The effects of transaction costs on depth and spread*

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Abstract

This paper develops a model of depth and spread setting by a monopolistic dealer under asymmetric information to investigate the effect of imposing a transaction cost on the dealer's quotes. Increasing the transaction cost affects the depth and the spread non-linearly. Under some distributional assumptions, when market conditions are favourable to the dealer, the spread responds less than proportionally to an increase in the transaction cost while the depth actually increases. In contrast, when market conditions are unfavourable to the dealer, the spread widens more than proportionally and the depth decreases, potentially to zero, in response to an increase in the transaction cost.

* I thank Vincent Reinhart for helpful discussions. This paper is an extension of my paper "market making, prices, and quantity limits" to which the reader is referred for more details. The views expressed herein are the author's and do not necessarily reflect those of the Board of Governors or the Federal Reserve System.

1. Introduction

This paper develops a theoretical set-up to examine how the liquidity of a quote-driven market is affected by the introduction of a transaction cost, such as a tax. Three different measures of liquidity are examined: the spread, the depth and the mean trading volume. The depth, or quantity limit, is the maximum amount the dealer stands ready to sell or buy at the posted prices. The paper features a one-period model where a monopolistic market maker facing a risk-neutral informed trader and a liquidity trader posts firm prices and depths on the bid and ask sides. The informed trader observes a private signal correlated with the true value of the asset. The demand of the liquidity trader is decreasing in price and increasing in the liquidity shock. Three cases are analysed. First, the asset value, the informed trader's signal and the liquidity shock each take two values. Second, the asset value is normally distributed with mean 0. Third, the asset value is lognormally distributed with mean 1. In the last two cases, the private signal and the liquidity shock are normally distributed. In the model, some level of asymmetric information has to exist for the risk-neutral dealer to impose finite quantity limits. Information asymmetries are generally believed to play a lesser role in bond markets than in equity markets, but need not be entirely absent. For example, prices of government securities move on the release of macroeconomic news. In these markets, information asymmetry stems from the potential ability by some agents to forecast the data more precisely. Even if the fundamental value of Treasury securities is assumed to be public knowledge, private information may still exist about incoming order flow, which could have short-term effects on prices.

Several conclusions are drawn from the model:

Introducing a transaction cost has non-linear effects on the spread and the depth. When the random variables follow discrete distributions, imposing a finite depth is not necessary if the asymmetry in information is low enough. In that case, a small increase in the transaction cost pushes up the spread but does not affect the depth, while a larger change may result in the imposition of a finite quantity limit but may not translate into a spread increase of equal magnitude. In contrast, when the information asymmetry is more severe, quantity limits are imposed, and increasing the transaction cost has a proportional effect on the spread. When the asset value is normally or lognormally distributed, the effect depends on whether market conditions are favourable or unfavourable to the dealer.

- When market conditions are favourable to the dealer, introducing a transaction cost increases the spread by less than the amount of the transaction cost and creates a larger depth. However, the spread effect dominates and trading volume falls.
- When market conditions are unfavourable to the dealer, introducing a transaction cost pushes the price up by more than the transaction cost and narrows the depth. Those two effects combine to greatly depress the trading volume, ultimately pushing it to zero.

The strength of the liquidity demand plays a central role in determining the equilibrium spread and depth. When studying the market making process, one can focus on the liquidity conditions the dealer faces rather than on information asymmetry. This is particularly suited to the U.S. government bond market. For example, spreads are typically narrower and depth larger for securities that are the most recent issues in their maturity class (the on-the-run Treasuries) than for similar securities issued just before (the off-the-run Treasuries). Bid-ask spreads on coupon Treasury securities are traditionally about twice as high for off-the-run issues than for comparable on-the-run issues, while quoted depth is lower. Moreover, during the recent bouts of market volatility, bid-ask spreads have widened and depths have contracted proportionally more for off-the-run than for on-the-run coupon securities. The asymmetric information argument cannot easily account for this fact since the quality of private information should be equal across the two market segments. However, agents trading bonds prefer to trade in on-the-run securities, therefore creating a stronger liquidity demand in that market. The more fundamental question as to why agents prefer to trade in the on-the-run segment is not addressed here, although self-fulfilling expectation arguments could be made.

