Inflation expectations and a model-based core inflation measure in Colombia

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Abstract

Empirical evidence following conventional tests suggests that inflation expectations in Colombia might not be rational, although the period of disinflation included in the sample makes it difficult to verify this conclusion. Inflation expectations display close ties with observed past and present headline inflation and are affected by exogenous shocks in a possibly non-linear way. A model-based core inflation measure is computed that addresses the shortcomings of traditional exclusion measures when temporary supply shocks have widespread effects and are persistent.

Keywords: Inflation expectations, core inflation, supply shocks, monetary policy

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1. Introduction

Since mid-2014, the external conditions of the Colombian economy have changed dramatically. With the sudden and sharp fall in oil prices, the country's terms of trade have rapidly deteriorated, while international financial conditions have tightened. As a result, the currency has experienced a strong depreciation, of around 60% in annual nominal terms. At the same time, weather-related shocks substantially increased food prices twice in 2015. The latest round of price rises is driven by an abnormally strong El Niño phenomenon that is causing an intense drought in the country that is affecting both food and (hydroelectric) energy prices. The coincidence of large currency and supply shocks has pushed CPI inflation beyond 6.5%, well above the 3% target.

This poses a big challenge to the monetary authorities, not only due to the concurrence of two large shocks, but also because, unlike in past depreciation episodes, a reversion of the Colombian peso (COP) to pre-shock values is highly improbable this time, as the currency adjustment follows a persistent and indefinite fall in international oil prices, as well as a long-lasting global liquidity retrenchment process. In contrast, weather-related relative food price shocks tend to be followed by large reversions produced, in part, by a "cobweb-like" behaviour of food prices and quantities. Hence, monetary policymakers must deal with a combination of large inflation shocks of differing characters and persistence.

In this context, the appropriate policy response in an inflation targeting regime crucially depends on the behaviour of inflation expectations. As long as these remain in line with the 3% target, the shocks could be treated as purely transitory events that require only a small tightening of monetary policy. Indeed, if the terms of trade shock produced a contraction of expenditure beyond what is required to maintain a sustainable path for the current account deficit, anchored inflation expectations would allow an expansionary monetary policy response. This is why understanding and monitoring the behaviour of inflation expectations has become a centrepiece of monetary policy analysis and discussion in Colombia. Are inflation expectations formed "rationally", as assumed in our macroeconomic models? If not, how are they formed? How do they respond to the exogenous shocks that have hit the economy? How has this response changed over time (especially since the long-term inflation target was reached)? How to assess the probability of their de-anchoring from target? These are some of the questions that will be addressed in this note on the basis of work done recently at the Bank of the Republic.

Of importance too is the measurement of core inflation in the context of the above-mentioned shocks. Disentangling the core and shock components of rising inflation in the midst of ongoing large and diverse shocks is technically challenging, yet crucial if the evolution of "macro" inflation is to be ascertained and suitable policy responses determined. In the presence of coinciding shocks with differing durations and channels of transmission, exclusion measures may not adequately represent the behaviour of core inflation. Widespread temporary supply shocks (like the COP depreciation shock) may affect a significant portion of the price index basket. Hence, exclusion measures may fail to filter them. Moreover, if the supply shocks are persistent, separating the direct impact of the shocks from their macroeconomic consequences (ie activation of indexation mechanisms, effects of expectations or monetary policy responses etc) becomes increasingly difficult with time. Technically, the derivation of adequate core inflation measures depends on the identification of the supply shocks. Consequently, this note presents a model-based core inflation measure, defined as observed inflation minus the model-identified supply shocks.

2. Characterisation of inflation expectations

In Colombia, inflation expectations are measured on the basis of a monthly survey of professional forecasters, a quarterly survey of a broader set of agents that includes some businesses, academics and labour unions, and bond-derived break-even (BI) and forward break-even (FBEI) inflation rates. Table 1 summarises the main features of these measures, while Graph 1 shows their time series along with the corresponding realised future annual inflation. Both realised inflation and inflation expectations exhibit a downward trend that reflects their gradual convergence to the long-term 3% target. Recently inflation has risen sharply as a result of the aforementioned shocks.

FBEI measures have generally been above realised inflation. This may be due to the fact that, although the 3% long-term target was announced as early as 2001, the exact convergence path was not defined. Thus, the FBEI for two years ahead or more seemingly imply a slower expected convergence path than the actual one. This is also consistent with the findings of González and Hamann (2011), who argue that the high and stable inflation persistence observed in Colombia is related to imperfect information of agents about the inflation target, rather than to indexation.

Monthly survey expectations for annual inflation one year ahead display a low coefficient of variation across respondents (10% on average since 2003), suggesting a small degree of dispersion for this measure (Graph 2). Quarterly survey expectations for annual inflation one year ahead exhibit a slightly greater dispersion (coefficient of variation of 15 % on average since 2003), a feature that may be explained by the more diverse set of respondents (Graph 2). Within sectors of the quarterly survey, the dispersion is also low, with the highest average coefficient of variation corresponding to labour unions (17%). However, the dispersion of expectations of the quarterly survey has reached high levels in some periods, especially around the end of 2009 and the beginning of 2010, after inflation fell steeply and the long-term inflation target was reached.

Are inflation expectations rational?

Table 2 presents the results of the conventional tests for rationality of inflation expectations (see for example Mankiw et al (2003) and Huertas et al (2015) for the case of Colombia). Expectations are deemed as rational if (i) they are co-integrated with realised inflation,[2] (ii) they are unbiased predictors of realised inflation, (iii) they are efficient, ie no further information helps improve their forecast of inflation. As seen in Table 2, co-integration is observed for all measures, but F2BEI3. Survey expectations and BEI1 are found to be unbiased, while all FBEI measures are biased. The lack of co-integration of F2BEI3 and the bias found for FBEI measures are not surprising, given the short sample and the mentioned uncertainty about the convergence path toward the long-term inflation target.

[2] In the sample inflation and inflation expectations appear to be integrated of order 1.

The efficiency requirement is not fulfilled, since there is strong auto-correlation of co-integration residuals. Moreover, in some cases, lagged values of the deviation of inflation from target, the output gap and the change in the policy rate are significantly associated with the co-integration residuals.

Hence, in general, it seems that expectations measures are not rationally formed, at least according to the conventional definition. An additional indication in this regard can be obtained from the comparison of the inflation expectation measures and the rational inflation expectations that are derived from DSGE models estimated with Colombian data.[3] In general, model-based rational expectations are closer to realised future inflation than inflation expectation measures, as suggested by their higher correlation coefficients and lower root mean square errors (RMSE). Conversely, inflation expectation measures seem to have a tighter relationship with contemporaneous inflation than model-based rational inflation expectations do, according to the same indicators (Table 3). These results point to a large influence from present observed inflation in the formation of inflation expectations.

In short, the evidence presented cast doubts about the rationality of inflation expectation measures. However, as stressed by Andolfatto et al (2008), conventional rationality tests may be plagued by short sample problems, and seemingly non-rational expectations may actually be formed rationally in a context of imperfect information about the inflation target and short-term learning dynamics. Indeed, some of the estimations for Colombia are based on short samples (especially for FBEI), while the work of González and Hamann (2011) supports the hypothesis of rational expectations under imperfect information about the inflation target for a significant part of the sample period.

If not rationally, how are inflation expectations formed?

If inflation expectations are not formed rationally, there are several alternative hypotheses regarding their determination. Huertas et al (2015), explore two sets of hypotheses. One states that inflation expectations follow adaptive learning by agents (Pfajfar and Santoro (2010))[4] and the other postulates that measured inflation expectations result from combinations of rational and adaptive expectations, or combinations of the inflation target and adaptive expectations (Heinemann and Ullrich (2006) and Oral et al (2011)).

Under adaptive learning, agents establish a rule to forecast inflation and update it with their forecast error once new data are observed. For the purpose of this note, a simple rule linking inflation expectations to past observed inflation is used (as in Huertas et al (2015)). If there is learning, the coefficient of past inflation will be updated through time. If not, it will be a constant. Table 4 shows adaptive learning (positive learning coefficient, v) for the monthly and quarterly survey expectations, as well as for the F1BEI1 and F2BEI3. The latest estimates of the coefficient of observed past inflation (ϕ_I) range from 0.33 (monthly survey) to 0.70 (BEI1), suggesting again

[3] Three DSGE models with nominal rigidities and "hybrid" Phillips curves are used. The first one, "Patacon", is a complex, open economy model regularly used for policy analysis, simulation and forecast (González et al (2011)). The second one is a simpler tradable/non-tradable model with nominal rigidities. The third one is a traditional textbook, closed economy New Keynesian model.

[4] See Appendix 1 for a brief description of the adaptive learning model.

an important influence of observed inflation on expectations (Table 4). Interestingly, for the expectation measures that exhibit learning, this coefficient declined after 2007 and stabilised around 2009–10, after the long-term inflation target was reached (Graph 3).

If measured inflation expectations were a mix of rational and adaptive expectations, the adaptive component would generally be dominant, as illustrated by the regression results presented in Table 5.[5] The weight of the adaptive part is lower for FBEI indicators, a result that is not surprising, as they forecast inflation at longer horizons. The pre-eminence of the adaptive component remains when measured inflation expectations are expressed as a combination of the relevant inflation target and adaptive expectations (Table 6).[6] This combination fits the data better than the combination of rational and adaptive expectations (higher adjusted R^2).[7]

In sum, inflation expectation measures in Colombia do not seem to conform with the rational expectations paradigm, although the caveats of the conventional tests in this regard are relevant, given the disinflation process experienced during part of the period examined. There is some evidence in favour of adaptive learning and, generally, contemporaneous and past observed inflation have a strong influence on all measures of inflation expectations.

