# A spectral perspective on natural interest rates in Asia-Pacific: changes and possible drivers<sup>1</sup>

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#### Abstract

I study the evolution of the equilibrium real interest rate, also known as the natural or neutral interest rate, in Asia-Pacific. Simple estimates based on a statistical approach suggest that, except for China, and Thailand since 2005, the natural interest rate may have declined substantially in Asia-Pacific economies since the early or mid-1990s, by over 4 percentage points on average. In many economies the rate has turned negative. The tendency has become more accentuated in the 2000s, especially following the onset of the Global Financial Crisis. The natural interest rate appears to vary significantly over time and across economies. Nevertheless, simple natural interest rate estimates are unreliable, and large uncertainties and sizeable heterogeneity in the estimates of the equilibrium real interest rate call for caution as well as monetary policy rules which are robust to such uncertainties.

I use frequency-domain techniques to examine the relationship between the long-run component of real interest rate and those of population characteristics, globalisation, and a range of macroeconomic and financial variables (eg credit and asset prices). I estimate spectral and cospectral densities, coherency and the frequency-specific coefficients of correlation and regression proposed by Zhu (2005). The association seems to be broad and strong between the natural interest rate and the low-frequency trend components of demographic and global factors in Asia-Pacific, but weak between the natural interest rate and trends in asset prices, credit-to-GDP ratio and trend growth in many economies in the region. In most cases, the natural interest rate seems to be correlated with broad measures of long-term financial sector development, and trends in savings rate and investment ratio. Understanding the underlying factors driving changes in each economy's natural interest rate is important for the correct calibration and implementation of monetary policy.

Keywords: asset price, credit, demography, equilibrium real interest rates, frequencydomain methods, globalisation, natural interest rates, population ageing, trend growth.

JEL classification: E43, E44, E52, F62, J11.

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# I. Introduction

Lower growth rates and slow global recovery, especially following the recent Global Financial Crisis and the Great Recession, have raised concerns on whether the global economy has arrived at a "new normal" with lower trend growth and higher unemployment. The slower-than-expected trend growth has been accompanied by an extended period of very low or negative real interest rates, prompting questions on whether we also live now in a world of "new neutral" with lower equilibrium real interest rates.

The century-old concept of an equilibrium real interest rate, also known as the natural or neutral interest rate, dates back to Knut Wicksell (1898), who suggests that "there is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods". Friedman (1968) extends the idea, proposing a "natural rate of unemployment", and in his words, using "the term 'natural' for the same reason Wicksell did – to try to separate the real forces from monetary forces". More recently, Woodford (2003) proposes a neo-Wicksellian or New Keynesian framework for the analysis of monetary policy, where the natural rate of interest plays a key role in the fluctuations of output and inflation.

The equilibrium real interest rate could potentially serve as an important benchmark for policymakers, who have long been searching for reliable indicators to help formulate and guide monetary policy.<sup>3</sup> This is even more relevant considering that most central banks conduct monetary policy by setting a target for a short-term nominal interest rate, eg an overnight money market rate. The equilibrium real interest rate provides a standard benchmark against which the policymakers can directly measure the policy rates and evaluate the policy stance.<sup>4</sup> Furthermore, the rate is also of great relevance to budgetary decisions and longer-term fiscal planning.

IMF (2014) finds that "real interest rates worldwide have declined substantially since the 1980s". King and Low (2014) estimate a "world real interest rate" and find that the weighted rate has declined from a peak of 4.93% in the first quarter of 1992 to -0.48% in the second quarter of 2013. Bernanke (2015a) observes that the exceptionally low global interest rates, both short- and long-term, are not a "short-

- <sup>3</sup> As Greenspan (1993) puts it, "One important guidepost is real interest rates, which have a key bearing on longer run spending decisions and inflation prospects. In assessing real rates, the central issue is their relationship to an equilibrium interest rate, specifically, the real rate level that, if maintained, would keep the economy at its production potential over time. Rates persisting above that level, history tells us, tend to be associated with slack, disinflation, and economic stagnation — below that level with eventual resource bottlenecks and rising inflation." Canzoneri, Cumby and Diba (2015) suggest that "it is most important for monetary policy to be able to track the (unobserved) natural rate of interest in environments in which that rate takes big and sustained swings away from its long run equilibrium". To achieve full employment of resources, in Bernanke's (2015a) words, the Federal Reserve needs to push market interest rates "toward levels consistent with the equilibrium rate, or – more realistically – its best estimate". In a recent speech, Yellen (2015) suggests that "as the equilibrium real funds rate continues to rise, it will accordingly be appropriate to raise the actual level of the real federal funds rate in tandem, all else being equal."
- <sup>4</sup> Indeed, the constant term in the alternative benchmark Taylor (1993, 1997) rules, is a real policy interest rate consistent with full employment and stable prices, where the output gap is eliminated and inflation is equal to its long-run equilibrium value.

term aberration" but a long-term trend. Low real interest rates may matter to policymakers for various reasons. When they stay low in normal times, adverse shocks are more likely to lead policymakers to drive real rates lower or even negative in their quest for full employment and price stability. In addition, low nominal and real interest rates may lead to an accumulation of financial imbalances and undermine financial stability. According to Summers (2014), low rates may increase investors' risk-taking, promote irresponsible lending and make Ponzi financial structures more attractive.

The downward trend in real interest rates has been accompanied by an apparent decline in estimated equilibrium real interest rates, which have again been at the centre stage in recent policy debates. Barsky, Justiniano and Melosi (2014) find the natural rate negative in the last three recessions and "has remained persistently depressed since 2008". Williams (2015) identifies a "moderate secular decline (in the US natural rate) over the two decades preceding the Great Recession", and a "more substantial decline during the Great Recession". Yellen (2015) points out that some current external estimates of a close-to-zero equilibrium real federal funds rate fall well below the FOMC's assessment of longer-run levels. Moreover, there is also a discrepancy between, eg the recently updated Laubach-Williams (2003) estimate of – 0.16% for the fourth quarter of 2014, and the 1–2% range provided by Hamilton, Harris, Hatzius and West (2015).

There are several competing theories behind the decline in equilibrium real interest rates. First, globalisation, especially trade and financial integration have helped forge a global market where domestic factors have begun to play a less prominent role. Financial integration implies that a larger share of global savings is channelled into cross-border financing of investment. In this vein, Bernanke (2005) proposes the "global saving glut" hypothesis, whereby the real interest rate falls to equilibrate the market for global savings as desired savings outstrip desired investment, and savings originating in China and other emerging economies hold down long-term interest rates. Caballero (2006) suggests the existence of a "safe asset shortage" due to rising global demand, as in emerging economies with rapid growth and high savings, there is only a limited availability of local safe assets in these undeveloped capital markets. Nevertheless, it is argued that these alone could not explain Greenspan's (2005) "conundrum" of anomalously low long-term interest rates. Although China's current account surplus has shrunk from over 10% in 2007 to 1.6% in 2013 and 2.1% in 2014, global interest rates have further declined (see Graph B.1.2, left-hand panel).

Second, many economists link the apparent decline in equilibrium real interest rates to a "new normal" world of lower potential output and trend growth, manifested in sluggish growth persisting in the major economies following the financial crisis. This has often been attributed to, among other factors, a secular deficiency in aggregate demand, significant financial frictions, unfavourable demographic trends, ebbing innovations, debt overhang and insufficient structural policies.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup> Lo and Rogoff (2014) survey alternative explanations for the sluggish economic growth persisting in many advanced economies after the onset of the financial crisis, and find it difficult to quantify and discern the effects of different long-term factors on economic growth until the post-financial-crisis debt overhang significantly abates.

