Matrix modeling of clearing and settlement operations in payment systems.

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Abstract

The article is devoted to modeling of settlement relations and their description in the form of matrix mathematical objects, graphic and tabular images. The matrix way of expressing settlement methods is aimed at formalizing clearing and settlement operations carried out in payment systems. Payment instructions and settlement methods are presented as a system of vector-matrix formulas which can be useful for the uniform understanding of functional characteristics in clearing and settlement operations, as well as for conducting research on the more effective use of payment systems participants' liquidity and, finally, for forecasting of the results of payment-settlement interaction between economic subjects.

Introduction

The purpose of the given article is defining formalized descriptions reflecting payment orders and settlement methods, as well as presenting clearing and settlement operations on the basis of vector-matrix formulas, graphic and tabular images.

Reliability and effectiveness of financial interaction between all subjects of an economic system largely depends on the effectiveness of all payment, clearing and settlement operations carried out within it. Payment systems unite participants of production-financial interaction. Implementing of payment, clearing and settlement operations connects accounting and production-financial activity of economic subjects to form a single economic system as the methods of transferring settlement assets from one economic subject to another and the rules of reflecting of these operations in accounting reports are interrelated phenomena. Consequently, it appears to be important to present these payment, clearing and settlement operations in a formalized way since only such a presentation can provide a compact expression of the relations between real, financial and state sectors of economy for the subsequent analysis and forecast.

Matrix modeling of settlement relations is an objective and universal method of researching characteristics of payment systems; this method allows to formulate assumptions in the area of payments and settlements in a compact and uniform fashion. These formulations, in their turn, can influence the existing practice of payment-settlement operations. Payment instructions, matrix models and clearing operations represented as matrix models acquire more specific and clear images which allow to analyze advantages and disadvantages of the functional characteristics in gross and net settlement methods.

To develop this theme, the article is logically structured in the following way: first it gives the review of the international terminology used in payment and settlement operations. Then vector-matrix formulas and transformations expressing clearing and settlement operations, are considered. After that, the formulas and transformations are illustrated using the example of solving the problem on modeling clearing and settlement operations, as well as graphic and tabular presentation of payment operations between settlements participants. The main conclusions of the article are formulated using the results of the problem's solution.

Terms and definitions

The internationally recognized terminology provides definitions of all basic terms used in implementing payment-settlement relations[1].

The definition of a payment system implies a certain number of payment instruments, banking operations and interbank funds transfer systems which enable effective money circulation.

The concepts of a payment instrument, payment order, payment and transfer are very important elements of payment and settlement operations which are sometimes used as synonyms in describing payment operations.

A payment instrument is any instrument which enables a bearer or user to transfer money.

[1] Committee on Payment and Settlement Systems. A glossary of terms used in payments and settlement systems, BIS, March 2003.

A payment order is an order or a message containing a requirement to transfer funds. This order can refer either to credit or debit transfer.

Payment is a transfer of funds by a payer to a party which is acceptable for a payee. As a rule, payment requirements take the form of banknotes or deposit balances placed in a financial establishment or in a central bank. Payment can be interpreted as a process during which transfer of funds or deposit requirements is fulfilled between two parties to complete the operation.

A payment instrument initiates a payment instruction which can be regarded as an order on payment and can apply either to credit or debit transfer.

Credit transfer is an electronic or paper order confirmed by a payer which instructs the settlement institution keeping the payer's account to transfer funds from the payer's account to the account of a named beneficiary in the same or different bank. A payment instrument (payment order), or, alternatively, a series of payment orders are transferred with an aim of providing a named beneficiary with funds. Both the payment orders and the funds indicated in them are replaced from a payer\initiator bank to a beneficiary bank, possibly, via a number of other intermediary banks.

Debit transfer is an electronic or paper order issued or confirmed by a payer which is sent from a beneficiary bank to a payer bank and which leads to debiting the latter's account.

Depending on the type of payment instrument used to initiate payment between a payer and a payment receiver, all payments can be divided into documentary and non-documentary ones. However, it should be noted that, regardless of the form of a payment instrument (documentary or non-documentary) and of the payment order (debit or credit) this instrument initiates in a payment operation, the flow of money is directed from a payer to a payee.

A settlement is a phase in a payment operation which stops obligations on transferring funds or securities between two or more parties. Settlements mean the end of a transaction or transfer processing aimed at fulfilling obligations by all parties involved, by means of transferring settlement assets (e.g. securities or funds).

A settlement asset is such an asset which is used for finishing settlement obligations according to the rules, instructions and common practice of a payment system.

A settlement system is a system which allows to fulfill transfers of funds or financial instruments.

According to the principles of performing transfer operations between the participants of payment systems one should distinguish between gross settlement and net settlement systems. Hybrid settlement systems are an example of the synthesis of both settlement systems – gross settlement and net settlement ones.

Gross settlement assumes that an individual operation is made through corresponding funds transfer pursuant to every payment order. Payments are made consecutively on an on-going basis, or in accordance with an established order of priority.

Gross settlement systems differ in the speed and order of fulfilling settlements. Gross settlements can be performed on the real-time (with continuous processing of payments during the day) or batch basis (with payments processing during a fixed period of time agreed on between the participants in advance). It determines the division of settlement systems into real time (in which payments are continuously processed) and batch settlements (in which payments are processed periodically).

A real time transfer is a transfer, processing and settlement according to the instruction on transfers of funds and securities at the moment of its initiation.