3. Anchoring of inflation expectations

As initially stated, the degree to which inflation expectations remain anchored to the target after an exogenous shock hits the economy conditions the corresponding monetary policy reaction. That is why it is useful to assess how far inflation expectations are anchored. This poses some technical challenges. First, the exogenous shock must be properly identified in order to avoid the possible bias that emerges when endogenous variables are used as regressors. Second, the shocks hitting the economy may differ in nature and persistence. Consequently, an estimated response of inflation expectations would be related to an "average" shock and it may not accurately reflect the response to a specific shock that deviates from the "average". In other words, the estimated response of inflation expectations reflects not only an

5 Following Huertas et al (2015), the regression model

$$\pi^e_{t+s/t} = c_1\pi_{t+s} + (1-c_1)\left[\pi^e_{t/t-s} + c_2\left(\pi_t - \pi^e_{t/t-s}\right)\right] + \varepsilon_t$$

was estimated for all inflation expectations measures $\pi^e_{t+s/t}$. The coefficient c_1 represents the weight of the rational expectations, $1-c_1$ denotes the weight of adaptive expectation and c_2 the speed at which past forecasting errors are corrected.

6 The regression model estimated in this case is similar to the one considered for the combination of rational and adaptive expectations, with the relevant inflation target in place of the realised future inflation:

$$\pi^e_{t+s/t} = c_1 Target_{t+s} + (1-c_1)\left[\pi^e_{t/t-s} + c_2\left(\pi_t - \pi^e_{t/t-s}\right)\right] + \varepsilon_t$$

7 Even though both inflation and the inflation expectations measures are I(1) series, the residuals of the regressions presented in Tables 5 and 6 are generally stationary. Hence, the probability of spurious correlation is small.

"inherent" characteristic of the expectation formation process, but also a combination of that process *and* the particular realisation of shocks throughout the sample.[8]

To address the first issue, two alternative tacks are pursued. First, the deviation of inflation expectation measures from the relevant inflation target is regressed against some exogenous variables that are known to affect the Colombian economy.[9] Secondly, the same deviation is regressed against food supply, general supply, demand and policy shocks that are obtained from a simple semi-structural macro model estimated with Colombian data.[10] The results of these estimations are subject to the second issue mentioned, namely that the estimated responses reflect both the nature of the expectations formation mechanism and the realisation of the shocks themselves. This is especially relevant for shorter-horizon inflation expectations (eg one year ahead).

To address both issues at the same time and account for possible non-linearities in the expectations formation process, a third exercise based on Guarín et al (2015) is presented in which the probability that long-term FBEI expectations will become de-anchored is estimated as a function of exogenous variables.

The relationship between inflation expectations and some exogenous variables and shocks

Changes in the international oil price and the intensity of El Niño phenomenon[11] are associated with deviations of survey-based expectations from the relevant inflation target in the period 2003–15 (Table 7). Increases in the oil price are negatively related with deviations of inflation expectations from the target. This could be due to the currency appreciation that follows a rise in the oil price (oil is a major Colombian export). Increasing intensity of El Niño phenomenon is positively associated with the deviation of survey expectations from the inflation target. This is probably the consequence of the direct and indirect effects of droughts on inflation and one-year-ahead inflation expectations. These results are clear for the quarterly survey at the aggregate and sectoral level, although less so for the monthly survey. Other exogenous variables, such as an international food price index or the intensity of La Niña phenomenon,[12] are not significantly associated with deviations of survey expectations from the inflation target. No significant effects of exogenous variables on BEI or FBEI measures were found.

For the second exercise, food supply, general supply, demand and policy shocks are obtained from a small semi-structural model estimated for Colombia by Bejarano

8 For example, a supply shock that permanently shifts the price level upwards would produce a response of annual inflation expectations that differs from their reaction to a shock of the same initial size that increases the price level for only a few months.

9 Estimated regression: $\pi^e_{t+s/t} - Target_{t+s} = a_1 + a_2 \Delta Oil\ Price_t + a_3 El\ Niño_t + \varepsilon_t$

10 Estimated regression: $\pi^e_{t+s/t} - Target_{t+s} = a_1 + a_2\ Supply\ Shock_t + a_3\ Demand\ Shock_t + a_4\ Policy\ Shock_t + a_5\ Food\ Shock_t + \varepsilon_t$

11 This intensity index is taken from NOAA (National Oceanic and Atmospheric Administration of the United States Department of Commerce).

12 La Niña is the opposite of El Niño, ie excessive rain and floods in Colombia.

et al (2015)[13] and are used as independent variables in the regressions for the deviation of expectation measures from the inflation target between 2003 and 2015. Although the shocks are model-dependent, their use helps minimise endogeneity-related bias in the estimation.[14]

For the quarterly survey inflation expectations, a significant positive effect from the general supply shocks on the deviation of expectations from target is obtained. Moreover, this effect has been rising since 2014 (Graph 4). The latest estimate indicates that a 1% general supply shock produces a deviation of quarterly expectations from the target of 0.38% (Table 8, second column). Other shocks do not significantly affect the anchoring of this expectations measure (Graph 4). Similar results are obtained for the sectoral components of the survey (Table 8).

Estimations for the monthly survey inflation expectations point in the same direction (Table 8). Interestingly, positive interest rate shocks reduced the deviations of the inflation expectations from target in part of the sample period (Graph 5). For BEI1 general supply shocks have a significant "de-anchoring" effect only by the end of the sample (Table 8 and Graph 6), while for F2BEI3 this effect is larger (a 1% supply shock increases the deviation of expectations from target by 0.68%). Also, estimations for F2BEI3 yield a significantly *negative* impact of demand shocks on the deviation of expectations from target (Table 8 and Graph 7).

In sum, exogenous shocks seem to have affected the anchoring of inflation expectations. Survey expectations are influenced by changes in the international price of oil and by the El Niño phenomenon, while a robust, positive and recently increasing "de-anchoring" effect of general supply shocks was detected. The latter may be due to a loss of credibility of monetary policy in the past year, the realisation of atypically persistent supply shocks (eg the sharp depreciation of the COP), or both.

Assessing the probability that long-term inflation expectations will become de-anchored

Following Guarín et al (2015), the probability that long-term inflation expectations for Colombia would become de-anchored between 2003 and March 2016 is estimated. This probability is computed for zero, three- and six-month horizons as a function of a set of exogenous variables. By focusing on long-term inflation expectations, the issue of disentangling changes in the credibility of monetary policy from the particular sample realisation of exogenous shocks becomes less severe.

A Bayesian model averaging (BMA) of logistic regression is used to estimate the probability that long-term inflation expectations will become de-anchored. This approach is suitable for dealing simultaneously with both model and parameter uncertainty.[15] The empirical exercises consider monthly data from two sets of information. The first set includes the annual inflation rate of CPI, the F2BEI3 as a proxy of long-term inflation expectations, the inflation target and its range. These

13 See a brief description of the model in Appendix 2.

14 The semi-structural macro model is estimated with quarterly variables and yields quarterly series of shocks. Since inflation expectations measures and the target refer to annual inflation, cumulative four-quarter shock series are used in the regressions.

15 Appendix 3 presents a brief description of Bayesian model averaging.

time series are used to build the proxy for the de-anchoring of long-term inflation expectations. A de-anchoring episode is identified when the FBEI rate is greater than the upper bound of the target range for two consecutive months (Graph 8).[16]

The second set of data considers exogenous variables used as possible explicative factors of the probability of de-anchoring. This set includes annual variations in the international food price index (Spot Index Food, SIF) and the Brent oil price, as well as intensity indexes for the El Niño and La Niña phenomena. By using exogenous variables, endogeneity bias in the estimation is avoided. A dummy variable $\mathrm{D_{IT}} = 1_{\{t<\mathrm{Jan\ 2010}\}}$ to discriminate between periods before and after achieving the long-term inflation target is also included.

The estimated episodes of de-anchoring for zero, three and six months ahead[17] exhibit a very good fit and anticipation of the historical events (Graph 9). Three main results are obtained from this exercise. First, significant effects of exogenous variables (climate and international food and oil prices) on the probability of inflation expectations de-anchoring are found. Table 9 reports statistics of the BMA logistic regression, such as the posterior inclusion probability (PIP),[18] the posterior mean and standard deviation of the coefficients, and their positive sign probability.[19] Only variables with the highest PIP are reported. In general, international food prices and the La Niña phenomenon affect the probability of de-anchoring with shorter lags than those of the oil price or the El Niño.[20] The dummy $\mathrm{D_{IT}}$ has a positive coefficient, which implies a larger probability of de-anchoring before the long-term inflation target was reached.

Second, there seems to be a non-linear effect of exogenous shocks on the de-anchoring of inflation expectations. Whereas no significant relationship between exogenous variables and deviations of FBEI from target were found with linear regression over the whole sample period in the previous section, that relationship appeared when critical, de-anchoring episodes were identified in the estimation of the probability of de-anchoring. Moreover, significant coefficients for exogenous variables were obtained with a non-linear logistical probability function specification. This implies that the sensitivity of the probability of de-anchoring to a shift in an exogenous variable will depend on the particular values of other exogenous variables.

Third, a rapid increase in the probability that long-term inflation expectations would become de-anchored in the second-half of 2015 and the beginning of 2016 for the six-month time horizon is detected, although the predicted probability is still below its threshold (Graph 9 and Appendix 3). This indicates an increasing probability of de-anchoring long-term inflation expectations after the strong depreciation and food price shocks mentioned above. Interestingly, this signal is picked up from the behaviour of exogenous oil price and climate shocks, and not from the behaviour of any endogenous variable.

16 The specific choices of the F2BEI3 and two months in our definition of de-anchoring are based on available data and several exercises on the consistency and robustness of results.

17 These periods correspond to those time spans when the probability for each time horizon is higher than the cut-off probability (see Appendix 3).