One notable thesis, that of "secular stagnation", was first proposed by Alvin Hansen (1934, 1939) and recently resurrected by Summers (2013, 2014a, 2014b) and Krugman (2013, 2014). The idea is that a low and declining rate of population growth and a slower pace of technological advance result in lower returns, less investment and consumer spending, creating a situation of persistently inadequate demand.<sup>6</sup> This leads to a declining natural rate of interest. Hansen (1939) considers that the *essence* of secular stagnation is the "sick recoveries which die in their infancy and depressions which feed on themselves and leave a hard and seemingly immovable core of unemployment",<sup>7</sup> a scenario of slow jobless recoveries that the world has grown accustomed to in more recent times.

Related to this new-normal slow growth scenario is the "new neutral" thesis focusing on the exceedingly low real policy rates in many advanced and emerging economies alike. McCulley (2003) considers the US natural rate much lower than commonly assumed. In Clarida's (2014) view, central banks now operate in a world where average policy rates are set well below their pre-crisis levels, a direct consequence of the "global leverage overhang and moderate rates of potential trend growth". Clarida (2015) suggests that global factors have played a key role, with the lower US neutral policy rate driven by a slowdown in "global potential growth", and "a persistent excess of global saving relative to desired investment opportunities".

Other economists have cast doubts on the relevance of the hypothesis of secular stagnation and its inevitability. Eichengreen (2014) opines that the global economy is not predestined to suffer from secular stagnation, and "if the US experiences secular stagnation, the condition will be self-inflicted".<sup>8</sup> Eichengreen (2015) points out that, based on the life cycle theory, slower population growth and greater life expectancy actually imply lower savings rates. Bernanke (2015a) criticises the proponents' lack of consideration of the international dimension and global factors affecting domestic spending, and he questions whether an economy's equilibrium real interest rate can stay negative for an extended period. Gordon (2014a, b) attributes the diminished long-run growth potential to a return of US technological progress to its low historical norm, besides the structural headwinds of stagnant population and average US education level, rising inequality, productivity slowdown and elevated public debt. Hamilton, Harris, Hatzius and West (2015) argue that the recent slow growth is likely due more to temporary "headwinds" which may be dissipating, as the balance sheet repair continues with the help of an easy monetary policy.

All of the arguments above are quite relevant to the Asia-Pacific economies. Real interest rates have trended down in this region too, as growth has slowed and inflation has dropped in many Asian economies in recent years. This has posed

<sup>&</sup>lt;sup>6</sup> According to Keynes (1937), "in an era of a declining population, ... demand tends to be below what was expected and a state of over-supply is less easily corrected. Thus a pessimistic atmosphere may ensue".

<sup>&</sup>lt;sup>7</sup> Eichengreen (2015) defines secular stagnation as a "downward tendency of the real interest rate, reflecting an excess of desired saving over desired investment, resulting in a persistent output gap and/or slow rate of economic growth".

<sup>&</sup>lt;sup>8</sup> According to Eichengreen (2014), a US secular stagnation would reflect the country's failure "to address its infrastructure, education and training needs", "to take steps to repair the damage caused by the Great Recession and support aggregate demand in an effort to bring the long-term unemployed back into the labour market." In his view, "these are concrete policy problems with concrete policy solutions".

significant challenges for policymakers seeking guidance from estimates of the natural rates of interest and unemployment to implement monetary policy. Yet there have been so far very few attempts to estimate and assess equilibrium real interest rates for the emerging economies, and even fewer for emerging Asia.<sup>9</sup> This paper attempts to fill this lacuna by providing some simple estimates and, in so doing, shed light on the evolution of equilibrium real interest rates in a number of Asia-Pacific economies. In particular, I examine the roles of demographic trends, globalisation, financial intermediation and trend growth in the evolution of natural interest rates in the region to determine whether these factors may account for the changes over time and differences across countries in the natural rate estimates.

Several results emerge. First, I find that except for China, and also Thailand since 2005, the natural interest rate has declined substantially in the Asia-Pacific economies since the early or mid-1990s, by over 4 percentage points on average. In many economies the rate has turned negative. The tendency has become more accentuated in the 2000s, especially since the onset of the Global Financial Crisis and the Great Recession. Second, the natural interest rate estimates vary significantly over time and across economies. Third, the association seems to be broad and strong between the natural interest rate and the low-frequency trend components of demographic and global factors in Asia-Pacific, but it appears to be weak between the natural interest rate and trends in asset prices, credit-to-GDP ratio and trend growth in many economies in the region. In most cases, the natural interest rate does seem to be correlated with broadly measured long-term financial sector development, and trends in savings rate and investment ratio.

Nevertheless, large uncertainties and sizeable heterogeneity in the estimates of the equilibrium real interest rate call for caution as well as monetary policy rules that are robust to such uncertainties. In addition, understanding the underlying factors driving changes in the each economy's natural interest rate is important for the correct calibration and implementation of monetary policy.

The rest of the paper is structured as follows. In the next section, I review the existing approaches to the estimation of equilibrium real interest rates. In Section III, I give a detailed account of the empirical methodology, namely the estimation of frequency-domain indicators. Section IV presents results on equilibrium real interest rate estimates in Asia-Pacific based on spectral time series analyses. Section V discusses the monetary policy implications and Section VI concludes. The appendices include a detailed description of tools for frequency domain analysis; graphs of the estimates of the natural interest rate and its relationship with macro, financial, demographic and global factors; and a description of data.

# II. Estimating the equilibrium real interest rate

The equilibrium real interest rate, or the natural or neutral rate, has been defined in various ways based on economic theory. Often the natural rate is taken as the rate that equates saving and investment; or the one that is equal to the marginal productivity of capital. In fact, the natural rate is most meaningful in a general equilibrium, and is more appropriately defined as the rate that is consistent with full

<sup>&</sup>lt;sup>9</sup> There are exceptions, see, for example, Goyal (2008) and Goyal and Arora (2013).

employment and aggregate price stability. For Woodford (2003), the natural rate is the "equilibrium real rate of return when prices are fully flexible". Williams (2003) terms it as the "real fed funds rate consistent with real GDP equalling its potential level (potential GDP) in the absence of transitory shocks to demand", with the potential GDP defined as the "level of output consistent with stable price inflation, absent transitory shocks to supply". Similarly, Ferguson (2004) defines the natural rate as the "real interest rate consistent with the eventual full utilization of resources". Bernanke (2015b) calls it "the real interest rate consistent with full employment of labor and capital resources, perhaps after some period of adjustment".

The equilibrium real interest rate is unknown and has to be estimated. There are four broad approaches to its estimation. The purely statistical approach is modelindependent and based on Wicksell's idea that the "natural interest rate" is essentially a rate that prevails in the long run, so it is necessarily slow-moving. To measure the natural rate, one deflates a nominal interest rate by a suitable measure of inflation or its expectations, and then extracts the trend component of the resulting real rate. Under the assumption that, on average, actual rates are at or near their equilibrium values, one takes simple historical averages or moving averages of the real interest rate series over an appropriate time span. The measures rely on data and on the timeseries trend-cycle decomposition techniques, taking an agnostic view on the underlying theories. One therefore avoids conflicting estimates of the natural rate that can be derived from the same set of data but differ due to disagreement on its definition and model assumptions. Hamilton, Harris, Hatzius and West's (2015) follow this approach.

Second, financial market-based approaches extract information about the equilibrium real interest rate from the yield curve. Bomfim (2001) points out that a major drawback of the approach is that the long rates may move relative to short rates "for reasons other than a changing differential between actual and equilibrium short-term interest rates". Indeed market participants may misprice interest rate risks, and the long rates may also reflect changes in monetary policy, eg large-scale asset purchases in the major advanced economies. He proposes to estimate a forward-looking natural rate using yields on the US Treasury's inflation-indexed securities (TIPS), as the TIPS yields are not distorted by inflation expectations or inflation risk premiums. Yet such securities exist only in a few advanced markets, and it remains difficult to apply this technique more broadly.