A batch processing is a transfer, or processing of grouped payment orders and(or) instructions on a batch transfer of securities at discrete time intervals.

A real-time gross settlement is a continuous (real-time) settlement of incoming money or securities transfers on an individual basis (without netting).

A gross settlement system is a transfer system in which settlements of money or securities transfers are made individually, on every incoming payment order.

A net settlement system is a system in which a final interbank settlement on individual transfer orders is made on a net basis, at one or more discrete time interval(s) agreed on in advance, during a processing day.

Netting is an agreed settlement of trade obligations or positions between trade partners or settlement participants. Netting reduces a big number of individual payments to a smaller number of requirements and obligations (debit and credit) of net positions.

A net credit (or debit) position of a participant of a netting system can be presented as a sum of costs of all transfers received by them by a certain moment of time, after deduction of the costs of all transfers sent by them. If the difference is positive, the participant has a credit net position, if it is negative — a debit net position. A credit or debit net position during a settlement period is called a settlement net position. They can be calculated on a bilateral or multilateral basis.

The systems of net settlements differ in the method of calculating net position requirements and obligations; according to it, they are divided into bilateral and multilateral netting.

Bilateral netting is an agreement between two parties to carry out netting of their bilateral obligations. The obligations covered by the agreement can refer to financial agreements, transfers or to both.

A bilateral net settlement system is a settlement system in which the settlement on bilateral settlement net positions of participants is fulfilled between each pair of participants.

A multilateral net settlement position is a sum of costs of all transfers which a participant of a net settlement operation received during a certain period of time, after deduction of the costs of transfers sent by that participant to other participants. If the sum is positive, the participant has a multilateral credit net position; if the sum is negative, the participant is in the multilateral debit net position.

Multilateral netting is an agreement between two or more parties to do netting on their obligations. The obligations covered by the agreement can refer to financial agreements, transfers or to both. Multilateral netting of payment liabilities is fulfilled, as a rule, within the framework of a multilateral netting system.

A multilateral net settlement system is a system in which every settlement operation participant is carrying out a settlement (as a rule, through making or receiving a payment) on a multilateral net position which is initiated by transfers sent and received by participants, at their own expense and on behalf of their clients or participants not taking part in the settlement operation.

A hybrid system is a system which combines a quick payment of gross settlement systems with a more efficient use of liquidity typical of netting systems. The main feature of these systems is frequent settlements of payments during an operation day with an immediate settlement accomplishment.

Clearing/clearance is a process of transfer, check-up and in some cases confirmation of payment orders or instructions on the transfer of securities before the completion of the settlement, possibly, by means of including instructions netting and setting the final settlement positions. Sometimes the term is used (not quite properly) to imply settlement.

A clearing system is a set of operations through which financial institutions exchange data and (or) documents referring to the transfer of funds or securities to other financial institutions in one place (clearing organization). These operations often include the mechanism of calculation of bilateral and (or) multilateral net positions of participants with the aim of simplification of settlements on their obligations.

One should distinguish between the clearing of obligations on agreements in financial markets, the subjects of which can be goods, foreign exchange, securities and other financial instruments, and the clearing of payment orders in non-cash settlements on transactions.

Summarizing the review of the terminology in the area of payment and settlement systems, one should pay attention to the fact that textual definitions of elements and components of payment and settlement systems are not always unambiguous in defining important characteristics of the considered objects, therefore, the article further presents formalized definitions of payment orders, clearing and settlement operations made on the basis of matrix modeling.

Matrix modeling of payment orders and settlement methods

The most common forms of presenting models are graphs and tables which often complement each other. However, this way of expressing clearing and settlements does not imply formalized definitions and transformations of settlement methods for further research of payment-settlement relations.

Mathematic data presented in tables correspond to structures named matrices, which, according to the classical definition, are rectangular tables of numbers or other objects including a certain number of lines m and columns n. Numbers m and n are called matrix orders and they determine its size. A matrix is considered square if $m=n$, that is, the number of lines is equal to the number of columns. To label matrices, we usually use capital letters, and the objects which are placed at the intersections of lines and columns, are called their elements, the number of an element's line being identified as (I= 1, 2, 3, …, i), the number of a column - (J= 1, 2, 3, …, j).

The research papers by O. Kolvakh[2] present the system of matrix modeling and analysis of financial relations which has a minimal number of original axiomatic assumptions which could explain the reflection of factual objects (original data) as mathematic symbols. By means of formulating two axioms these research papers determine the correspondence between original data and mathematic symbols reflecting these data. A correspondence matrix reflects the object of relations, and a transaction matrix reflects a quantitative indicator of this relation. The subjects of these relations are found at the intersections of lines and columns of correspondence matrices. As a model-forming matrix we take a matrix sized as equal to a number of subjects and consisting of numeric values of transaction matrices. All other properties of financial relations are derived by means of matrix transformations. The axioms and the model developed by O.Kolvakh also allow to formulate settlement methods in payment systems, besides being used in accounting, for which this model was originally intended.

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[2] Kolvakh O. Computer accounting for everybody. Rostov-on-Don. Publishing house "Phoenix",1996.

The matrix form of expression of payment orders allows to research problems of relations in payment systems as a result of the analysis of solving matrix equations. Such a matrix presentation of payment-settlement relations between settlement participants which connects with each other not just separate numbers, but various number structures organized as table analogues such as matrices, vectors (individual lines and columns) and individual numbers — scalars — allows to express the main calculation methods in a compact and, simultaneously, systematic way.