18 PIP is the probability that a given variable is included in the regression.

19 The probability that sign of coefficient is positive.

20 The i lags of the regressors are denoted by $\mathrm{L_i}$.

4. A model-based core inflation measure

When short-lived, localised supply shocks hit the economy, exclusion core inflation measures[21] are good proxies of "macroeconomic" inflation and could be trusted as relevant indicators for the macroeconomic diagnostic and forecast, and for the determination of monetary policy responses. However, in the presence of widespread, persistent shocks (such as the large depreciation shock experienced recently in Colombia), the exclusion core inflation measures have shortcomings. In this case, the shock temporarily affects a large fraction of prices in the economy, so that the exclusion measures cannot adequately filter the shock. Furthermore, if the shock is persistent, separating the direct impact of the shock from its macroeconomic consequences (ie activation of indexation mechanisms, effects of expectations or monetary policy responses etc) becomes increasingly difficult with time.

This difficulty is compounded if, as at the current juncture in Colombia, other shocks with different durations and channels of transmission hit the economy. So, not only must policymakers filter out the COP depreciation shock, but they must also distinguish the impact of the El Niño-related droughts and the macroeconomic consequences of both shocks. In this context, a model-based approach may be useful in identifying the "pure" supply shocks and computing a core inflation measure that simply subtracts those shocks from headline inflation. This has the drawback of tying the core measure to a particular model, but it does help address the aforementioned issues.

For this purpose, the small semi-structural macroeconomic model introduced in Section 3 and described in Appendix 2 is used, following Bejarano et al (2015). As mentioned above, the model allows for the existence of non-processed food supply shocks, general supply shocks, demand and monetary policy shocks. The model-based core inflation measure is defined as inflation without non-processed food minus the general supply shock identified with the model. By construction, such measure incorporates all the macroeconomic effects and responses to the supply shocks, but not the shocks themselves.

Graph 10 shows a comparison of the model-based core inflation and the average of four conventional exclusion measures monitored at the Bank of the Republic. The model-based indicator is generally higher than the average of exclusion core inflation rates. The distance between the two measures is notably larger in periods of strong demand pressures (eg 2006–07 or 2011). However, in the last part of the sample, the model-based indicator is below the average of exclusion measures, suggesting that the direct impact of the recent depreciation shock may be overestimated by the latter.

5. Conclusion

Based on the results presented in this note, it may be concluded that conventional core inflation measures in Colombia might be overstating true "macro-economic" inflation at present, due perhaps to the widespread effects of the depreciation shock

[21] Exclusion core inflation measures are subbaskets of the CPI or other price index that exclude specific components known to be affected by transitory supply shocks (eg inflation excluding foodstuffs or energy).

that hit the economy. Given this feature, it is possible that traditional exclusion core inflation measures fail to filter the temporary impact of the exchange rate on local prices. However, the risk of de-anchoring inflation expectations following recent, strong supply shocks is a concern that policymakers must bear in mind. The evidence shows that inflation expectations are closely tied to observed past and present headline inflation. They are also affected by exogenous shocks in a possibly non-linear way such that the combination of large shocks greatly increases the probability of de-anchoring.

Market-based inflation expectations are widely used by market participants and policymakers for decision-making and for inferring the likely monetary policy decisions of central banks. Survey-based inflation expectations are also widely used but are not suitable given the lower frequency of available data. Market-based inflation expectations can be determined in several ways but perhaps the most popular method resorts to the market prices of zero-coupon inflation swaps (Antunes (2015)). Policymakers and finance professionals often use the term structure of Treasury yields to infer expectations of inflation and real interest rates (Haubrich (2012)). Forward inflation compensation – defined as the difference between forward rates on nominal and inflation-indexed bonds – provides us with a high-frequency measure of the compensation that investors require to cover the expected level of inflation, as well as the risks associated with inflation, at a given horizon. If far-ahead forward inflation compensation is relatively insensitive to incoming economic news, then one could reasonably infer that financial market participants have fairly stable views regarding the distribution of long-term inflation outcomes. This is precisely the outcome one would hope to observe in the presence of an explicit and credible inflation target.

Financial indicators for inflation expectations offer two key advantages over survey measures: they are available at a much higher frequency and over a larger number of horizons. Nowadays readings of inflation expectations are available at trading frequency via the break-even inflation rates (BEI rates) computed either through the yield spread between nominal and inflation-linked bonds or from the strongly growing inflation-linked (IL) swap market. This higher frequency of observation, as we will show below, is crucial to identifying shifts in inflation expectations when they occur, changes that may only be seen after an interval in survey data due to their lower frequency of collection. In addition, financial instruments allow for collecting readings of inflation expectations over a large number of horizons, both at short- and long-term maturities, which allow for monitoring developments in inflation expectations at more horizons than survey indicators, and therefore identify the horizons at which relevant changes take place.

Indicators of inflation expectations extracted from financial instruments should be better interpreted as measures of inflation compensation rather than simple measures of inflation expectations, for they incorporate a premium component that compensates investors not only for the expected level of inflation over the horizon of the contract but also for the uncertainty and risks surrounding the level of future inflation. Fluctuations in inflation risk premia are also very relevant for monetary policy: the analysis of financial indicators of inflation expectations offers additional insights beyond the level of long-term inflation expectations (Ciccarelli and Garcia (2005)).

Inflation expectations measures

Table 1

Name	Description	Periodicity	Abbreviation
Survey of experts	Applied to analysts of financial sector (credit banks, pension funds, insurance companies, etc.). The relevant question is: What will annual inflation be in the same month of next year?	Monthly 2003–15	SE
Survey of some sectors	Applied to representatives of the financial sector, industry, retailers, transport and communications, labour unions and academics. The relevant question is: What will annual inflation be in the same month of next year?	Quarterly 2000–15	SSQ
One-year breakeven inflation	"Expected inflation" extracted from the prices of Government bonds indexed to inflation (TES UVR) and fixed nominal rate bonds (TES fixed rate).	Monthly 2003–15	BEI1
Forward break-even inflation 1-1	"Expected inflation" one year after one year extracted from the prices of Government bonds indexed to inflation (TES UVR) and fixed nominal rate bonds (TES fixed rate).	Monthly 2003–15	F1BEI1
Forward break-even inflation 2-3	"Expected inflation" on average for three years after two years extracted from the prices of Government bonds indexed to inflation (TES UVR) and fixed nominal rate bonds (TES fixed rate).	Monthly 2003–15	F2BEI3
Forward break-even inflation 2-1	"Expected inflation" one year after two years extracted from the prices of Government bonds indexed to inflation (TES UVR) and fixed nominal rate bonds (TES fixed rate).	Monthly 2003–15	F2BEI1

Table 2

TESTS OF RATIONALITY OF INFLATION EXPECTATIONS

	SE	SSQ	BEI1	F1BEI1	F2BEI3	F2BEI1
Panel A: Is there a long relationship between observed inflation and expecations?						
Johansen Cointegration test						
Ho: r<=1	2,42	2,85	3,25	4,45	8.91*	7,14
H0: r=0	48.07***	36.28***	24.9***	23.14**	36.21***	16,83
Panel B: Testing for bias Ho: $\alpha=0,\ \beta=1;\quad \pi_t = \alpha + \beta\,\pi^{e}_{t/t\text{-}j} + \mu_t$						
α	0,025	0,016	0,021	0,038	0.019***	0,039
	(0.047)	(0.029)	(0.158)	(0.032)	(0.007)	(0.041)
β	0,425	0.631***	0.471***	0,090	0.353***	0,048
	(0.948)	(0.326)	(0.136)	(0.29)	(0.092)	(0.339)
Adj R^2	0,05	0,40	0,26	0,00	0,27	0,00
test p.value	0,911	0,429	2,977	0,000	0,000	0,000
Reject H0?	NO	NO	NO	YES	YES	YES
Panel C: Are forecasting errors not autocorrelated?						
Box-Ljung test						
Test statistic lag=1	274,18	338,21	261,31	271,85	172,17	251,02
P.value	0,00	0,00	0,00	0,00	0,00	0,00
Test statistic lag=12	816,40	883,18	620,89	889,23	590,03	832,58
P.value	0,00	0,00	0,00	0,00	0,00	0,00
Reject H0?	YES	YES	YES	YES	YES	YES
Panel D: Are the expectations efficient?. Are macroeconomic data fully exploited?						
Ho: $\alpha 0 = \alpha 1 = \alpha 2 = \alpha 3 = 0;\ \ \mu_t = \alpha 0 + \alpha 1\,(\pi_{t\text{-}j\text{-}1} - \pi^{T}_{t\text{-}j\text{-}1}) + \alpha 2\ GAP_{t\text{-}j\text{-}4} + \alpha 3\ \Delta i_{t\text{-}j\text{-}1} + \eta_t$						
α0	0,000	0,001	0,001	0,002	-0,001	0,000
	(0.006)	(0.009)	(0.005)	(0.007)	(0.005)	(0.018)
α1	-0,238	-0,170	-0,249	-0.578*	0,145	-0,021
	(0.802)	(0.751)	(0.484)	(0.267)	(0.322)	(0.486)
α2	1,730	0,337	1,148	0,535	-2.327**	-0,476
	(1.212)	(1.292)	(1.425)	(1.831)	(0.877)	(2.098)
α3	0.092*	0,068	0.083*	0,061	0.124***	-0,046
	(0.042)	(0.087)	(0.039)	(0.048)	(0.038)	(0.05)
Adj R^2	0,23	0,05	0,16	0,15	0,41	0,02
test p.value	0,264	0,921	0,443	0,680	0,116	0,986
Reject H0?	NO	NO	NO	NO	NO	NO
Sample	Sep 2003 - Nov 2015	Mar 2000 - Sep 2015	Jan 2003 - Nov 2015	Jan 2003 - Nov 2015	Jan 2003 - Nov 2015	Jan 2003 - Nov 2015
Periodicity	Monthly	Quarterly	Monthly	Monthly	Monthly	Monthly
T	135	59	143	131	95	119

(Newey-West standard errors in parentesis, correcting for autocorrelation up to one year)