A third, hybrid method is to align a carefully chosen econometric method with economic theory to identify and estimate the natural interest rate as an unobserved component in time-series models. The reduced-form models range from simple univariate regressions on the main determinants of the equilibrium real interest rate, to more elaborate vector autoregressive (VAR) models. A common reaction-function approach is to use Taylor-type feedback rules and take the estimated intercept as the natural rate. Taylor (1993) suggests a natural interest rate estimate of 2% for the US economy. Rotemberg and Woodford (1997) embed a feedback rule within a structural VAR model and estimate the US natural rate to be 3% from Q1 1980 to Q4 1995. In a more sophisticated approach, Laubach and Williams (2003) evaluate the natural rate using the Kalman filter. Specifically, they analyse US inflation and output dynamics in a restricted VAR model, jointly estimating the time-varying natural interest rate, potential output, and trend growth rate. They find a close link between the natural rate and trend growth, yet they concede that the natural rate estimates are "very imprecise and subject to considerable real-time measurement error". Garnier and

Wilhelmsen (2005) apply the method to the euro area and find that its natural rate has declined gradually since the early 1960s.

A fourth, more elaborate approach relies on structural models to better identify an economy's unobservable natural interest rate, in which the structural shocks likely to drive the evolution of the natural rate are well specified. Bomfim (1997) uses the MIT-Penn-SSRC (MPS) Keynesian model of the US economy to obtain an equilibrium federal funds rate series. The concept of the natural interest rate is especially appealing in Woodford's (2003) neo-Wicksellian framework: in the New Keynesian dynamic stochastic general equilibrium (DSGE) models with nominal rigidities,<sup>10</sup> the natural rate is defined as the real interest rate that would prevail when prices are fully flexible. In a situation of "divine coincidence" without the trade-off between the stabilisation of inflation and output gap, monetary policy simply tracks the estimated natural rate as the benchmark in every period. Edge, Kiley and Laforte (2008) rely on the evolution of natural rates of output and interest estimated from a DSGE model for the US economy to explain macroeconomic fluctuations. Based on estimated DSGE models for the US and euro area economies, Andrés, López-Salido and Nelson (2009) find that real money balances are valuable in anticipating future variations in the natural interest rate.

The structural general equilibrium approach has the advantage of allowing for the accounting of the sources of fluctuations in the equilibrium real interest rate. One significant drawback, more so in the aftermath of the Global Financial Crisis, is that the DSGE models have yet to better account for non-linearities and provide a convincing and realistic description of the functioning of financial intermediation in the economy.

The natural interest rate is often taken as constant, eg the original Taylor rule assumes an equilibrium real rate of 2%. Yet Wicksell (1898) points out that the natural rate is "never high or low in itself, but only in relation to the profit which people can make with the money in their hands, and this, of course, varies. In good times, when trade is brisk, the rate of profit is high, and, what is of great consequence, is generally expected to remain high; in periods of depression it is low, and expected to remain low." Laubach and Williams (2003), Mésonnier and Renne (2007) and Trehana and Wu (2007) explicitly take into account time variation in the natural rate estimates.

I take the first, statistical, approach and estimate the natural interest rate for a number of Asia-Pacific economies by identifying a time-varying trend in short-term policy rates, isolating the long-run component in the time series. The paper acknowledges the time-varying nature of the natural interest rate and treats it as such in the estimation.

# III. Methodology and data

The equilibrium real interest rate is unknown and unobservable; it can only be estimated, and unfortunately, the existing literature indicates that the estimates tend to be imprecise. This is particularly the case when economies face large shocks or go through structural changes. This section illustrates two empirical approaches I use to estimate the time-varying natural interest rates in Asia-Pacific, and to assess their

<sup>10</sup> See, for example, Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005).

relationship with trend growth rates, demographic trends, financial developments and an index of globalisation. I first apply the well known time-domain Hodrick-Prescott (1980, 1997) filter to these series to obtain the trend components for these variables and examine their correlations; I then use various different tools of spectral analysis of time series in the frequency domain to evaluate the relationship between natural interest rate estimates and the above-mentioned potential drivers.

#### III.1. Data and variables<sup>11</sup>

The empirical analysis is based on annual and quarterly data for 13 Asia-Pacific economies: Australia, China, Hong Kong SAR, India, Indonesia, Japan, Korea, Malaysia, New Zealand, the Philippines, Singapore, Thailand and the United States, spanning the period Q1 1950–Q4 2014. To get the data as early in time as possible, the empirical analysis focuses on annual data, with the downside of having fewer data points available.

The set of data series comprises macroeconomic real and price variables (real GDP, real private consumption, employment, unemployment rate, consumer price index or CPI, GDP deflator, Consensus Economics © CPI forecasts and real effective exchange rate or REER), demographic variables (growth in total and working age population, shares of those aged between 39 and 64 years and of those aged above 64 years in total population, total and old age dependency ratios, and life expectancy), financial variables (real equity and housing prices, total bank lending, total credit to the private sector, financial development index of Sahay et al (2015) which measures the depth, access and efficiency of financial institutions and financial markets), and external or global variables (KOF globalisation indices of Dreher (2006) and Dreher, Gaston and Martens (2008); King-Low (2014) world real interest rate, global official liquidity, G7 and G20 aggregate policy rate).

The key variable is obviously the real interest rate, which is obtained by deflating a nominal interest rate by a suitable measure of inflation or its expectations. The choice of the type and maturity of the nominal interest rate is relatively straightforward: central banks are most interested in a natural rate benchmark towards which the policymakers can adjust the policy rates. Therefore I focus on short-term nominal interest rates that are either policy rates or their closest market counterparts. For most economies, this means an interbank overnight rate. In China's case, I use an average of interbank overnight, seven-day repo and three-month deposit rates. For Hong Kong SAR and New Zealand, the one-month HIBOR rate and 30-day Bank Bill rate are used, respectively. To obtain longer historical series, I supplement the series with less appealing alternatives, such as discount rates.

To compute an *ex ante* real interest rate, it is important to have an appropriate measure of inflation expectations. One measure is the difference between the interest rates on nominal US Treasuries and on US Treasury's inflation-indexed securities, yet inflation-indexed bonds are uncommon in the region. Another measure is private sector forecasts, which are scarce for most economies in our sample. While Consensus Economics © forecasts are available, they only start in the last quarter of 1989 or later for the economies in the region. A more practical alternative is to use the forecasts from a simple autoregressive model of actual inflation to proxy the expected inflation, as in Blanchard and Summers (1984) and Hamilton, Harris, Hatzius and West (2015). I

<sup>&</sup>lt;sup>11</sup> Annex C provides details of the definition, construction and sources of data used in this paper.

take a similar approach. I first use the Hodrick-Prescott filter and other spectral methods to decompose the inflation rate into the trend and cyclical components, I then use the trend component as the expected inflation to obtain the *ex ante* real interest rate. The approach has the advantage that trend inflation can be seen as a proxy for the long-run equilibrium inflation rate, a target for central banks pursuing price stability.

#### III.2. Empirical methods

Spectral analysis of time series is appealing, as covariance stationary processes can be uniquely decomposed into mutually uncorrelated components, each associated with a specific frequency (band). Spectral or frequency-domain methods have a long tradition in the economic analysis of time series. Granger (1966) provides evidence that macroeconomic time series tend to be persistent and have a "typical spectral shape" with much of the power of the time series concentrating in the very low frequencies, ie the long run. Granger and Rees (1968) apply spectral methods to analyse the term structure of interest rates. Hannan (1963a, 1963b) pioneered spectral regression analysis, which was introduced to economics by Engle (1974, 1978, 1980). Phillips (1991) applies it to integrated time series to obtain asymptotically median unbiased estimates of cointegrating coefficients. Spectral regression allows us to focus on specific frequency bands, and permit a non-parametric treatment of regression errors.

