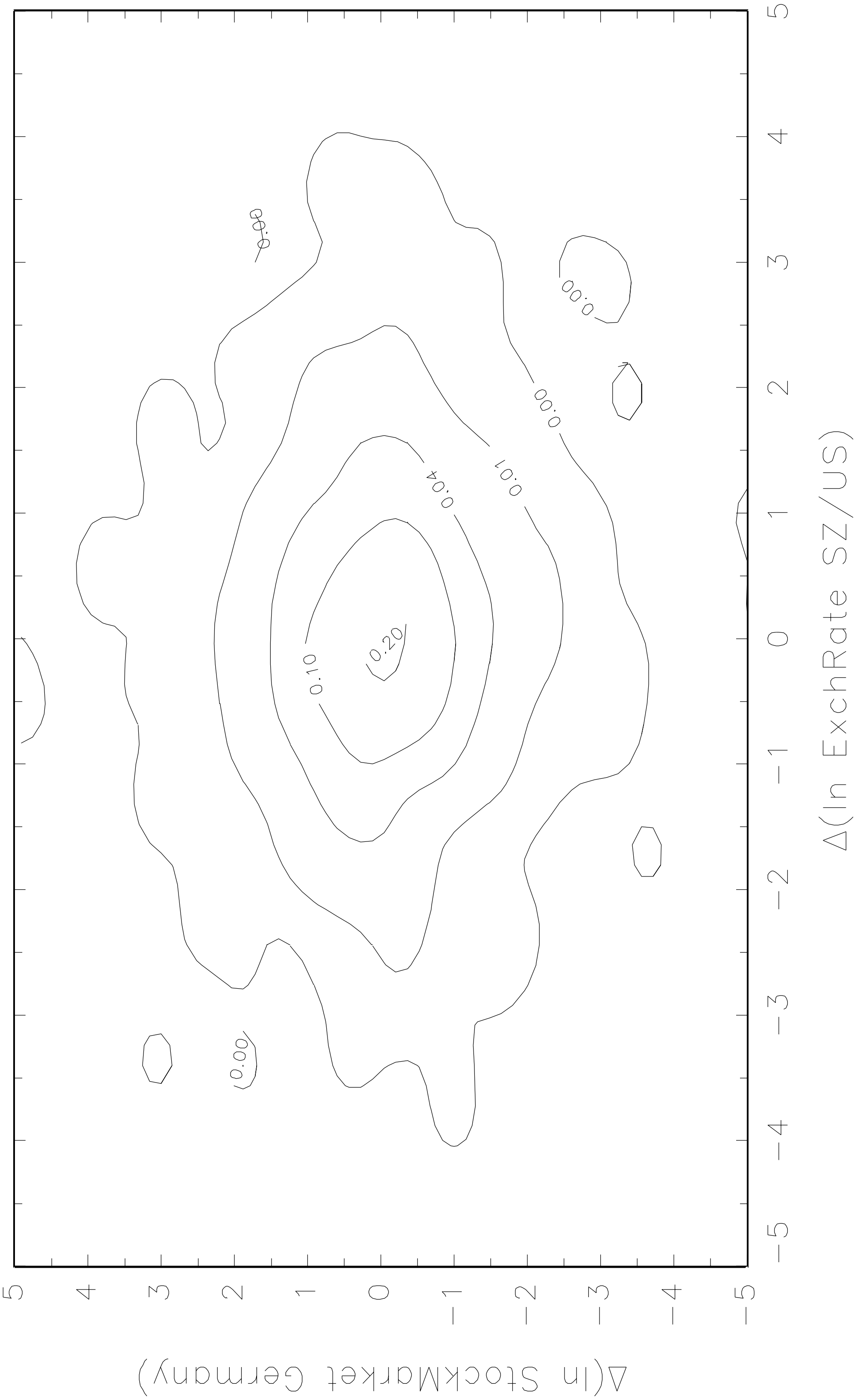
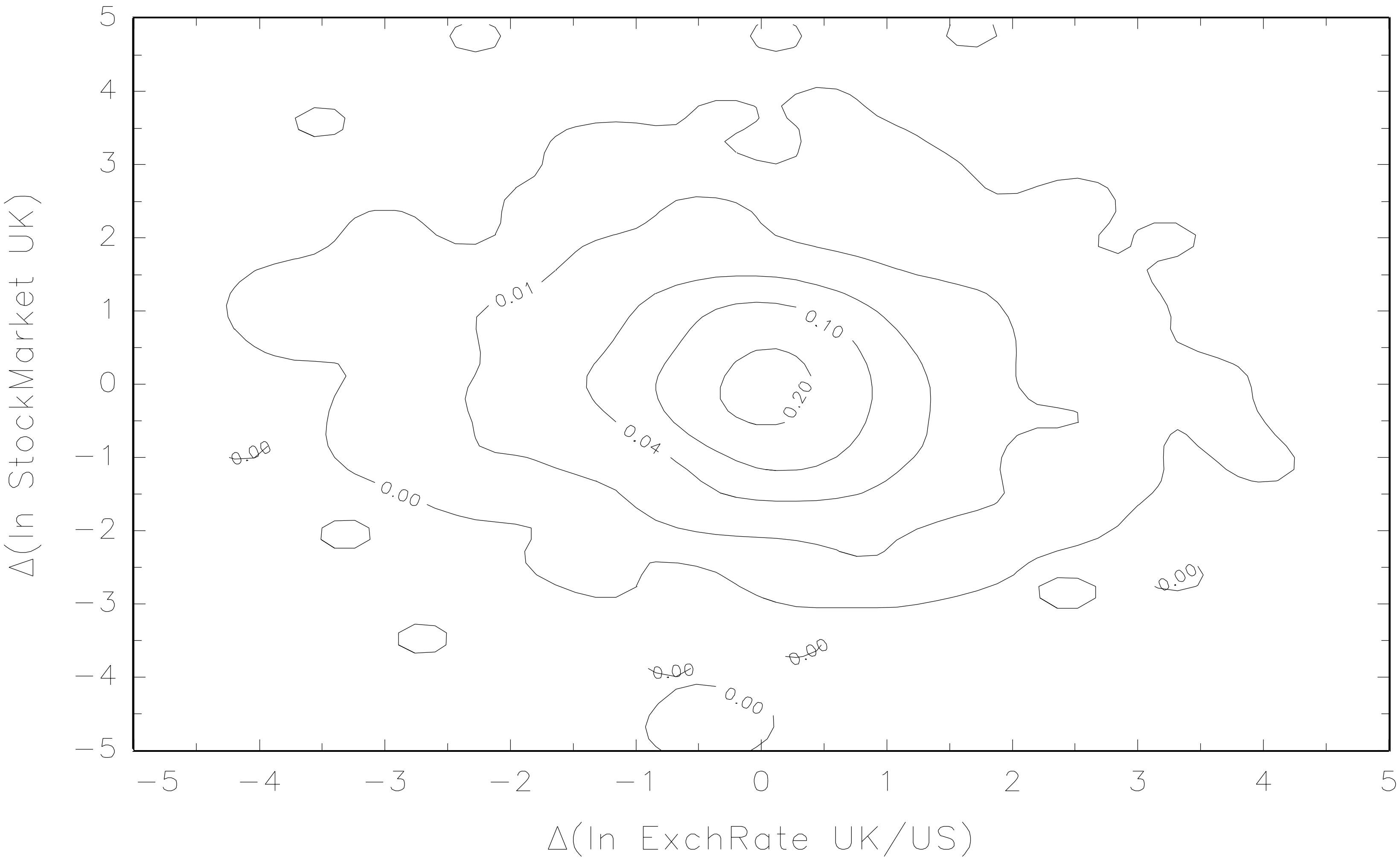


Figure 2
Nonparametrically Estimated Bivariate Density
UK/US Exchange Rate vs. British Stockmarket



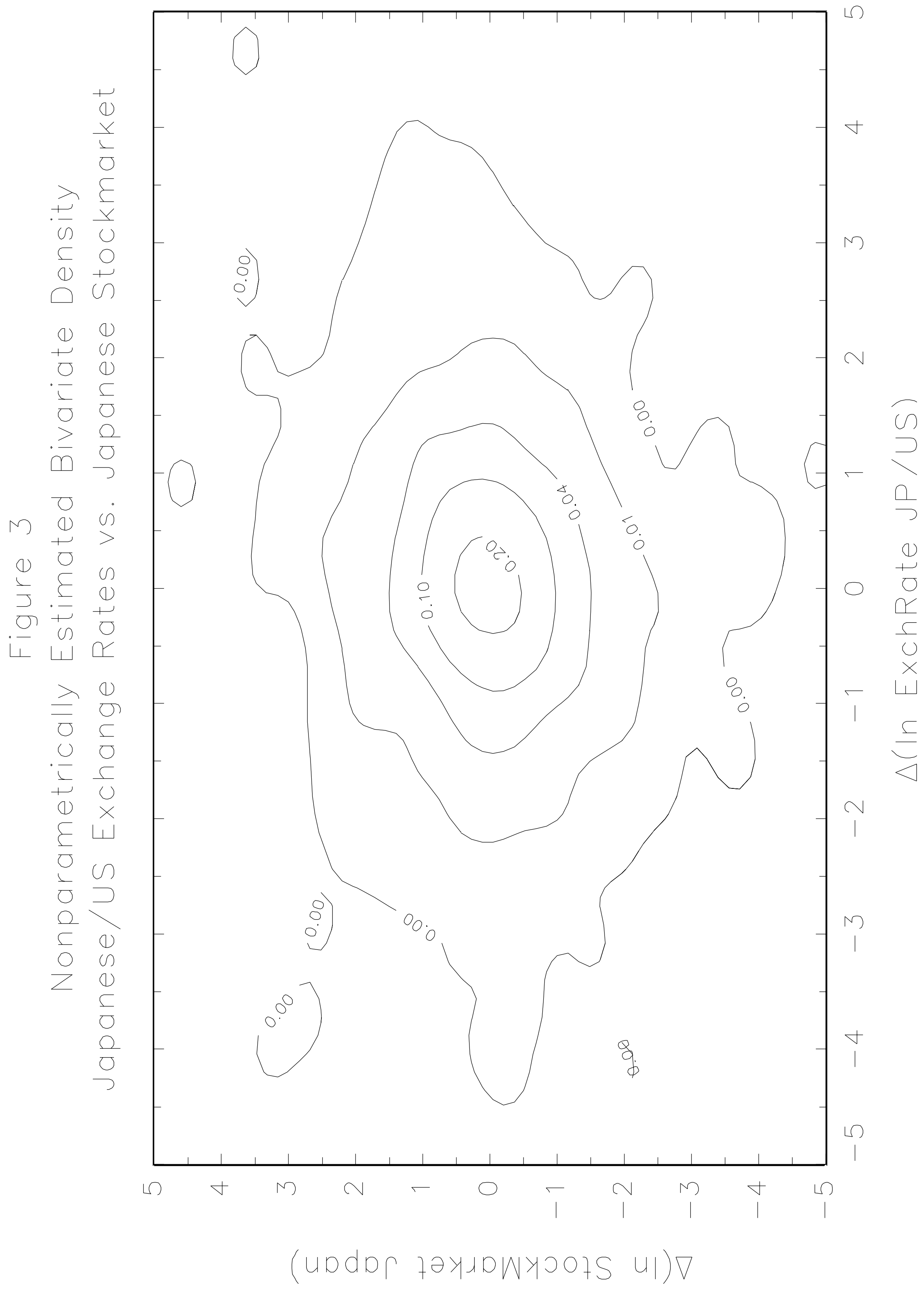


Figure 3
Nonparametrically Estimated Bivariate Density
Japanese/US Exchange Rates vs. Japanese Stockmarket
Δ(ln ExchRate JP/US)
Δ(ln StockMarket Japan)
0.00
0.01
0.04
0.10
0.20

Figure 4

Nonparametrically Estimated Bivariate Density

Changes in 1-year and 30-year T-Bond Interest Rates

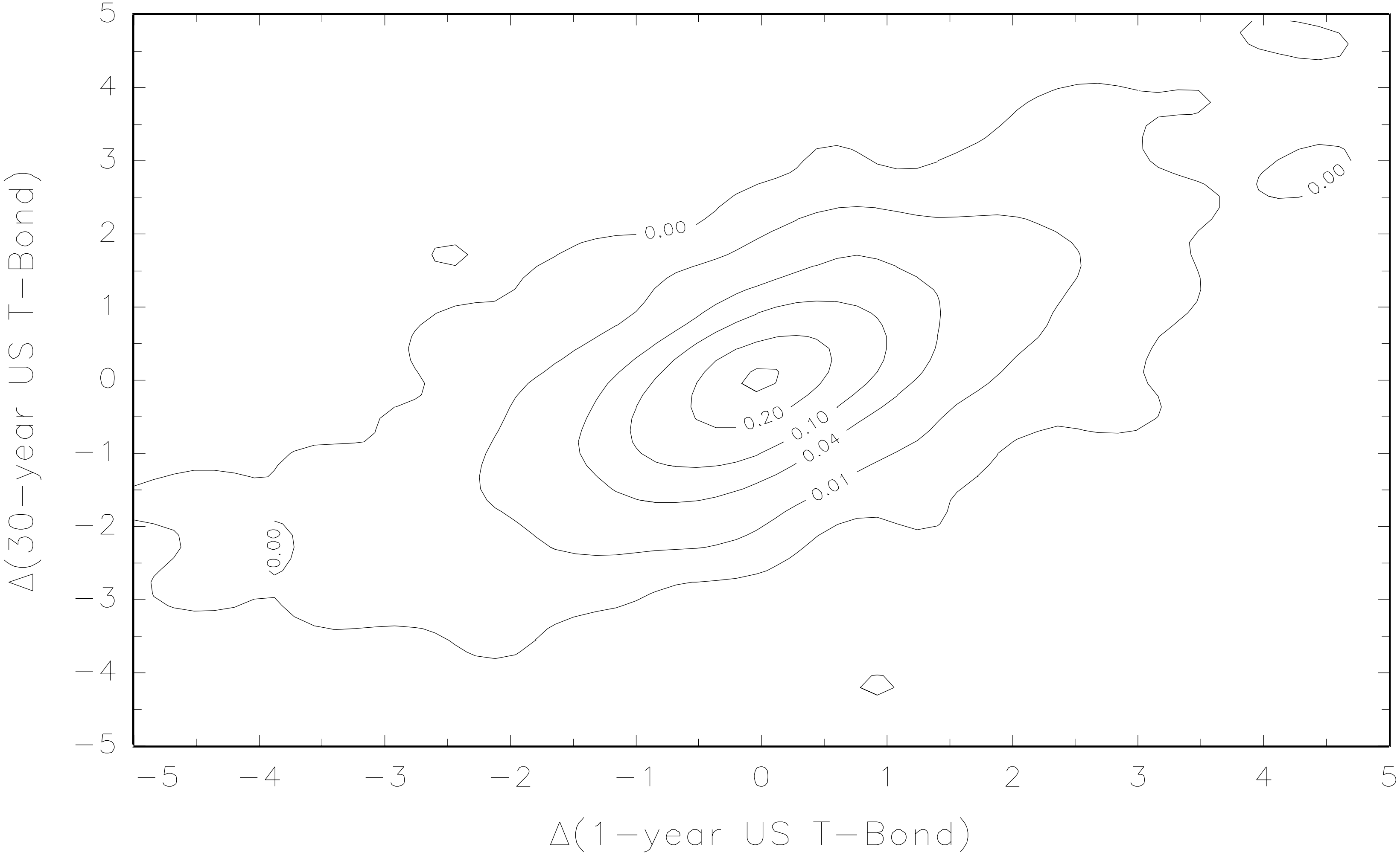


Figure 5
Nonparametrically Estimated Bivariate Density
Returns on Nasdaq-Composite and S&P-500

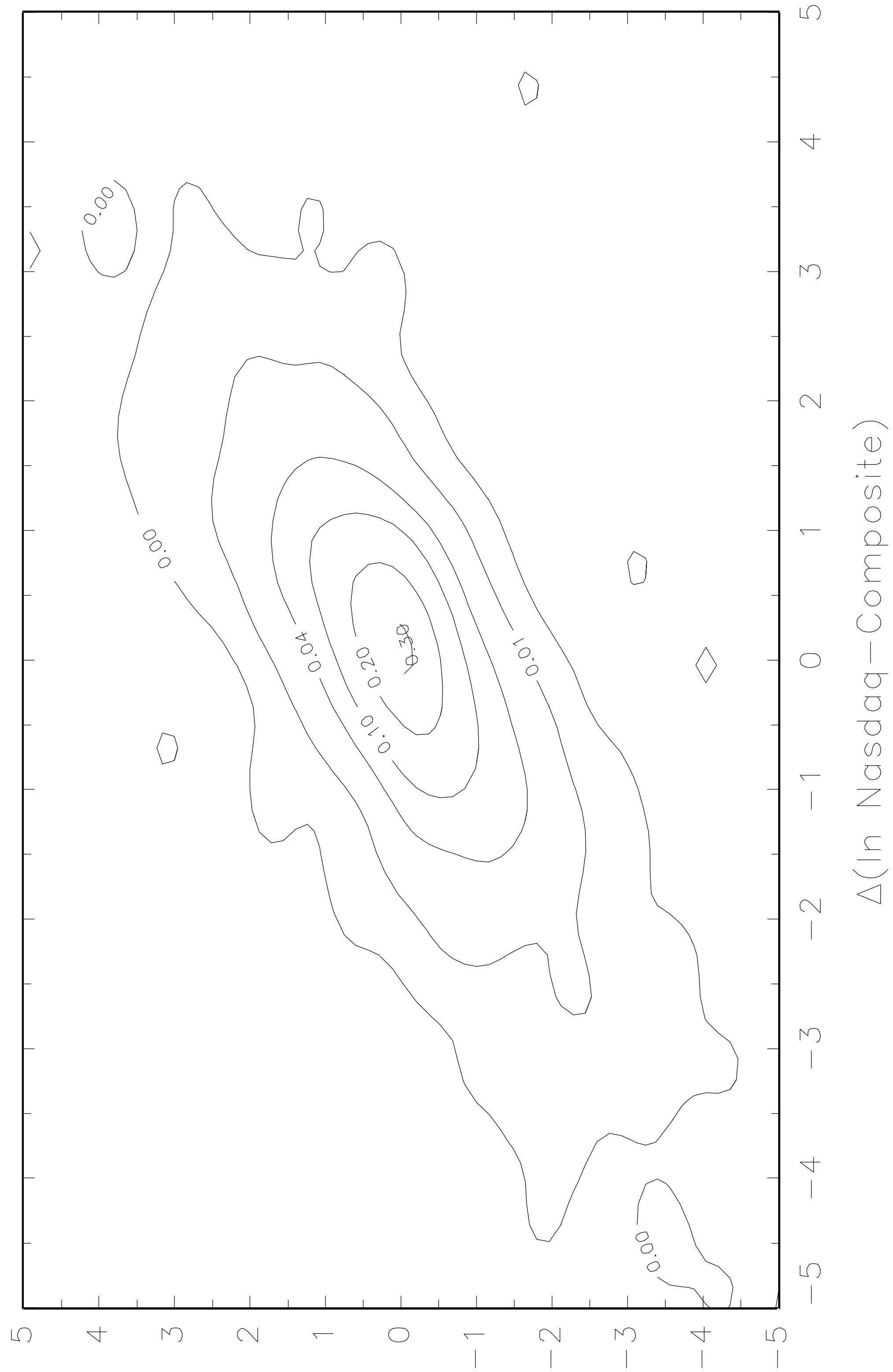
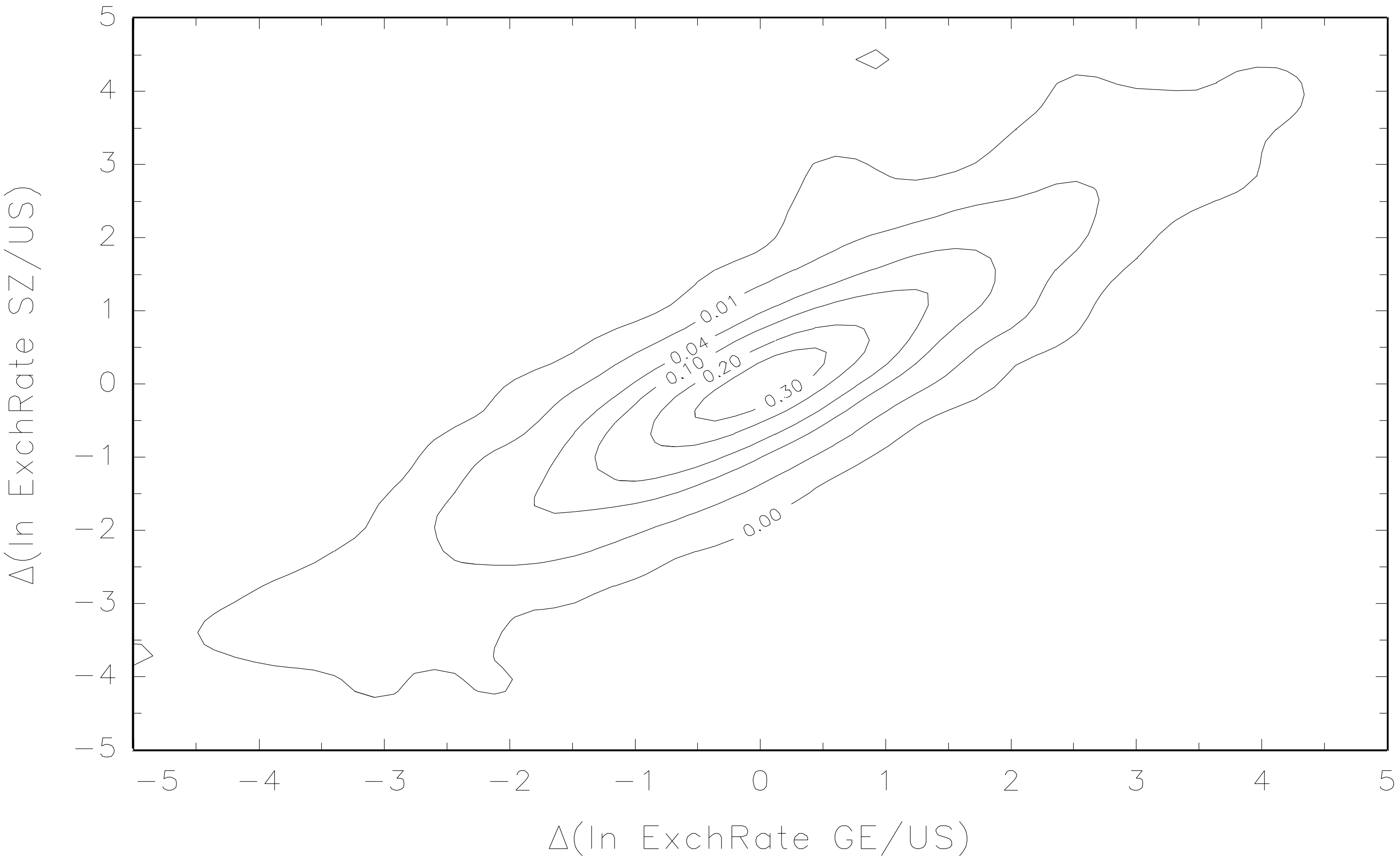


Figure 6
Nonparametrically Estimated Bivariate Density
German and Swiss Exchange Rates



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Term structure and volatility shocks

by

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Abstract

This paper presents methodologies for generating scenarios for term structure and volatility shocks from historical data. These methodologies are selected to provide good approximations to data and to be easy to replicate. The approaches are based on principal components estimated from return data.

Government bond returns from seven countries are used to illustrate the methods. The country level data suggest that three to four principal components are sufficient to capture most variation in individual country term structures. The first principal components for bond returns are somewhat correlated across countries; there is less evidence of correlation for other components. Cross-country evidence suggests that correlations between components changed from 1990-93 to 1994-96. Generalised autoregressive conditional heteroskedasticity (GARCH) variance models for these principal components are estimated for each country. In several country models asymmetric responses of volatility to return surprises are detected.

The evidence suggests that variation in particular country term structures can be well described by relatively few common components. However, considerably more components are required to jointly describe movements in several country term structures, in part because returns of short maturity government securities were not highly correlated across the sample countries in 1990-1996.

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1. Introduction

Why measure exposure to asset prices? Management of financial firms must know their firm's exposure to asset price changes both to understand the riskiness of the firm's business and to appreciate which price changes would generate large losses at the firm. In order to understand the exposure of a financial firm, it is necessary to identify asset price movements that could cause large changes in market value of the firm or in the value of the firm's trading book.

Measures of aggregate exposure over a set of major market-making firms could identify events that would cause large losses at many firms. Aggregated exposure might also identify asset price movements that could generate unusual market dynamics by, for example, inducing additional hedging-related transactions that could exacerbate the initial price movement.

Whether at the firm level or aggregated over many firms, exposure could be measured in at least one of three ways: The first is historical, using large historical movements in asset prices to define scenarios. In this method, firms would price their books of actively traded instruments at each scenario. The change in value of their books would measure exposure to each scenario's shocks. The second approach to measuring exposure is based on firm sensitivities to asset price changes. Here firms estimate the sensitivities (defined as the derivatives) of the value of their books to changes in a set of prices. The third approach begins with the development of hypothetical future events that might create large losses. Plausible asset price responses to each event are constructed with the aid of market experts and used to define the scenario associated with the event. The implied change in value of firms' books defines the possible exposure generated by the event.

Each of these three approaches can generate scenarios that help evaluate how the market value of firms might change in response to asset price shocks. The first approach may provide a better measure of responses to large price changes than the second approach which probably better describes the impact of small asset price changes. The value of the third approach depends critically on the selection of the particular stressful events and the definition of their asset market consequences.

The two sections below outline methodologies for generating scenarios for term structure and volatility shocks from historical data. These methodologies are selected both to provide good approximations to data and to be easy to replicate. Both approaches are based on principal components estimated from return data.

The methodology are illustrated using returns across the government bond term structure from seven countries. The country level data suggest that three to four principal components are sufficient to capture most variation in individual country term structures. The first principal components are somewhat correlated across countries while there is less evidence of correlation for other components. Cross-country evidence suggests that correlations between components changed from 1990-93 to 1994-96. GARCH (generalised autoregressive conditional heteroskedasticity) models for the variance of these principal components are estimated for each country. In several country models asymmetric volatility responses to positive and negative return surprises are important for the components.

2. Term structure shocks

This section describes a methodology for summarising historical movements in interest rates across government term structures. The methodology could be used to construct scenarios of large historical changes in term structures. These scenarios could be used both for computing the exposure of a financial firm's trading book to term structure movements as well as for stress testing purposes. The outputs from the scenarios could also be aggregated over market-making firms to analyse the impact of large term structure shifts on aggregate profit & loss statements.

Introduction

Term structure research has established that much of the variability in government bond returns can be summarised by movements in a few underlying factors.[1] Further, if these factors have high predictive power across maturities, the factors may also capture extreme historical movements in the term structure.[2] I suggest using principal components to extract these factors from government zero coupon bond returns.[3] I also discuss methods for using extrema in the observed principal components to construct extreme historical scenarios for changes in rates across government term structures.

One disadvantage of principal components is that they are identified by a statistical procedure, and thus may be harder to interpret than factors defined by specific economic variables. However, most evidence suggests that principal components can provide a useful summary of variation across a term structure. Moreover, the principal components could also be used in modelling volatility across the term structure.

Zero coupon rates are natural building blocks to describe interest rate movements in a government securities market. Scenarios defined by changes in government zero coupon rates can easily be converted into scenarios for prices of other non-callable government securities. Finally, zero coupon rates can provide a dimension reduction if interpolation is used to give zero rates for maturities between those in the scenario.

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1 See Kahn (1989) and Litterman and Scheinkman (1991) for an introduction to US evidence; Steeley (1990) for UK evidence; and Murphy and Won (1995) and Chaumeton et al. (1996) for discussions of cross-country evidence.

2 Kambhu and Rodrigues (1997) present an example where excluding factors that account for variation across the term structure can lead to scenarios that miss substantial variation in the value of trading books.

3 This methodology and factor analysis are used extensively in the term structure literature and in practice. See Campbell et al. (1997) for a general introduction.

Other methodologies that could be used to summarise term structure movements include a variety of factor modelling approaches. While the principal components and factor approaches can give very similar results when variances are constant,[4] some form of time-varying variance model is more plausible for most asset returns. Consequently, factor models incorporating time-varying variances could give a more precise approximation to financial data than principal components approaches. Moreover, an explicit factor model might provide powerful tests of whether factor sensitivities are constant over time and whether an adequate number of factors has been specified to account for cross-maturity correlations. However, a potential drawback to factor approaches is that they must be estimated with computationally-intensive nonlinear optimisation and, thus, may be somewhat hard to implement and to replicate. Largely for computational simplicity, the current analysis uses a principal components approach.

Data

The analysis below uses data from 7 countries including Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), and the United States (US). Individual country analyses use returns on zero coupon securities derived from J.P. Morgan daily zero yields measured from January 1994 through September 1996 (see Murphy (1991) for a description of the methodology). The zero coupon maturities are 1, 2, 3, 4, 5, 7, 10, 15 and 20 years for all countries. In the recent period, the data include 25 and 30 year maturities for all countries but Japan and the United Kingdom. I also use data from January 1990 through September 1996 to conduct an aggregate analysis across countries. Maturities over 10 years are not available for Germany, Italy, and Japan in 1990-93.

There are two sources of measurement error in these returns: First, the zero coupon yields are estimated from bond prices and, thus, are not measured exactly. This error arises both because the estimated zero curve generally does not exactly fit all coupon bond prices and because some government bonds may not be traded frequently so their quotes may not indicate current market valuation. Second, the returns are computed from successive daily yields at the same maturity and, thus, do not reflect the return on a specific zero coupon security.

Country results

The principal component results are presented in Table 1. Three general conclusions can be drawn from the country data: First, all country term structures can be summarised by a small number of principal components. Second, two or three components are not always sufficient if we wish to describe close to 100% of term structure movements. Third, the natural ordering of principal

[4] Campbell et al. (1997) discuss similarities between principal components and factor analysis. Loretan (1996) outlines conditions when principal component analysis might be expected to provide a valid summary of a multivariate distribution.

components does not always correspond to the most interesting economic order. Thus, some judgement is required in selecting principal components.

The same statistics are presented for each country. The first matrix reports the percentage of total variance of each maturity zero coupon return that is explained by a principal component. Each column in the matrix corresponds to a principal component - the columns are ordered from left to right; each row corresponds to the labelled maturity. The last row reports a measure of the overall explanatory power of the component for returns.[5] The second matrix for a country presents estimates of the responses of the zero coupon yields to a one-standard deviation increase in a particular principal component. As in the first matrix, columns correspond to principal components and rows correspond to maturities.

Starting with Canada results, the first (leftmost) principal component explains a large fraction of variance for each maturity, from a low of 46% up to 99% for the 25 year maturity. The second principal component adds to the explanatory power particularly for shorter maturity zeros. (Since the principal components are uncorrelated, these fractions can be added to determine the total fraction of variance explained by a set of components.) Note that the third component also improves explanatory power for short maturities but by less than the fourth component. The first, second, and fourth component together could explain at least 82% of variance at every maturity. More components would be required to cover 90% of variation in every maturity. The sensitivities at the bottom of the page, along with similar sensitivities for returns, give a method to convert extreme values of the principal components into extreme yield changes or returns. The sensitivities suggest that the first component represents shifts in the term structure, that the second component represents term structure steepening, and that the third component (and the fourth component except at 30 years) reflects greater curvature in the term structure.

The other country results are somewhat similar. In every case the first component accounts for considerable variance across the term structure and reflects a term structure shift. However, interpretation of the other components is not always identical. Further, for several countries (Germany, Italy, Japan, and the United Kingdom), four components are required to describe over 80% of variation at all maturities. In order to account for most of the variation across the term structure, this country-level analysis suggests that at least three or four principal components per country are required.

Results of an analysis of standardised returns (defined as the return for each maturity divided by its sample standard deviation) are shown in Table 2. The standardised returns lead to slightly neater results: Over 90% of variation in most countries standardised returns at all maturities are described by the first three components. As we observed in the analysis of unstandardised returns, the components for each country are associated with similar movements in yields across the term

[5] The measure of overall or total explanatory power is the ratio of variance of the component to the trace of the covariance matrix of returns. These values sum to one.

structure. Specifically, the first component for each country's data seems to represent a shift in the term structure while the second represents a twist and the third introduces greater curvature in the term structure. Note that the sensitivity of yields at various maturities to the components derived from standardised returns is often similar in size to sensitivities derived from unstandardised returns, implying that scenarios derived from both approaches will be similar.

Aggregate results

Combining several country term structures offers the possibility of further dimension reduction because there may be consistent, common influences across the markets. I consider two approaches to measuring possible gains by combining countries: First, I investigate correlations between principal components estimated for different countries. Second, I analyse combined data for the seven countries described above. These approaches both suggest that some dimension reduction is possible from combining data across countries - this reduction reflects correlations in returns across countries.

Evidence from correlations between country-level principal components suggests that there is some correlation between the first components extracted from different country bond return data. There is less evidence for correlation between second and third components. Table 3 compares correlations between first, second, or third principal components across countries over two time periods (1990-93 and 1994-96). Generally speaking the first principal components exhibit the highest correlations across countries with the second components less correlated and the third components typically weakly correlated. There is some evidence that correlations changed after 1993, with stronger correlations often observed between first principal components in 1994-96. Table 4 presents the same comparison estimated from standardised returns. While correlations between components are often more positive with the standardised returns, the standardised returns seem to give qualitatively similar results to the unstandardised returns.

Combining country data gives a different perspective on correlations across country bond returns. These calculations suggest that there are minor additional reductions in dimension from combining data across countries. The major positive result is that relatively few (5) principal components describe a large fraction of variance in the long ends of most country term structures. Japan is an exception; movements in Japanese rates are described by components that are not highly correlated with other country term structures. Moreover, term structure returns in the 1-5 year maturity range are not well described by the first 5 components that account for variation at long maturities. In fact, many principal components would be required to explain 1-5 year maturity variance across these countries. Thus, if our goals are both to reduce the number of variables that define scenarios as well as to choose variables that describe most of the movement in the term structures, joint modelling of term structures in these countries may require almost as many principal components as would be required for separate country models.

Two views of the explanatory power of the principal components illustrate these results: Table 5 presents the variance of each maturity's return explained by the first 5, 10, or 20 principal components estimated over 1994-96. These principal components were computed jointly from the covariances of all 73 returns. The first 5 principal components describe most of the variation in returns of maturities over 10 years with the exceptions of the U.K. returns (where around 60% is captured) and of the Japanese returns (where very little variation is captured). The first 10 principal components explain longer maturities well but only account for 30-50% of the variation in short maturities. Moving to the first 20 principal components helps but in five countries (Canada, Germany, Italy, Japan, the United Kingdom) is not sufficient to describe 80% of variation in short rates.

However, analysis of the 1990-93 period suggests that the relatively high recent correlation in returns may be exceptional. Table 6 reports the explanatory power of the first 5, 10, or 20 principal components for each maturity's return in 90-93. Note that the 15-30 year maturities are not available for Germany, Italy, and Japan in this earlier sample so the number of reported maturities is smaller for those countries. While the first 5 principal components continue to describe long maturity returns well in several countries, they capture less variation in German and Italian returns. Although the first 10 components describe variation in most country returns, the first 20 are required to also capture variation in Japanese returns. This comparison of 1990-93 to 1994-96 suggests that the role of the components changed somewhat from the first to the second period. These results are reinforced by a graphical analysis.

A second, graphical illustration of association across countries provides additional insight into return correlations across countries. Chart 1 is a contour plot for 1994-96 data that circles the combinations of principal components and of securities where a component explains at least 10% of a particular security's return variation. Thus, tight clusters of curves highlight components that account for substantial return variation in particular securities. In contrast, areas in the grid without any marking indicate components that do not describe those returns well. The chart illustrates that the first five components describe return variation for all countries except Japan. Component 6 describes several U.K. maturities while components 7 through 9 help explain the Japanese term structure. The small marks for higher numbered components indicate components that account for particular country shorter maturity returns. There is at least one unique "short-term" component for each country. An analysis of the return variation matrix suggests that at least 10 principal components are required to account for at least 90% of 1 year return variation in each country.

Chart 2 repeats this analysis for the 1990-93 data and illustrates that the correlations of returns were somewhat different than in 1994-96. The first five components no longer seem as important across countries with Japanese and Italian returns somewhat unrelated to other country returns. As in the later period, shorter and longer maturities are occasionally related to different sets of principal components.

Constructing term structure scenarios

Having derived a set of principal components that account for most variation in term structure returns, there are several approaches to deriving scenarios:[6] The first approach involves identifying large values of individual principal components and mapping those into a set of historical scenarios. In this approach, the large values of the components could be defined either by tail percentiles in the components' empirical distribution or, if the components are distributed symmetrically, by multiples of the components' standard deviations. Each actual scenario would be derived by multiplying the corresponding component value by the yield sensitivities reported earlier.

A second approach would combine movements in principal components to produce scenarios that might correspond more closely to actual term structure movements. Specifically, the approach involves creating a separate scenario for each possible combination of large increase, large decrease, or no change in the principal components.[7] Thus, with N principal components there would be 3^N-1 possible scenarios, illustrated below for a two component example:

		Component 1		
		–	0	+
Component 2	–	Scenario 1	Scenario 2	Scenario 3
	0	Scenario 4		Scenario 5
	+	Scenario 6	Scenario 7	Scenario 8

The extreme outcomes for each principal component could be selected using either observed values in the tails of the component's empirical distribution or multiples of the component's standard deviation.

A third approach would identify specific historical episodes and use the observed component shocks during those episodes to derive a scenario. An alternative to this approach would simply use the observed term structure movements during these episodes as a scenario.

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6 The analysis in this section assumes an objective of measuring the change in trading book value induced by large movements in term structures. A more detailed grid of scenarios would be required when the objective is measuring how trading book value varies with changes in interest rates over the whole range of historically observed rate movements.

7 This is a simplified version of the approach proposed by Jamshidian and Zhu (1996). A simpler alternative would construct scenarios using just large increases and large decreases in the components. This would create 2^N-1 possible scenarios. Jamshidian and Zhu (1996) present evidence that division of the historical range of component outcomes into several categories can lead to a set of scenarios that cover many likely term structure outcomes. The examples in Kambhu and Rodrigues (1997) imply that these risk measures should be constructed using all components whose risk is priced in the market. A separate analysis of portfolio sensitivity to shocks in sources of residual risk would also be appropriate if exposure to some sources of residual risk might be large.

Other topics

Several other issues need to be considered when constructing interest rate scenarios. These include how to measure credit quality shocks, whether some scenarios should include shocks that data suggest occurred over more than one day, and if the statistical models for returns are stable over the sample.

The discussion above has focused on modelling government security term structures. However, term structures for private sector debt with different credit quality are more relevant for pricing many instruments including swaps and most forwards and futures. Furthermore, academic and practitioner literature suggests that risky debt spreads over comparable government security yields vary somewhat predictably with the business cycle and with the level of short rates.[8] The limited data on private sector rates for different credit quality borrowers suggests that some compromises are necessary to model and generate scenarios for shocks to private sector rates. One approach would generate shocks for private sector rates by applying typical credit spreads to the government term structure shocks or by assuming that private sector rates retain their typical correlations with government rates. A major drawback of this approach is that large asset price movements might also lead to significant changes in credit spreads. A more realistic approach would use data on private sector rates by country in an analysis like that carried out above for government term structures.

My analysis has focused on modelling daily returns. An extension of this analysis would construct some scenarios from shocks over several days. While the results of applying this type of scenario could be misleading (because firms adjust hedges during an event), they may also represent a different type of stressful event from the single-day-shock. Another reason to consider multiple-day returns is that measured correlations would be less affected by non-synchronous daily measurements of yields.

Finally, a more detailed analysis of the stability of the principal components representations used to construct the scenarios is necessary.[9] If the models are not stable or if the models do not describe most of return variance, the scenarios may fail to map out large historical events.

3. Volatility shocks

Financial firms that trade options or financial instruments with option components will have typically have exposures both to asset prices and to volatility. Thus, analysis of their exposures

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8 See Duffee (1996), Knez, Litterman and Scheinkman (1994) or Litterman and Iben (1991) for examples.

9 The simplest test for stability, measuring whether the covariance matrix of returns is approximately constant over extended samples, is problematic with asset prices because conditional volatility of returns typically varies over time. However, it is possible that the responses of yields or returns to principal components or factors are stable. This could arise if variation in the conditional volatility of the principal components is the source of variation in the conditional volatility of returns.

requires measuring changes in the value of their trading books as both volatility and underlying prices and interest rates move.

There are several interpretations of changes in volatility: For example, a trader's view might be a change in the quote sheet of implied volatilities for options on a particular underlying by strike price and time-to-expiration. This is essentially equivalent to a change in the price of a group of options; without further structure such a change could be associated with a change in the variance of the underlying distributions of future prices or even with a change in the shape of underlying future distributions.[10]

Although volatility changes could be derived from options price data, I focus on models of the underlying variance of asset prices and specifically GARCH models. Such models provide estimates of the volatility of the underlying asset price and have the advantage of being easy to estimate.[11] Other models, such as stochastic volatility models, also provide good descriptions of movements in asset price volatilities.[12]

Introduction

This section outlines a fairly simple approach to measuring volatility movements, based on GARCH models. Specifically, if the set of underlying asset prices are well-described by a factor model, volatility models for the factors could describe most of the variation in volatility of the asset returns.[13] Volatility models for factors could provide a more parsimonious description of volatility and would require estimating far fewer models than direct modelling of volatility for each asset price.

The empirical section below illustrates an approximation to a full factor model. Rather than estimating a full factor model with GARCH volatilities, principal components are extracted from a set of asset returns and GARCH models are estimated for the principal components. This approach is easy to reproduce but would only provide consistent parameter estimates of the underlying factor model under quite restrictive assumptions. Future work is required to compare these estimates with factor model estimates to illustrate the size of estimation error arising from this approximation.

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[10] There is a large literature proposing methods for extracting the implied distribution of the underlying price from a set of options prices. For examples, see Bates (1991), Derman and Kani (1994), Malz (1996), Melick and Thomas (1994), Neuhaus (1995), or Rubinstein (1994).

[11] Andersen and Bollerslev (1997) suggest that evidence of poor volatility forecasts at high frequencies by ARCH class models may reflect the noise in daily volatility estimators and that ARCH models provide good forecasts of underlying volatility.

[12] See Melino and Turnball (1991) or Campbell et al. (1997) for an introduction to stochastic volatility models. Nelson (1992) and Nelson and Foster (1994) demonstrate that ARCH models provide an approximation to stochastic volatility models. Jacquier et al. (1994) present a computationally attractive method of estimating stochastic volatility models.

[13] This might correspond to the Factor ARCH model of Engle et al. (1990) or to a factor model with different volatility models for each factor as in Harvey et al. (1992) or King et al. (1994). If the sources of idiosyncratic risk also display time-varying volatility, then their volatilities must also be modelled.

When a set of volatility models have been derived, there are several approaches for developing scenarios for large historical changes in volatility: First, the models could be used directly to generate predicted changes in volatility in-sample; the extreme values of these changes could be used as scenarios. Second, scenarios for large factor movements could be combined with the GARCH models to generate predicted changes in volatility that are consistent with the changes in underlying prices. Finally, particular the predicted volatility changes during specific historical episodes could be computed. I present examples below of the first type of scenario.

Models

To illustrate this methodology, I estimated GARCH models for the first three principal components for government term structures of Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. The principal component procedure is described in the previous section. These models were estimated on daily J.P. Morgan zero coupon government bond return data from January 1994 through September 1996. (See Table 7.) The estimated models were GARCH(1,1) models with allowance for possible asymmetry in conditional variance (sometimes called the leverage effect in the literature). That is, negative returns are allowed to imply different future predicted volatility than positive returns. Results varied across term structure models but in many cases the models implied slowly decaying volatility in the first (shift) component; some country models also showed slow decay for the second (twist) and third (curvature) component. Leverage effects were most often important for the third component but in some countries are also observed in volatility of the first and second principal components.

Chart 3 displays several views of the predicted volatility across the term structures of each of the seven countries. The mean and range of predicted volatility across the maturities is reported in the top chart. As expected, volatility of returns typically rises with maturity. The volatility models appear to capture much of the underlying term structure variance because the mean variance predicted by the model is similar to an estimate of the mean unconditional variance. The middle and bottom charts show the largest one and five day changes in volatilities. Note that in some countries the largest five-day changes are slightly smaller than the one-day changes.

Scenarios

The predicted volatility changes provide one set of volatility scenarios. A full scenario would include changes in volatility for forecast horizons up to several months in addition to the daily changes exhibited in Chart 3. Another set of scenarios could be constructed from scenarios for the principal components by conditioning on the principal component shocks when constructing volatility scenarios. This second set of scenarios would be consistent with the scenarios for the term structure and so may be preferable to the first method. A third set of scenarios could be constructed from estimated conditional variance movements during specific historical episodes.

Future work

One check on the realism of this approach would be to compare volatility shocks constructed from GARCH models to movements in implied volatilities derived from options on the relevant asset. Practitioners often report that implied volatility tends to be less variable than historical volatility. Since GARCH estimates could be considered a type of historical volatility estimator, it is possible that the GARCH models generate volatility changes that are too large.[14]

Future work should compare volatility model estimates derived from principal components to factor model estimates. Such a comparison would provide an estimate of the likely size of estimation error from using the simpler principal component procedure.

4. Conclusions

This paper presented methodologies for summarising large movements in term structures and in volatilities. Empirical results suggest that several country government bond term structures can be described by three or four principal components. The principal components appear to have simple interpretations similar to those found by other researchers who have modelled the U.K. and U.S. term structures. The first principal component, which accounts for a large fraction of return variance at most maturities and which reflects roughly parallel shifts in the term structure, is more highly correlated across countries than other principal components. However, the cross country correlations seem to have shifted after 1993, suggesting that multiple country models are less stable than single country models. A methodology is also presented for constructing term structure shocks for risk measurement purposes.

The paper also proposes a standard GARCH methodology to derive conditional variances for the principal components. These models capture some aspects of the data well. Large changes in predicted variances from the GARCH models could be used to generate volatility scenarios.

[14] However, Andersen and Bollerslev (1997) present evidence that GARCH models may measure exchange rate volatility well even though they are not highly correlated with high frequency historical estimators of instantaneous volatility.

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Table 1a

Principal component analysis:
term structure returns in Canada (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	46%	23%	5%	13%	3%	7%	0%	1%	0%	0%	0%
2	58%	24%	6%	8%	1%	3%	0%	0%	0%	0%	0%
3	65%	23%	5%	6%	1%	0%	0%	0%	0%	0%	0%
4	71%	22%	4%	2%	0%	1%	0%	0%	0%	0%	0%
5	75%	21%	3%	1%	0%	1%	0%	0%	0%	0%	0%
7	82%	15%	2%	1%	0%	0%	0%	0%	0%	0%	0%
10	89%	7%	2%	2%	0%	0%	0%	0%	0%	0%	0%
15	91%	2%	6%	0%	0%	0%	0%	0%	0%	0%	0%
20	97%	0%	2%	0%	0%	0%	0%	0%	0%	0%	0%
25	99%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	97%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%
Total	95%	3%	2%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	7.7	- 5.4	- 2.7	- 4.1	- 1.8	2.9	0.5	- 1.3	0.1	- 0.7	0.7
2	7.5	- 4.9	- 2.4	- 2.8	- 1.2	1.7	0.2	- 0.3	- 0.2	- 0.1	- 0.4
3	7.5	- 4.4	- 2.1	- 2.2	- 0.7	0.2	- 0.2	0.6	- 0.1	0.4	0.1
4	7.3	- 4.1	- 1.8	- 1.2	- 0.1	- 0.7	- 0.2	0.2	0.2	- 0.3	0.0
5	7.3	- 3.8	- 1.4	- 0.6	0.2	- 0.8	0.0	- 0.4	- 0.2	0.1	0.0
7	6.8	- 2.9	- 1.1	0.7	0.3	0.2	0.1	- 0.1	0.2	0.1	0.0
10	6.5	- 1.8	- 0.9	0.9	0.2	0.2	0.0	0.1	- 0.1	- 0.1	0.0
15	5.8	- 0.9	1.5	0.3	- 0.4	- 0.1	0.0	0.0	0.0	0.0	0.0
20	5.8	- 0.1	0.9	- 0.2	0.2	0.1	- 0.1	0.0	0.0	0.0	0.0
25	5.7	0.4	0.1	- 0.2	0.1	0.0	0.1	0.0	0.0	0.0	0.0
30	5.6	0.8	- 0.5	0.1	- 0.1	0.0	0.0	0.0	0.0	0.0	0.0

Table 1b

Principal component analysis:
term structure returns in France (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	26%	25%	30%	2%	3%	0%	11%	3%	0%	0%	0%
2	55%	22%	14%	1%	2%	0%	5%	0%	0%	0%	0%
3	66%	20%	12%	1%	1%	0%	0%	0%	0%	0%	0%
4	73%	17%	9%	0%	0%	0%	0%	0%	0%	0%	0%
5	79%	14%	5%	0%	0%	0%	1%	0%	0%	0%	0%
7	92%	5%	0%	1%	1%	0%	0%	0%	0%	0%	0%
10	95%	4%	0%	1%	0%	0%	0%	0%	0%	0%	0%
15	97%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%
20	99%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	99%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	98%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	4.5	- 4.4	4.8	- 1.1	- 1.6	0.6	- 2.9	1.5	0.4	- 0.2	0.2
2	4.8	- 3.1	2.4	- 0.7	- 1.0	0.1	- 1.5	- 0.1	0.3	- 0.1	- 0.1
3	5.3	- 2.9	2.3	- 0.6	- 0.5	- 0.1	- 0.2	- 0.3	- 0.4	0.3	0.0
4	5.6	- 2.7	1.9	- 0.4	- 0.2	- 0.1	0.4	- 0.1	- 0.2	- 0.3	0.0
5	5.7	- 2.4	1.5	- 0.2	0.1	0.0	0.6	0.2	0.2	0.1	0.0
7	5.9	- 1.4	0.0	0.6	0.6	0.2	- 0.1	0.3	- 0.1	0.0	0.0
10	5.9	- 1.2	0.0	0.5	0.3	0.0	- 0.1	- 0.3	0.0	0.0	0.0
15	6.0	- 0.7	- 0.5	0.3	- 0.3	- 0.2	0.0	0.0	0.0	0.0	0.0
20	6.1	- 0.4	- 0.4	- 0.2	- 0.1	0.2	0.0	0.0	0.0	0.0	0.0
25	6.3	0.0	- 0.1	- 0.2	0.1	- 0.2	0.0	0.0	0.0	0.0	0.0
30	6.5	0.6	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1c

Principal component analysis:
term structure returns in Germany (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	24%	0%	23%	22%	1%	16%	9%	1%	4%	0%	1%
2	44%	2%	35%	10%	0%	7%	1%	0%	1%	0%	0%
3	54%	2%	29%	12%	0%	2%	0%	0%	1%	0%	0%
4	65%	1%	26%	6%	0%	0%	1%	0%	0%	0%	0%
5	69%	2%	20%	7%	0%	1%	0%	0%	0%	0%	0%
7	79%	5%	15%	0%	2%	0%	0%	0%	0%	0%	0%
10	88%	0%	9%	2%	0%	0%	0%	0%	0%	0%	0%
15	92%	7%	1%	0%	0%	0%	0%	0%	0%	0%	0%
20	90%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	98%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	86%	14%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	90%	8%	2%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	2.5	0.2	2.4	- 2.3	- 0.4	2.0	1.5	0.4	- 1.0	0.2	- 0.5
2	3.3	0.6	3.0	- 1.6	- 0.3	1.4	0.4	- 0.3	- 0.4	- 0.1	0.2
3	4.1	0.8	3.0	- 1.9	- 0.1	0.8	0.0	0.2	0.5	0.0	0.0
4	5.1	0.6	3.2	- 1.6	- 0.3	- 0.1	- 0.7	0.2	- 0.2	0.0	0.0
5	4.8	0.7	2.6	- 1.5	- 0.4	- 0.6	0.4	- 0.1	0.0	0.0	0.0
7	5.2	1.3	2.2	0.0	0.8	- 0.1	0.1	0.0	0.0	0.0	0.0
10	5.5	0.3	1.8	0.9	- 0.1	0.1	- 0.1	- 0.2	0.0	0.0	0.0
15	6.1	- 1.7	0.5	0.5	- 0.1	0.0	0.1	0.1	0.0	0.0	0.0
20	6.1	- 2.0	- 0.2	- 0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	5.8	- 0.8	- 0.4	- 0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	5.1	2.0	- 0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1d

Principal component analysis:
term structure returns in Italy (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	41%	11%	3%	22%	5%	13%	2%	2%	1%	0%	0%
2	53%	16%	9%	14%	2%	4%	2%	0%	0%	0%	0%
3	55%	16%	12%	12%	1%	1%	1%	2%	0%	0%	0%
4	61%	15%	12%	9%	1%	1%	0%	1%	0%	0%	0%
5	64%	15%	15%	4%	1%	1%	1%	0%	0%	0%	0%
7	50%	31%	16%	0%	2%	0%	0%	0%	0%	0%	0%
10	69%	12%	18%	1%	0%	0%	0%	0%	0%	0%	0%
15	93%	1%	7%	0%	0%	0%	0%	0%	0%	0%	0%
20	94%	5%	1%	0%	0%	0%	0%	0%	0%	0%	0%
25	98%	2%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	95%	3%	3%	0%	0%	0%	0%	0%	0%	0%	0%
Total	92%	4%	3%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	7.7	4.0	2.3	- 5.6	- 2.6	- 4.4	- 1.8	1.7	- 1.3	0.4	0.0
2	7.8	4.3	3.2	- 3.9	- 1.6	- 2.2	- 1.3	0.5	0.6	- 0.2	0.0
3	7.4	4.0	3.5	- 3.5	- 1.1	- 0.8	1.1	- 1.3	0.0	0.0	0.0
4	8.3	4.0	3.7	- 3.1	- 1.0	1.1	0.6	0.8	0.0	0.0	0.0
5	8.2	3.9	3.9	- 2.0	0.8	0.8	- 0.8	- 0.4	0.0	0.0	0.0
7	7.6	6.0	4.3	0.0	1.4	- 0.4	0.3	0.2	0.0	0.0	0.0
10	8.3	3.4	4.2	1.0	- 0.5	0.0	0.0	0.0	0.0	- 0.1	0.0
15	9.7	- 0.7	2.6	0.4	- 0.2	0.0	0.0	0.0	0.0	0.1	0.0
20	10.4	- 2.4	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	10.5	- 1.6	- 0.4	- 0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0
30	9.8	1.7	- 1.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1e

Principal component analysis:
term structure returns in Japan (1994-96)

Component	1	2	3	4	5	6	7	8	9
Fraction of variance explained by component									
1	40%	7%	13%	15%	19%	4%	0%	0%	0%
2	64%	12%	7%	10%	7%	1%	0%	0%	0%
3	72%	11%	7%	7%	1%	0%	0%	0%	0%
4	78%	11%	6%	5%	0%	0%	0%	0%	0%
5	82%	9%	5%	3%	1%	0%	0%	0%	0%
7	89%	5%	2%	4%	0%	0%	0%	0%	0%
10	93%	5%	1%	1%	0%	0%	0%	0%	0%
15	89%	0%	10%	1%	0%	0%	0%	0%	0%
20	91%	8%	1%	0%	0%	0%	0%	0%	0%
Total	89%	6%	4%	1%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	3.5	- 1.4	2.0	2.1	2.4	1.2	- 0.4	0.3	0.0
2	3.9	- 1.7	1.3	1.5	1.3	0.5	0.0	- 0.1	0.0
3	4.5	- 1.8	1.4	1.4	0.6	- 0.3	0.3	0.0	0.0
4	4.8	- 1.8	1.3	1.2	- 0.1	- 0.4	- 0.2	0.0	0.0
5	4.9	- 1.6	1.2	0.9	- 0.6	0.2	0.0	0.0	0.0
7	5.3	- 1.3	0.8	- 1.2	0.1	0.0	0.0	0.0	0.0
10	4.0	- 0.9	- 0.5	- 0.4	0.0	0.0	0.0	0.0	0.0
15	3.0	- 0.1	- 1.0	0.2	0.0	0.0	0.0	0.0	0.0
20	3.2	1.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0