The paper seems to point to two regimes as far as transaction cost is concerned. When market conditions are favourable, the dealer pays part of the transaction cost himself (by increasing the spread less than the cost) and quotes a larger depth to attract order flow in order to make up for the loss in demand due to the transaction cost. The increase in the depth offsets, albeit partially, the effect on trading volume of the wider spread. When market conditions are unfavourable, increasing the transaction cost leads to a drastic reduction in the liquidity provided by the market maker, enticing him to exit the market.

Although the implications of the model have been introduced by presenting the effect on the spread and the depth of an increase in the transaction cost, symmetric conclusions hold for a reduction in this cost. As a consequence, a decision to lower taxes on transactions in the hope of improving market liquidity might actually lead to smaller depths and a less-than-proportional reduction in the spread. This is because, when market conditions are rather favourable to the dealer, a tax – in so far as it is at least partially reflected in the bid and ask prices – reduces the probability of the informed trader's buying at the ask or selling at the bid. This additional protection entices the market maker to quote a larger depth than he would without tax.

2. Model

A monopolistic dealer posts firm prices and depths on the bid and ask sides. He faces a price-sensitive liquidity trader and a trader possessing private information about the value of the asset. To simplify computations, we assume that the informed trader is risk neutral. We study only the ask side of the dealer's activity, the bid side being symmetrical. Call x the true value of the asset, c the transaction cost, a the ask price inclusive of cost, z the quantity limit, G the informed trader's private signal, $v = E[x|G]$ his valuation of the traded asset, η the liquidity shock (η and G are independently distributed), $d(a)$ the liquidity trader's demand, $q(a)$ the informed trader's demand. When x is normally distributed $d(a) = -a + \eta$, when it is lognormally distributed d(a) = - log(a) -. In all cases, the distribution of (x, η,G) is common knowledge. The orders of the liquidity trader and of the informed trader are pooled together and passed on to the dealer. To simplify, when the quantity demand is binding, we assume that the informed trader's orders are satisfied first. By convention, the dealer pays the transaction cost but, ceteris paribus, the transaction cost could be levied on the customers. Let P be the dealer's profit before subtracting the costs of transaction, and I the indicator function. Since the informed trader is risk neutral, $q(a)$ is zero if $v \le a$, infinite if $v > a$. Hence,

$$P(a,z,v,\eta) = I(v \le \text{a})\, I(d(a) \ge 0)\, (a - x) \min(d(a),z) + I(v > a)\, (a - x)\, z \tag{1}$$

To acquire one unit of the traded asset, informed and liquidity traders pay a, but the dealer receives only $(a - c)$ if the transaction cost is a fixed amount per transaction (fixed transaction cost), or $(\frac{a}{1+c})$ if it is a fixed percentage of the price (proportional transaction cost). Let Π be the dealer's profit net of transaction costs and S the transaction volume on the ask side.

$$S(a,z,v,\eta) = I(v \le \text{a})\, I(d(a) \ge 0) \min(d(a),z) + I(v > a)\, z \tag{2}$$

$\Pi = P - cS$ with a fixed transaction cost and $\Pi = P - \frac{c}{1+c}\, aS$ with a proportional transaction cost.

3. Equilibrium with discrete distributions

Let x and G take the values 1 or –1 and η take the values $\overline{\eta}$ or $-\overline{\eta}$ with probability $\frac{1}{2}$. The informed trader's valuation v takes values $\overline{v} = E[x|\text{G} = 1]$ and $\underline{v} = -\overline{v}$ with probability $\frac{1}{2}$. Table 1 displays the optimal price and quantity limit as a function of $\overline{v}, \overline{\eta}$ and c. When the precision of the trader's information is low, the market maker does not need to impose quantity limits. After this precision reaches some threshold, a quantity limit is imposed, after it reaches another threshold, the dealers exits the market by setting its depth to zero. The effect of an increase in the transaction cost c on the spread and the quantity limit depends on the precision of the informed trader's information. Figure 1 graphs the equilibrium half-spread (which is also the ask price) as a function of $\bar{v}$ for different value of c. When the precision of the informed trader's signal is between two values determined by the strength of the liquidity demand and the transaction cost, an increase in c has no marginal effect on the spread; outside these two values, increasing c by an infinitessimal amount raises the selling price by half this amount, resulting into a proportional widening of the spread. A higher transaction cost also shrinks the region of admissible values for the precision of the informed trader's signal: when the degree of asymmetric information is high, raising c incites the dealer's to exit the market by contracting the posted depth to zero.

