The term structure of interest rates, volatility and risk premia: evidence from the eurolira spot and option markets

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Introduction

This paper investigates the relation between interest rate volatility and risk premia in the eurolira market. The expectations hypothesis of the term structure (EHTS) constitutes a convenient benchmark for assessing the importance of time-varying volatility and risk premia in driving interest rates movements. Most empirical studies (see Shiller (1990) for a survey) have often found that nominal interest rates are non-stationary stochastic processes. Under these circumstances, a necessary condition for the EHTS to hold is that the spread between short and long-term interest rates be stationary; a sufficient condition would also require the spread to be approximately constant. As is well known, time-varying risk premia can be a source of EHTS violation; time varying volatility may account for time variation in term premia. This relationship can be studied by modelling the term structure rates with respect to its fundamentals, which allows a joint and consistent treatment of spot and derivative markets in estimating volatility and risk premia. For simplicity, our modelling strategy is based on the Cox, Ingersoll and Ross (CIR, 1985) one-factor model; volatility and risk premia are estimated for the eurolira spot and option markets and standard measures of implied volatility, based on Black and Scholes option pricing, are brought to bear on the issue of volatility measurement. The paper is organised as follows; Section 1 looks at the theoretical implications of the EHTS; Section 2 deals with testing the econometric restriction implied by the EHTS for the eurolira interest rates; Section 3 introduces the CIR model for spot and swap rates and Section 4 extends it to the pricing of options on the three-month eurolira futures rate; Section 5 contains estimates of volatility - and associated risk premia - based on spot and option markets; conclusions are set out in the final section.

1. The expectations hypothesis of the term structure of interest rates and time-varying risk premia

The EHTS states that long-term interest rates should be determined by an average of current and expected future short-term interest rates plus a time invariant - albeit maturity dependent - term premium. Interest rates are expected to move so that expected returns on short and long-term investment strategies do not change over time - and are equalised, in the absence of term premia (the pure version of EHTS) - for comparable investment horizon. Under rational expectations, EHTS has the testable implication that movements in the excess return on long-term bonds over short bonds are unforecastable.

As is well known, stochastic trends are pervasive in financial data. Stock prices, exchange rates, forward and future prices and, often, interest rates are known to have stochastic trends. However, the implication of the presence of unit roots in restricting the testing of financial theory are yet to be acknowledged fully; many popular models and tests are inappropriate in the presence of stochastic trends. The sharing of a common stochastic trend by two or more bond returns - cointegration - has recently deserved much attention. Whether or not interest rates have a stochastic trend is perhaps still open to question. There is substantial evidence that (nominal) interest rates do

1 Bank of Italy, Research Department. Views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Italy.

have a stochastic trend,[2] but there is also substantial evidence that they do not.[3] While a rapidly growing body of empirical literature on cointegration in financial markets is available, comparatively little examination of the theoretical reason for cointegration in financial markets has been provided. Arguably, while learning phenomena, noise trading and peso problems may justify sample-based non-stationarity, the ultimate reason for stochastic trends in asset prices is likely to be found in stochastic trends driving long-run market fundamentals. For example, as derived in Campbell and Shiller (1987), equilibrium (real) stock prices, based on a present value model with a constant (real) discount rate, would embody the stochastic trend driving the future income stream (whose present value determines the price); therefore, dividends and stock prices must be cointegrated. Similarly, if bonds of different maturities are priced according to a stochastic discount factor kernel,[4] they would share a common stochastic trend underlying the pricing kernel. A candidate for the underlying factor may, in fact, be embodied in the inflation rate; at least for some countries - for example Italy and Canada - there is much evidence that inflation has a high degree of persistence, especially over much of the postwar history, which makes it very difficult to reject the hypothesis of a unit root in the inflation rate process.

The existence of a unit-root in the process governing interest rates has far-reaching implications for the decomposition of changes in the yield curve slope between expected movements in future short rates and time variation in the risk premia. Under the no-arbitrage assumption, cointegration restricts to 1 the number of common trends - e.g. factors - determining (long-run) bond pricing. Moreover, the conditions under which the EHTS holds require the interest rate spread to be stationary across the whole maturity spectrum (see Campbell and Shiller (1991)).

2. Some empirical evidence of cointegration for eurolira interest rates

The starting-point for the empirical analysis is the well-known (linearised) rational expectation version of the EHTS. The basic idea is that, with the exception of a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short-term bonds. As a result, returns on long-term bonds should be an average of current and expected future short-term interest rates plus a time-invariant (but maturity dependent) term premium. Specifically, the return on a long-term bond of maturity τ, $Y_t(\tau)$, will obey

$$Y_t(\tau) \equiv \frac{1}{s}\sum_{i=0}^{s-1} E_t Y_{t+\mu i} + \varphi(\tau,\mu), \tag{1}$$

where $Y_{t+\mu i}(\mu)$ is the μ period bond return at date $t+i\mu$, E_t is the conditional expectations operator over time t information, and $\varphi(\tau,\mu)$ is the term premia between the τ and the μ period bonds. In equation (1), $s=\tau/\mu$ is restricted to be an integer. If we now consider s pure discount bonds with maturity $[1, \tau_2, \tau_3,..., \tau_s]$, then all pairs of yields $[Y_t(1), Y_t(\tau_2)]$, $[Y_t(1), Y_t(\tau_3)]$,...., $[Y_t(1), Y_t(\tau_s)]$, fulfil equations of type (1), and

2 Cf. Engle and Granger (1987), Bradley and Lumpkin (1992), Hall, Anderson and Granger (1992), Engsted and Tanggaard (1994), Arshanapalli and Doukas (1994) and Gonzalo and Granger (1995) for evidence favouring stochastic trends and cointegration for interest rates of different maturities.