***, ** and * denote statistical signiticance at the 1%, 5% and 10% levels, respectively

Table 3

RMSE with respect to realized future inflation	
Monthly Survey	1,45
Quarterly Survey	1,51
BEI 1y	1,41
Tradable/ Non Tradable DSGE	1,41
PATACON	1,29
Small New Keynesian DSGE	1,24

RMSE with respect to contemporaneous inflation	
Monthlyy Survey	0,92
Quarterly Survey	0,64
BEI 1y	0,84
Tradable/ Non Tradable DSGE	1,33
PATACON	0,98
Small New Keynesian DSGE	1,09

Correlation coefficient with realized future inflation	
Monthly Survey	0,39
Quarterly Survey	0,42
BEI 1y	0,54
Tradable/ Non Tradable DSGE	0,61
PATACON	0,62
Small New Keynesian DSGE	0,58

Correlation coefficient with contemporaneous inflation	
Monthlyy Survey	0,88
Quarterly Survey	0,96
BEI 1y	0,87
Tradable/ Non Tradable DSGE	0,78
PATACON	0,80
Small New Keynesian DSGE	0,77

Adaptive Learning

Table 4

Expectation measures	MSE	v	ϕ0	std. Error	ϕ1	std. Error
SE	0,000	0,048	0,028	0,004	0,330	0,081
SSQ	0,001	0,034	0,017	0,004	0,695	0,072
BEI1	0,000	0,000	0,010	0,005	0,701	0,132
F1BEI1	0,001	0,045	0,025	0,005	0,411	0,133
F2BEI3	0,001	0,055	0,034	0,009	0,361	0,176
F2BEI1	0,000	0,000	0,013	0,002	0,610	0,079

Inflation expectations as a combination of rational and adaptive expectations Table 5

$$\pi^e_{t+s/t} = c_1\pi_{t+s} + (1 - c_1)\left[\pi^e_{t/t-s} + c_2\left(\pi_t - \pi^e_{t/t-s}\right)\right] + \varepsilon_t$$

Expectation measures	C1	p.value	C2	p.value	R^2	AIC
SE	0,151	0,035	0,434	0,000	0,746	–8,204
SSQ	0,294	0,000	0,647	0,000	0,899	–7,947
BEI1	0,217	0,031	0,602	0,000	0,761	–7,241
F1BEI1	0,526	0,001	1,040	0,001	–0,251	–6,057
F2BEI3	0,306	0,004	0,470	0,014	0,111	–6,390
F2BEI1	0,448	0,000	0,683	0,000	–0,492	–5,724

Inflation expectations as a combination of the inflation target and adaptive expectations Table 6

$$\pi^e_{t+s/t} = c_1 Target_{t+s} + (1 - c_1)\left[\pi^e_{t/t-s} + c_2\left(\pi_t - \pi^e_{t/t-s}\right)\right] + \varepsilon_t$$

Expectation measures	C1	p.value	C2	p.value	R^2	AIC
SE	0,413	0,000	0,391	0,000	0,939	–9,633
SSQ	0,254	0,000	0,625	0,000	0,977	–9,434
BEI1	0,325	0,005	0,643	0,000	0,790	–7,370
F1BEI1	0,552	0,000	0,516	0,000	0,384	–6,715
F2BEI3	0,207	0,084	0,684	0,000	0,420	–5,918
F2BEI1	0,430	0,000	0,643	0,000	0,061	–6,023

Regressions for the difference between inflation expectations and the inflation target Table 7

$$\pi^e_{t+s/t} - Target_{t+s} = a_1 + a_2 \Delta Oil\ Price_t + a_3 El\ Niño_t + \varepsilon_t$$

	Inflation Expectations Measure							
	SSE	**SSQ Total**	**SSQ Manufacturing**	**SSQ Financial**	**SSQ Retail**	**SSQ Transportation & Communications**	**SSQ Academics**	**SSQ Labor Unions**
Constant	**0,004**	**0,0068**	**0,0069**	**0,006**	**0,0079**	**0,0068**	**0,0063**	**0,0122**
Std. Error	0,0011	0,0018	0,0019	0,0015	0,0021	0,0019	0,0019	0,0028
Δ Brent Price	**-0,0028**	**-0,0146**	**-0,0153**	**-0,0143**	**-0,014**	**-0,0149**	**-0,0168**	**-0,0174**
Std. Error	0,0027	0,0063	0,0065	0,0058	0,0062	0,0063	0,0066	0,009
Intensity of El Niño	**0,0021**	**0,0105**	**0,0105**	**0,0105**	**0,01**	**0,0098**	**0,01**	**...**
Std. Error	0,0012	0,0052	0,0052	0,0052	0,0052	0,0053	0,0057	...
Adj. R^2	0,1138	0,3516	0,3593	0,3953	0,3068	0,3341	0,3539	0,1696

Regressions for the difference between inflation expectations and the inflation target

Table 8

$$\pi^e_{t+s/t} - Target_{t+s} = a_1 + a_2\ Supply\ Shock_t + a_3\ Demand\ Shock_t + a_4\ Policy\ Shock_t + a_5\ Food\ Shock_t + \varepsilon_t$$

	Inflation Expectations Measure									
	SSE	SSQ Total	SSQ Manufacturing	SSQ Financial	SSQ Retail	SSQ Transportation & Communications	SSQ Academics	SSQ Labor Unions	BEI 1	F2BEI3
Constant	0,0057	**0,0080**	0,0080	**0,0071**	**0,0094**	**0,0078**	0,0073	0,0099	**0,0046**	**0,0133**
Std. Error	0,0010	0,0013	0,0014	0,0012	0,0014	0,0012	0,0014	0,0015	0,0017	0,0016
Supply Shock	**0,3481**	**0,3807**	**0,3793**	**0,3658**	**0,4351**	**0,4118**	**0,4345**	**0,4459**	0,27082	**0,6745**
Std. Error	0,1519	0,1360	0,1373	0,1414	0,1283	0,1323	0,1474	0,1515	0,1575	0,0825
Demand Shock	**-0,3052**	**-0,1764**	**-0,1710**	**-0,1954**	**-0,2133**	**-0,1839**	**-0,1918**	**-0,0736**	**0,0439**	**-0,6997**
Std. Error	0,1471	0,1205	0,1199	0,1121	0,1284	0,1128	0,1312	0,1274	0,2099	-0,0946
Policy Shock	**-0,1063**	**-0,1042**	**-0,1153**	**-0,1072**	**-0,0929**	**-0,1051**	**-0,0732**	**-0,1668**	**-0,0564**	**0,1311**
Std. Error	0,0763	0,0926	0,0956	0,0776	0,1039	0,0886	0,1004	0,0987	0,170338	0,0631
Food Supply Shock	**0,0056**	**0,0064**	**0,0066**	**0,0052**	**0,0055**	**0,0057**	**0,0078**	**0,0105**	**0,0089**	**0,0047**
Std. Error	0,0068	0,0058	0, 095606	0,0056	0,0060	0,0053	0,0063	0,0060	**0,0101**	0,0059
Adj. R^2	0,3087	0,3079	0,3055	0,3189	0,3451	0,3539	0,3218	0,4047	**0,1303**	**0,5714**

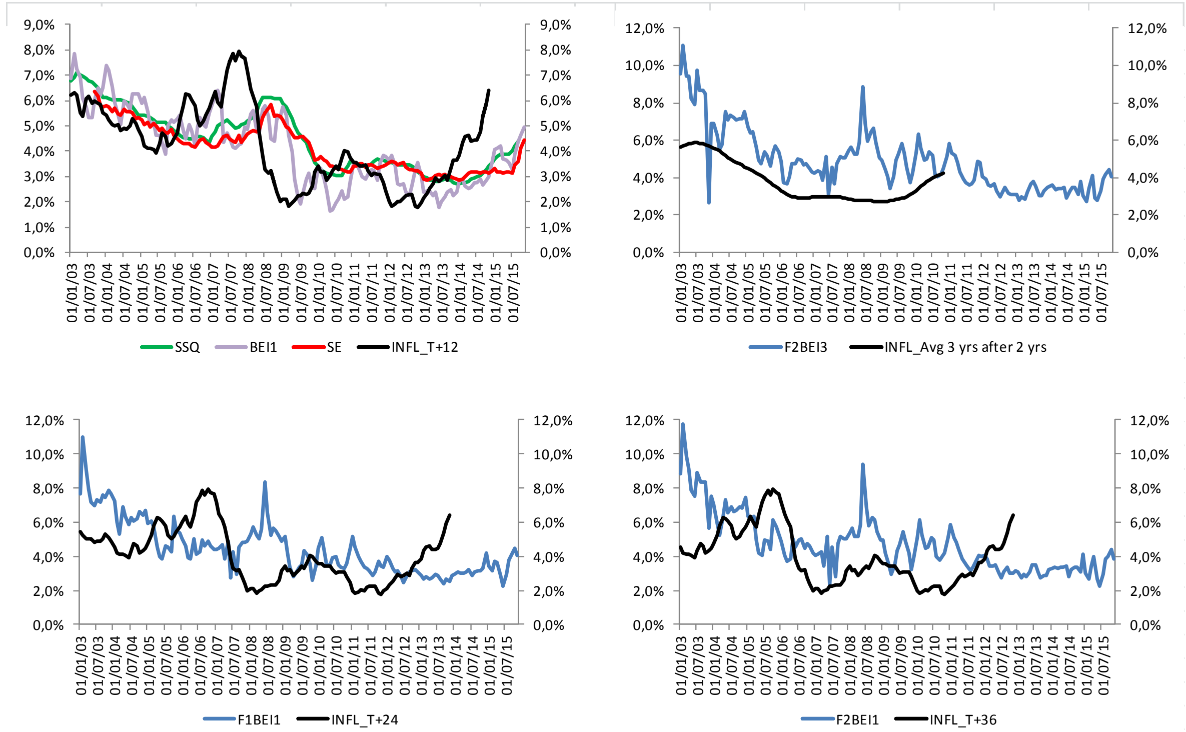
Probability of de-anchoring of inflation expectations

