How does tranching create economic value?

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Abstract

Tranching a portfolio into multiple securities with a market value greater than the original portfolio is possible when there differences in the non-credit components of spreads between segmented markets. This paper documents that the ability of a tranching to generate cash flows that fund structuring fees is driven simply by an arbitrage between high non-credit components of spread for fixed-income securities and low non-credit components of spread for equity securities.

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1. Introduction

Consider an arbitrageur that buys new cars, takes them apart, and sells the parts in a market where they are viewed as perfect substitutes for those produced by a parts manufacturer. If the total value of parts is larger than the cost of purchasing the car and breaking it up into parts, this process generates economic profit. Alternatively consider an arbitrageur that purchases a portfolio of corporate bonds from the secondary market, re-packages this portfolio into a senior debt tranche and a subordinated equity tranche of a collateralized bond obligation (CBO), and sells these securities to investors at a price higher than the cost of acquiring the portfolio.

In the first transaction, the arbitrageur is taking advantage of market power by an auto parts manufacturer which has the ability to price discriminate between consumers in two segmented markets. This paper is an attempt to explain why the second entrepreneur is able to earn economic profit through the simple re-packaging of credit risk.

Structural asset pricing models are typically built on the assumption that this type of arbitrage should not generate economic profit. In particular, a financial security establishes ownership to an investor over a series of state-contingent cash flows. The absence of arbitrage in financial markets requires that cash flows in each state at each time have a unique value. When the price of a security is simply the present discounted value of expected cash flows using the unique probability measure, investors should not be able to break apart a security into its components and sell the parts for more than the whole.

It immediately follows that the existence of structured finance products like the collateralized bond obligation mentioned above is a challenge to the efficacy of pricing models based on a no arbitrage assumption. Moreover, it seems completely inappropriate to use a model to price the tranches of a structured finance product when the model cannot reconcile the existence of that product in the first place.

Of course a charge that asset pricing models fail to explain real world data is neither new nor shocking. A recent paper by Huang and Huang (2003) documents that a broad array of asset pricing models are unable to explain observed credit spreads on corporate bonds. In particular, observed market spreads are typically much higher than those predicted by structural models, especially at the high quality end of the ratings spectrum.

This paper begins with the view that there significant non-credit components to spreads on fixed-income instruments. If these non-credit components of these spread vary significantly between financial markets, one might imagine an arbitrage where securities are purchased in markets where these non-credit components are high and sold in markets where these non-credit components are low. In this view, a structured finance product is simply a way of arbitraging the differences between non-credit components of spreads between segmented markets.[1]

The idea that the ability of tranching to create value is connected to the mis-pricing of funded credit risk is reinforced by an investigation of the ability of asset pricing models to explain spreads for unfunded credit protection. In particular, a recent paper by Hull and White (2004) documents that a similar pricing model to the one which under-predicts spreads on corporate bonds actually does quite well in explaining market spreads on iTRAXX EUR tranches. Given that structuring fees are minimized in these single-tranche CDOs, this fact only seems to reinforce the claim that the source of economic value behind structuring fees is mis-pricing.

In this paper, I demonstrate that like corporate bonds, a structural asset pricing model is not able to explain either the level of market credit spreads or the market slope of the spread-rating schedule for funded structured finance tranches. Using data on mis-pricing developed by Huang and Huang (2003), I document that the only way that tranching a portfolio can create value is if the non-credit component of spreads is much smaller for equity investors than it is for fixed-income investors.

The paper proceeds as follows. Section 2 investigates the ability of an asset pricing model to explain observed spreads on funded and unfunded structured finance tranches. Section 3 investigates how tranching creates value given non-credit components of spreads that are observed in the data. Section 4 concludes.

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[1] This view implies that researchers should accept the failure of structural models of credit risk to explain spreads on funded instruments like corporate bonds and work toward better understanding the non-credit risk component of spreads.

2. Answers from structural models of credit risk.

Pricing structured finance tranches with a structural model is conceptually straightforward. Given a probability distribution over different states of nature at each point in the future, one can easily compute the expected cash flows allocated to each tranche at each point in time, and then discount these expected cash flows back to the present using the risk-free rate of interest. The difficult part of pricing is computing the probability of loss in each state at each future point in time, but this is typically accomplished by putting structure on the uncertainty faced by each of the obligors in the portfolio of collateral.

2.1 Structure of uncertainty[2]

Consider an underlying portfolio of N obligors with equal face values normalized to 1/N at equal maturities of 5 years. Interest is paid quarterly so that the number of time periods T is equal to 20. Asset returns X_i of each obligor are driven by two independent normal random variables, a market factor M which affects all credits and an idiosyncratic factor Z_i that unique to each credit.

(1) $$X_i = M*\varrho^{1/2}+Z_i*(1-\varrho)^{1/2}$$

The loading $\varrho^{1/2}$ on the market factor corresponds to the correlation between the firm's returns X_i and the market factor M, and affects the amount of correlation there is in the underlying portfolio of collateral. The parameter ϱ is chosen as a function of the one-year probability of default (pd) that appears in the revised Basel Accord (2004):

(2) $$\varrho = 0.12*[1-e^{-50*pd}]/(1-e^{-50})+0.24*[1-(1-e^{-50*pd})]/(1-e^{-50})$$

The main features of this function are that it is decreasing in the probability of default with a ceiling of 24 percent and a floor of 12 percent. The risk-neutral probability of default q_t is a simple exponential function of time t in years.

[2] This analysis here is follows Gibson (2004), who outlines a simple approach for computing spreads on synthetic CDO tranches.

(3) $q_t = 1\text{-}e^{-\alpha^* t}$

The parameter α is chosen in order to match one-year probabilty of default (pd) associated with each originator:

(4) $\alpha = -\ln[1\text{-pd}]$

The firm defaults on its debt when asset returns are below the threshold γ, which is determined by the risk-neutral probability of default and the distribution of X_i:

(5) $\gamma_t = \Phi^{-1}[q_t]$

The probability that an individual obligor defaults conditional on the market factor can thus be written:

(6) $q_{t,m} = \Phi[(\gamma_t\text{-}M^*\varrho^{1/2})/\ (1\text{-}\varrho)^{1/2}]$

The probability distribution of the number of losses given the market factor Pr(l|t=j,m) is derived using a recursive procedure described by Gibson (2004) from the set of probabilities $\{q_{t,m}\}$.

With these conditional probabilities of loss in hand, the risk-neutral probabilities of portfolio loss are computed using numerical integration.

(7) $\Pr(l|t) = \int \Pr(l|t,m)\phi(m)dm$

This integration is done for every future date t, summing across an approximation to the support of the systematic factor.[3]

[3] The pricing program is written using Stata/SE 8.2. The first program computes the loss distribution for a particular portfolio at each future date until maturity. For a portfolio of 100 securities, 20 payment dates, and a 100-point approximation of the normal distribution, the program takes about 5 minutes. The second program uses this loss distribution in order to price different structures almost instantaneously. Program code is available upon request.

Structured finance tranche spreads are calculated by equating the expected present discounted value of cash flows from the protection seller to the protection buyer to the expected present discounted value of cash flows from the protection buyer to the protection seller.