3 See Fama and Bliss (1987), Sanders and Unal (1988) and Chan, Karolyi, Longstaff and Sanders (1992), among many others, for evidence contrary to the non-stationarity of interest rates.

4 The stochastic discount factor kernel depends on the adopted asset pricing model. For models studied, among others, by Lucas (1979), Cox, Ingersoll and Ross (1985) and Epstein and Zin (1989) it would coincide with indirect marginal utility of money (intertemporal marginal rate of substitution).

$$Y_t(\tau) \equiv \frac{1}{\tau_i}\sum_{i=0}^{s-1} Y_{t+i}^1 + \varphi(\tau)$$
$$Y_t^1 \equiv Y_t(1), \tag{2}$$

where the yield at time t can be expressed as an average of expected 1-period yields.

If interest rates behave like integrated stochastic processes, this equation has a number of cointegration implications. These can be derived by considering a generic cointegrating vector $[\beta_1, \beta_2, \beta_3, \ldots, \beta_s]$:

$$\sum_{i=1}^{s} \beta_i Y_t(\tau_i). \tag{3}$$

If we insert (2) into this expression, we obtain

$$\sum_{i=1}^{s} \beta_i Y_t(\tau_i) = Y_t^1 \sum_{i=1}^{p} \beta_i + \frac{\beta_2}{\tau_2}\sum_{i=1}^{\tau_2 - 1} E_t(Y_{t+i}^1 - Y_t^1) + \frac{\beta_3}{\tau_3}\sum_{i=1}^{\tau_3 - 1} E_t(Y_{t+i}^1 - Y_t^1) + \ldots.. + \frac{\beta_s}{\tau_s}\sum_{i=1}^{\tau_s - 1} E_t(Y_{t+i}^1 - Y_t^1) + \sum_{i=1}^{s} \beta_i \varphi_t(\tau_i). \tag{4}$$

If Y_t^1 is a non-stationary I(1) process, i.e. a process which needs first-differencing to become stationary, then $E_t\left(Y_{t+i}^1 - Y_t^1\right)$ is stationary; therefore the right-hand side of (4) is stationary if and only if

$$\sum_{i=1}^{s} \beta_i = 0. \tag{5}$$

This implies cointegration in the system of s yields and that the sum of the cointegration coefficients should equal zero. Moreover, as this implication is valid for any $s>1$, there should be s-1 independent cointegration vectors, all of which must fulfil the zero-sum restriction. It can be shown that for the (s-1) restrictions the cointegration space is also spanned by the columns of

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & & 1 \\ -1 & 0 & 0 & 0 & & 0 \\ 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & & -1 \end{bmatrix} \tag{6}$$

Therefore, under the EHTS, the s-1 spreads,

$$S_t(\tau_i) \equiv Y_t(\tau_i) - Y_t^1 \qquad \forall i = 2,s \tag{7}$$

should be stationary, which is a testable assumption implied by the EHTS.[5]

The existence of s-1 cointegrating vectors implies that there is 1 common non-stationary component (trend) driving (the long-run dynamics of) interest rates (see Johansen (1991)); with nominal rates, inflation is a natural candidate as the common factor driving the nominal interest rate structure. The duality between the existence of (s-1) stationary relations (cointegrating vectors) and 1 non-stationary common trend is very useful for characterising the generating mechanism behind the chosen data. It implies a one-factor model representation for interest rates

$$Y_t(\tau_i) = \Phi_t + \Omega_t(\tau_i) \qquad \forall i = 1,s,$$

where Φ_t is a non-stationary (I(1)) scalar variable (common trend) and $\Omega_t(\tau)$ a vector of (I(0)) stationary variables; Gonzalo and Granger (1995) show how to identify econometrically the two components of the common-trend representation.

The cointegration restrictions implied by the EHTS are tested using the Johansen methodology with an n-order VAR model:

$$\Delta X_t = \mu_0 + \sum_{j=1}^{n} \Gamma_j \Delta X_{t-j+1} + \Pi X_{t-1} + \varepsilon_t \tag{8}$$
$$X_t \equiv \left[Y_t(1), Y_t(\tau_2), \ldots, Y_t(\tau_s) \right].$$

The results of testing the cointegration implications from a sample of eurolira weekly data on 1-3-6-12 month rates and the overnight rate - for the period 1979 to 1995 - suggest that there is some evidence that Italian short rates are driven by one common trend. However, the zero-sum restriction (5) on the predictive power of the four spreads, taken with respect to the overnight rate, is rejected. For longer maturities, the cointegration restriction for the existence of only one common trend, tested for eurolira swap rates up to five years and the one-month short rate, is rejected. Hence, more than one factor - two to three common trends are identified - would account for the long-term behaviour of interest rates. Fairly similar results seem to hold for German interest rates - measured on the euromarket - as well. Even for short maturities, more than one factor may be needed to account for the long-run behaviour of German rates. These conclusions are in contrast with the evidence gathered for US rates in Engsted and Taggart (1994) and partly contradict some of the conclusions drawn by Gerlach and Smets (1995) on the empirical evidence supporting the EHTS for short rates.