Instead of working directly in the frequency domain, economists often rely on linear filters that decompose data into trend and cycle components converted back into the time domain. These include the Lucas (1980) exponential smoothing filter, the Hodrick-Prescott (1980, 1997) and the Baxter-King (1999) band-pass filter. Due to finite data length, these filters are only approximations to the ideal filters, and filter leakage, compression and exacerbation are inevitable. Moreover, simple correlation and regression analyses average relationships within each frequency band, this could mask possibly large variations within any pre-specified band.

We use frequency-domain methods to study how estimated equilibrium real interest rates may behave in Asia-Pacific economies, and how they relate to potential natural rate drivers such as demographics, globalisation, financial developments and growth at different frequencies in different economies. Engle (1974) points out that "there is little discussion of whether the same model applies to all frequencies. It may be too much to ask of a model that it explain both slow and rapid shifts in the variables, or both seasonal and non-seasonal behaviour. It is at least reasonable to test the hypothesis that the same model applies at various frequencies." Zhu (2005) uncovers significant difference in the inflation-output trade-offs across the spectrum, from the short to the long run. Zhu (2012) finds that the credit and output relationship varies greatly from the short to the long run, being strong in the very low frequencies but rather weak in business-cycle and higher frequencies.

Significant cross-frequency differences in how the natural rates relate to the potential drivers may have important implications for policymaking. In this paper, I take a more direct approach and examine such linkages in the entire spectrum. I first estimate conventional spectral indicators including spectral and cross-spectral densities, coherency, transfer function, gain and phase-to-frequency ratio. I then assess the natural interest rate linkages estimating frequency-specific coefficients of

correlation (FSCC) and regression (FSCR) proposed by Zhu (2005).<sup>12</sup> To obtain the coefficient estimates, I apply a data extraction procedure based on Fourier and inverse Fourier transforms. I convert the data back into the time domain, where conventional statistics can be calculated.

The FSCC is superior to traditional indicators, such as coherence and cospectrum, by providing a real-valued, normalised and signed measure of the strength of multiple correlation. Unlike coherence, the FSCC is signed. Compared with the cospectrum, it is standardised taking values in the [-1,1] range, providing a clear indication of the strength of correlation independent of data scale. The FSCR has the advantage of being easily applied to any specific frequency, and the statistical inference with both FSCC and FSCR estimates is straightforward.

# IV. Estimating equilibrium real interest rates

A visual inspection of the equilibrium real interest rates, estimated based on the Hodrick-Prescott (1981) filtering technique, suggests both commonality and diversity (Graphs B.2.1 and B.2.2 in Annex B.2). First, the estimated natural interest rates have fallen significantly since the early and mid-1990s. In most cases, the decline has been sizeable, above 4 percentage points. In some economies, the decline started earlier, eg Singapore and Thailand in the early 1980s and the United States in the late 1980s. In addition, the fall has apparently been accentuated in the aftermath of the 2007–09 Global Financial Crisis. In contrast, the estimated equilibrium real interest rate has been on an upward trend in China since 1993, when data became available. In Thailand's case, the natural rate apparently started to rebound from a low of 0.61% in 2005 to reach 1.60% in 2014.

Second, natural rate estimates in the Asia-Pacific economies show different patterns of evolution, and the rates can differ substantially from each other at any single point of time. In particular, Japan's natural interest rate has evolved in a notably different way. It rose significantly from the low, negative levels in the first years of the 1970s to a peak in the early 1980s, but has experienced a long secular decline since then.

Notably, while negative equilibrium real interest rates were less common in the past, they have become a standard feature in many regional economies in the aftermath of the Global Financial Crisis. Out of Australia, China, Malaysia and Thailand, the natural rates are now at historically low levels. While the natural rates in New Zealand, Singapore and Thailand have stayed positive throughout the sample period, all other economies have experienced negative natural interest rates in the past.

The frequency-domain analysis focuses on the likely factors behind the secular decline in equilibrium real interest rates, namely, changing demographic trends, rising globalisation, financial sector developments and slowing trend growth. The analysis is based on the estimates of a set of traditional frequency-domain indicators: spectral density, or power spectrum, estimated using Welch's method, which records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process; cospectral density, also known as

<sup>&</sup>lt;sup>12</sup> Annex A provides an exposition of frequency-domain analysis used in this paper. For further details, see Zhu (2005, 2012).

cross-power spectral density or cospectrum, estimated using Welch's averaged modified periodogram method, which represents the covariance between the inphase components of two stochastic processes at a specific frequency or frequency band; squared coherence, or coherency, estimated using Welch's averaged periodogram method, which, like the coefficient of determination  $R^2$ , measures the strength of linear association between two stochastic processes at a specific frequency or frequency band; gain or transfer is analogous to a standardised regression coefficient at a given frequency; and the phase measures the timing or average phase lead of one series over another at different frequencies, which incorporates all relevant information about leads and lags.

The estimates of traditional spectral indicators are presented in Annex B.3, along with the estimates of Zhu's (2005) frequency-specific coefficients of correlation and regression. In all graphs, the shaded grey area indicates the business-cycle frequency range of between six quarters (marked by "H") and 32 quarters (marked by "L"). The area between 0 and L is the low-frequency range which contains the trend component, which is of primary interests to the analysis when low-frequency, long-run relationships are examined in comparison with their business-cycle- and high-frequency counterparts.

Spectral density estimates indicate that almost all demographic, global, macroeconomic and financial variables under study have the "typical spectral shape" of Granger (1966). The spectral density estimates indicate that most of these variables are very persistent in Asia-Pacific economies, with much of the power of their spectral density concentrating in the very low frequencies, ie the long run of beyond 32 quarters. This suggests that understanding trends are crucial to the analysis. However, one major exception is asset prices, of which the power is more evenly distributed across the frequency range, or more heavily concentrated in higher frequencies. This is the case for real housing price inflation in Australia, and for real equity price inflation in China, India, Indonesia and Japan. In a number of cases, eg real housing price inflation in Korea and real equity price inflation in Malaysia, asset prices. This implies that cycles in equity or housing prices may carry more power than trends. It is no wonder that trend equity or housing price inflation turns out to have a relatively weak association with the equilibrium real interest rate in the analysis.

Similarly, estimates of cross-spectral density, coherency, gain and frequencyspecific coefficients of correlation and regression suggest that association between real interest rates and many of the demographic, global, macroeconomic and financial variables I examine are often strongest in the very low frequencies (ie the long run) in Asia-Pacific, but such an association is not always strong in absolute terms. In particular, real interest rates seem to be more associated with many financial variables in higher frequencies, implying that real GDP growth and asset market or credit market developments are not as crucial to the understanding of evolution of the equilibrium real interest rate as many of us have assumed, and other factors, especially globalisation and demographic trends may matter more.

The following subsections give a detailed account of the correlations of these factors with the estimates of equilibrium real interest rates in the Asia-Pacific economies.

### IV.1. Demographic trends

As the world population continues to expand, it has been ageing rapidly (Graph B.1.1 in Annex B.1). Population ageing is likely to reduce per capita GDP growth and prompt changes in the savings and investment dynamics. Ageing tends to decrease saving in the advanced economies and boost the equilibrium real interest rate, but it tends to make labour scarcer relative to capital, reducing the interest rate. In the emerging economies, at higher growth rates, saving is higher as the population is expected to age, which depresses the world equilibrium interest rate.

Empirical work suggests that demographic trends and changes in population structure may have an important impact on the equilibrium real interest rate. For example, simulations by Börsch-Supan (2004) suggest a 1.5 percentage-point decline in the European Union's natural rate in the next 30 years. Population ageing has already profoundly affected the Japanese economy.<sup>13</sup> Indeed Japan's working-age population began to decline in absolute terms in the mid-1990s, and both Korea and China are expected to face similar challenges in the coming decades. According to its National Bureau of Statistics, China's working age population has continued to decline since 2012, accompanied by rising labour costs.