Consider a structure with attachment point κ_s for a senior debt tranche and κ_m for a mezzanine debt tranche, implying that the equity tranche investors absorb losses up to κ_m, while the mezzanine tranche investors cover losses greater than κ_m but less than κ_s, and the senior tranche investors assume losses greater than κ_s. The face value of the senior tranche f_s is equal to $1-\kappa_m$, while the face value of the mezzanine tranche f_m is equal to $\kappa_m-\kappa_s$, and the face value of the equity tranche f_e is equal to κ_m.

Using a constant recovery rate of R, the expected portfolio loss at time t l^c_t can be written

(8) $l^c_t = \Sigma Pr(l|t)*(1-R)*l$

The expected loss at future date t allocated to each tranche is computed:

(9) $l^s_t = \Sigma_l Pr(l|t)*0.6*max(l-\kappa_s,0)$

(10) $l^m_t = \Sigma_l Pr(l|t)*0.6*max(min(l,\kappa_s)-\kappa_m,0)$

(11) $l^e_t = \Sigma_l Pr(l|t)*0.6*min(l,\kappa_m)$

Note that this does not correspond to the expected loss on the tranche, which would be calculated by taking the ratio of the loss allocated to tranche x denoted by l^x_t to the face value of tranche x, denoted by f^x.

2.2 Pricing funded structured finance tranches

Computing and understanding funded spreads

First, consider the purchase of funded protection by the owner of a portfolio of collateral. An investor purchases a tranche security at face value f_x at time 0. A no arbitrage restriction that the present discounted value of expected cash flows from the security equals its time 0 price implies:

$$f_x = (s_x+r_f)\Delta t^*\Sigma_t\beta^t(f_x-l^x_t)+\beta^T(f_x-l^x_T) \quad (12)$$

When the discount factor is equal to inverse of the gross risk-free rate ($\beta = 1/[1+r_f]$), it follows that the periodic spread on a funded instrument can be written:

$$\Delta t^*s_x = [\beta^T{}^*l^x_T+r_f\Delta t^*\Sigma_t\beta^t l^x_t]/[\Sigma_t\beta^t(f_x-l^x_t)] \quad (13)$$

When spreads are paid quarterly, the time adjustment factor Δt in front of the annualized spread s_x is simply equal to 0.25. The numerator on the right-hand side of this equation corresponds to the present discounted value of expected losses faced by the investor, where this is split into the loss of principal in the first term and loss of risk-free interest in the second term. The denominator on the right-hand side of this equation corresponds to the present discounted value of principal on which interest is paid each period. Naturally, the model implies that the credit spread is a ratio of expected loss to expected principal. In this model, credit spreads depend only on the amount and timing of expected losses absorbed by each tranche.

Funded spreads and structure leverage

Tables 1 illustrates how expected loss and model-implied spreads vary for a simple two-tranche structure with the size of the equity tranche (e). The underlying portfolio of collateral contains 100 obligors, each with a one-year probability of default of 1 percent, corresponding to a portfolio correlation of 19.2 percent and an expected loss at maturity equal to 292.3 basis points. The table illustrates how portfolio expected loss is allocated between the debt and equity tranches. Note that once the structure has equity of at least 2 percent, the equity tranche absorbs a majority of the expected loss. Moreover, the expected loss on the senior tranche is less than one basis point when there is 20 percent equity in the structure.

The table also illustrates how spreads on funded structures change with leverage. Note how the spread and expected loss on each of the debt and equity tranche are increasing in leverage. The last column contains an important lesson: that the face-weighted average spread on funded tranches $(1-e)*s_d+e*s_e$ has little to do with the economic viability of the structure. In particular, for every two-tranche structure, the face-weighted average spread is greater than or equal to the spread on collateral (which inferred from the all-equity structure). This fact does not imply that tranching is not feasible. Instead, this phenomenon implies that tranching is associated with negative carry which must be financed by the protection buyer at the risk-free rate. In particular, as the collateral matures over time, negative carry disappears as losses on collateral are absorbed by the equity tranche. At some point, the weighted average-spread on the tranches is smaller than the spread on collateral, creating positive carry which permits the repayment of the original financing.

Pricing examples

Consider a portfolio of bank loans, of which 80 percent are rated BB and 20 percent are rated B. Using the estimated one-year corporate bond credit rating transition matricies from Table 2, the one-year probability of default (pd) associated with this portfolio is either 242 basis points (using cohort estimates from panel a) or 143 basis points (using time-homogenous Markov estimates in panel b).

The ϱ associated with this portfolio is 15.6 percent for the pd-based on cohort estimates and 17.9 percent for the pd based on Markov estimates. Using these parameters to describe the structure of uncertainty, I follow the methodology described above in order to compute the portfolio loss distribution at each point in the future $Pr(l|t)$.

To fix ideas, I consider three-tranche structures where the senior tranche rating attains a rating of AAA and the mezzanine tranche attains a rating of BB. In order to find the appropriate attachment points, I convert the one-year ratings transition matricies from Table 2 into expected loss rates by initial rating and time horizon in Table 3, using a recovery rate of 51.31 percent following Huang and Huang (2003). I use the five-year expected loss figures from table in order to identify the right attachment points. For example, in order to attain a AAA rating using the cohort estimates of PD, investors can absorb about 2.4 basis points of expected loss. In contrast, a AAA rating based on Markov estimates of PD would require investors absorbing about 1.4 basis points of expected loss.

Results are displayed in Table 4. In the first two columns, using the cohort estimates and the Basel-calibrated value for ϱ, in order to attain a AAA rating an attachment point of 25 percent is necessary for the senior tranche have an expected loss of 2.3 basis points. For the mezzanine tranche, 11 percent subordination is necessary in order to achieve an expected loss of 579.2 basis points and an associated BB rating. The spread on a funded senior tranche is only 0.4 basis points while the spread on a funded mezzanine tranche is 112.6 basis points.

Note that using the Markov estimates associated with a lower PD, the feasible structure is actually more conservative. In particular, the senior tranche requires an attachment point of 20 percent in order to attain an expected loss of 1.8 basis points while the mezzanine tranche requires an attachment point of 8 percent for an expected loss of 437 basis points. This feature is driven by the fact that the lower PD is associated with a higher ϱ, and the modest increase in correlation has a big effect on the tail of the distribution where tranches are attached.

In order to make this point clear, I also consider structures for these pd's under the restriction that $\varrho = 0.24$, the upper bound of the Basel formula. The senior tranche using the cohort estimates now has an attachment point of 32 percent while the senior tranche using the mezzanine tranche has an attachment point of 25 percent. This example demonstrates that both the quality of collateral (through pd) and portfolio correlation (through ϱ) have important effects on feasible tranche structure.

The performance credit spread models of funded credit protection

Market spreads on tranched and corporate debt securities are reported in Table 5, taken from JP Morgan's *Global ABS/CDO Weekly Market Snapshot* on 10/22/2004. Panel (a) focuses on short-term ABS (credit cards, prime auto loans, near prime auto loans, equipment loans, and stranded assets), panel (b) focuses on long-term ABS (home equity, manufactured housing, and MBS) as well as corporates, while panel (c) focuses on CDOs.