4. Equilibrium with continuous distributions

Assume now that the asset value is normally or lognormally distributed. The spread can be thought as the dollar spread in the first case, and the percentage spread in the latter (i.e. a percentage of the mean asset value). The key parameter determining the spread and the depth is $\rho \frac{\sigma_x}{\sigma_\eta}$ when x is normally distributed and $\rho \frac{\sigma_y}{\sigma_\eta}$ when x is lognormally distributed. The denominator of this ratio measures the strength of the liquidity demand insofar as the amount the dealer can expect the liquidity trader to buy at the ask or sell at the bid is increasing in σ_η. Figure 2 displays the effects of introducing a fixed transaction cost – equal to $0.1\sigma_\eta$ – when the asset is normally distributed.[1] Figure 3 shows the effect of introducing a proportional transaction cost equal to 1% of the pre-tax sale price when the asset value is lognormally distributed. Introducing a fixed transaction cost has very similar effects. Note that the effects are much larger than when x was supposed to be normally distributed.

5. Conclusion

The model shows that introducing a transaction cost could affect market liquidity differently depending on the market conditions facing the dealer. If the asset value, the informed trader's signal, and the liquidity shock each take two values, there is an interval for the precision of the private signal for which increasing the transaction cost has no or little effect on the spread (no quantity limit is necessary then). If the asset value is normally or lognormally distributed, the market maker widens the spread by less than the transaction cost and actually increases the depth when the degree of information asymmetry is subdued or liquidity demand is strong. In contrast, when market conditions are unfavourable to the dealer, he increases the spread by more than the transaction cost and reduces the depth. For all distributions, introducing a transaction cost may induce the dealer to exit the market before he would have done so in the absence of a transaction cost. This suggests that a transaction tax could aggravate liquidity loss in periods of market stress.

[1] We do not study the effect of a proportional transaction cost, because in that case the mean value of the asset cannot be normalized to zero.

References

Dupont, D, 1998: "Market making, prices, and quantity limits," working paper, Federal Reserve Board.

Table 1

Equilibrium price and quantity limit with risk-neutral informed trader and discrete distributions

$\overline{V}$	$\overline{V} \leq \frac{1}{2}(\overline{\eta}+c)$	$\frac{1}{2}(\overline{\eta}+c) < \overline{V} \leq \frac{3}{5}(\overline{\eta}+c)$	$\frac{3}{5}(\overline{\eta}+c) < \overline{V} \leq 3(\overline{\eta}+c)$	$\overline{V} > 3(\overline{\eta}-c)$
optimal *a*	$\frac{1}{2}(\overline{\eta}+c)$	$\overline{V}$	$\frac{1}{2}\overline{\eta}+\frac{1}{2}c+\frac{1}{6}\overline{V}$	n.a.
optimal *z*	n.a.	n.a.	$\frac{1}{2}\overline{\eta}-\frac{1}{2}c-\frac{1}{6}\overline{V}$	0
max $(E[\pi])$	$\frac{1}{8}(\overline{\eta}-c)^2$	$\frac{1}{2}(\overline{V}-c)(\overline{\eta}-\overline{V})$	$\frac{1}{48}(3\overline{\eta}-3c-\overline{V})^2$	0
maximum *c*	$\overline{\eta}$	$\frac{2}{3}\overline{\eta}$	$\frac{1}{4}\overline{\eta}$	0

When $a \geq \overline{V}$, imposing a quantity limit is not necessary; the optimal *z* is undefined and "n.a." is entered in the corresponding cell. When $\overline{V} > 3(\overline{\eta} - c)$, the expected profit is never positive and the dealer exits the market by setting *z* to 0; the optimal *a* is undefined and "n.a." is entered in the correspoding cell.