Table 1f

Principal component analysis:
term structure returns in the United Kingdom (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Fraction of variance explained by component									
1	32%	23%	17%	11%	11%	4%	2%	0%	0%
2	62%	23%	7%	5%	3%	1%	0%	0%	0%
3	72%	18%	6%	3%	1%	0%	0%	0%	0%
4	80%	15%	4%	1%	0%	0%	0%	0%	0%
5	85%	12%	3%	0%	1%	0%	0%	0%	0%
7	92%	7%	0%	0%	0%	0%	0%	0%	0%
10	96%	2%	0%	1%	0%	0%	0%	0%	0%
15	98%	0%	1%	0%	0%	0%	0%	0%	0%
20	97%	2%	0%	0%	0%	0%	0%	0%	0%
Total	96%	3%	1%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	3.9	- 3.3	2.8	- 2.2	- 2.3	1.3	- 0.9	- 0.4	0.3
2	5.2	- 3.2	1.8	- 1.4	- 1.2	0.5	- 0.3	0.0	- 0.2
3	5.8	- 2.9	1.7	- 1.2	- 0.6	- 0.2	0.4	0.3	0.0
4	6.4	- 2.7	1.4	- 0.6	0.2	- 0.3	0.2	- 0.3	0.0
5	6.6	- 2.5	1.1	- 0.2	0.6	- 0.2	- 0.3	0.1	0.0
7	7.2	- 1.9	0.0	0.5	0.2	0.4	0.1	0.0	0.0
10	7.4	- 1.1	- 0.5	0.6	- 0.2	- 0.2	0.0	0.0	0.0
15	6.5	0.1	- 0.8	- 0.4	0.0	0.0	0.0	0.0	0.0
20	6.1	1.0	0.4	0.1	0.0	0.0	0.0	0.0	0.0

Table 1g

Principal component analysis:
term structure returns in the United States (1994-96)

Component	1	2	3	4	5	6	7	8	9	10	11
Fraction of variance explained by component											
1	52%	34%	0%	10%	1%	3%	0%	0%	0%	0%	0%
2	64%	30%	0%	4%	0%	1%	0%	0%	0%	0%	0%
3	71%	26%	0%	3%	0%	0%	0%	0%	0%	0%	0%
4	78%	21%	0%	1%	0%	0%	0%	0%	0%	0%	0%
5	81%	18%	0%	0%	0%	0%	0%	0%	0%	0%	0%
7	88%	12%	0%	0%	0%	0%	0%	0%	0%	0%	0%
10	93%	6%	0%	0%	0%	0%	0%	0%	0%	0%	0%
15	98%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	99%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
25	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
30	99%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	98%	2%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	4.7	3.8	- 0.1	2.0	0.5	1.2	0.3	- 0.4	- 0.1	- 0.1	0.1
2	5.3	3.6	- 0.1	1.4	0.3	0.6	0.2	- 0.1	- 0.1	0.0	- 0.1
3	5.7	3.4	- 0.2	1.1	0.2	0.1	0.0	0.2	0.1	0.1	0.0
4	5.9	3.1	- 0.2	0.7	0.0	- 0.2	- 0.1	0.0	0.0	- 0.1	0.0
5	6.0	2.8	- 0.2	0.4	0.0	- 0.3	- 0.1	- 0.1	0.0	0.0	0.0
7	6.1	2.2	- 0.3	- 0.3	- 0.2	0.1	0.0	0.0	0.1	0.0	0.0
10	5.9	1.5	- 0.3	- 0.3	- 0.1	0.1	0.0	0.0	0.0	0.0	0.0
15	5.7	0.6	0.3	- 0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0
20	5.5	- 0.1	0.4	0.0	0.0	0.0	- 0.1	0.0	0.0	0.0	0.0
25	5.2	- 0.3	0.1	0.1	- 0.1	0.0	0.1	0.0	0.0	0.0	0.0
30	5.1	- 0.4	- 0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 2a

Principal component analysis:
standardised term structure returns in Canada (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	73%	20%	6%	0%	1%	0%	0%	0%	0%	0%	0%
2	85%	14%	1%	0%	0%	0%	0%	0%	0%	0%	0%
3	90%	8%	0%	0%	1%	1%	0%	0%	0%	0%	0%
4	93%	4%	1%	0%	1%	0%	0%	0%	0%	0%	0%
5	95%	2%	3%	0%	0%	0%	0%	0%	0%	0%	0%
7	95%	0%	3%	0%	1%	0%	0%	0%	0%	0%	0%
10	95%	1%	2%	0%	1%	0%	0%	0%	0%	0%	0%
15	84%	10%	0%	6%	0%	0%	0%	0%	0%	0%	0%
20	85%	12%	1%	1%	0%	0%	0%	0%	0%	0%	0%
25	85%	13%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	81%	14%	1%	4%	0%	0%	0%	0%	0%	0%	0%
Total	87%	9%	2%	1%	1%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	9.7	- 5.1	- 2.8	- 0.3	- 0.9	0.6	0.2	0.2	- 0.2	0.0	0.0
2	9.1	- 3.7	- 1.1	0.0	- 0.2	- 0.4	- 0.2	- 0.3	0.3	0.0	0.0
3	8.8	- 2.7	0.1	0.1	0.9	- 0.7	- 0.1	0.0	- 0.3	0.0	0.0
4	8.4	- 1.7	1.0	0.2	0.9	0.2	0.2	0.3	0.2	0.0	0.0
5	8.2	- 1.1	1.4	0.0	0.5	0.6	0.0	- 0.4	- 0.1	0.0	0.0
7	7.3	0.0	1.4	0.1	- 0.8	0.0	- 0.1	0.1	- 0.1	- 0.2	0.0
10	6.7	0.7	1.0	0.4	- 0.8	- 0.2	0.0	0.0	0.0	0.2	0.0
15	5.5	1.9	- 0.1	- 1.5	- 0.1	- 0.1	0.3	- 0.1	0.0	0.0	0.0
20	5.4	2.0	- 0.6	- 0.5	0.2	0.1	- 0.3	0.1	0.0	0.1	- 0.1
25	5.2	2.0	- 0.6	0.4	0.2	0.1	- 0.1	0.0	0.0	0.0	0.1
30	5.1	2.1	- 0.7	1.1	0.0	0.0	0.2	- 0.1	0.0	0.0	- 0.1

Table 2b

Principal component analysis:
standardised term structure returns in France (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	51%	44%	3%	0%	0%	0%	0%	0%	0%	0%	0%
2	81%	18%	1%	0%	1%	0%	0%	0%	0%	0%	0%
3	89%	8%	1%	0%	0%	0%	0%	0%	0%	0%	0%
4	93%	4%	2%	0%	0%	0%	0%	0%	0%	0%	0%
5	96%	1%	3%	0%	0%	0%	0%	0%	0%	0%	0%
7	94%	3%	0%	2%	0%	0%	0%	0%	0%	0%	0%
10	95%	3%	0%	1%	0%	0%	0%	0%	0%	0%	0%
15	92%	7%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	91%	8%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	90%	9%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	86%	10%	2%	1%	0%	0%	0%	0%	0%	0%	0%
Total	87%	10%	1%	1%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	6.3	- 5.9	- 1.6	0.3	0.6	- 0.2	- 0.1	0.1	0.0	0.0	0.0
2	5.8	- 2.7	- 0.5	0.1	- 0.6	0.1	0.2	- 0.2	0.0	0.0	0.0
3	6.2	- 1.9	0.6	- 0.4	- 0.3	0.3	- 0.4	0.1	0.1	0.0	0.0
4	6.3	- 1.2	1.0	- 0.5	0.1	- 0.1	0.1	0.1	- 0.3	0.0	0.0
5	6.3	- 0.6	1.0	- 0.2	0.4	- 0.3	0.2	- 0.1	0.2	0.0	0.0
7	6.0	1.0	0.4	0.9	0.4	0.1	- 0.1	0.0	0.0	0.0	0.0
10	5.9	1.1	0.2	0.5	- 0.1	0.3	0.1	- 0.1	0.0	0.0	0.0
15	5.8	1.6	- 0.1	0.3	- 0.4	- 0.3	0.1	0.4	0.0	0.0	0.0
20	5.9	1.7	- 0.4	- 0.1	- 0.2	- 0.3	- 0.2	- 0.2	0.0	0.2	0.0
25	6.0	1.9	- 0.6	- 0.4	0.0	0.0	- 0.1	- 0.2	0.0	- 0.2	0.0
30	6.1	2.1	- 0.9	- 0.7	0.4	0.3	0.1	0.2	0.0	0.1	0.0

Table 2c

Principal component analysis:
standardised term structure returns in Germany (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	51%	33%	10%	0%	0%	0%	0%	0%	0%	0%	0%
2	77%	21%	0%	1%	0%	0%	0%	0%	0%	0%	0%
3	85%	12%	0%	1%	1%	0%	0%	0%	0%	0%	0%
4	91%	4%	1%	1%	0%	0%	1%	0%	0%	0%	0%
5	92%	3%	1%	1%	2%	0%	0%	0%	0%	0%	0%
7	92%	0%	5%	1%	0%	1%	0%	0%	0%	0%	0%
10	91%	3%	1%	3%	0%	0%	0%	0%	0%	0%	0%
15	80%	14%	4%	0%	0%	0%	0%	0%	0%	0%	0%
20	74%	18%	8%	0%	0%	0%	0%	0%	0%	0%	0%
25	80%	17%	2%	1%	0%	0%	0%	0%	0%	0%	0%
30	76%	7%	9%	0%	0%	0%	0%	0%	0%	0%	0%
Total	81%	12%	4%	1%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	3.6	2.9	- 1.6	0.3	0.6	- 0.2	- 0.1	0.1	0.0	0.0	0.0
2	4.4	2.3	- 0.5	0.1	- 0.6	0.1	0.2	- 0.2	0.0	0.0	0.0
3	5.1	1.9	0.6	- 0.4	- 0.3	0.3	- 0.4	0.1	0.1	0.0	0.0
4	6.0	1.2	1.0	- 0.5	0.1	- 0.1	0.1	0.1	- 0.3	0.0	0.0
5	5.5	0.9	1.0	- 0.2	0.4	- 0.3	0.2	- 0.1	0.2	0.0	0.0
7	5.6	0.0	0.4	0.9	0.4	0.1	- 0.1	0.0	0.0	0.0	0.0
10	5.6	- 1.1	0.2	0.5	- 0.1	0.3	0.1	- 0.1	0.0	0.0	0.0
15	5.7	- 2.4	- 0.1	0.3	- 0.4	- 0.3	0.1	0.4	0.0	0.0	0.0
20	5.5	1.7	- 0.4	- 0.1	- 0.2	- 0.3	- 0.2	- 0.2	0.0	0.2	0.0
25	5.2	1.9	- 0.6	- 0.4	0.0	0.0	- 0.1	- 0.2	0.0	- 0.2	0.0
30	4.8	2.1	- 0.9	- 0.7	0.4	0.3	0.1	0.2	0.0	0.1	0.0

Table 2d

Principal component analysis:
standardised term structure returns in Italy (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	67%	11%	19%	0%	2%	0%	0%	0%	0%	0%	0%
2	85%	10%	4%	0%	0%	0%	0%	1%	0%	0%	0%
3	87%	8%	0%	0%	2%	2%	0%	0%	0%	0%	0%
4	91%	4%	0%	0%	2%	0%	1%	0%	0%	0%	0%
5	92%	3%	2%	0%	0%	1%	1%	0%	0%	0%	0%
7	80%	7%	10%	1%	1%	0%	0%	0%	0%	0%	0%
10	89%	0%	7%	0%	2%	0%	1%	0%	0%	0%	0%
15	85%	12%	0%	2%	0%	0%	0%	0%	0%	0%	0%
20	73%	24%	0%	1%	0%	0%	0%	0%	0%	0%	0%
25	74%	25%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	79%	11%	0%	9%	0%	0%	0%	0%	0%	0%	0%
Total	82%	10%	4%	1%	1%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	9.9	- 4.1	- 5.3	0.2	1.5	0.2	- 0.1	- 0.6	- 0.3	0.0	0.0
2	9.9	- 3.4	- 2.1	- 0.1	0.5	0.2	0.1	0.9	0.5	0.0	0.0
3	9.3	- 2.8	- 0.4	- 0.6	- 1.5	- 1.5	0.6	- 0.1	- 0.1	0.0	0.0
4	10.0	- 2.2	0.5	- 0.6	- 1.7	0.6	- 1.3	- 0.3	0.2	0.0	0.0
5	9.8	- 1.7	1.6	- 0.3	- 0.6	1.2	0.8	0.3	- 0.4	0.0	0.0
7	9.5	- 2.8	3.4	1.3	0.9	0.1	0.5	- 0.6	0.4	0.0	0.0
10	9.4	- 0.4	2.7	- 0.2	1.4	- 0.7	- 0.7	0.3	- 0.3	0.0	0.0
15	9.4	3.4	0.6	- 1.5	0.7	- 0.1	- 0.1	0.0	0.0	- 0.1	0.0
20	9.2	5.3	- 0.8	- 1.3	0.1	0.1	0.3	- 0.2	0.1	0.0	0.0
25	9.1	5.3	- 1.2	0.2	- 0.3	0.1	0.2	- 0.1	0.1	0.0	0.0
30	9.0	3.3	- 0.6	3.0	- 0.6	- 0.2	- 0.2	0.2	- 0.1	0.0	0.0

Table 2e

Principal component analysis: standardised term structure returns in Japan (1994-96)

Component	1	2	3	4	5	6	7	8	9
Fraction of variance explained by component									
1	65%	28%	5%	0%	1%	0%	0%	0%	0%
2	88%	11%	0%	0%	0%	0%	0%	0%	0%
3	93%	5%	1%	0%	1%	1%	0%	0%	0%
4	95%	1%	2%	0%	1%	0%	0%	0%	0%
5	95%	0%	2%	1%	1%	1%	0%	0%	0%
7	88%	4%	2%	1%	4%	0%	0%	0%	0%
10	88%	8%	1%	1%	1%	0%	0%	0%	0%
15	76%	16%	1%	7%	1%	0%	0%	0%	0%
20	73%	12%	10%	5%	0%	0%	0%	0%	0%
Total	85%	10%	3%	2%	1%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	4.4	- 2.9	- 1.2	- 0.3	0.5	- 0.2	0.1	- 0.1	0.0
2	4.6	- 1.6	- 0.2	- 0.3	0.1	0.0	- 0.1	0.2	0.0
3	5.1	- 1.1	0.4	0.1	- 0.4	0.5	- 0.1	- 0.1	0.0
4	5.3	- 0.6	0.8	0.3	- 0.6	0.1	0.3	0.1	0.0
5	5.3	- 0.2	0.8	0.4	- 0.6	- 0.6	- 0.1	- 0.1	0.0
7	5.3	1.1	0.8	0.7	1.2	0.1	0.0	0.0	0.0
10	3.9	1.2	0.4	- 0.5	0.4	0.0	0.0	0.0	0.0
15	2.7	1.2	- 0.3	- 0.8	- 0.2	0.0	0.0	0.0	0.0
20	2.8	1.2	- 1.0	0.7	- 0.2	0.0	0.0	0.0	0.0

Table 2f

Principal component analysis:
standardised term structure returns in the United Kingdom (1994-96)

Component	1	2	3	4	5	6	7	8	9
Fraction of variance explained by component									
1	60%	36%	4%	0%	0%	0%	0%	0%	0%
2	88%	11%	0%	0%	0%	0%	0%	0%	0%
3	94%	4%	1%	1%	0%	0%	0%	0%	0%
4	97%	1%	2%	1%	0%	0%	0%	0%	0%
5	97%	0%	2%	0%	0%	0%	0%	0%	0%
7	95%	3%	1%	1%	0%	0%	0%	0%	0%
10	91%	7%	0%	1%	0%	0%	0%	0%	0%
15	85%	12%	1%	0%	1%	0%	0%	0%	0%
20	80%	14%	4%	1%	0%	0%	0%	0%	0%
Total	88%	10%	2%	1%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)									
1	5.3	- 4.1	- 1.3	- 0.3	- 0.2	- 0.1	0.0	- 0.1	0.1
2	6.3	- 2.3	- 0.1	- 0.2	0.2	0.0	0.0	0.1	- 0.2
3	6.6	- 1.4	0.5	0.5	0.5	0.3	- 0.1	0.1	0.1
4	7.0	- 0.5	0.9	0.5	0.0	- 0.1	0.1	- 0.3	- 0.1
5	7.1	0.0	1.0	0.3	- 0.4	- 0.4	0.1	0.2	0.1
7	7.3	1.3	0.7	- 0.7	- 0.4	0.1	- 0.3	- 0.1	0.0
10	7.2	2.0	0.2	- 0.8	- 0.1	0.3	0.3	0.0	0.0
15	6.1	2.3	- 0.7	- 0.3	0.6	- 0.4	0.0	0.0	0.0
20	5.5	2.3	- 1.3	0.7	- 0.3	0.1	0.0	0.0	0.0

Table 2g

Principal component analysis:
standardised term structure returns in the United States (1994-96)

Component	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Fraction of variance explained by component											
1	78%	19%	3%	0%	0%	0%	0%	0%	0%	0%	0%
2	88%	11%	0%	0%	0%	0%	0%	0%	0%	0%	0%
3	93%	7%	0%	0%	0%	0%	0%	0%	0%	0%	0%
4	96%	3%	1%	0%	0%	0%	0%	0%	0%	0%	0%
5	97%	2%	1%	0%	0%	0%	0%	0%	0%	0%	0%
7	98%	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%
10	98%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%
15	95%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	89%	10%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	87%	12%	1%	0%	0%	0%	0%	0%	0%	0%	0%
30	86%	13%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Total	91%	7%	1%	0%	0%	0%	0%	0%	0%	0%	0%
Sensitivity to component (basis points)											
1	5.8	- 2.9	- 1.1	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0
2	6.2	- 2.2	- 0.4	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0
3	6.5	- 1.7	0.1	- 0.2	- 0.2	- 0.2	0.1	0.0	0.0	0.0	0.0
4	6.6	- 1.2	0.5	- 0.3	- 0.2	0.1	0.0	0.0	- 0.1	0.0	0.0
5	6.6	- 0.8	0.6	- 0.2	- 0.2	0.2	- 0.1	0.0	0.0	0.0	0.0
7	6.5	- 0.1	0.8	0.3	0.2	0.0	0.1	0.0	0.0	0.0	0.0
10	6.0	0.5	0.6	0.3	0.1	0.0	0.0	0.0	0.0	- 0.1	0.0
15	5.6	1.2	0.0	- 0.2	0.3	- 0.1	- 0.2	0.0	0.0	0.0	0.0
20	5.2	1.7	- 0.4	- 0.3	0.2	0.1	0.1	- 0.1	0.0	0.0	0.0
25	4.8	1.8	- 0.4	0.0	- 0.1	0.0	0.1	0.1	0.0	0.0	0.0
30	4.7	1.9	- 0.3	0.2	- 0.3	0.0	- 0.1	0.0	0.0	0.0	0.0

Table 3

Correlation between principal components

(derived from returns)

Correlations between first principal components (1990-93)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.63	0.20	-0.14	-0.09	-0.16	-0.21
CA	0.63	1.00	0.19	-0.16	-0.10	-0.20	-0.16
FR	0.20	0.19	1.00	-0.44	-0.24	-0.18	-0.28
GE	-0.14	-0.16	-0.44	1.00	0.16	0.26	0.13
IT	-0.09	-0.10	-0.24	0.16	1.00	0.08	0.23
JP	-0.16	-0.20	-0.18	0.26	0.08	1.00	0.13
UK	-0.21	-0.16	-0.28	0.13	0.23	0.13	1.00

Correlations between second principal components (1990-93)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.27	0.01	-0.04	-0.01	0.07	0.06
CA	0.27	1.00	-0.05	0.02	0.03	0.00	0.10
FR	0.01	-0.05	1.00	-0.07	-0.08	0.01	-0.13
GE	-0.04	0.02	-0.07	1.00	0.05	0.09	0.14
IT	-0.01	0.03	-0.08	0.05	1.00	-0.01	0.25
JP	0.07	0.00	0.01	0.09	-0.01	1.00	-0.01
UK	0.06	0.10	-0.13	0.14	0.25	-0.01	1.00

Correlations between third principal components (1990-93)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	-0.03	-0.02	-0.11	-0.16	-0.11	-0.01
CA	0.08	1.00	-0.05	-0.01	-0.01	-0.02	0.01
FR	0.02	-0.05	1.00	-0.02	-0.04	0.06	0.02
GE	-0.09	-0.01	-0.02	1.00	-0.09	-0.01	-0.02
IT	0.01	-0.01	-0.04	-0.09	1.00	0.07	0.11
JP	-0.01	-0.02	0.06	-0.01	0.07	1.00	-0.05
UK	-0.02	0.01	0.02	-0.02	0.11	-0.05	1.00

Correlations between first principal components (1994-96)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.72	0.30	0.30	0.19	-0.02	0.39
CA	0.72	1.00	0.30	0.29	0.24	0.02	0.38
FR	0.30	0.30	1.00	0.60	0.40	0.09	0.65
GE	0.30	0.29	0.60	1.00	0.39	0.11	0.54
IT	0.19	0.24	0.40	0.39	1.00	-0.03	0.38
JP	-0.02	0.02	0.09	0.11	-0.03	1.00	0.02
UK	0.39	0.38	0.65	0.54	0.38	0.02	1.00

Correlations between second principal components (1994-96)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	-0.23	-0.06	0.06	0.03	-0.05	-0.16
CA	-0.23	1.00	0.15	0.00	-0.04	0.01	0.11
FR	-0.06	0.15	1.00	0.07	-0.05	0.09	0.19
GE	0.06	0.00	0.07	1.00	-0.14	-0.08	0.07
IT	0.03	-0.04	-0.05	-0.14	1.00	-0.03	-0.05
JP	-0.05	0.01	0.09	-0.08	-0.03	1.00	0.10
UK	-0.16	0.11	0.19	0.07	-0.05	0.10	1.00

Correlations between third principal components (1994-96)

	USA	CA	FR	GE	IT	JP	UK
USA	1.00	0.14	0.09	-0.03	0.00	0.00	0.09
CA	0.14	1.00	0.14	-0.08	0.03	0.00	0.00
FR	0.09	0.14	1.00	0.03	-0.04	0.03	0.05
GE	-0.03	-0.08	0.03	1.00	-0.10	-0.10	0.03
IT	0.00	0.03	-0.04	-0.10	1.00	0.00	0.07
JP	0.00	0.00	0.03	-0.10	0.00	1.00	0.07
UK	0.09	0.01	0.05	0.03	0.07	0.07	1.00

Note: The table reports correlations between the first, second and third principal components derived from each country's government security zero coupon returns.

Table 4

Correlation between principal components

(derived from standardised returns)

	Correlations between first principal components (1990-93)								**Correlations between first principal components (1994-96)**						
	USA	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**		**USA**	**CA**	**FR**	**GE**	**IT**	**JP**	**UK**
USA	1.00	0.62	0.21	0.07	0.06	0.14	0.18	USA	1.00	0.76	0.30	0.35	0.20	0.01	0.42
CA	0.62	1.00	0.20	0.11	0.07	0.14	0.16	CA	0.76	1.00	0.33	0.37	0.30	0.03	0.41
FR	0.21	0.20	1.00	0.44	0.31	0.16	0.33	FR	0.30	0.33	1.00	0.64	0.54	0.13	0.66
GE	0.07	0.11	0.44	1.00	0.22	0.16	0.20	GE	0.35	0.37	0.64	1.00	0.49	0.15	0.60
IT	0.06	0.07	0.31	0.22	1.00	0.07	0.30	IT	0.20	0.30	0.54	0.49	1.00	-0.01	0.48
JP	0.14	0.14	0.16	0.16	0.07	1.00	0.14	JP	0.01	0.03	0.13	0.15	-0.01	1.00	0.05
UK	0.18	0.16	0.33	0.20	0.30	0.14	1.00	UK	0.42	0.41	0.66	0.60	0.48	0.05	1.00
	Correlations between second principal components (1990-93)								**Correlations between second principal components (1994-96)**						
USA	1.00	0.30	0.04	-0.06	-0.03	-0.08	0.10	USA	1.00	0.35	0.13	-0.14	0.05	0.02	0.22
CA	0.30	1.00	0.08	-0.09	-0.10	-0.07	0.12	CA	0.35	1.00	0.13	-0.09	0.06	0.01	0.12
FR	0.04	0.08	1.00	-0.22	-0.01	-0.04	0.12	FR	0.13	0.13	1.00	-0.24	0.05	0.01	0.27
GE	-0.06	-0.09	-0.22	1.00	0.01	0.17	-0.17	GE	-0.14	-0.09	-0.24	1.00	0.06	0.04	-0.05
IT	-0.03	-0.10	-0.01	0.01	1.00	0.01	-0.35	IT	0.05	0.06	0.05	0.06	1.00	0.08	0.03
JP	-0.08	-0.07	-0.04	0.17	0.01	1.00	-0.01	JP	0.02	0.01	0.01	0.04	0.08	1.00	0.00
UK	0.10	0.12	0.12	-0.17	-0.35	-0.01	1.00	UK	0.22	0.12	0.27	-0.05	0.03	0.00	1.00
	Correlations between third principal components (1990-93)								**Correlations between third principal components (1994-96)**						
USA	1.00	0.08	0.02	-0.09	0.01	-0.01	-0.02	USA	1.00	0.14	0.09	-0.03	0.00	0.00	0.09
CA	0.08	1.00	-0.05	-0.01	-0.01	-0.02	0.01	CA	0.14	1.00	0.14	-0.08	0.03	0.00	0.01
FR	0.02	-0.05	1.00	-0.02	-0.04	0.06	0.02	FR	0.09	0.14	1.00	0.03	-0.04	0.03	0.05
GE	-0.09	-0.01	-0.02	1.00	-0.09	-0.01	-0.02	GE	-0.03	-0.08	0.03	1.00	-0.10	-0.10	0.03
IT	0.01	-0.01	-0.04	-0.09	1.00	0.07	0.11	IT	0.00	0.03	-0.04	-0.10	1.00	0.00	0.07
JP	-0.01	-0.02	0.06	-0.01	0.07	1.00	-0.05	JP	0.00	0.00	0.03	-0.10	0.00	1.00	0.07
UK	-0.02	0.01	0.02	-0.02	0.11	-0.05	1.00	UK	0.09	0.01	0.05	0.03	0.07	0.07	1.00

Note: The table reports correlations between the first, second and third principal components derived from each country's standardised government security zero coupon returns.

Table 5

Explanatory power of principal components (1994-96)

(percent of variance explained)

Country	**First 5**	**First 10**	**First 20**	**Country**	**First 5**	**First 10**	**First 20**	**Country**	**First 5**	**First 10**	**First 20**
Canada 1	47%	48%	79%	**Italy 1**	43%	56%	78%	**UK 1**	27%	39%	65%
2	59%	59%	89%	**2**	57%	78%	92%	**2**	45%	65%	87%
3	67%	67%	94%	**3**	61%	83%	95%	**3**	51%	74%	91%
4	74%	74%	98%	**4**	68%	89%	97%	**4**	57%	82%	95%
5	77%	78%	98%	**5**	69%	95%	98%	**5**	60%	86%	97%
7	84%	84%	99%	**7**	59%	98%	98%	**7**	64%	93%	99%
10	91%	91%	99%	**10**	75%	99%	100%	**10**	64%	97%	99%
15	90%	91%	99%	**15**	94%	100%	100%	**15**	60%	99%	99%
20	97%	97%	100%	**20**	95%	100%	100%	**20**	58%	97%	100%
25	99%	100%	100%	**25**	98%	100%	100%				
30	97%	98%	100%	**30**	95%	100%	100%	**US 1**	52%	53%	87%
France 1	29%	31%	81%	**Japan 1**	4%	44%	63%	**3**	72%	73%	97%
2	56%	57%	91%	**2**	5%	68%	85%	**4**	78%	79%	99%
3	66%	67%	97%	**3**	5%	76%	92%	**5**	82%	82%	99%
4	74%	75%	98%	**4**	5%	82%	96%	**7**	88%	88%	100%
5	80%	81%	98%	**5**	5%	85%	97%	**10**	93%	94%	99%
7	92%	93%	98%	**7**	4%	90%	96%	**15**	97%	98%	100%
10	94%	95%	99%	**10**	4%	95%	99%	**20**	98%	99%	99%
15	96%	97%	99%	**15**	3%	89%	98%	**25**	98%	100%	100%
20	97%	99%	100%	**20**	4%	91%	100%	**30**	98%	99%	100%
25	98%	100%	100%								
30	97%	99%	100%								
Germany 1	26%	27%	54%								
2	45%	48%	83%								
3	56%	59%	87%								
4	66%	67%	93%								
5	71%	73%	92%								
7	79%	84%	98%								
10	88%	88%	98%								
15	91%	99%	100%								
20	90%	100%	100%								
25	98%	99%	100%								
30	85%	100%	100%								

Note: The table reports the percent of total variance of returns for government zero coupon bonds of the specified country and maturity explained by either the first 5, first 10, or the first 20 principal components derived from returns for all the securities.

Table 6

Explanatory power of principal components (1994-96)

(percent of variance explained)

Country	First 5	First 10	First 20
Canada 1	14%	57%	76%
2	25%	80%	90%
3	33%	88%	95%
4	42%	93%	98%
5	46%	94%	98%
7	56%	95%	99%
10	71%	95%	100%
15	89%	93%	100%
20	98%	98%	100%
25	99%	100%	100%
30	97%	100%	100%
France 1	21%	37%	68%
2	34%	60%	88%
3	40%	66%	91%
4	47%	72%	94%
5	50%	75%	96%
7	61%	82%	95%
10	74%	88%	97%
15	85%	90%	99%
20	94%	95%	99%
25	98%	98%	100%
30	97%	98%	100%
Germany 1	5%	18%	58%
2	8%	26%	70%
3	10%	32%	78%
4	13%	40%	85%
5	15%	43%	87%
7	20%	53%	94%
10	22%	55%	98%

Country	First 5	First 10	First 20
Italy 1	15%	80%	82%
2	27%	88%	90%
3	42%	89%	93%
4	62%	91%	93%
5	77%	90%	92%
7	87%	89%	100%
10	80%	99%	100%
Japan 1	2%	16%	43%
2	3%	35%	63%
3	5%	47%	72%
4	5%	64%	83%
5	6%	75%	87%
7	7%	94%	97%
10	7%	94%	98%

Country	First 5	First 10	First 20
UK 1	11%	66%	90%
2	19%	80%	95%
3	26%	84%	99%
4	33%	88%	99%
5	41%	90%	99%
7	61%	93%	98%
10	78%	89%	100%
15	81%	86%	100%
20	93%	99%	100%
US 1	28%	39%	72%
2	39%	50%	86%
3	48%	57%	92%
4	56%	64%	96%
5	62%	69%	97%
7	73%	77%	99%
10	86%	88%	99%
15	94%	95%	98%
20	99%	99%	99%
25	99%	100%	100%
30	99%	99%	100%

Note: The table reports the percent of total variance of returns for government zero coupon bonds of the specified country and maturity explained by either the first 5, first 10, or the first 20 principal components derived from returns for all the securities.

Table 7: Garch Models

(Maximum Likelihood Estimates, January 1994 to September 1996)*

Country	**Principal Component**	**Constant**	**Lagged Conditioning Variables**		
			Squared Error	**Variance**	**Squared Error (Error < 0)**
Canada	1	0.326	0.206	0.556	-0.091*
	2	0.488	0.296	0.285	-0.026*
	3	0.875	0.129	0.194*	-0.135
France	1	0.105	0.214	0.750	-0.067*
	2	0.713	-0.014*	0.178*	0.219
	3	0.620	0.071	0.125	0.405
Germany **	1	0.190	0.400	0.476	0.261
	2	0.003	0.264	0.710	0.573
	3	0.302	0.212	0.417	0.342
Italy	1	0.023	0.135	0.880	-0.012*
	2	0.003	0.411	0.790	-0.205
	3	0.094	0.235	0.757	-0.068*
Japan	1	0.085	0.228	0.705	0.139
	2	0.285	0.375	0.359	0.059*
	3	0.260	0.162	0.555	0.111
United Kingdom	1	0.238	0.249	0.621	-0.160
	2	0.369	0.368	0.299	-0.017*
	3	0.111	0.214	0.693	0.074*
United States	1	1.560	-0.001*	-0.495	-0.104
	2	0.814	0.284	-0.051*	-0.034*
	3	0.131	-0.012*	0.774	0.296

* significant at the 5% level
** Germany's coefficients are estimated from March 1994 to September 1996.

CHART 1: EXPLANTORY POWER OF PRINCIPAL COMPONENTS (94-96)
Contour Lines Represent Explanatory Power in Steps of 10%

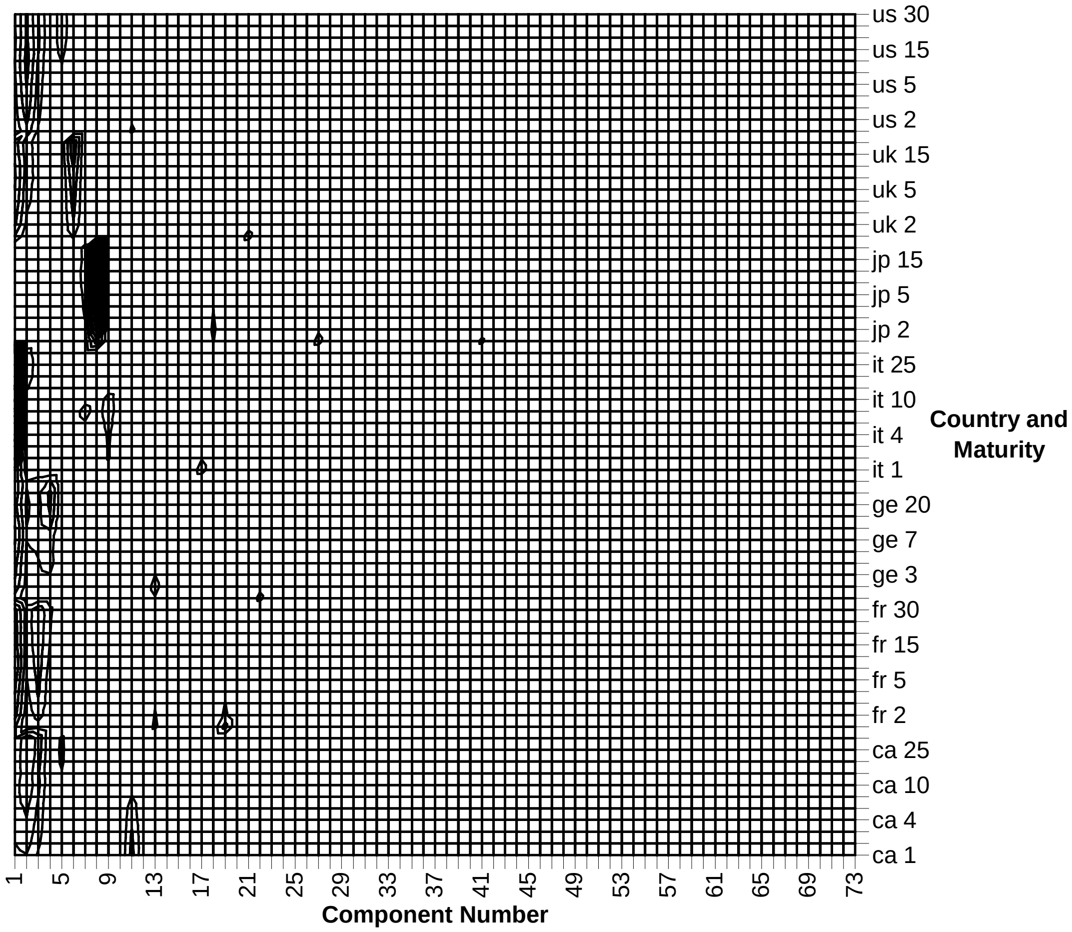


CHART 2: EXPLANATORY POWER OF PRINCIPAL COMPONENTS (90-93)
Contour Lines Represent Explanatory Power in Steps of 10%

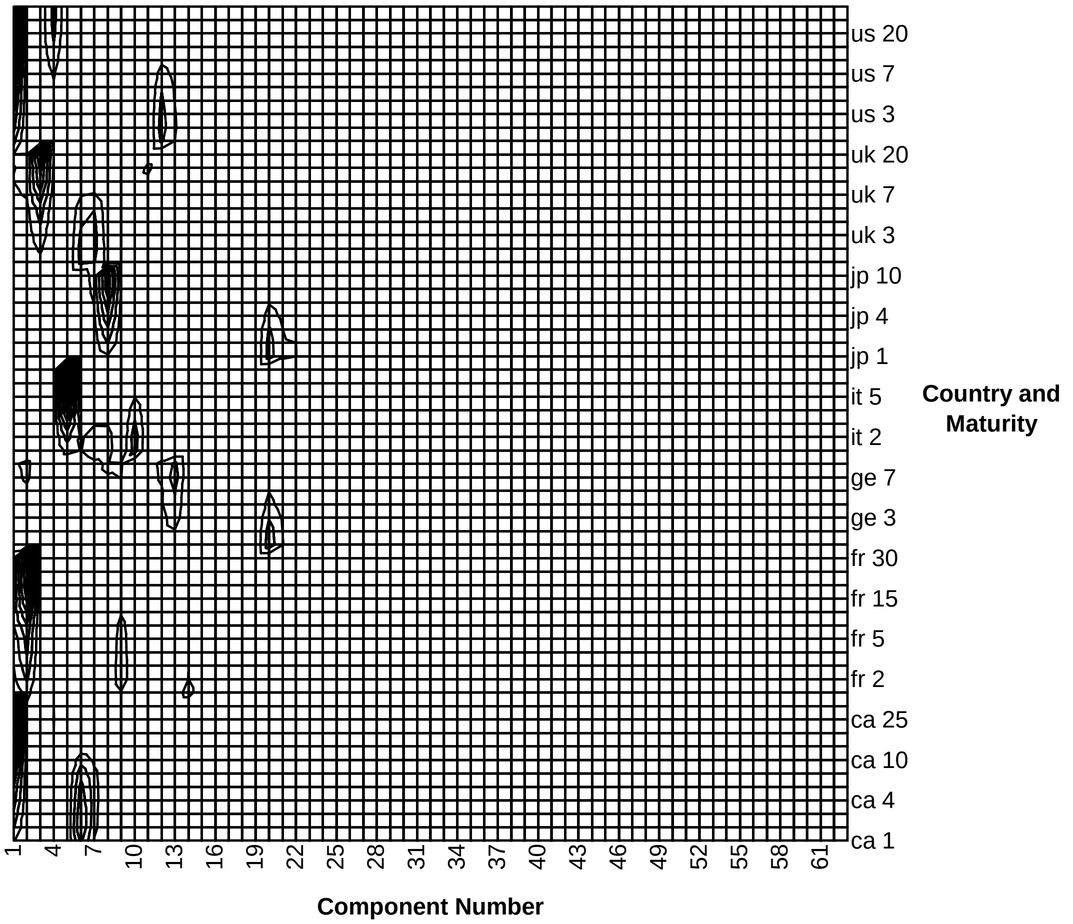
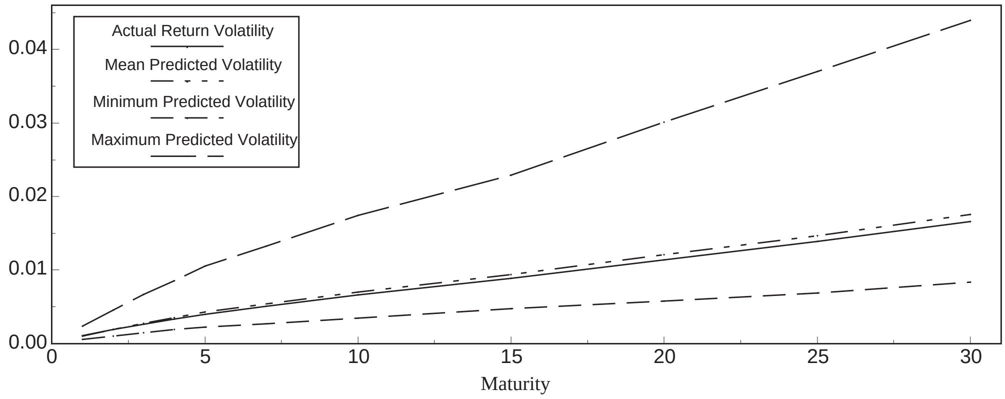
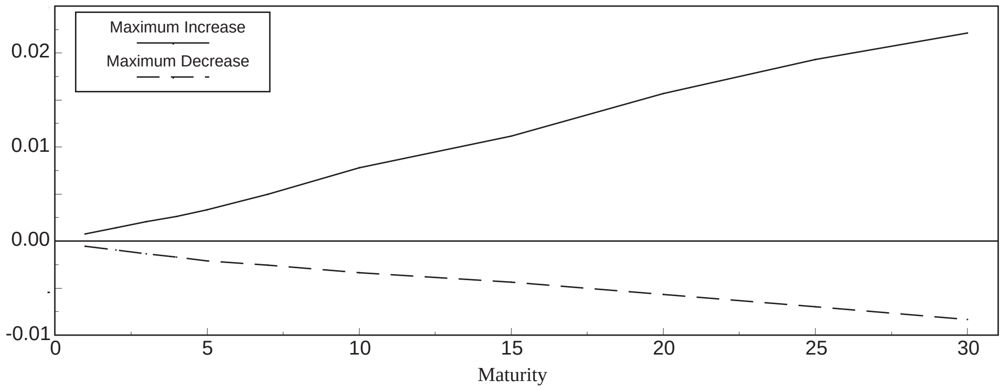


Chart 3a: Volatility Predictions, Canada

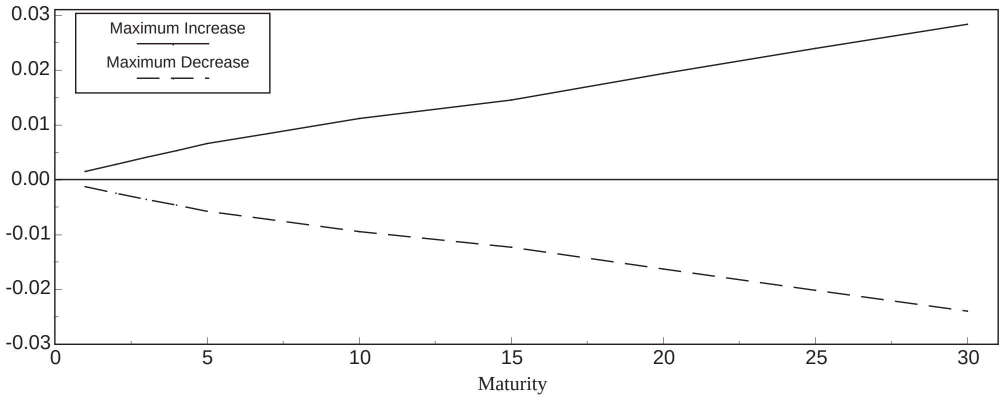
Canada Zero Return Volatilities (1994 to 1996)



One-Day Changes in Canada Zero Return Volatilities (1994 to 1996)



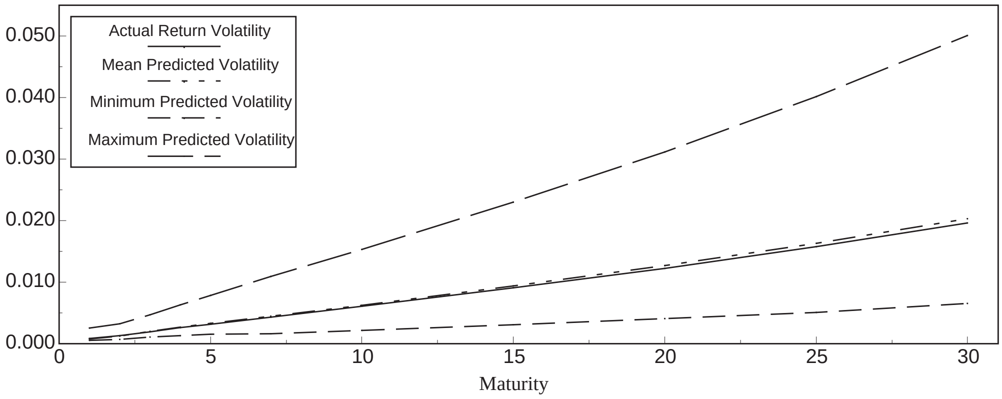
Five-Day Changes in Canada Zero Return Volatilities (1994 to 1996)



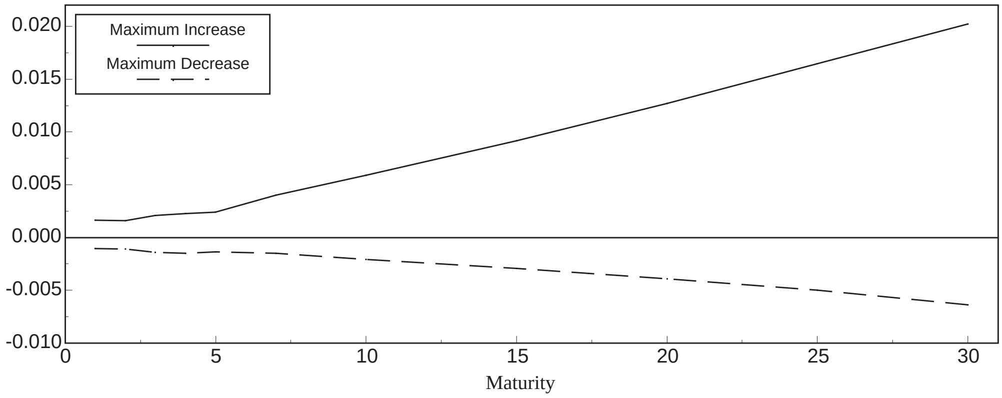
Volatility

Chart 3b: Volatility Predictions, France

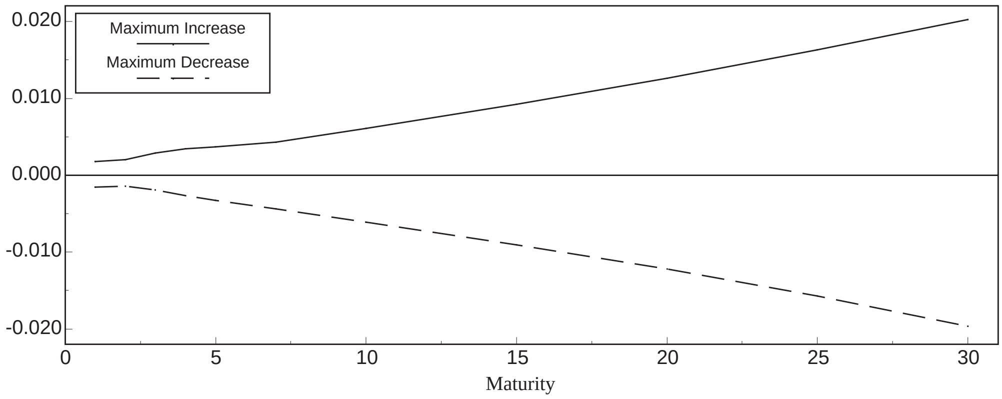
France Zero Return Volatilities (1994 to 1996)



One-Day Changes in France Zero Return Volatilities (1994 to 1996)

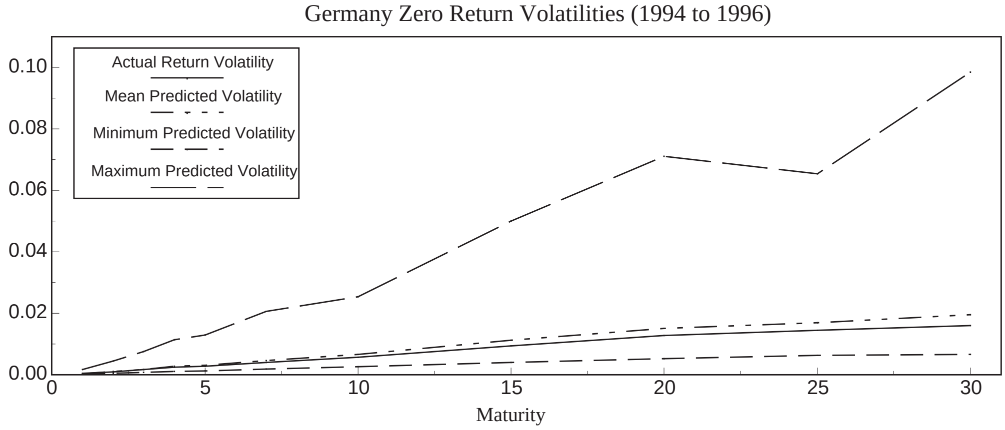


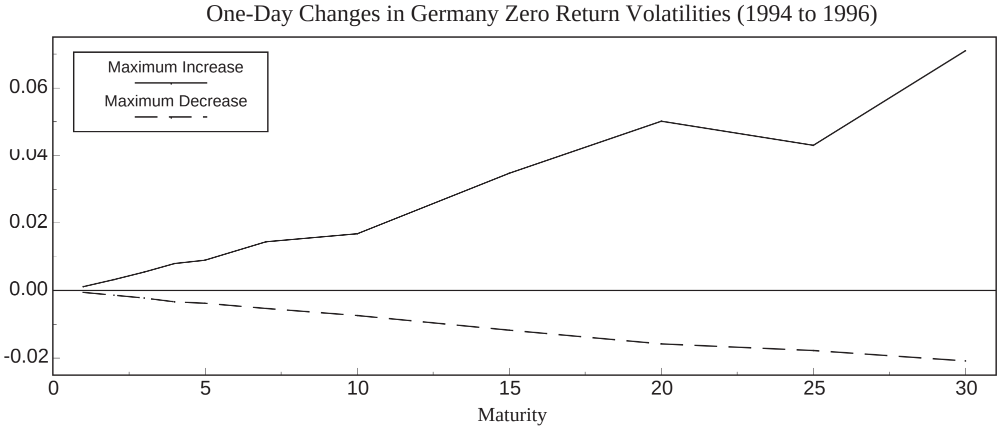
Five-Day Changes in France Zero Return Volatilities (1994 to 1996)