I use several variables to describe population dynamics. First, the shares of those aged between 39 and 64 years and of those aged above 64 years in total population, which correspond to the population groups that save most for retirement or dissave in retirement, respectively. The patterns of their saving and consumption have a direct impact on an economy's saving-investment dynamics, and hence the equilibrium real interest rate. Second, the total and old age dependency ratios indicate an economy's ability to sustain non-working age population. Third, rising life expectancy may influence individual decisions on work, consumption and saving. As life expectancy rises, retirement is postponed yet periods of retirement lengthen. Savings can rise in the aggregate as workers save more or they can fall due to the dissaving of retirees. Fourth, growth in the working age population has slowed or turned negative in many advanced economies, and the secular trend has started to affect some emerging economies. This reduces an economy's long-term growth potential and its natural interest rate.

As Graph B.1.1 shows, the King-Low (2014) world real interest rate has followed a broad downward trend, falling from a peak of 4.80% in 1993 to a trough of –0.08% in 2012. This has been accompanied by the continued rise in people aged between 39 and 64 years and of those aged above 64 years, and the groups' shares in the world population. At the same time, the growth rate in the world working age population has slowed to less than half its previous rate in less than two decades. Spectral analysis provides further interesting details of the relationship between real interest rate and population dynamics in Asia-Pacific (Graphs in Annex B.3). First, while spectral density estimates indicate that trend is the most important component in the share of people aged 39–64 in total population and old age dependency ratio, there are clear four distinct cycles in higher frequencies of less than six quarters, for all economies in the sample. The demographic trend and cycles may then be translated into their relationships with the real interest rate, as indicated by the cospectral density estimates.

<sup>&</sup>lt;sup>13</sup> See Shirakawa (2011) and Shirai (2012).

Second, a strong association between demographic trends and the equilibrium real interest rate is seen in most spectral indicators in many economies. This is most apparent in the cospectral density and coherency estimates. However, correlation patterns can differ, and in some cases, the sign of correlation, raising important issues regarding economic reasoning behand the relationship between demographic trends and the natural interest rate. Two important exceptions may be China and Korea, where correlation coefficient estimates at around 32 quarters point to statistically insignificant correlations between the equilibrium real interest rate and the population share of those aged 39-64 and the old-age dependency ratio; but they indicate a statistically significant, strong and negative correlation of -0.75 with working age population growth in China, but a large positive correlation in Korea. The old-age dependency ratio correlates negatively with the natural interest rate in Japan but positively in the United States, while working age population growth has a strong positive correlation with the natural interest rate in both countries. Hence population ageing and shrinking working age population lower the equilibrium real interest rate in Japan, but the more complex US population dynamics, largely due to immigration, may imply a different relationship here.

#### IV.2. Globalisation

Another potential driver of the decline in equilibrium real interest rates is globalisation. The gradual elimination of barriers to cross-border trade and financial transactions has led to an ever more integrated global market for goods and services, labour and financing. As a consequence, the potential output, CPI and asset price dynamics of one economy have become increasingly dependent on those of other economies. Indeed, as the degree of globalisation rises in the major economies, the King-Low (2014) world real interest rate has continued to decline (Graph B.1.2). One key element might have been China's integration into the world economy. Measured by the KOF economic globalisation index, China's globalisation level surpassed that of Japan in 1995, and again in the early 2000s after China's accession to the World Trade Organisation in December 2001. In addition, China's savings rate began a rapid rise in 2001, which probably more than compensated for the decline in the Japanese and US savings rates (Graph B.1.3). These coincided with a steep descent in the world real interest rate starting in 2001.

Global financial integration implies a decreased home bias, as savings in one economy can be used to finance investment in another and domestic interest rates become more closely related. Focusing on the surge of interest rates in the early 1980s, Blanchard and Summers (1984) highlight the global nature of such rate increases. Barro and Sala-i-Martin (1990) postulate a global capital market where the interest rate is determined by global investment demand and desired saving. Bosworth (2014) considers capital markets highly integrated globally, so that it makes little sense to study interest rates within a closed-economy framework. Moreover, global official liquidity could have played a major role with the advent of large-scale asset purchase programmes by central banks in major advanced economies since late 2008, prompting a second steep descent in the world real interest rate (Graph B.1.3, left-hand panel).

For want of a more appropriate indicator of global trade and financial integration, I use the KOF overall globalisation index developed by Dreher (2006) and Dreher, Gaston and Martens (2008), which summarises the economic, social and political dimensions; and the KOF index of economic globalisation, and sub-indices of the actual trade and capital flows, and of trade and capital account restrictions. I also use an indicator of global official liquidity, which is the sum of the total assets of the central banks in the advanced economies and the foreign reserves of the major emerging economies. Other global liquidity indicators, such as the G7 and G20 aggregate (weighted average) policy rate, are considered, and it is well known that domestic interest rates have over time become more strongly correlated with interest rates in the advanced economies.

Spectral density estimates indicate a large persistence in KOF globalisation indices (Graphs in Annex B.3). Estimates of cospectral density, squared coherence and frequency-specific correlation coefficient suggest that globalisation, overall or economic, is more strongly associated with the equilibrium real interest rate than many other variables in most economies in the region. There is again sizeable heterogeneity: while the evidence is strong in Australia, Malaysia, Japan and the United States, the case is weaker among some emerging Asian economies. In particular, estimates of frequency-specific correlation coefficient indicate that the correlation of KOF overall and economic globalisation indices with the equilibrium real interest rate is not significantly different from zero in China, Indonesia, the Philippines and Thailand, in some cases contradicting evidence provided by other spectral indicators such as cospectral density and coherency. One possibility is that the emerging economies, albeit more globalised than before, are still some distance away from the level of globalisation achieved by the advanced economies, and their interest rates may be more prone to various domestic factors or controls, leading to a weaker association.

#### IV.3. Financial sector developments

A third driver may be changes in the functioning of financial intermediation due to sizeable financial frictions in the wake of the crisis. There are reasons why financial sector developments may matter for the evolution of the equilibrium real interest rate. As Hamilton, Harris, Hatzius and West (2015) point out, the natural rate may vary according to how monetary policy is transmitted through longer-term interest rates, credit availability and various asset prices including exchange rates. Sustained changes to regulatory policy, such as those we see in the aftermath of the Global Financial Crisis, may change the financial infrastructure and affect the natural rate.

There is evidence that recessions and financial crises may cause permanent damage to an economy and dislocate its equilibrium real interest rates.<sup>14</sup> In addition, financial market frictions may matter for the natural interest rate. De Fiore and Tristani (2011) find that natural rate dynamics may react to exogenous shocks in qualitatively different ways, depending on the underlying model assumptions. While financial market turbulence leads to a fall in the natural rate in their model, which is characterised by nominal rigidities with borrower-lender information asymmetry, it has no macroeconomic impact if financial markets are frictionless. Hamilton, Harris,

<sup>&</sup>lt;sup>14</sup> Cerra and Saxena (2008) and Reinhart and Rogoff (2009) find highly persistent output effects from deep recessions around the world. Cecchetti and Zhu (2009) suggest that, most often, output level falls permanently and growth rate rises after financial crises. Ball (2014) examines 23 OECD countries and finds that the potential output loss from the Great Recession varies greatly across countries, but is large in most cases. The long-term effects of recessions seem to be consistent with the hysteresis hypothesis of Blanchard and Summers (1986). Reifschneider, Wascher and Wilcox (2013) argue that a recession reduces an economy's potential output.

Hatzius and West (2015) consider a more robust policy rule, which reacts to changes in financial conditions when policymakers have doubts about the true value of the natural interest rate. The rule may be helpful if, eg changes in the natural rate reflect variations in the marginal product of capital, information on which may be contained in equity prices.