Any type of payment order can be found a correspondence in its matrix equivalent – a transaction matrix (payment). Various types of instructions on funds transfers can be regarded as factual analogues of settlement operations which are made using payment instruments.

A square matrix sized *m*=*n* which accommodates a unity at the crossing point located on the line corresponding to settlement participant X and the column corresponding to settlement participant Y, while all its other elements are equal to zero, is called a correspondence matrix.

A correspondence matrix is indicated as E(X,Y), and its non-zero element is always equal to 1, E(X,Y)=1. In accordance with the definition all other elements E(I,J)=0 for all cases of I not equal to X, and J not equal to Y.

A transaction matrix (payment instruction (relation) — R) can be presented as a product of the sum of a transaction multiplied by a correspondence matrix:

$$\mathbf{R\ (X,\ Y) = \lambda_{X,Y} \cdot E(X,Y)} \qquad \textbf{(1)}$$

The subjects of these relations (settlement participants) are determined by intersections of lines and columns of correspondence matrices. The horizontal elements express obligations (obl) between participants, vertical elements present their requirements (req).

For instance, for the sum of a settlement operation $\lambda_{A,B} = 80$ units of settlement assets and the correspondence between participants ***E****(A, B)* is as follows:“Settlement assets are transferred from settlement participant A to settlement participant B”. As a result, we obtain the following transaction matrix:

$$
\mathbf{R}(\mathbf{A},\mathbf{B}) = 80 \times \left[\begin{array}{c|ccccc}
\textbf{obl/req} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\
\hline
\textbf{A} & 0 & 1 & 0 & 0 & 0 \\
\textbf{B} & 0 & 0 & 0 & 0 & 0 \\
\textbf{C} & 0 & 0 & 0 & 0 & 0 \\
\textbf{D} & 0 & 0 & 0 & 0 & 0 \\
\textbf{E} & 0 & 0 & 0 & 0 & 0
\end{array}\right] = \left[\begin{array}{c|ccccc}
\textbf{obl/req} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\
\hline
\textbf{A} & 0 & 80 & 0 & 0 & 0 \\
\textbf{B} & 0 & 0 & 0 & 0 & 0 \\
\textbf{C} & 0 & 0 & 0 & 0 & 0 \\
\textbf{D} & 0 & 0 & 0 & 0 & 0 \\
\textbf{E} & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

When multiplying scalar λ by matrix A all its elements increase by λ. All matrix elements except for ***E****(A, B) =1*, are equal to zero. Consequently, a scalar value – a sum of transaction $\lambda_{A,B} = 80$ gets the value due to a corresponding matrix position at the intersection of line A and column B, that is, ***R****(A, B) = 80* while all other elements of a transaction matrix will be equal to zero[3].

Henceforth, it is not obligatory to perform matrix transformations which is time-and labour-consuming and, moreover, occupies a lot of place, that's why in recording examples we will use symbolic equivalents of payments, and the final results will be presented as a model-forming matrix.

A square matrix having the size equal to a number of participants, which consistently accumulates matrix equivalents of payments between payment participants, is taken as a module-forming (basic) matrix.

The matrix formula (2) expresses the method of gross settlements in a real-time mode (Real-time Gross Settlement — RTGS); in it the sums of operations determined in corresponding correspondence matrices between settlement participants are presented in the chronological order.

$$
\mathbf{RTGS} = \sum_{i=1}^{n} \lambda_i \cdot \mathbf{E}_i(X_i, Y_i), \qquad \mathbf{(2)}
$$

Matrix formula (3) presents settlements for a certain period of processing or the method of batch gross settlements (BSG): its sums ($S_{X,Y}$) are total sums consisting of separate transactions defined on similar correspondences between the participants.

[3] In formulas and their numeric values which express practical implementation of settlement methods formulas, the following system if indications is accepted: symbols «X» and «Y» may take any values with a multitude of settlement participants, symbols other than «X» and «Y» , such as «A», «B», «C», «D», «E» are used to identify a specific settlement participant.

$$\mathbf{BGS} = \sum_{X,Y} S_{X,Y} \cdot \mathbf{E}\ \ (X,Y), \qquad \textbf{(3)}$$

where the coefficients of a linear transformation will be presented by the sums of summary settlement operations $S_{X,Y}$ (X, Y denote a multitude of settlement participants).

Reduction of similar transaction-matrices for a certain processing period can be made using the following formula:

$S_{X,Y} = \sum_{i_{XY}=1}^{n_{X,Y}} \lambda_{i_{XY}}$, where $\lambda_{i_{XY}}$ is a sum of an individual transaction between participants X and Y, and $n_{X,Y}$ is a number of transactions between participants X and Y during the period of settlement processing . If there were no settlements between any of participants during this period, then $S_{X,Y} = 0$.

Matrix formula (4) is a formula which reflects the settlement method based on bilateral netting (BN).

$$\mathbf{BN = BGS - BGS'}. \qquad \textbf{(4)}$$

Vector formula (5) is a formula of multilateral netting (mn): it is obtained by means of the consecutive transformations of formulas (2), (3) and (4).

$$\mathbf{mn = BN \cdot e} \qquad \textbf{(5)}$$

Matrix transformations which correspond to transitions from one settlement system (method) to another, are defined as follows:

1) for the transition from the real time gross settlement system to the system of the batch settlement , it’s necessary to reduce similar transaction matrices during the processing period; as a result of this reduction payment flows between corresponding settlement participants consisting of individual payment orders, will be formed.