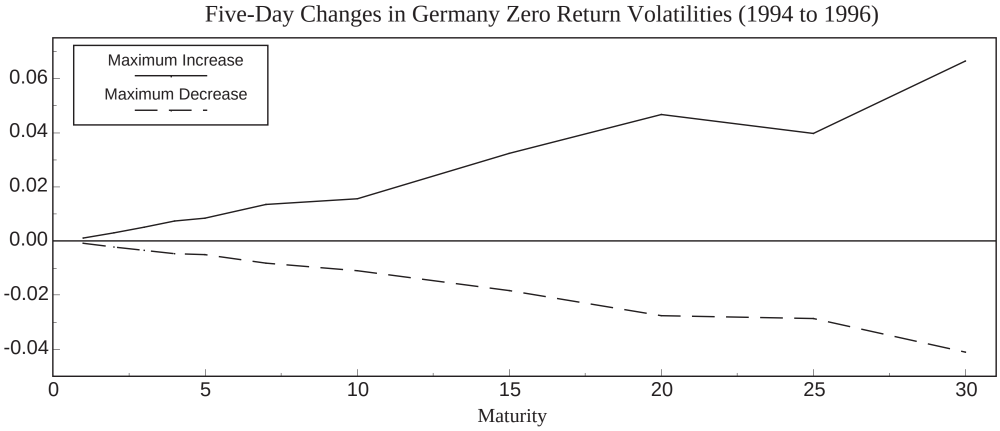


Volatility

Chart 3c: Volatility Predictions, Germany

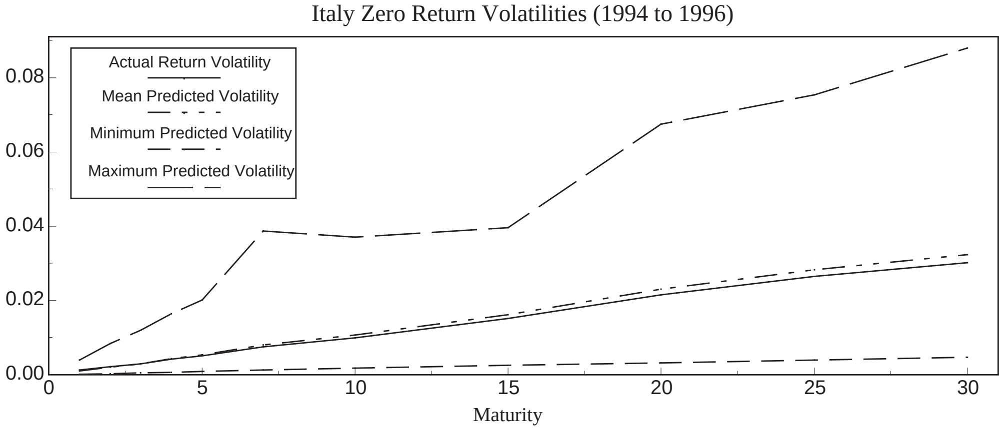


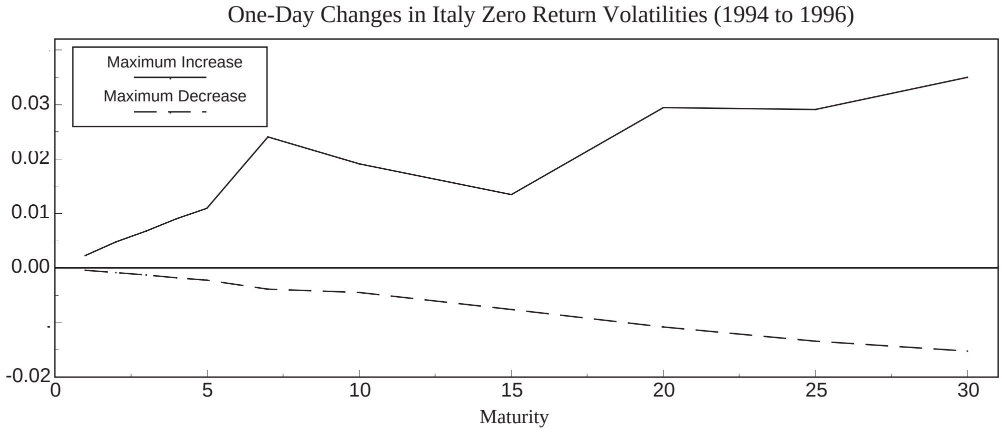


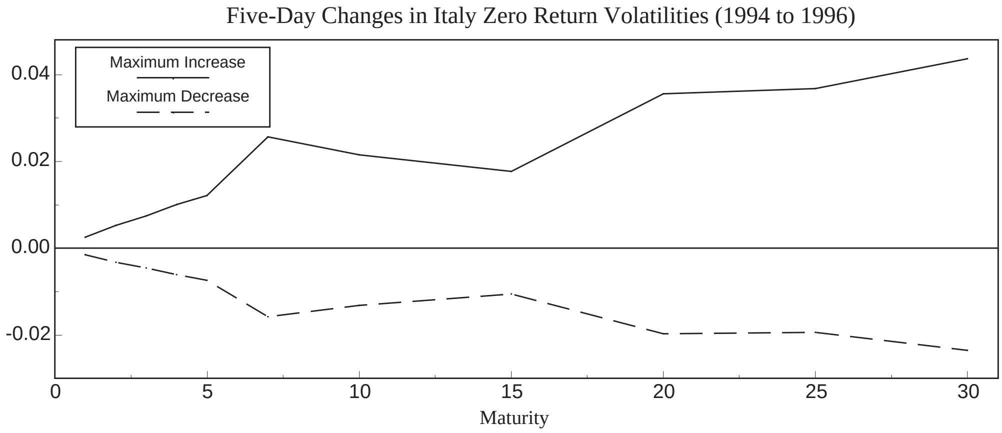


Volatility

Chart 3d: Volatility Predictions, Italy

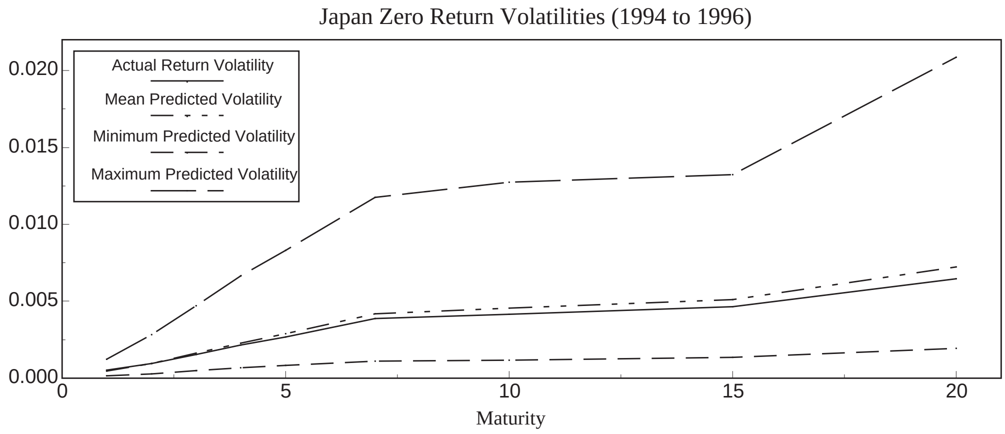


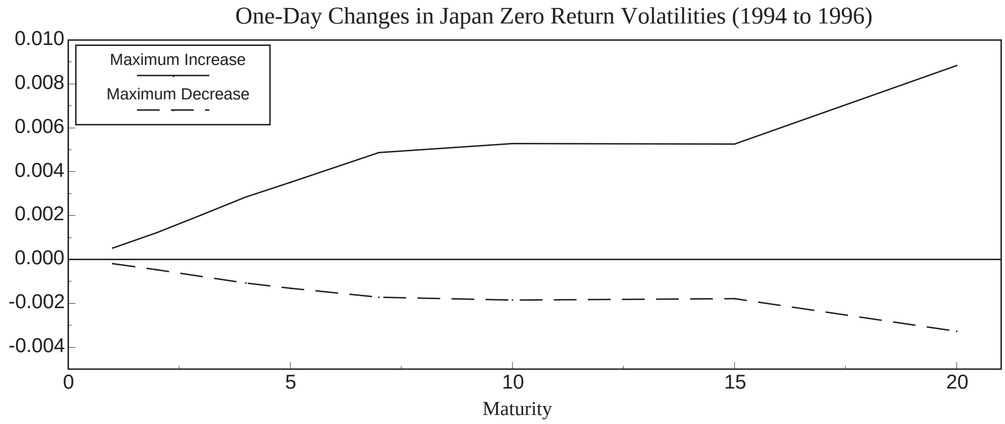




Volatility

Chart 3e: Volatility Predictions, Japan





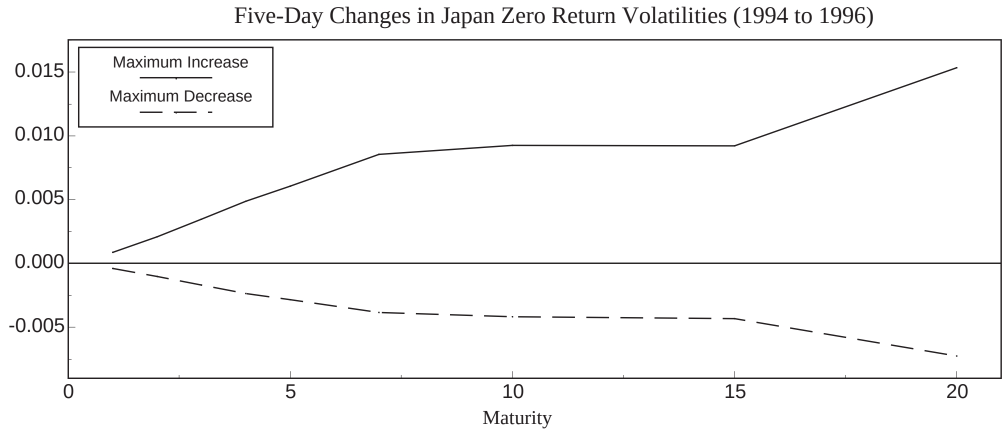
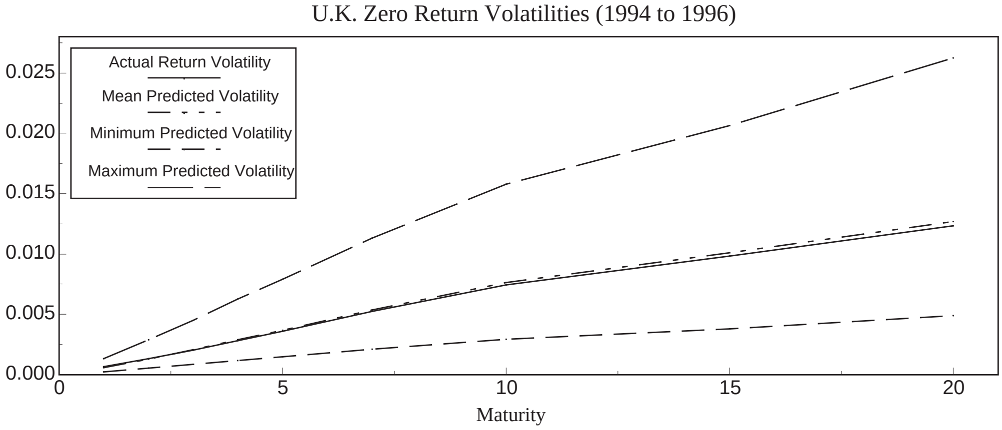
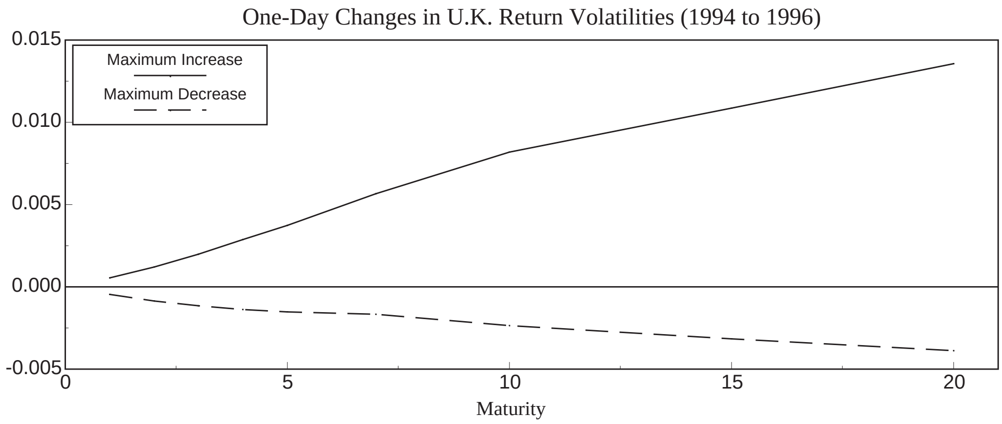


Chart 3f: Volatility Predictions, United Kingdom





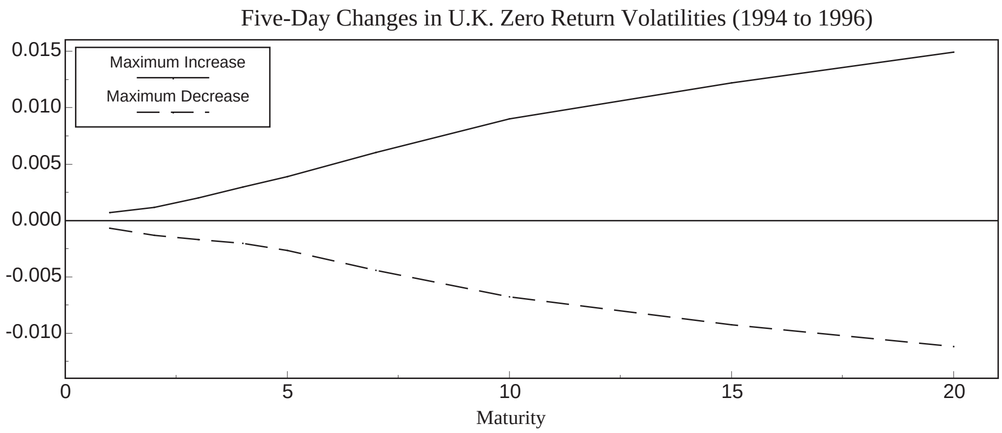
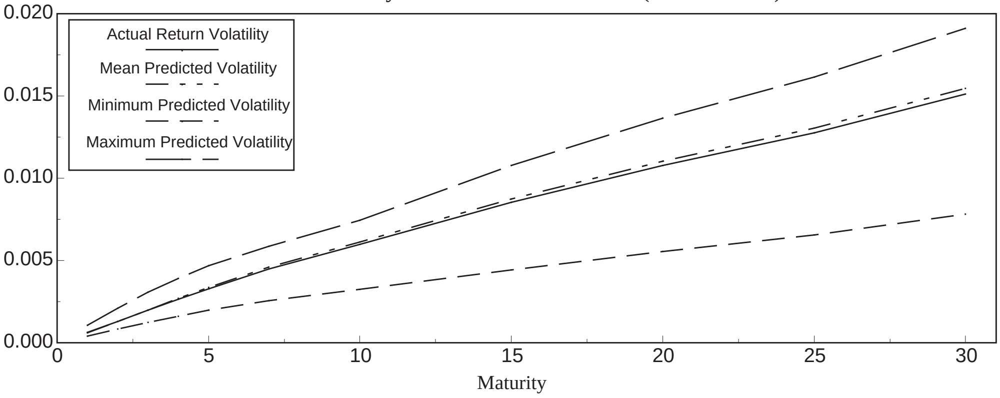
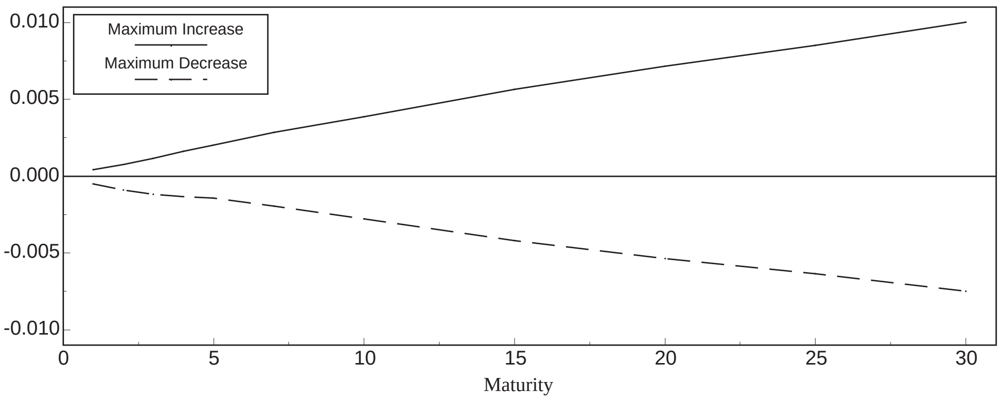


Chart 3g: Volatility Predictions, United States

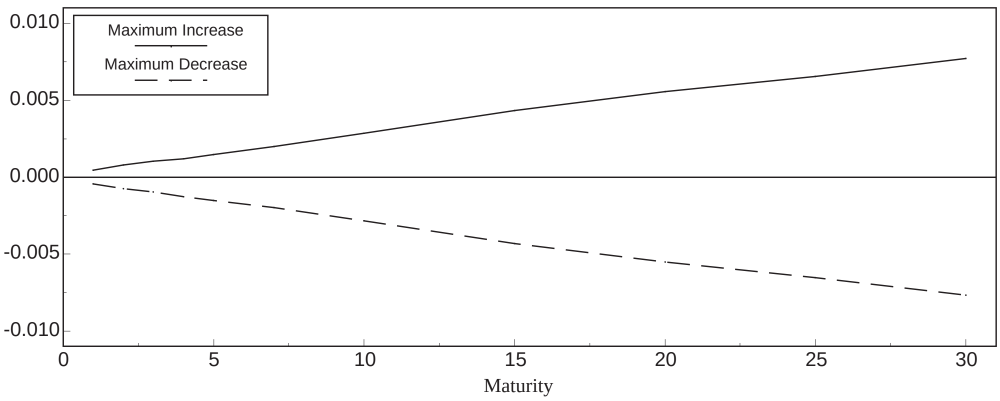
U.S. Treasury Zero Return Volatilities (1994 to 1996)



One-Day Changes in U.S. Treasury Zero Return Volatilities (1994 to 1996)



Five-Day Changes in U.S. Treasury Zero Return Volatilities (1994 to 1996)



Volatility

Approximation of changes in option values and hedge ratios: how large are the errors?

by

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Federal Reserve Bank of New York

March 1997

Abstract

When changes in portfolio values must be calculated over asset price shocks of different sizes such as in stress tests over a variety of scenarios, constraints imposed by computation speed as well as database structure sometimes lead analysts to use approximations to portfolio values instead of exact portfolio revaluations. This paper examines the effect of time to maturity and moneyness of an option on the magnitude of the approximation error. In addition, the approximation error is also examined in a portfolio consisting of the outstanding stock of options on Eurodollar interest rate futures on the Chicago Mercantile Exchange.

* The views expressed in this paper are the authors' and do not necessarily reflect the positions of the Federal Reserve Bank of New York, the Federal Reserve System, the Bank for International Settlements, or the Eurocurrency Standing Committee.

Approximation of changes in options values and hedge ratios: how large are the errors?

When changes in portfolio values must be calculated over asset price shocks of different sizes such as in stress tests over a variety of scenarios, constraints imposed by computation speed as well as database structure sometimes lead analysts to use approximations to portfolio values instead of exact portfolio revaluations (see Gibson 1997). Given this tradeoff between computation and database costs and accuracy, this paper examines the magnitude of the approximation error in the most common approximation methods. Other papers that provide insight on this issue would include papers by Estrella (1996), Pritsker (1996), Robinson (1996), and Rouvinez (1997).

The paper examines first and second order Taylor series approximations of option values and hedge ratios. For option values, the second order approximation is the option's delta plus gamma, and for the hedge ratio, the second order approximation is the gamma plus the third derivative of the option value. The approximations in the paper are estimates around a single price (the initial price). Better approximations can be obtained by means of piecewise approximations over a number of different "initial" prices. However, such approaches would be more demanding in computation time and might undercut one principal motivation for approximations: computation speed.

In Section 1, the effect of time to maturity and moneyness on the approximation error are examined. To consider the practical significance of the maturity and moneyness effects, the magnitude of the approximation error in a market portfolio is addressed in Section 2. Using the open interest of options on Eurodollar interest rate futures on the Chicago Mercantile Exchange, some insight to the size of the approximation error in practice can be obtained. In both sections, the option parameters of interest are the value of the option and the hedge ratio. The hedge ratio and its change may be of interest to analysts or risk managers who need to anticipate the volume of hedge transactions required after a sharp change in the price of the underlying asset.

The options examined are interest rate options, and the approximation error is examined for interest rate changes equal to a largest one day change and a 99th percentile two-week change. In both cases, movements in 3-month Eurodollar rates over the period 1/91 through 12/95 were expressed in percentage changes, and the largest and 99th percentile changes were applied to the initial interest rate. The option valuation function used was Black's forward interest rate option model (Hull 1993).

1. Effects of maturity and moneyness

Change in option value

As expected, a first order approximation (delta only) leads to large approximation errors across a wide range of maturities and strike prices. Errors relative to the true change are above 35% across a wide range (Figure 1, left panel).

Second order approximations (delta and gamma) produce relatively small approximation errors across a wide range of strikes and maturities. The error relative to the true change, however, are large in the case of deep out-of-the money options with short maturities (Figure 1, right panel). For a three month option with a strike 80 bp out-of-the money, the error is 10% of the true change, while for a six month option the error is less than 5%.

Figure 2 shows that in absolute terms, without adjusting for relative size, the largest approximation errors occur in options that are close-to-the-money.

Change in hedge

Relative to the true change in the hedge ratio, the approximation error is large across a wide range of maturity and strike price combinations -- for both the first order (gamma only) and second order (gamma and third derivative) approximations (Figure 3). In the case of the gamma only approximation, for a three month option with a strike 80 bp in the money (strike=0.052), the error is 100% of the true change in hedge, and for a six month option the error is 50%.

Relative to the initial hedge, the approximation error is large for deep out-of-the money options with short maturities (Figure 4). In the case of the gamma only approximation, for a two month option with a strike 80 bp out of the money (strike=0.068), the approximation error is 160% of the initial hedge position, while for a six month option the error falls to 15%.

Figure 5 shows that in absolute terms, without adjusting for relative size effects, the largest approximation errors occur in options that are close-to-the-money.

2. Portfolio effects

This section examines the approximation error for a portfolio consisting of the outstanding stock of options on Eurodollar interest rate futures traded on the Chicago Mercantile Exchange. The data for each option consists of: the strike price, the option maturity, the amount outstanding (open interest), and the market value of the option. Table 1 lists some features of this option portfolio. This portfolio does not include contracts whose open interest was very small relative to other contracts.

Estimation

For each option, the implied volatility that returns the observed market price was estimated using Black's forward interest rate option model (Hull 1993). Given this estimated implied volatility, the approximations to the change in each option's value and the change in its hedge ratio were calculated. Finally, the value of each option was weighted by its amount outstanding, and summed to produce a portfolio value. For the option value, this weighted sum is the value of the option portfolio, for the hedge ratio, the weighted sum is the weighted average of the hedge ratios of each option.

Table 1

Portfolio of interest Rate Caps
(Puts on Eurodollar Futures)

Maturity	Number of Strikes	Range of Strikes	At-the-money Rate
2 months	10	4% - 6.25%	5.65%
5 months	14	4% - 7.5%	5.72%
8 months	19	3.5% - 8.25%	5.87%
11 months	18	4.5% - 9%	6%
14 months	18	4.5% - 9.25%	6.17%
17 months	12	4.25% - 9%	6.24%

Data as of October 14, 1996

Change in portfolio value

In the case of second order approximations (delta and gamma), approximation errors relative to the true values of interest rate caps are large for decreases in interest rates (Table 2). A similar, but opposite, relationship holds for interest rate floors because of the reversal of the moneyness relationship between puts and calls. While approximation errors would be the same for puts and calls with the same strike because of put call parity, the cap and floor portfolios have different strike distributions.

Table 2

Relative Approximation Errors of Portfolio Value
Interest Rate Cap Portfolio
99th percentile two-week change in rates

	Fall in rates	Rise in rates
Relative to change in value	22%	2%
Relative to initial portfolio value	16%	4%

In comparison to the figure in Table 2, the largest one-day change in rates produces an error of 6% relative to both the change and level of portfolio value, in the case of a fall in rates.

Change in portfolio hedge

For first order approximations (gamma only) to the change in the portfolio hedge ratio, approximation errors relative to true values are large for interest rate decreases (Table 3).

Table 3

Relative Approximation Errors of the Portfolio Hedge
Interest Rate Cap Portfolio
99th percentile two-week change in rates

	Fall in rates	**Rise in rates**
Relative to hedge adjustment	58.1%	2.0%
Relative to initial hedge position	39.1%	2.1%

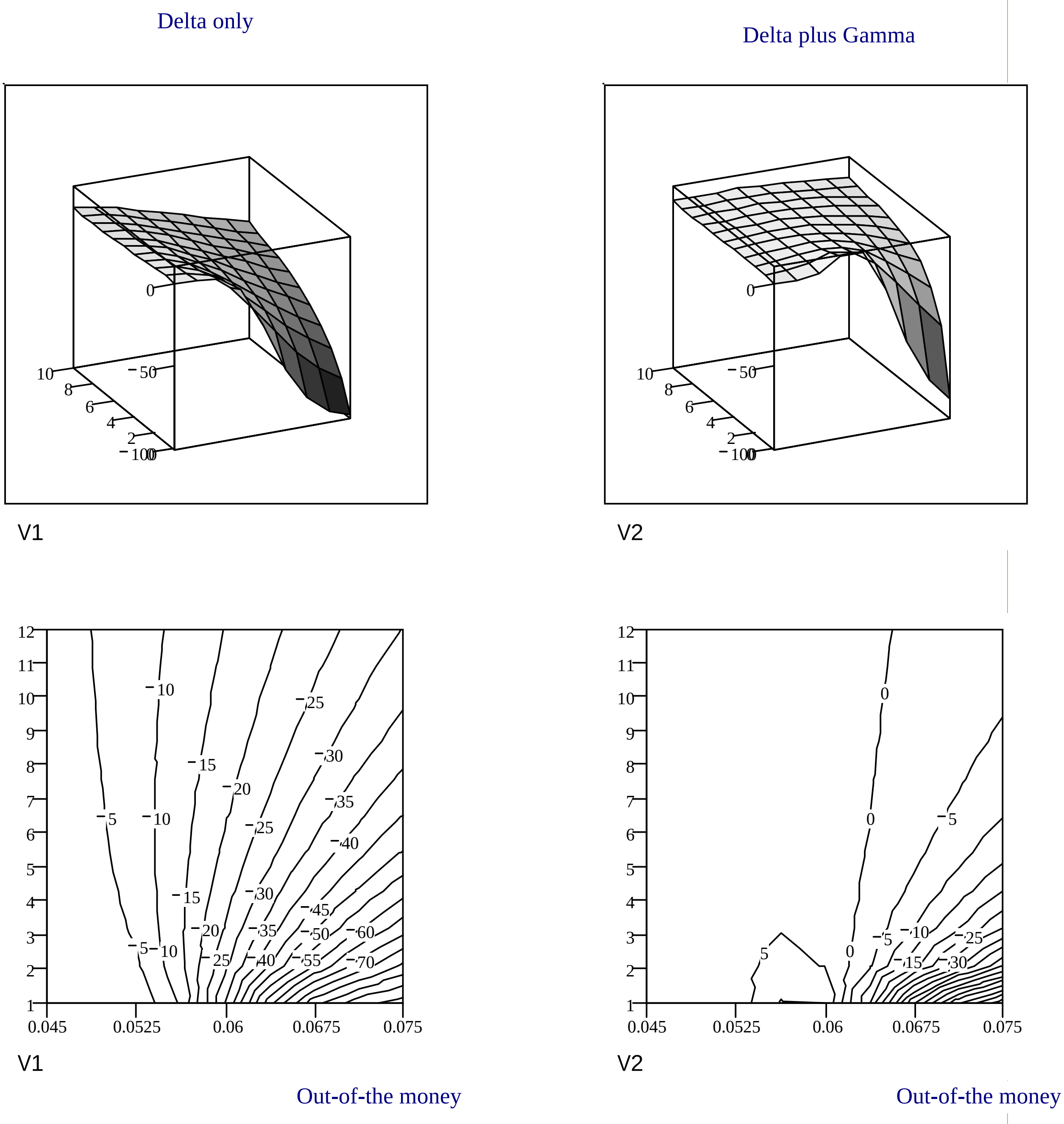
For comparison with the figures in Table 3, the largest one-day change in rates produces an approximation error of 30% relative to the hedge adjustment and 14% relative to the initial hedge in the case of a fall in rates. The approximation error in hedge adjustments is not monotonically increasing in the size of the shock, because the sign of gamma changes at the at-the-money strike, and the delta function is bounded from above.

FIGURE 1

Change in option value: Approximation error relative to true change (%)

Maturity and strike variations

IR Caps



Out-of-the money

Out-of-the money

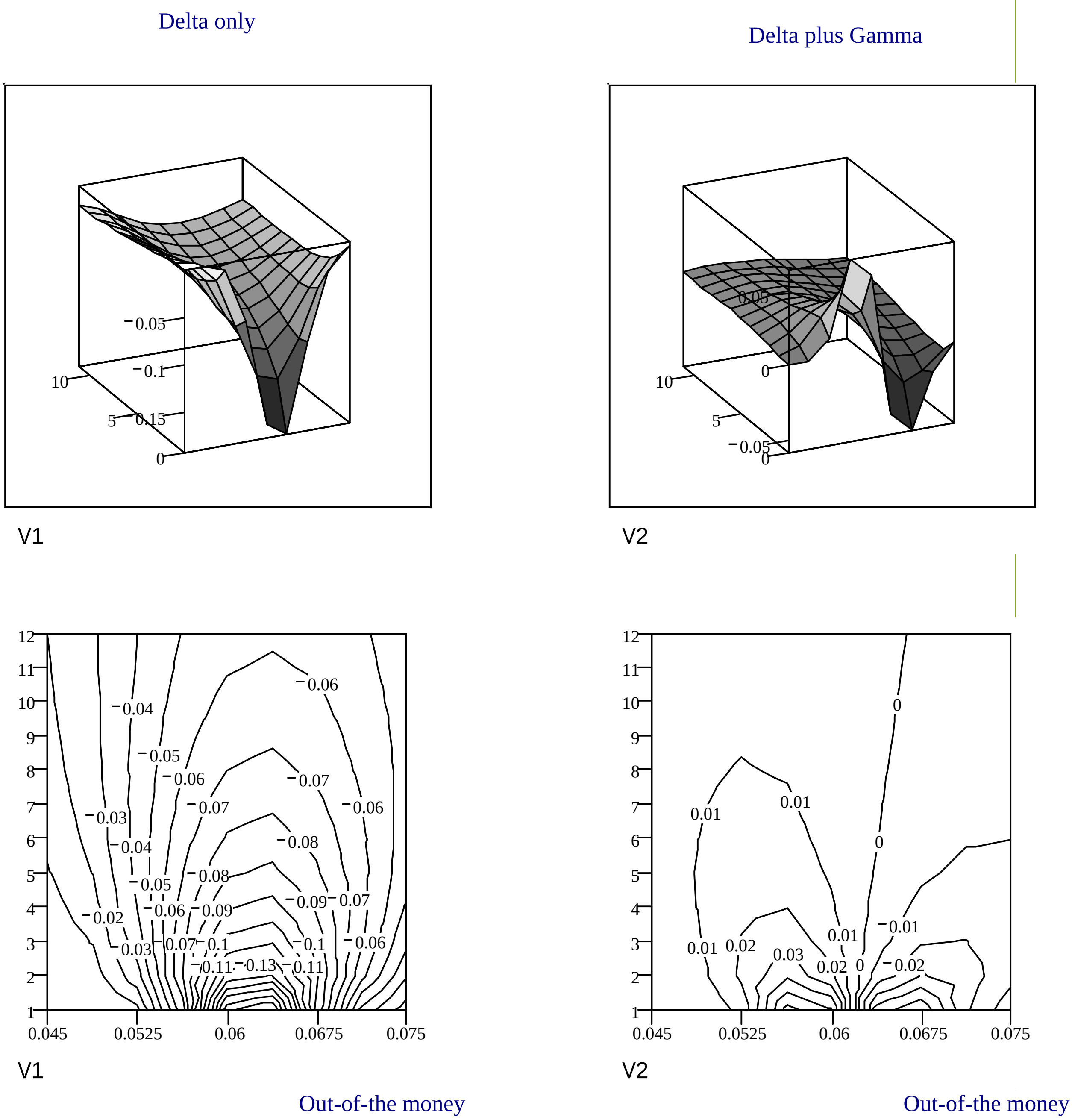
Strike rates range from 0.045 to 0.075

at-the-money strike is 0.06

Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 2
Option value: Approximation error
Maturity and strike variations
IR Caps



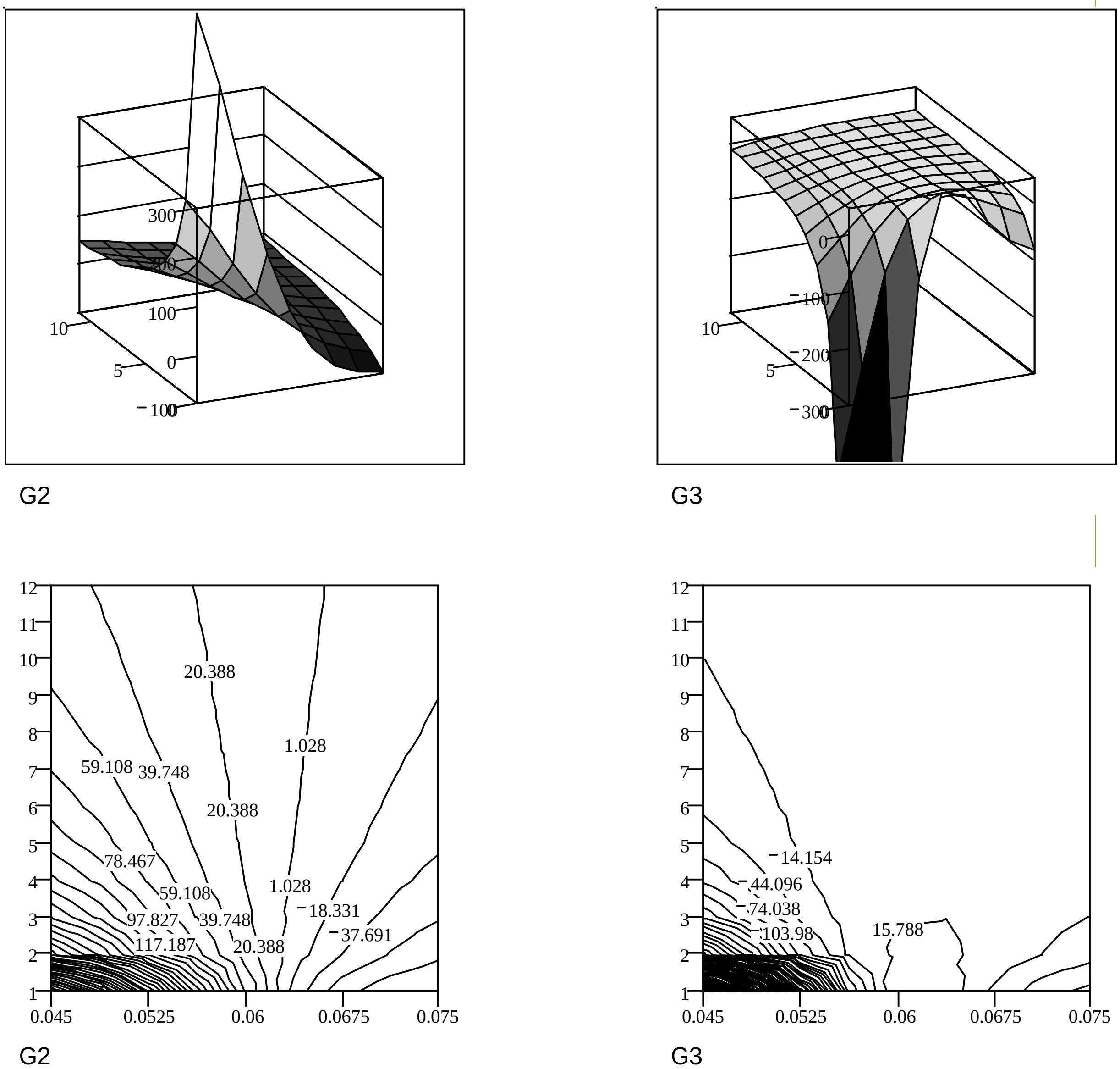
Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 3
Change in hedge: Approximation error relative to change in hedge (%)
Maturity and strike variations
IR Caps

Gamma only

Gamma and third derivative



Out-of-the money

Out-of-the money

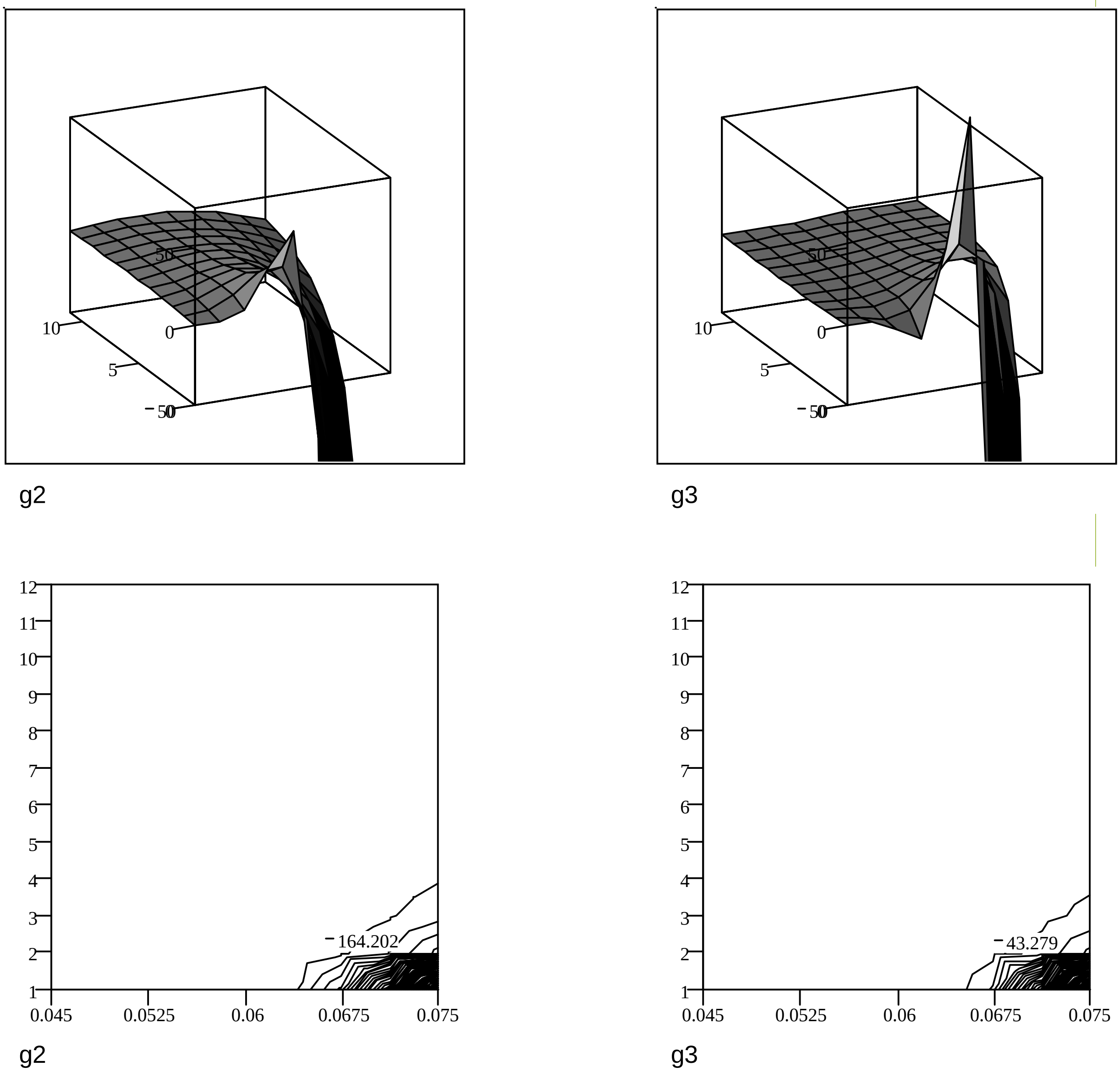
Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 4
Change in hedge: Error relative to (old) hedge position (%)
Maturity and strike variations
IR Caps

Gamma only

Gamma and third derivative



Out-of-the money

Out-of-the money

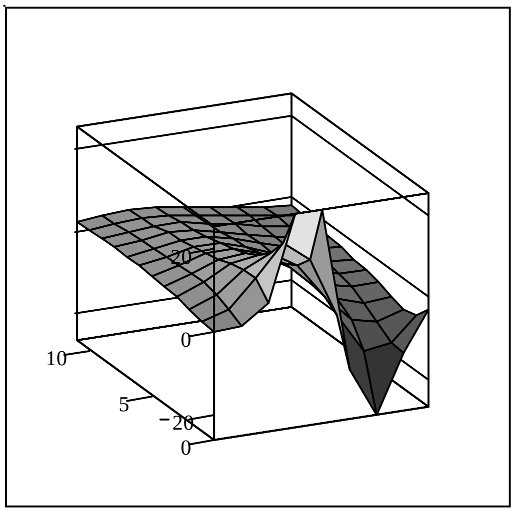
Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

FIGURE 5
Change in hedge: Approximation error
Maturity and strike variations
IR Caps

Gamma only

Gamma and third derivative



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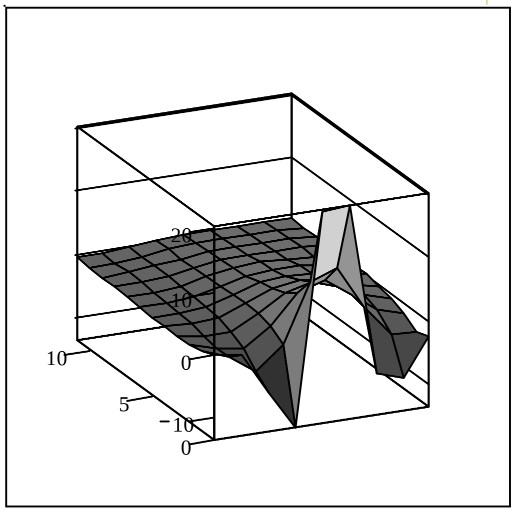
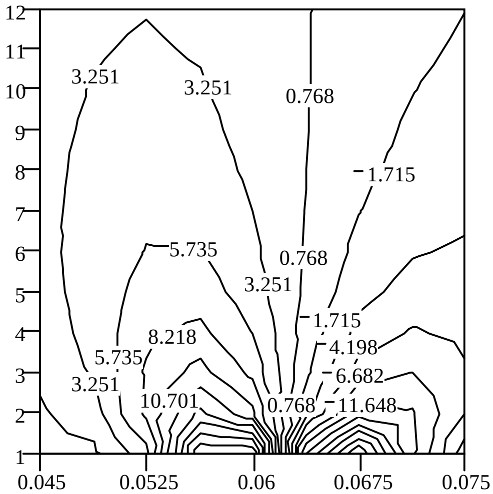


G3



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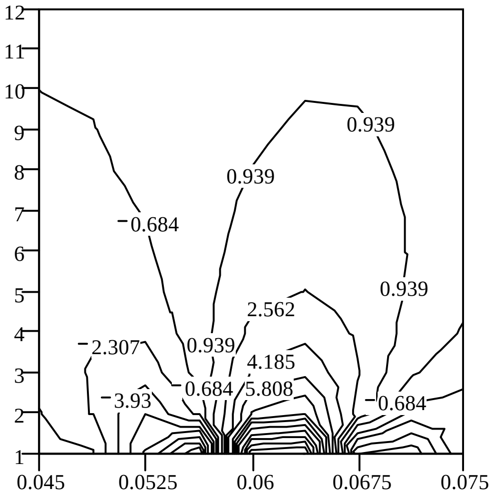

G3

Out-of-the money

Out-of-the money

Strike rates range from 0.045 to 0.075
at-the-money strike is 0.06
Maturity range from 1 month to 12 months

99th percentile 2 week IR change

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Information systems for risk management

by

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Abstract

Risk management information systems are designed to overcome the problem of aggregating data across diverse trading units. The design of an information system depends on the risk measurement methodology that a firm chooses. Inherent in the design of both a risk management information system and a risk measurement methodology is a tradeoff between the accuracy of the resulting measures of risk and the burden of computing them. Technical progress will make this tradeoff more favorable over time, leading firms to implement more accurate methodologies, such as full revaluation of nonlinear positions. The current and likely future improvements in risk management information systems make feasible new ways of collecting aggregate data on firms' risk-taking activities.

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1. Introduction

To lay a foundation for a discussion of the role of information systems in risk management, we must first define the business needs that drive financial firms to implement risk management functions. We see three such needs:

1. To better understand the risks it is taking, a firm wants to measure them. Risks that lend themselves to quantification, which are the only risks discussed in this paper, include market risk (the sensitivity of a firm's value to financial market variables like interest rates, exchange rates, volatilities, etc.) and credit risk (the sensitivity of a firm's value to default by its counterparties).[1]

2. To provide better incentives to its business units and to individual employees, a firm wants to reward good risk-adjusted performance. The firm must measure its risk before it can adjust performance for risk.

3. To provide its shareholders with a consistent and optimal risk-return tradeoff over time, a firm wants to accurately match the amount of capital it employs with the risks it takes.

To meet these needs, firms have developed sophisticated risk measurement methodologies and have made substantial investments in risk management information systems.

Managers expect a risk management information system to provide them with the data they need to meet the above three business needs. Currently, most managers want four things from their risk management information system:

1. calculate Value at Risk;

2. perform scenario analyses;

3. measure current and future exposure to each counterparty;

4. do all three of the above at varying levels of aggregation, across various groupings of risks, across product types, and across subsets of counterparties.

With these four goals met, a manager can measure risk at the firmwide level, which is the level of aggregation that shareholders care about, and at the individual desk, product, or trader level, where decisions on risk positions and risk-adjusted compensation are taken.

This paper describes what a risk management information system must do to meet these four goals[2]. In section 2, we use two simple examples to frame the problem as one of *aggregation*. We go

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[1] Risks which are less easily quantified, such as legal risk (the sensitivity of a firm's value to legal judgements) and operational risk (the sensitivity of a firm's value to the performance of operational tasks), are outside the scope of this paper.

[2] Most large financial firms are moving toward firmwide risk measurement, but many are not there yet. Accordingly, some of the descriptions of risk management information systems in this paper apply not to current practice but to the information systems we expect to see 3 to 5 years hence.

into some detail of two approaches to calculating Value at Risk to see what is required of information systems under each approach. In section 3, we discuss several problems or choices that have to be addressed along the way. We highlight the interdependence between risk measurement approach and information system architecture in these two sections. In section 4, we discuss measuring aggregate market risk and the potential limitations that risk management information systems might impose. Section 5 concludes.

As an aside, we note that the risk management information systems processing requirements of a typical large financial trading firm are not huge compared with information systems that large firms in other industries have already implemented. Banks with large trading operations maintain large databases of transaction data, but phone company MCI has a three-terabyte customer database, growing at 100 gigabytes a month.[3] The Frontier risk management system of Canadian bank CIBC gathers data from 160 organizational units worldwide daily to produce risk management reports, but retailer Wal-Mart, in its Arkansas headquarters, accesses weekend sales data from each of its 3,017 stores on Monday morning.[4] Large trading banks maintain trading floors with dozens (or hundreds) of trader workstations, but each Wal-Mart store has dozens of cash registers, each of whose software is updated on average 24 times a year.[5] Each trader may do dozens of trades a day, but Wal-Mart does 65 million transactions each week.[6] However, financial trading firms may face an organizational (not technological) hurdle from their traditionally decentralized structure that makes it difficult to convince disparate trading units why a central risk management function needs their data.

2. Information systems requirements and risk measurement methodologies

The problem of designing an information system for risk management is a problem of *aggregation*. Data from each of a firm's trading locations worldwide must be aggregated to calculate Value at Risk or to perform a scenario analysis on the firm's worldwide portfolio. Different methodologies for calculating Value at Risk will require different slices of each trading unit's portfolio data to be aggregated across trading units, imposing different requirements on the risk management information system. In this section, we discuss some interactions between risk measurement methodology and information systems requirements.

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[3] "Towering Terabytes," Information Week, 30th September, 1996. (One terabyte = 1,000 gigabytes = 1 million magabytes).

[4] "Wal-Mart Ups the Pace," Information Week, 9th December, 1996.

[5] Ibid.

[6] Ibid.

Table 1

Two simple examples

	Example 1	**Example 2**
Question to be answered	How many financial instruments does the firm have on its books?	How many counterparties does the firm currently have?
Information required from each trading unit	Number of financial instruments the trading unit has on its books	List of the trading unit's counterparties, identified uniformly across trading units
Calculation required to compute firmwide answer	Sum across trading units	Sum across trading units with duplicate counterparties removed

To clarify what we mean by aggregation, consider two simple examples, summarized in Table 1. In the first example, a firm wants to count the number of financial instruments in its firmwide portfolio. The centralized information system can take the number of financial instruments in each trading unit's portfolio and do a simple sum to get the firmwide answer. There are two ways the centralized information system can get the number of financial instruments in each trading unit's portfolio: either the trading unit can compute the number and provide it, or the centralized information system can use the trading unit's position data to do the count itself. Which way is chosen will depend on how flexible the trading unit's systems are and how easy it is for the centralized system to access the trading unit's data.

In the second example, the firm wants to count the number of counterparties to which it has current or potential future exposure. For this slightly more complicated question, the centralized information system needs a list of counterparties from each trading unit, and the counterparty identification scheme must be uniform across trading units. Again there are two ways the centralized information system can get the trading unit-level information: either the trading unit can provide it or the centralized information system can use the trading unit's position data to get the information itself. Again the choice will depend on the relative flexibility of the trading unit's systems and accessing its data.

In both examples, the centralized and decentralized approaches can give identical, correct answers to the question of interest. While we are reluctant to make absolute statements about how a particular firm would make these choices, we are comfortable making two relative claims about the tradeoff between the flexibility of trading units' systems and the ability of a central risk management function to access position data.

1. Comparing a simple query (Example 1) with a more complicated query (Example 2), the second example's more specific information needs make a centralized solution more likely than in the first example. Producing a list of counterparties with a uniform counterparty identification scheme requires more flexibility from the trading unit's systems than simply counting the number of financial instruments on the books.

2. A centralized solution can handle different queries with the same data, while a decentralized solution requires each trading unit to provide different data for different queries. In other words, a centralized solution is more open-ended.

2.1 Aggregation and Value at Risk

Computing Value at Risk on a firmwide basis is also an exercise in aggregation, though more complicated than the simple examples presented above. The data on portfolio composition that the risk management information system needs to access and the types of calculations it has to perform will depend on the methodology chosen to calculate Value at Risk.

Many methodologies exist to calculate Value at Risk, defined as a portfolio's maximum loss over a given time period with a given probability. Each methodology combines an assumption on the future distribution of market risk factors and current data on portfolio positions to approximate the distribution of the change in portfolio value. Certain methodologies can severely test the ability of the risk management information system to gather the necessary data and do the necessary calculations in the time frame required. Consequently, firms often trade off accuracy and computational demands when computing Value at Risk.[7] We discuss two Value at Risk methodologies: delta-normal and full revaluation Monte Carlo.

2.2 Delta-normal

The delta-normal methodology (also called the J.P. Morgan RiskMetrics™ methodology) stipulates that (1) the future distribution of changes in market risk factors is assumed to be multivariate normal; (2) portfolio positions are summarized by the "deltas" of each position with respect to each market risk factor, where "delta" is the change in the position's value for a one-unit change in the market risk factor; and (3) the distribution of the change in portfolio value is approximated linearly by the sum of the products of the firmwide delta for each market risk factor and the assumed distribution of that market risk factor.

Any Value at Risk methodology must make some assumption on the future distribution of changes in market risk factors. Possible assumptions include multivariate normality, another multivariate distribution, or using the historical distribution of changes to proxy for the future distribution. If a parametric distribution is used, the parameters must be estimated from some combination of historical data and current data such as futures, forwards and options. One important burden on the risk management information system is to maintain a database of historical time series on the relevant market risk factors to be used to estimate the covariance matrix of future changes.

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[7] See Pritsker (1997) for an evaluation of the tradeoff.

Because this burden does not vary significantly with the choice of methodology, and because it can easily be contracted out,[8] it will not be discussed in what follows.

For each trading unit, "deltas" must be computed for each market risk factor. The RiskMetrics™ way to compute these deltas is to decompose each instrument into a sum of positions in some subset of the market risk factors. (This is equivalent to taking a linear approximation or a first-order Taylor expansion.) For example, if the set of market risk factors includes zero coupon bonds but not coupon bonds, a coupon bond would be decomposed into a set of zero coupon bonds of different maturities. An option on a stock market index would be decomposed into a position in the index itself. Deltas for each instrument in the trading unit's portfolio are summed to give the trading unit's deltas; these are summed across trading units to yield the portfolio's deltas. One advantage of the delta-normal method is that deltas can be easily aggregated. Finally, estimating Value at Risk requires taking the square root of a weighted sum of variances and covariances of market risk factors, where the weights are simple functions of the portfolio's deltas.

Calculating a trading unit's "deltas" requires access to the cash flows of each instrument in the trading unit's portfolio. As in the two simplified examples given above, there are two ways this can be done. If the trading unit calculates its own deltas, it must do so using a uniform set of market risk factors so the results can be aggregated at the firmwide level. For a system with N market risk factors this would involve computing and passing N "deltas" for each trading unit. However, any task that involves specifying new computations to be performed by each trading unit's systems will be more difficult to carry out the more diverse those systems are. If the centralized risk management information system calculates the deltas, accessing the portfolio data of each trading unit becomes a burdensome requirement of the delta-normal method; the difficulties of accessing transaction data stored in disparate systems often lead firms to duplicate their transaction data and store it centrally. While the location of the computation would differ, the estimated Value at Risk would be unaffected by the choice of centralized or decentralized processing.

The normality assumption and the delta (i.e., linear) approximation together imply that the distribution of the change in portfolio value is easy to compute. It will be normally distributed, and its variance will be a weighted sum of the variances and covariances of the market risk factors on which its value depends, with the weights depending on the "deltas" of the portfolio's positions. Since the probability percentiles of the normal distribution are well known, once the variance of the change in portfolio value is known the Value at Risk can be computed immediately.

To summarize, the delta-normal method requires that the risk management information system know the "deltas" of each trading unit's portfolio, which requires access to a large amount of

[8] Currently J.P. Morgan does this for free, providing a covariance matrix each day on its Internet site (http://www.jpmorgan.com/RiskMeasurement/RiskMetrics/RiskMetrics.html). Olsen and Associates also provides a covariance matrix via the Internet (http://www.olsen.ch/cgi-bin/w3risk-menu).

data - the cash flows of each instrument in the portfolio. The computational burden is relatively light, requiring a large number of simple calculations to be performed.

2.3 Full revaluation Monte Carlo

When using the full revaluation Monte Carlo methodology to calculate Value at Risk, the future distribution of market risk factors need not be assumed to be multivariate normal. Some parametric distribution must be chosen, and its parameters estimated, but because normality is not required in this methodology an alternate distribution that better captures the statistical features of financial time series can be used.[9] Two examples of such distributions are Student's *t* distribution and a mixture of multivariate normal distributions. The distribution of changes in portfolio value is approximated by taking a large number of draws from the assumed distribution of changes in market risk factors and revaluing the portfolio for each draw. For N draws, the 5 percent Value at Risk would be the (.05N)th largest loss.[10] Value at Risk can be computed in the same way, using the same N draws, for an individual trader, individual desk, or firmwide.[11]

Each trading unit's portfolio must be revalued for each of the N Monte Carlo draws. These revaluations will be computationally burdensome for some instruments. In particular, complex derivatives that cannot be valued analytically are typically solved by computation-intensive numerical methods. Valuing such a derivative for, say, N=10,000 Monte Carlo draws would require a significant amount of computing power, much more than the linear approximation of the delta-normal methodology. On the other hand, two factors mitigate the computational burden of full revaluation Monte Carlo and one factor makes it easier to deal with. First, firms have an important business need to speed up the valuation of complex derivatives, for trading purposes as well as for risk management. There is no reason to think that research into faster numerical option pricing methods will not continue to be fruitful.[12] Second, as discussed in section 3.4 below, "smart" valuation techniques, such as using an analytic approximation to value a complex option, could significantly reduce computational burden with only a small or no reduction in accuracy. Finally, parallel processing techniques will not reduce the computational burden of multiple portfolio revaluations, but such a burden would be relatively easy to divide across many processors and thus reduce computational time at the expense of additional computer hardware.

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[9] The high dimensionality of the set of market risk factors rules out a nonparametric approach in nearly all circumstances. An alternative methodology, not discussed in this paper, takes a nonparametric approach by using historical data to represent the future distribution of market risk factors.

[10] Pritsker (1997) shows that an advantage of full revaluation Monte Carlo over delta-normal is that a confidence interval on the estimate of Value at Risk can be computed at no additional computational cost.

[11] This may not be the most useful way to measure the marginal market risk of a trading unit, since it does not account for diversification across trading units.

[12] To give one example of such research, Carverhill and Clewlow (1994) describe how to speed up Monte Carlo valuation of options by a factor of 70 with a martingale variance reduction technique.

The calculations needed to revalue the trading unit's portfolio can be done either at the trading unit or at the central risk management function. If the trading unit revalues its portfolio for each Monte Carlo draw, the only burden on the central risk management function is to ensure that each trading unit uses the same N Monte Carlo draws to revalue its portfolio. Each trading unit would then simply pass N numbers, representing the changes in the value of its portfolio under the N Monte Carlo draws, to the central risk management function for aggregation. If the central risk management function does the revaluation of each trading unit's portfolio, it will again require complete position data from each trading unit along with a valuation model for each instrument. This last requirement is nontrivial for some complex derivatives, for which no "market standard" valuation technique exists.[13] Again, the choice between centralized and decentralized processing need not affect the estimate of Value at Risk.

Once the change in portfolio value for each of the N Monte Carlo draws has been calculated, these N changes in portfolio value will approximate the distribution of the change in portfolio value and can be treated as an empirical distribution function. The Value at Risk at confidence level a can be read off the ordered list of N changes in portfolio value as the αN^{th} largest loss.

The full revaluation Monte Carlo methodology makes greater demands on the risk management information system than the delta-normal. Randomly drawing a large number of changes in market risk factors from a multivariate distribution can be done easily and quickly,[14] but computing the change in portfolio value by revaluing each trading unit's portfolio for each draw from the assumed distribution of changes in market risk factors will be computationally burdensome for some instruments.

2.4 Choosing a risk measurement methodology

In choosing between a delta-normal or full revaluation Monte Carlo methodology to measure Value at Risk, a firm will trade off the accuracy of its Value at Risk estimates with the computational burden required to compute the estimates. The tradeoff can be represented as a curve, as shown in Figure 1. The delta-normal method would be a point like A, with relatively low accuracy and low computational burden. The full revaluation Monte Carlo methodology would be a point like

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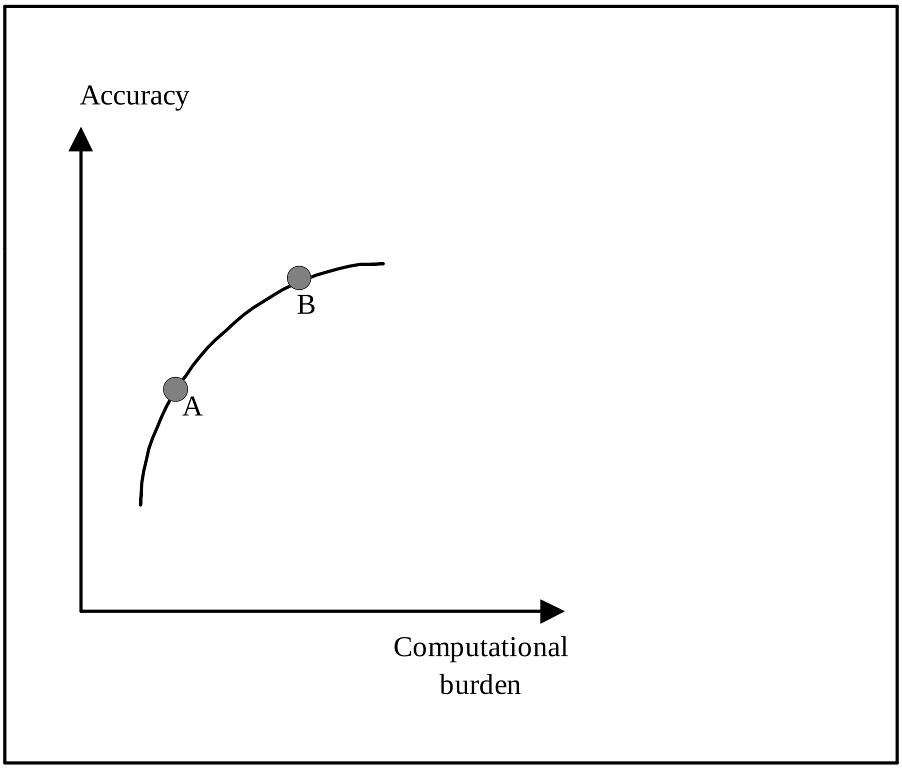
[13] See Bernardo and Cornell (1997) for an example of an auction of mortgage-backed securities and the diverse valuations of large broker dealers and institutional investors. Pierides (1996) shows that the price of interest rate derivatives can be sensitive to the stochastic process chosen to model the short-term interest rate.

[14] Drawing large numbers of random vectors from a high-dimensional space presents its own computational problems. Press et al (1992, p. 277) point out that typical computer random number generators cannot fill up a high-dimensional space. A discussion of random number generation is beyond the scope of this paper, but we conjecture that drawing truly random points in a high-dimensional space will require some thought at the time of system design but not a significantly increased number of calculations.