I use a number of financial variables, namely, the financial development index of Sahay et al (2015), real credit growth and the growth rates in real housing and equity prices to examine how they correlate with equilibrium real interest rates in the long run. The financial development index developed by Sahay et al (2015) measures the depth, access and efficiency of financial institutions and financial markets. A visual examination shows that, while the decline in world real interest rate seems to be associated with the sustained improvement in the financial development index in China and Japan, its relationship with global credit growth is less clear (Graph B.1.3, right-hand panel).

Estimates of spectral indicators provide a number of interesting findings (Graphs in Annex B.3). First, growth in real equity or housing prices is generally uncorrelated with the estimated equilibrium real interest rate, and much of the correlation between asset prices and the real interest rate occurs at high or very high frequencies, well below four quarters. One exception may be Japan, where the estimated frequencyspecific coefficients of correlation point to a positive correlation of close to 0.6 in the low-frequency range, which was contradicted by a very low reading in the squared coherence estimate. Second, compared with real equity and housing prices, the credit-to-GDP ratio tends to have a stronger association with the equilibrium real interest rate estimate at very low frequencies, eg in Australia, India, Indonesia, Malaysia and the Philippines, but the association is generally weak. Long-term credit market developments may therefore have a mild relationship with the evolution of equilibrium real interest rate.

Consistent with earlier observations, broad financial sector development as measured by the financial development index developed by Sahay et al (2015), in the very low frequencies (long run), is indeed more strongly associated with the equilibrium real interest rate estimate in most economies in the region. Again there are exceptions, eg India, Indonesia and Thailand. And in several cases, while cospectral density and coherency estimates indicate the existence of association, this is contradicted by evidence provided in the estimates of frequency-specific correlation coefficients.

To sum up, long-term developments in the financial sector, especially when defined broadly, may be associated with the evolution of equilibrium real interest rate, but credit market per se, and asset prices in particular, are estimated to have rather weak association with the natural interest rate. Improvement in a country's financial infrastructure could play a role in the evolution of the equilibrium real interest rate.

#### IV.4. Trend growth

One issue that has recently attracted much attention is how closely trend growth is associated with the equilibrium real interest rate. Estimation of the natural rates, eg Laubach and Williams (2003), often embodies a close relationship between the two, with the estimation becoming predicated on this relationship. Yet Bosworth (2014) identifies only a weak relationship between real interest rates and economic growth.

Leduc and Rudebusch (2014) examine private sector professional forecasts and historical data, but they find little evidence that declines in the long-run potential growth rate of the economy translate into lower interest rates. A visual examination of the relationship between the world real interest rate and growth rates in the advanced and emerging economies does not provide a clear-cut pattern and a careful analytical study may help (Graph B.1.1, right-hand panel).

Evidence from the spectral analysis is mixed, and several messages emerge (Graphs in Annex B.3). First, in a number of economies, the estimated correlation between the equilibrium real interest rate and trend growth is not statistically significant: this is the case in Australia, China, Indonesia, and the Philippines. Indeed a stronger relationship between the real interest rate and real GDP growth tends to happen in the high-frequency range of less than four quarters, eg in Australia, India, Korea, Malaysia, the Philippines and the United States, as indicated by coherency estimates. In some other economies, such as Japan, Korea and Thailand and the United States, the estimated correlation between the equilibrium real interest rate and trend growth is strong. The overall evidence is rather mixed, with significant cross-economy heterogeneity. Second, many economies observe a statistically significant correlation of their savings rate and investment ratio with the equilibrium real interest rate. The notable exceptions are China and India, and also Indonesia (investment ratio) and Thailand (savings rate). The correlation is estimated to be stronger in Japan, Korea and Malaysia.

The results suggest that the relationships between the equilibrium real interest rate and trend growth and other macroeconomic variables are more complex than once assumed, and the large disparity in the estimated relationships across the Asia-Pacific economies suggest that more work needs to be done to update economic theory so as to improve our understanding of the equilibrium real interest rate and provide a sound foundation for monetary policymaking.

# V. Implications for monetary policy

Can we rely on the estimated equilibrium real interest rates to guide monetary policy? Orphanides and Williams (2002) suggest that, for central banks pursuing price stability and full employment through the adjustment of short-term interest rates in response to economic developments, they need "accurate, quantitative, contemporaneous readings of the natural rate of interest and the natural rate of unemployment". Yet Friedman (1968), the proponent of the concept of "natural rate of unemployment", points out that the policymakers would not know the true natural rate.

One key challenge is that the natural rate estimates are known to be imprecise and subject to a high degree of uncertainty, which greatly reduces their practical usefulness. Laubach and Williams (2003) find the natural interest rate estimates very imprecise and subject to considerable real-time measurement error. Clark and Kozicki (2005) conclude that the natural rate estimates are prone to a "high degree of specification uncertainty, an important one-sided filtering problem, and considerable imprecision due to data uncertainty", and conclude that "statistical estimates of the equilibrium real rate will be difficult to use reliably in practical policy applications". Obtaining good estimates is even harder in most Asia-Pacific economies, as data tend to be of short span and less reliable, and data and model uncertainties are typically greater. There is significant heterogeneity in terms of the stage of economic and financial sector development, political, socio-economic and market institutions, and monetary policy framework, and different economies also tend to face shocks which may differ substantially in terms of their nature and persistence.

Our analysis suggests that the estimates of the equilibrium interest rates in Asia Pacific in general vary substantially over time and across economies. The perceived degree of imprecision and uncertainty is sizeable. For instance, the HP-filtered estimates of the natural interest rate in all economies vary significantly depending on the value of the smoothing parameter  $\lambda$  (Graph B.2.1). Indeed, for annual data, while Hodrick and Prescott (1981) propose a value of 100 for the smoothing parameter  $\lambda$ , Ravn and Ulhig (2002) suggest that  $\lambda = 6.25$  is more appropriate. I experimented with different values for the smoothing parameter, namely,  $\lambda = \{6.25, 100, 400, 800, 1600, 6400, 25600, 129600\},$  the resulting equilibrium real interest rate estimates turn out to be very different, with the difference going well above 2 percentage points. In the case of Thailand, for example, the difference in the 2014 estimates with  $\lambda = 6.25$  and  $\lambda = 6400$  is almost 4 percentage points; and more seriously, when one estimate points to a strong recent upward trend, the other estimate suggests a secular decline in Thailand's natural interest rate. Indeed this is a common issue with many filters, where the choice of the degree of smoothing belongs to the researcher, but monetary policy obviously cannot reply on the subjectivity of such decisions.

Some economists argue that the estimation precision may improve with those estimates based on carefully specified structural models tailored to each economy and a well implemented estimation technique. Yet such estimates may depend on specific model assumptions, which may be too strong or lack empirical foundations. For instance, the natural rate estimates of Laubach and Williams (2003) predicate a strong relationship between trend growth and the natural rate. Yet, spectral estimates in this paper suggest that such a relationship is quite weak in many economies in the region. Existing natural interest rate models tend to be closed-economy and ignore demographic trends; this paper shows that global factors and population dynamics do matter for changes in the natural interest rate. Model uncertainty is aggravated by data uncertainty: besides the difficulties in correctly specifying a convincing model for an emerging economy, data tend to be too short or imprecise to best serve the quantification of any economy in emerging Asia. Therefore the use of the current natural rate frameworks to implement monetary policy runs many risks, given the uncertainties surrounding the natural rate estimates and the underlying factors driving the natural rate movements.<sup>15</sup>

Understanding how different factors may influence the equilibrium real interest rate is important for monetary policy considerations. First, some factors, such as population dynamics, may be structural and out of the monetary policymaker's direct control, and can be better addressed with structural measures. This may have been the case of Japan in the last two decades, where the natural interest rate estimate is

<sup>&</sup>lt;sup>15</sup> As Friedman (1968) put it, "what if the monetary authority chose the 'natural' rate – either of interest or unemployment – as its target? One problem is that it cannot know what the 'natural' rate is. Unfortunately, we have as yet devised no method to estimate accurately and readily the natural rate of either interest or unemployment. And the 'natural' rate will itself change from time to time. But the basic problem is that even if the monetary authority knew the 'natural' rate, and attempted to peg the market rate at that level, it would not be led to a determinate policy. The 'market' rate will vary from the natural rate for all sorts of reasons other than monetary policy."

shown to have a strong positive correlation with trends in working age population growth and economic globalisation and a strong negative correlation with the trend in the old-age dependency ratio (Graph B.3.5.4). In fact, if Japan's natural interest rate did fall substantially due to population ageing and a sustained reduction in the working force, then even a very low or near-zero policy rate in a low-inflation or deflationary environment may not necessarily imply a real interest rate close to the natural rate, with the policy stance being tighter than perceived. Furthermore, monetary accommodation itself would do little to amend the structural deficiency; only structural measures encouraging labour force growth and participation may address the problem.