In all, the results show that the cointegration implications of the EHTS seem to hold only in part, at least for nominal rates. Further investigation may be required to identify the source of rejection of the EHTS (cointegration) implications. Relaxing the assumption of a constant term premium seems a natural candidate to start with; relating the time-varying property of term premia to volatility changes may throw some light on this by disentangling different sources of interest rate movement.

3. Deriving measures of volatility from models of the term structure

Non-stationarity of interest rates is at odds with standard assumptions for models of the term structure, which require the (nominal) short-term rate to be a stationary process, in order to generate a finite long-term yield; based on this assumption, pricing relationships incorporating arbitrage or general equilibrium conditions are derived.

5 This assumption underlies the Campbell and Shiller (1987, 1991) analysis.

These two instances cannot be easily reconciled: on the one hand, a literal interpretation of the stationarity tests would exclude the type of stochastic processes which are usually adopted in term structure modelling. On the other hand, an application of non-stationary interest processes to term structures has led to estimations with undesirable features.[6] Given the short estimation period adopted in the subsequent empirical analysis, it is reasonable to assume that the long-run component may play a minor role.

In term structure models the volatility of interest rates affects bond prices in an indirect way. First, it is assumed that the short-term rate - the instantaneous rate in continuous time models - evolves according to a specified process, usually a diffusion. Then, by arbitrage or equilibrium considerations, bond prices are derived, where the parameters of the short-term rate process, including volatility, enter the pricing equation.

CIR (1985) assume that the yield, r, on an instantaneously maturing riskless bond has the following equilibrium dynamics

$$dr = (\alpha - \kappa)dt + \sigma\sqrt{r}dz, \tag{9}$$

where α, κ and σ are positive parameters and $\{z(t), t>0\}$ is a standard Wiener process; α corresponds to the product term $\kappa\theta$ in CIR (1985), eq. 17, where κ is the speed of adjustment of r to its long-run mean, θ.

A no-arbitrage or equilibrium condition, based on the stochastic differential equation for the riskless rate, implies that a τ-maturity bond yield can be expressed in terms of the discount function:

$$P(\tau) = F(\tau)e^{-G(\tau)r}$$

$$F(\tau) \equiv \left\{ \frac{\phi_1 e^{\phi_1 \tau}}{\left[\phi_2 (e^{\tau\phi_2} - 1) + \phi_1\right]} \right\}^{\phi_3} \tag{10}$$

$$G(\tau) \equiv \frac{\left(e^{\tau\phi_2} - 1\right)}{\left[\phi_2\left(e^{\tau\phi_2} - 1\right) + \phi_1\right]}$$

$$\phi_1 \equiv \sqrt{\beta^2 + 2\sigma^2}$$

$$\phi_2 \equiv 0.5(\beta + \phi_1)$$

$$\phi_3 \equiv 2\alpha / \sigma^2$$

$$\alpha \equiv \kappa\theta$$

$$\beta \equiv \kappa + \lambda,$$

6 This is the case, for instance, of the Ho and Lee model and of gaussian models without mean reversion. For a comment on this topic, see Backus, Foresi and Zin (1995).

where the parameter λ determines the risk premium for a τ-maturity bond:

$$-\lambda G(\tau)r \tag{11}$$

with

$$G(\tau)\sigma\sqrt{r} \tag{12}$$

expressing its price volatility (standard deviation of the rate of change of the price).[7] As is apparent, bond prices depend on the parameters of the short-term rate and on a risk premium. It is possible to prove that bond prices are positively affected by an increase in the volatility parameter. In the CIR model, this effect is interpreted as deriving from uncertainty and risk-aversion.

Yield to maturity, $Y(\tau)$, can be specified as

$$Y(\tau) \equiv -\frac{\log P(\tau)}{\tau} = \frac{-\log F(\tau) + rG(\tau)}{\tau}. \tag{13}$$

As the bond nears maturity, the yield-to-maturity approaches the current interest rate; as we consider longer and longer maturities, the yield approaches a limit independent of the current rate:

$$RL \equiv \lim_{\tau\to\infty} Y(\tau) = \frac{2\alpha}{\phi_1 + \beta}. \tag{14}$$

4. The valuation of options on yields with the CIR model of the term structure

We consider option contracts on the eurolira futures price, which are essentially options on yield (see Longstaff (1992)); a payoff function for a call option on a yield Y, with time to maturity τ, is $MAX[0,Y-K]$, where K denotes the strike, or exercise, yield. The value of this contingent claim, $C[Y,K,\tau]$, can be obtained by taking expectation with respect to the risk-neutral probability measure of Y:

$$C[Y,K,\tau] = E\{MAX[0,Y-K]\}. \tag{15}$$

Following Barone and Mengoni (1995), let $H(t;s+T_1,s+T_2)$ be the futures price at time t of a contract maturing at time s, written on a euro-deposit 1 with settlement date $s+T_1$ and maturity date $s+T_2$. At LIFFE, the conventional futures yields are quoted as 100 - H. Consider the futures yield evaluated at the option expiration date T, e.g. $\tau = 0$:

$$Y=1 - H(T;s+T_1,s+T_2)/100. \tag{16}$$

As a matter of fact, when the settlement date of the underlying futures coincides with the expiration date of the option contract, the option pricing formula on the yield agrees with that on the futures price. Defining the associated strike price in terms of the futures yield $K_H = 100 - K$, (15) can be rewritten as

$$E\{MAX[0,H-K_H]\} = [(1+\omega)-K]\chi^2(\varphi r^*,2\phi_3,\eta r)\xi - \omega M(\tau)\exp(N(\tau)r)\chi^2(\varphi_1 r^*,2\varphi_3,\eta_1 r), \tag{17}$$