Some facts jump immediately off of the table. First, structured finance spreads are typically higher than corporate spreads of the same rating. In particular, spreads on long-term ABS in panel (b) as well as CDO tranches in panel (c) are signicantly larger than spreads on corporate bonds of equivalent rating. This structured finance premium cannot be completely be attributed to liquidity, because while funded CDO tranches are typically less liquid than corporate bonds and syndicated loans, the funded ABS market is typically more liquid. One answer is provided in a recent paper by Perraudin and Landschoot (2004), who document some evidence that returns on ABS tranches are more volatile than returns on corporate bonds of the same rating. When investors are unable to diversify away all idosyncratic risk, unexpected losses can show up in prices. Another answer might lie in the greater complexity of a structured finance instrument.

A second clear fact from the table is that model-implied credit spreads for funded protection from Table 4 are much smaller than observed market spreads. In particular, the model-implied spreads on an AAA-rated tranche never become larger than 1 basis point while market spreads on AAA-rated CDO tranches always exceed 30 basis points. For BB-rated tranches, model-implied spreads are no larger than 112 basis points, but market spreads are between 600 and 775 basis points.

The under-prediction of market spreads by the model is another restatement of the well-known "credit spread puzzle" described by Amato and Remolona (2003). Table 6 replicates a table from the authors' paper, documenting spreads and expected loss on corporate bonds from January 1997 to August 2003. The most striking fact from the table is that expected loss seems to explain a small fraction of the spread, especially for highly-rated corporate debt. For a AAA-rated bond with maturity of 3 to 5 years, the spread is more than 350 times expected loss. On the other hand, for a BB-rated bond with the same maturity, the spread is more than 2.8 times expected loss. Compare these figures to the ratio of model spreads to expected loss from panel (d) of Table 4. This ratio in the first column is 0.235 for AAA-rated bonds and 0.244 for BB-rated bonds. It appears that not only is the level of spreads predicted by the model too low, the slope of the spread-ratings schedule is opposite what one sees in the data. In particular, the ratio of spread to expected loss tends to increase as the credit rating improves in market data, but this ratio tends to decrease as the rating improves when using at model-implied spreads.

The inability of the above model to explain market credit spreads turns out to be a common feature of a broad range of models. Table 7 is taken from a recent paper by Huang and Huang (2003), which computes a 10-year corporate-treasury spread for eight different structural models.

Since credit ratings are largely based on expected loss and spreads on funded CDO and long-term ABS tranches are higher than the observed spreads on corporate bonds, it is no surprise that a structural model of credit risk would perform poorly in explaining the level of funded structured finance tranche spreads.

Scope for structuring fees

The requirement that losses on collateral are absorbed by the tranches at every time t implies it is possible to sum across all possible dates of the present discounted value of tranche losses each period:

(14) $l^c_T+r_f\Delta t\Sigma_t\beta^t l^c_t = (l^e_T+r_f\Delta t\Sigma_t\beta^t l^e_t)+(l^m_T+r_f\Delta t\Sigma_t\beta^t l^m_t)+(l^s_T+r_f\Delta t\Sigma_t\beta^t l^s_t)$

Using equation (13), this can easily be re-written as the following:

(15) $s_c\Delta t\Sigma\beta^t(1-l^c_t) = s_e\Delta t\Sigma\beta^t(e-l^e_t)+s_m\Delta t\Sigma\beta^t(m-l^e_t)+s_s\Delta t\Sigma\beta^t(s-l^e_t)$

The equation above states that the present discounted value of expected spread received from collateral is equal to the present discounted value of spread paid out in the form of spreads to investors.

It is straightforward to show that this equation implies that the spread on collateral is simply a linear combination of the spread on the tranches, where the weight on the tranche spread θ_x in this linear combination is equal to $\Sigma\beta^t(f_x-l^x_t)/\ \Sigma\beta^t(1-l^c_t)$:

(16) $s_c = \theta_e*s_e+\theta_m*s_m+(1-\theta_e-\theta_m)*s_s$

Writing θ_x as the ratio of $\Sigma\beta^t(f_x-l^x{}_t)/T$ to $\Sigma\beta^t(1-l^c{}_t)/T$, we have the ratio of an average across all times of the discounted expected principal balance on a tranche relative to an average across all times of the discounted expected principal balance on collateral. As with spreads, this ratio not only depends on the amount of expected loss allocated to a particular tranche, but also on the timing of expected losses. The first column of Table 8 illustrates how this weight varies with leverage. One interesting feature of θ_e is that it is smaller than the face value of equity, reflecting the fact that equity absorbs expected loss more quickly than the debt tranches.

An immediate problem with this equation is that all of the cash flows that are generated by collateral seem to be exhausted by payments to investors, implying that there are no cash flows left over to fund origination fees for creating the structure. For example, consider structuring fees δ that are paid up-front as a percent of the face value of collateral.[4] The spread available to pay investors is only equal to $(1-\delta)s_c$, which violates the asset pricing equation. Since these fees are an essential part of structuring funded tranches, it follows that the tranching of structured products is not compatible with the underlying structural model of asset pricing.

A careful reader might note that the presence of structuring fees would change model-implied spreads. In particular, up-front fees equal to δ are equivalent to a time 0 loss on the portfolio of the same amount. Perhaps one has accounted for the implicit losses associated with structuring fees, the model implies funded spreads more similar to market quotes. The fallacy of this approach is that implicit losses through structuring fees affects expected tranche loss and thus the feasible attachment points.

As an example, consider the attachment points (25 percent and 11 percent) in the first column of Table 4, but add up-front structuring fees of 2 percent. It is straightforward to compute spreads (0.7 and 166.4 basis points) and expected loss (3.9 and 843.6 basis points) for these tranches. While spreads on these securities have increased considerably, so has expected loss, implying rating downgrades. In fact, the structure requires more conservative attachment points (27 percent and 13 percent) for these tranches to attain the same rating. With these new attachment

[4] I ignore the presence of periodic fees to an asset manager because these would typically be paid in a pooled structure and I want to focus on value created by tranching. Moreover, the presence of an asset manager who actively manages the portfolio of collateral would change the probability distribution over losses at future dates.

points, it is straightforward to compute spreads (0.4 and 105 basis points) and expected loss (1.9 and 541.2 basis points), which are similar to those calculated in the first column of Table 4.

2.3 Pricing unfunded (synthetic) tranches

In contrast to a funded structure, consider a synthetic structure where the owner of a portfolio of collateral buys unfunded credit loss protection from investors. The protection buyer makes regular payments to investors each period in the form of spreads on any remaining collateral. In the event of a loss related to the default of an obligor in the portfolio, investors compensate the protection buyer for these losses with a cash payment.

Computing and understanding unfunded spreads

Discounting the expected cash flows from the protection seller to protection buyer associated with credit loss by the appropriate factor yields the value of the "contingent leg" of the transaction for tranche x:

(17) $CL_x = \Sigma_t \beta^t \Delta l^x_t$

In each period, the cash flows from the protection seller to the protection buyer are expected to be the change in expected loss from one period to the next allocated to that investor Δl^x_t. I use a risk-free interest rate of 5 percent in computing the discount factor β.