Second, as the spectral analysis shows, factors such as globalisation are likely to be relevant for the determination of natural rate of interest in most economies (Graphs in Annex B.3). This complicates a central bank's task of monetary policy evaluation, as the underlying natural rate has grown increasingly dependent on external factors. Policymakers may need to pay greater attention to external developments, beyond the narrow focus on domestic price and output gaps.

Third, a close relationship between the natural interest rate and financial sector developments in some economies would imply that monetary policy tracking the natural rate benchmark needs to carefully monitor the emergence and evolution of possible financial imbalances and take into account financial conditions. Concerns with financial stability and the impact of regulatory changes may therefore factor into monetary policy considerations. Yet correlations of the natural interest rate and asset price trends are shown to be weak in many economies in Asia-Pacific (eg Australia, India, Indonesia, Japan, Korea, Malaysia, the Philippines and Thailand). This is true, to a lesser extent, in terms of long-term credit market developments, although trends in a broad financial development index do seem to correlate with natural interest rates in many economies. Therefore each central bank needs to carefully assess the likely impact of financial sector components on its equilibrium real interest rate, and avoid the temptation of overreacting to asset market fluctuations.

Last but not least, Clark and Kozicki (2005) and Hamilton, Harris, Hatzius and West (2015) consider the link between trend growth and the equilibrium real interest rate to be quite weak. This turns out to be the case for a number of Asia-Pacific economies, eg Australia, India, Indonesia, Malaysia, the Philippines and the United States, as the estimates of many spectral indicators reveal. The weak linkage between the equilibrium real interest rate and trend growth in many economies may undermine many structural natural rate estimates predicated on such linkages, increasing the uncertainty surrounding some existing equilibrium real interest rates. However, the equilibrium real interest rate's linkages with savings rate and investment ratios tend to be stronger in many economies.

One important caveat about my estimates of equilibrium real interest rates is that they are filtered trends rather than true "equilibrium" values, under the assumption that the trends may faithfully reflect the equilibrium. Indeed it is possible that the actual real interest rate may deviate from their equilibrium values for an extended period of time, especially for an economy hit by large and persistent shocks. Yet there is no assurance that structural estimates, many of which are based on linear models, would fare any better under these circumstances. Another caveat is that the estimates of spectral indicators tend to be less accurate with small sample sizes, and in the cases of frequency-specific correlation and regression estimates for some Asia-Pacific economies, the data are too short to provide any reliable reading of the relationship between the real interest rate and its possible driving factors in the frequency range of beyond eight years.

The heightened uncertainties surrounding the natural interest rate estimates call for policy rules which are robust to such uncertainties. Canzoneri, Cumby and Diba (2015) find that, for a central bank, tracking the natural interest rate is important for household welfare, especially "in an environment where interest rates take large and persistent swings around their long run equilibrium values, making it difficult for the policy rate to catch up with its natural rate". Orphanides and Williams (2002) conclude that "uncertainty about natural rates in real time recommends against relying excessively on these intrinsically noisy indicators when making monetary policy decisions". Drawing lessons from the 1970s and the late 1990s, they advise that "the policy rule should incorporate a biased protection against measurement error and respond only modestly to estimates of the natural rates of interest and unemployment". Similarly, Hamilton, Harris, Hatzius and West (2015) reach the conclusion that it pays to inject more inertia into the monetary policy reaction function when the uncertainty around the natural interest rate is high, by placing higher weights on the lagged values of the policy rate. To guard against misconceptions about the natural rate, they defend the adoption of a later but steeper path for US federal funds rate normalisation.

In addition to the difficulties in obtaining accurate and timely estimates of the equilibrium interest rates, the recurrent binding zero lower bound on nominal interest rate may pose a significant challenge for central banks intending to implement interest rate targets against natural rate benchmarks. In Asia-Pacific, this has been a serious issue for the Bank of Japan in the last two decades and for the Federal Reserve more recently, and it may potentially become an issue for other central banks, although most economies in the region still have quite some room for manoeuvre in the foreseeable future.

### VI. Conclusion

This paper estimates the equilibrium real interest rate for a number of Asia-Pacific economies, and studies its relationships with demographics, the process of globalisation, indicators of financial sector developments, and macroeconomic variables such as trend growth. I take a purely statistical approach and focus on the empirics, applying the time-domain Hodrick-Prescott (1981, 1997) filter and frequency-domain tools, including both traditional spectral indicators and the frequency-specific correlation and regression techniques proposed by Zhu (2005).

The spectral analysis uncovers a number of interesting empirical facts for the Asia-Pacific economies. First, the estimated natural interest rates started to trend down in some economies in the region as early as the 1980s, and the tendency has become more accentuated in the 2000s, especially since the onset of the Global Financial Crisis and the Great Recession. Second, the natural interest rate estimates vary significantly over time and across economies, ranging from -1.32%, -0.47% and -0.37% to 4.20%, 5.27% and 5.96% in Indonesia, Japan and Korea, respectively. Third, the association seems to be broad and strong between the natural interest rate and the low-frequency trend components of demographic and global factors in Asia-Pacific, but it appears to be weak between the natural interest rate and trends in asset prices, credit-to-GDP ratio and trend growth in many economies in the region. In

most cases, the natural interest rate does seem to be correlated with broadly measured long-term financial sector development, and trends in savings rate and investment ratio.

Nevertheless, large uncertainties and sizeable heterogeneity in the estimates of the equilibrium real interest rate call for caution as well as monetary policy rules which are robust to such uncertainties. In addition, understanding the underlying factors driving changes in the each economy's natural interest rate is important for the correct calibration and implementation of monetary policy.

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# Annex A: Frequency domain analysis

In this annex, we illustrate the method we use to estimate frequency-wise correlation and regression estimates, which is based on a simple frequency-specific data extraction procedure.

#### A.1. A frequency-specific data extraction procedure

Consider a time series vector  $x = [x_1, x_2, ..., x_T]^T$ . For s = 1, ..., T, define the fundamental frequencies as  $\omega_s = 2\pi s/T$ . The discrete Fourier transform of x at frequency  $\omega_s$  is

$$w_s x = T^{-1/2} \sum_{t=1}^T x_t e^{(t-1)i\omega_s}$$

where

$$w_s = T^{-1/2} \begin{bmatrix} 1 & e^{i\omega_s} & \cdots & e^{(T-1)i\omega_s} \end{bmatrix}$$

Define

$$W = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{T-1} \end{bmatrix}$$

W is a unitary matrix such that  $W^*W = WW^* = I$ , where \* indicates the Hermitian conjugate (ie transpose of the complex conjugate). Then  $\tilde{x} = Wx$  is the vector of discrete Fourier transform of time series x at all fundamental frequencies  $\omega_s$ , s = 1, ..., T - 1.