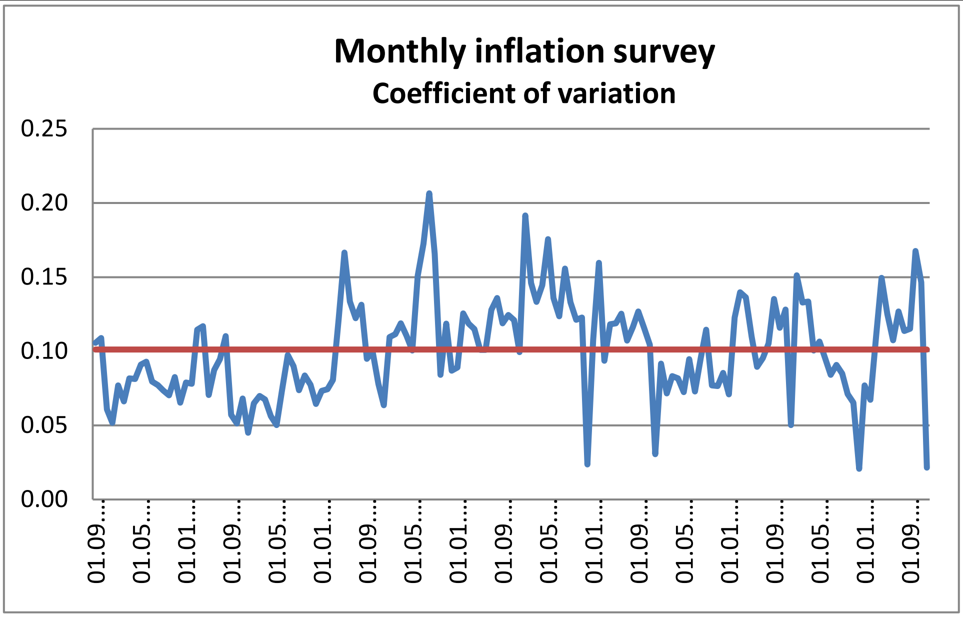
BMA estimation statistics

Table 9

Forecasting horizon to														
h=0 months ahead					h=3 months ahead					h=6 months ahead				
Variable	PIP	Posterior Mean	Posterior SD.	Sign + Prob.	Variable	PIP	Posterior Mean	Posterior SD.	Sign + Prob.	Variable	PIP	Posterior Mean	Posterior SD.	Sign + Prob.
Niña,L2	0,96	7,53	2,72	1,00	SIF,L0	1,00	20,9	5,50	1,00	Niño,L6	1,00	6,72	2,24	1,00
Niño,L6	0,90	6,34	3,36	1,00	Niña,L6	0,95	-6,92	2,70	0,00	SIF,L0	0,99	16,4	6,34	1,00
SIF,L3	0,77	9,92	7,55	1,00	SIF,L5	0,83	11,1	7,58	1,00	Brent,L2	0,94	–8,78	3,56	0,00
SIF,L6	0,69	7,67	6,71	1,00	Brent,L5	0,70	-5,01	4,17	0,00	Brent,L6	0,90	–6,93	3,44	0,00
Brent,L6	0,58	-2,87	2,89	0,00	SIF,L4	0,52	5,45	6,55	1,00	SIF,L2	0,81	12,6	8,54	1,00
SIF,L5	0,54	5,64	6,50	1,00	SIF,L1	0,44	3,85	5,66	1,00	SIF,L1	0,69	8,02	7,37	1,00
Niño,L3	0,54	3,46	3,75	1,00	Dummy,L6	0,38	2,10	3,03	1,00	SIF,L3	0,64	7,17	7,06	1,00
SIF,L2	0,47	4,23	5,70	1,00	Niño,L4	0,38	1,48	2,28	0,98	Niña,L4	0,59	–2,53	2,40	0,00
Niño,L4	0,45	2,98	3,78	1,00	SIF,L6	0,36	2,94	4,87	1,00	Dummy,L6	0,53	4,00	4,19	1,00

Inflation expectations measures Graph 1




Monthly inflation survey
Coefficient of variation
0.25
0.20
0.15
0.10
0.05
0.00
01.09…
01.05…
01.01…
01.09…
01.05…
01.01…
01.09…
01.05…
01.01…
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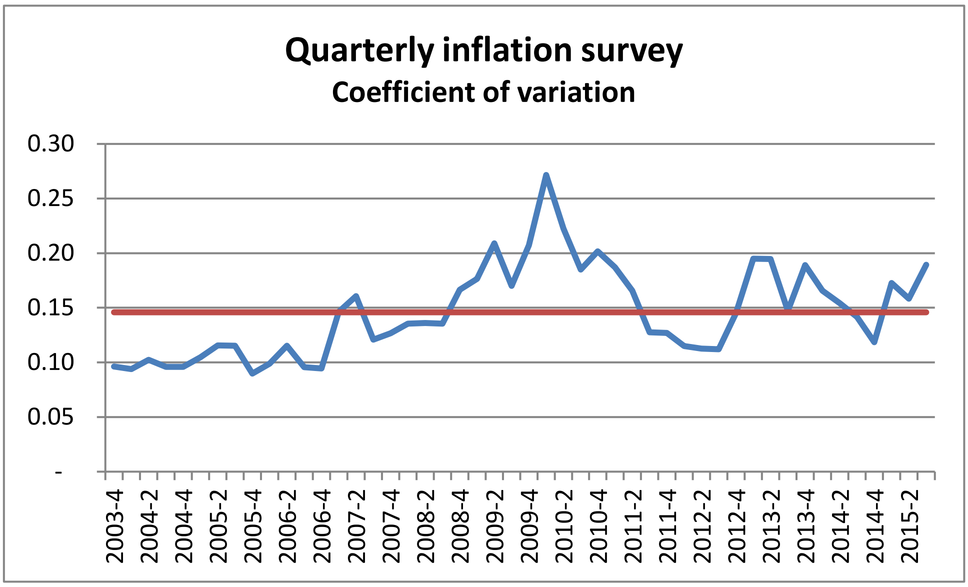
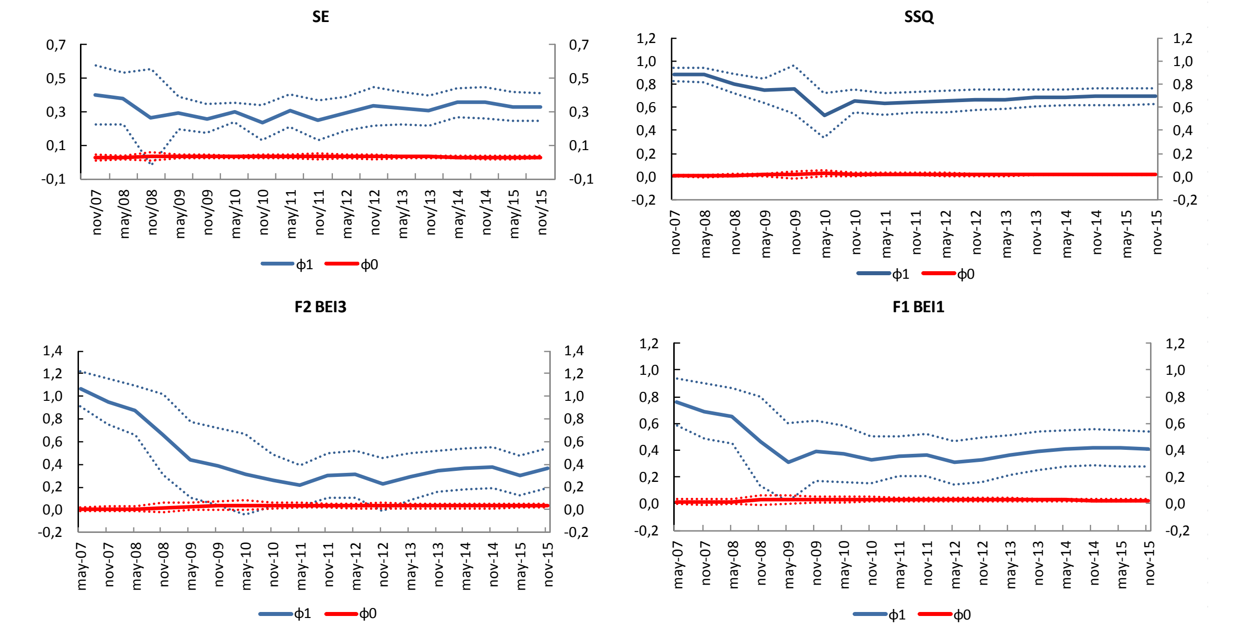
01.09…
01.05…
01.01…
01.09…


Quarterly inflation survey
Coefficient of variation
0.30
0.25
0.20
0.15
0.10
0.05
-
2003-4
2004-2
2004-4
2005-2
2005-4
2006-2
2006-4
2007-2
2007-4
2008-2
2008-4
2009-2
2009-4
2010-2
2010-4
2011-2
2011-4
2012-2
2012-4
2013-2
2013-4
2014-2
2014-4
2015-2