7 Yield volatility can be obtained dividing (12) by τ.

where

$$N(\tau) \equiv \frac{\eta(G^2 - G^1)}{\varphi - 2(G^2 - G^1)}; \qquad M(\tau) \equiv \frac{F^1}{F^2}\left[\frac{\eta(G^2 - G^1)}{\varphi - 2(G^2 - G^1)}\right]^{\phi_3};$$

$$\varphi \equiv \frac{4\beta}{\sigma^2\left[1 - e^{-\beta\tau}\right]}; \qquad \eta \equiv \varphi e^{-\beta\tau}; \qquad \varphi_1 \equiv \varphi - 2(G^2 - G^1); \qquad \eta_1 \equiv \frac{\varphi\eta}{\varphi - 2(G^2 - G^1)}$$

$$F^1 \equiv \left\{\frac{\phi_1 e^{\phi_1 T_1}}{\left[\phi_2(e^{\phi_2 T_1} - 1) + \phi_1\right]}\right\}^{\phi_3}; \qquad F^2 \equiv \left\{\frac{\phi_1 e^{\phi_1 (T_1 + T_2)}}{\left[\phi_2(e^{\phi_2 (T_1 + T_2)} - 1) + \phi_1\right]}\right\}^{\phi_3}$$

$$G^1 \equiv \frac{\left[e^{\phi_2 T_1} - 1\right]}{\left[\phi_2(e^{\phi_2 T_1} - 1) + \phi_1\right]}; \qquad G^2 \equiv \frac{\left[e^{\phi_2 (T_1 + T_2)} - 1\right]}{\left[\phi_2(e^{\phi_2 (T_1 + T_2)} - 1) + \phi_1\right]}$$

and r^*, such that:

$$(1 + \omega) - \omega M(0)\exp(r^* N(0)) = K_H, \tag{18}$$

with $\omega = (1/T_1)(360/365)$; $T_1 = 0.25$, $T_2 = 2/365$ (for the three-month eurolira estimation) and where $\chi^2(\varphi, \nu, \eta)$ denotes a non-central chi-square distribution with ν degrees of freedom and non-centrality parameter η. An accurate algorithm for computing $\chi^2(\varphi, \nu, \eta)$ is given by Sankaran (1963),[8] whereby the density of a transformation of a chi-square distributed random variable

$$\left\{\frac{\left[\chi^2 - (1/3)(\nu - 1)\right]}{(\nu + \eta)}\right\}^{0.5}$$

is approximately normal with expected value

$$\left\{1 - \frac{[(\nu - 1)]}{3(\nu + \eta)}\right\}^{0.5}$$

and variance $(\upsilon + \eta)^{-1}$.

5. Volatility estimates based on the CIR model of the term structure

In the empirical application, the CIR model has been estimated using different econometric approaches. The estimation based on bond prices allows the identification of the volatility parameter σ. However, extracting the volatility parameter from bond prices may be

8 The algorithm approximates the non-central chi-square distribution by a normal distribution. See Johnson and Kotz (1970), p. 140, eq. 23.3.

problematic since the likelihood function might depend too tenuously upon σ; therefore, the precision of the estimate could turn out to be very poor.

We estimate volatility parameter extracted from short-rate deposits and swap rates in the eurolira market. In addition, we try to compare the resulting volatility estimation with the volatility implied by observed option prices in the three-month eurolira future. As is well known, option prices are more sensitive to volatility in the underlying variable since their payoff is convex.

Our sample is restricted to the time span of the option market for the three-month eurolira future, whose trading activity started at LIFFE on 17th May 1995. A total of 109 daily observations, up to October 1995, were included. Results of estimating parameters for euro-deposits and swap rates, using a non-linear least-square algorithm,[9] are reported in Tables A2. Estimation is based upon equations (10) to (14); the former were applied to observed eurolira deposit rates (Libor) with a one to twelve-month maturity, the latter to swap rates, quoted in London, with a maturity from two to ten years. As is customary for econometric implementation, a measurement error is added to both equations; the usual assumption of independence across equations and over time is adopted. The instantaneous short rate, r, and volatility, σ, are jointly estimated with (α, β, RL). The estimation results are relatively encouraging; the standard errors, robust to heteroscedasticity and autocorrelation according to the White procedure, are small and the parameter values appear to be meaningful. r seems reasonably underpinned at some 10.5%; the implied long rate, RL, would be close to 9% and the asymptotic short rate, for small λ (e.g. term premia), would be at around 12%. The implied volatility parameter, σ, equals 0.7, when only short rates are included in the estimation; as a result, the standard deviation of short-rate (instantaneous) changes, $\sigma\sqrt{r}$, would be of the order of 20% (=0.7*$\sqrt{10.5}$). Interestingly, if swap rates are included[10] in the estimation, the implied volatility would drop to some 15% (=0.52*$\sqrt{10.5}$), whereas the long-term rate would rise to close to 11%. This parameters instability may signal problems in estimating volatility based on spot rates. In addition, the volatility estimate appears be much larger - 7 times - than the standard deviation which can be derived from the Black and Scholes (BS) volatility estimate (see Figure 1), σ_{BS}. The volatility of interest rate changes implied by the BS model can be approximated by multiplying σ_{BS} by the level of the three-month interest rate; hence the comparable BS figure would average some 2% (=22*0.11) for the period under consideration. However, a comparison with CIR-based estimates for one conditional volatility of the three-month yield with time to maturity over the remaining life of the option contract suggests a slightly closer link. The CIR-based conditional volatility for the time interval [t, (t+s)] is given by [11]