The discounting of fees paid to the protection seller by the appropriate discount factor yields the value of the so-called "fee leg" of the transaction for tranche x:

(18) $FL_x = s_x * \Delta t * \Sigma_t \beta^t (f_x - l^x_t)$

In each quarter, an insurance premium of FL_x is paid to investors in tranche x, where this cash flow is equal to the quarterly spread $s_x * \Delta t$ times the amount of principal expected to remain. This latter object is simply the difference between the face value of the tranche f_x and the expected losses absorbed by the tranche at time t denoted by l^x_t. The absence of arbitrage requires that the value of the fee leg be equal to the value of the contingent leg, and this defines the appropriate spread on each synthetic tranche x:

$$(19)\qquad s_x = \Sigma_t\beta^t\Delta l^x{}_t/[\Delta t^*\Sigma_t\beta^t(f_x\text{-}l^x{}_t)]$$

To better understand this equation, ignore the effect of discounting with the assumption that β = 1 and the role of the annualization factor Δt by moving it to the left-hand side of the equation.

$$(20)\qquad \Delta t^*s_x = (l^x{}_T/f_x)/[\Sigma_t(1\text{-}(l^x{}_t/f_x))]$$

In a synthetic instrument, the investor absorbs expected losses each period. On the other hand, in a funded instrument, the bulk of expected losses are absorbed at maturity when principal is returned. In addition, some losses are absorbed over time as risk-free interest is not paid on principal reduced by expected losses. In absence of discounting (r_f = 0 and β = 1), note from equation (13) that the spreads on funded and unfunded instruments are identical. It follows that any differences in model-implied spreads across funding type are explained completely by differences in the timing of losses.

Unfunded spreads and leverage

In Table 8, one might immediately note that the spread an all-equity unfunded structure (59.4 basis points) is smaller than the spread on an all-equity funded structure from Table 1 (60.3 basis points). This is not a mistake, and reflects the fact that the spread on a funded instrument must be higher than the spread on the unfunded instrument in order for each instrument to pay out the same expected present discounted value of cash flows.

Pricing examples

The second four columns of Table 4 illustrate spreads on unfunded tranches given the structures from the first four columns. Using the cohort estimates of pd and the Basel formula for portfolio correlation, the spread on the senior and mezzanine tranches are 0.43 and 111.1 basis points, respectively.

The performance of credit spread models of unfunded protection

While structural models seem to under-predict spreads on funded structured finance tranches, there is evidence that these models perform much better for unfunded instruments. In particular, Table 9 replicates two tables from Hull and White (2004). Market quotes are documented in the first row of each panel correspond to the spread in basis points for each debt tranche. The quote for an equity tranche represents an up-front payment as a percent of notional in addition to 500 basis points paid per year.

Panel (a) of Table 9 uses a single-factor normal copula approach similar to the one developed above. On a positive note, the range of model-implied spreads across reasonable measures of correlation include the actual market quotes for iTRAXX EUR tranches. However, note that the correlation which makes the model price equal to the market quote varies considerably across the tranches. In particular, the tranche-implied correlation increases from 5.5% percent for the 3-6% tranche to 31.2% for the 12-22% tranche. Since the portfolio correlation is fixed across tranches, this suggests that the model is not performing well.

Upon closer examination, the data require a higher correlation for the more senior tranches in order to push the model-implied spreads on these tranche up to market quotes. At the same time, using the implied correlation for the 3-6% tranche, the model over-predicts the up-front payment to equity. In contrast to the debt tranches, a higher correlation actually reduces the spread paid to equity tranche investors by reducing the up-front payment. This pattern seems similar to what is observed in the data for funded credit protection above, where the model seems to under-predict spreads for senior protection more than junior protection.

The authors fix this problem by using t-distribution for the factors in panel (b), which has more weight in the tails of the asset return distribution. Given the limited up-side of a debt instrument, this tends to increase the model-implied spreads. At the same time, given the limited down-side of an equity instrument, tends to decrease the model-implied spreads for the equity tranche. The bottom two rows of the table illustrate that a single portfolio correlation does a much better job of explaining the data.[5]

Scope for structuring fees

The requirement that losses on collateral are absorbed by the tranches implies it is possible to write the change in expected loss between two dates as the sum of the change in expected loss on each of the tranches:

(21) $\Delta l^c_t = \Delta l^e_t + \Delta l^m_t + \Delta l^s_t$

Taking a sum across all possible dates of the present discounted value each change in expected losses, and then using equation (14) to substitute $s_x \Delta t \Sigma \beta^t (f_x - l^x_t)$ for $\Sigma_t \beta^t \Delta l^x_t$ motivates the following useful expression:

(22) $s_c \Delta t \Sigma \beta^t (1 - l^c_t) = s_e \Delta t \Sigma \beta^t (e - l^e_t) + s_m \Delta t \Sigma \beta^t (m - l^e_t) + s_s \Delta t \Sigma \beta^t (s - l^e_t)$

The equation implies that the present discounted value of expected spread received from collateral is equal to the present discounted value of spread paid out in the form of spreads to investors. It is straightforward to show that this equation implies a restriction that the spread on collateral is simply a linear combination of the spread on the tranches, where the weight on the tranche spread θ_x is equal to $\Sigma \beta^t (f_x - l^x_t) / \Sigma \beta^t (1 - l^c_t)$:

(23) $s_c = \theta_e * s_e + \theta_m * s_m + (1 - \theta_e - \theta_m) * s_s$

Note that θ_x is the same for funded and unfunded structures. Recall that this equation implies that there is no scope for structuring fees, but since the model seems to fit market quotes and there are no structuring fees in an unfunded structure, one can conclude that the model fits the data well.

[5] The claim by the authors that the model explains spreads might be a little too strong because the authors do not investigate tranches more senior than the 12-22% tranche. In principle, the model should be able to

Without the appropriate data, I do not present evidence that implicit structuring fees on unfunded tranches are close to zero. The right exercise to conduct is to compare the implied correlation from the portfolio of credit default swaps referenced in the iTRAXX EUR with the correlation necessary for the model spreads on unfunded tranches to match market quotes.[6]

3. Answers from real data

3.1 The value of tranching into debt and equity

How does tranching create value in a funded structure?

Consider a structured finance product where actual spreads on collateral and tranches are denoted by v_x. For simplicity, focus on a structure with a debt tranche and a single equity tranche. The only restriction I use is a weak cash flow constraint: the expected present discounted value of spreads on collateral must fund structuring fees as well as the expected present discounted value of spreads on the tranches.

(24) $(1-\delta)v_c\Delta t\Sigma\beta^t(1-l^c_t) = v_e\Delta t\Sigma\beta^t(e-l^e_t)+v_d\Delta t\Sigma\beta^t(m-l^d_t)$

As above, it is straightforward to re-arrange this equation using the weights on model spreads from above:

(25) $(1-\delta)v_c = \theta_e * v_e+(1-\theta_e) * v_d$

Note that even when spreads deviate from model spreads, a restriction that the expected present discounted value of spreads paid out to tranches is funded by collateral implies restrictions on tranche spreads. A sufficient condition for a structured finance product to be able to fund any structuring fees is:

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explain market prices for all of the tranches.

[6] The idea that purchasing funded credit protection from AAA investors is expensive is corroborated by recent developments in structured finance markets. In particular, commercial banks often purchase unfunded credit protection for the super-senior tranche of a collateralized loan obligation (CLO) instead of issuing another AAA tranche because of the cost advantage.