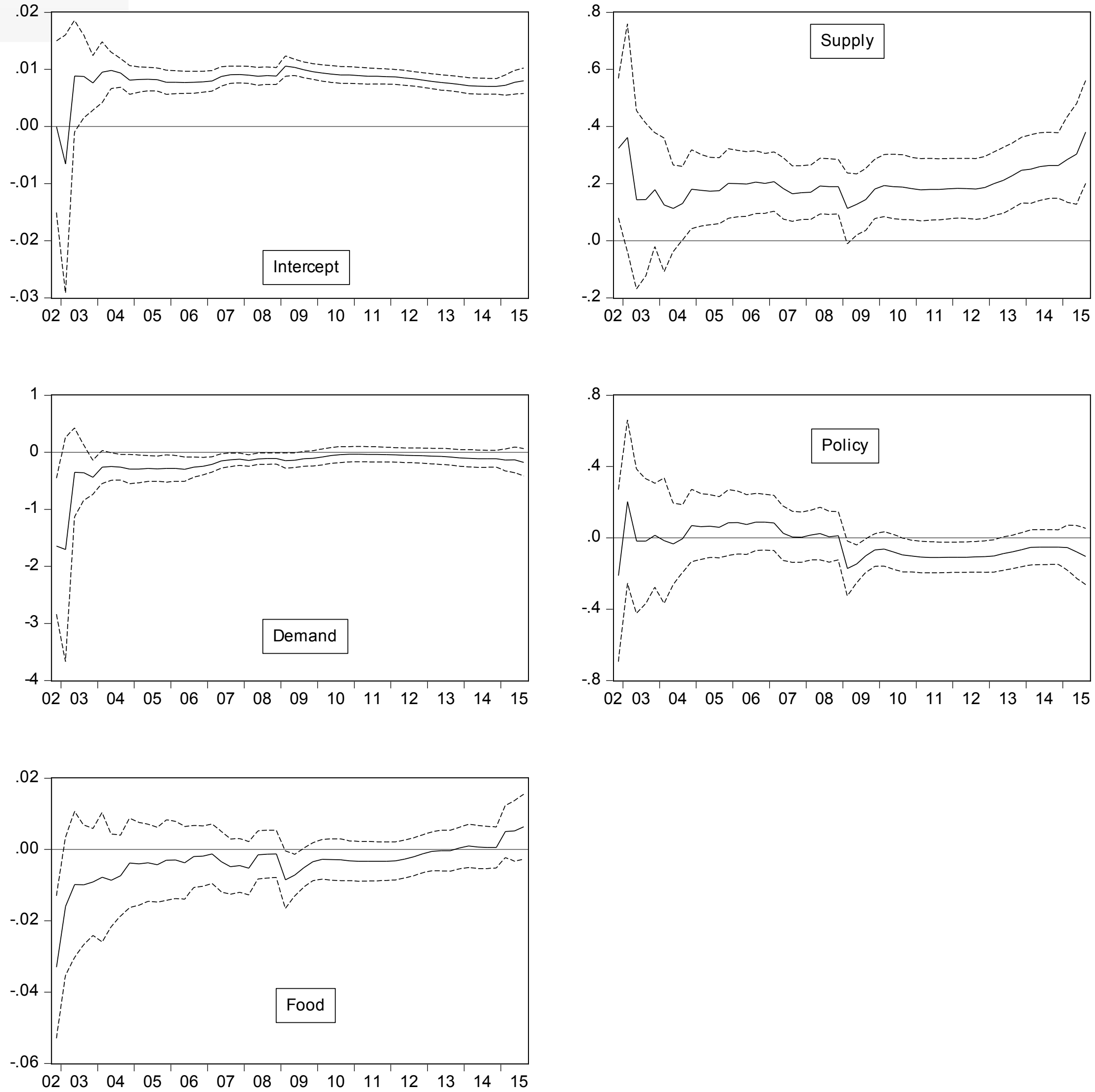
Adaptive learning

Coefficient of observed inflation (ϕ_1) Graph 3



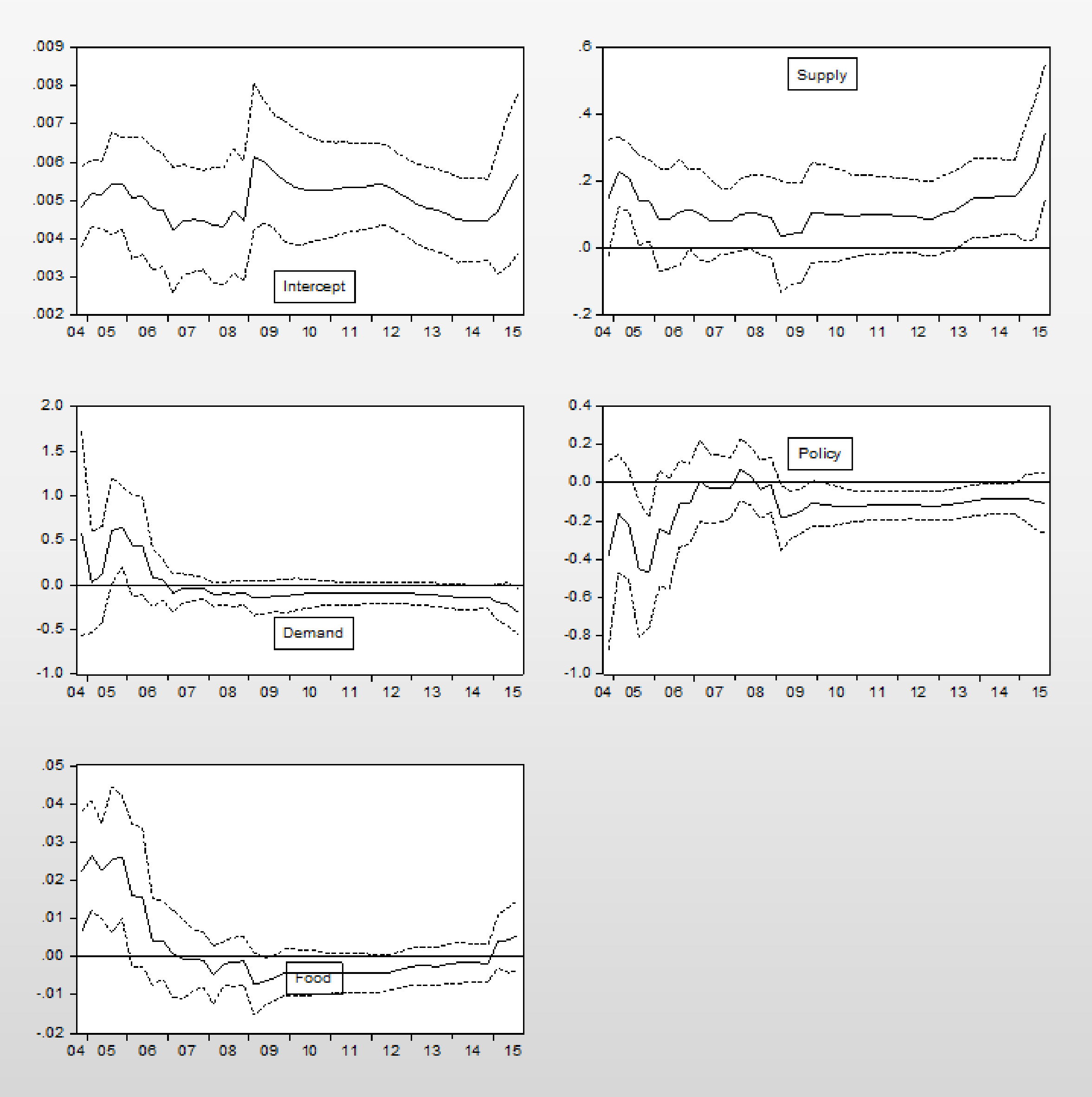
Coefficients of macroeconomic shocks in the regression for the deviation of quarterly survey inflation expectations from target

Graph 4



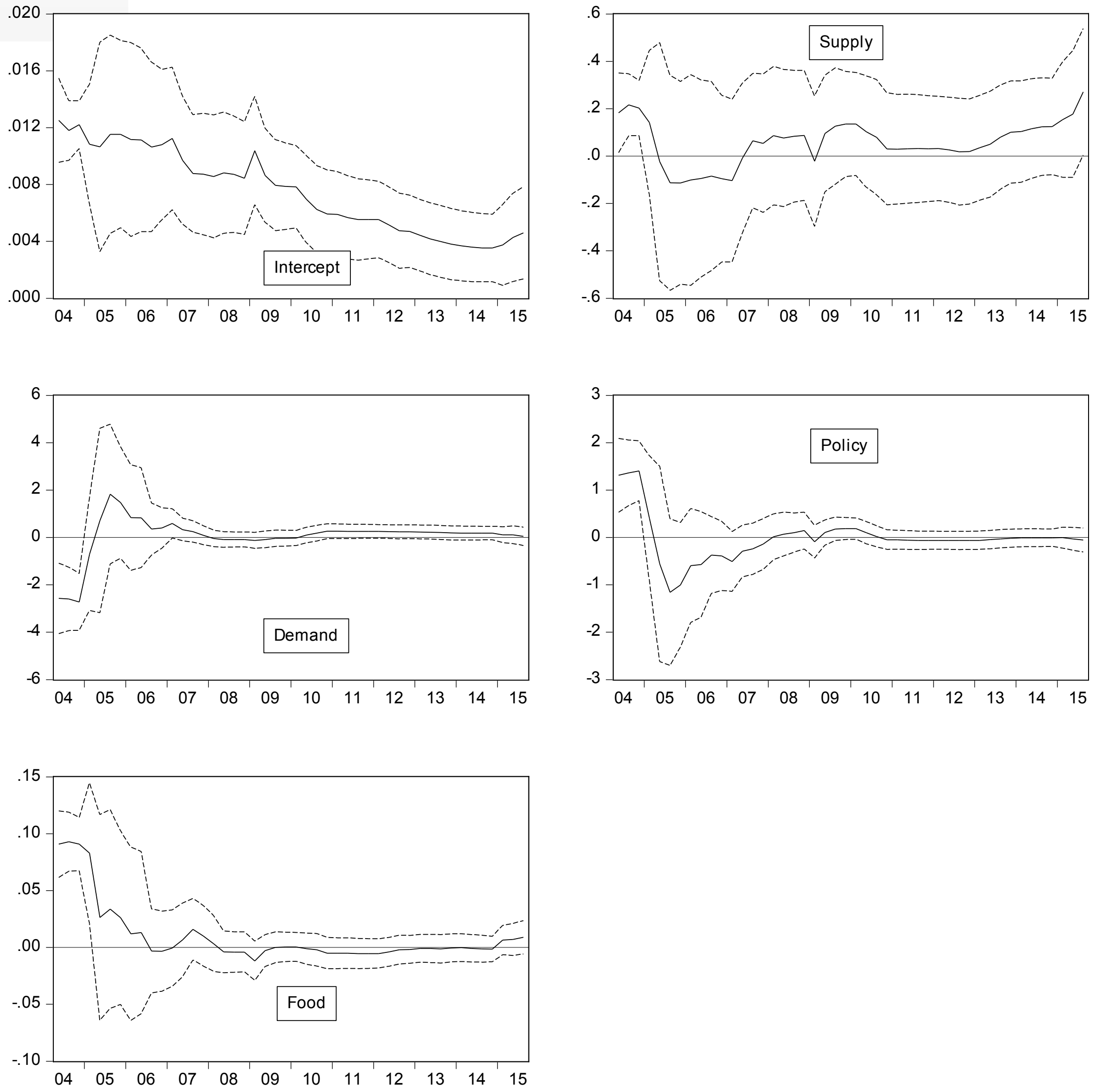
Coefficients of macroeconomic shocks in the regression for the deviation of monthly survey inflation expectations from target