$$\sigma_{\text{CIR}}(Y_{t+s}^{0.25} \mid Y_t^{0.25}) = (1/G^{0.25})\sqrt{Y_t^{0.25} + \log F^{0.25})(\sigma^2/\kappa)(e^{-\kappa s} - e^{-2\kappa s}) + \theta(\sigma^2/2\kappa)(1-e^{-\kappa s})}, \tag{19}$$

where $F^{0.25}$, $G^{0.25}$ and $Y^{0.25}$, defined in equation 10 and 13, are evaluated at the three-month maturity. Evaluating expression (19) for s equal to 45 days (the average time to option contract expiration) and a current three-month yield at 11%, the estimated CIR conditional volatility would equal 5.5%, more than five times larger than the BS conditional implied volatility.[12]

9 All estimates were carried out in TSP, version 4.3.

10 Also, swap yields, C_t^m can be expressed in terms of the discount function (10):

$$C_t^m = \frac{1 - P(\tau_m)}{\sum_{j=1}^{m} P(\tau_j)}$$

11 See CIR (1985), eq. 19.

12 BS volatility assumes that yield changes follow a stochastic lognormal diffusion process.

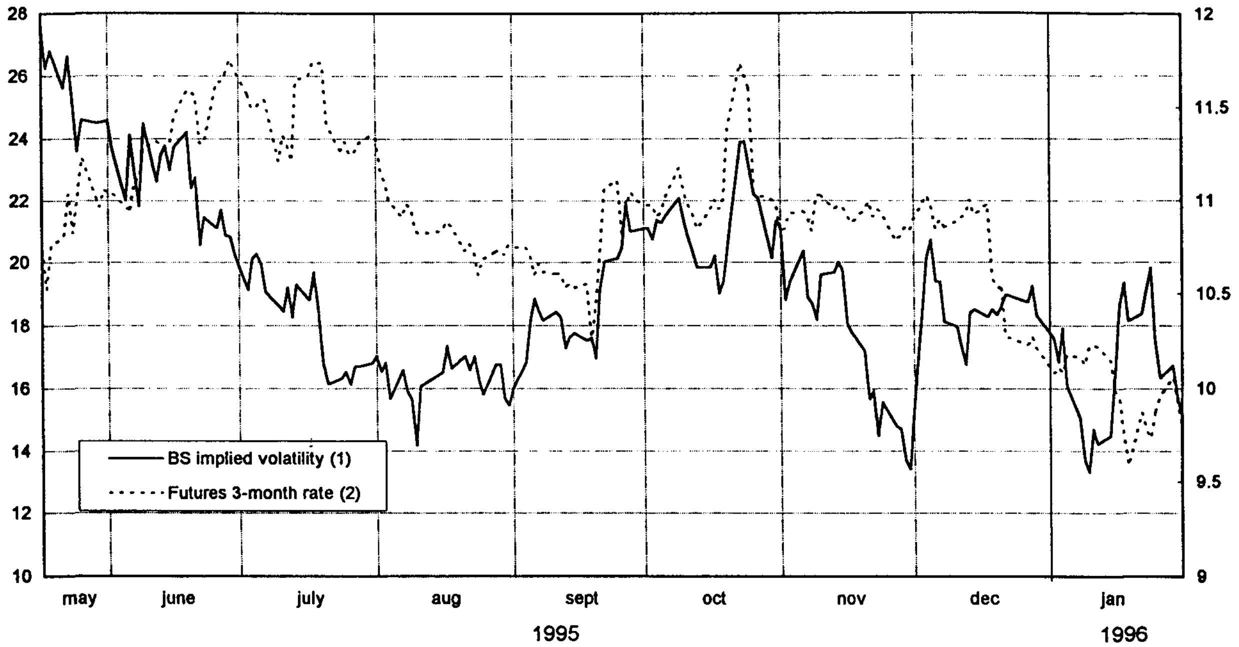
$$\sigma_{BS}(Y_{t+s}^{0.25}|Y_t^{0.25}) = E(Y_{t+s}^{0.25}|Y_t^{0.25})\sqrt{e^{s\sigma^2_{BS}} - 1} \qquad (20)$$

which is approximately 1%. Since (20) refers to spot yield volatility rather than future rate volatility, such estimates still suffer a small bias, which should vanish as the option contract approaches the expiration date. Further problems may also arise if the assumption of constant instantaneous rate as well as volatility were rejected. Econometric evidence suggests that this assumption may not be warranted; Table A2.3 reports parameter estimation where r and σ are constant only within each of the 23 weeks of the sample. While weekly variations of r are most of the time within the range of the 2-standard-error band, changes in σ are not.