(26) $v_c > \theta_e{*}v_e{+}(1{-}\theta_e){*}v_d$

Using the spreads predicted by the asset pricing model, define Δ_x as the ratio of the actual spread on a security v_x relative to the model-implied spread on the security s_x. Moreover, define the weight on each tranche ψ_x as $s_x\theta_x/s_c$, which can be re-written as $\Sigma\beta^t\Delta l^x{}_t/\ \Sigma\beta^t\Delta l^c{}_t$.

(27) $\Delta_c > \psi_e{*}\Delta_e{+}(1{-}\psi_e){*}\Delta_d$

This equation says that in order for a structure to be able to fund any structuring fees, the mis-pricing of collateral must be larger than the weighted-average mis-pricing of tranches. In other words, the non-credit component of spreads on tranches must be smaller than the non-credit component of spreads on collateral. The weight on the non-credit component of spreads is now an average across all time periods of the discounted change in expected loss on a tranche relative to an average across all time periods of the discounted change in expected loss on collateral. Note when there is no discounting so that $\beta = 1$ we have the result $\psi_x = l^x{}_T/l^c{}_T$, so that the weights correspond to the fraction of expected loss at maturity allocated to each tranche. The first two columns of Table 10 illustrate the relationship between leverage e and the mis-pricing weight on equity ψ_e. Note that this weight is 28.3 percent at only 1 percent equity in the structure and increases to more than 90 percent with only 8 percent equity.

Given the existence of evidence on the mis-pricing of debt in Table 7, re-arrange this equation as a restriction on the maximum non-credit component of spreads on the equity tranche:

(28) $\Delta_e < [\Delta_c{-}\Delta_d]/\ \psi_e{+}\Delta_d$

For the moment, assume that the non-credit component of spreads for the tranche debt is the same as that for collateral ($\Delta_c = \Delta_d$), so that the equation suggests that the only way for tranching to create value is for the non-credit component of spreads for equity to be smaller than the non-credit component of spreads for the debt tranche.

Since credit ratings are in part based on expected loss, the presence of an equity tranche implies that the credit rating of the debt tranche must be higher than the credit rating of the collateral. The evidence described by Huang and Huang (2003) implies that the non-credit component of spreads increases as the credit rating improves, so it follows that the non-credit components spreads for the debt tranche must be larger than those for collateral, implying that $\Delta_c - \Delta_d < 0$. The point is significant it implies that equation (28) not only requires the non-credit component of equity tranche spreads to be less than that of the debt tranche, but less than an expression which is significantly less than the non-credit component of debt tranche spreads.

With the restriction ($\Delta_c - \Delta_d < 0$) in mind, equation (28) implies that as the fraction of expected loss allocated to the equity tranche increases (ψ_e-> 1) the upper bound on the non-credit component of equity spreads approaches the mis-pricing of collateral. On the other hand, as the fraction of expected loss allocated to the equity tranche decreases (ψ_e-> 0), the upper bound goes to negative infinity.

To fix ideas, assume that credit risk model explains about 20 percent of the level of spreads for highly-rated bonds (the average percent of the credit spread explained for investment-grade corporate bonds is 19.9% in column 1 of Table 7) so that the mis-pricing of debt is Δ_d is equal to five. Further assume that credit risk explains about 80 percent of the level of spreads for low-rated bonds so that Δ_c is 1.25. With these numbers, the third column of Table 10 illustrates that as long as equity is less than 4 percent (ψ_e is less than 75 percent), the upper bound is actually negative.

When equity investors accept model-implied spreads (so $\Delta = 1$) , it is possible to tranche the portfolio using equity of 9 percent, where debt tranche investors receive a premium of 5 times the model-implied spread. On the other hand, when equity investors also require a premium over model-implied spreads of 7 percent, it is necessary to increase equity to 10 percent.

In order to get a gauge for the magnitude upper bound on non-credit components of equity spread that is permissible in the presence of structuring fees, one can re-write equation (21) as follows:

(29) $\Delta_e < [(1-\delta)\Delta_c - \Delta_d]/\psi_e + \Delta_d$

When structuring fees are 2 percent of the underlying portfolio, the fourth column of Table 10 indicates no structure with an equity weight smaller than 10 percent is attainable when equity investors accept model-implied spreads.

It should be obvious from the empirical observation that the non-credit component of debt tranche spreads is greater than that of collateral $\Delta_c - \Delta_d < 0$ that the relatively low non-credit components of equity tranche spreads are crucial to the ability of the transaction to fund structuring fees.

Why is there less mis-pricing of the equity tranche?

A natural question of course is why should there be less mis-pricing in the equity tranche. One answer comes from a recent paper by Amato and Remolona (2003), who offer an explanation for excess spreads on corporate bonds. In particular, the limited number of corporate debt issuers combined with the limited upside of bond returns creates skewness in the investor's loss distribution which makes it relatively difficult for fixed-income investors to diversify credit risk, implying that corporate-treasury spreads are much higher than predicted by models which take diversification for granted. On the other hand, the large number of public equity issuers combined with the symmetric distribution of equity returns makes it relatively easy for equity investors to diversify credit risk, implying that spreads paid to equity investors are close those predicted by models which take diversification for granted. From this line of argument, it naturally follows that models which presume diversification is complete should do a much better job of explaining the price of equity than the price of corporate debt.

Note that differences in the price of credit risk between the equity and corporate debt markets create an opportunity for arbitrage that is realized in the tranching of structured finance products, where credit risk is purchased at a low price in debt markets and sold at a high price in equity markets. In this sense, some of the value created in a structured finance product simply reflects differences in how corporate credit risk is priced between these two markets.

Unfortunately this explanation is not complete. In particular, this story does not seem to explain why the non-credit components should be different for bonds of different credit ratings. This fact seems to be important because structured finance products are typically tranched into more than one rated debt instrument, as we explore below. Moreover, this fact does not seem to explain the difference in performance of the structural model between funded and unfunded tranches.

3.2 The value of further tranching the debt tranche

How does tranching create value?

Most structured finance products have more than one debt tranche. Why should one split up a debt tranche into two tranches? If such a transaction reduces the average non-credit component of spread of the debt tranches Δ_d, equation (28) suggests that this will increase the maximum mis-pricing of the equity tranche, which in turn creates extra cash flow that can be claimed by an underwriter through higher fees or by the equity tranche investor through a higher spread.

Consider splitting the debt tranche into a senior debt tranche and a mezzanine debt tranche. In order for this splitting to create value, it follows:

$$(30) \qquad \Delta_d > (\psi_s/\psi_d)^*\Delta_s+(\psi_m/\psi_d)^*\Delta_m$$

To fix ideas, consider the collateral referred to in the first four columns of Table 10 and a structure that has 4 percent equity (corresponding to $\psi_e = 72.4\%$). The expected loss of the debt tranche is 88 basis points, about half-way between the expected loss for the A and BBB ratings, translating into debt tranche mis-pricing of $\Delta_d = 4.15$. It is not possible for this structure to compensate equity investors with model-implied spreads using this two-tranche structure.