Graph 5



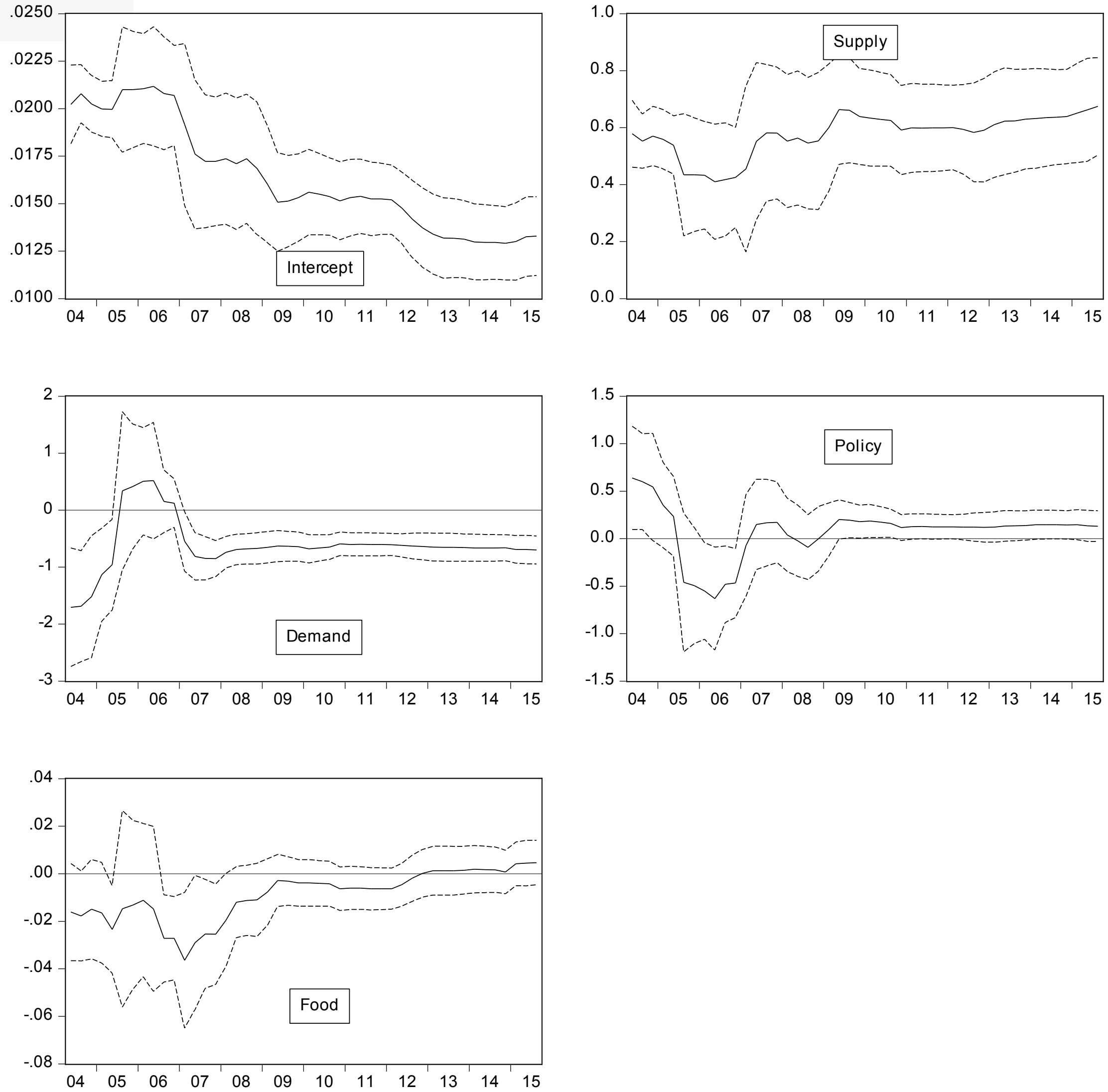
Coefficients of macroeconomic shocks in the regression for the deviation of BEI 1 inflation expectations from target

Graph 6



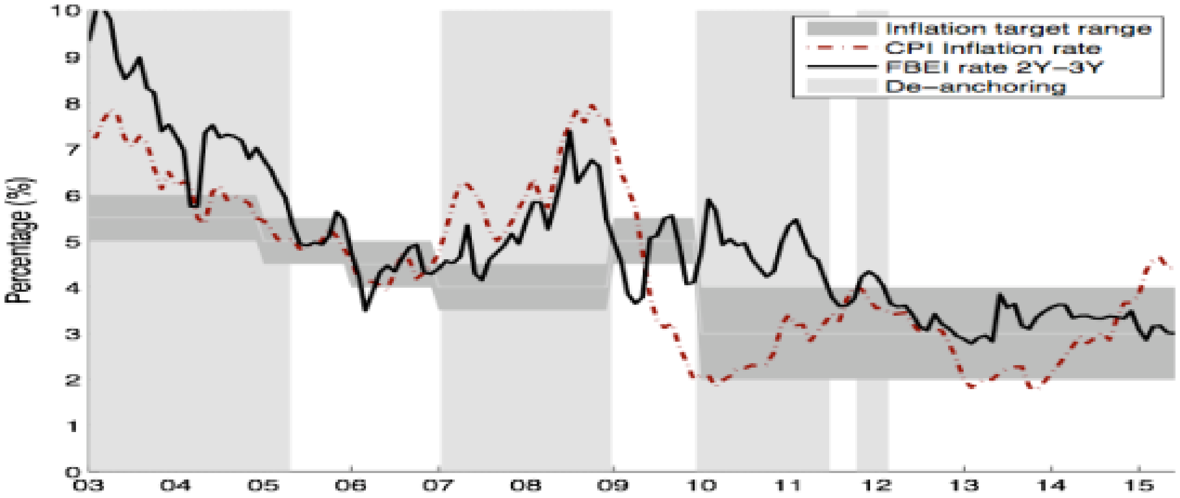
Coefficients of macroeconomic shocks in the regression for the deviation of F2BEI3 inflation expectations from target

Graph 7



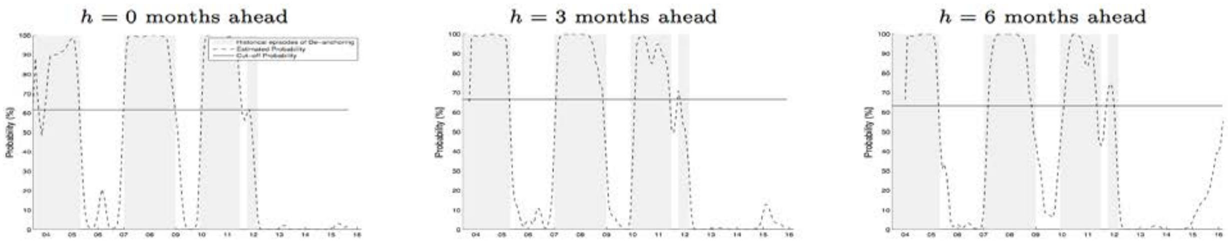
De-anchoring of inflation expectations: Historical events