B, with relatively high accuracy and high computational burden. Each firm's choice will depend on the relative importance of the two factors for that firm.[15]

Figure 1



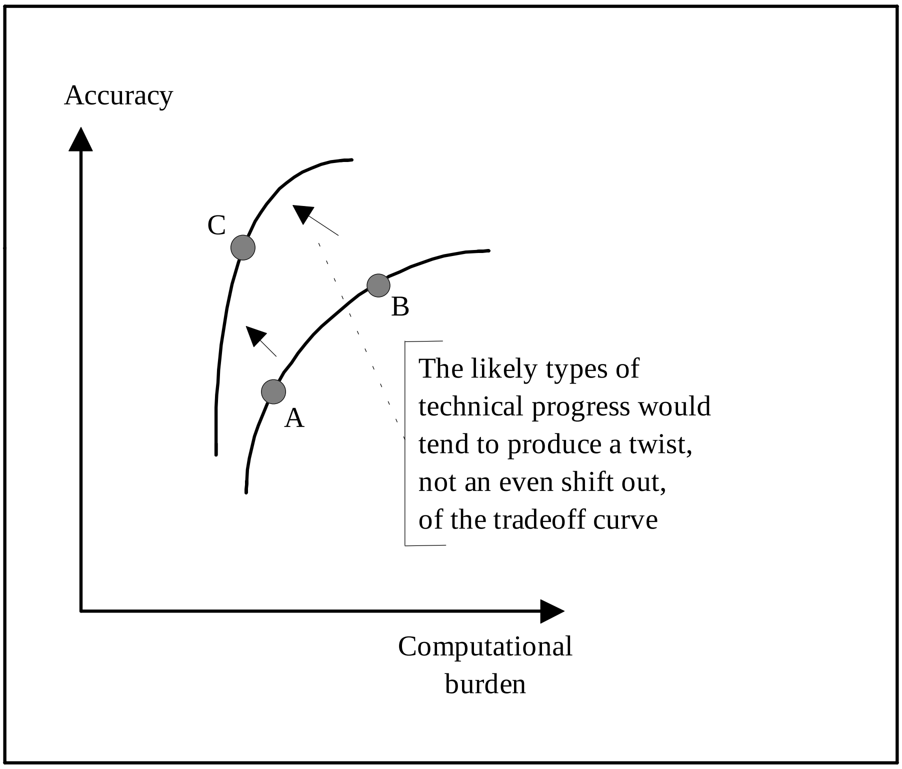
While we cannot predict where an individual firm will choose to be on the tradeoff curve in Figure 1, we do feel comfortable predicting how the curve will shift over time. Technical progress - including faster option price techniques, cheaper computer hardware, and advances in computer networking - will cause the curve to shift out, making the tradeoff more favorable. Such a shift is shown in Figure 2.

In addition, we can predict what the shift of the tradeoff curve will look like, as Figure 2 shows. Financial theorists will likely continue to produce faster option pricing models, and faster computer hardware will make a large number of Monte Carlo draws less burdensome to handle. Because the delta-normal methodology is already quite simplified, neither of these advances would reduce its computational burden by much. On the other hand, the scope for reductions in the computational burden of the full revaluation Monte Carlo methodology from these advances is large. For this reason, we predict that the reduction in computational burden, holding accuracy constant, will be greatest for those methodologies that achieve high accuracy, producing a twist in the tradeoff curve as shown in Figure 2. As this twist occurs over time, firms are likely to switch away from a

[15] We know that this tradeoff captures a meaningful choice, since there are firms using each of the two methodologies.

low-accuracy delta-normal methodology to a high-accuracy full revaluation Monte Carlo methodology.

Figure 2



3. Other issues to be addressed

Our discussion of the information systems requirements of the two Value at Risk methodologies has touched on some interactions between methodology and information systems design. In this section, we discuss some problems or choices that any firm setting up a risk management information system must confront.

3.1 Centralized or decentralized?

A basic decision that must be made when designing a risk management information system is whether to choose a centralized model or decentralized model. As the discussion above of two simple examples and two Value at Risk methodologies should make clear, either model can estimate Value at Risk or do scenario analysis. The two models will give identical answers if given identical data. The difference lies in where the most burdensome calculations are done. Which model is chosen will depend on several factors: the flexibility of trading units' systems, the degree of uniformity among trading units' systems, the availability of a large central database to hold position data, and the ability of the central risk management function to handle and revalue all instruments traded by all trading units.

The centralized model has several strong points. The risk management function can more easily monitor that the risk calculations (computing "deltas" or doing revaluations) are being done properly if they are being done centrally. In particular, if risk measurement calculations are being used in risk-adjusted compensation calculations, the centralized model avoids moral hazard. It is easier to "upgrade" to a more rigorous risk measurement methodology (from delta-normal to full revaluation Monte Carlo, for example) if a centralized model has already been adopted so that position data is already being accessed centrally. Finally, the centralized model is more open-ended, and a firm may find other uses for a centralized database of position information beyond risk measurement.

A decentralized model also has advantages. There is no need for position data and the analytics needed for revaluation to be duplicated at the central risk management function. The analytics for complex derivatives may be particularly costly to duplicate. Because there is no need to access position data or do revaluation in the central risk management function, there is no obstacle to introducing new products. For example, a data architecture that provides one field to identify the underlying security for an option would have a hard time handling a compound option whose value depends on two or more underlyings. If the analytics behind a compound option are new and complicated, it may be costly to duplicate them at the trading unit ***and*** central risk management level.

3.2 Mapping

Choosing a set of market risk factors on which to base a risk measurement methodology is an important decision. A firm making the common choice to use the RiskMetrics™ set of market risk factors or the default set of market risk factors in the risk management software it has purchased is likely not recognizing the importance of this decision. The choice of the set of market risk factors is equivalent to defining the "market risk" to be measured; risk left unmeasured, which can be termed "basis risk," is not guaranteed to be small and depends on how the set of market risk factors is chosen.

Consider a firm that has some exposure to the NLG/USD and DEM/USD exchange rates. If the firm's set of market risk factors includes both exchange rates, it will be able to measure its exposure to all possible combinations of moves in the two exchange rates, including moves in opposite directions. If the firm's set of market risk factors includes only the DEM/USD exchange rate and NLG/USD exchange rate risks are mapped into DEM/USD exposures for risk measurement purposes using the historical correlation between the two exchange rates, no market risk measurement technique will be able to reflect the possibility that the correlation could (with low probability) change. That possibility will fall under basis risk, not market risk, in the second case. Delta-normal Value at Risk will still be measured correctly, since with that methodology the correlation is held to be constant over the time period during which Value at Risk is calculated. However, the output of both stress tests and sensitivity analysis with respect to individual market risk factors, two commonly used supplements to Value at Risk, will depend on the mapping. Two firms with identical positions but different sets of market risk factors will get different stress test or sensitivity outcomes.

A firm chooses the set of market risk factors on which to base its risk management methodology with several considerations in mind. If fewer market risk factors are chosen, the methodology will be easier to work with, to understand, and to explain, and fewer computing resources will be needed. If more market risk factors are chosen, the approximation error involved in mapping a position onto a limited number of market risk factors can be reduced.[16] If more market risk factors are chosen, (unmeasured) basis risk can be reduced and (measured) market risk expanded. A firm will be able to minimize basis risk if it includes in its set of market risk factors every market risk factor to which it has exposure. Of course, exposures change over time, while it may be costly to update information systems to expand the set of market risk factors, so it is unlikely that basis risk can be eliminated as a concern.

3.3 Legacy systems

Another problem to be addressed when building a risk management information system is that of legacy systems. The term "legacy system" here refers to a trading unit's information system that cannot be easily integrated with a central risk management system. Although the typical legacy system is an old system that lacks features that have only recently been invented, the problem of legacy systems can include newer software as well, if that software cannot easily be set up to interface with a central risk management system. For example, if the pricing model for a complex option exists only in a trader's spreadsheet, it may be impossible for the central risk management system to request that the option be repriced for N Monte Carlo draws, as a full revaluation Monte Carlo Value at Risk calculation might require.

While the problems of legacy systems are serious, they should not significantly hamper a firm's ability to do firmwide risk management. Because legacy systems are so widespread, not just in financial services but in all industries, solutions to legacy systems problems are common. Many consulting firms exist solely to provide these solutions. One common solution is to write a "wrapper" program that acts as a mediator between the central risk management function and the trading unit's legacy system. For example, a "wrapper" could translate the legacy system's position data into a format that can be sent to and understood by the central risk management function. A financial firm with many legacy systems may face a higher cost of implementing a firmwide risk management function, because of the need for specialized "wrappers" for each legacy system, but the task is not impossible.

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[16] The magnitude of the approximation error involved in Value at Risk and stress test calculations based on mapping actual portfolio instruments onto a limited set of market risk factors could be large. For example, the error of mapping a 10-year bond with semiannual coupons onto a set of zero coupon bonds (say, 1, 2, 3, 5, 7, 9 and 10-year zeros) will vary with the shapes of the term structures of interest rates and volatility and the accuracy of the interpolations that must be made. As far as I am aware, this error has not been quantified in the risk measurement literature.

Many firms choose to replace their legacy systems rather than work around their limitations. A need to manage risks on a firmwide basis leads firms to insist that trading units have software that can interface smoothly with a central risk management function. Since many legacy systems lack this ability, many firms choose to replace them.[17] As discussed briefly below, the European Union's Capital Adequacy Directive's requirements for firmwide measurement of market and credit risk have also led many firms to replace legacy systems.

3.4 "Smart" data structures

The use of "smart" data structures can significantly improve the cost-benefit tradeoff in favor of a rigorous (i.e., full revaluation Monte Carlo) risk measurement system. A "smart" data structure for storing financial transactions would have some or all of the following characteristics:

1. the financial instrument knows to which market risk factors its value is sensitive;
2. the financial instrument knows both a "more exact" and a "less exact" valuation method for itself;
3. the financial instrument knows what error is introduced at different times by its different valuation methods.

Exploiting such a "smart" data structure could significantly reduce the computational burden of revaluing each instrument in a portfolio N times to calculate a Monte Carlo Value at Risk.

If a financial instrument knows to which market risk factors its value is sensitive, it can avoid recalculating its value for some of the N Monte Carlo draws. If a DEM/USD currency swap is asked to revalue itself for several draws in which all market risk factors that would affect its value (presumably USD and DEM interest rates and the DEM/USD exchange rate) are identical, the value will be the same for all such draws and the computational burden can be reduced accordingly.

Many derivatives are typically valued using numerical methods. A simple example would be an American option, which could be priced exactly using a lattice as in Cox, Ross, and Rubenstein (1979). For risk measurement purposes, if there are many American options in the portfolio, it may be too time-consuming to value each American option on a lattice. Barone-Adesi and Whaley (1987) give an approximate analytic valuation method for an American option. A "smart" American option would know both valuation methods and would know how much error is introduced by the approximation at different times. This last feature would allow the risk management application to track how much uncertainty has been added to the estimate of firmwide Value at Risk by using "less exact" valuation techniques, as well as to set up a threshold for approximation error that would force the use of a "more exact" technique if a "less exact" technique gave a particularly bad

[17] The information systems demands of risk management are driving thirty percent of UK banks to replace legacy systems, according to a recent survey. "Risk Management is Driving Banks to Replace Legacy Systems," *Risk Management Operations*, 16th December, 1996.

approximation for a certain set of changes in market risk factors.[18] Our impression is that firms have begun to adopt "smart" techniques on an ad hoc basis, but their use has not yet become standard practice.

3.5 Credit risk measurement

There is no single measurement concept for credit risk, unlike Value at Risk for market risk, which has become widely accepted in the market. The current view of "best practice" for credit risk, as expressed in the Group of Thirty's (1993) report, is to measure both current exposure and potential future exposure, the latter calculated using statistical analysis of the future credit exposure and a broad confidence interval (two standard deviations). There are many ways to measure credit risk; current market practice can be divided into two groups: transaction methods and portfolio methods.[19]

Transaction methods compute a potential exposure for each transaction as notional principal times a multiplier that reflects the transaction's form (option, swap, etc.), maturity, and the inherent riskiness of the transaction's underlying market risk factor(s). The multipliers are calculated in advance for all combinations of transaction form, maturity, and riskiness of underlying to which the firm anticipates having exposures. The information systems requirements of transactions methods are small; to measure potential exposure, each transaction's multiplier must be looked up in the tables of multipliers that have been calculated in advance. Total credit risk with a counterparty is simply the sum of current and potential exposure of each transaction with the counterparty in the firm's portfolio.

Portfolio methods compute the potential exposure of all the firm's transactions with a counterparty at once, considering correlations between potential exposures of multiple transactions with the counterparty as well as netting arrangements. Portfolio methods of measuring potential credit risk exposure are conceptually close to methods of calculating Value at Risk to measure market risk, with the additional complication of identifying each transaction's counterparty and netting status. The information systems requirements would be similar to those for calculating Value at Risk. In particular, a similar choice of methodology (delta-normal or full revaluation Monte Carlo) exists for credit risk measurement, and the choice will have a strong influence on information systems requirements just as it does for market risk.

Portfolio methods are superior to transactions methods because they incorporate the correlation of credit risk exposure among all transactions with a counterparty and netting arrangement that serve to reduce credit risk exposure. Firms pursuing "industry best practice" either have adopted or hope to adopt a portfolio approach to measuring credit risk. Since the information systems

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[18] For example, if an at-the-money American option close to expiration were subject to unacceptably high approximation error, the numerical method would be used.

[19] This discussion is based on two articles describing Citibank's credit risk measurement system: Picoult (1996) and Lawrence (1995).

requirements of such an approach are similar to the requirement for calculating Value at Risk, firms may enjoy a synergy if they can design a single information system to meet both needs.

4. Information systems and measuring aggregate market risk

To measure aggregate market risk, a central bank would be faced with many of the same problems vis-a-vis individual firms that firms are faced with vis-a-vis trading units when setting up a central firmwide risk management function. Since a central bank will presumably not be interested in a "centralized" solution, we must look to the decentralized solutions already discussed to explore this analogy further.

Compare the "decentralized" solutions for computing delta-normal and full revaluation Monte Carlo Value at Risk. In the former, each trading unit must supply information on its "deltas" with respect to a uniform set of market risk factors. To use this methodology to measure aggregate market risk, a central bank would have to specify a common set of market risk factors for all firms to use. Specifying a common set of market risk factors presents several problems. For those firms that use a delta-normal methodology to calculate Value at Risk, their set of market risk factors is likely to be different from the common set, requiring reprogramming of the mapping procedure. Those firms using a different methodology, one that does not require mapping, would have to devote information systems resources to devising a mapping scheme that is irrelevant for their own business needs.

In full revaluation Monte Carlo Value at Risk, each trading unit computes the change in portfolio value for each Monte Carlo draw. To use this methodology to calculate aggregate market risk, a central bank would have to specify a set of Monte Carlo draws for all firms to use. Firms using a full revaluation Monte Carlo methodology, whether centralized or decentralized, could compute their change in portfolio value for each Monte Carlo draw in the same fashion as they compute it when measuring Value at Risk with their own Monte Carlo draws. Firms using another methodology, such as delta-normal, will also be able to calculate their portfolio's change in value if they are able to do stress tests or scenario analyses. The common set of Monte Carlo draws could be processed as "stress scenarios" by such firms. A potential problem with this approach to measuring aggregate market risk is the burden of computing a large number of scenarios, on top of the calculations need to compute a firm's own risk measures.

4.1 Heterogeneous mappings and aggregate market risk

In section 3.2 above, we discussed the problem a firm faces in choosing a set of market risk factors to summarize its position data for computing Value at Risk and conducting stress tests and sensitivity analyses. Additional problems arise when combining the output of individual firms' risk management information systems to measure aggregate market risk.

If two firms use different mappings for the same position, they implicitly have different definitions of market risk and basis risk, making aggregation of market risk across firms problematic. For example, suppose two firms each have made a contract for delivery of NLG for USD at a certain exchange rate in 30 days. One firm has the appropriate NLG/USD risk factor in its set of market risk factors, so no mapping is required. The other firm, with little NLG/USD exposure, chooses to map its exposure onto a DEM/USD risk factor, implicitly categorizing changes in the correlation between NLG/USD and DEM/USD exchange rates as basis risk. Suppose a range of scenarios of exchange rate changes are provided to both firms, and a portfolio revaluation is requested for each scenario. Given the limitations of its risk management information system, the second firm will use only the DEM/USD exchange rate to evaluate the NLG/USD forward contract. The two firms will likely give different answers to the question "How much does the value of the forward contract change under each scenario?" because their mappings are different. The variance in the answers will be greatest for those scenarios that incorporate moves in the NLG/USD and DEM/USD exchange rates that are farthest from their historical correlation.

Similar problems would arise when using individual firm "deltas" with respect to particular market risk factors as a basis for measuring aggregate market risk. A firm's sensitivity to DEM/USD risk may incorporate other exchange rate risks that have been mapped onto the DEM/USD risk factor; if each firm has a different mapping, each firm's sensitivity will be measuring a different concept, again making aggregation problematic.

Different firms' business needs may lead them to choose different sets of market risk factors to build into their risk management information systems. This variation across firms creates a problem for aggregating measures of market risk across firms, since different sets of market risk factors imply different definitions of what is encompassed by "market risk." Investigation of how serious a problem, in practice, this variation across firms would create for measuring aggregate market risk and how to get around the problem is left as an exercise for further research.

4.2 Data on counterparty type

Depending on the use to which the data is to be put, data on aggregate market risk may need to be computed by counterparty type. An information system designed solely to measure market risk will not necessarily be able to produce a breakdown by counterparty of either cashflow mappings or changes in portfolio value from a Monte Carlo draw. However, an information system designed to jointly measure market and credit risk could track counterparty type. The merits of such systems have been discussed in industry trade journals, but it is unclear how many firms are implementing such systems.[20]

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[20] See "Risk Where Credit's Due", *Risk* 9:6 (June 1996) and "Together They Stand", *Risk Firmwide Risk Management Supplement* (July 1996).

5. Conclusion

To measure risk, a financial firm needs sophisticated information systems. The information systems must combine data from disparate trading units in a structured way to estimate the aggregate risk of the firm. In this paper we have outlined some of the issues firms face when setting up such systems. We have described two of the many risk management methodologies currently in use in the market, and shown how methodology and information system design interact.

Efforts by firms to construct information systems that measure their risk on a firmwide basis have enable us to consider the possibility of aggregating risk data across firms in a meaningful, timely way. If the many hurdles could be overcome, this could represent a revolutionary new way to construct market oversight information. To measure aggregate market risk, risk data must be combined across firms. Any such effort would face many of the same issues of information systems capabilities faced by firmwide risk management in an individual firm. While the issues are similar, the limited ability to impose coordination across firms raises new problems. How to overcome these problems is left as a task for future research.

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Residual risk factors, portfolio composition and risk measurement

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Abstract

When risk managers develop firm-wide measures of risk, the efforts can impose substantial costs both of developing information systems and, on an ongoing basis, of computation and aggregation. These costs can lead risk managers to base risk measures on a set of risk factors (including asset prices) of lower dimension than the dimension of underlying sources of risk. Such truncation of the set of risk factors, however, could cause risk measures to systematically underestimate a portfolio's risk. This paper presents examples where risk aversion leads a firm to hedge risk factors that have high explanatory power for many asset returns; however, the firm may remain exposed to other, less-important risk factors if their market price of risk is sufficiently high. Statistical techniques for identifying sources of risk that choose risk factors based on the variability of asset prices without taking account of the market price of risk could systematically underestimate portfolio risks.

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Residual risk factors, portfolio composition and risk measurement

1. Introduction

Two questions regarding measures of portfolio risk that are not equivalent are:

– What price shocks would lead to large losses?

– What losses would be caused by the class of price shocks that have occurred with sufficient regularity in historical data to be identified by statistical techniques?

The price shocks identified by the first question may be different from the price shocks used in the second question. At least two reasons for the difference can be mentioned.

– Selection bias: dealers' trading and hedging strategies may be conditioned on the same statistical regularities identified by the designers of stress tests. The dependence of observed statistical regularities on the sample period could cause stress tests to fail to identify the shock in the first question. See, Mahoney, 1996.[1]

– Portfolio composition and aggregation of shocks to risk factors.

This note addresses the last point, how portfolio composition determines the aggregation of shocks to the risk factors that drive asset price volatility. Computation burden and the information-system costs of firm-wide aggregation of risk can cause risk managers to construct risk measures that parsimoniously reduce the number of risk factors to a smaller dimension than the dimension of asset prices. Such truncation of the set of risk factors, however, could cause risk measures to systematically underestimate a portfolio's risk. This underestimate can occur even when the exercises uses the full dimensionality of the portfolio's sensitivity to asset prices (i.e. the sensitivity to every asset price is accounted for).

If a firm's risk aversion causes it to hedge risk factors that have high explanatory power in the variability of asset returns, smaller risk factors that appear to be less consequential may remain unhedged if those factors have a market price of risk. Statistical techniques that summarise the variability of asset prices without taking into account the sizes of market prices of risk could then systematically underestimate portfolio risks if the techniques ignore some factors with a market price of risk. While this claim should not be surprising, its implications in the measurement of portfolio risk should not be overlooked by designers of risk measures who may need to reduce the dimensionality of the measurement exercise due to constraints of data availability or computation burden. The following two examples illustrate this claim.

[1] "Empirical-based versus model-based approaches to value-at-risk: An examination of foreign exchange and global equity portfolios", James Mahoney; in *Risk measurement and systemic risk: Proceedings of a joint central bank research conference*. Board of Governors of the Federal Reserve System (1996).

2. An example with financing constraints

Our first example considers portfolio risks in a setting where the portfolio manager faces financing constraints.

Asset prices

Consider a portfolio that contains the four assets, whose returns are described by four independent random risk factors, f_j , j=1,..,4,

$$
\begin{aligned}
c_1 &= r^* + (\lambda_1 + f_1) \\
c_2 &= r^* + a_1(\lambda_1 + f_1) + (\lambda_2 + f_2) \\
y_1 &= r^* + (\lambda_1 + f_1) + a_2(\lambda_2 + f_2) + (\lambda_3 + f_3) \\
y_2 &= r^* + b_1 (\lambda_1 + f_1) + (\lambda_2 + f_2) + a_3(\lambda_3 + f_3) + (\lambda_4 + f_4)
\end{aligned}
$$

where the remaining terms are constants. The constant terms λ_j are the market price of risk of the risk factors, f_j, and r* is the riskless interest rate. Each asset return is influenced by its "own" risk factor, and some are also influenced by other assets' risk factors as well. The market price of risk of each risk factor is assumed to be determined by its volatility and correlation with the return of a market portfolio consisting of equal amounts of all four assets.

Assume that the risk factors f_1 and f_2 explain more than 90% of the variability of each of the four asset prices, as is the case with the following parameter values: $a_1 = 0.5$, $a_2 = 0.25$, $a_3 = 0.25$, $b_1 = 0.75$, $\sigma_1 = 0.01$, $\sigma_2 = 0.01$, $\sigma_3 = 0.0033$, and $\sigma_4 = 0.004$, where σ_i is the standard deviation of the risk factor f_i , $E(f_i) = 0$, and $Cov(f_i,f_j) = 0$.

With the assumed parameter values, the two risk factors f_1 and f_2 would explain more than 90% of the variability of y_1 and more than 90% of the variability of y_2, while explaining 100% of the variability of c_1 and c_2 (see Table 1). In other words, the variability of all asset returns can be very well described by only two risk factors.[2] With this excellent explanatory power of only two risk factors, an analyst might be tempted to dismiss the remaining risk factors as unimportant residual terms. Can such residual risks be ignored in considering specific risk?

Table 1

Explanatory power of risk factors in *asset* returns

	c_1	c_2	y_1	y_2
Variance of asset returns	*0.01*	*0.0112*	*0.0108*	*0.0131*
Proportion of variance explained by: f_1 and f_2	1	1	0.91	0.90
Proportion of variance explained by: f_1 and f_3	1	0.2	0.95	0.33
Proportion of variance explained by: f_1, f_2, and f_3	1	1	1	0.91

2 The correlations between the asset prices are, $corr(c_1, y_1) = 0.93$, $corr(c_1, y_2) = 0.76$, $corr(c_1, c_2) = 0.45$ and $corr(c_2, y_2) = 0.94$.

Portfolio allocation

The risk factors f_1 and f_2 can be interpreted as market risk factors, since they have a large influence on all asset prices. The other risk factors represent spread risk. Consider the portfolio return described by,

$$y \;=\; [y_1 A + y_2(1\text{-}A) - c_1 B - c_2(1\text{-}B)]\, N, \tag{1}$$

where A and B are choice variables that determine portfolio composition, 0≤A≤1 and 0≤B≤1, and N is the portfolio size. This profit function has the following interpretations.

> *Interpretation 1*: A bank can invest and fund its positions in different markets (countries). $\mathbf{y}_j$ is the investment return in market j, and $\boldsymbol{c}_j$ is the funding cost in market j, for j = 1,2. In contrast to the model in section 3, here the bank does not have access to funding at the risk-free rate. Hence, its portfolio must earn a return above the risk-free rate with the corresponding risk.
>
> *Interpretation 2*: A bank takes long positions in two assets with returns y_j, j=1,2, and short positions in two assets with returns c_j, j=1,2.

Variance of portfolio return

Depending on portfolio composition, the risk factors that appear to have high explanatory power in asset returns can have much smaller explanatory power in portfolio returns. Table 2 compares portfolio risk and estimates of that risk based on an incomplete model (with the portfolio size variable N=100). Panel A shows results in terms of portfolio variance for contrast with Table 1, while Panel B shows results in terms of portfolio standard deviation. The portfolio weights in Table 2, are optimal portfolios corresponding to different values of the risk aversion parameter (ρ) in the utility function,

$$U = E(y(A, B, f_1, f_2, f_3, f_4)) \; - \; \frac{\rho}{2} V(y(A, B, f_1, f_2, f_3, f_4)) \tag{2}$$

where *E* and *V* denote expected value and variance, and *y* is portfolio returns as defined in (1), where portfolio size is held constant.

While the first two risk factors explain more than 90% of the variability of all asset prices, their ability to describe the risks in portfolio returns can be much smaller. In the case of the portfolio chosen by a moderately risk averse firm, the first two risk factors explain only 31% of the variance of portfolio returns (56% in terms of portfolio standard deviation). Moreover, adding the third risk factor would increase the explanatory power to only 47% of the variance of portfolio returns (69% in terms of portfolio standard deviation).

Table 2

Explanatory power of risk factors in *portfolio* returns

Panel A: ***Variance*** **of portfolio returns**				
	Risk neutrality: A=0 B=1	Slightly risk averse: A=0 B=0.4	Moderate risk aversion: A=0.3 B=0.4	Extreme risk aversion: A=0.7 B=0.6
Portfolio variance	*1.23*	*0.33*	*0.15*	*0.10*
Proportion of variance explained by: f_1 and f_2	0.86	0.49	0.31	0.21
Proportion of variance explained by: f_1, f_2, and f_3	0.87	0.51	0.47	0.85
Panel B: ***Standard deviation*** **of portfolio returns**				
	Risk neutrality: A=0 B=1	Slightly risk averse: A=0 B=0.4	Moderate risk aversion: A=0.3 B=0.4	Extreme risk aversion: A=0.7 B=0.6
Portfolio Standard Deviation, σ(y)	*1.11*	*0.57*	*0.39*	*0.31*
Proportion of σ(y) explained by: f_1 and f_2	0.93	0.70	0.56	0.46
Proportion of σ(y) explained by: f_1, f_2, and f_3	0.93	0.72	0.69	0.92

Another feature of these results is that the amount of risk left unaccounted for does not always decline as portfolio risk decreases. Table 3 shows the amount of portfolio risk (as measured by portfolio standard deviation), as well as modelled risk (using f_1 and f_2 only) and the amount of risk left unmeasured. The last row of Table 3 shows that the amount of risk left unaccounted for need not decrease as portfolio becomes risk becomes smaller.

Table 3

The effect of neglected risk factors: portfolio risk left unmeasured

	Risk neutrality: A=0 B=1	Slightly risk averse: A=0 B=0.4	Moderate risk aversion: A=0.3 B=0.4	Extreme risk aversion: A=0.7 B=0.6
Portfolio Standard Deviation, σ(y)	*1.11*	*0.57*	*0.39*	*0.31*
Modelled Standard Deviation using f_1 and f_2	*1.03*	*0.40*	*0.22*	*0.14*
Risk left unmeasured	*0.08*	*0.17*	*0.17*	*0.17*

Stress test results

As with the variance of portfolio returns, risk factors that have high explanatory power in asset prices can have much smaller explanatory power in stress tests of portfolio returns. The results in Table 4 apply to the moderately risk averse portfolio (A=0.3, B=0.4).

Table 4

The effect of neglected risk factors: stress tests of portfolio returns

Actual shock in risk factors	**Stress test specification**	**Proportion of actual change predicted by the stress test**
$f_i = \sigma_i$, i = 1 to 4	$f_i = \sigma_i$, i = 1,2 $f_j = 0$, j = 3,4	0.41
Same as above	$f_i = \sigma_i$, i = 1,2,3 $f_4 = 0$	0.62

In contrast to their higher explanatory power in the space of asset prices, the truncated set of risk factors has much weaker explanatory power in stress tests of portfolio returns. For example, while the first two risk factors account for 79% of the change in y_1 and 78% of the change in y_2 due to a one standard deviation shock to all risk factors, they account for only 41% of the actual change in portfolio value.[3] Adding the third risk factor to the stress test, would account for all the change in y_1 and 82% of the change in y_2, but yet account for only 62% of the true change in portfolio value.

3. Portfolio risk

Like the first example, our second example investigates sources of portfolio risk when market returns follow the restrictions of linear arbitrage pricing theory and the portfolio is selected by a risk averse portfolio manager. The difference between the two examples is the unconstrained choice in the second example, where the portfolio manager can choose a portfolio with the risk-free return. We demonstrate that portfolio risk will likely be mismeasured by risk-management methodologies that do not include all sources of non-diversifiable risk in asset returns. Specifically, the example suggests that an analysis of portfolio risk that uses only factors accounting for a large fraction of return variance or that leaves out factors with high expected returns will often understate portfolio risk.

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[3] The first two risk factors account for 100% if the variability of c_1 and c_2.

Asset returns

We assume that returns on the n risky assets follow a k factor model:

$$r = r^e + \beta f + \varepsilon$$

where returns above the risk-free rate (r), expected excess returns (r^e) above the risk-free rate, and idiosyncratic errors (ε) are n x 1 vectors; the factors (f) are a k x 1 vector; and the matrix of factor loadings (β) is a n x k matrix. The expected value of the factors and idiosyncratic errors is zero. For convenience, we assume that the factors are uncorrelated both with each other and with the idiosyncratic errors, that the factors are normalised so $\beta'\beta = \mathrm{I}_k$ (the k dimension identity matrix), and that the variances of the idiosyncratic errors are equal so $Var(\varepsilon) = \sigma_\varepsilon^2 \mathrm{I}_n$ (i.e. proportional to the n x n identity matrix).

Thus, the variance of excess returns is given by:

$$Var(r) = \beta\, Var(f)\, \beta' + Var(\varepsilon) = \beta\, Var(f)\, \beta' + \sigma_\varepsilon^2 \mathrm{I}_n$$

where $Var(f)$ is a diagonal matrix. The normalisation assumption on β and the correlation assumptions for f imply that β is the matrix of eigenvectors of $\beta\, Var(f)\, \beta'$ corresponding to the (positive) eigenvalues on the diagonal of $Var(f)$. The full decomposition is given by:

$$\beta Var(f)\beta\beta' = \begin{bmatrix}\beta & \tilde{\beta}\end{bmatrix}\begin{bmatrix}Var(f) & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\beta & \tilde{\beta}\end{bmatrix} \text{ and } \begin{bmatrix}\beta & \tilde{\beta}\end{bmatrix}\begin{bmatrix}\beta'\\ \tilde{\beta}'\end{bmatrix} = \begin{bmatrix}\beta'\\ \tilde{\beta}'\end{bmatrix}\begin{bmatrix}\beta & \tilde{\beta}\end{bmatrix} = \mathrm{I}_n$$

where $\tilde{\beta}$ is the n x (n-k) matrix of eigenvectors corresponding to the zero eigenvalues of $\beta Var(f)\beta'$. With these assumptions, the variance matrix of returns can be expressed as:

$$Var(r) = \sum_1^n \beta_j\beta_j' \; (\sigma_j^2 + \sigma_\varepsilon^2)$$

where $\sigma_j^2 = \begin{cases} Var(f_j), & j \le k \\ 0, & j > k \end{cases}$, either the variance of the j-th factor or 0, and β_j is the j-th eigenvector of $Var(r)$, a column of either β or $\tilde{\beta}$.[4] The inverse of the variance matrix of returns is:

$$Var^{-1}(r) = \sum_1^n \beta_j\beta_j' \; (\sigma_j^2 + \sigma_\varepsilon^2)^{-1}$$

A portfolio is defined by the shares (ω) held in the n risky assets. Returns on a portfolio are:

$$\omega'(r + r^0 i) + r^0(1-\omega' i) = \omega' r + r^0$$

where r^0 is the risk-free rate and i is a n x 1 vector of ones. Expected portfolio returns and the variance of returns are:

$$E(\omega' r + r^0) = \omega' r^e + r^0 \text{ and}$$

$$Var(\omega' r + r^0) = \omega'\beta(Var(f)\beta'\omega + \sigma_\varepsilon^2\, \omega'\omega.$$

Arbitrage profits could be obtained with these assets unless riskless portfolios earn the risk-free rate of interest (i.e., have a zero expected excess return). When the number of risky assets is large, idiosyncratic risk can be largely eliminated by diversification. For example, a portfolio whose

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[4] The decompositions of A and A + bI are closely related, where A is a n x n matrix, b is a number, and I is the n x n identity matrix. It can be shown that the eigenvalues of A + bI equal b plus the eigenvalues of A while the eigenvectors of the two matrices are identical.

weights are $\tilde{\omega} = \tilde{\beta}\, i\,/\,(n-k)$ has variance $\sigma_{\varepsilon}^{2}\,/\,(n-k)$ and will have little risk when the number of assets, n, is large. Thus, portfolios whose risk arises only from idiosyncratic risk will have a return equal to the risk-free rate.[5] A portfolio whose risk arises from idiosyncratic risk has weights on risky assets satisfying $\omega'\beta = 0$, with not all weights equal to zero. If these portfolios have zero expected excess return then $\omega' r^e = 0$. These conditions imply that the vector of expected excess returns, r^e, must be a linear combination of the columns of β. Thus, absence of arbitrage opportunities implies that expected returns satisfy:

$$r^e = \beta\lambda$$

where the elements of λ represent the marginal expected excess return from additional investments in portfolios reproducing particular factors.

Portfolio choice

To investigate the possible implications of choosing too few factors to describe returns and the risk of a portfolio, we consider portfolios formed by mean-variance optimisation. Mean-variance optimises generally choose portfolios to reduce risk unless the returns to bearing risk are sufficiently attractive. We use mean-variance optimisation to illustrate possible tradeoffs that might occur in real portfolio choices.

We assume that the portfolio manager selects the portfolio weights on risky assets, ω, to maximise an objective function that rewards expected return and penalises variance of returns:

$$E(\omega' r + r^0) - 0.5\,\rho\, Var(\omega' r + r^0).$$

In this objective function, the coefficient ρ describes risk aversion; managers with higher values of ρ are more risk averse. The optimising portfolio weights are:

$$\omega^{OPT} = (\rho Var(r))^{-1} r^e.$$

This well-known result shows that the portfolio manager tends to give higher weights to assets with high expected excess returns and lower weights to assets with high variances. This portfolio has realised return equal to:

$$r^e + \rho^{-1} r^{e\prime} Var(r)^{-1} r^e + \rho^{-1} r^{e\prime} Var(r)^{-1} (\beta f + \varepsilon)$$

with variance:

$$\rho^{-2} r^{e\prime}\, Var(r)^{-1} r^e.$$

If all market participants are mean-variance optimisers and if there are a large number of assets (none of which is a large part of the market portfolio), then in market equilibrium:

$$r^e = (\rho^{avg}\, Var(r))\, \omega^{market} \approx \beta\left[Var(f)\, \beta' \omega^{market} \rho^{avg}\right] = \beta\,\lambda$$

5 As the discussion suggests, this property holds exactly only in a limiting case as the number of assets grows. See J. Ingersoll, *Theory of financial decision making*, (Rowman & Littlefield, 1987), Chapter 7 for a more detailed discussion. We assume that the result holds exactly here to simplify the algebra that follows.

where ω^{market} represents the portfolio weights in the market portfolio and ρ^{avg} is a wealth-weighted average of market participants' risk aversion.[6] It follows that the elements of λ are proportional to the factor variances, or $\lambda_l = \sigma_l^2\ \kappa_l$, where the constant of proportionality, κ_l, will be large either if the factor has large β for many assets or if the large elements of β correspond to assets with large shares in the market portfolio. This second condition could occur if the relative factor loadings corresponded closely to the shares in the market portfolio.

Factor contributions to return and portfolio variance

We can combine these results to compare factor contributions to the variance of returns or to the variance of the portfolio. These contributions will suggest when a factor is more important to return or portfolio variances.

One measure of the contribution of a factor to a set of returns is the share of summed return variances that can be attributed to the factor. With the structure assumed above, this calculation is very easy because each factor is uncorrelated with the other sources of risk. Specifically, the fraction of total return variances contributed by factor l is given by:

$$\frac{trVar(\beta_l f_l)}{trVar(r)} = \frac{tr\beta_l \beta_l'\ \sigma_l^2}{tr\sum_{j=1}^{n}\beta_j\ \beta_j'\ (\sigma_j^2 + \sigma_\varepsilon^2)} = \frac{\sigma_l^2}{\sum_{j=1}^{n}(\sigma_j^2 + \sigma_\varepsilon^2)}$$

where tr represents the trace of a matrix - the sum of the diagonal elements.

Turning to portfolio returns, the variance of realised returns contributed by factor l is:

$$Var(\rho^{-1}\cdot(r^e)'\cdot Var(r)^{-1}\cdot\beta_l f_l) = \rho^{-2}\cdot(r^e)'\cdot Var(r)^{-1}\cdot Var(\beta_l f_l)\cdot Var(r)^{-1}\cdot r^e =$$
$$\rho^{-2}\cdot\sigma_l^2\cdot(r^e)'\cdot Var(r)^{-1}\cdot\beta_l\cdot\beta_l'\cdot Var(r)^{-1}\cdot r^e$$

This expression can be simplified considerably by using the expression for the inverse of $Var(r)$ derived above, the property that $\beta_j\ \beta_l = 0$, $j \neq l$, and the arbitrage-free value of r^e to obtain:

$$\rho^{-2}\cdot\sigma_l^2\cdot\frac{\lambda_l}{\left(\sigma_l^2 + \sigma_\varepsilon^2\right)^2}$$

where λ_l is marginal expected excess return for factor l.

The contribution of factor l to portfolio variance can be compared to the overall variance of the portfolio. Recall that the variance of the optimising portfolio is:

$$\rho^{-2}\ r^{e\prime}\ Var(r)^{-1}\ r^e$$

Substituting for the inverse of $Var(r)$ and for the arbitrage-free value of r^e gives the following expression for the portfolio variance:

$$\rho^{-2}\cdot(\beta\cdot\lambda)'\cdot\sum_{j=1}^{n}\beta_j\beta_j'\left(\sigma_j^2 + \sigma_\varepsilon^2\right)^{-1}\cdot\beta\cdot\lambda = \rho^{-2}\cdot\sum_{j=1}^{n}\lambda_j^2\left(\sigma_j^2 + \sigma_\varepsilon^2\right)^{-1}$$

where $\lambda_l = 0, l>k$

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6 Specifically, $\rho^{avg} = (\sum \rho_i^{-1}\ W_i/W)^{-1}$ where ρ_i and W_i are risk aversion and wealth, respectively, of participant i and W is total wealth of all market participants.

Thus, the fraction of portfolio variance that is attributable to a factor is:

$$\frac{\dfrac{\sigma_l^2 \cdot \lambda_l^2}{\left(\sigma_1^2+\sigma_\varepsilon^2\right)^2}}{\displaystyle\sum_{j=1}^{n}\frac{\lambda_j^2}{\left(\sigma_j^2+\sigma_\varepsilon^2\right)}} = \frac{\dfrac{\lambda_l^2}{\left(\sigma_1^2+\sigma_\varepsilon^2\right)} \cdot \dfrac{\sigma_l^2}{\left(\sigma_1^2+\sigma_\varepsilon^2\right)}}{\displaystyle\sum_{j=1}^{n}\frac{\lambda_j^2}{\left(\sigma_j^2+\sigma_\varepsilon^2\right)}} = \frac{\dfrac{\sigma_l^4\kappa_l^2}{\left(\sigma_1^2+\sigma_\varepsilon^2\right)} \cdot \dfrac{\sigma_l^2}{\left(\sigma_1^2+\sigma_\varepsilon^2\right)}}{\displaystyle\sum_{j=1}^{n}\frac{\sigma_j^4\kappa_j^2}{\left(\sigma_j^2+\sigma_\varepsilon^2\right)}}$$

Note that the fractions do not add up to one because the idiosyncratic, asset specific, sources of risk also contribute some risk to a portfolio.

Contributions to portfolio and return variances

A factor will contribute substantially to portfolio variance either if κ_l is large or if σ_l^2 is large. Note that the first case, when κ_l is large, is one where the factor may account for a larger share of portfolio variance than of overall return variance. (This is most likely when κ_l is large and σ_l^2 is small.) The second case highlights that factors with large variance will contribute usually substantially to both portfolio variance and to the variance of returns, $\sigma_l^2 / \sum_{j=1}^{n}\left(\sigma_j^2+\sigma_\varepsilon^2\right)$.

4. Conclusion

This paper presents two simple but somewhat realistic examples of portfolio exposure to the sources of risk in the underlying assets. Both examples suggest that all factors with priced risk should be included in risk measurement systems.

The first example considers a risk-averse portfolio manager who optimises subject to some financing constraints. The example shows that the resulting portfolio will include investments with high market prices of risk; these investments may generate exposure to factors that do not account for a substantial part of asset return variance.

The second example also models a risk-averse portfolio manager selecting a portfolio, but without financing constraints. The model shows that the market risk of a portfolio may be determined by factors that contribute fairly little to asset return variance. Thus risk measurement methodologies that do not include all sources of priced risk could substantially understate the risk faced by some participants.

Liquidity risk and positive feedback

by

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Abstract

This paper reviews some of the literature on market liquidity and feedback trading in the context of a simple multi-asset rational expectations general equilibrium model. The determinants of market liquidity and feedback trading are discussed, and the effect of feedback trading on price volatility and on market liquidity is examined in the context of the model.

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1. Introduction

This paper provides a brief summary of some of the relevant issues raised in the literature on feedback and market liquidity for our research on market stress. Rather than presenting an extensive review of all of the literature on this topic, I provide a static model which contains what I believe are the most important features of models of liquidity and feedback. I discuss the relevant literature and the issues that are raised in the context of the model. A bibliography provides a more extensive list of readings than is discussed in this summary.[1] Finally, the last section of this summary discusses areas for further research.

2. A basic model

In this section of the paper we model how the prices of assets in an economy are determined by the trading strategies of its market participants, and by the informational structure of the market. Specifically, we will consider an economy endowed with a fixed supply of N fundamental assets $A_1,\ldots A_N$, where P and V denote Nx1 vectors of their prices and long run fundamental values. The economy also contains derivative securities that are in zero net supply; the value of these securities is determined by the prices of the underlying fundamental assets.

Although derivative securities are in zero net supply, we will assume that some participants buy and hold derivative securities while others hedge the risk associated with their derivatives position. I will make the simplifying assumption that derivatives dealers hedge their net derivatives exposures, while other participants do not. H(P) denotes the value of dealers' net derivative securities positions as a function of underlying asset prices.

There are three types of participants in the markets for the underlying assets: noise traders, derivatives dealers and value investors. The trades of noise traders are uncorrelated with other participants objectives. We will denote their net trades by ε, and make the additional simplifying assumption that

$$\varepsilon \sim N(O, \Sigma_\varepsilon). \tag{1}$$

The second type of traders are derivatives dealers. Derivatives dealers maintain positions in the underlying assets to hedge their net positions in the derivatives market. For simplicity we will maintain the assumption that derivatives dealers choose their positions in the underlying assets to remain delta-neutral overall. To derive the implications of this assumption for derivative dealers net asset demands, let X_{DD} denote the net position of derivatives dealers in the underlying assets at time t.

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1 The article by Hebner (1996) provides a good review of the market microstructure liquidity literature. Damodaran and Subrahmanyam (1992) provide a summary of the literature on the effects of introducing options and futures markets.

The value of the derivatives dealers positions in the fundamental assets and the derivative securities is given by

$$X^{1}_{DD}P + H(P).$$

If the current time is time period 0, and P is initially P_0 but changes to P_1 at time 1, then the change in the value of the derivative dealers position is approximately:

$$\left[X_{DD} + H_P(P_0)\right](P_1 - P_0).$$

To minimise the change in the value of the derivatives dealers position, X_{DD} should be chosen so that $X_{DD} = -H_{P_0}(P)$. As P goes from P_0 to P_1, the hedges will have to be readjusted to maintain delta neutrality. In this case, dynamic hedging requires that

$$\Delta X_{DD} = -H_{P,P}(P_0)(P_1 - P_0). \tag{2}$$

The third type of traders are value investors. Value investors are modelled as long-term players in the securities markets who are willing to take positions to exploit deviations of price P from long run fundamental value V. More specifically, we assume that the net position of value investors in the underlying assets can be described by the equation:

$$X_{VI} = X_{VI}(0) + K(V^* - P)$$

where X_{VI} is the value investors net position, $X_{VI}(0)$ is the desired position conditional on $V = P_0, V^*$ is the *E(V\P,I)* based on the observed price P and the information set (I) of the value investors. As *E(V\P,I)* and as P changes, the desired position of value investors also changes. The equation for the change is

$$\Delta X_{VI} = K\left(V_1^* - V_0^* - (P_1 - P_0)\right). \tag{3}$$

Finally, assume that the change in fundamental value V conditional on P is distributed normally as follows:

$$V_1^* - V_0^* \sim N(0, \Sigma_v).$$

Equilibrium prices in the model are those prices which make the net changes in all three participants positions sum to 0, i.e. the equilibrium condition requires that:

$$\Delta X_{VI} + \Delta X_{DD} + \varepsilon = 0.$$

Solving for $P_1 - P_0$ yields:

$$P_1 - P_0 = \left[H_{PP}\left(P_0\right) + K\right]^{-1}\left[K\left(V_1 - V_0\right) + \varepsilon\right]. \tag{4}$$

This implies that Σ_P the variance of P is given by:

$$\Sigma_P = \left[H_{PP}\left(P_0\right) + K\right]^{-1}\left(K\Sigma_V K^1 + \Sigma_\varepsilon\right)\left[H_{PP}\left(P_o\right) + K^{'}\right]^{-1}. \tag{5}$$

The liquidity of the market is measured by its ability to accommodate liquidity shocks without prices being driven far away from fundamentals. One measure of the deviation of prices from fundamentals is the variance of price relative to the variance of fundamentals. It is useful to illuminate what determines this variance since it is these same variables that need to be and to some extent have been modelled on papers on liquidity.

The above model is very simple and stylised. Its main contribution is that it contains important features of feedback and liquidity models that appear in the academic literature. The next sections discuss the liquidity and feedback aspects of the model in more detail.

3. Liquidity

Aspects of market liquidity include the time involved in acquiring or liquidating a position and the price impact of this action. Beyond these aspects market liquidity is difficult to define in practice. However, it is sensible to talk about it in the confines of specific models.

Liquidity of the derivatives market

Many OTC derivatives markets are not highly liquid which is part of why dealers in these markets make profits. In markets which are not highly liquid, the main determinants of liquidity are dealers' willingness to bear risk, and their ability to hedge risk. This latter ability depends on the liquidity of the underlying markets which is the focus of most of our analysis.

Liquidity of the underlying markets

There are two main sources of liquidity in the underlying markets. The first source of liquidity is the liquidity provided by market-makers. Market-makers quote prices at which they are willing to buy and sell fixed, typically small quantities of underlying assets. The spread between these prices (Bid-Ask spread), or the price impact associated with making a trade (often denoted λ) with a market-maker is an appropriate measure of liquidity in normal market conditions. However, it is probably not reasonable in abnormal conditions.

I make this distinction between normal and abnormal conditions because I view market-makers as providers of immediacy; i.e. market-makers provide immediate temporary liquidity to the market to absorb short-term order imbalances which they believe will disappear when the other side of the market eventually (relatively soon) emerges. In abnormal market conditions, this other side of the market may be small or non-existent; in these abnormal circumstances market-makers will provide very little liquidity to the market. More importantly, any analysis of liquidity that is based on market-makers ability and willingness to absorb volume in normal markets will undoubtedly generate erroneous implications about market liquidity in abnormal market conditions.

The most important determinant of market liquidity in the event of abnormal market conditions is not market-makers, but value investors who will presumably be willing to eventually take the other side of the positions that market-makers hold temporarily. Value investors willingness to provide liquidity is represented by the matrix K in our stylised model because K measures the propensity of value based investors to push prices back towards what they perceive as fundamental values when prices appear to deviate from fundamentals. Equation (5) shows that as K goes to infinity, the variance of asset prices goes to Σ_V, i.e. asset prices do not deviate from fundamentals as K becomes large. This corresponds to the case of infinitely liquid markets.

As K becomes small, there is a distinct absence of value investors. In this case, variations in price are due to noise traders and derivatives dealers, but are not due to long-run fundamentals. This creates a scenario where prices could wander far from long-run asset value.

Determinants of K

In noisy rational expectations models, the standard functional form for K is

$K = \tau \Pi(V \backslash I, P)$

where $\Pi(V \backslash I, P) = (\Sigma_V^{-1} \backslash I, P)$.

$\Pi(V \backslash I, P)$ is known as the precision of value investors estimates of V given their information set I and observed asset prices P, and τ is the average value of value investors coefficient of risk tolerance. The formula for K shows that markets will be more liquid the greater is value investors tolerance for risk or the more precise are value investors estimates of V.

The precision of value investors information is partially determined ex-ante and partially determined ex-post. From an ex-ante perspective, value investors have an incentive to expend effort learning about V because this will be useful in exploiting differences between realised securities prices and value investors estimates of V.

The desire to learn about V also depends on the amount of ex-ante expected dynamic hedging in the market by dealers, and on the ex-ante expected amount of noise trading. The larger are these latter two, the greater is the incentive to learn about V to exploit potential mispricing. However, if value investors do not ex-ante know the amount of dynamic hedging interest in a market, or set of markets, then they will choose their amount of information gathering based on the unconditional average amount of dynamic hedging that takes place in a market. This means they will gather too little information in some markets and too much information in others. This may lead to suboptimal risk-sharing and too much price volatility. Grossman (1988) makes a similar argument (mine is based on his) to illustrate the distinction between exchange traded and OTC options. The open interest in exchange traded options is publicly known which implies that demands for dynamic hedging are known as well. This generates the right incentives to engage in liquidity provision. By contrast, if an option is OTC, then the amount outstanding is not known, and thus incentives for liquidity provision may be suboptimal.

In a framework with asymmetric information, K could potentially be determined by ex-post as well as ex-ante factors, although modelling this is very difficult. For example, suppose there are two types of value investors. One type only has public information about V and the other has private information. Under these circumstances, if a value investor with public information observes a change in P, he knows it could be because of dynamic hedging, noise traders, or news about V that other value investors received. Under these circumstances, a large change in P may cause the value investor to revise downward the precision of his assessment of V.

Determinants of *E(V\I, P)*

The other important determinant of liquidity demand is *E(V\I, P)*. For value investors with only public information, if *E(V\I, P)* declines for sufficiently large decreases in P, then these value investors will tend to buy less as prices decline. Moreover, their beliefs may place them in a position where once price declines are steep enough, it looks better to sell into a decline than to buy. This is the scenario for the crash in Gennotte and Leland (1990).

Put slightly differently, after a large decrease in P, value investors with public information may be very hesitant to purchase stock because they do not know whether the change in P is due to a change in V or not. If it is due to a downward move in V, they should not want to purchase, and may want to sell. But, if it is due to a liquidity shock, they should want to purchase.

The determinants of *E(V\I, P)* depend on the signal extraction problem solved by publicly informed value investors. If publicly informed value investors are not aware of liquidity trades, or hedging trades by derivatives dealers, they may mistakenly attribute these price movements to information. The key to avoiding this particular problem is the ability of value investors to distinguish to some extent among various reasons for trade. This will be discussed more below.

4. Feedback and liquidity demand

Liquidity demand comes from two sources, noise traders and derivatives dealers. The demands from noise traders, as modelled here, are not price sensitive which means they represent liquidity demand, but they do not have potential feedback effects on prices. By contrast, derivatives dealers hedging trades are contingent on price. Thus, they have a feedback effect into the market. The strength of this feedback is measured by $H_{PP}\left(P_0\right)$. Roughly speaking, this is the slope of dynamic hedgers excess demand curve for the underlying assets. If the diagonal elements of H_{PP} are negative this would imply that the dynamic hedgers (on net) positive feedback trade by selling as prices go down and buying as prices go up. Alternatively, if the diagonal elements are positive, then the dynamic hedgers (on net) sell as prices rise and buy as prices fall. $K + H_{PP}\left(P_0\right)$ is the slope of the excess demand curve for dynamic hedgers and value investors together. Since value investors tend to buy if the asset is underpriced, it is probably safe to assume that the elements of K (or at least its diagonals) are positive. This implies that those markets which are the most illiquid, i.e. those with the greatest amount of price volatility will be those with $K + H_{PP}\left(P_0\right)$ close to zero; i.e. it will be those where there is negative feedback trading. If this negative feedback trading is so severe that the slope of the excess demand curve approaches 0, then a shock from noise traders or a shock from liquidity traders will generate price volatility that is nearly infinite.

Sunshine trading

Dynamic hedgers (derivative dealers) and value investors have much to gain by trade with each other since value investors are generally liquidity providers and (derivatives dealers) are liquidity demanders. One way that hedgers can minimise their own price impact is by taking steps to increase K. This can be done by making some aspect of their trades or trading intentions known to the market in advance. They can also indicate that the trades are not informationally motivated. This practice is known as Sunshine Trading and is discussed extensively in Admati and Pfleiderer (1991). The advantage of preannouncing planned large trades is twofold. First, it gives value investors some time to investigate market conditions (acquire information) before they provide liquidity. Second, since the trades are informationless, preannouncing the trades reduces the chance that the ensuing price movement will be misconstrued as information, potentially triggering a very large price move. Admati and Pfleiderer (1991) show that preannouncing trades can have an important effect in reducing price volatility. Gennotte and Leland (1990) take this reasoning a step further. They show that in a rational expectations model, with asymmetric information, if some participants are not aware of the presence of hedging trades, then very large price drops may be misconstrued as information. This can cause prices to drop much further, i.e. this can cause the market to crash. Furthermore, they show fairly convincingly that this type of reasoning is probably needed to explain the crashes of 1929 and 1987.

Simulation results on liquidity and feedback

To gain a better understanding of the model presented in section II and expanded upon in the appendix, two types of model simulations were conducted. In the first we assumed that there is only one type of value investor in the market, that these represent a fixed proportion of market participants, and that these value investors have a common private signal of the asset's underlying value. We refer to this as the symmetric information case since all value investors have the same information. We then examined how varying the intensity of dynamic hedging (as measured by H_{PP} or Derivatives Dealers Net Gamma) affected price volatility in the underlying asset market (shown in Figure 1), and how it affected the sensitivity of asset prices to liquidity trades (shown in Figure 2).[2] In the second type of simulation we kept the same number of value investors, but divided them into two types, informed value investors who observe a common signal about v, and publicly informed value investors who base their decisions on the information about v revealed by market price. We refer to this as the asymmetric information case since value investors do not have the same information. The result for the second type of simulation is also contained in figures 1 and 2.