However, it is possible to issue a senior tranche at a AAA rating where $\Delta_s = 6.3$ (note the first column of Table 7 reports that the model explains 15.6 percent of observed spreads). Panel (a) of Table 3 suggests that investors in the senior tranche require expected loss of no more than 2.3 basis points, which requires an attachment point of no less than 15.5 percent for the senior tranche. At this point, the mezzanine tranche has expected loss of 709.8 basis points. If the senior attachment point is pushed up to 18.4 percent, the expected loss on the mezzanine tranche is reduced to 578.7 basis points, well within the constraint for a BB-rated tranche. The weighted-average non-credit component of debt tranche spreads has fallen to Δ_d = 1.70, which now makes a structure with 4 percent equity feasible if equity investors accept model-implied prices.

4. Conclusions

This paper has documented that the credit spread puzzle that exists in the corporate bond market also exists in the structured finance market, but only in the context of funded as opposed to unfunded credit protection. The existence of high non-credit components to spreads in the rated debt market relative to those on equity tranches creates an arbitrage opportunity that is taken advantage of by structured finance products, realized by the presence of structuring fees.

The next revision of this paper will document the performance of the model by Hull and White (2004) for all tranches and compare the implied correlation of the underlying portfolio to the implied correlation that equates model spreads to market quotes. The claim in this paper that structuring fees are minimized for unfunded protection requires evidence that these two correlations are very close to each other.

References

Amato and Remolona (2003): "The credit spread puzzle," BIS Quarterly Review, December.

Basel Committee (2004): “Third Consultative Paper,” Basel Committee on Banking Supervision.

Gibson, Michael (2004): “Understanding the Risk of Synthetic CDOs,” Finance and Economics Discussion Series 2004-36.

Goodman, Laurie and Fabozzi, Frank (2002): Collateralized Debt Obligations. John Wiley & Sons (Hoboken, NJ).

Huang, Jing-zhi and Huang, Ming (2003): “How Much of Corporate-Treasury Yield Spread Is Due to Credit Risk?: A New Calibration Approach,” unpublished working paper.

Hull, John and White, Alan (2004): “Valuation of a CDO and an n^{th} to Default CDS Without Monte Carlo Simulation,” unpublished working paper.

Jafry, Yafry and Til Schuermann (2004): “Measurement, Estimation and Comparison of Credit Migration Matrices,” *Journal of Banking & Finance*, 28(11): 2603-2639

Oldfield, George (2000): “Making Markets for Structured Mortgage Derivatives,” Journal of Financial Economics, Volume 57, Issue 3, pp. 445-471.

Tavakoli, Janet (2003): Collateralized Debt Obligations & Structured Finance, John Wiley & Sons (Hoboken, NJ).

Table 1: Leverage, Expected Loss, and Speads on Funded Tranches								
e	**l^d**	**l^e**	**l^d/d**	**l^e/e**	**s_d**	**s_e**	**$e*s_e+d*s_d$**	**θ_e**
0%	292.3	0.0	292.3	n/a	60.3	n/a	60.3	0.0%
1%	213.3	79.0	215.5	7902.5	43.4	3701.5	80.0	0.5%
2%	155.5	136.8	158.7	6840.6	31.5	2530.1	81.5	1.1%
3%	113.6	178.7	117.1	5956.9	23.1	1920.7	80.0	2.0%
4%	84.1	208.2	87.6	5205.7	17.1	1532.1	77.7	2.8%
5%	62.2	230.1	65.5	4602.2	12.7	1272.1	75.7	3.8%
6%	46.0	246.3	48.9	4105.6	9.5	1083.8	73.9	4.7%
7%	34.3	258.0	36.9	3686.3	7.1	939.4	72.4	5.7%
8%	25.5	266.9	27.7	3335.9	5.3	827.0	71.0	6.7%
9%	18.8	273.5	20.7	3039.0	4.0	737.0	69.9	7.7%
10%	14.0	278.4	15.5	2783.6	3.0	663.0	69.0	8.7%
20%	0.4	292.0	0.5	1459.8	0.1	320.2	64.1	18.8%
30%	0.0	292.3	0.0	974.4	0.0	208.2	62.5	28.9%
40%	0.0	292.3	0.0	730.8	0.0	154.1	61.7	39.1%
50%	0.0	292.3	0.0	584.7	0.0	122.4	61.2	49.2%
60%	0.0	292.3	0.0	487.2	0.0	101.4	60.9	59.4%
70%	0.0	292.3	0.0	417.6	0.0	86.6	60.6	69.5%
80%	0.0	292.3	0.0	365.4	0.0	75.6	60.5	79.7%
90%	0.0	292.3	0.0	324.8	0.0	67.1	60.4	89.8%
91%	0.0	292.3	0.0	321.2	0.0	66.3	60.3	90.9%
92%	0.0	292.3	0.0	317.8	0.0	65.6	60.3	91.9%
93%	0.0	292.3	0.0	314.3	0.0	64.9	60.3	92.9%
94%	0.0	292.3	0.0	311.0	0.0	64.2	60.3	93.9%
95%	0.0	292.3	0.0	307.7	0.0	63.5	60.3	94.9%
96%	0.0	292.3	0.0	304.5	0.0	62.8	60.3	95.9%
97%	0.0	292.3	0.0	301.4	0.0	62.1	60.3	97.0%
98%	0.0	292.3	0.0	298.3	0.0	61.5	60.3	98.0%
99%	0.0	292.3	0.0	295.3	0.0	60.9	60.3	99.0%
100%	0.0	292.3	n/a	292.3	n/a	60.3	60.3	100.0%

Notes: the table refers to a CDO with an equity tranche and a single debt tranche. The columns describe: the fraction of equity funding, the expected losses at maturity allocated to the debt tranche, the expected losses at maturity allocated to the equity tranche, the expected loss of the debt tranche, the expected loss of the equity tranche, and the face-weighted average spread on the tranches. The first and last columns are in percentage terms while other columns are in terms of basis points.
Source: author’s calculations.

Table 2: Corporate Bond Credit Rating Transition Matricies

A. Cohort Estimates

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	93.20%	6.05%	0.58%	0.08%	0.08%	0.00%	0.00%	0.00%
AA	0.60%	90.89%	7.57%	0.70%	0.07%	0.12%	0.03%	0.01%
A	0.07%	2.08%	91.44%	5.60%	0.52%	0.19%	0.04%	0.06%
BBB	0.04%	0.22%	4.54%	89.07%	4.81%	0.83%	0.17%	0.33%
BB	0.04%	0.10%	0.45%	6.08%	82.41%	8.65%	0.86%	1.42%
B	0.00%	0.09%	0.26%	0.35%	5.36%	82.69%	4.82%	6.42%
CCC	0.12%	0.00%	0.36%	0.71%	1.78%	9.96%	54.57%	32.50%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

B. Time-homogenous Markov Estimates

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	93.45%	5.68%	0.71%	0.07%	0.08%	0.01%	0.00%	0.00%
AA	0.58%	91.33%	7.27%	0.64%	0.09%	0.07%	0.01%	0.01%
A	0.08%	1.89%	91.85%	5.49%	0.49%	0.18%	0.01%	0.01%
BBB	0.04%	0.25%	4.20%	89.53%	5.03%	0.77%	0.09%	0.09%
BB	0.04%	0.10%	0.58%	5.07%	83.74%	9.12%	0.76%	0.59%
B	0.00%	0.07%	0.26%	0.49%	4.44%	84.14%	5.83%	4.76%
CCC	0.07%	0.01%	0.29%	0.52%	1.13%	7.94%	45.46%	44.58%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Notes: the table reports estimated one-year ratings transition matricies for rated US corporate bonds over 1981-2000.
Source: Jafy and Schuermann (2004).