2) for the transition from the gross settlement system with batch processing of payments to the system of bilateral netting, it is necessary to subtract from the matrix of obligations between the participants the matrix of requirements transposed to it;

the result of this transformation will be the matrix in which each vector-line consists of bilateral net positions of certain settlement participants.

3) for the transition from the system of bilateral netting to the system of multilateral netting the matrix of bilateral netting must be multiplied by a unit vector, the result of this multiplication being a vector-column consisting of multilateral net positions of each settlement participant.

Hybrid systems are a special case of net settlement systems which are modeled by means of successive transformations of the gross settlement system which can end in either bilateral or multilateral netting.

The notional expression of the relations between settlement participants depending on the methods of settlements used can be formulated as follows: in carrying out real time gross settlements an operation is performed on every payment order between two settlement participants denoted in this order. To carry out batch gross settlements, a summary payment between two settlement participants is formed; it consists of all payment orders which are accumulated during a certain period of time and then processed in batches. A bilateral net settlement is carried out by means of transferring funds on the basis of a net position which is defined as a difference of all requirements and obligations between two settlement participants, which are accumulated during a certain period (a clearing cycle). In a multilateral net settlement calculating a net position based on the results of a clearing cycle is fulfilled on the basis of determining the difference between obligations and requirements of one settlement participant and all other participants, that is, bilateral relations of one settlement participant with another is replaced by the relations between one participant, on the one side, and all other participants, on the other side.

Illustration of matrix modeling of clearing and settlement operations

For the purpose of illustrating matrix modeling of clearing and settlement operations a practical numeric example is given: it's an example of solving the task on modeling payment orders and settlement methods between the settlement participants, then gross and net settlement methods are presented in graphic and tabular forms.

According to the problem statement, for the period of time t1–t2 using the data of payment orders on the basis of which the settlements between 5 participants (denoted as banks *A, B, C, D, E*) are being fulfilled, it is necessary to build the following models:

— of real time gross settlements (RTGS);

— of batch gross settlement (BGS);

— of bilateral netting (BN);

— of multilateral netting (MN).

As initial data we can use numeric values of the real time gross settlement formula on the following payment instructions:

$$\boldsymbol{RTGS}_{t2\text{-}t1} = 40\cdot E(A,B) + 80\cdot E(A,C) + 50\cdot E(A,D) + 30\cdot E(A,E) + 70\cdot E(B,A) + 50\cdot E(B,C) + 40\cdot E(B,D) + 100\cdot E(B,E) + 110\cdot E(C,A) + 40\cdot E(C,B) + 90\cdot E(C,D) + 60\cdot E(C,E) + 100\cdot E(D,A) + 120\cdot E(A,B) + 70\cdot E(D,C) + 140\cdot E(D,E) + 130\cdot E(E,A) + 20\cdot E(E,B) + 170\cdot E(E,C) + 30\cdot E(E,D) + 90\cdot E(A,B) + 190\cdot E(D,C) + 80\cdot E(B,D),$$

where the sums indicated in the payment instructions are multiplied by proper correspondence matrices and recorded in chronological order during the processing period (t1–t2).

It should be noted that during the processing period settlement participant A makes three funds transfers to participant *B*, and participants *D* and *B* transfer settlement assets to, correspondingly, participants *C* and *D* twice, while participant *D* does not make any transfers to participant *B*. Consequently, the numeric value of batch gross settlement formula, after reducing similar transaction matrices, will look as follows:

$$\mathbf{BGS}_{t2\text{-}t1} = 250 \cdot E(A,B) + 80 \cdot E(A,C) + 50 \cdot E(A,D) + 30 \cdot E(A,E) + 70 \cdot E(B,A) + 50 \cdot E(B,C) + 120 \cdot E(B,D) + 100 \cdot E(B,E) + 110 \cdot E(C,A) + 40 \cdot E(C,B) + 90 \cdot E(C,D) + 60 \cdot E(C,E) + 100 \cdot E(D,A) + 0 \cdot E(D,B) + 260 \cdot E(D,C) + 140 \cdot E(D,E) + 130 \cdot E(E,A) + 20 \cdot E(E,B) + 170 \cdot E(E,C) + 30 \cdot E(E,D).$$

That is, the transition from the real time gross settlement systems to the batch gross settlement systems is realized through the "reduction of similar" transaction matrices for the period of processing.

To illustrate further transformations of settlement methods, let's write down a numeric value of the symbolic image of the batch gross settlement system in the form of a matrix:

$\mathbf{BGS}_{t2\text{-}t1}$ =

obl/req	A	B	C	D	E
A	0	250	80	50	30
B	70	0	50	120	100
C	110	40	0	90	60
D	100	0	260	0	140
E	130	20	170	30	0

Let's assume that **BGS** is a matrix of obligations between settlement participants, and $\mathbf{BGS}' = (\mathbf{BGS})'$ is a matrix of participants' requirements and payments received transposed to it, that is, a matrix in which lines and columns are replaced – inverted with respect to the original matrix **BGS**. To obtain the difference between obligations (obl) and requirements (req) one should subtract the matrix of requirements transposed to the matrix of obligations, from the matix of obligations. The result of this operation will be the bilateral netting matrix representing the difference between requirements and obligations which simultaneously shows the flow of funds transfer form a payer to a payee for performing settlement operations.