Figure 1

Implied volatility and futures three-month rate

(daily data; in percentages)



1 Left-hand scale. 2 Right-hand scale.

In all, CIR parameter estimation raises the question as to whether implied interest rate volatility, inferred from spot rates, can be made consistent with observed option prices. The latter seem to suggest a lower, though perhaps more reactive, volatility than the one extracted from spot yields. Part of this discrepancy may be due to the systematic deviation of interest rate changes from the lognormal diffusion hypothesis, underlying the BS volatility model, as well as to the measurement of implied volatility for at-the-money option contract. Well-known smile effects and non-flatness of the volatility term structure have cast doubt on the ability of the BS model to capture the market's assessment of assets volatility,[13] favouring an option pricing model that incorporates stochastic volatility.

The CIR model-based theoretical option price (17) was used for a preliminary estimation of parameters, based on the observed option price for the three-month eurolira futures contract quoted at LIFFE since 17th May 1995; our sample was restricted only to call options with positive turnover,

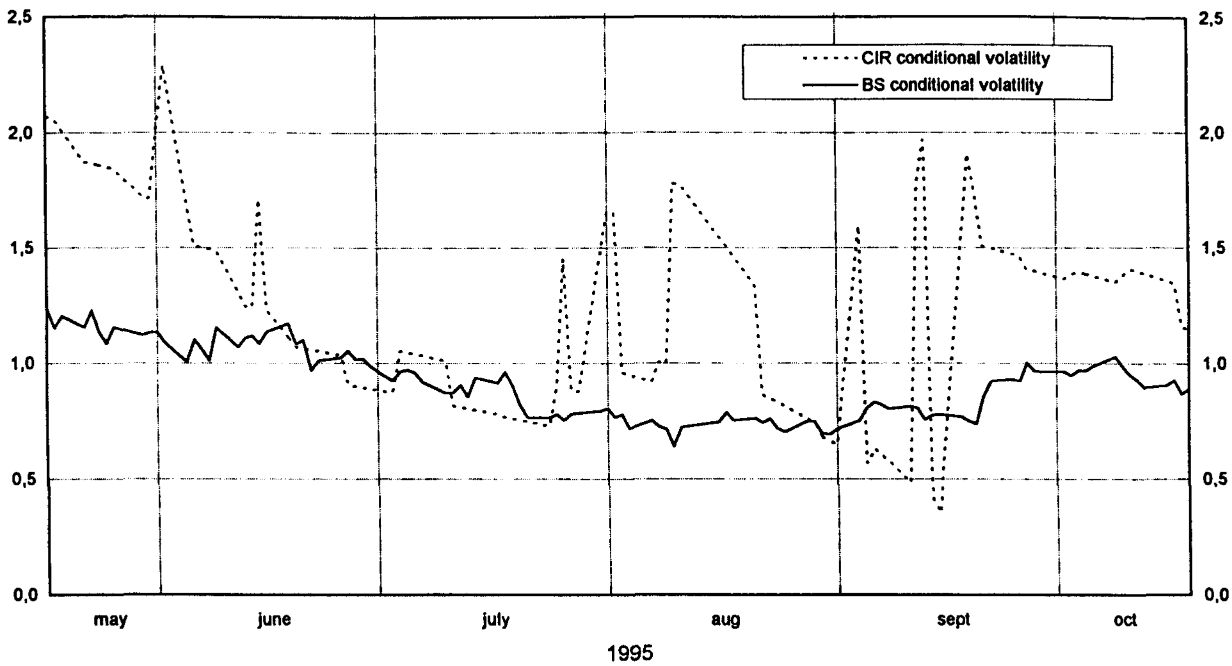
13 See Sheikh (1993) and, more recently, Hynen, Kemna and Vorst (1994).

up to the end of October 1995, for a total of 216 observations. The volatility parameter, σ, varies over time, albeit assumed to remain constant within each of the 23 weeks of the sample period. Results of the parameter estimation, reported in Table A2.4, differ significantly from those derived from spot rate estimation. The quality of the estimates also deteriorates; parameters are smaller and, especially for α and β, less precisely estimated. Estimates of σ imply that the conditional volatility, the long-term rate and the speed of adjustment to the steady state short rate are all smaller. Figure 2 contains both the conditional volatility of interest rate changes, the one generated by CIR estimates extracted from option price and the corresponding BS-based implied volatility. It is perhaps interesting that both measures match more closely, at least in terms of levels, than the estimated conditional volatility based on the spot rate, by varying between 1 and 3%. Risk premia, however, for a price of risk, λ , equal to -0.0165 - as estimated Cesari (1992) - would not differ much across estimates for short rates; for the three-month spot yield, reckoned by equation (11), they range between 15-20 basis points.

Figure 2

Conditional volatility of three-month interest rate: BS and CIR model

(daily data; in percentages)



Concluding remarks

Preliminary econometric evidence suggests that eurolira (nominal) interest rates contain a unit root and are cointegrated. However, spreads with respect to short rates still display non-stationarity, implying the rejection of the EHTS and pointing to sizable volatility in risk premia, which causes currently observed long-term rates to deviate from the discounted future path of short interest rates. The inability of interest rate spreads to predict changes in future short rates is exacerbated by the possibility of non-stationary term premia, which may result in excessively volatile long-term rates. For the short end, there is evidence that a one-factor (common trend) model can capture most of the long-run behaviour of interest rates; for longer maturities - eurolira swap rates up to five years - more than one common (long-run) factor - perhaps up to three years - is needed to account for the long-run behaviour of interest rates. Fairly similar results seem to hold for euro-DM interest rates.