Table 3: Expected Loss by Rating and Maturity

A. Cohort Estimates

	Maturity in Years						
	1	**2**	**3**	**4**	**5**	**7**	**10**
AAA	0.0	0.1	0.5	1.3	2.4	6.3	17.1
AA	0.8	2.7	5.9	10.2	15.8	31.2	65.7
A	3.1	8.6	16.6	27.4	40.9	76.6	151.5
BBB	16.8	41.0	72.3	110.5	155.1	260.5	448.8
BB	72.8	176.7	302.8	443.4	592.3	896.8	1331.6
B	329.6	686.5	1032.9	1352.9	1641.5	2127.5	2671.0
CCC	1667.7	2612.0	3164.9	3503.6	3723.1	3986.2	4203.3
D	5131.0	5131.0	5131.0	5131.0	5131.0	5131.0	5131.0

B. Time-homogenous Markov Estimates

	Maturity in Years						
	1	**2**	**3**	**4**	**5**	**7**	**10**
AAA	0.0	0.1	0.3	0.8	1.4	3.7	10.3
AA	0.3	1.1	2.4	4.4	7.1	15.0	34.9
A	0.4	2.0	5.0	9.8	16.7	37.2	86.8
BBB	4.8	14.4	29.9	51.9	80.3	155.6	306.5
BB	30.4	95.9	190.3	306.1	436.2	717.7	1141.9
B	244.3	584.5	935.5	1264.8	1562.3	2061.7	2617.6
CCC	2287.5	3347.3	3856.9	4117.6	4263.7	4419.9	4545.9
D	5131.0	5131.0	5131.0	5131.0	5131.0	5131.0	5131.0

Notes: the table reports the expected loss (in basis points) of holding corporate bond of specified rating as an investment for maturity of years, using the transition matricies from Table 1 and a recovery rate of 51.31 percent.
Source: author's calculations.

Table 4: Examples of Feasible Structures								
Funding	**Funded Protection**				**Unfunded Protection**			
Collateral	**pd = 242**		**pd = 143**		**pd = 242**		**pd = 143**	
Correlation	**15.6%**	**24.0%**	**17.9%**	**24.0%**	**15.6%**	**24.0%**	**17.9%**	**24.0%**
a. Model-implied Tranche Spreads								
AAA	0.43	0.41	0.35	0.30	0.43	0.41	0.34	0.30
BB	112.61	117.19	85.23	84.68	111.10	115.60	84.08	83.53
Equity	1621.67	1472.48	1220.29	1134.87	1598.11	1451.03	1202.59	1118.37
b. Expected Loss								
AAA	2.3	2.2	1.9	1.6	2.3	2.2	1.9	1.6
BB	579.3	593.7	437.4	430.5	579.3	593.7	437.4	430.5
Equity	5518.7	5118.0	4502.5	4240.6	5518.7	5118.0	4502.5	4240.6
c. Tranche Attachment Points								
κ_s	0.25	0.32	0.20	0.25	0.25	0.32	0.20	0.25
κ_m	0.11	0.11	0.08	0.08	0.11	0.11	0.08	0.08
d. Ratio of Spread to Expected Loss								
AAA	0.187	0.187	0.183	0.187	0.185	0.184	0.181	0.184
BB	0.194	0.197	0.195	0.197	0.192	0.195	0.192	0.194
Equity	0.294	0.288	0.271	0.268	0.290	0.284	0.267	0.264

Notes: the portfolio of collateral refers to a mix of 80 percent BB-rated obligors and 20 percent B-rated obligors. The first four columns refer to funded instruments while the second four columns refer to unfunded instruments. For each funding type, the first two columns refer to a one-year pd based on cohort estimates of the transition matrix while the second two columns use a one-year pd based on Markov estimates. Tranche spreads and expected loss are reported in basis points.

Source: author's calculations

Table 5: Spreads on Tranched Securities and Corporates

A. Short-term Asset-backed securities (maturity of 3 years)

	Credit Cards	Prime Auto	Near Prime Auto	Ag/Const Equipment	Medical Equipment	Stranded Assets
AAA	0	4	16	16	50	4
AA						
A	21	18		28		
BBB	45			85		

B. Long-term Asset-backed securities and Corporates (maturity of 5 to 10 years)

	Home Equity	Mfg Housing	CMBS	MBS	Industrials	Banks	Finance
AAA	80	140	31		10		
AA	120	200	39	97	1	-6	-5
A	165		48	117	10	23	29
BBB	190	400	81	162		71	41
BB				287			
B				782			

C. Collateralized debt obligations (weighted average life of 7 to 11 years)

	High-yield CLO	SF CDO	Synthetic CDO	High-yield CBO
AAA	33	39	53	45
AA	65	85	100	85
A	105	155	150	160
BBB	200	315	300	300
BB	600			775

Notes: Spreads are calculated in basis points over Swaps/LIBOR. Credit cards and prime auto in panel (a) as well as home equity in panel (b) refer to fixed-rate collateral.
Source: JP Morgan's *Global ABS/CDO/Weekly Market Snapshot* (10/22/2004).

Table 6: Corporate-Treasury Spreads and Expected Loss								
	1 to 3 years		**3 to 5 years**		**5 to 7 years**		**7 to 9 years**	
	Spread	**Expected Loss**	**Spread**	**Expected Loss**	**Spread**	**Expected Loss**	**Spread**	**Expected Loss**
Aaa	49.50	0.06	63.86	0.18	70.47	0.33	73.95	0.61
Aaa	58.97	1.24	71.22	1.44	82.36	1.86	88.57	2.70
A	88.82	1.12	102.91	2.78	110.71	4.71	117.52	7.32
Baa	168.99	12.48	170.89	20.12	185.34	27.17	179.63	34.56
Ba	421.20	103.09	364.55	126.74	345.37	140.52	322.32	148.05
B	760.84	426.16	691.81	400.52	571.94	368.38	512.43	329.40

Note: the table reports option-adjusted spread bond indicies from Merrill Lynch using data from January 1997 to August 2003 and expected loss based on the one-year ratings transition matrix based on historical Moody's data as reported in Altman and Kishore (1998). Spreads and expected loss are in basis points.
Source: Amato and Remolona (2003).