Graph 8



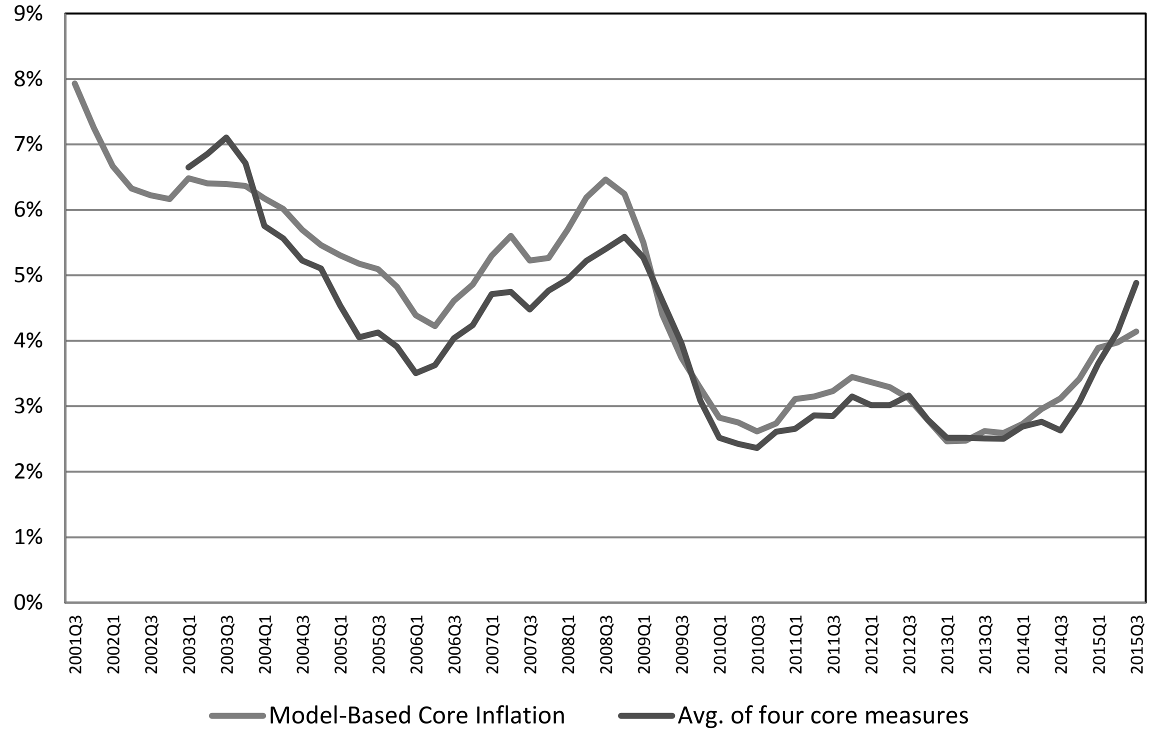
Probability of de-anchoring of inflation expectations: Direct estimation and prediction

Graph 9



Core inflation measures

Graph 10



Appendix 1

The Adaptive Learning Model (Huertas et al 2015)

The adaptive learning hypothesis assumes that agents act as econometricians to produce inflation forecasts. Since they do not know the structure of the economy, they need to establish a forecast rule known as perceived movement law (PML). Based on this law, they estimate the coefficients of the rule and update them when they observe new information and compute the forecast error.

To explore the relevance of this expectation formation mechanism, a test on whether inflation expectations can be estimated by an adaptive learning algorithm with a constant gain coefficient is performed (Pfajfar and Santoro (2010)). Suppose that agents have the following PML:

$$\pi^{g}_{t/t-j} = \phi^{g}_{0,t-1} + \phi^{g}_{1,t-1}\pi_{t-(j+1)} + \varepsilon_t$$

In this equation, agent g forms his forecast of inflation for period t in period $t-j$, ($\pi^{g}_{t/t-j}$), based on the observed inflation in the previous period ($\pi_{t-(j+1)}$). When headline inflation is published in period $t-j$, agent g updates the estimation of $\phi^{g}_{0,t-1}$ and $\phi^{g}_{0,t-1}$ with a constant gain law (CGL).

Let $X_t = (1, \pi_{t-j})$ y $\widehat{\phi}_t = \left(\widehat{\phi}_{0,t}, \widehat{\phi}_{1,t}\right)'$. Then, if a least square updating method is used, the estimated coefficient will follow this rule:

$$\widehat{\phi}^{g}_{t} = \widehat{\phi}^{g}_{t-1} + vR^{-1}_{t-1}X'_{t-(2j+1)}\left(\pi_{t-j} - X_{t-(2j-1)}\widehat{\phi}^{g}_{t-(j+1)}\right)$$

$$R_t = R_{t-1} + v\left(X_{t-(2j-1)}X'_{t-(2j-1)} - R_{t-1}\right)$$

R_t is the matrix of second moments of X_t and v is the constant gain. When the gain is positive, the parameters are updated with their forecast error and the new available information. If the gain is zero, the coefficients are not updated and there is no learning.

To test for the existence of learning, the methodology used by Pfajfar and Santoro (2010) is followed. They propose this PML:

$$\pi^{s}_{t/t-j} = \phi_{0,t-1} + \phi_{1,t-1}\pi_{t-(j+1)} + \varepsilon_t \qquad j = \{1,12\}$$

Where s represent a simulated series. The method consists of calculating simulated series by combining estimates of v and ϕ. The idea is to find a combination of initial values of the coefficients and a gain parameter to replicate a measure of inflation expectations as close as possible.

Appendix 2

A small semi-structural macroeconomic model for Colombia (Bejarano et al (2015))

A semi-structural model is estimated for Colombia. It is based on the basic closed-economy New Keynesian monetary policy model and includes an IS curve, an ARMA equation for non-processed food (an important source of inflation shocks in Colombia), a "hybrid" Phillips curve for non-food inflation and a Taylor rule. The "hybrid" Phillips curve captures the effects of inflationary inertia. Each one of these equations is subject to shocks. Hence, there are four types of shocks: A food price shock that is associated with the food inflation equation, a "supply" shock that is related to the Phillips Curve, a "demand" shock that is associated with the IS curve and a policy shock that is linked to the Taylor rule. Being a closed economy model, the direct inflationary impact of exchange rate shocks is picked by the "supply" shock.

Phillips curve:

$$\pi_t^{sa} = \phi_1 \pi_{t-1} + (1-\phi_1) E_t(\pi_{t+1}) + \kappa x_t + z_t^{\pi}$$

π_t is headline inflation, π_t^{sa} is non-food inflation, x_t is the output gap, z_t^{π} is an AR(1) supply shock.

Food Inflation:

$$\pi_t^A = \beta_1 \pi_{t-3}^A + \beta_2 \pi_{t-5}^A + \gamma_1 \varepsilon_{t-2} + \gamma_2 \varepsilon_{t-3} + \gamma_3 \varepsilon_{t-4} + \varepsilon_t$$

π_t^A is food inflation, ε_t is a shock associated with an ARMA(5,4) process that captures the dynamics of non-processed food prices. It includes the possibility of "cobweb-like" price behaviour.

IS Curve:

$$x_t = E_t(x_{t+1}) - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1}^{sa})] + z_t^u$$

z_t^u is an AR(1) demand shock and i_t is the nominal interest rate.

Policy Rule:

$$i_t = \rho^i(i_{t-1}) + \left(1-\rho^i\right)(\varphi^{\pi}\pi_t + \varphi^x x_t) + z_t^i$$

z_t^i is an iid policy shock, ρ^i is a monetary policy "smoothing" parameter, φ^{π} es represents the strength of the monetary policy response to deviations from the inflation target and φ^x is the degree to which policy reacts to the output gap.

The model is estimated for the period Q4 2000–Q4 2015.[22] The output gap series used in the estimation is the Central Bank Staff measure presented in the quarterly inflation reports. The parameters of the model are estimated with Bayesian methods.

Parameters	
ρ^{z_π}	0.2874
ρ^{z_u}	0.8758
ρ^{i}	0.9100
σ^{z_π}	0.0098
σ^{z_u}	0.0038
σ^{z_i}	0.0069
σ^{ε}	0.1474
φ^{π}	3.8626
φ^{x}	1.4277
ϕ_1	0.2759
σ	2.59
κ	0.0956

22 Central Bank staff short-term forecast were used for the 2015 Q4 data on inflation, GDP gap and the policy interest rate.

Appendix 3

Bayesian model averaging

BMA takes into account model uncertainty by going through all the combinations of models that can arise within a given set of variables (Green (1995); Raftery et al (1997)). Consider a dummy variable y_{t+h} as proxy of de-anchoring of long-term inflation expectations such that

$$y_{t+h} = \begin{cases} 1 \text{ if there is de} - \text{anchoring at time } t+h \\ 0 \text{ otherwise.} \end{cases} \quad (1)$$

for $t = 1, \ldots, T$ and $h \geq 0$. The parameter h denotes time horizons for direct estimation.

The BMA methodology assumes that there is a set of possible models $M_1, \ldots, M_k$ for estimating a quantity $y_{t+h} = 1$ from the set of variables, D_t. The k^{th} model, M_k, is defined by a subset of covariates of D_t. Instead of using a single model for performing inference on $y_{t+h} = 1$, BMA constructs $P(y_{t+h} = 1 \mid D_t)$, the posterior density of $y_{t+h} = 1$ given the data D_t, not conditional on any particular model. Many possible models are considered, so that model uncertainty is accounted for.

The posterior probability of $y_{t+h} = 1$ given data D_t is

$$P^{BMA}(y_{t+h} = 1 \mid D_t) = \sum_{k=1}^{k} \int P(y_{t+h} = 1 \mid \theta^k, M^k, D_t)\ P(\theta^k, M^k|D_t) d\theta^k \ (2)$$

$P(y_{t+h} = 1 \mid \theta^k, M^k, D_t) = F(\theta^k, M^k, D_t)$ denotes the probability of being in an episode of de-anchoring of inflation expectations at time $t + h$, θ^k is one of the possible parameter sets of the M^k model and F is the cumulative logistic distribution function. On the other hand, $P(\theta^k, M^k|D_t)$ is the joint posterior probability of θ^k and M^k given data D_t. Therefore, Eq. (2) is a weighted average of probabilities $P(y_{t+h} = 1 \mid \theta^k, M^k, D_t)$ whose weights are given by $P(\theta^k, M^k|D_t)$.

We also compute a cut-off probability $\tau \in [0{,}1]$ above which the probability $P^{BMA}(y_{t+h} = 1 \mid D_t)$ for $t = 1, \ldots, T$ and $h \geq 0$ provides a signal of de-anchoring. The value τ is computed as the solution to the minimisation problem

$$Min\ \varphi\ (\tau)\ subject\ to\ \gamma(\tau) \leq \bar{\gamma} \quad (3)$$
$$\tau \in [0{,}1]$$

$\varphi(\tau)$ and $\gamma(\tau)$ are the percentages of de-anchoring's false alarms and undetected events, respectively. The parameter $\bar{\gamma}$ corresponds to the maximum value of γ admitted by the policymaker. Guarín et al (2015) presents technical details of the derivation of the probability $P^{BMA}(y_{t+h} = 1 \mid D_t)$ in Eq. (2) and the minimisation problem in Eq. (3).

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