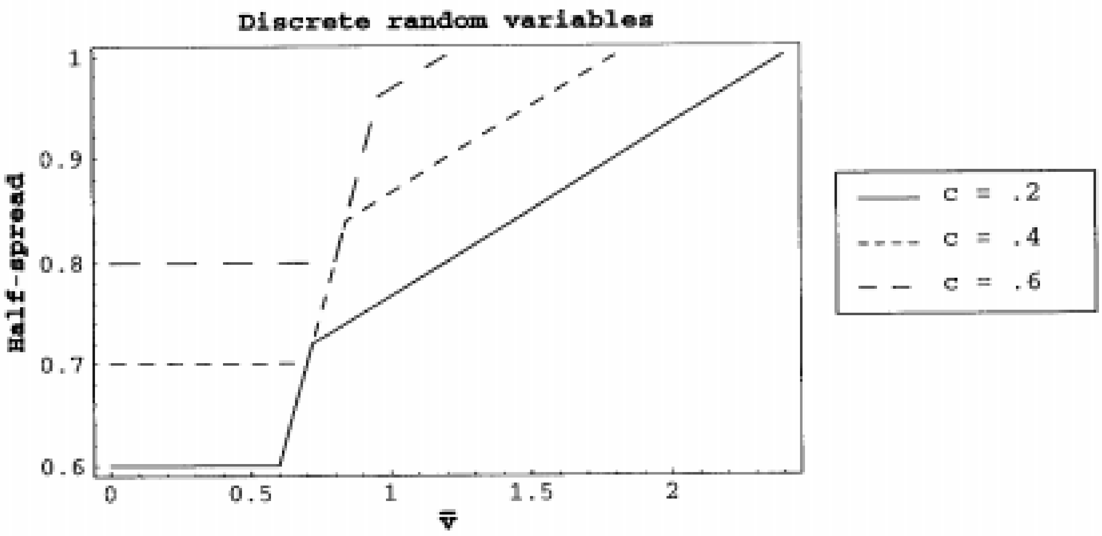

Figure 1: Half-spread as a function of $\bar{v}$ for the following values of the transaction cost: $c = .2$ (solid line), $c = .4$ (short-dashed line), $c = .6$ (long-dashed line). In this graph, half-spread and transaction cost are divided by $\bar{\eta}$.