Table 7: The Fraction of Corporate-Treasury Spreads Explained by Credit Risk								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Aaa	15.8	9.6	58.6	49.5	18.2	20.8	17.4	84.3
Aa	15.6	9.4	37.9	36.6	16.4	20.0	17.3	79.8
A	19.0	11.8	31.3	32.1	18.3	23.4	21.2	82.8
Baa	29.1	19.9	30.6	30.6	26.9	33.6	31.7	79.7
Ba	60.1	48.1	51.8	N/A	57.1	63.3	62.2	82.3
B	82.5	72.8	86.9	N/A	79.1	83.6	84.0	97.0

Notes: the table reports the fraction of the 10-year corporate-treasury spread explained by each of eight structural models: Longstaff and Schwartz (1995) in column (1); adding stochastic interest rates in column (2); Leland-Toft (1996) endogenous default in column (3); Anderson-Sundaresan-Tychon (1996) strategic default in column (4); Collin-Dufresne and Goldstein (2001) mean-reverting leverage in column (5); counter-cyclical market risk premium in column (6); Anderson, Benzoni, and Lund (2001) jump-diffusion firm value process in column (7); and adding extreme parameters to jump-diffusion in column (8).
Source: Huang and Huang (2004)

Table 8: Leverage and Spreads for Unfunded Tranches

e	s_d	s_e	$e^*s_e+d^*s_d$
0%	59.4	n/a	59.4
1%	42.8	3645.8	78.8
2%	31.1	2492.6	80.3
3%	22.7	1892.5	78.8
4%	16.9	1509.7	76.6
5%	12.6	1253.6	74.6
6%	9.3	1068.1	72.9
7%	7.0	925.8	71.3
8%	5.2	815.1	70.0
9%	3.9	726.4	68.9
10%	2.9	653.5	68.0
20%	0.1	315.6	63.2
30%	0.0	205.2	61.6
40%	0.0	151.9	60.8
50%	0.0	120.6	60.3
60%	0.0	100.0	60.0
70%	0.0	85.4	59.8
80%	0.0	74.5	59.6
90%	0.0	66.1	59.5
91%	0.0	65.4	59.5
92%	0.0	64.6	59.5
93%	0.0	63.9	59.5
94%	0.0	63.2	59.4
95%	0.0	62.6	59.4
96%	0.0	61.9	59.4
97%	0.0	61.3	59.4
98%	0.0	60.6	59.4
99%	0.0	60.0	59.4
100%	n/a	59.4	59.4

Notes: the table refers to a CDO with an equity tranche and a single debt tranche. The columns describe the spread on an unfunded debt tranche, the spread on an unfunded equity tranche, and the face-weighted average spread on unfunded tranches.
Source: author's calculations.

Table 9: Model Performance for Unfunded Tranches

a. Normal copula

Tranche	0-3%	3-6%	6-9%	9-12%	12-22%
Quote	**27.6%**	**168**	**70**	**43**	**20**
ϱ = 0.0%	44.3%	69	0	0	0
ϱ = 5.0%	39.7%	161	10	1	0
ϱ = 10.0%	35.4%	222	36	6	0
ϱ = 15.0%	31.5%	258	64	18	2
ϱ = 20.0%	27.9%	281	90	33	6
ϱ = 25.0%	24.5%	294	110	49	11
ϱ = 30.0%	21.2%	300	127	64	18
ϱ = 40.0%	15.2%	299	151	86	34
Tranche implied ϱ	20.4%	5.5%	16.1%	23.3%	31.2%
Base implied ϱ	20.4%	28.8%	33.7%	36.9%	44.8%

b. t-copula

Tranche	0-3%	3-6%	6-9%	9-12%	12-22%
Quote	**27.6%**	**168**	**70**	**43**	**20**
ϱ = 0.0%	43.7%	66	0	0	0
ϱ = 5.0%	41.0%	107	9	3	1
ϱ = 10.0%	37.9%	133	23	10	4
ϱ = 15.0%	34.8%	150	37	18	8
ϱ = 20.0%	31.7%	161	49	26	13
ϱ = 25.0%	28.6%	167	60	35	18
ϱ = 30.0%	25.5%	171	69	42	23
ϱ = 40.0%	19.5%	173	84	56	34
Tranche implied ϱ	26.6%	25.8%	30.3%	30.4%	27.0%
Base implied ϱ	26.6%	26.6%	26.0%	25.3%	24.1%

Notes: the table reports market quotes and model prices for the iTraxx EUR tranches. Market data are reported from August 4, 2004. The quote for the equity tranche is an upfront payment as a percent of notional paid in addition to 500 basis points per year. Quotes for all other tranches are in basis points per year. Model prices in panel (a) are computed using a single-factor model with a nomal copula while panel (b) are computed using a t-copula with 4 degrees of freedom.
Source: Hull and White (2004)

Table 10: Mis-pricing and Feasible Structures							
e	ψ^e_f	Δ_e	Δ_e **fees**	**s**	ψ^m_f/ψ^d_f	Δ_m	$l^m/(s\text{-}e)$
0%	0.0%	-∞	-∞	**4.0%**	0.0%	-∞	n/a
1%	28.3%	-8.25	-8.34	**5.0%**	25.5%	-5.01	2203.9
2%	48.3%	-2.76	-2.82	**5.9%**	44.4%	-0.19	1922.1
3%	62.5%	-1.00	-1.04	**6.9%**	58.2%	1.36	1686.0
4%	72.4%	-0.18	-0.21	**7.8%**	68.8%	2.13	1496.5
5%	79.7%	0.29	0.26	**8.8%**	76.6%	2.55	1334.4
6%	85.0%	0.59	0.56	**9.8%**	82.5%	2.82	1199.4
7%	88.9%	0.78	0.75	**10.7%**	87.0%	3.00	1084.9
8%	91.8%	0.92	0.89	**11.7%**	90.3%	3.13	986.5
9%	93.9%	1.01	0.98	**12.6%**	92.9%	3.22	902.4
10%	95.5%	1.07	1.05	**13.6%**	94.8%	3.28	829.1
20%	99.9%	1.25	1.22	**14.6%**	96.3%	3.32	765.5
30%	100.0%	1.25	1.23	**15.5%**	97.3%	3.36	709.8
40%	100.0%	1.25	1.23	**16.5%**	98.1%	3.38	660.7
50%	100.0%	1.25	1.23	**17.4%**	98.7%	3.40	617.4
60%	100.0%	1.25	1.23	**18.4%**	99.1%	3.41	578.7
70%	100.0%	1.25	1.23	**19.4%**	99.4%	3.42	544.3
80%	100.0%	1.25	1.23	**20.3%**	99.6%	3.43	513.4
90%	100.0%	1.25	1.23	**21.3%**	99.8%	3.43	485.6
91%	100.0%	1.25	1.23	**22.2%**	99.9%	3.43	460.5
92%	100.0%	1.25	1.23	**23.2%**	99.9%	3.43	437.7
93%	100.0%	1.25	1.23	**24.2%**	100.0%	3.44	417.0
94%	100.0%	1.25	1.23	**25.1%**	100.0%	3.44	398.1
95%	100.0%	1.25	1.23	**26.1%**	100.0%	3.44	380.9
96%	100.0%	1.25	1.23	**27.0%**	100.0%	3.44	365.0
97%	100.0%	1.25	1.23	**28.0%**	100.0%	3.44	350.4
98%	100.0%	1.25	1.23	**29.0%**	100.0%	3.44	337.0
99%	100.0%	1.25	1.23	**29.9%**	100.0%	3.44	324.5
100%	100.0%	1.25	1.23	**30.9%**	100.0%	3.44	312.9

Table notes: the first four columns refer to a CDO with a single debt and equity tranche. The third and fourth column refer to the maximum mis-pricing of the equity tranche that is possible in order for the structure to exist with zero and two percent structuring fees, respectively.
Source: author’s calculations