Using the CIR (1985) one-factor model of the term structure, volatility and risk premia were estimated for the eurolira spot and option markets; standard measures of implied volatility, based on BS option pricing, were brought to bear on the issue of volatility measurement. Estimated volatility levels extracted from spot rates and the BS measure tend to differ systematically. Comparing these volatility estimates with a measure derived from an estimated CIR-based option pricing model raises the question whether implied volatility can be made consistent with observed option prices. An option price-based conditional volatility measure exhibits systematically lower levels than those estimated from spot rates. However, estimated risk premia for short spot yields appear to be relatively stable across various volatility estimates.

APPENDIX

A1. COINTEGRATION ANALYSIS

(weekly data)

Table 1

Eurolira[1]

Short rates — Period: 1978 (4) to 1995 (32); number of lags of the VAR: 11

Eigenvalues	Loglik for rank	
	7,147.44	0
0.0856975	7,189.10	1
0.0432766	7,209.67	2
0.0300449	7,223.86	3
0.0161832	7,231.44	4
0.0035064	7,233.08	5

Ho:rank=p	λmax[1]	95%	λtrace[1]	95%
p == 0	74.81**	33.5	153.8**	68.5
p <= 1	36.94**	27.1	78.97**	47.2
p <= 2	25.47*	21.0	42.03**	29.7
p <= 3	13.62	14.1	16.56*	15.4
p <= 4	2.933	3.8	2.933	3.8
Likelihood ratio test for H matrix restriction: χ^2 (4)=25.84**				

Swap rates — Period: 1991 (3) to 1995 (32); number of lags of the VAR: 14

Eigenvalues	Loglik for rank	
	3,027.87	0
0.3969680	3,089.07	1
0.2306900	3,120.81	2
0.1651770	3,142.65	3
0.1292380	3,159.40	4
0.0301203	3,163.10	5
0.0111720	3,164.46	6

Ho:rank=p	λmax[1]	95%	λtrace[1]	95%
p == 0	119.4**	42.5	266.4**	104.9
p <= 1	61.89**	36.4	147**	77.7
p <= 2	42.61**	30.3	85.13**	54.6
p <= 3	32.66**	23.8	42.53*	34.6
p <= 4	7,218	16.9	9.869	18.2
p <= 5	2,651	3.7	2.651	3.7

[1] (*) (**) Significant at 5% and 1% levels respectively.

Table 2

Euro-Deutsche Mark

Short rates — Period: 1978 (4) to 1995 (32); number of lags of the VAR: 20

Eigenvalues	Loglik for rank	
	14,284.0	0
0.0504949	14,308.1	1
0.0447820	14,329.4	2
0.0214334	14,339.5	3
0.00998374	14,344.2	4
0.00651495	14,347.2	5

Ho:rank=p	λmax[1]	95%	λtrace[1]	95%
p == 0	43.01**	36.4	112.8**	77.7
p <= 1	38.03**	30.3	69.76**	54.6
p <= 2	17.98	23.8	31.74	34.6
p <= 3	8.328	16.9	13.75	18.2
p <= 4	5.425*	3.7	5.425*	3.7
Likelihood ratio test for H matrix restriction: χ^2 (4)=20.6**				

Swap rates — Period: 1991 (13) to 1995 (32); number of lags of the VAR: 9

Eigenvalues	Loglik for rank	
	3,911.32	0
0.2883310	3,950.77	1
0.2117330	3,978.37	2
0.1055240	3,991.31	3
0.0664912	3,999.29	4
0.0335288	4,003.24	5
0.0183238	4,005.39	6

Ho:rank=p	λmax[1]	95%	λtrace[1]	95%
p == 0	74.83**	42.5	178.4**	104.9
p <= 1	52.34**	36.4	103.6**	77.7
p <= 2	24.53	30.3	51.24	54.6
p <= 3	15.14	23.8	26.71	34.6
p <= 4	7.503	16.9	11.57	18.2
p <= 5	4.069*	3.7	4.069*	3.7

No trend in the DGP

Ho:rank=p	λmax[1]	95%	λtrace[1]	95%
p == 0	74.75**	39.4	168.00**	94.2
p <= 1	49.66**	33.5	93.29**	68.5
p <= 2	25.05	27.1	43.63	47.2
p <= 3	11.82	21.0	18.58	29.7
p <= 4	6.028	14.1	6.754	15.4
p <= 5	0.7262	3.8	0.7262	3.8

[1] (*) (**) Significant at 5% and 1% levels respectively.