$$
\boldsymbol{BN}_{t2\text{-}t1} = \left[\begin{array}{c|ccccc}
\textbf{obl/req} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\
\hline
\textbf{A} & 0 & 250 & 80 & 50 & 30 \\
\textbf{B} & 70 & 0 & 50 & 120 & 100 \\
\textbf{C} & 110 & 40 & 0 & 90 & 60 \\
\textbf{D} & 100 & 0 & 260 & 0 & 140 \\
\textbf{E} & 130 & 20 & 170 & 30 & 0
\end{array}\right] -
$$

$$
- \left[\begin{array}{c|ccccc}
\textbf{req/obl} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\
\hline
\textbf{A} & 0 & 70 & 110 & 100 & 130 \\
\textbf{B} & 250 & 0 & 40 & 0 & 20 \\
\textbf{C} & 80 & 50 & 0 & 260 & 170 \\
\textbf{D} & 50 & 120 & 90 & 0 & 30 \\
\textbf{E} & 30 & 100 & 60 & 140 & 0
\end{array}\right] =
$$

$$
= \left[\begin{array}{c|ccccc}
\textbf{b-netting} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\
\hline
\textbf{A} & 0 & 180 & -30 & -50 & -100 \\
\textbf{B} & -180 & 0 & 10 & 120 & 80 \\
\textbf{C} & 30 & -10 & 0 & -170 & -110 \\
\textbf{D} & 50 & -120 & 170 & 0 & 110 \\
\textbf{E} & 100 & -80 & 110 & -110 & 0
\end{array}\right].
$$

Pay attention to the net position symbols expressed by the elements constituting a vector-line: positive and negative. As is well known, the symbols «–» (minus) and «+» (plus) may mean either quantity or action (manipulation). In this case, while interpreting the symbols one should take them as symbols of actions: «+» — transfer of funds, «–» — receiving of funds.

The reduction of the matrix of obligations, requirements and bilateral netting into a final column (vector[4]) is obtained by means of multiplication on the right by unit vector **e** (the column which consists of ones). The transformation $\mathbf{r}_{obl} = \mathbf{BGS}\cdot\mathbf{e}$ reduces **BGS** into the total vector of obligations, and the transformation $\mathbf{r}_{req} = \mathbf{BGS}'\cdot\mathbf{e}$

[4] Vectors, unlike matrices, are usually denoted with small letters.

— into the total vector of requirements. According to the data of our sum, numeric values of these transformations will be written as follows:

$$\boldsymbol{r}_{t2\text{-}t1obl} = \left[\begin{array}{c|ccccc} \textbf{obl/req} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\ \hline \textbf{A} & 0 & 250 & 80 & 50 & 30 \\ \textbf{B} & 70 & 0 & 50 & 120 & 100 \\ \textbf{C} & 110 & 40 & 0 & 90 & 60 \\ \textbf{D} & 100 & 0 & 260 & 0 & 140 \\ \textbf{E} & 130 & 20 & 170 & 30 & 0 \end{array}\right] \times \begin{bmatrix} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \textbf{obl} \\ 410 \\ 340 \\ 300 \\ 500 \\ 350 \end{bmatrix}.$$

$$\boldsymbol{r}_{t2\text{-}t1req} = \left[\begin{array}{c|ccccc} \textbf{req/obl} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\ \hline \textbf{A} & 0 & 70 & 110 & 100 & 130 \\ \textbf{B} & 250 & 0 & 40 & 0 & 20 \\ \textbf{C} & 80 & 50 & 0 & 260 & 170 \\ \textbf{D} & 50 & 120 & 90 & 0 & 30 \\ \textbf{E} & 30 & 100 & 60 & 140 & 0 \end{array}\right] \times \begin{bmatrix} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \textbf{req} \\ 410 \\ 310 \\ 560 \\ 290 \\ 330 \end{bmatrix}.$$

The numeric value of the net positions vector, after multiplication of a bilateral settlement matrix by a unit vector, can be expressed as follows:

$$\boldsymbol{mn}_{t2\text{-}t1} = \left[\begin{array}{c|ccccc} \textbf{b-net} & \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \\ \hline \textbf{A} & 0 & 180 & -30 & -50 & -100 \\ \textbf{B} & -180 & 0 & 10 & 120 & 80 \\ \textbf{C} & 30 & -10 & 0 & -170 & -110 \\ \textbf{D} & 50 & -120 & 170 & 0 & 110 \\ \textbf{E} & 100 & -80 & 110 & -110 & 0 \end{array}\right] \times \begin{bmatrix} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \textbf{m-net} \\ 0 \\ 30 \\ -260 \\ 210 \\ 20 \end{bmatrix}.$$