The vertical axes in figures 1 and 2 are the natural logarithm of the variance of price and the natural logarithm of liquidity sensitivity respectively. This scaling was chosen because when gamma becomes negative enough, the market's excess demand curve becomes nearly vertical leading to near price indeterminancy and major price volatility which is too large to appear in a graph on a different scale. That said, Figures 1 and 2 display two striking features. The first is that when gamma becomes negative enough price volatility increases very sharply. For example Figure 1 shows that in the symmetric information case, when net gamma is near -2200, a small decrease in gamma increases price volatility by a factor of about e^8, which is a 3000-fold increase in price volatility. The same feature appears for the asymmetric information case, but it occurs much earlier, i.e. in the asymmetric information case price volatility increases far faster as gamma increases. This occurs for two reasons, first, value investors with lower quality information are more hesitant to provide liquidity to the market, i.e. they are less willing to provide an offset to liquidity trades. Secondly, and more importantly, publicly informed value investors are making inferences about value from prices, thus when prices go down, they are more likely not to purchase because it may indicate a decline in assets underlying value and not a good time to buy underpriced assets. Figure 2 shows essentially the same story as measured by the sensitivity of price to noise traders demands. The figure shows this sensitivity is increasing as derivative dealers gamma positions become more negative and also as we move from the symmetric to asymmetric information cases.

A single-asset static model lacks the richness associated with multi-asset models. Figure 3 illustrates some of the richness of the multi-asset setting. Specifically, in Figure 3 we examine a six asset economy in which derivative dealers net gamma position in one of the assets (asset one) becomes progressively negative. This has implications for the price volatility of asset 1, but also has

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[2] Sensitivity is measured as the change in asset price due to a change in noise trading (ε).

major general equilibrium spillover effects that occur as value investors adjust their entire portfolios to accommodate the large amounts of dynamic hedging that take place in asset 1. More specifically, the Figure shows that as derivative dealers net gammas become more negative in asset 1, the volatility of the other assets (shown in markets 2 through 6) rise as well. This effect appears to be more pronounced the greater is the asymmetric information in the market, as shown by the dashed line.

A feature of Figure 3 that is puzzling at first is that price volatility appears to begin falling again as derivative dealers net gamma becomes sufficiently negative. This is because the economy's excess demand curve for these assets goes from near vertical to backward sloping as gamma becomes negative enough. This is a very perverse case which I do not believe should be taken seriously but does illustrate some of the weaknesses involved in using a Walrasian model.[3]

5. Items for further study

Many important details have been left out of this broad-brush treatment. The purpose of this section is to highlight additional areas for further research.

The first important item is further study of hedging behaviour. While one of our primary feedback concerns is dynamic hedging by dealers, little is known about how this hedging is actually done in practice. This is very important for our analysis on liquidity. I think this can be studied in two ways. First, as part of this effort some of us should go to dealers and ask them details about how and where they hedge their risk, the frequency with which hedges are adjusted, etc. Second, our simulations exercises should impose a variety of assumptions about hedging behaviour. Then we can test the sensitivity of our results to these assumptions.

A second important area for more research involves an examination of the role of risk sharing among dealers. The basic model that I have presented is based on net aggregate positions of all derivative dealers. However, the distribution of these positions across the dealer community is probably very important for determining hedging needs. For example, if derivative dealers net position is equivalent to one position in a put option with a huge open interest, then if this is distributed evenly across derivatives dealers, the need for each dealer to dynamically hedge may be very small. However, if the net position is concentrated with one or a very small number of dealers, the needs for dynamic hedging become larger. We need to inquire more about how these risks are shared.

A third area for study involves transparency of option positions. More specifically, most of the papers that I have discussed make the point that knowledge of the amount of liquidity motivated trade (or trades for hedging purposes) may reduce price volatility. To some extent, market forces

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[3] To illustrate the perversity, in this case the market's response to a liquidity induced purchase requires asset prices to fall so that derivative dealers will sell enough assets to clear the market. This is probably an unrealistic representation of how the market clears.

already provide this transparency via sunshine trading. We should study: what determines the prevalence of sunshine trading, and why do not we observe it more often? Second, even without sunshine trading, knowledge of open interest in options may provide some information on hedging demands. This may explain why the introduction of option markets does not tend to increase volatility in the underlying assets. The role of information in option open interest can be studied empirically. This should be an intermediate goal of this project. Finally, we should study the role of disclosure of some aggregates of risk exposure. In particular, we should study the extent to which this potential disclosure substitutes for knowledge of open interest or other hedging demand proxies in OTC markets. We also need to study the potential for a moral hazard problem if it is perceived that the government will monitor market liquidity and take steps to maintain it.

A fourth area for additional effort involves studying the behaviour of value investors. Two tasks need to be carried out. First, we need to figure out who these investors are. My guess is they are pension funds, life insurers, and others (?). Second, we should get additional information on their demands. Part of this could involve studying which investors are perceived to have long horizons. A small, so-so, literature exists on this subject.

Last, but not least, the relationship between market stress and market liquidity should be studied more. With disclosures about stress in various scenarios, central banks will have better measures of markets in stress even if the markets themselves have not broken down. However, we do not know how conventional measures of liquidity and market function are related to these measures of stress. Acquiring the data on stress, but not releasing it, may provide an avenue to study these relationships further.

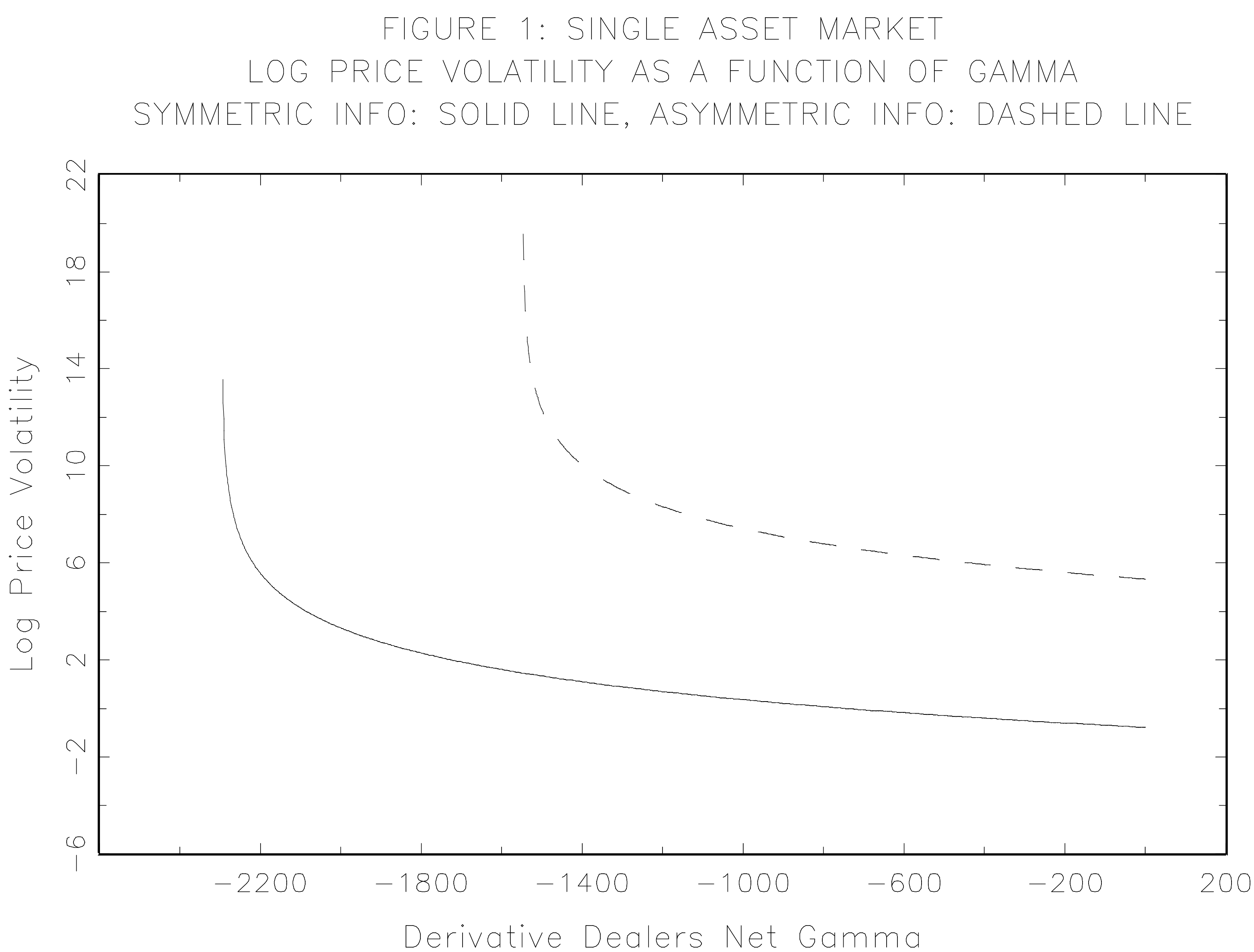


FIGURE 1: SINGLE ASSET MARKET
LOG PRICE VOLATILITY AS A FUNCTION OF GAMMA
SYMMETRIC INFO: SOLID LINE, ASYMMETRIC INFO: DASHED LINE
Log Price Volatility
22
18
14
10
6
2
−2
−6
−2200
−1800
−1400
−1000
−600
−200
200
Derivative Dealers Net Gamma

FIGURE 2: SINGLE ASSET MARKET
LOG LIQUIDITY SENSITIVITY AS A FUNCTION OF GAMMA
SYMMETRIC INFO: SOLID LINE, ASYMMETRIC INFO: DASHED LINE

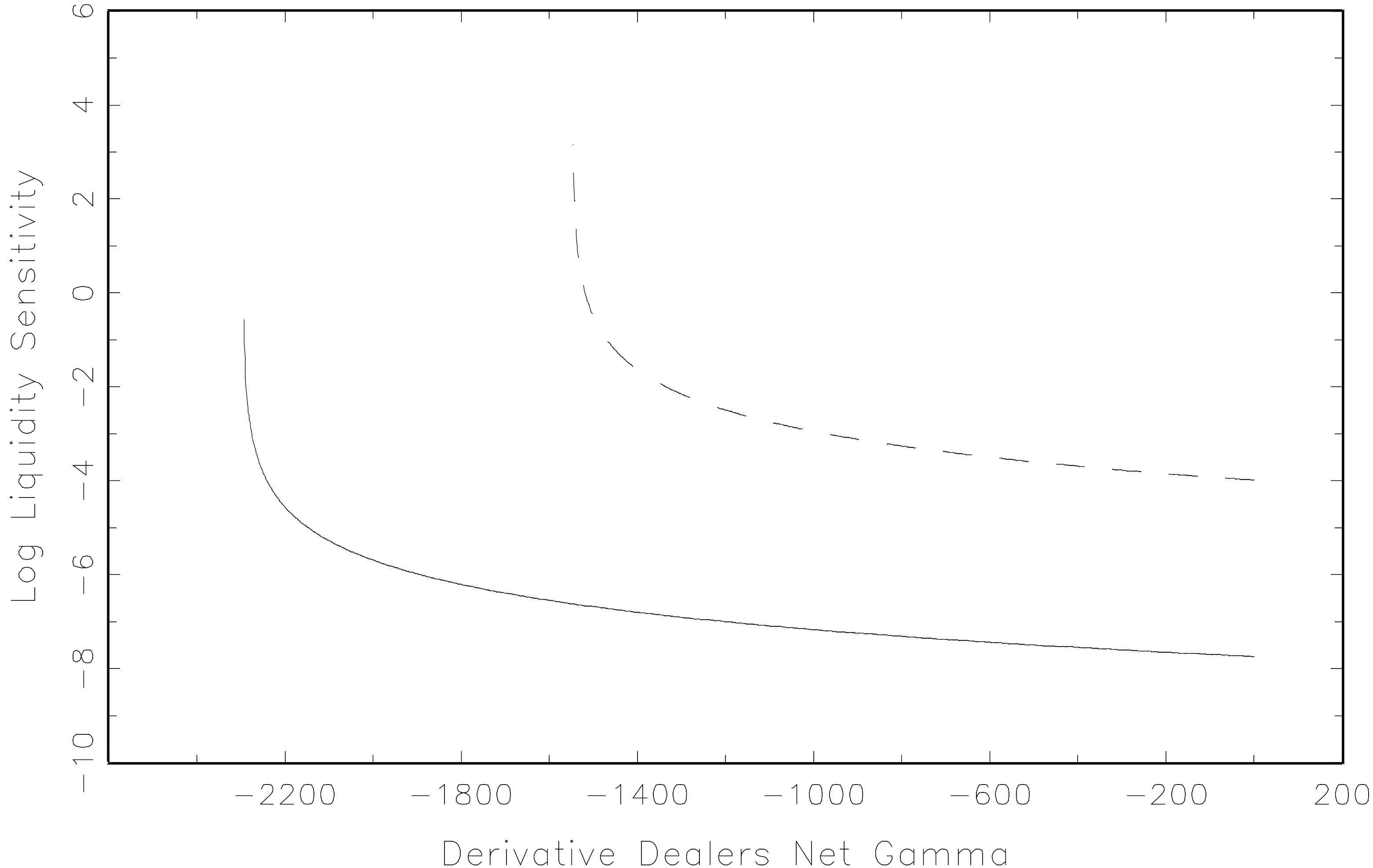
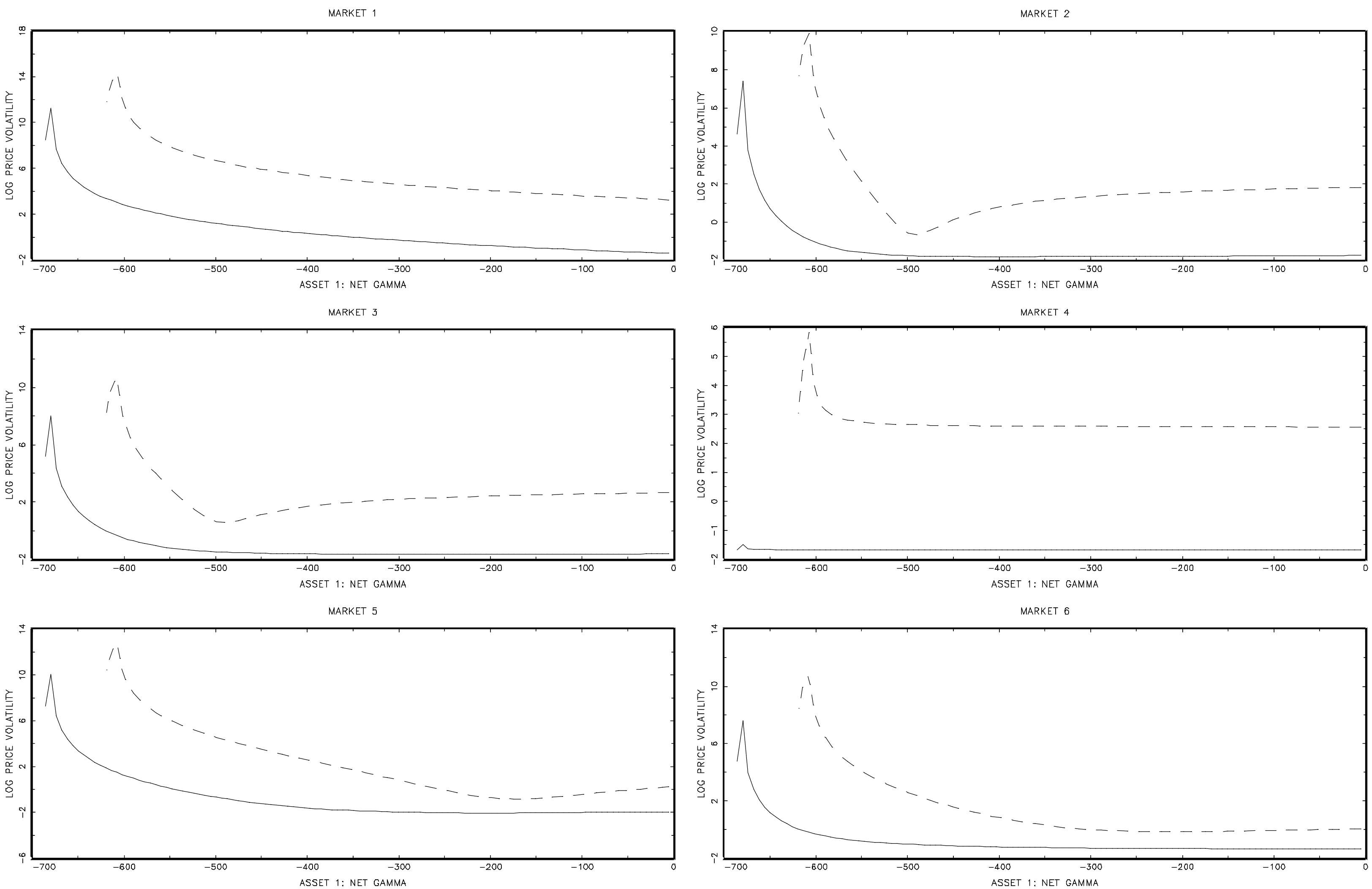


FIGURE 3: MULTI-ASSET MARKET MODEL
LOG MARKET PRICE VOLATILITY VS. GAMMA IN MARKET 1
SYMMETRIC INFO: SOLID LINE, ASYMMETRIC INFO: DASHED LINE



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Appendix

The purpose of this appendix is to further extend the model presented in the text and to provide additional details on the model's derivation and results.

1. An extended model

The assets

Assume there are N financial assets indexed $A_1,\dots A_N$, represented by the Nx1 vector *A*. The net supply of the assets is represented by the Nx1 vector X_T. The price of the assets is represented by the vector *P*, and the price of the assets last period is denoted P_0. The liquidation value of the assets is represented by the random vector

$$\tilde{v} = \tilde{\theta} + \tilde{u}$$

where:

$\tilde{\theta} \sim N(\bar{\theta},\Sigma_\theta)$ and $\tilde{u} \sim N(0,\Sigma_u)$.

Market participants

There are four types of participants in the market, noise traders, derivatives dealers, informed value investors, and uninformed value investors. Noise traders objectives are uncorrelated with other participants objectives. We will denote their net trades by $\tilde{\varepsilon}$ and make the assumption that:

$$\tilde{\varepsilon} \sim N(0,\Sigma_\varepsilon).$$

The second type of traders are derivatives dealers. Derivatives dealers are assumed to choose their net positions X_{DD} in the underlying assets so as to remain delta-neutral. It is assumed that they were delta-neutral in the previous period at $P = P_0$, with net asset holdings represented by the vector $X_{DD}(P_0)$ and with a delta of the net derivatives portfolio of $H_P(P_0)$. Therefore, their position in the underlying market must change as *P* changes in order to maintain delta-neutrality. In particular, to a linear approximation their desired position as function of *P* is:

$$X_{DD} = X_{DD}(P_0) - H_{P,P}(P_0)(P - P_0).$$

The third type of traders are informed value investors. Informed value investors are "informed" because they know the realisation of $\tilde{\theta}$. We assume that the mass of informed traders is μ_I ("I" denotes informed), that each informed trader has CARA utility with risk tolerance parameter τ, and initial wealth W_0. Borrowing and lending take place without constraint at the riskless rate of interest, which is normalised to *0*. This allows each informed trader to choose his portfolio positions $X_1(P,\tilde{\theta})$ to maximise:

$$E\left(-exp^{-\frac{\tilde{W}}{\tau}}\middle|\tilde{\theta}\right)$$

such that $\tilde{W}=W_0+X_I^{'}\left(\tilde{V}-P\right)$.

It is well known that the optimal $X_I\left(P,\tilde{\theta}\right)$ satisfies:

$$X_I\left(P,\tilde{\theta}\right) = \tau Var\left(\tilde{v}\middle|\tilde{\theta}\right)^{-1}\left(E\left(\tilde{v}\middle|\tilde{\theta}\right)-P\right)$$

$$= \tau\Sigma_u^{-1}\left(\tilde{\theta}-P\right).$$

The fourth type of participant is uninformed value investors. The mass of these investors is μ_{UI} ("UI" denotes uninformed). They have the same utility function as informed value investors but choose their positions $X_{UI}\left(P\right)$ without knowledge of $\tilde{\theta}$. They do, however, know the structure of the model, and they observe *P* (which reveals some of the information of the informed) and condition their choices on *P*. In particular, the uninformed choose $X_{UI}\left(P\right)$ to maximise:

$$E\left(-exp^{-\frac{\tilde{W}}{\tau}}\middle|P\right)$$

such that $\tilde{W}=W_0+X_U^{'}\left(\tilde{v}-P\right)$.

It is well known that the optimal X_U satisfies:

$$X_{UI}\left(P\right) = \tau Var\left(\tilde{v}|P\right)^{-1}\left(E\left(\tilde{v}|P\right)-P\right).$$

However, the optimal $X_{UI}\left(P\right)$ depends on $Var\left(\tilde{v}|P\right)$ and $E\left(\tilde{v}|P\right)$. Both of these depend on the informativeness of *P* for $\tilde{v}$. This needs to be solved for as part of the overall general equilibrium of the model. We do this below. More specifically, the exposition that follows is designed to accomplish four goals: 1. Relate *P* to the private information; 2. Solve for $E\left(\tilde{v}|P\right)$; 3. Solve for $Var\left(\tilde{v}|P\right)$; and 4. Solve for *P* and do various comparative statics exercises.

Information about $\tilde{v}$ revealed by *P*

Solving for the information revealed by *P* is a two-step procedure. First, we will solve for the information revealed by *P* as a function of $E\left(\tilde{v}|P\right)$ and $Var\left(\tilde{v}|P\right)$. Second, given the information revealed by *P*, we will solve for $E\left(\tilde{v}|P\right)$ and $Var\left(\tilde{v}|P\right)$. This section is only concerned with the first of these steps.

The market clearing condition in these markets requires that prices be set so that supply equals demand. This implies:

$$X_T=\mu_{UI}X_{UI}(P)+\mu_I X_I(\tilde{\theta},P)+X_{DD}(P)+\varepsilon$$

$$=\mu_{UI}\tau Var\left(\tilde{v}\middle|P\right)^{-1}\left(E\left(\tilde{v}\middle|P\right)-P\right)+\mu_I\tau\Sigma_u^{-1}\left(\tilde{\theta}-P\right)+X_{DD}\left(P_0\right)-H_{P,P}\left(P_0\right)\left(P-P_0\right)+\varepsilon.$$

Rearranging the above equation shows that a nonlinear function of P and other known parameters is equal to $\tilde{\theta}$ plus a function of "noise". More specifically, rearrangement produces:

$$S\left(P\right)=\tilde{\theta}+\frac{\Sigma_u\varepsilon}{\mu_I}$$

where:

$$S\left(P\right)=\frac{-1}{\mu_I}\Sigma_\mu\ [\mu_{UI}\tau Var\left(\tilde{v}\middle|P\right)^{-1}\left(E\left(\tilde{v}\middle|P\right)-P\right)-\mu_I\tau\Sigma_u^{-1}P+X_{DD}\left(P_0\right)-H_{P,P}\left(P_0\right)\left(P-P_0\right)-X_T\ ]$$

Since *S(P)* is a function of *P* and publicly known variables, the above equation shows that knowledge of *P* is equivalent to having a noisy signal of $\tilde{\theta}$, the information of the informed value investors. Uninformed value investors can condition on this information. In particular, since $\tilde{v}$, $\tilde{\theta}$ and $\tilde{\varepsilon}$ are normally distributed, we know that:

$$E\left(\tilde{v}\middle|P\right)=E\left(\tilde{v}\middle|S\left(P\right)\right)$$

$$=\tilde{v}+Cov\left[S\left(P\right),\tilde{v}\right]\left[Var\left(S\left(P\right)\right)\right]^{-1}\left[S\left(P\right)-\overline{S\left(P\right)}\right]$$

$$=\overline{\theta}+\Sigma_\theta\left[\Sigma_\theta+\frac{\Sigma_u\Sigma_\varepsilon\Sigma_u}{\mu_I^2}\right]^{-1}\left[S\left(P\right)-\overline{\theta}\right].$$

We also know:

$$Var(\tilde{v}|P) = Var(\tilde{v}|S(P))$$

$$= \Sigma_v - Cov(\tilde{v}, S(P))\left[Var(S(P))\right]^{-1} Cov(\tilde{v}, S(P))'$$

$$= \left[\Sigma_\theta + \Sigma_u\right] - \Sigma_\theta\left[\Sigma_\theta + \frac{\Sigma_u \Sigma_\varepsilon \Sigma_u}{\mu_I^2}\right]^{-1} \Sigma'_\theta.$$

At this point we have solved for $Var(\tilde{v}|P)$, and we have solved for $E(\tilde{v}|P)$ as a function of *S(P)*; however, we have not fully solved for $E(\tilde{v}|P)$ since *S(P)* depends on $E(\tilde{v}|P)$. The next section presents the complete solution for $E(\tilde{v}|P)$.

Complete solution for $E(\tilde{v}|P)$

The complete solution is complicated, because it is derived in a general equilibrium setting. That said, the answer is:

$$E(\tilde{v}|P) = M_0 + M_1\bar{v} + M_2 P$$

where:

$$M = \left[I + Cov(S(P), \tilde{v})\left[Var(S(P))\right]^{-1} \frac{\tau\mu_{UI}}{\mu_I} \Sigma_u Var(\tilde{v}|P)^{-1} \right]$$

$$M_0 = M^{-1}\left(Cov(S(P), \tilde{v})\left[Var(S(P))\right]^{-1} \frac{\Sigma_u}{\mu_I}\left[-X_{DD}(P_0) + X_T - H_{PP}(P_0)P_0\right]\right)$$

$$M_1 = M^{-1}\left(\left[I - Cov(S(P), \tilde{v})\left[Var(S(P))\right]^{-1}\right]\right)$$

$$M_2 = M^{-1}\left(Cov(S(P), \tilde{v})\left[Var(S(P))\right]^{-1} \frac{\Sigma_u}{\mu_I}\left[\mu_{UI}\tau\left[Var(\tilde{v}|P)\right]^{-1} + \tau\mu_I\sigma_u^{-1} + H_{P,P}(P_0)\right]\right).$$

The basic form of the above answer is that $E(\tilde{v}|P)$ is a linear combination of *P* and the unconditional expectation $\bar{v}$. This is a standard result in the rational expectations literature.

Finally, given solutions for $E(\tilde{v}|P)$ and $Var(\tilde{v}|P)$, it is possible to solve for *P*. We do this in the next section.

The solution for *P*

The solution for *P* is found by substituting the expressions for $E(\tilde{v}|P)$ and $Var(\tilde{v}|P)$ into the market clearing condition, and then solving for *P*. This yields the following expression:

$$P = \Pi^{-1}\left(X_T - \varepsilon - \tau\mu_{UI} Var(\tilde{v}|P)^{-1}(M_0 + M_1\bar{v}) - \tau\mu_I \Sigma_u^{-1}\tilde{\theta} - X_{DD}(P_0) - H_{P,P}(P_0)\right)$$

where $\Pi = \tau\mu_{UI} Var\left(v|P\right)^{-1}\left(M_2 - I\right) - \tau\mu_I \Sigma_u^{-1} - H_{P,P}\left(P_0\right)$.

Similarly, we can solve for the unconditional variance of *P*. This yields:

$Var(P) = \Pi^{-1}\left[\Sigma_\varepsilon + \tau^2\mu_1^2\Sigma_u^{-1}\Sigma_\theta\Sigma_u^{-1}\right]\Pi^{-1}$.

Dynamic macro stress exercise including feedback effect

by

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Abstract

The goal of this study is to illustrate a viable way to explore macro risk in markets, not only from a static viewpoint but also from a dynamic one. In this paper, I focus mainly on the feedback effect caused by a market stress and try to present a possible analytical framework to incorporate the effect into a macro stress exercise. I discuss how to take into account feedback effects employing two approaches to the estimation of market participants' behaviors in response to a stress. One approach assumes typical portfolio rebalancing of each agent based on the available information, including the agent's trading strategy and its loss cutting rules, etc. The other approach involves the extraction of a pattern of portfolio rebalancing of each agent based on the historical data on its portfolio profile, such as sensitivity to risk factors, by utilizing a neural network. A dynamic stress exercise taking into account any feedback effect will provide us with more useful and vivid information on the macro market risk profile under stress and enable us to prepare for stress in a more efficient and effective way.

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1. Introduction

1.1 Framework of Macro Stress Exercise

The inputs for the Macro Stress Exercise which will be a possible tool for market perticipants to comprehend the macro market risk profile: a stress scenario provided by central banks and portfolio sensitivity data for reporting institutions (Figure 1). The output is an aggregate risk measure covering the institutions. There could be two approaches to aggregating micro risk figures into a macro risk measure. One can be called as the "Revaluation Approach," in which firms are expected to report expected loss amounts in a given stress scenario. The other can be called as the "Sensitivity Approach," in which firms are expected to report summary data on their sensitivity to market risk factors.

My research goal is to illustrate a viable way to explore macro risk in markets, not only from a static viewpoint but also from a dynamic one. In this paper, I focus mainly on the feedback effect caused by a market stress and try to present a possible analytical framework to incorporate the effect into the exercise. I discuss how to take into account feedback effects basically employing the Sensitivity Approach, which provides us with more flexibility in aggregating risk. The Revaluation Approach, however, could be dealt with by constructing the actual stress scenario obtained by employing an initial stress scenario via the feedback effect.

1.2 Issues to be discussed under Dynamic Stress Exercise

There are various factors to be considered when aggregating micro risk figures into a macro risk measure. Especially when conducting a macro stress exercise, we have to take into account not only static risk profiles in individual institutions but also the dynamic effects caused in them by those institutions' reactions against an initial shock. Feedback effect and liquidity effect are the key issues to be discussed in this context.

In a static world, if Bank-i's expected loss amount is R_i under a stress scenario, $\sum R_i$ could be an aggregate risk measure. However, once initial stress occurs in a market, traders in each bank begin to rebalance their portfolios or hedge their positions to minimize future losses. Along the way, the initial stress scenario could be either exacerbated or alleviated by such reactions. Feedback effect is defined as the impact on market price caused by traders' trading behaviors towards market price movements, which are realized as a result of traders' trading strategies and traders' needs for portfolio rebalancing. Liquidity effect including market impact caused by position liquidation behaviors is another important issue to be explored in order to capture a dynamic picture of macro risk under a stress. In the following chapters, I mainly focus on feedback effect, by presenting a viable analytical framework and discussing the implications of a dynamic macro stress exercise which takes into account the feedback effect.

2. Analytical Framework

2.1. Expanding a Framework of Static Stress Exercise

We need to expand the static framework in two dimensions in order to take the feedback effect into consideration. The first dimension is time horizon. I employ a sequential framework to conduct a dynamic stress exercise. My purpose in this study is not to aggregate micro risk figures at a static point, but to construct a model through which we can comprehend the dynamics of a macro risk profile that is affected by institutions' reactions at intervals (a multiperiod model). There are many impressive studies employing a multiperiod model in market microstructure. The Glosten and Milgrom model (GM model) [1985] employs multiperiod models to analyze the dynamic features of market impacts.

Secondly, we need to expand our model to correspond with a variety of agents. Each agent's behavior varies, depending on its portfolio mix and trading strategy. Glosten and Milgrom assume that there are three types of agents in a market, namely, informed trader, uninformed trader, and market maker. They analyze the decision-making process of a market maker by modeling price mechanisms in a market. The Gennottee and Leland model (GL model) [1990] is another example which takes the variety of agents' behaviors into consideration. The GL model takes into account the factor of supply from hedge traders who employ a portfolio insurance strategy. The model allows them to review a market meltdown mechanism caused by the hedging behaviors of a significant proportion of market participants, as was the case on Black Monday in 1987.

In the following chapters, I examine two approaches to the estimation of various behaviors of agents. One approach assumes that each agent employs a typical trading strategy and that we know it a priori. In the other approach, I derive the trading pattern of each agent from historical data on market movements and individual portfolio profiles.

2.2 Multiperiod Model

I develop a multiperiod model in order to take the feedback effect into account when I conduct the stress exercise described in this chapter. Bank-i's expected loss under a provided stress scenario is R_i at time t+1. Each agent reacts against the shock at t+2, and we finally obtain an aggregate macro risk measure after considering the feedback effect at t+3 (Figure 2). When we employ the Sensitivity Approach, we can estimate the aggregate risk measure at t+3 by assuming agents' reactions and calculating the risk amount based on sensitivity. Using finally realized prices to construct an actual stress scenario, we can take the feedback effect into account, even if we adopt the Revaluation Approach, by applying this scenario to reporting institutions.

2.2.1 *Framework of Multiperiod Model*

The portfolio value of the i-th agent at time t is expressed by F_{it}. F_{it} is a function of the portfolio mix f_{it} and risk factor prices x_t. Here, we need to consider sequential movements not only in risk factor prices but also in the portfolio mix.

$$F_{it} = f_{it}(x_t)$$

When an initial stress scenario (ISS, S_0) is provided, the portfolio value at t+1 can be shown as below. I assume that there is only one risk factor x_t and that f_{it} is consistent from t to t+1.

$$S_0 = dx$$

$$F_{it+1} = f_{it+1}(x_{t+1}) = f_{it}(x_t + dx)$$

We can obtain static aggregate risk R_s

$$R_s = \sum R_i = \sum dF_i = \sum \frac{\partial f_i}{\partial x} dx$$

Then, I take into account the i-th agent's portfolio rebalancing ($f_{it+1} \rightarrow f_{it+2}$). Portfolio value at t+2 is

$$F_{it+2} = f_{it+2}(x_t + dx)$$

Assume that there are n banks in our model and that they react to the ISS individually. Kawahara [1996] argues that the market impact on the risk factor price (dx') caused by an agent's trading can be expressed as a function of the macro trade imbalance, i.e., net supply in the market.

$$dx' = G\left(\sum_{i=1}^{n} \frac{dF_i}{x}\right)$$

The actual stress scenario (ASS, S_1) is provided as follows.

$$S_1 = dx + dx' = dX$$

Dynamic aggregate risk including feedback effect R_d is

$$
\begin{aligned}
R_d &= \sum \left(F_{it+2} - F_{it}\right) \\
&= \sum \left(f_{it+2}\left(x_t + dX\right) - f_{it}\left(x_t\right)\right) \\
&= \sum \left(\frac{\partial f_i}{\partial x}\left(dx + dx'\right) + \frac{\partial f_i}{\partial t} dt\right)
\end{aligned}
$$

2.3 Considering the variety of agents' behaviors

The multiperiod model described in Chapter 2.2 includes agents' portfolio rebalancing behaviors towards ISS, which depend on each agent's portfolio mix at time t and its trading strategy. We can take the variety of their behaviors into consideration by providing various types of $\frac{\partial f_i}{\partial t} dt$ in equation (1) corresponding to agents' types. In this chapter, I examine two alternatives:

1) assuming typical portfolio rebalancing of each agent based on the available information including its trading strategy and loss cutting rules, etc.
2) extracting a pattern of portfolio rebalancing of each agent based on the historical data on its portfolio profile - sensitivity to risk factors.

2.3.1 Assuming Typical Portfolio Rebalancing

I present a simple example to show how we obtain R_d, taking into account the feedback effect . Assume that there are three agents in a market where only one tradable risk factor, x, exists and that each agent has the portfolio mix described as follows:

Agent 1 : holding a_t units of asset x at t. $a_t = a$ constant

Agent 2 : holding b_t units of asset x at t. $db_t = b / x_t$ b is constant.

Agent 3 : holding c_t units of asset x at t. $dc_t = c \times dx_t$ c is constant.

Each agent has the trading strategy described below:

Agent 1 never trades

Agent 2 buys a constant amount (b dollars) of x every period. This strategy is the so-called "dollar-cost-average strategy".

Agent 3 buys x after x has risen or sells after it has fallen. The trading amount depends on the magnitude of the change in x in the previous period. If an agent employs a portfolio insurance strategy involving dynamic hedging, we can observe trading behavior which is the same as that of Agent 3.

I can explore dynamic aggregate risk based on the multiperiod model including the three agents described above. Each agent has 100 units of x at time t.

$$x_t = 100$$

$$S_0 = -6.5$$

$$x_{t+1} = 93.5$$

$$F_{it} = f_{it}(x_t) = 100 \times x_t = 100 \times 100 = 10000$$

$$F_{it+1} = f_{it+1}(x_{t+1}) = 100 \times x_{t+1} = 100 \times 93.5 = 9350$$

Static aggregate risk R_s can be calculated as

$$R_s = \sum R_i = \sum dF_i = 1950$$

Then, I consider agents' reactions against ISS.

$$F_{2t} = f_{2t}(x_t) = 100 \times x_t = 100 \times 100 = 10000$$

$$b = 2000$$

$$F_{2t+1} = f_{2t+1}(x_{t+1}) = (100 + b / x_{t+1}) \times x_{t+1} = (100 + 2000 / 93.5) \times 93.5 = 11350$$

Here, Agent 2 is assumed to buy $\dfrac{dF_2}{x_{t+1}} = +21$ units of x.

$$F_{3t} = f_{3t}(x_t) = 100 \times x_t = 100 \times 100 = 10000$$

$$c = 5$$

$$F_{3t+1} = f_{3t+1}(x_{t+1}) = (100 + c \times dx) \times x_{t+1} = \{100 + 5 \times (-6.5)\} \times 93.5 = 6311$$

Here, Agent 3 is assumed to sell $\dfrac{dF_3}{x_{t+1}} = -32.5$ units of x.

Assume that the market impact caused by three agents' reactions can be obtained as a linear function of the trade imbalance. I can get ASS;

$$
\begin{aligned}
dx' &= G\left(\sum_{i=1}^{n} \frac{dF_i}{x}\right) \\
&= k \cdot (21 - 32.5) \\
&= -11.5k
\end{aligned}
$$

If we assume k=0.2

$$S_1 = dx + dx' = -6.5 - 2.3 = -8.8$$

Dynamic aggregate risk R_d is calculated as follows;

$$
\begin{aligned}
R_d &= \sum \left(F_{it+2} - F_{it}\right) \\
&\cong \sum F_{it+2}\left(1 - \frac{x_{it}}{x_{it+2}}\right) \\
&= \left|-880 - 1065 - 544\right| \\
&= 2539
\end{aligned}
$$

Dynamic aggregate risk is 1.3 times static aggregate risk. We also observe that ISS will cause a further price decline in x. This information that we obtain via a dynamic stress exercise is more useful than that from a static exercise. However, we should be careful about the probability and accuracy of the behavioral assumptions that I employ in the model and further study is necessary.

2.3.2 Extracting a trading pattern of portfolio rebalancing of each agent

I present another approach assuming more realistic trading behavior in this chapter. Utilizing a neural network system, it would be possible to extract trading patterns of agents from historical risk factor price data and corresponding changes in their portfolio profiles (see Appendix).

First of all, we have to decide on a set of data to be used as inputs and outputs to and from the neural network. Since my purpose is to estimate the probable reaction of an agent against risk factor price movements, inputs must be factors which affect a trader's decisions and outputs are some indicators of its trading behavior. Candidates for inputs would be;

1) the agent's portfolio mix at time t,

2) business circumstances surrounding the agents, such as profit/loss conditions and adequacy of risk capital, and

3) risk factor price movements.

The portfolio rebalancing behaviors that I am trying to estimate can be expressed as movements in agents' portfolio mixes, in other words, agents' positions. Since it does not seem possible for us to gather historical data on agents' portfolio mixes, I have to regard changes in portfolio sensitivity to major risk factors or actual profit and loss figures as proxies for portfolio rebalancing behaviors and select them as outputs of neural network analysis.

Inputs to a neural network can vary because of the flexibility of the neural network system. At this stage I use data on risk factor price movements and news as inputs for the analysis. Other economic or financial measurements could be candidates for analytical inputs, and I will continue to explore the selection of suitable and effective inputs and outputs.

I conduct a set of simulations using historical market data and trading data during a certain period. Trading data is obtained by letting one of the staff of the Institute, who has nine years experience as a bond trader in a bank, simulate daily trading based on market data during a set historical period. Details of simulations are described below.

Risk factors

Yen interest rates are selected as risk factors in our simulation. Three points on a yield curve, 3-year, 5-year, and 10-year swap rates, are regarded as factors of yen interest rate risk.

Simulation period

It is essential to provide learning data which include stress periods for neural networks in order to make networks capable of estimating agents' behaviors under stress. I chose the period from October 1993 to March 1994 as a learning period which includes market stress. As shown in Figure 3, we experienced several sudden rises of yen interest rates at the beginning of 1994. After the strong downward trend in interest rates toward the end of 1993, both futures and cash JGB markets faced significant short trading triggered by the MOF's operation of selling JGB of 14th January 1994. I have also conducted out-of-sample simulations using market and trading data from April to September 1994.

Characteristics of agents

There are three agents in the simulation. They trade JGB in three maturities: 3-year, 5-year and 10-year. They decide their trading volume and direction based on their own trading strategies, and their portfolio sensitivity to the three risk factors and actual P/L are assumed to be available for us. Each agent has its own typical trading pattern shown in Table 1. Then, we observe daily interest rate data, news of economic and financial events, their realized P/L, and sensitivity to the risk factors. Reported data on sensitivity are converted to the equivalent positions of 10-year JGB. Each agent has its own target level of profit, position limit, and loss cutting rules. If the level of accumulated loss reaches the targeted profit level, the agent has to close his/her position at once.

Inputs for learning

I adjust the data mix of learning inputs in the following sequence.

First I provide a set of daily returns for four risk factors and interest rate volatility to a network as inputs for learning. In this case, the level of estimation accuracy of network which has learned agent 1's trading pattern is only 71.67% (see Figure 4-1).

Second, I include realized P/L data among the inputs, since each agent's appetite for trading is constrained by its accumulated P/L condition. The estimation power thus increases to 76.67%. from 71.67% (see Figure 4-2). However it still cannot follow the movements in actual data during the periods particularly when it changes in an accelerating fashion, such as around the 20th, 45th, and 103rd sample data.

Finally, I include three days recent market movements and news of financial and economic events among the inputs in order to improve the estimation power of networks (see Figure 4-3, Figure 5). Among the market movements and news, I put more weight on the more recent information. This adjustment improves the network's estimation level to 85.42%. Regarding Agent 3, an effective way of improving the network's estimation power further is to add input data on a market trend over a longer horizon, since he/she has a view of longer horizon than the other agents (See Figure 6). Table 2 shows the process of improving the estimation power regarding each agent by adjusting the inputs.

I conduct out-of-sample simulations using the networks which have learned the trading patterns of the agents. According to the simulation results, it seems relatively easy to follow the trading patterns of Agent 1 and Agent 2 (see Figure 7-1 and Figure 7-2). However, the network cannot follow Agent 3 very well (see Figure 7-3). The reason why the network fails in the case of Agent 3 is the difference in trading patterns between a dealer-type of agent and an investor-type of agent. During the former sub-period, Agent 3 has almost fixed its position to the long side because of market trend shows the interest rate falling. Since the market trend has drastically changed in the latter sub-period, it is very difficult for the network, which has only learned the agent's trading pattern in the period when the market trend was only moving in one direction, to predict position changes from short to long during a period which includes fluctuating market trends.

Stress scenario

I pick up the largest daily change in both upward and downward directions in each risk factor during the period and construct stress scenarios by combining these figures. In Figure 3, we can see which days are picked up as a stress on each risk factor. Regarding 3-year and 10-year swap rates, the largest changes in both directions occurred in March 1994. The period from the end of November 1993 to the middle of January 1994 was selected as a stress period for 5-year swap rates.

The magnitude of these market changes are shown in Figure 8-1 to Figure 8-3 and Table 3. A pair of parallel lines in each figure show 2 standard deviations during the period.

By combining these figures, I provide 4 types of stress scenario: parallel shift of the yield curve; stress at the shorter end of maturity; the middle zone of the yield curve; and the longer end of maturity. Each type of scenario has two directions, upward and downward. We therefore obtain altogether eight stress scenarios (see Table 4). In each scenario, volatility level is also set as the level equivalent to its largest change during the period.

Feedback effect

According to the simulation results (see Table 5), the downward stress in the yield curve cause larger changes in the agents' positions (scenario 2, 4, and 8) than the upward one. Directions in total demand/supply differ between a stress at the shorter end and a stress in the longer end. Downward stress at the shorter end of maturity causes a fair amount of supply to the JGB market, which can produce negative feedback, offsetting the initial stress. On the other hand, a downward stress at the longer end of maturity causes a fair amount of demand to the market, and it would produce positive feedback to the initial stress.

In order to determine the magnitude of the feedback effect, I regard total demand/supply, a figure summed up all agents' delta changes as a measurement. The average of daily total demand/supply during the period is - 0.2 billion yen, and its standard deviation is 12 billion yen (see Figure 9).

Parallel shift scenario (scenarios 1 and 2)

Regarding the directions of the feedback effects under the scenarios, both of them would cause negative feedback and offset the initial stress. Upward parallel shift pushes Agent 2 into buying a relatively large amount of its position, 26 billion yen. Total supply to the market, however, is only 12 billion, which is equivalent to the standard deviation of total demand/supply changes during the simulation period, and the direction of the feedback effect would be negative. Although downward parallel shift would cause a fair amount of selling of JGB, it also produces negative feedback to the stress under this stress scenario. These results could be interpreted to mean that a parallel-shift type of yield curve stress is less harmful from the view point of the macro feedback effect.

Stress at the shorter end of maturity (scenarios 3 and 4)

Simulation results show that upward stress at the shorter end of maturity causes a fair amount of demand for JGBs. On the other hand, downward stress causes a significant supply of such bonds. As same as the results in the parallel shift scenarios, stress at the shorter end of maturity would also cause negative feedback effect to the yield curve.

Stress in the middle zone of the yield curve (scenarios 5 and 6)

Stress in the middle zone causes a smaller magnitude of change in total demand/supply than the other stress scenarios. The direction of the feedback effect cause by upward stress in the zone, however, would be positive. It means that if we face upward stress in the middle zone, the longer maturity interest rate (10-year JGB price) is expected to increase (fall) further as a consequence of stress.

Stress at the longer end of maturity (scenarios 7 and 8)

Downward stress causes a more significant effect than does a downward one. Increase in the JGB price causes a fair amount of demand for JGB. It means that there could be positive feedback effect under this scenario.

Risk profiles of the agents

Assume that the magnitude of the feedback effect on JGB prices depends on the total demand/supply volume which is realized by agents' response to a stress. If market price changes can be described as a linear function of total demand/supply volume as follows, we can estimate the level of loss each agent will face because of the feedback effect. Estimated risk profiles of each agent are shown in Table 6. Figures in Table 6 show agents' loss amounts determined by the additional market price change as a result of feedback effect and delta position after the feedback trading. Further discussion on the definition of estimated loss caused by feedback effect will be necessary, particularly whether implicit profit/loss of feedback trading is taken into account.

$$\Delta P = \alpha\sigma_p \cdot \beta$$

ΔP : JGB price change

α : (total D/S) / (standard deviation of total D/S during the period)

σ_p : standard deviation of JGB price changes during the period

β : ratio of price change caused by market demand and supply conditions (in this simulation, $\beta = 1$)

Among the eight scenarios, scenario 2, the downward parallel-shift type of stress, has the potential to cause the most severe feedback effect in relation to the level of the macro risk. According to the simulation result, I can say that all of these scenarios would have low potential to produce a severe feedback effect, because each agent would not face a loss which exceeds VaR at time t, just before stress. I can explore which kind of stress scenarios can produce severe feedback effect and which type of agents play a critical role in stress by analyzing the results of a simulation, as we have shown in this chapter.

I am improving the estimation power of the network by umbundling the output, delta, into direction and volume in its changes. The prediction on whether an agent increses or decreases its delta is essential to estimate the direction of feedback effect. The predicton on tradeing volume is necessary to capture the magnitude of feedback effcet. The unbundlled outputs will provide us with more accurate approximations of the feedback effect. The tentative results of the simulation with new outputs give better estimation of both agent's trade direction and its volume. Figure 10-1 shows the estimation results of trade direction, where if agent increases/decreases its delta, the parameter is set +0.5/-0.5. The network fails to predict agent's trade direction only twice out of 120 data. Figure 10-2 shows the estimation results of trading volume. Comparing to Figure 10-3, which shows the results obtained via the simulation with the former output, i.e. delta itself, the estimation error is reduced to 44 from 78.

I also attempt to explore the effect of the variation in stress scenario. As I show in this chapter, the stress scenario provided here do not produce severe impact on macro risk profiles. I am now conducting a stress simulation, which assumes a stress not only in the yield curve but also with news. The tentative results of the simulation show that if agents face a stress with news which exacerbate the stress, the feedback effect becomes greater than that in a stress without any news.

3. Implication for implementing Dynamic Stress Exercise

3.1 Necessary data for Macro Stress Exercise

Under the Revaluation Approach, each institution reports an expected loss amount under a provided stress scenario. The Sensitivity Approach requires institutions to report summary data on their sensitivity to market risk factors.

If we try to conduct a dynamic macro stress exercise as described in this note, no matter which approach we employ, risk profile data on sensitivity will be necessary. Even under the Revaluation Approach, data which show reactions against an initial shock will be required to construct an ASS.

As I pointed out in 2.3.2, historical data on institutions' risk profiles need to be provided to networks so that they can learn those institutions' portfolio rebalancing patterns. Further study on the candidates for inputs will be necessary in order to estimate their trading behavior more effectively. Historical P/L data would be another candidate as an output of network simulation, since P/L data seems to be more available than data on sensitivity.

It is true that the availability of these data, such as daily sensitivity and P/L, is not full enough for us to be able to ask that they be reported at this moment. However, since those data constitute fundamental information for internal risk management in financial institutions, banks which actively conduct trading business will in the near future come to use these kind of data more frequently as a tool for daily risk management.

3.2 How can exercise results be utilised?

Information we obtain via a stress exercise will vary according to the choice of approach employed at every stage, such as the Revaluation Approach or the Sensitivity Approach, and according to whether a neural network is used or whether assumptions on trading behavior are made. The information which obtained from static and dynamic exercises are listed as follows:

1) Static Stress Exercise : ISS and static aggregate risk
2) Dynamic Stress Exercise : ISS, static aggregate risk, institutions' reaction against ISS, ASS, and dynamic aggregate risk

The difference between the Revaluation Approach involving a stress scenario, and the Sensitivity Approach is the flexibility of scenario used for calculating the aggregate risk measure. The choice of assumptions concerning institutions' reactions also affects the informational content of exercise results. If we compare the two alternatives for estimating traders' behavior examined in this note, it is safe to point out that the approach which uses an AI system for learning the trade pattern can provide the exercise results with a more realistic shape.

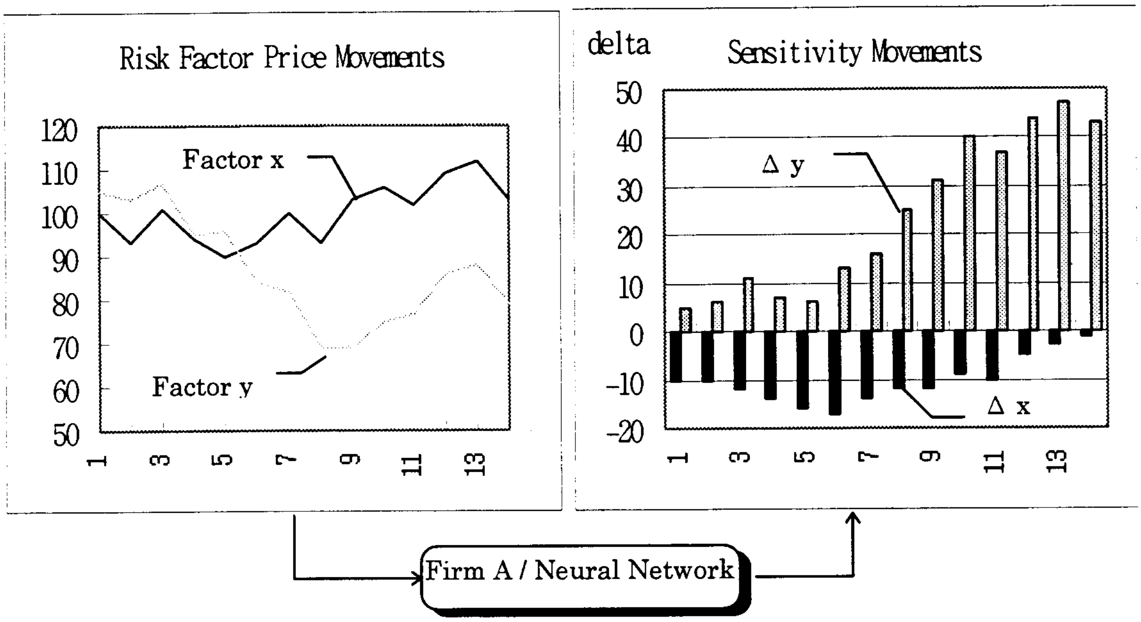
Great attention must be paid to the way that stress exercise results are used. A dynamic exercise taking into account any feedback effect will provide us with more useful and vivid information on the macro market risk profile under stress. If we obtain the information listed above via a dynamic stress exercise, we can prepare for stress in a more efficient and effective way. The information that most institutions will begin to sell the asset in reaction to the initial crash makes us more secure than in the case when we don't have any idea what their reaction might be. For example, as the simulation results in this note show, if we know that the upward parallel shift of the yen yield curve has a higher possibility of causing a positive feedback effect which could trigger systemic risk than the other stress scenarios, we should pay more attention to yield curve movements in this direction in our daily market monitoring. On the other hand, the prediction of a negative feedback effect under a stress at the shorter end of maturity affords us more room for making the political decision to conduct a necessary operation, such as supplying liquidity to a market, when we face a sudden crash of the short term interest rate.

When we expand the time horizon of the exercise, its result will have useful implications for the framework of financial or trading systems. For example, we can discuss in which situations a circuit breaker system works well or not from the point of view of systemic risk. My model can be a tool to simulate systemic meltdown in markets by expanding its time horizon and examining cases where price equilibrium disappears.

APPENDIX

Macro dynamic simulation using neural networks

1. Inducing information on a firm's behavioural pattern



$X_t = (x_t, y_t)$

Firm's portfolio value at t: $f_t(X_t) \rightarrow$ Sensitivity data at t: $\dfrac{\partial f_t(X_t)}{\partial x}, \dfrac{\partial f_t(X_t)}{\partial y}$, . . .

Firm's portfolio value at $t+1$: $f_{t+1}(X_{t+1}) \rightarrow \dfrac{\partial f_{t+1}(X_{t+1})}{\partial x}, \dfrac{\partial f_{t+1}(X_{t+1})}{\partial y}$, . . .

If we have daily data on a firm's sensitivity and risk factor price movements, we could estimate how each firm rebalances its portfolio in response to market movements. Daily change in a firm's sensitivity data is caused by

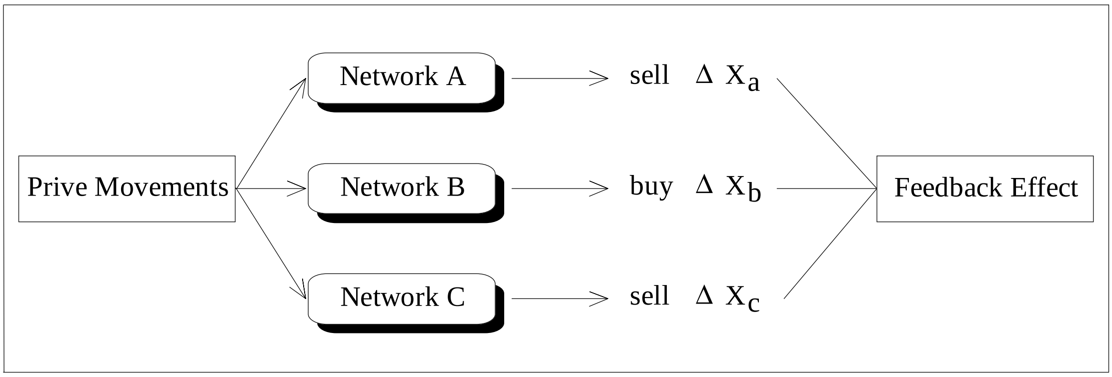
(a) risk factor price movements $\dfrac{\partial f_t(X_t)}{\partial x} \rightarrow \dfrac{\partial f_t(X_{t+1})}{\partial x}$, and

(b) portfolio rebalancing $\dfrac{\partial f_t(X_{t+1})}{\partial x} \rightarrow \dfrac{\partial f_{t+1}(X_{t+1})}{\partial x}$.