A2. TERM STRUCTURE AND IMPLIED VOLATILITY ESTIMATION

Table 1

Dependent variable: spot short rates
Log of likelihood function: 2,612; number of observations: 109

Parameter	Estimate	St. error[1]	t-statistic
α	0.086008	0.022734	3.78319
β	0.699078	0.205234	4.99257
R	0.102898	0.196694E-03	523.140
RL	0.897430	0.505099E-02	17.7673
σ	0.705010	0.089646	10.0003
σ_{CIR}	0.070234	0.472588E-02	14.8617

Table 2

Dependent variable: spot short rates
Log of likelihood function: 5,623; number of observations: 109

Parameter	Estimate	St. error[1]	t-statistic
α	0.086681	0.221401E-02	39.1509
β	0.618978	0.017895	34.5898
R	0.108099	0.157579E-03	685.998
RL	0.109658	0.232586E-03	471.475
σ	0.520672	0.012488	41.6932
σ_{CIR}	0.054139	0.122801E-02	44.0867

Table 3

Dependent variable: spot short rates
Log of likelihood function: 2,889; number of observations: 109

Parameter	Estimate	St. error[1]	t-statistic
α	0.117092	0.18148	6.45206
β	0.974110	0.162381	5.99894
R	0.108099	0.157579E-03	685.998
RL	0.109658	0.232586E-03	471.475
R_1	0.098870	0.252109E-03	392.174
R_2	0.102994	0.231748E-03	444.420
R_3	0.106737	0.23770E-03	450.236
R_4	0.106574	0.236231E-03	451.142
R_5	0.106167	0.235157E-03	451.474
R_6	0.105026	0.233945E-03	448.933
R_7	0.104306	0.233017E-03	447.633
R_8	0.104070	0.232951E-03	446.746
R_9	0.102703	0.232709E-03	441.335
R_{10}	0.103793	0.233571E-03	444.376
R_{11}	0.103429	0.233695E-03	442.582
R_{12}	0.103181	0.234176E-03	440.613

Table 3 (cont.)

Parameter	**Estimate**	**St. error[1]**	**t-statistic**
R_{13}	0.101602	0.233355E-03	435.397
R_{14}	0.101033	0.231228E-03	436.943
R_{15}	0.100586	0.231888E-03	433.770
R_{16}	0.101688	0.234101E-03	434.376
R_{17}	0.101360	0.237242E-03	427.242
R_{18}	0.101935	0.240662E-03	423.560
R_{19}	0.102764	0.237845E-03	432.061
R_{20}	0.102424	0.238494E-03	429.463
R_{21}	0.102948	0.239491E-03	429.861
R_{22}	0.102797	0.238894E-03	430.304
σ_1^2	0.481555	0.97112	4.95878
σ_2^2	0.522370	0.090081	5.79889
σ_3^2	0.495383	0.781000	6.34296
σ_4^2	0.452823	0.74418	6.08489
σ_5^2	0.406518	0.71123	5.71572
σ_6^2	0.440753	0.076863	5.73430
σ_7^2	0.436402	0.78169	5.58279
σ_8^2	0.475393	0.082561	5.75805
σ_9^2	0.612488	0.100853	6.07310
σ_{10}^2	0.551648	0.091257	6.04502
σ_{11}^2	0.604689	0.097967	6.17239
σ_{12}^2	0.657901	0.104624	6.28822
σ_{13}^2	0.756408	0.120742	6.26465
σ_{14}^2	0.700007	0.115663	6.05213
σ_{15}^2	0.770399	0.125510	6.13817
σ_{16}^2	0.782578	0.123632	6.32989
σ_{17}^2	0.927904	0.142639	6.50527
σ_{18}^2	0.996284	0.149498	6.66419
σ_{19}^2	0.847401	0.128299	6.60487
σ_{20}^2	0.894458	0.135519	6.61781
σ_{21}^2	0.892910	0.133345	6.69623
σ_{22}^2	0.883788	0.132671	6.66149
RL	0.109658	0.232586E-03	471.475

[1] Standard errors computed from heteroskedastic-consistent matrix (Robust-White).

Table 4

Dependent variable: option prices
Log of likelihood function: 15.6; number of observations: 216

Parameter	Estimate	St. error[1]	t-statistic
α	0.517575E-02	0.425830	0.121473
β	0.586700	0.424703	0.138143
R	0.110502	0.149759E-02	73.7867
σ_1^2	0.118650	0.294822E-02	4.02439
σ_2^2	0.010120	0.242847E-02	4.17090
σ_3^2	0.915679E-02	0.230767E-02	3.96798
σ_4^2	0.746466E-02	0.181787E-02	4.10628
σ_5^2	0.545558E-02	0.141720E-02	3.84954
σ_6^2	0.433479E-02	0.113714E-02	3.81200
σ_7^2	0.335719E-02	0.871484E-03	3.85228
σ_8^2	0.495751E-02	0.122755E-02	4.03855
σ_9^2	0.327262E-02	0.968835E-03	3.37790
σ_{10}^2	0.321474E-02	0.938254E-03	3.42630
σ_{11}^2	0.500436E-02	0.126234E-02	3.96435
σ_{12}^2	0.678163E-02	0.172550E-02	3.93024
σ_{13}^2	0.837116E-02	0.222148E-02	3.76828
σ_{14}^2	0.021315	0.571127E-02	3.732214
σ_{15}^2	0.933033E-02	0.223855E-02	4.16803
σ_{16}^2	0.827818E-02	0.202429E-02	4.08942
σ_{17}^2	0.011191	0.260676E-02	4.29308
σ_{18}^2	0.013818	0.340406E-02	4.05927
σ_{19}^2	0.879490E-02	0.204497E-02	4.30074
σ_{20}^2	0.822055E-02	0.211657E-02	3.88390
σ_{21}^2	0.882256E-02	0.281293E-02	3.13643
σ_{22}^2	0.986831E-02	0.231327E-02	4.26596
σ_{23}^2	0.742682E-02	0.221910E-02	3.34678

[1] Standard errors computed from heteroskedastic-consistent matrix (Robust-White).

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