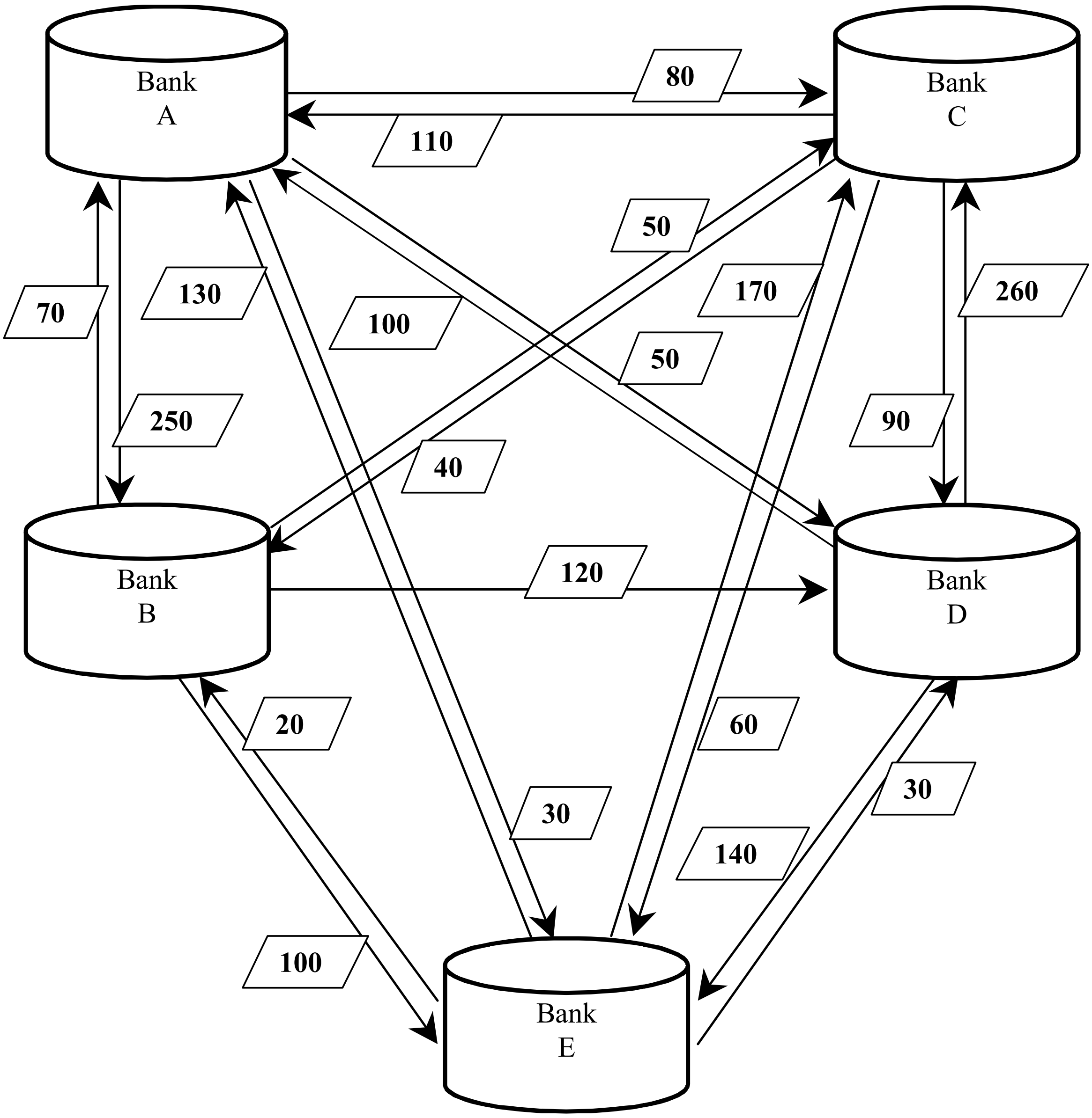
Hence, it follows that for the transition from the system of bilateral netting to the system of multilateral netting it is necessary to multiply a matrix of bilateral netting by a unit vector with the result in the form of multilateral net positions of every settlement participant. The symbols of the elements of a multilateral net position denote the same actions as in bilateral netting: «+» — transfer of funds, «–» — receiving of funds.

As is well known, a matrix and a vector containing positive and negative elements can be expanded into two objects, one of which will be negative, and the other — positive. To illustrate this provision, we will perform this transformation, then the vector of multilateral netting will be presented as follows:

$$\mathbf{mn}_{t2-t1} = \begin{bmatrix} \mathbf{par} \\ \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{m\text{-}net} \\ \mathbf{0} \\ \mathbf{30} \\ \mathbf{-260} \\ 210 \\ 20 \end{bmatrix} = \begin{bmatrix} \mathbf{obl} \\ \mathbf{0} \\ \mathbf{30} \\ 0 \\ 21\mathbf{0} \\ \mathbf{20} \end{bmatrix} - \begin{bmatrix} \mathbf{req} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{260} \\ 0 \\ 0 \end{bmatrix}.$$