When we exclude (a) the effect of risk factor price movements from changes in sensitivity, we could obtain information on (b), the effect of the firm's portfolio rebalancing, $f_t \rightarrow f_{t+1}$.

Using neural networks, which can learn changing patterns of non-linear functions, we could make quasi firms by providing each network with the corresponding firm's portfolio rebalancing information based on the data set of daily changes in the firm's sensitivity and risk factor price movements.

2. Simulating a feedback effect of the rebalancing of portfolios by firms



Each network which learns a corresponding firm's portfolio rebalancing pattern functions as a quasi firm in our simulation. When we provide an initial price movement of asset X to networks, they decide whether and how much to buy/sell X based on the trading pattern they have learned. Their trade orders are aggregated and a new equilibrium found in the quasi market. If the price at the new equilibrium is significantly below/above the initial price, we can say that negative/positive feedback could be caused by the price movement. Furthermore, using this quasi market model, we could simulate a market price movement taking into account a feedback effect without incurring any reporting burden on firms.

Figure 1

Stress Exercise Structure

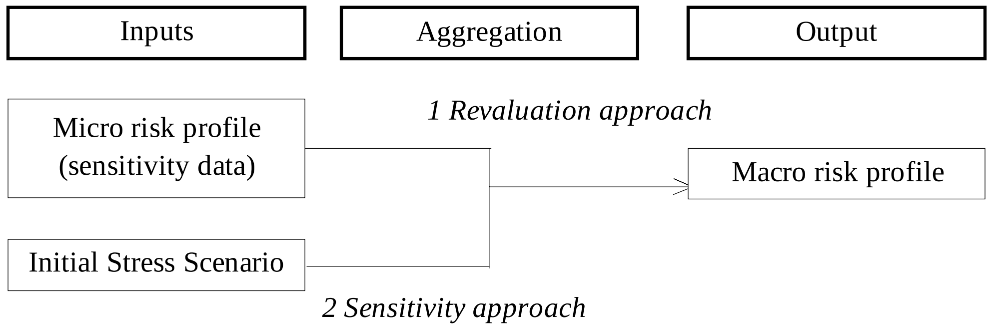


Figure 2

Multiperiod Model

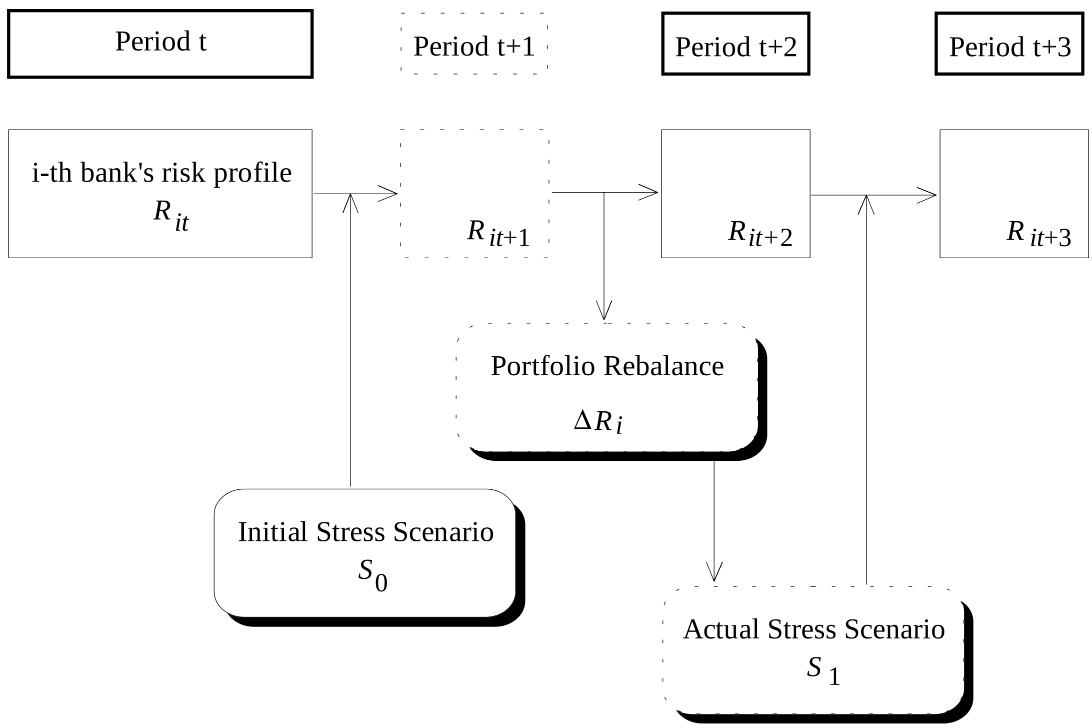


Figure 3

Figure 4-1

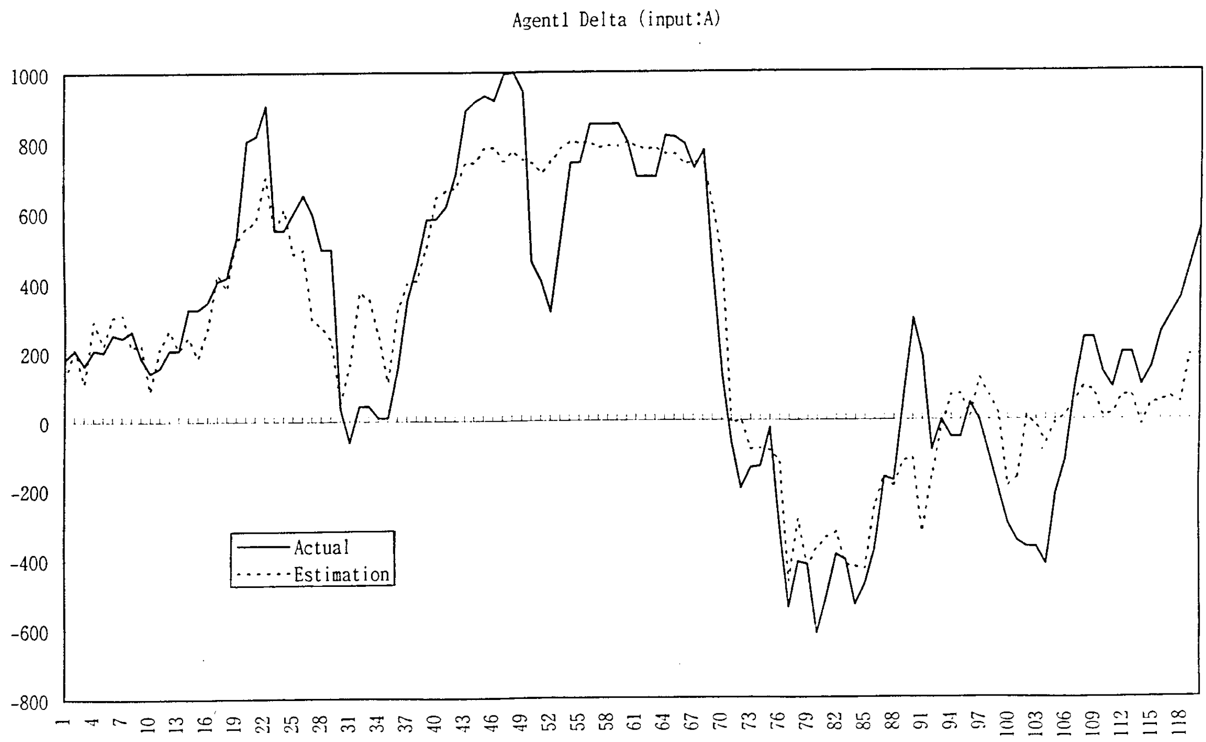


Figure 4-2

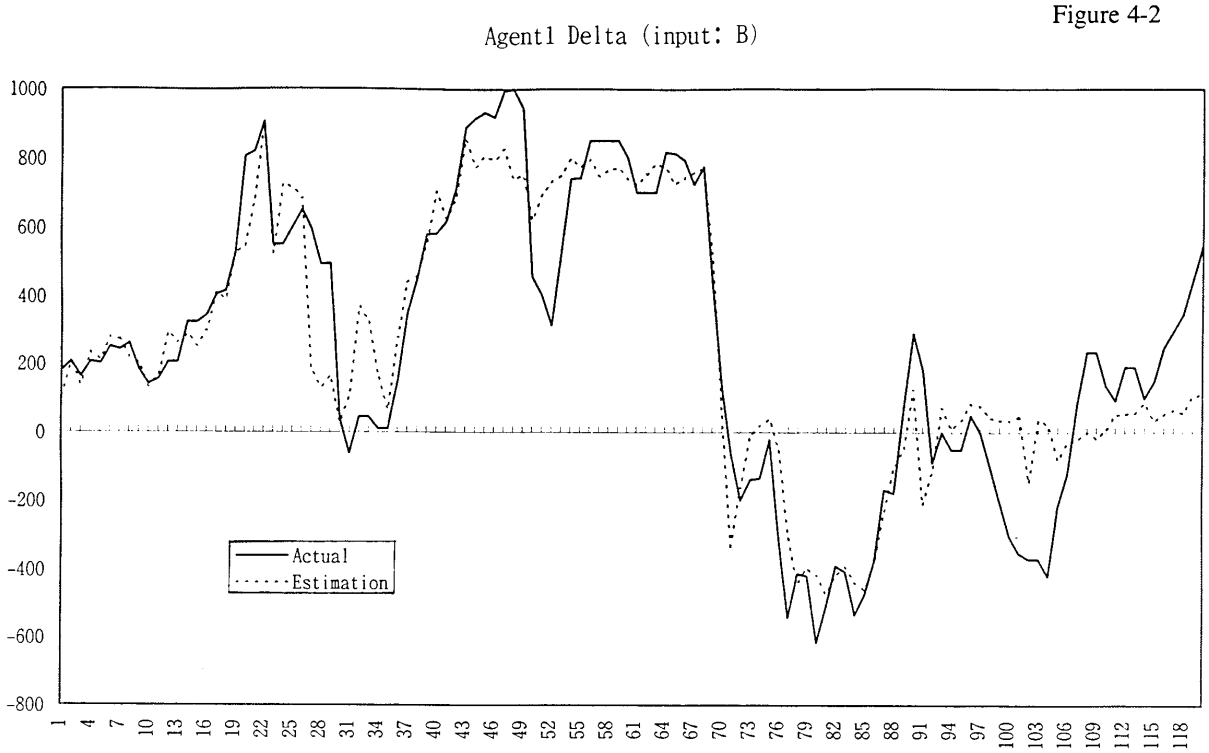


Figure 4-3

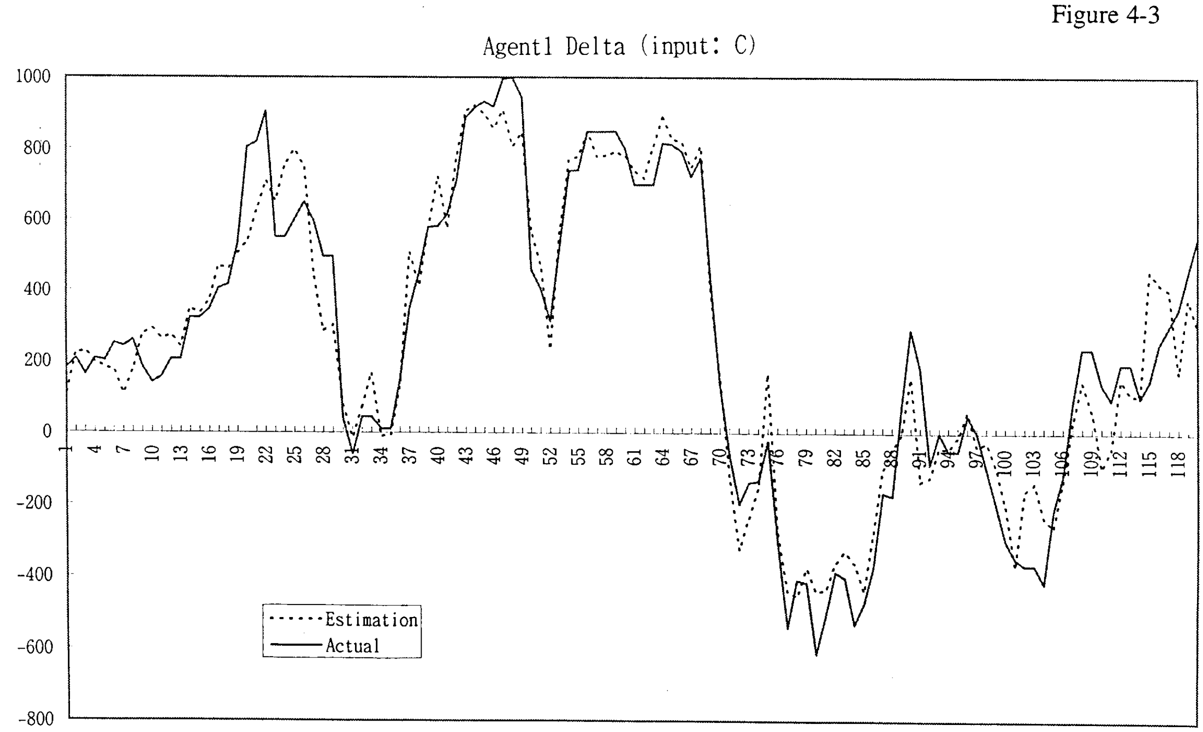


Figure 5

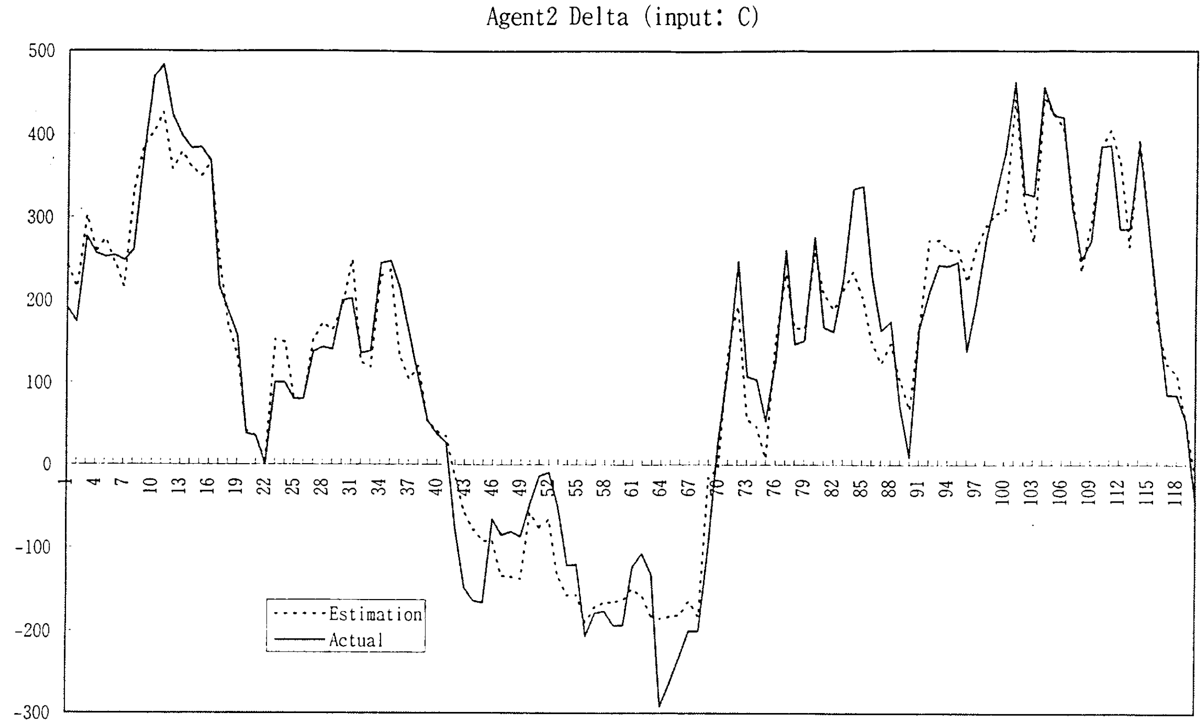


Figure 6

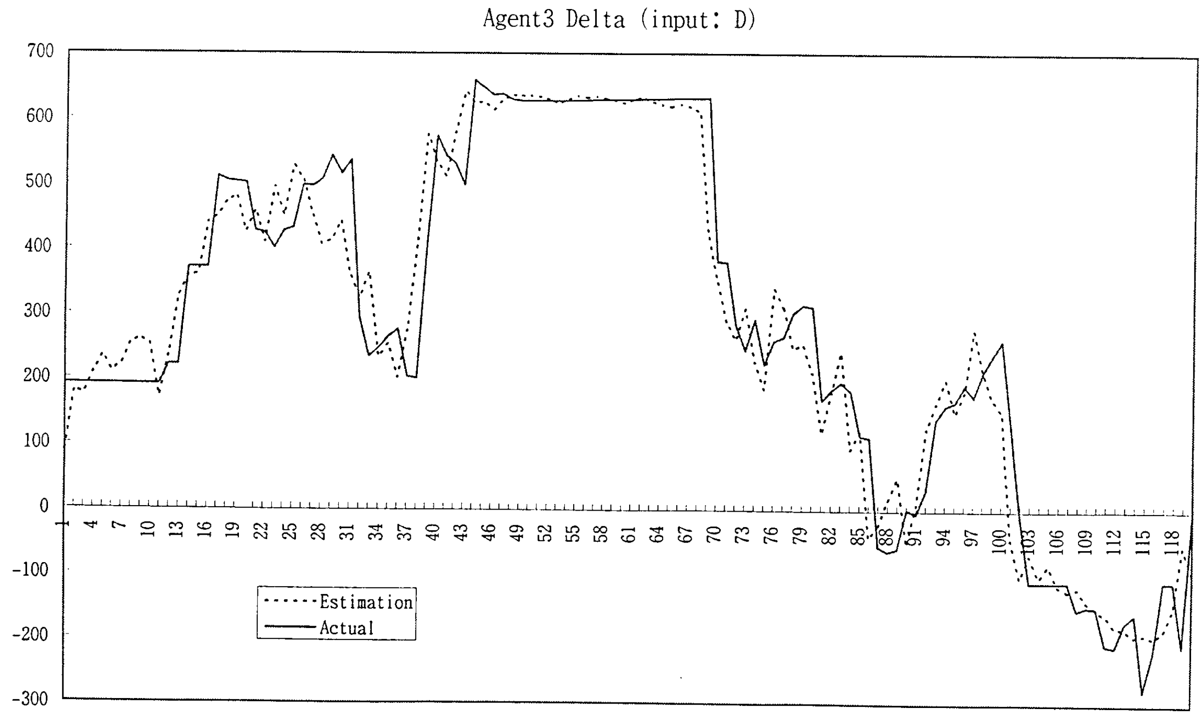


Figure 7-1

Agent1 Delta (out-of-sample simulation)

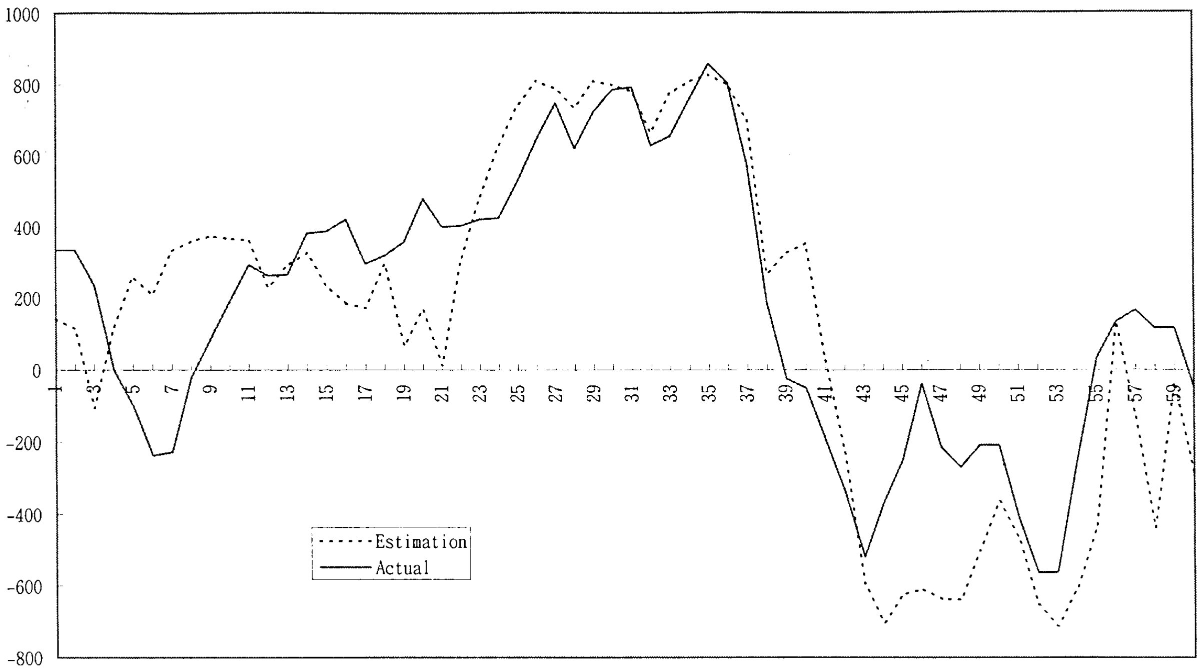


Figure 7-2

Agent2 Delta (out-of-sample simulation)

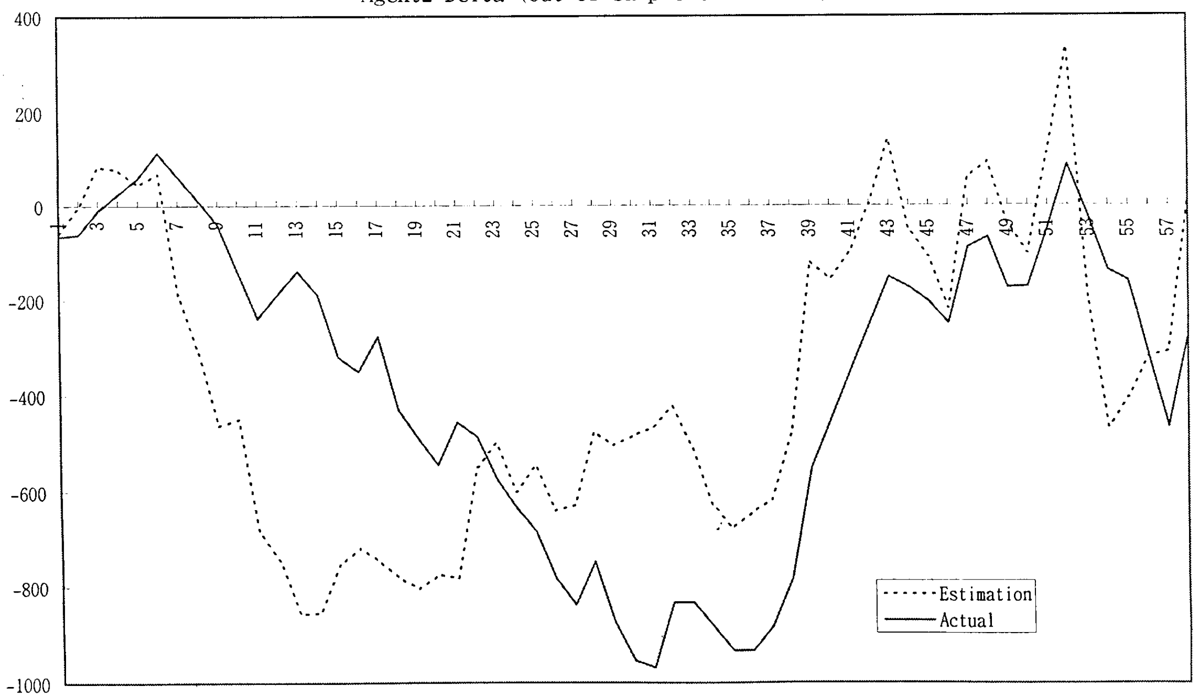
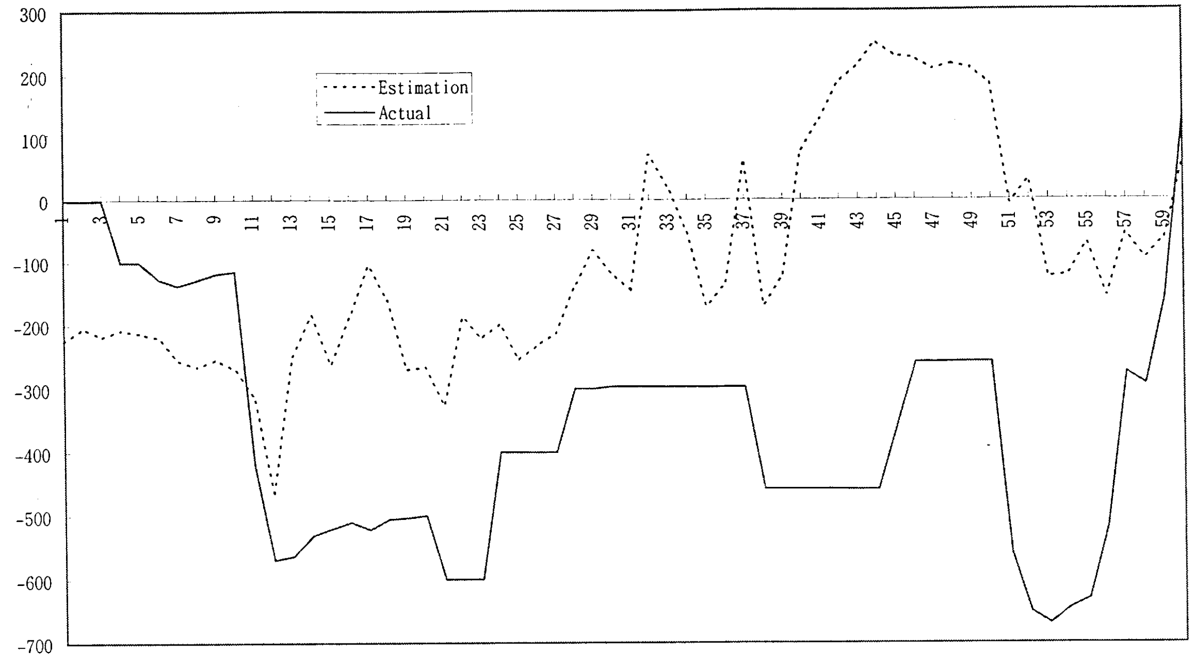


Figure 7-3

Agent3 Delta (out-of-sample simulation)



3y Swap Rate Daily Return

Figure 8-1

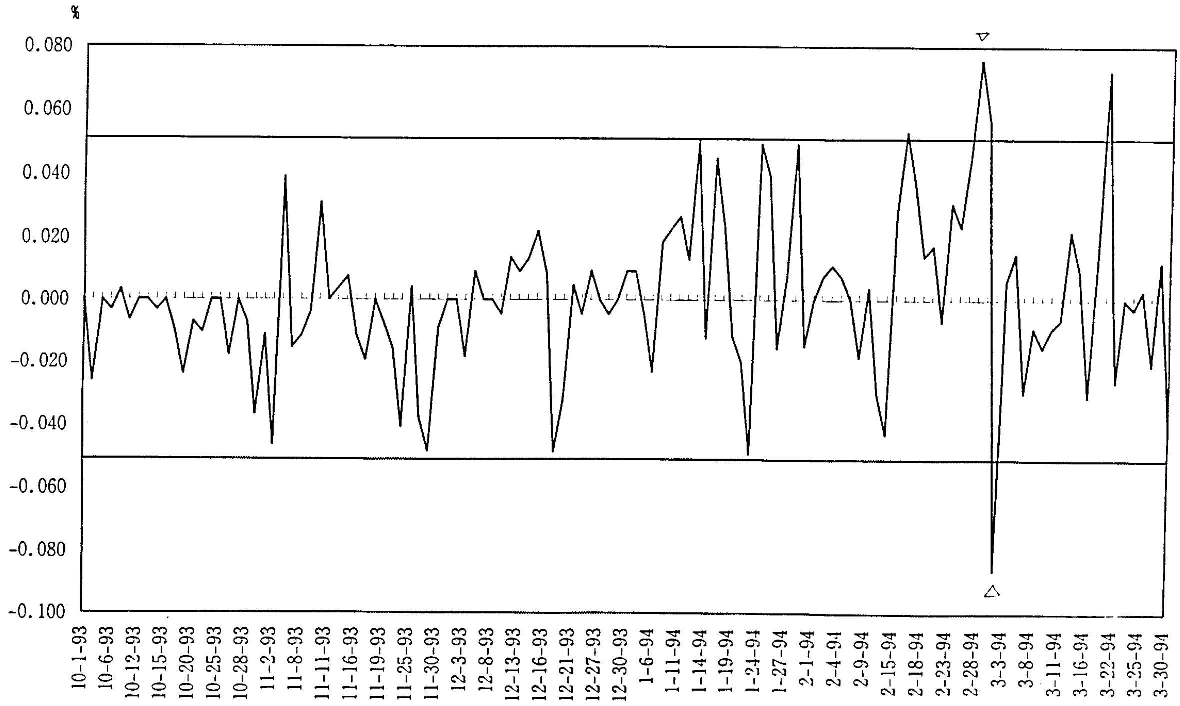


Figure 8-2

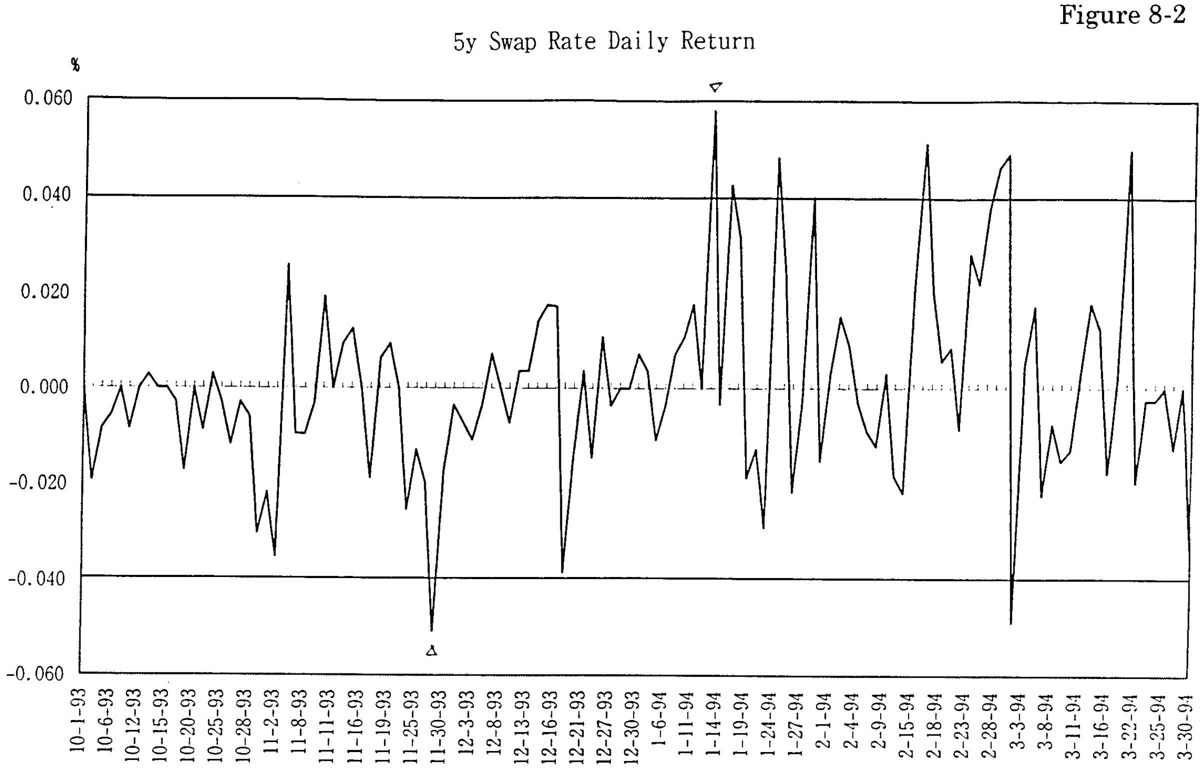


Figure 8-3

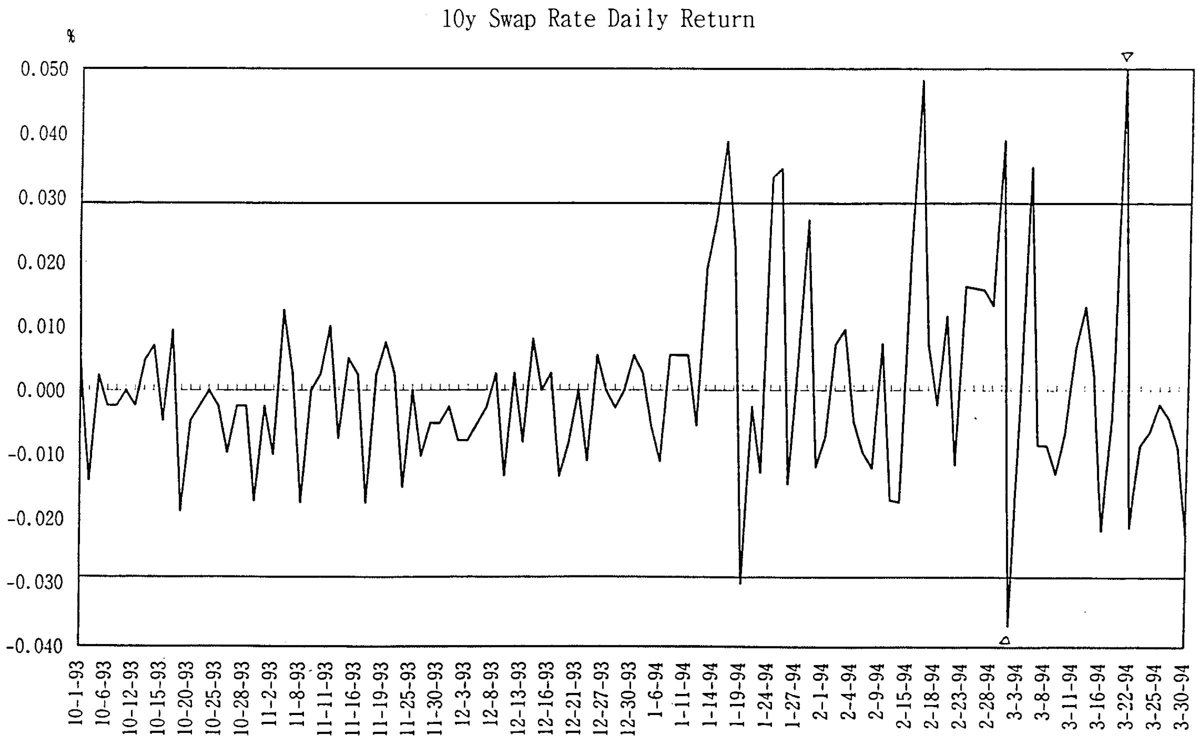
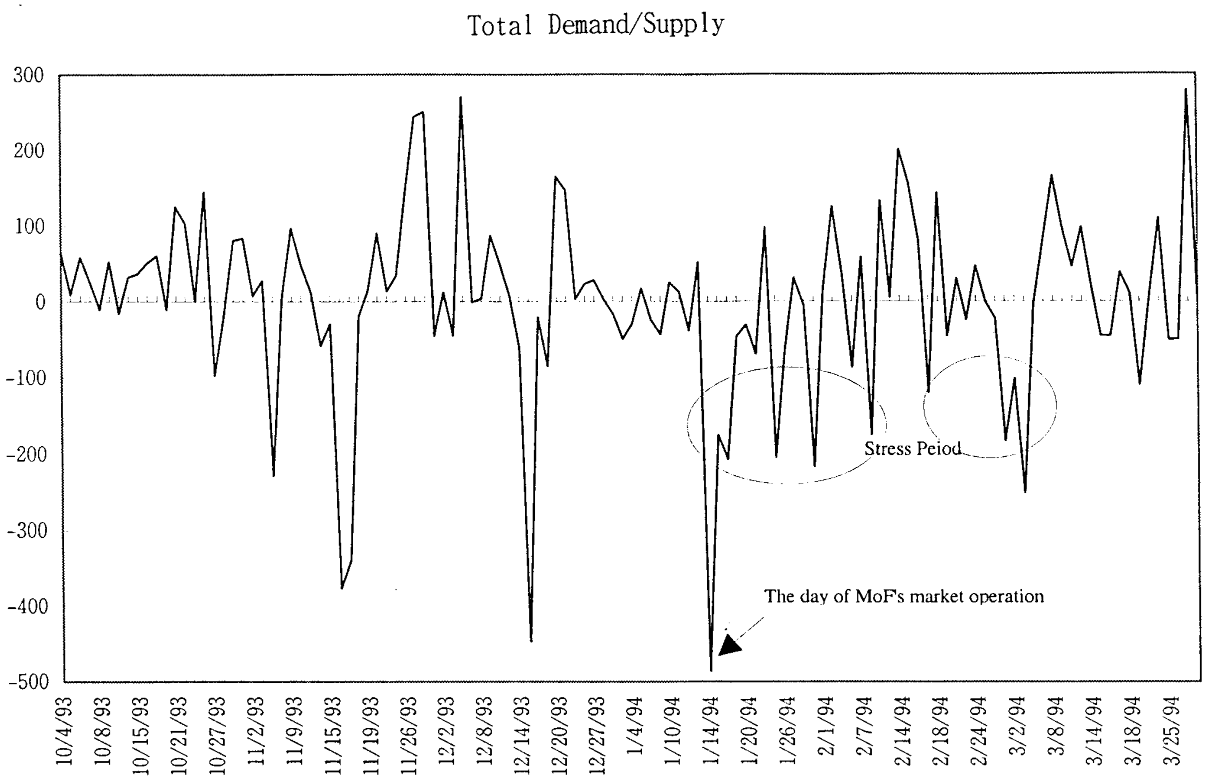


Figure 9


Total Demand/Supply
300
200
100
0
-100
-200
-300
-400
-500
Stress Peiod
The day of MoF's market operation
10/4/93
10/8/93
10/15/93
10/21/93
10/27/93
11/2/93
11/9/93
11/15/93
11/19/93
11/26/93
12/2/93
12/8/93
12/14/93
12/20/93
12/27/93
1/4/94
1/10/94
1/14/94
1/20/94
1/26/94
2/1/94
2/7/94
2/14/94
2/18/94
2/24/94
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3/14/94
3/18/94
3/25/94

Figure 10-1

Agent 1 Trade Direction

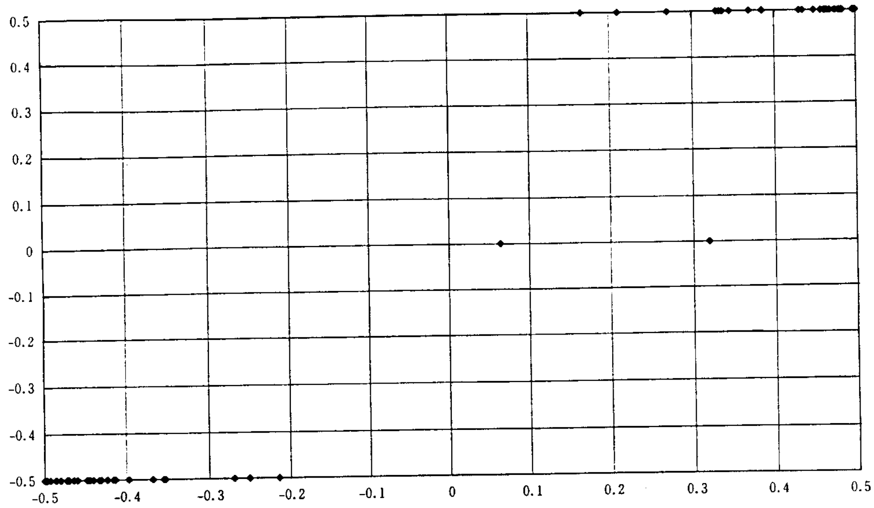

Figure 10-2

Agent 1 Trade Volume
(in-sample simulation)

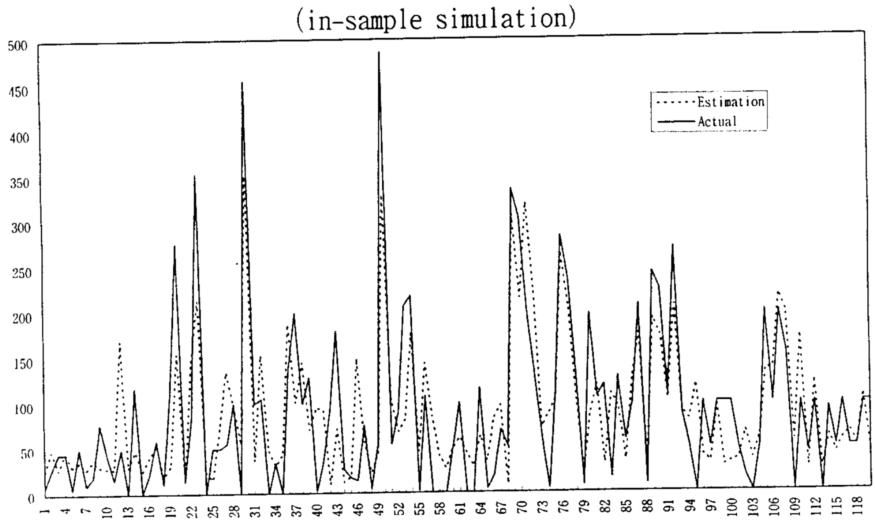

Figure 10-3

Agent 1 Trade Volume
(in-sample simulation)

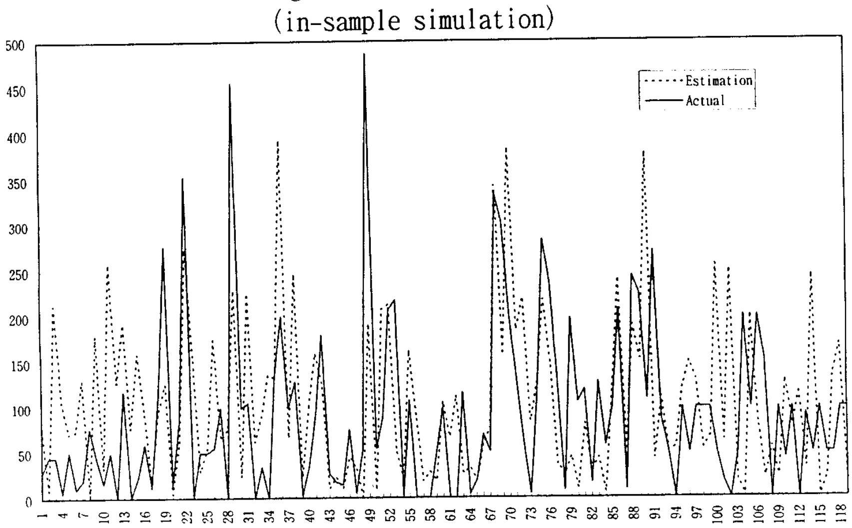

Table 1

Characteristics of the agents

	Key for trading	Fortrend/contrarian	Other characteristics	Targeted profit (loss limit), Position limit
Agent 1	Charts of market movements (chartist)	Fortrend	Positive correlation b/w P/L condition and trading volume	3 billion yen/half year, delta limit:± 100 billion
Agent 2	Charts of market movements (chartist)	Contrarian	Frequent writer of options	3 billion yen/half year, delta limit:± 100 billion
Agent 3	Fundamental events (fundamentalist)		Trading horizon is longer than the others (more an investor-type of trader than a dealer)	2 billion yen/half year, delta limit:± 70 billion

Table 2

Steps for inputs adjustments

Inputs	Agent 1	Agent 2	Agent 3
A: Market data (change on a trading day)	71.67%	82.08%	80.00%
B: A + accumulated P/L	76.67	82.08	82.92
C: B + recent market movements + news	85.42	87.92 (Figure 5)	87.92
D: C + market trend over a longer horizon (Agent 3)			90.83 (Figure 6)

Table 3

The magnitude of largest daily change

	Upward change	**Downward change**
3-year swap rate	3.01σ	-3.43σ
5-year swap rate	2.96σ	-2.59σ
10-year swap rate	3.46σ	-2.54σ

Table 4

Stress scenarios

	3y	**5y**	**10y**
Period t	3.33%	3.98%	4.49%
Scenario 1	3.59%	4.22%	4.72%
Scenario 2	3.05%	3.78%	4.33%
Scenario 3	3.59%	3.98%	4.49%
Scenario 4	3.05%	3.98%	4.49%
Scenario 5	3.33%	4.22%	4.49%
Scenario 6	3.33%	3.78%	4.49%
Scenario 7	3.33%	3.98%	4.72%
Scenario 8	3.33%	3.98%	4.33%

Table 5

Changes of each agent's delta under scenarios

(in billions of yen)

	Agent 1	**Agent 2**	**Agent 3**	**Total D/S**
Scenario 1	-8	26	-6	12
Scenario 2	-15	-9	-10	-34
Scenario 3	-3	33	-6	20
Scenario 4	-43	-3	9	-37
Scenario 5	-42	27	30	-15
Scenario 6	4	1	-4	1
Scenario 7	-4	14	-4	6
Scenario 8	21	20	31	72
Standard deviation				12

Table 6

Risk profiles of each agent

(in billions of yen)

	Agent 1	**Agent 2**	**Agent 3**	**Total**
Scenario 1	0.20	0.09	-0.03	0.26
Scenario 2	-0.51	0.18	0.13	-0.28
Scenario 3	0.45	0.25	-0.05	0.65
Scenario 4	-0.16	0.10	-0.12	-0.18
Scenario 5	0.08	0.13	0.18	0.39
Scenario 6	0.04	-0.00	-0.00	0.03
Scenario 7	0.11	0.02	-0.01	0.13
Scenario 8	1.98	0.40	0.80	3.18
VaR(t)	0.58	0.05	0.22*	

* Agent 3's VaR figure is measured at period t-1, since its position at period t is square.

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Measurement of liquidity risk in the context of market risk calculation

by

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Abstract

This paper aims at shedding light on liquidity risk, which has been left behind in the pursuit of more sophisticated market risk measurements both by market practitioners and by central banks. We first define liquidity risk and show that it can be divided into execution cost and opportunity cost. In the light of stylized facts regarding tick-by-tick dynamics of market liquidity and price/spread movements, which have been documented previously by finance literature, we propose several modified market risk measures reflecting intraday liquidity patterns and price movements. We then demonstrate, by applying those risk measures to the Japanese equity market, to what extent the quantified liquidity effects could effect conventional measurement of market risk represented by VaR.

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* The views expressed in the paper are those of the authors and do not necessarily reflect those of the Bank of Japan, the Institute for Monetary and Economic Studies or the Euro-currency Standing Committee. Although the paper benefited from comments by central bank economists participating in the research project on the aggregation of market risk, the authors are solely responsible for any remaining errors.

I. Introduction

The quantification of market risk has been a major issue for financial institutions as well as central banks over the past few years. Considerable technical efforts have been made to measure market risk as accurately as possible.[1]

One of the areas which has attracted a great deal of attention is how one can appropriately make assumptions about future movements in market risk factors. One direction of such research is studying the characteristics of ***daily price changes*** more carefully. Mori, Ohsawa, and Shimizu (1996), for instance, proposed techniques for explicitly taking account of fat-tail distributions of market prices and correlation breakdown between various risk factors to see to what extent these characteristics of market prices affect value-at-risk (VaR) calculations. Another type of research [e.g. Fallon (1996) and Alexander (1996)] evaluates the effectiveness of various types of volatility models such as the moving average method, GARCH, and implied volatility, for improving the accuracy of the VaR model.

Another potential direction of research on future price movements used as an input for market risk calculation, which we follow in this paper, is to analyze ***intraday price changes*** more carefully, since the prices at which one can trade are not necessarily end-of-day trade prices or mid-prices – as assumed in the ordinary framework for market risk calculation, but, as a matter of fact, reflect bid or ask prices which fluctuate during the day. As quite a few pieces of academic literature after Black Monday documented, the key to understanding second-by-second price changes is the investigation of a relationship between trading activity and price changes. In other words this is the price impact of trading activity, which can be labeled "liquidation or liquidity risk".

Although it is generally recognized among market practitioners that liquidity risk is a very serious concern for firms, especially in a stressful market situation, quantitative techniques measuring liquidity risk both in normal and stressful situations appear to be very premature.[2] In marking to market positions, the current standard practice is to use mid-rates (or the latest trade prices, which tend to be determined somewhere within bid-ask spreads); a more realistic and conservative method, i.e. using bid and ask prices for long and short positions respectively, does not in practice prevail. Even if bid/ask prices are used, the decision regarding which bid/ask prices should be used is usually left to individual traders and is not centrally monitored and controlled by middle offices. With regard to the quantification of liquidity risk in the context of daily risk management, the effort to incorporate the liquidity effect into market risk and a stress test is still at an embryonic stage.

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[1] Two recent additions to the literature by private and central bank communities, which provide a comprehensive review of the measurement of market risk, are Wilson (1996) and Session 4 of Part II in Board of Governors of the Federal Reserve System ed. (1996), respectively.

[2] The following description is based on informal interviews with several leading Japanese and U.S. trading houses conducted in the summer of 1996.

Recently, however, we observe some advancement in this direction. In order to take account implicitly of the abrupt leaps in observed market prices which often occur in less liquid markets (e.g. emerging markets), a few firms use worst-case scenarios in which worst historical price movements over relatively longer periods such as 5-10 days are assumed.[3] Some firms are considering the adjustment of holding periods of their positions in value-at-risk calculations based on the liquidities of individual products. Certain firms regularly check the difference between the necessary liquidation time for less liquid positions forecast by traders and the actual time needed to close the positions. The more complex part of measuring liquidity risk is how to measure the impact of trading volume on price. Currently, no firms appear to take this effect into account in daily risk management. However, the prevailing use of computer-driven asset management and the growing interest in evaluating the performance of pension funds more accurately tend to increase the need to quantify the price impact.

The objective of this paper is to propose several measures for quantifying liquidity risk, based on intraday price and trading data, which can be of some value for more accurate market risk quantification. First, Section 2 of this paper defines the scope of liquidity risk in our discussion, making clear distinction between execution and opportunity cost components of liquidity risk. Section 3 surveys stylized facts regarding intraday price and bid-ask spread movements and the price impact of trading activity, which are documented in finance literature. In light of the stylized facts Section 4 proposes several different modified market risk measures reflecting intraday liquidity patterns and price movements and demonstrates to what extent the quantified liquidity effects could affect measurement of market risk – represented by VaR – for the Japanese equity market. Section 5 discusses areas for future research.

II. Definition of liquidity risk

Liquidity risk in this paper is defined as the risk of being unable to liquidate a position in a timely manner at a reasonable price.[4] Theoretically, liquidity risk in this sense can be divided into the variability of ***execution cost*** (the cost of immediacy) and that of ***opportunity cost*** (the cost of waiting). As Chart 1 shows, the execution costs, comprising (1) "bid-ask spreads" representing transaction cost and (2) the "price impact of trading activity" (hereafter called "market impact"), both of which are sometimes not easily separable from each other, decrease with the time needed to

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3 The difficulty deriving historical volatility in illiquid markets lies in the fact that effective quotations of the bid/ask prices at which one can actually trade disappear and cannot be observed for quite a long period (which sometimes lasts for several days). If we calculate daily historical volatility by extrapolating effective quotes from the data obtained before the trade halt, it is highly likely that we will end up with an underestimation of underlying market volatility.

4 One might imagine ***funding risk*** from the term "liquidity risk". As a matter of fact, funding risk could trigger a rise in market rates, in particular under the assumption of there being no lender of last resort, which could lead to losses in trading positions. However, the scope of our analysis in this paper does not include the measurement of funding risk.

complete an intended transaction. In contrast, opportunity cost, i.e. the cost of being forced to postpone trading, tends to increase with execution time. What traders need to do is to strike a balance between these two costs so as to minimize liquidity risk. Chart 1 shows that the sum of both costs is usually regarded as liquidity risk, which traders implicitly try to minimize in their daily trading activity.

As a number of pieces of market microstructure literature show [e.g. Glosten and Harris (1988)], the bid-ask spread, which is an important component of the execution costs faced by investors, is divided into (1) order-processing cost and (2) adverse selection cost. The adverse selection component exists because a market maker may trade with investors who possess superior information. This component is believed to represent the market maker's profits from uninformed traders who compensate him for the expected losses to informed traders [Glosten and Milgrom (1985)]. An empirical study by George, Kaul, and Nimalendran (1991) using daily and weekly data of both AMEX/NYSE and NASDAQ stocks found out that the predominant component of quoted spread is order-processing cost, while the adverse selection component comprises only 8-13% of quoted spreads.

With regard to the opportunity cost component of liquidity risk, the only theoretical effort worth noting here seems to be the work by Longstaff (1995). Since liquidity premium is basically determined by price changes during the period of restricted trade, the maximum value of the liquidity premium is derived in his model as a premium for a lookback option, the underlying asset price path of which is determined by the optimal price path calculated under conditions of perfect foresight of the future price path and market timing, and the strike price of which is determined by a Black-Scholes stochastic process. In practice, however, it is not at all easy to take explicit account of the variability of opportunity cost in market risk calculation. As mentioned in the Introduction to this paper, certain firms consider the arbitrary adjustment of holding periods of particular positions in order to reflect the opportunity cost component of liquidity risk, though there exist no objective and scientific ways to determine what is the appropriate holding period. Hence in the following analysis we focus mainly on the execution cost element of liquidity risk. This implies, however, that, as Chart 1 shows schematically, our measurement of liquidity risk tends to be understated, especially in a stressful situation in which shortage of liquidity prevents virtually all transactions for quite a long time period.

III. Stylized facts regarding liquidity effects documented by previous academic literature

This section reviews stylized facts regarding the relationship between trading activity (market liquidity) and bid/ask price changes as well as spread changes in different markets and time periods as well as theoretical explanations for them, based on previously published academic literature. Here we do not intend to conduct a comprehensive survey but rather focus on stylized facts which should be kept in mind in measuring liquidity risk in the context of market risk calculation in the ensuing chapter.

1. Volatility of intraday price movements

A first well-known fact documented by Amihud and Mendelson (1987) with regard to NYSE stocks' intraday price movement is that ***open-to-open return volatility is higher than close-to-close return volatility*** in the markets applying the call-clearing procedure at the opening session. There seem to be two streams of theoretical explanation about what causes this phenomenon. Stoll and Whaley (1990) argue that the wider bid-ask spreads caused by specialists using their monopoly position at the opening call makes prices more volatile, since transaction prices tend to bounce between bid and ask prices. In contrast Lee and Lin (1995) postulate that the specialist encourages trading by putting a smaller cost on immediacy in order to reveal more private information – accumulated during the overnight trading halt, which reduces the adverse-selection problem specialists face and makes subsequent trades in the continuous market more profitable. In any case this could have important implications for market risk management, since almost all the firms use closing price data in their firm-wide market risk calculation.

Another stylized fact documented also by Amihud and Mendelson (1987) is the ***greater deviation of opening returns from the normal distribution – with fatter tails – than closing returns***, which could also be worth noting in quantifying market risk.

2. Determinants of bid-ask spreads

A rather trivial fact about bid-ask spreads is that ***infrequently traded stocks are characterized by large bid-ask spreads***. There are several conjectured explanations for these large spreads.[5] The first explanation involves inventory and liquidity effects. A second reason is monopolistic market power exercised by a single market maker providing liquidity for inactive stocks. A third explanation is that the large spreads arise as the natural consequence of the greater risk of informed trading in illiquid stocks which market makers incur.

Another stylized fact about bid-ask spreads is ***the turn-of-the-year seasonal movement of bidask spreads***: A significant decline in the spreads of NYSE stocks from the end of December to the end of the following January, which is believed to cause excess January returns – since this tendency is pronounced especially for small or low-priced stocks, this seasonal anomaly is sometimes labeled "size-related anomaly". For instance, Clark, McConnell and Singh (1992) detects such a decline at the turn of the year by observing 1982-1987 NYSE stock price data. Ritter and Chopra (1989) attribute the January excess returns to portfolio rebalancing strategy – i.e. investment in riskier stocks – taken

5 The following summary is from Easley, Kiefer, O'Hara, and Paperman (1996).

by those who sold losers at year end for various reasons ranging from tax-loss selling, portfolio window dressing and parking-the-proceeds.