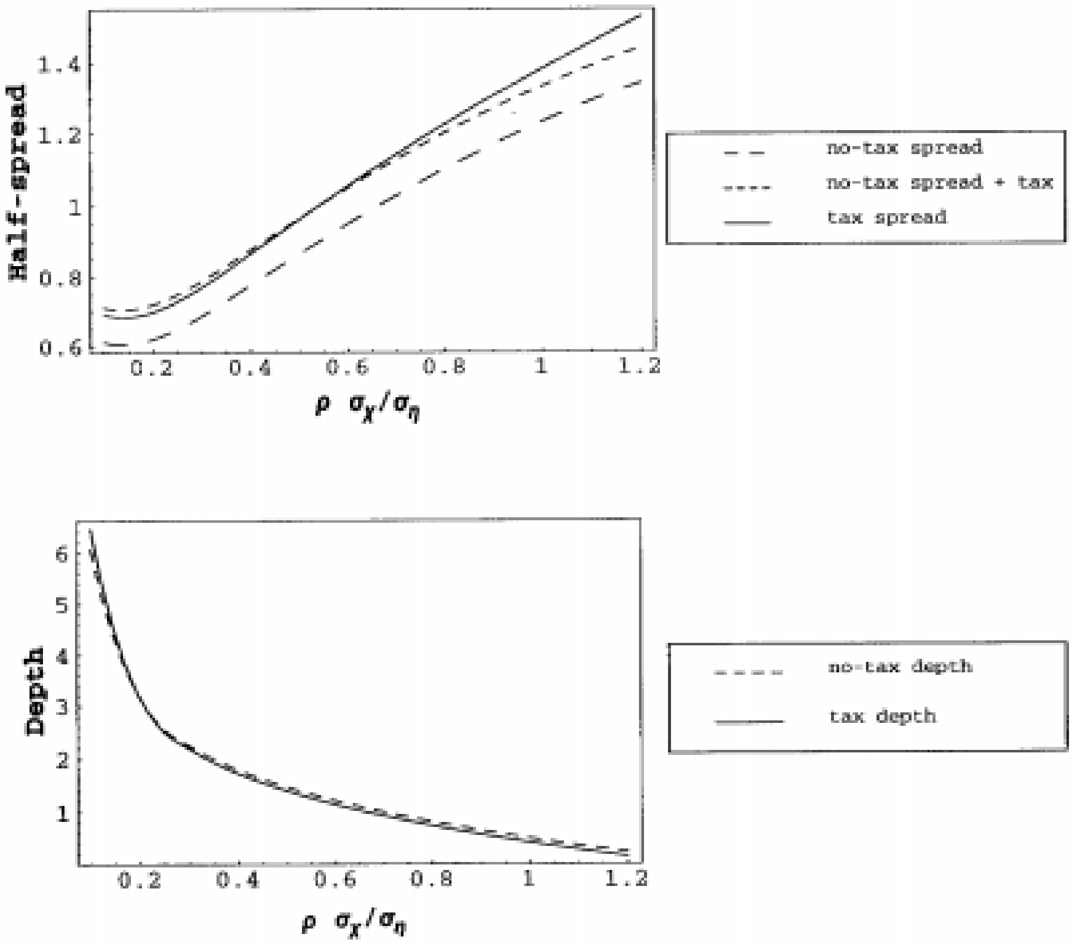

Figure 2: Half-spread and depth with a **fixed transaction cost** of $0.1\,\sigma_\eta$ (solid line) and without transaction cost (short-dashed line). The asset value, x, is **normally distributed**. In the top panel, the long-dashed line is the sum of the price without transaction cost and the transaction cost. Half-spread and depth are divided by σ_η.

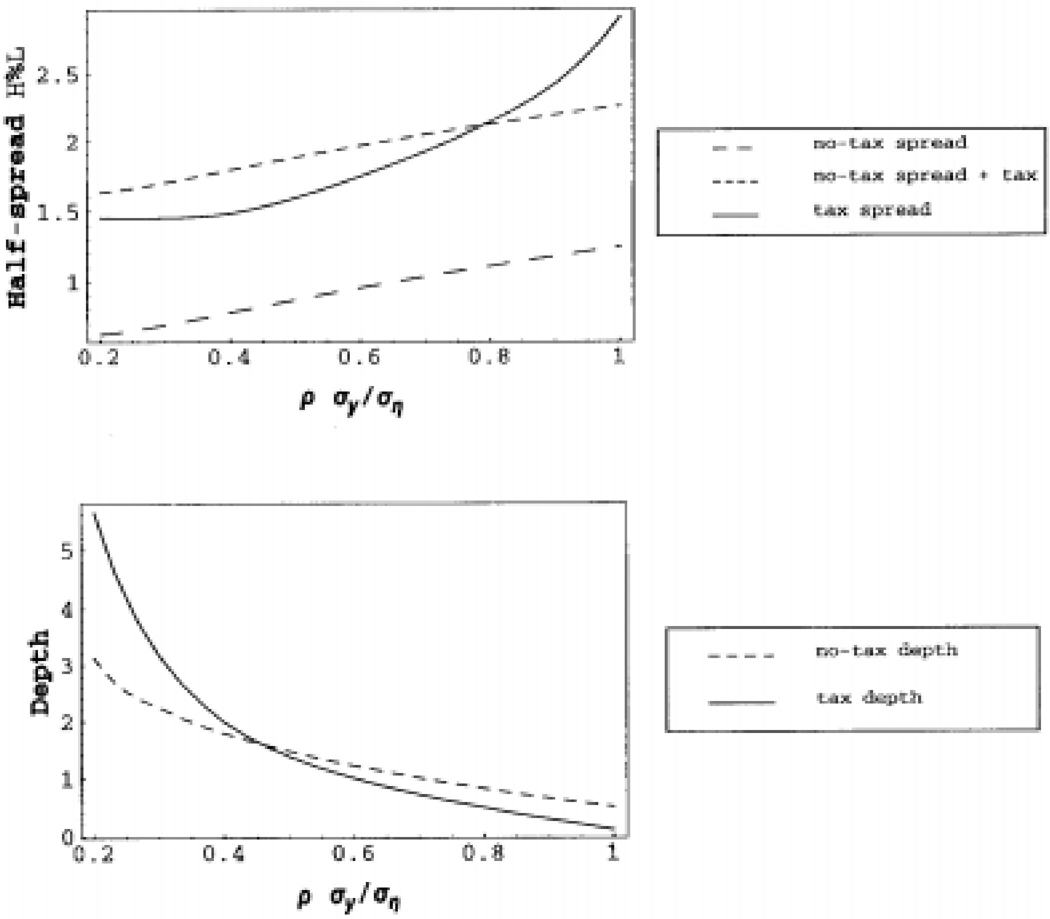

Figure 3: Half-spread (in percentage) and depth with a **proportional transaction cost of 1 percent** (solid line) and without transaction cost (short-dashed line). The asset value, x, is **log-normally distributed** and $y = \log(x)$. In the top panel, the long-dashed line is the sum of the price without transaction cost and the transaction cost. Half-spread and depth are divided by σ_η.