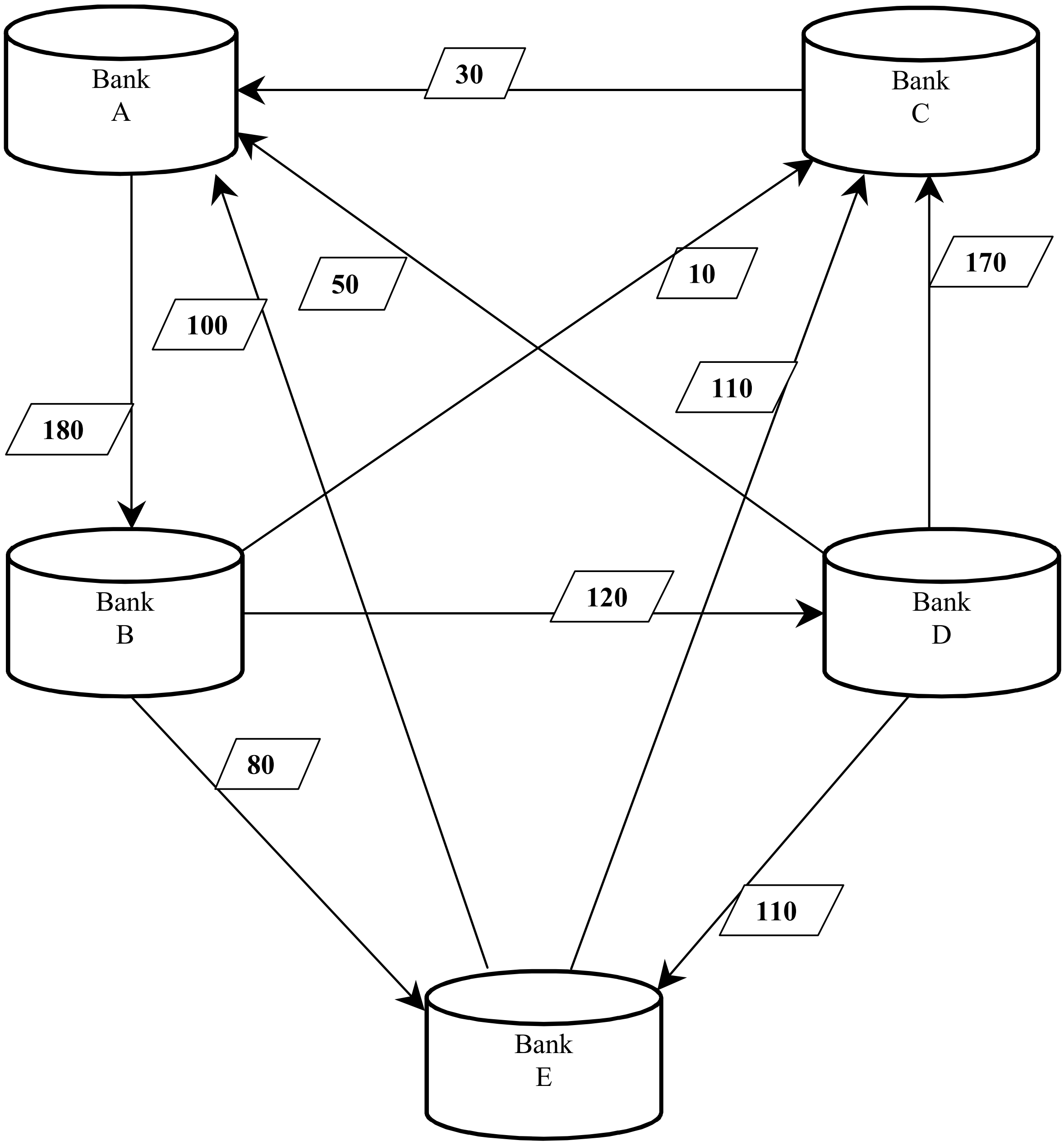
The review of the given example shows that, for carrying out gross settlements much more funds are required than for net settlement systems. The data of our sum shows that participant *A*, for instance, needs the amount of liquid funds equal to 410 units in gross settlement systems, and in multilateral netting systems they have a zero net position. In carrying out settlements on the basis of bilateral netting between participants *A* and *B*, participant *A* needs only 180 units of settlement assets instead of 250, and participant *B* does not spend any assets on bilateral settlements. Besides, the assets required for the settlement between all participants, when comparing the systems of gross settlement and bilateral and multilateral netting, reduce, correspondingly, from 1900 to 960 and 260 units of settlement assets.

The methods of settlement are presented in a graphic way in pictures 1, 2 and 3.



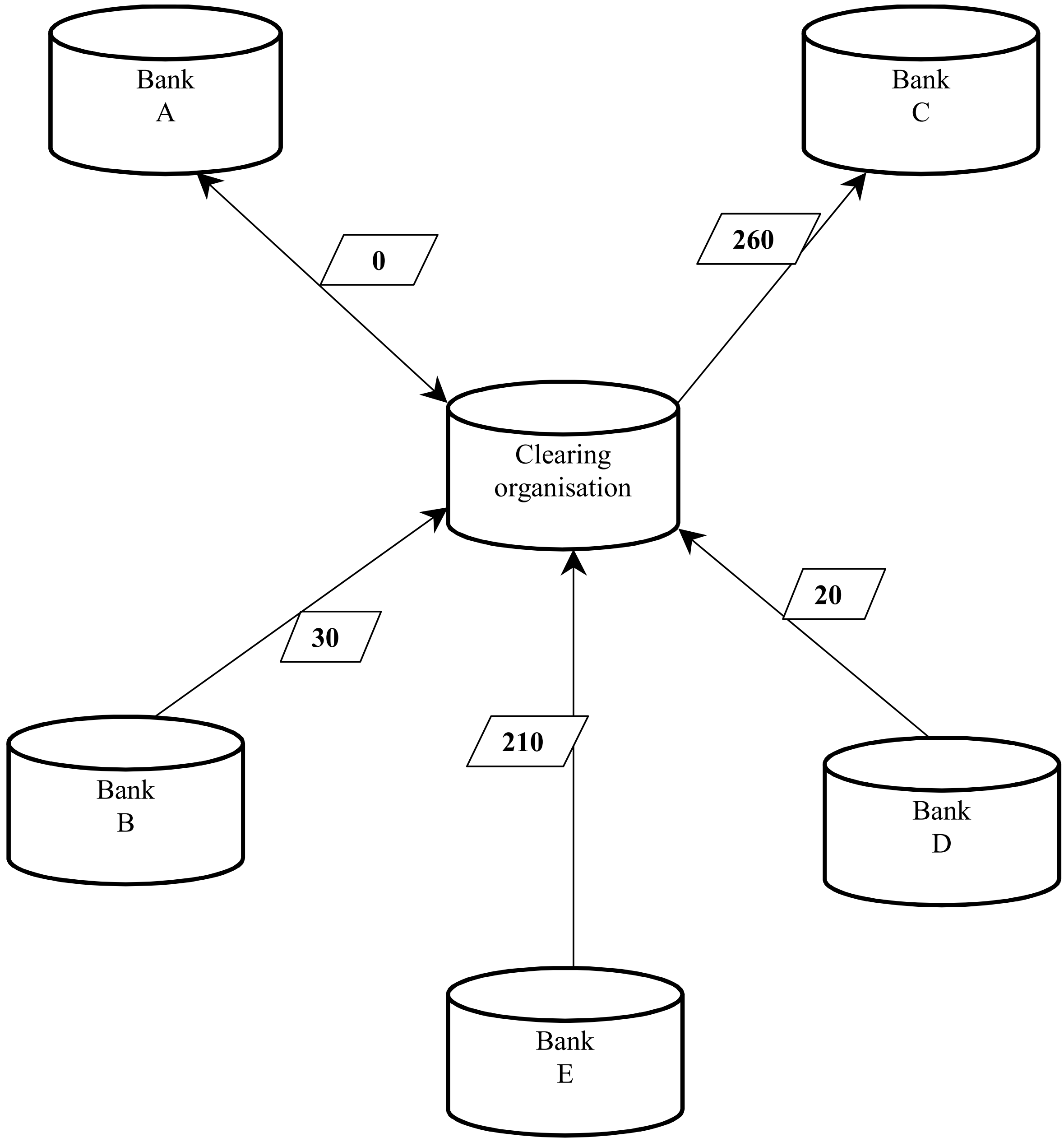
Picture. 1. Graphic expression of the batch gross settlement method.