In the Japanese stock market, which has two sessions a day, bid-ask spreads exhibit a ***W-shape pattern***, where they peak at the opening, just before and after lunch time, and during the closing session. This is in parallel with the W-shape of transaction volume and volatility movement during a day. This contrasts with U-shape movement of U.S. stock market, where there is only one session in a day. What Bollerslev and Melvin (1994) found for foreign exchange markets is that ***the size of bid-ask spreads in the foreign exchange market (DM/$ rates) is positively related to the underlying exchange rate (conditional) volatility***, by using ordered probit regression to cope with discreteness in the spreads data.

In contrast, however, Bollerslev and Domowitz (1993) suggest that ***quotation activity of foreign exchange does not influence bid-ask spread changes – seemingly contradicting the fact derived from stock markets*** – while spreads have a positive effect on return volatility. Glassman (1987) concluded that trading volume in foreign exchange markets is rather negatively correlated with bid-ask spreads. In fact, spreads widen a significant amount prior to weekends and holidays. Locke and Sarkar (1996) also show that bid-ask spreads at several futures markets do not increase even on higher volatility days. These results might suggest that there is a certain difference in the readiness of liquidity supply between the stock and forex/futures market.

3. Market impact

The stylized facts which are most important for dynamic liquidity analysis involve the characteristics of ***market impact,*** defined as the slope (sensitivity) of the bid or ask price schedule (hereafter called λ) to trading volume or order flow. The following features of λ for U.S. stock markets are identified by the numerous additions to the empirical literature which emerged after Black Monday [see a survey by Hebner (1996)]:

a. $\lambda > 0$, which is supported almost unanimously.

b. λ is concave in transaction size (Effects of stealth trading and more intensive brokerage search for larger transaction). However, if the ratio of informed/uninformed trading increases with transaction size, λ should be convex in transaction size.

c. λ is lower for more active securities.

d. λ is asymmetrical between buyer – and seller – initiated transactions. However, which of the two is larger varies from study to study.

Watanabe (1996), who analyzes intraday data of Japanese Government Bond Futures, finds that a significant causality from volume to volatility exists, while many other studies in this area either focus on contemporaneous relationships or find no strong causality.

IV. Modified market risk measures reflecting intraday liquidity pattern and price movements

Keeping the aforementioned stylized facts with respect to intraday price movement and trading activity in mind, we now propose several liquidity risk measures which could be useful for achieving more accurate market risk quantification. The market to which we will apply the proposed risk measures is the Tokyo Stock Exchange (TSE), tick-by-tick trade data of which have kindly been provided by Nikko Securities for the purpose of this research.[6] Although we have to confine ourselves to focusing on the Japanese equity market because of the data availability constraint, the following measures are, in principle, applicable to other markets in other countries. Simulations based on a hypothetical portfolio of stocks listed in the TSE will demonstrate to what extent those quantified liquidity risks could affect measurement of market risk obtained in the form of VaR.

The first modified measure for market risk we propose takes into account intraday tick-by-tick price movement which reflects intraday liquidity patterns. As a first attempt in this direction we will check whether opening sessions of the TSE, where actual trading is at its most active and significant quantities of trades of less liquid equities take place, also exhibit higher volatility and kurtosis, as evidenced for the NYSE. Second, in order to relate intraday price fluctuations to the intraday trading pattern more precisely and quantify "execution timing risk" during the day, we introduce the notion of "trade volume-weighted average prices (VWAP)" and construct a market risk measure reflecting the daily volatility of VWAP and intraday histograms of actual trade prices which clearly display the risk of deviating from average trade execution performance during a single day. The second modification we will propose in obtaining a more accurate measure for market risk is the explicit incorporation of intraday variability of bid-ask spreads into the price risk calculation. Finally we propose a market liquidity measure based on λ,which represents the market impact of trading activity, and the historical distribution of which could potentially be used for augmenting market risk calculation.

1. Modified market risk measure reflecting intraday price movement

Since many less liquid stocks on the TSE are traded mostly in the opening clearing process, it is important to take a close look at the volatility and distribution of opening prices in order to measure market risk more accurately. Using the daily data, we will check whether a stylized fact of higher volatility and kurtosis of opening sessions than closing sessions is also applicable to the Japanese equity market. Since there are two (morning and afternoon) sessions a day in the TSE, we compute two opening and closing volatility indices, respectively. The result presented in Chart 2 shows, in contrast with our conjecture, that there is no significant difference in volatility and kurtosis on average between opening sessions and closing sessions in the TSE, although 237 and 230 stocks

[6] The data are on quoted bid and ask prices as well as trade prices and volumes of over 1200 stocks listed in the first section of the TSE and the data observation period ranges from 2nd October 1995 to 30th September 1996.

out of 500 samples exhibit higher open-to-open volatility and kurtosis, respectively (the average open-to-open volatility and kurtosis of these equities are higher by 5.9% and 29.4%). This result clearly contrasts with the stylized fact observed in the NYSE. This could be partly because the TSE's closing sessions follow the exactly same call-clearing procedure as in the morning sessions, while in the case of the NYSE the closing sessions are characterized by "market-on-close (MOC) order"[7] which tends to prevent prices from moving by wider margins. In the next part of this section, therefore, we will take a more careful look at the tick-by-tick intraday price movement of the TSE-listed equities in order to reflect liquidity effects on market risk measurement.

Transaction prices move widely during a day, as shown in Chart 3 which shows a histogram of trade prices of a particular stock in our sample. The execution timing risk during one day can be defined as the risk of deviating from the average transaction cost for the day and ending up by trading at unfavorable prices compared with other participants in the market. Although the execution timing cost could arise simply from an inappropriate trading strategy, liquidity constraint prevalent in the market could also force traders to execute transactions with bad timing, particularly in stressful situations. In this sense the execution timing risk can be interpreted as also including opportunity cost.

Judging from Chart 3, the distribution of daily trading prices which are accumulated over the observation time period and standardized by daily volume-weighted average price (VWAP) can be roughly assumed to be a normal distribution. Assuming that the daily movement of VWAP follows a lognormal stochastic process, we now conduct a two-step Monte Carlo simulation (100,000 times) based on two normal distributions of daily VWAP changes and daily trade prices.[8] The

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7 A market-on-close order is guaranteed execution at the closing price according to prescribed pricing and order entry procedures. When there is an imbalance of MOC orders, the imbalance is executed against the prevailing bid or offer on the Exchange at the close of trading, thus setting the closing price. An excess of buy orders is executed against the offer and an excess of sell orders is executed against the bid. The remaining buy and sell MOC orders are then paired off at the price at which the imbalance was executed. When the aggregate size of the buy MOC orders equal the aggregate size of the sell MOC orders, the buy and sell orders are paired off at the price of the previous NYSE trade. The result of these pricing procedures is that all executed MOC orders receive the same closing price.

8 This simulation process can be mathematically expressed as follows:

$$P_{ex} = P_{VWAP}^{0} \exp\left(\sigma_{VWAP} \varepsilon_a \sqrt{t}\right) + \sigma_H \varepsilon_b$$

P_{ex} : expected execution price at the end of the holding period

P_{VWAP}^{0} : VWAP on the risk evaluation day (Sep 1996)

σ_{VWAP} : historical volatility of VWAP over the observation period

σ_H : SE of distribution of daily trade prices accumulated over the observation period (Oct. 1995-Sept. 1996) and standardized by each-day VWAP

t : holding period (one day in this simulation)

$\varepsilon_a , \varepsilon_b$: standard normal random numbers simulation result (Chart 4) based on a hypothetical portfolio comprising of five stocks chosen randomly from the sample (basic statistics of the portfolio are summarized in Chart 5) shows that VaR taking into consideration execution timing risk is significantly higher than both VWAP-based VaR and ordinary VaR using end-of-day mid-prices.

2. Modified market risk measure taking account of intraday variability of bid-ask spreads

As past market microstructure literature indicates, the bid-ask spreads of Japanese equities fluctuate widely even during a single day (Chart 6). By analogy with the simulation with respect to execution timing risk described above, we quantify the risk involved in the intraday variability of bid-ask spreads, by using the same portfolio introduced in the preceding section. We again assume a lognormal stochastic process for end-of-day mid-prices. Ask and bid prices are defined as mid-price plus and minus ($\frac{1}{2}$ *bid-ask spreads) respectively, where the probability density function of bid-ask spreads is based on a historical (non-parametric) simulation using the data covering the one-year observation period.[9] The result of the simulation for bid prices is presented in Chart 7. First, we note that the expected value of the distribution is lower than that of mid-prices. Moreover, the modified market risk measure represented by 99 percentile VaR turns out to be higher than ordinary VaR, which underscores the importance of bid-ask spreads in market risk calculation.[10]

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9 The simulation process can be formally described as follows:

$$P_{bid} = P_m^0 \exp\left(\sigma_m \varepsilon \sqrt{t}\right) - \frac{1}{2} f(u)$$

$$P_{ask} = P_m^0 \exp\left(\sigma_m \varepsilon \sqrt{t}\right) + \frac{1}{2} f(u)$$

P_{bid}	: bid price at the end of the holding period
P_{ask}	: ask price at the end of the holding period
P_m^0	: end-of-day mid-price on the risk evaluation day
t	: holding period (one day in this simulation)
σ_m	: volatility of mid-prices
$f(\bullet)$	: probability density function of bid-ask spreads
ε	: standard normal random numbers
u	: uniform random numbers

10 In conducting the historical simulation for bid-ask spreads we arbitrarily excluded equities, the spread distribution of which looks like Chart 8. The tail events in this case simply reflect the fact that either quoted bid or ask price disappear from the market. Although it is desirable for these illiquid market conditions to be investigated more carefully and incorporated into liquidity risk calculation, we do not deal with the issue in this paper. This might contribute to a spuriously small difference between ordinary and modified VaR.

3. Risk in λ for securities with different liquidity

As we saw in the preceding chapter, the market impact often defined by λ (sensitivity of bid or ask prices to trading volume) has attracted greater attention in market microstructure literature. While most of the previous literature focuses on characteristics of λ in a deterministic sense as described in Chapter 3, we try to proceed one step further and investigate the statistical distribution of λ in order to determine the implications of market impact for market risk calculation.

First we construct λ as a ratio of price impact to adjacent trading volume standardized by normal market size. Normal market size is approximated by a daily average of trading volumes per transaction, which is assumed to be constant during one day.[11] Price impact is measured by a change from before-trade quoted bid or ask price to after-trade quoted bid or ask price. If there is a difference between a change in ask price and that in bid price, we take the larger price change – if bid (ask) price change is larger, then we can interpret it as seller (buyer)-initiated market impact.[12] Charts 9 and 10 show that expected value of λ and 90 percentile point of λ distribution of are negatively correlated with the level of liquidity for securities. In obtaining λ we pool both seller-initiated and buyer-initiated transaction data, though there might be an asymmetry between two transactions, as documented by previous studies.

Chart 11 clearly demonstrates that market impact represented by λ cannot be neglected in market risk calculation. Hence, we conduct VaR simulation to show how λ affects ordinary market risk measurement. We again assume a lognormal stochastic process for mid-prices. In quantifying market impact, we additionally assume that our trader having the same hypothetical portfolio used for the previous simulations liquidates his position not all at once but only gradually. In this simulation he divides his position into a piece of transactions, each of which is assumed to be equal to the average trading amount per transaction of the evaluation date. This assumption means that a trader has to execute at least a few transactions before he liquidates his entire positions, each transaction of

[11] Normal market size might be better represented by time-varying orders in the market, which are not available in our data set.

[12] A conventional method of classifying trades compares the trading price to the quote prices in effect at the time of the trade. However, it should be pointed out that there are several shortcomings in this method. Another method relies solely on trading prices, avoiding data quality problems with quote prices. Lee and Ready (1991) provide a succinct review of this issue.

which causes downward price pressure. The simulation result is shown in Chart 12.[13] It is clear that the market risk measurement taking account of λ ("λ-augmented VaR") is much larger than the ordinary VaR. It is also notable that the distribution of the λ-augmented VaR is of fat-tail feature which gives rise to unproportionately large 99-percentile risk compared with 95-percentile risk. Another characteristic of this market risk measurement is that as the initial holding position increases in size, the respective risk amounts increase in an unproportionate fashion. For instance, if we double the initial position, the risk amount almost triples.

However, we should interpret this simulation result with caution. First, our λ is derived under the assumption of deterministic normal market size, though, as a matter of fact, normal market size seems to follow a definite stochastic process which should be incorporated into risk measurement. In stressful situations, in particular, outstanding orders (normal market size) could easily diminish, resulting in higher market impact. Secondly, although our λ is certainly a departure from a static risk calculation in the sense that it takes account of first-round price impacts of individual trades, it does not capture second-round price effects which would, for instance, arise from the herding behavior of other participants. If aggregated sensitivity data for the market as a whole is available, one possible way of capturing this kind of second-round effect is first to estimate future trading patterns from information on aggregated market sensitivities under certain scenarios about future price movements, the procedure of which is exemplified by Shimizu (1997), and then derive

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[13] The simulation process can be mathematically expressed as follows:

$$P^i_{bid} = P^{i-1}_{bid} \cdot \exp\left(-\frac{V^i g(u_2)}{NMS}\right) \quad \text{where} \quad P^0_{bid} = P^0_m \exp\left(\sigma_m \varepsilon\sqrt{t}\right) - \frac{1}{2}E[f(u_1)]$$

$$P^i_{ask} = P^{i-1}_{ask} \cdot \exp\left(\frac{V^i g(u_2)}{NMS}\right) \quad \text{where} \quad P^0_{ask} = P^0_m \exp\left(\sigma_m \varepsilon\sqrt{t}\right) + \frac{1}{2}E[f(u_1)]$$

P^i_{bid}	: after-trade quoted bid price
P^i_{ask}	: after-trade quoted ask price
P^0_m	: end-of-day mid-price on the risk evaluation day
t	: holding period (one day in this simulation)
σ_m	: volatility of mid-prices
$f(\bullet)$	: probability density function of bid-ask spreads
$g(\bullet)$	: probability density function of λ
ε	: standard normal random numbers
u_1, u_2	: uniform random numbers
V^i	: trading volume per transaction
NMS	: normal market size (a daily average of trading volumes per transaction assumed constant in this simulation).

the price impact by using λ. Although this issue is worth pursuing, it is beyond the scope of this paper.

V. Summary and areas for future research

This paper aims to shed light on liquidity risk, which has been left behind in the pursuit of more sophisticated market risk measurements both by market practitioners and by central banks. We first defined liquidity risk and showed that it can be divided into execution cost and opportunity cost. In the light of stylized facts regarding dynamics of liquidity and price/spread movements, which have been documented previously by finance literature, we proposed several modified market risk measures reflecting intraday liquidity patterns and price movements. We then demonstrated, by applying the measures to the Japanese equity market, to what extent the quantified liquidity effects affect conventional measurement of market risk represented by VaR.

Although our proposed market risk measures clearly reflect liquidity risk, we did not analyze the time-varying properties of market liquidity and the dynamic interaction between trading activity and price/spread movements. A possible next step would be to investigate the dynamics of market impact by applying time-series modeling. Furthermore, the market impact of trading on prices might not be one-time, as implicitly assumed in our measurement, but could trigger second-round effects, which might need to be analyzed in an experimental simulation as formulated by Shimizu (1997). Since the driving forces behind tick-by-tick price movements are not only trading activity but also the arrival of new information, identification of these two factors would be the key to understanding intraday price movements in a dynamic context.

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Chart 1

Components of liquidity risk

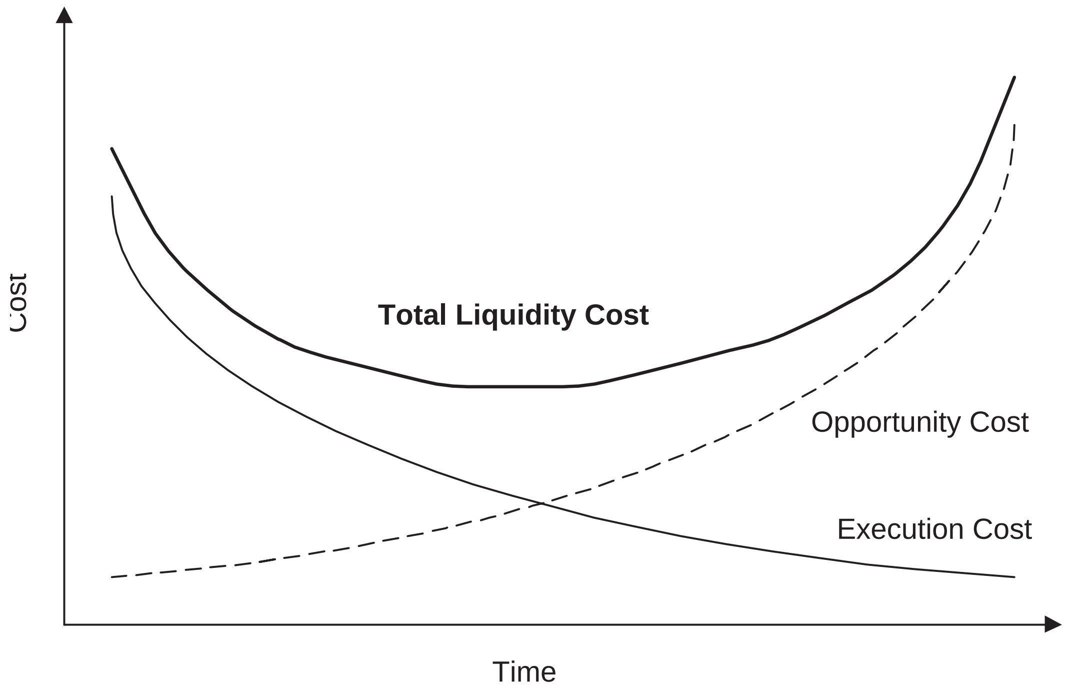
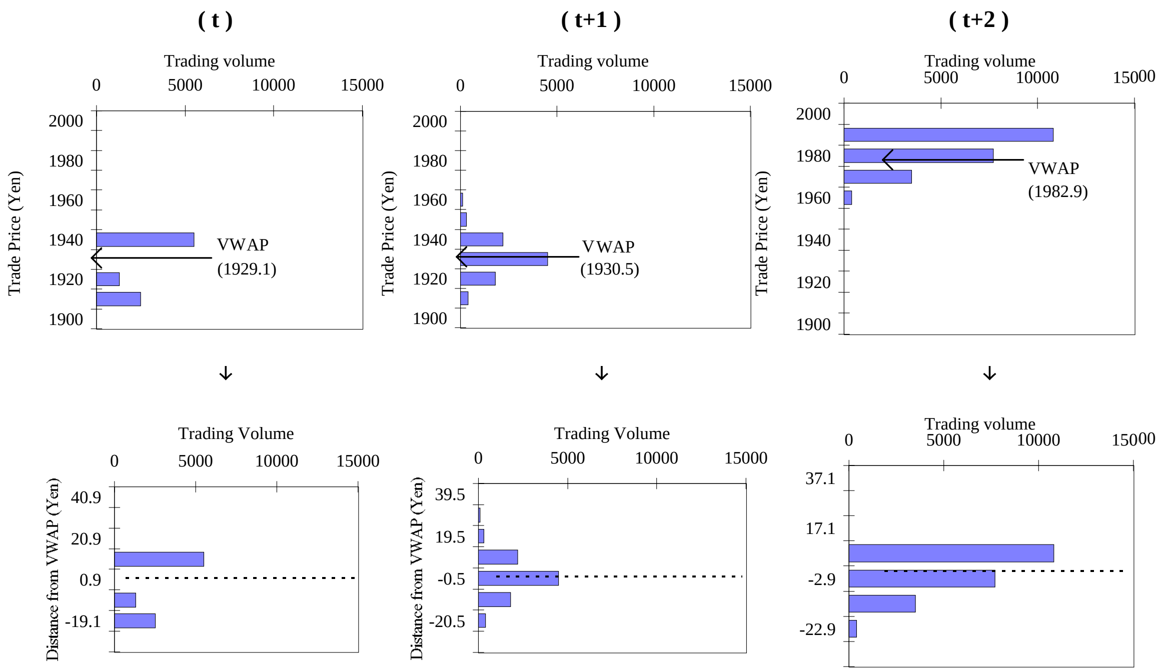


Chart 2

Volatility and kurtosis of daily return

	AM Open	AM Close	PM Open	PM Close
Volatility	0.02000	0.01930	0.01896	0.01999
Kurtosis	6.5503	6.7363	7.2008	6.5145

Chart 3
Histogram of Trade Prices



Accumulated Histogram

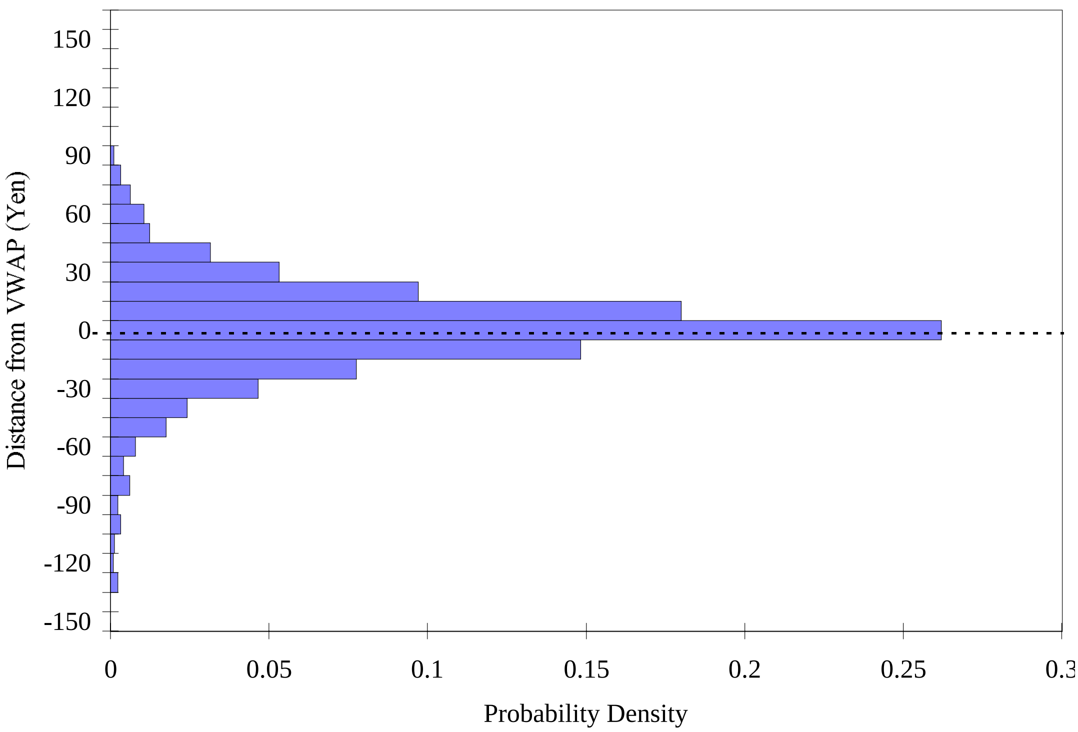
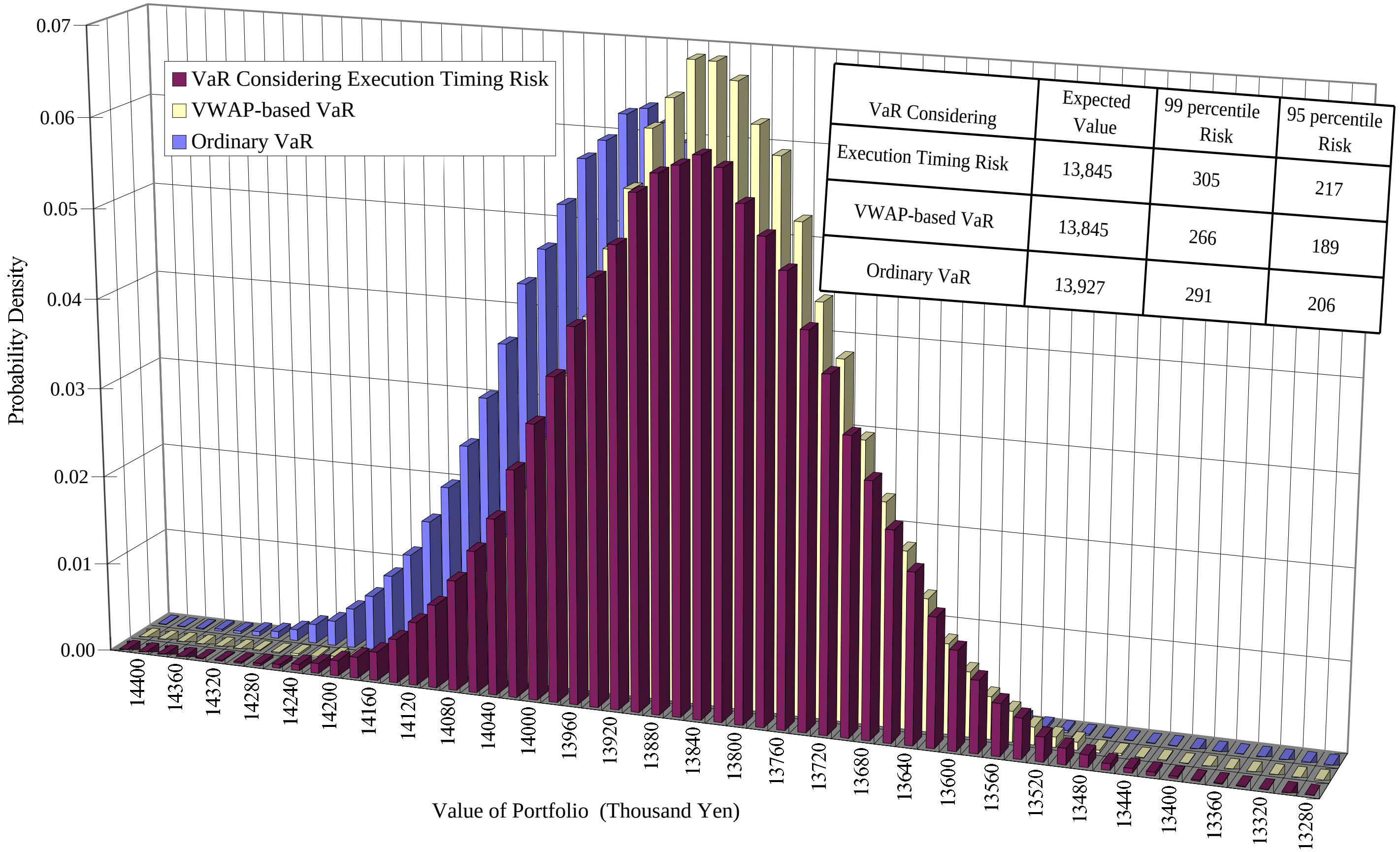


Chart 4 Market Risk Considering Execution Timing Risk



VaR Considering	Expected Value	99 percentile Risk	95 percentile Risk
Execution Timing Risk	13,845	305	217
VWAP-based VaR	13,845	266	189
Ordinary VaR	13,927	291	206

Chart 5

Basic statistics of the portfolio

	Stock A	Stock B	Stock C	Stock D	Stock E
Position (units of stocks)	+ 1,000	+ 1,000	+ 1,000	+ 1,000	+ 1,000
Mid-price on the day of risk evaluation (Yen)	2,275	5,730	1,580	2,355	1,985
HV of mid-price (%)	1.957	1.420	1.886	2.167	1.863
VWAP on the day of risk evaluation (Yen)	2,256	5,718	1,524	2,335	2,010
HV of VWAP (%)	1.749	1.314	1.567	1.867	1.683
Standard deviation of VWAP (Yen)	25.72	43.43	13.60	33.89	20.84
Average bid-ask spread (Yen)	22.62	45.90	17.33	25.60	21.76
Daily average of trading volumes per transaction	608	386	500	950	375

Chart 6

Histogram of bid-ask spread

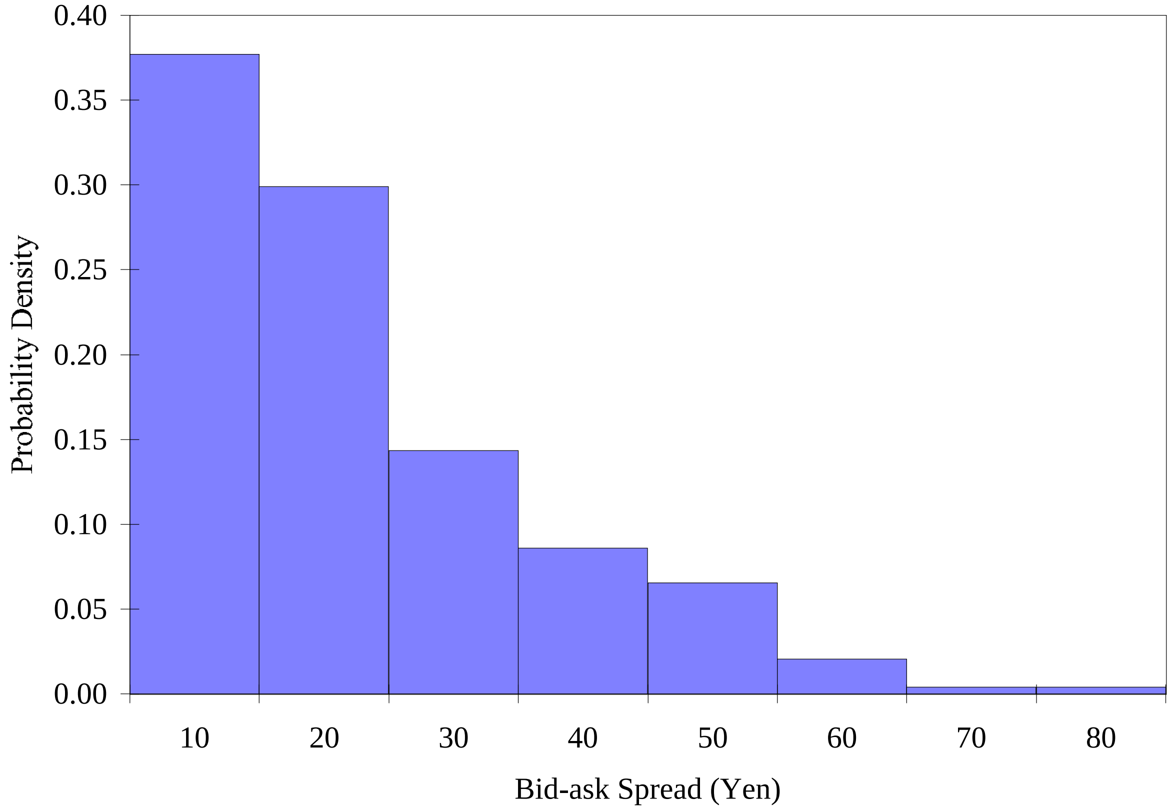
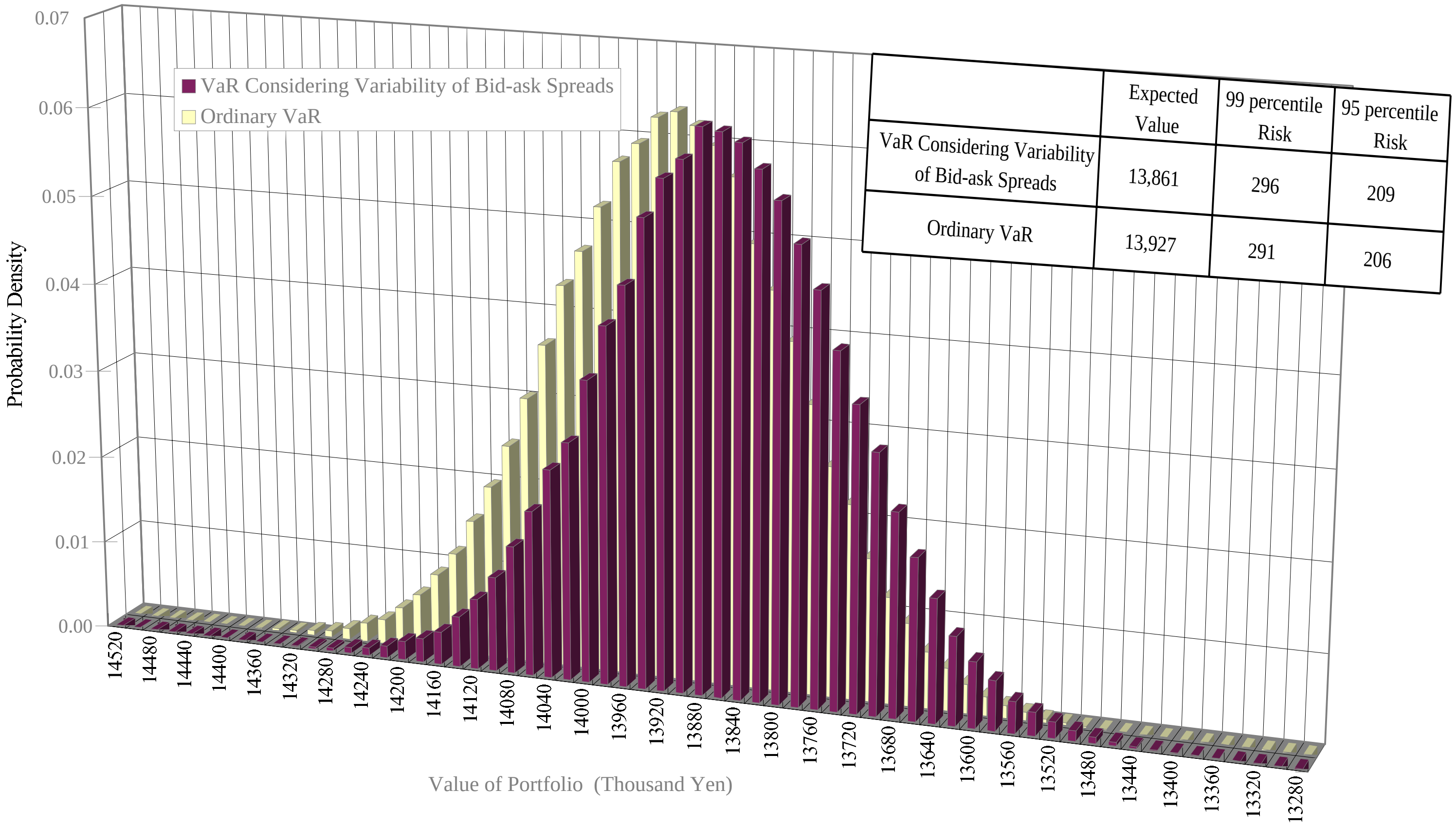


Chart 7 Market Risk Considering Intraday Variability of Bid-ask Spreads



	Expected Value	99 percentile Risk	95 percentile Risk
VaR Considering Variability of Bid-ask Spreads	13,861	296	209
Ordinary VaR	13,927	291	206

Chart 8

Tail events of bid-ask spreads

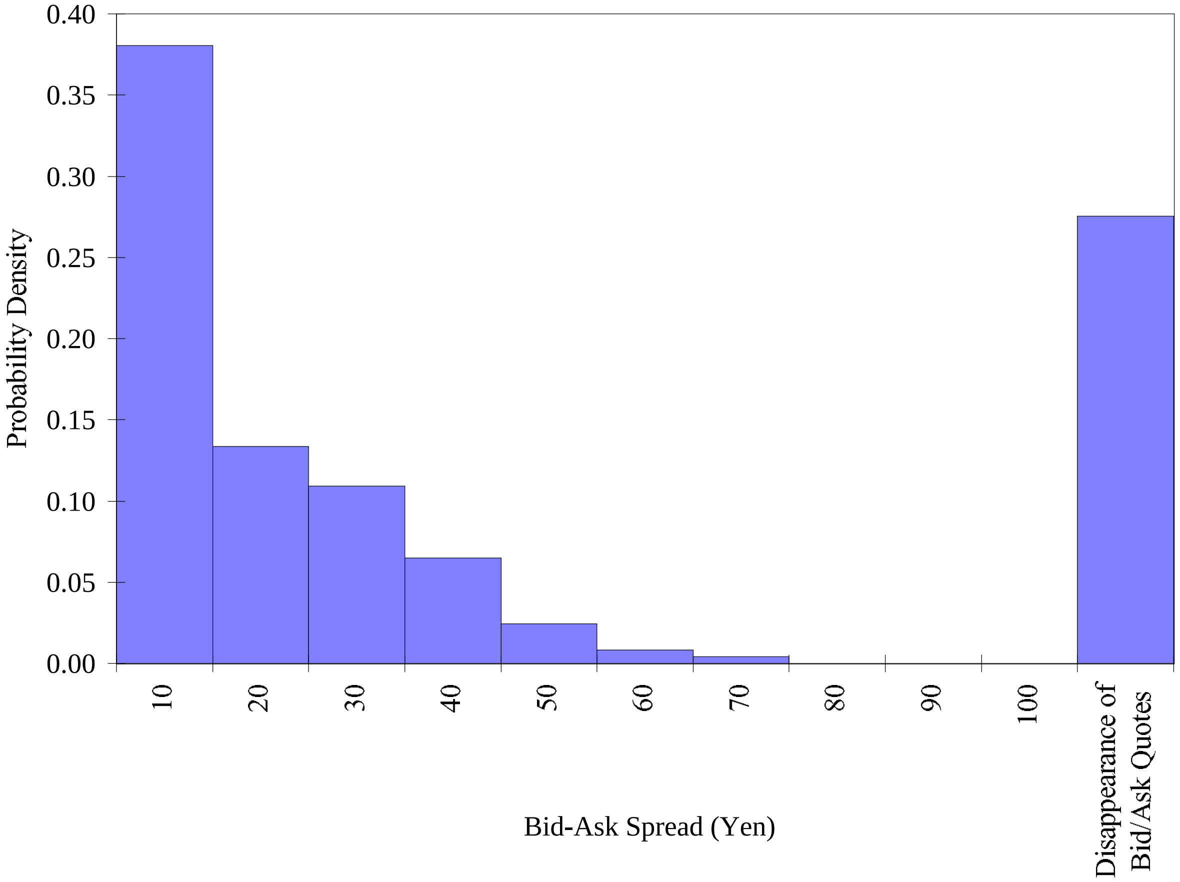


Chart 9

Relationship between λ and turnovers

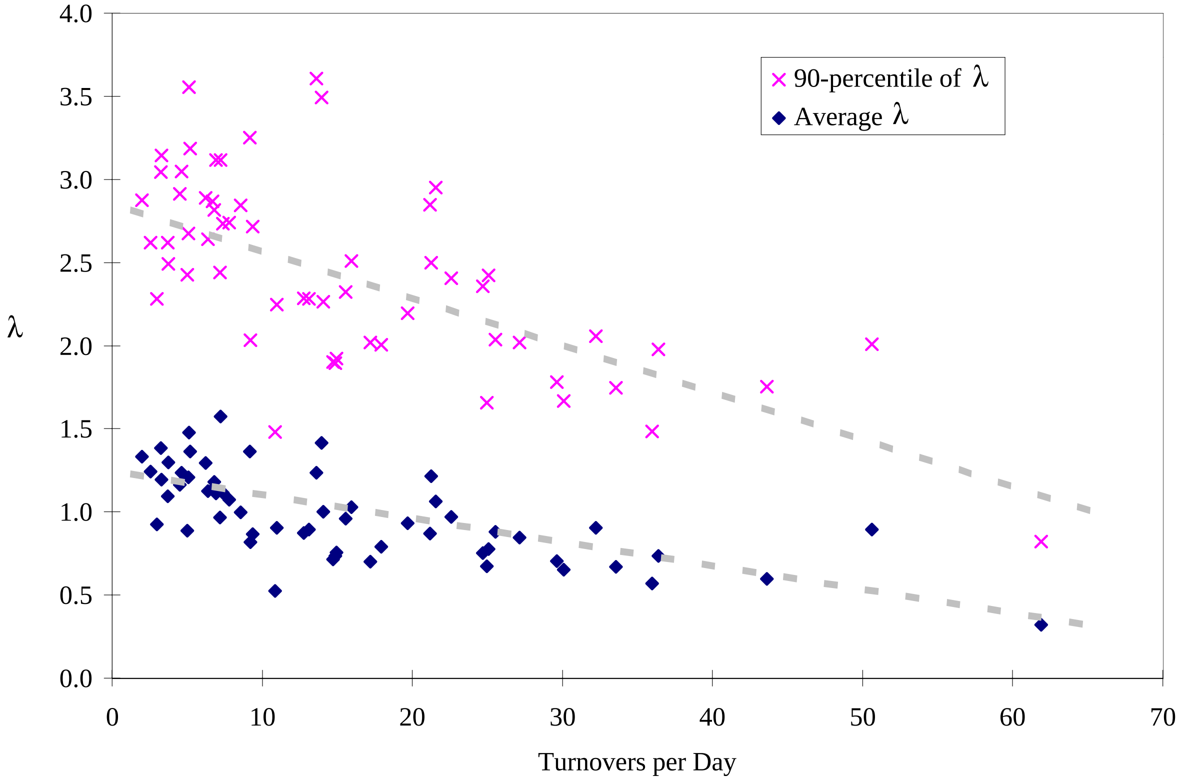


Chart 10

Relationship between λ and trading volume

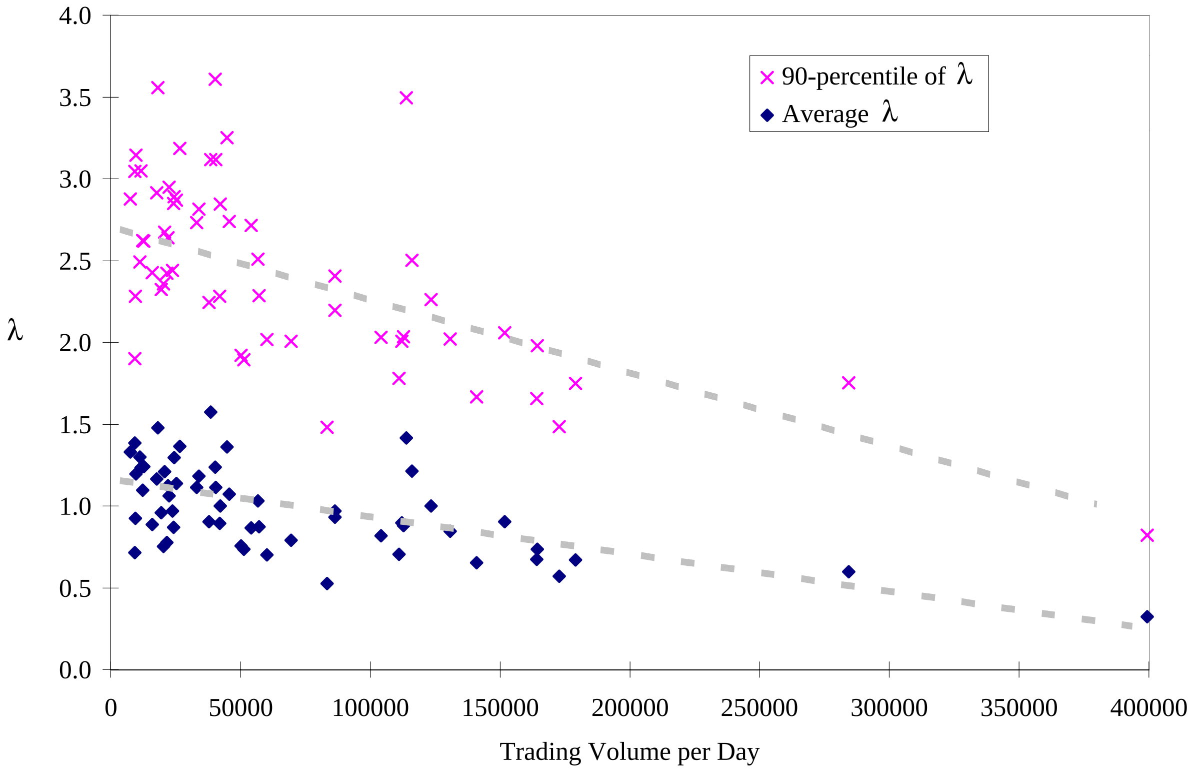


Chart 11

Histogram of λ

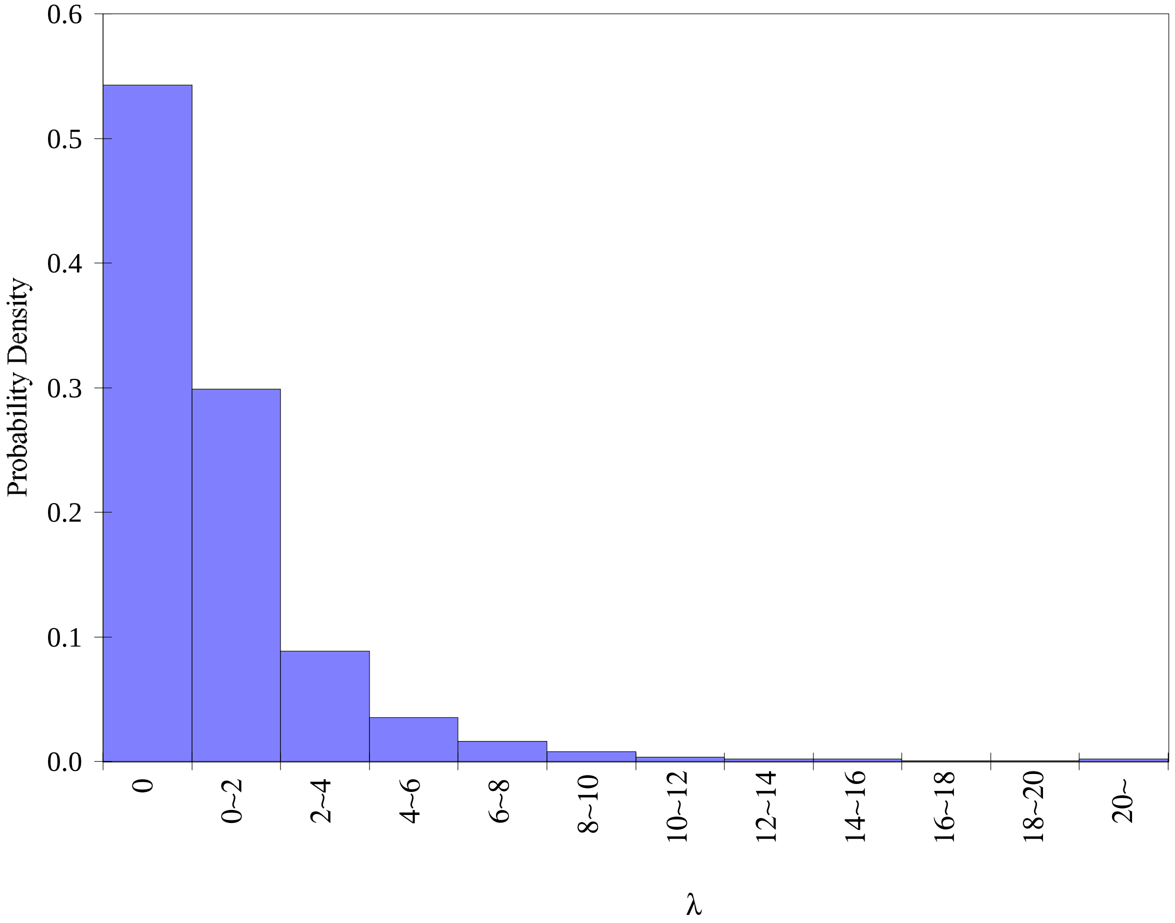
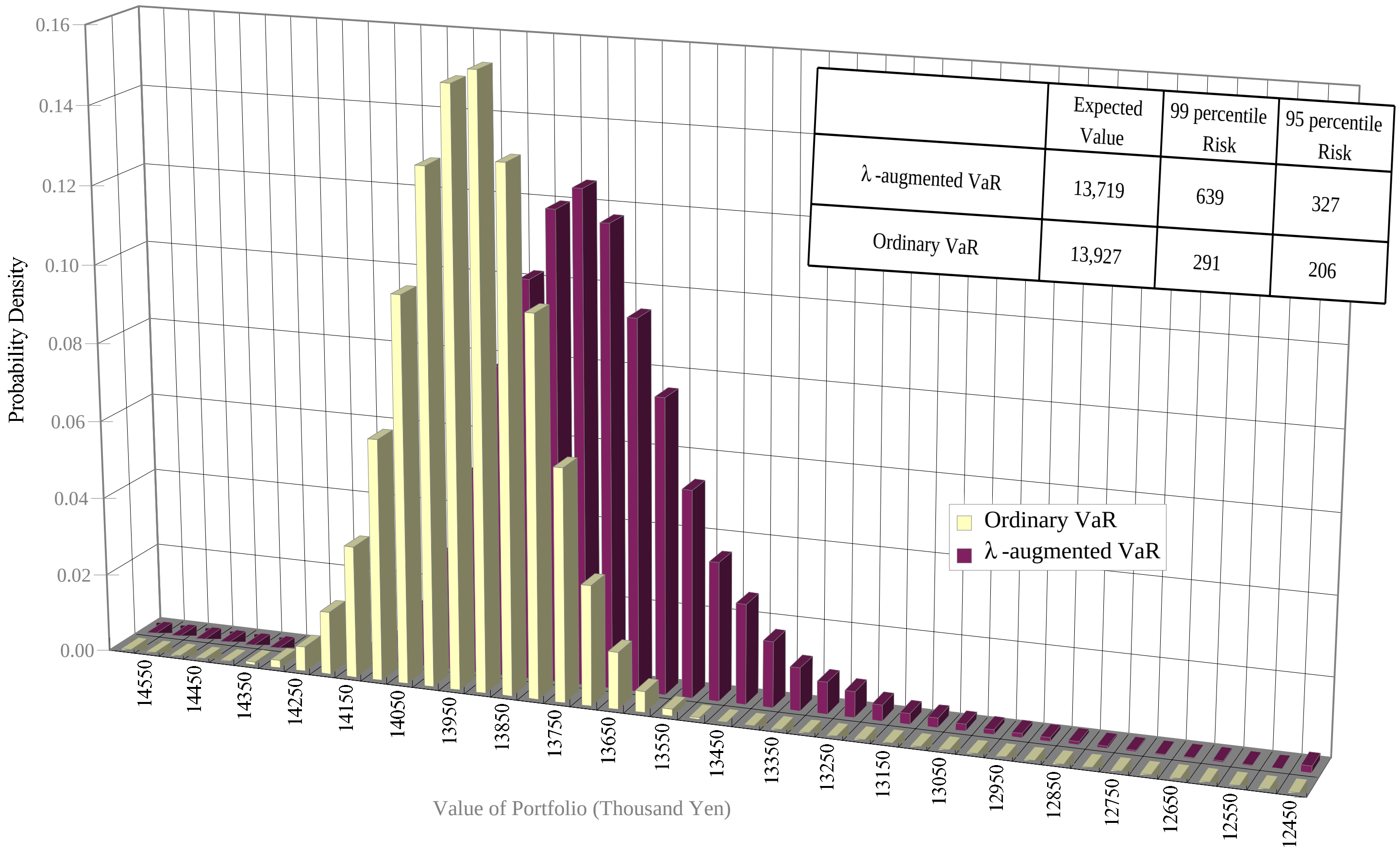


Chart 12　Market Risk Considering Variability of λ



	Expected Value	99 percentile Risk	95 percentile Risk
λ -augmented VaR	13,719	639	327
Ordinary VaR	13,927	291	206

Information collection and disclosure

by

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and

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Abstract

This paper examines the issues related to the public disclosure of information on aggregate market risk. While such disclosure might be expected to enhance market transparency, problems inherent in the proposed methods to construct such information and time delays in disclosing it would in fact reduce its informational value. Meanwhile, the direct cost of disclosure is identified as possible behavioural distortions through market participants misinterpreting the data or even an attempt to avoid regulatory reaction. Overall, the cost of collecting and disseminating the information would outweigh the potential benefit.

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* The views expressed in this paper are the authors' and do not necessarily reflect the position of the Bank of England, the Bank of Japan or the Euro-currency Standing Committee.

Introduction

This paper examines the issues related to the public disclosure of information on aggregate market exposures. Such disclosure might be expected to enhance market transparency to the extent that the information disseminated revealed anything about market dynamics and the robustness of the financial markets to shocks. We first ask whether the proposed methods such as those based on principal component analysis would succeed in capturing the important elements of market risk and whether it would convey information on robustness of markets. Furthermore, time delays in collecting and disseminating the information cannot be avoided. These and other problems would reduce the informational value of such aggregate information considerably whether or not it is disclosed to the market. On the other hand, when it is disclosed, the direct cost of disclosure could be various behavioural distortions. We discuss the conceivable measures to minimise market distortions, which might include the central banks' commitment to the data generation process. Even if commitment were possible, our overall assessment is that the cost entailed in collecting and disseminating aggregate market exposures would outweigh the potential benefit.

In this paper we look at the possible value which information on aggregate market exposures might have for the central banks and whether the market participants and market users would derive benefits if the data were published.

The data

This section looks at the issues which would arise if the central banks collected information from market participants on a periodic (but not frequent) basis on the following:

- The exposure of each firm to market moves given a number of scenarios covering the main markets (probably concentrating on market risk rather than spread or basis risk). The scenarios could either represent statistically significant changes in prices across a number of markets or could be, for example, sizeable moves seen in the past - eg October 1987. The third option would be for the central banks to specify forward looking stress tests.
- Alternatively the central banks could collect information on the sensitivities of the individual portfolios held by the firms to changes in prices. The effects of particular scenarios could then be calculated by the central banks.

The information which would be disclosed to the market would probably be the aggregate exposure of all entities operating in a particular market to a particular set of price shocks. One issue is whether the full details of the scenarios should be published. Another issue is whether some indication of the distribution regarding the results across individual firms should be published.

Usefulness to the central banks

One question is how useful this data would be to the central banks themselves. One area where it might possibly be helpful would be in giving a better indication over time of the behaviour of the market participants in terms of the quantum of risk being run and the exposure to particular types of scenario. In order to provide this time series, the scenarios would have to be consistent over time or at least calculated on a consistent basis. One danger would be that the level of risk being run could be understated and shifts in the type of risk being run could go unnoticed if the scenarios did not, as they almost certainly would not, cover spread risk and different types of basis risk at all or in a sufficiently complex way. In other words the scenarios would not be sufficiently sophisticated to highlight the exposures given the type of risk management we are now seeing. It might however be possible to specify more elaborate scenarios, in terms of the effect on a range of instruments in a particular market, by using past examples.

Another issue is whether the data would tell the central banks anything about the robustness of the markets to particular shocks. Because the central banks would have the information on the effect of shocks firm by firm it would be possible to look at the results in terms of the quantum of capital at risk and likelihood of particular players failing. The information would be a starting point but clearly would not encompass second or third round effects which in a crisis would determine the eventual outcome-for example the effect on clearing houses, or gridlock in markets. Conversely it could exaggerate the risk because exposures are constantly being altered in the light of market conditions. The scenarios would be independent of current conditions and therefore the books being held could be quite different from those which might be held preceding a market break of the type assumed in the scenario.

In terms of understanding the likely dynamics , the information on exposures would only be partial covering only the players at the centre of the market not the large investing institutions whose flows would dominate the market in a market break .

It is important to note that reporting burdens have been reaching the limit of tolerance at each institution and the central banks are expected to offer benefits for the markets as well as convincing reasons for carrying out such an exercise.

Disclosure to market participants

This section looks at the benefits to market participants (ie the trading firms) of the disclosure of such information.

The objective of disclosing the aggregate risk measures would be to help individual market participants and market users make more efficient decisions by reducing uncertainty and incomplete information. One issue is whether the disclosed information would provide useful material regarding the robustness of the financial markets, to the players and users-ie, whether it would contribute to mitigating the problem of asymmetric information. Clearly given the delay in releasing the information it would not improve the understanding of the current exposures in the market nor would aggregate information probably provide much illumination on the question of exposures of individual players. But knowledge of the past patterns of exposures, as well as the effect of given shocks (in aggregate on all participants) on those exposures, could possibly enhance participants' and users understanding of market dynamics thereby affecting expectations about the effects of future crisis.