In picture 1 the arrows denote the direction of the transfer of funds between settlement participants, the number in the parallelogram reflects the amount of a payment. In the given case, when using the gross settlement method between banks, it is necessary to transfer 19 payments accounting for 1900 units of settlement assets (the sum of obligations of all participants), on the basis of 23 payment orders.



Picture. 2. The chart showing the bilateral netting settlement method in a graphic way.

Picture 2 illustrates the operations made to obtain net positions of requirements and obligations in bilateral settlement. In this case, the difference between payments on obligations and requirements is calculated, then the settlement is carried out on the amount of this difference, which is called a bilateral net position. As is seen from the picture, in bilateral mutual settlements the number of interbank settlement payments is reduced to 10 operations, and the demand for liquidity is reduced to 960 units of settlement assets.



Picture 3. The chart showing the multilateral settlement method in a graphic way.

Picture 3 illustrates a payment transfer carried out by settlement participants through a clearing organization by means of the method of multilateral requirements and obligations settlement (calculations on the basis of the net multilateral position).

In multilateral netting the number of payment-settlement operations is reduced to 4, and the demand for liquidity accounts for 260 units of settlement assets. In this case Bank *A* has a zero net position , Bank *C* is a “net-payee” (it has a debit net position), and banks *B*, *D*, *E* are “net-payers” (they have a credit net position).

To present the methods of settlements as a table, flows of payments received after reduction of transaction matrices during the clearing cycle (t1–t2), can be recorded in the following way.

Banks Obl / req	**A**	**B**	**C**	**D**	**E**	Total on credit obligations	Net credit obligations
A	0	250	80	50	30	410	0
B	70	0	50	120	100	340	30
C	110	40	0	90	60	300	
D	100	0	260	0	140	500	210
E	130	20	170	30	0	350	20
Total on debit obligations	410	310	560	290	330	1900	
Net debit obligations	0		260				260

The horizontal cells of the table contain the values of payments from the settlement participant denoted on the left side, to the settlement participants denoted above. The sum of all payments in the horizontal cells makes the total of credit obligations.

The vertical cells reflect the values of payments which a settlement participant denoted above receives on payments from participants denoted in the left part of the table. The sum of all payments in the vertical cells makes the total of debit obligations.

The bilateral net-position between pairs of corresponding participants is calculated as a difference between the numbers located horizontally (obligations) and vertically (requirements) symmetrical to the main diagonal of the table which consists of zeros.

Net debit obligations are calculated as the difference between totals on credit and debit obligations; the difference obtained as a result of these calculations is a multilateral net position of a corresponding settlement participant.

The total on debit and credit obligations shows the amount of funds necessary to carry out gross settlement operations. The sum of net debit and credit obligations shows the amount of money necessary to carry out a multilateral netting settlement. According to the data of our example, the amount necessary to fulfill gross settlement operations accounts for1900 units; in a multilateral netting settlement this amount is 260 units of settlement assets.

Summary

The abovementioned allows for the following generalization to be made: any kinds of payment instructions correspond to a mathematical object in the form of a transaction matrix; as a consequence, formulas, methods of settlement and their transformations can express and analyze in a formalized way various quantitative characteristics of the relations between settlement participants. With the help of matrix models of payment instructions and settlement methods gross and net settlements can be presented as a system of vector-matrix formulas derived from each other, which can be useful for uniform understanding of clearing and settlement operations as well as for more effective use of liquidity of payment systems participants and minimizing financial risks emerging in payment systems.

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