If the aggregate data did convey information of this kind then the measures (if disclosed by currency, market segment and reporting institutions' nationality) might influence the allocation of economic capital between markets or at least the way in which a firm chose to deploy its capital in a particular market, in terms of the positions the firm was willing to run. For example, indicators of the aggregate exposures in a certain region could possibly help individual institutions to improve their strategy. However, this should not be overstressed given the snapshot nature of the data. In order for firms to make any use of the data the scenarios would have to be disclosed .It is probably desirable also to disclose the way in which they are generated (see below).

However, the aggregate data could be disclosing profitable strategies being run by some market participants, undermining their ability to continue to run those strategies. Although for this to be the case the market would have to be quite concentrated and the strategies would be quite long lived.

It is also possible that the disclosure of such aggregate information could distort the behaviour of market participants. Distortions could have multiple causes. Uncertainty about the procedures the central banks followed in deriving aggregate risk measures, variations in the disclosure schemes over time and so on could lead market participants to react in an inappropriate way. In other words the firms could read into the data messages which were not correct and adjust their behaviour inappropriately.

There could also be a danger that the market participants might misinterpret the implications of the data. For example, if players believed that the outcome of the stress scenarios showed that the authorities would have to provide support to the market or firms in certain circumstances (although it is not clear that they could be interpreted in this way) then some players might not make adequate efforts to avoid such an event by curtailing their risk exposures (moral hazard).It is not clear how real a danger this is nor, if it is real, whether it can be overcome.

In order to reduce adverse behavioural effects in terms of misjudging (or even judging all too accurately the intentions of the authorities) it might be necessary to fix the methodology used for the scenarios. The danger with changing the methodology would be that a change could be interpreted (quite possibly wrongly) as a signal from the central banks regarding their current concerns or inherent rate policy intentions, which would be potentially distortionary. This would clearly be exacerbated even further if the central bank set forward looking scenarios or specific ad-hoc scenarios which could become self fulfilling.

It would also have to be accepted that snapshot data of this kind might not always be immune from understatement which could create distortions. Some firms in some markets might be tempted to run lower exposures over reporting dates. But in terms of ***aggregate*** disclosure (across the whole market) a firm would have to regard itself as extremely large and atypical in order to fear that the aggregate information would convey unwanted information to the market and encourage other firms to counter-adjust their positions.

If the figures for any market revealed very large aggregate exposures to particular stress scenarios, then all the firms in that market could be viewed as equally risk taking. Firms would have to be able to signal their own internal position -ie publish what their own exposure was to that scenario. In order to do this they would have to know what the scenarios were.

Disclosure to market users

A separate question is whether investors would benefit from access to information on aggregate exposures of players in the market to particular scenarios.

They would clearly have an interest in the robustness of the players with whom they were dealing but the same arguments apply as for the market participants-that disclosure of aggregate statistics would provide little information on this outside very concentrated markets.

They also have an interest in information on likely market dynamics in stress periods if these dynamics could be construed from the aggregate data.

Some investors might also be interested in any information which revealed profitable trading strategies of the market players which could be imitated.

Evaluation of the two approaches

The critical task in the design of disclosure is to balance the two conflicting forces: enhancement of market transparency on the one hand and the danger of market distortion on the other.

Price sensitivity approach

Under this approach, the scenarios are not revealed to reporting institutions at the time of data collection. But when the aggregate measures were disclosed the decision would have to be taken whether to publish information on the scenarios or not.

For reasons of transparency and to reduce the likelihood of inappropriate behaviour adjustments it would be necessary to disclose the scenarios. There would also be reason (as set out above) to keep the scenarios, or at least the methodologies, fixed over time.

However, the incentives for market participants to fine-tune their exposures are probably reduced where the scenarios are unknown at the time of data reporting.

Theoretically, it might be possible to circumvent the problem of revealed scenarios by preparing ***multiple*** sets of scenarios to be used in the calculation and by ***randomising*** over the set. For example, suppose there are five sets of scenarios in the hands of the central banks. When the reporting institutions are required to submit their sensitivity data they are not informed about which set of scenarios is to be used in the current calculation of the aggregate risk measures. One of the sets of scenarios is chosen and the aggregate measures are calculated by the central banks using that set and the results are revealed to the market with the scenarios.

This may, however, not discourage firms from simply reducing all exposures to a very low level to reduce the results whatever the scenario.

Portfolio revaluations

Under this approach each reporting institution is given a set of scenarios on which to base their calculations of the change in value of their portfolio. As discussed above the scenarios should be common across institutions and should be fixed or at least the methodology should be fixed.

First thing to note is that the danger of fine-tuning exposures is greater under this approach. Provision to reporting institutions of fixed and detailed scenarios before they calculate (and hence report) the risk amounts will leave scope for them to fine tune their books. Although it seems impossible to tackle this point directly, we can still try to reduce this possible distortion by adopting multiple sets of scenarios and choosing randomly among the sets with regard to disclosure - ie, a number of scenarios would be calculated but only a sub-set would be released in the aggregate information. In this way, the incentive to manipulate behaviour could be weakened.

Further Issues

Commitment

One issue is therefore that of commitment by the central banks - to disclosure schemes, reporting requirements, the way regulation is implemented and so on. It is sometimes argued that if central banks do not make a commitment, the fear of their opportunistic behaviour would undermine the positive effects of enhanced information and resulting efficiency. Furthermore, discretionary changes in the procedures for the exercise may aggravate market uncertainty. It is not at all clear though that the central banks could make any commitment regarding the use to which they would put the data nor any regulatory action that might be triggered.

Unless the central banks' commitment is guaranteed, however, the distortion issues we have discussed so far remain significant. In particular, if the central banks chose to disclose the aggregate exposure in any way, the lack of commitment may even aggravate the market distortion. We therefore need to balance carefully the practical difficulty of making commitment and the possible distortionary consequences of it. Needless to say, commitment must be credible. It may therefore be helpful to investigate how we can design an optimal mechanism to make commitment credible, should we decide to proceed in this direction.

Timing of disclosure

Time delays in collecting data, calculating risk measures and disseminating them seem inevitable In highly competitive markets like financial markets, each participant can carry out a variety of transactions in varying volumes instantaneously and with little cost. Given this any delays in disseminating the results would reduce substantially the information content. Although this in turn could be helpful in encouraging reporting firms not to window dress.

More importantly, delayed dissemination gives rise to a risk that some market participants might erroneously interpret the results. Although improvements in the technology to process efficiently a huge amount of data could eventually mitigate this problem, delays are probably fundamental.

Global aggregation

One issue regarding global aggregation is the consistency of data among institutions. Data collected from reporting institutions - either sensitivity data or calculated exposures to stress scenarios - must be on the same basis and comparable across institutions and countries. Either measure could be fundamentally affected by assumptions made by particular firms - for example, the volatilities used. Without putting these assumptions on the same basis or at least having a consistent methodology across countries, the data thus obtained could show a distorted picture between markets. In sum, a consistent definition of sensitivity data and calculation of stress scenarios across countries could be indispensable if the exercise was to be meaningful.

Another issue though is that this common treatment across market could in itself be misleading. This is because a particular scenario could exactly highlight the risks being run in one market while completely missing the risks being run in another given the range of possible types of exposure - outright market exposure, basis risk, spread risk and so on.

Partial disclosure

One issue is whether there is a case for disclosing the aggregate measures only to central banks - both participating and non-participating. If the benefit of enhanced transparency (to the market in general) is outweighed by market distortions then partial disclosure (only among the central banks) could be an alternative with a commitment that the information would not be released more widely and also a commitment to limit the use to which the data would be put.

It would also be important to encourage non-participating central banks to participate in the exercise. This is because information disclosure can confer positive externalities.

The size of hedge adjustments of derivatives dealers' US dollar interest rate options

by

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June 1997

Abstract

The potential for the dynamic hedging of written options to lead to positive feedback in asset price dynamics has received repeated attention in the literature on financial derivatives. Using data on OTC interest rate options from a recent survey of global derivatives markets, this paper addresses the question whether that potential for positive feedback is likely to be realised. With the possible exception of the medium term segment of the term structure, transaction volume in available hedging instruments is sufficiently large to absorb the demands resulting from the dynamic hedging of US dollar interest rate options even in response to large interest rate shocks.

* I am grateful for helpful comments and suggestions of Young Ho Eom, James Mahoney, and participants in workshops at the Bank for International Settlements and the Federal Reserve Bank of New York. The views expressed in this paper are the authors' and do not necessarily reflect the positions of the Federal Reserve Bank of New York, the Federal Reserve System, the Bank for International Settlements, or the Euro-currency Standing Committee.

The size of hedge adjustments of derivatives dealers' US dollar interest rate options

Dealers' hedging transactions in underlying fixed income markets required for the management of the price risks of their options' business raises two questions. First, might dealers' hedging demands be so large as to disrupt the markets in the available hedging products? Second, is the dynamic hedging of dealers' residual exposures sufficiently large to justify a concern about positive feedback in price dynamics in the fixed income market?

The potential for dynamic hedging of written options positions to introduce positive feedback in asset price dynamics has received repeated attention in the literature on financial derivatives. A short and incomplete list would include, Grossman (1988), Gennotte and Leland (1990), Fernald, Keane and Mosser (1994), Bank for International Settlements (1986, 1995), and Pritsker (1997). Using data on OTC US dollar interest rate options from a survey of global derivatives markets, this paper assesses the likelihood of such positive feedback caused by dynamic hedging of options. The OTC interest rate options market is an interesting place to explore the positive feedback issue because dealers are net writers of these options (see Annex Table A2).

The estimates in this paper suggest that, with the possible exception of the medium term segment of the term structure, transaction volume in available hedging instruments is sufficiently large to absorb the demands resulting from the dynamic hedging of US dollar interest rate options. While a definitive answer to the positive feedback question would require data on investors' demand for interest rate products in addition to dealers' hedging demand arising from dynamic hedging of options (see Pritsker 1997), comparing potential hedging demand with transaction volume in typical hedging instruments might give a provisional assessment of the likelihood of positive feedback.

1. Introduction

The data in this paper are global market data for US Dollar OTC interest rate options from the April 1995 Central Bank Survey of Derivatives Markets (Bank for International Settlements, 1996). Using data on notional amounts and market values, strike prices were estimated such that when applied to the notional amounts, the strike prices generate the observed market values of the options. In particular, given maturity data (from the Survey) and market growth data (from ISDA[1]), estimates were generated of the notional amount of options by maturity and origination date (going back 10 years). Strike prices, based on historical interest rate data, were then assigned to the options originated at each point in time, such that the strike prices produced option values equal to those observed in the survey.

[1] International Swaps and Derivatives Association.

With the estimated strike prices and a postulated interest rate shock, we ask what would be the change in dealers' hedge positions that would restore the net delta of a (hedged) option portfolio to its initial level? This estimated hedge adjustment is the incremental net demand of dealers for hedge instruments, given the assumed interest rate shock. The estimated demand for hedge instruments might give some indication of the potential for positive feedback effects attributable to derivatives dealers' hedging of their OTC options portfolios.

2. Price sensitivity of the global dealer portfolio

Figure 1 shows the estimated price sensitivity of the global dealers' portfolio. The value at the prevailing forward rates is the amount reported in the Survey, and the values at the indicated changes in interest rates are estimated values. While dealers have sold more options than they have purchased, at the prevailing forward rates the bought options had higher market values and the net value of the global portfolio was positive (see Annex Tables A1 and A2). This relationship between the notional amounts and market values of bought and sold options implies that the options sold to customers had a lower degree of "moneyness" than options purchased from customers. The estimated strike prices are consistent with this relationship, as relative to swap rates at origination, sold options were found to be out-of-the money while options purchased from customers were estimated to be in-the-money.

Since dealers were net sellers of options, large interest rate shocks that drive the sold options into-the-money will cause the value of the sold options to dominate the portfolio value. Hence, the aggregate dealers' portfolio value becomes negative at interest rate shocks of more than 100 basis points. Figure 1 shows, however, that if the portfolio is hedged (but the hedge not dynamically adjusted) the value of the hedged portfolio would turn negative only after an extremely large interest rate shock. A rise of interest rates of almost 200 basis points would be required before the hedged portfolio value turns negative. Dynamically adjusting the hedge position as interest rates change would make such an adverse outcome even less likely.

The curverture of the option value function implies that the hedge position must be adjusted after an interest rate shock because the option values decrease at an increasing rate as interest rates rise. Without the hedge adjustment, the gain in value of the initial hedge position would no longer be sufficient to compensate for the declining option values. This need to dynamically adjust the hedge position as interest rates change introduces a potential for positive feedback. Since the required hedge is a short position in fixed income securities, the hedge adjustment would introduce additional sales into the market on top of the initial selling pressure that accompanied the initial interest rate shock.

Another feature of the aggregate dealer position is its exposure to rising interest rates: the negative slope of the option value curve at the prevailing forward rates in Figure 1. The conventional view of financial institutions' interest rate risk profile holds that these firms have a structural long position in the fixed income market. Namely, exposure to rising rates. Thus Figure 1 implies that, in the aggregate, dealers as a group can not hedge their net option exposures with offsetting structural exposures from other business lines. While some dealers may have offsetting exposures elsewhere in their firms that hedge their options position, Figure 1 suggests that not all dealers can fully hedge internally.

3. Dynamic hedging estimates

Dealers' options positions, especially of longer maturities, are most likely hedged with a variety of interest rate instruments. The market for US dollar interest rate products is sufficiently large and diverse that options dealers can choose from a wide range of hedging instruments, such as futures contracts, FRAs, interest rate swaps, Treasury securities, and interbank loans. While these instruments are not perfect substitutes because of differences in credit risks, transactions costs, and liquidity, economies of scale and diversification help dealers manage and intermediate these risks. If dealers have sufficient time to hedge a position or replace a hedge with a cheaper alternative, they are unlikely to encounter difficulty meeting their hedging needs. For immediate hedge adjustments in large volume, however, their alternatives may be more limited. Across the range of maturities that need to be hedged, the most liquid instruments available are Eurodollar futures, Treasury securities, and Treasury futures.

Eurodollar futures

The Eurodollar futures market appears to have transaction volume sufficiently large to accommodate the estimated hedge adjustments for small interest rate shocks. At shorter maturities, the Eurodollar futures market is more than large enough to accommodate dealers' hedging demands, even for large interest rate shocks. For hedging of longer maturity exposures, however, the Eurodollar futures market appears to be able to accommodate only the hedging of residual exposures (after the use of other hedging instruments) and marginal adjustments to hedge positions.

The largest daily turnover volume of Eurodollar futures contracts exceeds the estimated hedge adjustments: out to 10 year maturities, for a 10 basis point change in forward rates; out to 4 to 5 years, and also between 8 and 10 year maturities for a 25 basis point change in forward rates (Table 1); and, out to only 2 year maturities, for a 75 bp change in forward rates (Table 2). To put these figures in perspective, a 25 basis point change is slightly less than the largest daily change, and a 75 basis point change is slightly less than the largest two-week change, in forward rates in the 4 to 7 year segment of the yield curve (during the period 1991 to 1995).

The estimated hedge adjustments are smaller than the stock of outstanding futures contracts at all maturities. Even in the case of hedge adjustments to a 75 basis point change in forward rates, except for contracts between 7 and 8 years maturity, the estimated hedge adjustment in most cases is much less than half of outstanding futures contracts (Table 3.)

With respect to the estimated hedge position, rather than adjustments to the hedge position, for longer maturity exposures the Eurodollar futures market is not large enough to accommodate the entire hedge demands that would be generated by a fully delta neutral hedging strategy, especially for exposures beyond 4 or 5 years (Table 3.)

Treasury securities

To hedge exposures to forward rates between 5 and 10 years maturity, a possible hedge position in Treasury securities consists of a short position (sale of a borrowed security) in the 10 year note, and a long position in the 5 year note.

For adjustments to hedge positions, the on-the-run security turnover volume exceeds estimated dealers' dynamic hedging demands (Table 4, Panel A). For an extremely large shock to forward interest rates, however, such as a 75 basis point shock to forward rates beyond 5 years out, the estimated hedge adjustment in the 5 and 10 year note would be approximately half of average daily turnover.

With regard to the hedge position, the on-the-run issue volume appears to be too small to accommodate hedging demand if a fully delta neutral hedging strategy were attempted exclusively in the cash market in Treasury securities. For example, if dealers fully hedged their exposures beyond 5 years with 5 and 10 year on-the-run issues, the required hedge position would be approximately equal to the outstanding amount of the on-the-run 5 and 10 year notes (Table 4, Panel A).

Two means by which the Treasury market may accommodate this hedging demand exist. *First*, the existence of a large repo (collateralized security lending) market in Treasury securities allows a fixed stock of on-the-run Treasury securities to meet trading demands that exceed the size of the on-the-run issue. Through the repo market, a trader that establishes a short position enables another trader to establish a long position in the security. Hence, the size of market participants' long position in the security can be larger than the outstanding stock of the security. *Second*, off-the-run issues when available can also be used, further enlarging the pool of available hedging instruments.

Futures on Treasury securities

In addition to the cash market in Treasury securities, dealers can also hedge with futures contracts on Treasuries. As seen in Panel B of Table 4, open interest and turnover volume in the Treasury futures market exceeds estimated dealers' hedging demand.

While outstandings and turnover volume in the cash and futures markets in Treasury securities exceeds estimated dealers' hedging demands, that demand could be significant relative to the size of the market. For example, the estimated hedge adjustment to a 75 basis point shock could be large as 25% of the combined average daily turnover in both markets, while the estimated hedge position could be as large as a third of total outstanding in both markets (see Table 4, Panels A and B).

Interest rate term structure models

If dealers are willing to accept model risk (correlation risk), they could also hedge exposures beyond 5 years by spreading their hedging demands across a wider maturity range of securities than only the 5 and 10 year notes. For example, with the use of a two (or more) factor interest rate term structure model, a dealer could construct a hedge of exposures between 5 and 10 years using a position in one year bills and 30 year bonds that replicate the exposure to the term structure factors that drive forward rates between 5 and 10 years. Such hedges, however, would be vulnerable to atypical price shocks that the term structure model does not account for.

Conclusions

The estimated size of dealers' hedge positions of longer maturity exposures, suggests that dealers' hedges, especially of exposures beyond 4 years maturity, are distributed over a range of fixed income instruments. While outstanding Eurodollar futures contract volume is smaller than the estimated size of the hedge position beyond 5 years, the large size of the US dollar fixed income market suggests that the hedge positions can still be absorbed by the markets in other fixed income instruments. With regard to an immediate dynamic hedge adjustments to an interest rate shock, however, the ideal hedging instrument is one that is liquid and has low transactions costs, such as Eurodollar futures, on-the-run Treasury securities, or Treasury futures.

Impact on transaction volume

The Eurodollar futures, on-the-run Treasury securities, and Treasury futures markets together can easily absorb hedge adjustments to shocks to the forward curve as large as 25 basis points along the entire term structure (Tables 1 and 4). For example, the estimated hedge adjustment for 5 to 10 year exposures to a 25 basis point shock is approximately 10% of the combined turnover in the Treasury on-the-run cash and futures markets.

For an extremely large interest rate shock, however, such as a 75 basis point shock to forward rates, dealers' dynamic hedge adjustments would generate significant demand relative to turnover and outstanding in these hedging instruments (see Tables 2 and 4). In this case, by bearing the price risk of a partially hedged position and spreading the hedge adjustment over more than one day, the hedge adjustment could be broken into smaller pieces that would be small relative to daily turnover. The terms of this trade-off between price risk and the cost of immediacy or liquidity of course would depend on the volatility of interest rates, and volatility may rise at the same time that liquidity is most impaired.

These results suggest that dealers' inter mediation of price risks through market making in interest rate options is supported by liquidity in underlying markets that allow them to manage their residual price risks. Transaction volume in the standard hedging instruments appear to be large enough to accommodate dealers' hedge adjustments in all but the most extreme periods of interest rate volatility.

Price impact

With regard to the price impact of dynamic hedging our results are less clear. For a definite answer an analysis of demands of other market participants would be required (see Pritsker, 1997). For example, investors whose demands are driven by "fundamentals" could be expected to undertake transactions in the opposite direction of dealer's dynamic hedging flows if those transactions drove interest rates to levels that appeared unreasonable to the "fundamentals" investors." If these investors constitute a sufficiently large part of the market, then their transactions would stabilise prices and keep positive feedback dynamics in check. However, such stabilising investors are not the only other market participants. Other participants include traders who follow short term market trends either because of "technical trading" strategies or because they interpret short term changes to be driven by transactions of better informed "fundamentals" investors. The trades by these investors could amplify the price impact of dealers dynamic hedging. Thus, the ultimate impact of dealers' dynamic hedging would depend on the relative sizes of these types of market participants, as described in Pritsker (1997).

At shorter maturities, transaction volume and open interest of the most liquid trading instruments are so much larger than dealers' dynamic hedging flows that positive feedback driven by dealers' dynamic hedging seems unlikely, even with very large interest rate shocks. However, at longer maturities around 5 to 10 years, dynamic hedging in response to an extremely large interest rate shock could be of significant volume relative to total transaction volume and open interest in the most liquid trading instruments. Hence, at this segment of the yield curve, the positive feedback hypotheses in the case of a very large interest rate shock can not be dismissed. The dynamic hedging volume in response to an unusually large interest rate shock could be large enough to have a significant impact on order flows in the medium term segment of the yield curve-maturities between 5 and 10 years. Such order flows might have a transitory impact on this segment of the yield curve.

4. The data and estimation

Option characteristics

Option type

All options were assumed to be caps and floors on a 6-month interest rate. A cap payoff at period t is,

$$y_t = \max\left[f_t - x, 0\right]\frac{0.5}{1+0.5f_t}n, \quad t < M$$

where f_t is the interest rate at period t, x is the strike rate, n is the notional amount, and M is the maturity of the cap. The payoff on the 6-month rate between periods t and t+1 is paid at the beginning of period t.

Counterparty type

The Survey data has three counterparty types, and options are either inter dealer options, options bought from customers, or options sold to customers. Dealers are net writers of options, as they have sold significantly more options to customers than they have bought (see Annex Table A2).

Maturities

Options are assumed to have maturities up to 10 years, in 6 month increments. The first caplet in any cap has a maturity of 3 months (mid-point of the first 6-month maturity band): a 3-month option on the 6-month rate that applies between 3 months and 9 months. The last caplet in any cap has a maturity 6 months shorter than the maturity of the cap: an option on the 6-month rate that applies for the last 6 months of the cap's term.

Origination dates

Options are assumed to have been originated up to 10 years earlier.

Strike prices

Strike prices are derived from historical term structure data. For example, a 5 year cap originated at period p will have a strike proportional to the 5 year swap rate at period p. Thus, two caps originated at the same time may have different strikes if their maturities differ. The distinction between bought and sold options also implies that two caps with the same remaining maturity and origination date may have different strikes if one is a sold option and the other is a bought option - given that the options are not inter-dealer.

Maturity distribution

The maturity distribution of options originated at any date is assumed to be described by a quadratic function. The notional amount of options with t periods remaining maturity, originated p periods in the past is

$$n(t,p) = \left(\prod_{j=0}^{p} g_j\right)\left(a + b(t+p) + c(t+p)^2\right) \tag{1}$$

and,

$$n(t,p) = 0, \; for\; t+p < 1 year,$$

where t is remaining maturity, t < 10 years; p is the origination date (periods earlier), p < 10 years; $t+p$ is the original maturity, $t+p$ < 10 years; g_j is the market growth term at period j, where $g = \frac{1}{1+r}$, and r is the growth rate from period j-1 to period j. The growth rates r are growth rates of notional amounts outstanding of US dollar interest rate options obtained from ISDA's surveys. The restriction in (1) forces caps and floors to have maturities of at least one-year when originated. (Regardless of this restriction, the first caplet (option) in any cap or floor has a maturity of 3 months (the midpoint of the first 6-month time band). Estimates without this restriction are shown in Section 5.

The maturity distribution is found by solving for the parameters (a,b,c) of the quadratic function in:

$$\sum_{t \le 1yr} n(t,p) = N_1 \tag{2a}$$

$$\sum_{1yr < t \le 5yrs} n(t,p) = N_5 \tag{2b}$$

$$\sum_{5yrs < t \le 10yrs} n(t,p) = N_{10} \tag{2c}$$

where N_m are notional amounts in the survey's three maturity categories (see Annex Table A3), and the function $n(.)$ is as defined in equation (1).

Separate maturity distributions were estimated for interdealer options, options purchased from customers, and options sold to customers. The maturity data, however, were available only for all sold options and all bought options, where interdealer options were included in the maturity data of both bought and sold options. The maturity distribution of interdealer options was assumed to be the average of the bought and sold options' maturity distribution. Most outstanding contracts were of less than five years remaining maturity and were estimated to have been originated within three years of the survey date.

Option price function

The options are valued using Black's forward interest rate option model (see Hull 1993). The value of the period t payoff of a cap (floor) with strike rate x and notional amount n is

$$C(n,t,x) \;=\; e^{-r_t t}\left[f_t\, N\big(D1(t,x)\big) - x\, N\big(D2(t,x)\big)\right] \frac{\lambda}{1+\lambda f_t}\, n$$

$$F(n,t,x) \;=\; e^{-r_t t}\left[x\, N\big(-D2(t,x)\big) - f_t\, N\big(-D1(t,x)\big)\right] \frac{\lambda}{1+\lambda f_t}\, n$$

where, $D1(t,x) = \dfrac{1n\left(\dfrac{f_t}{x}\right) + \dfrac{\sigma_t^2 t}{2}}{\sigma_t \sqrt{t}}$, $D2(t,x) = D1(t,\ x) - \sigma_t \sqrt{t}$,

and λ is the length of the period for which the reference interest rate applies (6-months), f_t is the period t interest rate, σ_t is its volatility, and *N(.)* is the standard normal distribution function. The value of a cap (floor) with maturity *m* is,

$$V^c(n,m,x)=\sum_{t<m} C(n,t,x),$$

$$V^f(n,m,x)=\sum_{t<m} F(n,t,x),$$

The valuation used the term structure of forward rates and the term structure of implied volatilities at end-of-March 1995 (Derivatives Week, 1996)[2].The section at the end of the paper presents estimates using alternative implied volatility structures.

Strike prices

Strike prices were derived from historical yield curves. Because separate market values were not available for caps and floors, a relationship between the strikes of caps and floors was required in the estimation. The structure was chosen on the assumption that buyers (sellers) of caps and floors had similar preferences regarding their options' moneyness. Thus, if buyers of caps desired out-of-the money options because of their cheaper premia, then buyers of floors would also. This structure regarding the options' moneyness was implemented in three different ways. These implementations gave similar results as shown in Table 5.

First, a proportional displacement of the strike price from the swap rate. The strikes of caps and floors are,

$$X^{cap}(t,p,A) = h(t+p,p)A \tag{3.1a}$$

$$X^{fir}(t,p,A) = \frac{h(t+p,p)}{A} \tag{3.1b}$$

where, t is the remaining maturity of the cap, p is the origination period (periods earlier), $t+p$ is the cap's original maturity, $h(m,p)$ is the historical swap rate of p periods earlier for a m period maturity swap, and A is a scaling factor.[3]

2 The Derivatives Week forward rates and implied volatility data are consistent with those implied by Eurodollar futures prices and Eurodollar future options prices.

3 A complete 10 year time series for swap rates could not be found (data were available only from 1988). To complete the time series the missing values were assumed to equal the corresponding Treasury rate plus the last available swap spread.

Second, a cap and floor are assumed to have equal premia at origination,

$$v^{cap}\left(n,t,h(t,p)A^{cap}\right) = v^{flr}\left(n,t,h(t,p)A^{flr}\right), \tag{3.2a}$$

where the option values are evaluated at the term structures prevailing at origination, the strikes are defined as,

$$x^{cap}(t,p,A) = h(t,p)\,A^{cap}, and,\ x^{flr}(t,p,A) = h(t,p)A^{flr}, \tag{3.2b}$$

and A^{cap} and A^{flr} are separate scaling factors for caps and floors.

Third, caps and floors are assumed to have equal deltas at origination,

$$\Delta v^{cap}\left(n,t,h(t,p)A^{cap}\right) = \left|\Delta\ v^{flr}\left(n,t,h(t,p)A^{flr}\right)\right|, \tag{3.3}$$

where Δv^{cap} and Δv^{flr} are the deltas of a cap and floor (evaluated at the term structures prevailing at origination), and the strikes are defined as in (3.2b).

The scaling factors *(A)* are chosen so that the option values at the resulting strike prices equal the observed market values in the Survey. In each of the above specifications, the restrictions are applied to bought and sold options separately, with different scaling factors (*A*) for bought and sold options. In these strike price specifications, a cap will be out-of-the-money when a floor is out-of-the-money. Alternative strike price specifications are presented in Section 5.

Estimated strike prices and option values

Given the strike prices defined in (3), total values for bought and sold customer options, and interdealer options can be defined as functions of the scaling factors (A),

$$V_b\left(A^b\right) = \sum_t\sum_p v^c\left(B^c(t,p),t,x^c\left(t,p,A^b\right)\right) + \sum_t\sum_p v^f\left(B^f(t,p),t,x^f\left(t,p,A^b\right)\right) \tag{4a}$$

$$V_s\left(A^s\right) = \sum_t\sum_p v^c\left(S^c(t,p),t,x^c\left(t,p,A^s\right)\right) + \sum_t\sum_p v^f\left(S^f(t,p),t,x^f\left(t,p,A^s\right)\right) \tag{4b}$$

$$V_D\left(A^D\right) = \sum_t\sum_p v^c\left(D^c(t,p)t,x^c\left(t,p,A^D\right)\right) + \sum_t\sum_p v^f\left(D^f(t,p),t,x^f\left(t,p,A^D\right)\right) \tag{4c}$$

where *B* and *S* are notional amounts for bought and sold customer options, *D* is notional amount of interdealer options; and $v(n,t,x)$ is the value of a cap (floor) with notional amount *n*, maturity *t*, and strike price *x*. The index *t* represents remaining maturity, the index *p* is the origination date, where $t+p < 10$ years, and the superscripts *c* and *f* denote caps and floors.

On the basis of ISDA data we assume that caps amount to 73% of the options with the remainder being floors. The notional amounts of bought and sold options are derived from equations (1) and (2), and assigned to the caps and floors using the 73% ratio from the ISDA data. A small proportion of interest rate options are swaptions (19% at year-end 1994 in the ISDA data). However, for simplicity, we treat all options as either caps or floors.[4]

The value of each group of options in (4) is determined by the scaling factors in the strike rates - the parameter A in the strike price equations (3) and the value equations (4). The estimation is to find values of A^b, A^s, and A^D, such that:

$$V_b\left(A^b\right)+V_D\left(A^D\right)=V_b \tag{5a}$$

$$V_s\left(A^s\right)+V_D\left(A^D\right)=V_s \tag{5b}$$

subject to the restriction in (3.1, 3.2, or 3.3), where v_b (and v_s) is the observed market value of all US dollar options bought (and sold) by dealers including interdealer options.

Given the value of interdealer options (see below), in the case of the strike price structure (3.1), the estimation for bought options consists of solving for the single parameter A^b in equation (5a). In the strike price structure (3.2), however, the estimation for bought options consists of solving for the two parameters A^{cap}, A^{flr} in the two equations (3.2a) and (5a).

Interdealer options

Separate market values of interdealer US dollar options are unavailable. (The interdealer market values is available only in aggregate across all currencies, see Annex Table A1). For that reason, the problem in (5) is solved using four alternative assumptions: (1) inter-dealer options have strikes equal to the reference rate, $A^D=1$, in (3.1), (at-the-money strikes, relative to the swap term structure); (2) inter-dealer options have the same strikes as options bought from customers, $A^D=A^b$; (3) inter-dealer options have the same strikes as options sold to customers, $A^D=A^s$, and; (4) estimate the value of US Dollar interdealer options from the data in Annex Tables A1 and A2. The last estimation method (4) distributes the market value of interdealer options in Annex Table A1 between US dollar and other currencies so as to minimise the error in the ratios of market value to notional amounts relative to the margin ratios of the totals in Annex Tables A1 and A2.

The first and last alternatives produce comparable values for interdealer options. The at-the-money assumption (1) produces a value of interdealer options of $11.3 billion, while the

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4 This assumption is not likely to alter the paper's conclusions. For example, if a one year option on a five year swap were reported as a one year option, then the swaptions would appear as shorter maturity options in the data. Hence, the true exposures of shorter maturity would be less than assumed in the estimation, with the result that hedging demand for shorter maturity instruments would be smaller than estimated. This effect would only strengthen the conclusion that shorter maturity hedging volumes are small relative to transaction volume in Eurodollar futures. On the other hand, however, the swaptions would add to the estimated hedging demand at longer maturities Nevertheless, since swaptions are only 19% of the market, the net increment to estimated hedging demand would not significantly change the conclusions. The effect would be to strengthen the conclusions that longer maturity hedging demand could be significant relative to order flows in longer maturity hedge instruments but not so much larger as to overwhelm the market.

estimation in (4) results in a value of interdealer options of $10.9 billion. Table 7 shows the comparability of the hedge estimates with assumptions (1) and (4). Results using the other assumptions (2 and 3) were also similar to those in (1) and (4). The results reported in Sections 2 and 3 were derived using assumption (4).

An implication of the comparability of methods (1) and (4) is that inter-dealer options have strikes closer to at-the-money than customer options. This result is plausible, since dealers who use the interdealer market to hedge their net short volatility (negative gamma) position would obtain more hedging benefit from at-the-money options since such options have larger gamma.

Options sold to customers

In the strike rate equation (3), the value of the scaling factor that solves the sold options value equation (5b) is $A^s = 1.18$. Thus, for caps sold to customers, strike prices consistent with the observed market values are 18% higher than swap rates of comparable maturity at origination. These estimated strikes for sold options are predominantly deep out-of-the money (relative to swap rates of comparable maturity) at origination.

Options bought from customers

A solution to the bought options value equation (5a) requires a scaling factor in the strike rate equation (3) of, $A^b = 0.94$. For caps bought from customers, strike prices consistent with the observed market values are 6% smaller than swap rates of comparable maturity at origination. Thus, bought options are predominantly in-the-money (relative to swap rates of comparable maturity) at origination. In addition, strike rates for floors in this solution are higher than strike rates for caps. This relationship is the opposite of the relationship found for options sold to customers.

While this result might appear counterintuitive and could point to a problem in the estimation, it is consistent with market commentary in the early 1990s. An implication of this result is that customers looking for "yield-enhancement" during the low-interest rate regime of the early '90s, acquired "higher" yield by selling interest rate caps to dealers that were in-the-money relative to the swap term structure. While this "higher yield" is the market price or compensation for the expected pay out of the option, investors speculating on the path of interest rates would obtain higher investment returns (or losses) by selling in-the-money options. In addition, investors who believed that the forward curve was an overestimate of the future path of spot rates would sell options that were in-the-money relative to the forward curve. In retrospect, for positions that were not leveraged, the risks appear to have been moderate.

Assumptions regarding hedging

The analysis of dealers' hedging behaviour relies on the following assumptions.

(a1) Customers do not hedge their options positions.

Customers who have sold or bought options are assumed not to hedge, because doing so would negate what ever hedging or investment objective the options were used for. Customers who have sold options to dealers presumably did so for speculative "yield enhancement" or intertemporal income shifting. In which case, the costs of delta hedging the options would negate that investment objective. On the other hand, customers who have bought options from dealers for hedging purposes would not hedge the option since doing so would expose the underlying position the option was hedging.

If customers were to hedge their options, perhaps due to a reassessment of risks, then the market impact of dealers hedge adjustments would be smaller because they would be offset by customers' hedging. Since the predominance of our results support the claim that the market impact of dealers' hedging is small relative to the size of the market, dropping assumption (a1) would only strengthen the results.

(a2) Dealers restore the net delta of their position after an interest rate shock to its initial level.

Regardless of whatever hedge ratio they had initially, subsequent to an interest rate shock dealers are assumed to adjust their hedge position to bring the net delta of the portfolio back to its initial level. Dealers may or may not fully hedge the initial delta of the options book, and whatever hedging is initially done may be accomplished either internally with offsetting positions in the firm or with external hedging transactions. These initial offsetting positions, either internal or external, are assumed to have small gamma so that a change in the options' delta requires additional hedging transactions to return the portfolio's net delta to its original level.

(a3) An option exposure to a period t interest rate is hedged with an instrument that also has exposure to the period t interest rate - no basis risk in hedged positions.

With this assumption, a separate hedge ratio was calculated for each maturity's exposure.

Estimated hedge

The delta and the change in delta of the global dealers' portfolio was calculated given the notional amounts (from equation (2)) and estimated strike prices (from equation (5)). The estimated delta is the net hedge position of all dealers' (if they fully hedged) and the change in delta given an assumed interest rate shock is the change in the dealers' net hedge position. In response to an interest rate shock, if dealers are assumed to restore the net delta of their portfolios to their initial levels, then the change in delta of the global portfolio is the net dealer demand for hedge instruments. If hedging is executed with futures contracts, the estimated hedge adjustments are shown in Tables 1 and 2, and the hedge position (assuming complete hedging) is shown in Table 3. Table 4 shows the hedge adjustment and hedge position, if hedging of 5 to 10 year exposures is done with Treasury securities and futures on treasuries. These results are described in Section 3.

5. How robust are the results?

The results shown in Tables 1 to 4 are the results with the basic assumptions described above with the strike price restriction (3.1). To explore whether these results were sensitive to the assumptions, estimates were also performed using a variety of assumptions regarding the structure of strike prices, implied volatility, and other restrictions. The estimated hedge position and its change due to interest rate shocks were comparable across these different specifications and do not alter the conclusions. The results with these alternative assumptions are shows in Tables 5 through 8. The first column in these tables is the result under the basic assumptions, and the other columns are the results with the alternative assumptions.

Strike price variations

Distribution of strike prices

Instead of assuming that all options of a given maturity and origination date had the same strike rate, these options were distributed over two different strike prices, with the larger strike 22% higher than the smaller (10% above and below the reference rate for that option). Instead of equation (3), the strike prices were estimated using the following restrictions,

$$x^{cap}(t,p,A)_{high} = (1+\alpha)\, h\,(t+p,p)\,A \tag{6a.i}$$

$$x^{cap}(t,p,A)_{low} = (1-\alpha)\, h\,(t+p,p)\,A \tag{6a.ii}$$

$$x^{fir}\,(t,p,A)_{high} = (1+\alpha)\,\frac{h(t+p,p)}{A} \tag{6b.i}$$

$$x^{fir}\,(t,p,A)_{low} = (1-\alpha)\,\frac{h(t+p,p)}{A} \tag{6b.ii}$$

where $\alpha = 0.1$, and $h(m,p)$ is the historical swap rate of p periods earlier for a m period maturity swap. (The size of ∀ was chosen from inspection of the range of strike prices over which the bulk of Eurodollar futures options were distributed.)

Maturity variation in strike prices

For options bought from customers, instead of the strike price restriction in equation (3), the options' "moneyness" was assumed to vary with original maturity. In the first variation, the deviation of the strike from the swap reference rate decreased with maturity, and in the second the deviation increased with maturity.

Identical strike prices for caps and floors

Instead of the strike price structure in (3) for bought options, caps and floors were assumed to have identical strikes. This alternative specification produced in-the-money caps and out-of-the money floors. Applying a similar restriction for sold options was not meaningful, as it produced option values that exceeded the observed values. This result supports the use of equation (3) for sold options.

Implied volatility variations

Cap and floor implied volatilities

Instead of using a common implied volatility for both caps and floors, different implied volatilities were used. Caps were estimated using the Derivatives Week implied volatility data as in the basic assumptions, but implied volatilities for floors were adjustment upwards to conform with the difference between cap and floor implied volatility in DRI data. (The DRI implied volatility data are available only from January, 1996; while the Derivatives Week implied volatility data are derived from caps only).

Volatility smile

As an alternative to a common implied volatility across all degrees of "moneyness," results were also estimated using a volatility smile. A volatility smile consistent with Eurodollar futures options prices was constructed, and extrapolated across all maturities using the base volatility term structure as the at-the-money volatility.

Other variations

Options on 3-month interest rates

Instead of assuming that all options were on the 6-month interest rate, results were also derived on the assumption that the options were 3-month interest rate options. This variation doubles the number of individual options in a cap (floor).

Growth rate assumption in maturity distribution

The ISDA market size data for interest rate options contained a number of anomalous growth rates between certain dates. On the possibility that these growth rates were due to survey problems at those dates, alternative smoothed growth rates were derived by ignoring the market volumes at the anomalous dates. The notional amounts from the Central Bank Survey were then distributed across maturities and origination dates using these alternative growth rates in equations (1) and (2).

Unrestricted maturity distribution

As an alternative to the assumption that all caps (floors) have a maturity of at least one year when originated, the distribution of notional amounts across maturities and origination dates in (1) and (2) was estimated without the restriction in the maturity distribution (1).

Simultaneous volatility and interest rate shock

The results in Section 2 were estimated under the assumption that the volatility of interest rates remained constant while interest rates changed. In contrast, the hedge adjustments in Table 8 were estimated assuming simultaneous volatility and interest rate shocks. Interest rate volatility was assumed to increase by 25% relative to initial volatility levels, while the forward curve was assumed to increase by 75 basis points. While the estimated hedge adjustment is larger, the difference does not appreciably change the conclusions.

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Annex: Data

Table A1

Market values of OTC interest rate options

Billions of US dollars

	Bought			Sold		
	USD	**Other**	**Total**	**USD**	**Other**	**Total**
Dealer			22.4			21.6
Customer			15.2			14.6
Total	20.9	16.7	37.6	19.4	16.8	36.2

Table A2

National amounts of OTC interest rate options

Billions of US dollars

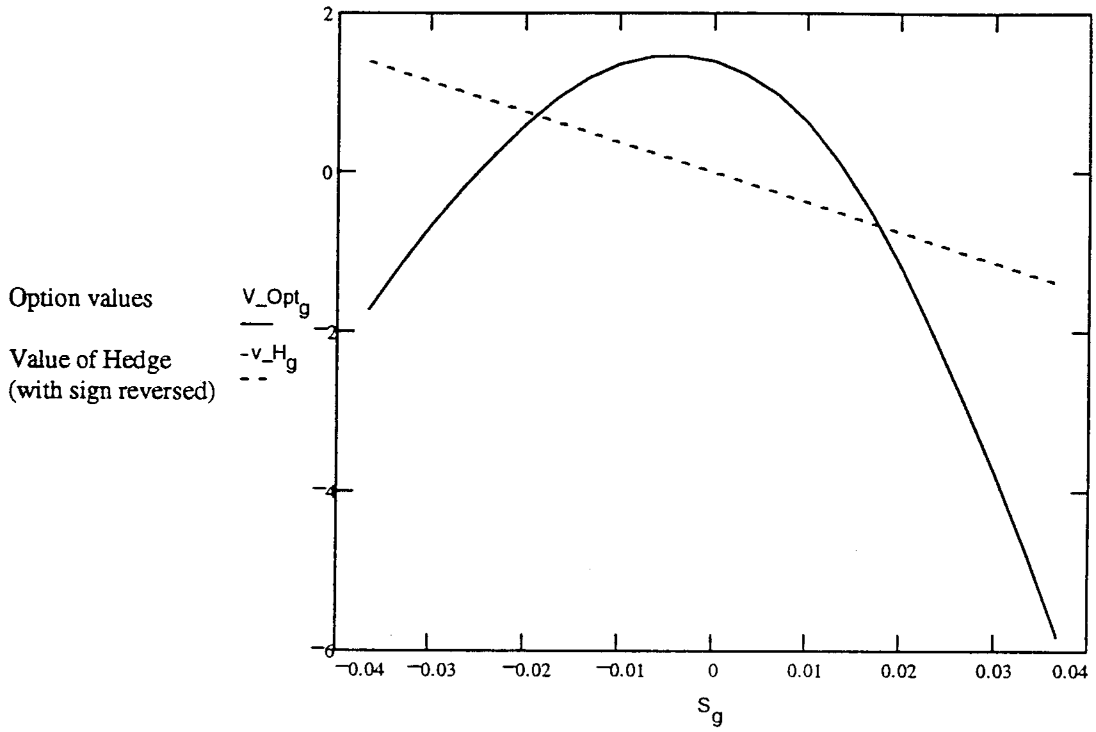
	Bought			Sold		
	USD	**Other**	**Total**	**USD**	**Other**	**Total**
Dealer	529.4	726.5	1255.9	576.1	681.9	1258.1
Customer	432.7	340.6	772.2	690.4	398.1	1088.4
Total	961.1	1067.1	2028.1	1266.5	1080.0	2346.5

Table A3

Maturity distribution of US dollars interest rate options

	Bought options	**Sold options**
Up to one year	30%	28%
Over one and up to five years	58%	56%
Over five years	12%	15%

Figure 1
Net Options and Hedge Values (-)



Notes:

(1) Vertical axis is market value in billions, and horizontal axis is interest rate change in percentage points (0.01 is 100 bp).

(2) The solid curve is the options value, and the dotted line is the mirror image of a hedge portfolio that delta hedges the options at the initial interest rate. The hedged portfolio has positive value when the solid curve (the options value) is above the dotted line (the hedge).

Table 1

Change in required hedge position compared to daily volume of Eurodollar futures 25 BP change in forward curve

Maturity (years)	Change in hedge position	Largest volume		Average volume	
		Volume of 1st contract	Volume of 2nd contract	Volume of 1st contract	Volume of 2nd contract
0.5	- 6.3	**374.0**	**334.1**	**115.73**	**148.36**
1.0	- 9.2	**260.9**	**135.2**	**92.05**	**35.81**
1.5	- 7.7	**55.1**	**39.7**	**19.99**	**14.00**
2.0	- 5.7	**26.9**	**18.9**	**9.40**	**5.96**
2.5	- 4.6	**9.2**	**7.5**	4.02	3.26
3.0	- 3.7	**7.3**	**4.5**	2.69	1.94
3.5	- 3.1	**3.9**	2.6	1.52	1.32
4.0	- 2.6	**2.7**	**3.3**	1.20	1.09
4.5	- 2.1	**2.4**	**2.3**	0.89	0.79
5.0	- 1.9	**2.0**	1.4	0.75	0.46
5.5	- 1.6	1.3	**2.4**	0.20	0.23
6.0	- 1.4	1.3	1.3	0.22	0.20
6.5	- 1.2	1.0	1.2	0.17	0.15
7.0	- 1.0	**3.3**	0.7	0.20	0.12
7.5	- 0.9	0.6	**1.2**	0.07	0.09
8.0	- 0.6	**0.8**	**3.7**	0.07	0.11
8.5	- 0.4	**1.2**	**1.2**	0.11	0.09
9.0	- 0.3	**1.2**	**1.7**	0.08	0.08
9.5	- 0.1	**1.0**	**0.7**	0.07	0.06
10.0		1.2	1.0	0.06	0.04

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) The second column is the change in hedged position by maturity exposure. (3) The middle columns are the largest daily volume of futures contracts (by maturity of contract) in the first half of 1995. (4) The right most columns are the average daily volume (by maturity) in the first half of 1995. (5) The first and second futures contracts in the futures volume columns represent the two back to back contracts on 3-month interest rates required to hedge a six month exposure. (6) Bold indicates contract volume in excess of change in hedge position. (7) Negative values indicate an increase in a short position.

Table 2

Change in required hedge position compared to daily volume of Eurodollar futures 75 BP change in forward curve

Maturity (years)	Change in hedge position	Largest volume		Average volume	
		Volume of 1st contract	Volume of 2nd contract	Volume of 1st contract	Volume of 2nd contract
0.5	- 31.9	**374.0**	**334.1**	**115.73**	**148.36**
1.0	- 31.2	**260.9**	**135.2**	**92.05**	**35.81**
1.5	- 23.7	**55.1**	**39.7**	19.99	14.00
2.0	- 17.2	**26.9**	**18.9**	9.40	5.96
2.5	- 13.6	9.2	7.5	4.02	3.26
3.0	- 11.0	7.3	4.5	2.69	1.94
3.5	- 9.0	3.9	2.6	1.52	1.32
4.0	- 7.6	2.7	3.3	1.20	1.09
4.5	- 6.2	2.4	2.3	0.89	0.79
5.0	- 5.5	2.0	1.4	0.75	0.46
5.5	- 4.7	1.3	2.4	0.20	0.23
6.0	- 4.1	1.3	1.3	0.22	0.20
6.5	- 3.5	1.0	1.2	0.17	0.15
7.0	- 3.0	**3.3**	0.7	0.20	0.12
7.5	- 2.4	0.6	1.2	0.07	0.09
8.0	- 1.9	0.8	**3.7**	0.07	0.11
8.5	- 1.3	1.2	1.2	0.11	0.09
9.0	- 0.7	**1.2**	**1.7**	0.08	0.08
9.5	- 0.3	**1.0**	**0.7**	0.07	0.06
10.0		1.2	1.0	0.06	0.04

See notes to Table 1.

Table 3

Required hedge position in Eurodollar futures contracts compared to contracts outstanding

Maturity (years)	**Hedge position**	**Open interest 1st contract**	**Open interest 2nd contract**	**Change in hedge position (75 BP Chg)**
0.5	38.3	**561.9**	**366.4**	- 31.9
1.0	23.9	**279.7**	**222.0**	- 31.2
1.5	2.8	**174.0**	**145.4**	- 23.7
2.0	- 4.0	**114.2**	**96.3**	- 17.2
2.5	- 9.8	**84.9**	**68.6**	- 13.6
3.0	- 13.4	**60.3**	**54.8**	- 11.0
3.5	- 16.4	**49.5**	**38.8**	- 9.0
4.0	- 17.9	**34.4**	**27.2**	- 7.6
4.5	- 20.2	**22.6**	14.5	- 6.2
5.0	- 18.9	12.9	9.5	- 5.5
5.5	- 18.8	7.5	7.7	- 4.7
6.0	- 18.4	6.2	5.9	- 4.1
6.5	- 17.5	6.7	6.8	- 3.5
7.0	- 15.1	6.8	4.5	- 3.0
7.5	- 12.6	3.8	2.5	- 2.4
8.0	- 9.6	1.6	2.2	- 1.9
8.5	- 6.2	1.8	1.8	- 1.3
9.0	- 3.4	1.7	2.0	- 0.7
9.5	- 1.4	0.8	0.9	- 0.3
10.0		0.8	0.0	

Notes: (1) Billions of USD. Hedge estimates and open interest at end of March 1995. (2) The second column is the hedge position by maturity of exposure. (3) The middle columns are the outstanding volume of futures contracts at end of March 1995. (4) The first and second futures contracts in the futures volume columns represent the two back to back contracts on 3-month interest rates required to hedge a six month exposure. (5) Bold indicates contract volume in excess of hedge position. (6) Negative values indicate a short position or an increase in a short position.

Table 4

Hedge position in bonds using 5 and 10 year securities

Panel A: Treasury securities						
	Hedge position	**Chg hedge (10 BP)**	**Chg hedge (25 BP)**	**Chg hedge (75 BP)**	**On-the-run treasury**	
					Outstanding	**Daily volume**
5 year	13.0	0.4	1.0	2.9	13.2	6.0
10 year	- 13.0	- 0.4	- 1.1	- 3.3	13.8	4.0

Panel B: Treasury futures							
	Hedge position	**Chg hedge (10 BP)**	**Chg hedge (25 BP)**	**Chg hedge (75 BP)**	**Treasury futures**		
					Open interest	**Large daily volume**	**Av. daily volume**
5 year	13.0	0.4	1.0	2.9	19.7	12.3	5.1
10 year	- 13.0	- 0.4	- 1.1	- 3.3	25.8	24.4	9.2

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) Treasuries outstanding at end of March 1995; daily volume is from GovPx only (Fleming, 1997). (3) Treasury futures are the 5 and 10 year note contracts. Open interest as of end of March 1995, and volume is over first half of 1995. (4) Negative values indicate a short position or an increase in a short position.

Table 5

Strike price variations:
change in required hedge position due to 75 BP change in forward curve

Maturity (years)	**Base**	**Equal premia**	**Equal delta**	**Strike distr.**	**Maturity vrtn. 1**	**Maturity vrtn. 2**	**Identical caps/floors**
Change in futures hedge							
0.5	- 31.9	- 38.3	- 33.3	- 34.9	- 38.5	- 24.8	- 55.8
1.0	- 31.2	- 32.9	- 30.5	- 27.1	- 33.3	- 29.6	- 42.2
1.5	- 23.7	- 24.2	- 22.8	- 21.3	- 24.3	- 23.4	- 29.3
2.0	- 17.2	- 17.3	- 16.6	- 15.9	- 17.4	- 17.2	- 20.1
2.5	- 13.6	- 13.6	- 13.2	- 12.7	- 13.6	- 13.7	- 15.4
3.0	- 11.0	- 10.9	- 10.7	- 10.4	- 10.9	- 11.1	- 12.1
3.5	- 9.0	- 8.9	- 8.8	- 8.6	- 8.9	- 9.2	- 9.8
4.0	- 7.6	- 7.5	- 7.4	- 7.2	- 7.5	- 7.7	- 8.1
4.5	- 6.2	- 6.2	- 6.1	- 6.0	- 6.2	- 6.4	- 6.6
Change in bond hedge							
5 year	2.9	2.9	2.9	2.8	2.9	3.0	3.1
10 year	- 3.3	- 3.2	- 3.2	- 3.2	- 3.2	- 3.3	- 3.4

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) Column headings indicate the assumption as described in the text. (3) Negative values indicate an increase in a short position.

Table 6

Volatility variations:
change in required hedge position due to 75 BP change in forward curve

Maturity (years)	Base	Cap/floor volatility	Volatility smile	Cap/floor and smile
Change in futures hedge				
0.5	- 31.9	- 31.5	- 27.2	- 26.8
1.0	- 31.2	- 31.2	- 27.8	- 27.7
1.5	- 23.7	- 23.7	- 21.1	- 21.0
2.0	- 17.2	- 17.2	- 14.6	- 14.5
2.5	- 13.6	- 13.6	- 11.4	- 11.3
3.0	- 11.0	- 10.9	- 9.1	- 9.0
3.5	- 9.0	- 9.0	- 7.4	- 7.4
4.0	- 7.6	- 7.5	- 6.2	- 6.2
4.5	- 6.2	- 6.2	- 5.3	- 5.3
Change in bond hedge				
5 year	2.9	2.9	2.6	2.6
10 year	- 3.3	- 3.3	- 2.9	- 2.9

See notes to Table 5.

Table 7

Other variations:
change in required hedge position due to 75 BP change in forward curve

Maturity (years)	Base	Dir. option at-the-m	Options on 3-month rate	Growth rate	Unrestr. mtry dstr.
Change in futures hedge					
0.5	- 31.9	- 25.2	- 32.5	- 38.6	- 35.9
1.0	- 31.2	- 28.6	- 30.4	- 32.2	- 30.9
1.5	- 23.7	- 22.6	- 23.4	- 25.5	- 24.4
2.0	- 17.2	- 16.6	- 17.0	- 19.2	- 18.1
2.5	- 13.6	- 13.2	- 13.5	- 15.4	- 14.5
3.0	- 11.0	- 10.7	- 10.9	- 12.5	- 11.7
3.5	- 9.0	- 8.9	- 9.0	- 10.2	- 9.6
4.0	- 7.6	- 7.5	- 7.5	- 8.3	- 7.9
4.5	- 6.2	- 6.2	- 6.2	- 6.6	- 6.4
Change in bond hedge					
5 year	2.9	2.9	2.9	2.4	2.8
10 year	- 3.3	- 3.3	- 3.3	- 2.7	- 3.1

See notes to Table 5.

Table 8

Change in required hedge position due to simultaneous volatility and forward rate shocks

Maturity (years)	I.R. shock only	Volt. shock only	I.R. and volt. shock
Change in futures hedge			
0.5	- 31.9	- 6.0	- 40.7
1.0	- 31.2	- 9.7	- 38.7
1.5	- 23.7	- 8.7	- 29.4
2.0	- 17.2	- 7.7	- 22.6
2.5	- 13.6	- 6.2	- 17.9
3.0	- 11.0	- 4.9	- 14.4
3.5	- 9.0	- 3.9	- 11.6
4.0	- 7.6	- 3.0	- 9.5
4.5	- 6.2	- 2.2	- 7.5
Change in bond hedge			
5 year	2.9	0.8	3.4
10 year	- 3.3	- 0.8	- 3.8

Notes: (1) Billions of USD. Hedge estimates based on data at end of March 1995. (2) Forward rates increase by 75 basis points, and volatility increases by 25% relative to initial volatility levels at short maturities, and by 8% at 10 years. (3) Negative values indicate an increase in a short position.