Define  $A_s$  as a  $T \times T$  selection matrix which selects the s-th element or row from any data vector or matrix, respectively. It has 1 as the s, s-th element and zeros elsewhere.<sup>16</sup> The data vector of the discrete Fourier transform of time series x at the s-th frequency  $\omega_s$  is

$$A_s \widetilde{x} = A_s W x$$

So there are T data vectors  $A_s \tilde{x}$ , extracted from the original time series x, each of length T. All but the s-th elements of the s-th data vector  $A_s \tilde{x}$  are zero. We then use inverse Fourier transform to convert the complex data vector  $A_s \tilde{x}$  back into the time domain. Write the frequency- $\omega_s$  inverse Fourier transform of the time series x as

$$\widetilde{x}(\omega_s) = L_s x = W^* A_s W x$$

<sup>16</sup> To select a frequency band  $[\omega_s, \omega_t]$ , let the *s*-th to *t*-th diagonal elements of *A* be one.

where  $L_s = W^*A_sW$  is a linear operator. Using Fourier and inverse Fourier transforms and the selection matrix  $A_s$ , from any data vector x, we can extract T time series  $x(\omega_s) = [x_1(\omega_s), x_2(\omega_s), ..., x_T(\omega_s)]^T$ , each corresponding to a specific frequency  $\omega_s$ , where s = 1, ..., T. Based on these frequency-specific data, we can then design frequency-wise correlation and regression coefficients in a conventional way.

#### A.2. Correlation analysis

For bivariate stochastic processes  $z_t = \begin{bmatrix} x_t & y_t \end{bmatrix}^T$ , which are assumed to be jointly weakly stationary with continuous spectra, we write the corresponding spectral density matrix as<sup>17</sup>

$$f_{zz}(\omega) = \begin{bmatrix} f_{xx}(\omega) & f_{xy}(\omega) \\ f_{yx}(\omega) & f_{yy}(\omega) \end{bmatrix}$$

where the spectral densities  $f_{xx}(\omega)$ ,  $f_{yy}(\omega)$  and the cross-spectral density  $f_{xy}(\omega)$  determine the relationship between  $x_t$  and  $y_t$  at frequency  $\omega$ . In Cartesian form, the cross-spectral density  $f_{xy}(\omega)$  can be written as

$$f_{xy}(\omega) = c_{xy}(\omega) - iq_{xy}(\omega)$$

where  $c_{xy}(\omega)$  and  $q_{xy}(\omega)$  are real-valued functions known as **cospectrum** (or **cospectral density**) and **quadspectrum** (or **quadrature spectral density**), respectively. The cospectrum  $c_{xy}(\omega)$  represents the covariance between coefficients of the in-phase components of two time series, while the quadspectrum  $q_{xy}(\omega)$  represents the covariance between coefficients. Cospectrum estimation is equivalent to studying the off-diagonal elements of the variance-covariance matrix between two time series, which are uniquely related to cospectra by Fourier and inverse Fourier transforms.

In polar form,

$$f_{xy}(\omega) = |f_{xy}(\omega)| \exp(i\varphi_{xy}(\omega))$$

where

$$\varphi_{xy}(\omega) = -\arctan\left(\frac{q_{xy}(\omega)}{c_{xy}(\omega)}\right)$$

The **phase**  $\varphi_{xy}(\omega)$  measures the average phase lead of  $x_t$  over  $y_t$ , and  $\varphi_{xy}(\omega)/\omega$  indicates the extent of time lag. The **gain**  $G_{xy}(\omega)$  is defined as

<sup>&</sup>lt;sup>17</sup> All concepts described in the Annex for bivariate time series can be easily generalised to multivariate stochastic processes, where exogenous variables can also be introduced.

$$G_{xy}(\omega) = \frac{\left| f_{xy}(\omega) \right|}{f_x(\omega)}$$

This is a standardised version of the regression coefficient of y on x at frequency  $\omega$ . A small  $G_{xy}(\omega)$  indicates that x has little effect on y at frequency  $\omega$ .

Define the **complex coherence**  $ccoh_{xy}(\omega)$  at frequency  $\omega$  as

$$ccoh_{xy}(\omega) = \frac{f_{xy}(\omega)}{[f_{xx}(\omega)f_{yy}(\omega)]^{1/2}}$$

The complex coherence  $ccoh_{xy}$  is the frequency domain analogue of the coefficient of correlation, but since  $f_{xy}$  and  $ccoh_{xy}$  are complex, it is hard to interpret this indicator in terms of the overall strength of linear correlation between x and y. **Real coherence**  $rcoh_{xy}$ , which we define as the real part of  $ccoh_{xy}$ , is the cospectrum  $c_{xy}$  standardised by the square root of the product of  $f_{xx}$  and  $f_{xy}$ . It is the coefficient of correlation between coefficients of the in-phase components of two time series x and y. However, a true frequency-specific correlation coefficient needs to account for both the real and complex parts of the complex coherence  $ccoh_{xy}$ .

One alternative is the **coefficient of coherence** of  $x_t$  over  $y_t$  at frequency  $\omega$ , defined as  $cohe_{xy}(\omega) = |ccoh_{xy}(\omega)|$ .<sup>18</sup> But although it delivers a real number, it fails to reveal the sign of linear correlation at frequency  $\omega$ . The **coherency**  $ccoh_{xy}(\omega)$  of  $x_t$  over  $y_t$  at frequency  $\omega$  is

$$coh_{xy}(\omega) = cohe_{xy}(\omega)^2 = |ccoh_{xy}(\omega)|^2$$

Analogous to the coefficient of determination (ie  $R^2$ ) in the time domain, the coherency  $coh_{xy}$  is the standardised modulus of cross spectral density. It measures the strength of linear association between two or more variables of interest across frequencies. By Schwarz Inequality,  $\forall \omega$ ,  $coh_{xy}(\omega) \in [0,1]$ . At frequencies for which  $f_{xx}(\omega)f_{yy}(\omega) = 0$ , we define  $coh_{xy}(\omega) = 0$ , so the two series  $x_t$  and  $y_t$  are completely unrelated at frequency  $\omega$ . If  $coh_{xy}(\omega) = 1$ , then one series is an exactly linearly filtered version of the other at frequency  $\omega$ . In general,  $coh_{xy}$  varies with the frequency  $\omega$ , indicating the changing pattern of linear association across frequencies. Regions of high coherence are of particular interest.

What we need is a frequency-domain analogue of the time-domain coefficient of correlation, corresponding either to a specific frequency  $\omega$ , or to a frequency band  $[\omega_l, \omega_m]$ , where  $0 \le \omega_l \le \omega_m \le 2\pi$ . One natural choice would be the complex coherence  $ccoh_{xy}$ . But although  $ccoh_{xy}$  is signed and normalised, since in general

<sup>&</sup>lt;sup>18</sup> Extending the conceptual construct of bivariate coherence to multiple time series, we have multiple and partial coherences.

 $f_{xy}$  is complex-valued, so is  $ccoh_{xy}$ . There is no easy way to graphically illustrate, and to interpret the interplay between the real and complex parts of the complex coherence, even if we are able to represent the indicator in a three-dimensional diagram. Our solution is to take advantage of the proposed simple frequency-domain data recovery procedure, and we define  $\rho(\omega)$ , the **frequency-specific coefficient of correlation** (FSCC) at frequency  $\omega$ , as follows

$$\rho(\omega) = \frac{Cov(\hat{x}(\omega), \hat{y}(\omega))}{\sqrt{Var(\hat{x}(\omega))}\sqrt{Var(\hat{y}(\omega))}}$$

where  $\hat{x}(\omega)$  and  $\hat{y}(\omega)$  are frequency-specific time series extracted from data vectors x and y, and  $Cov(\bullet)$  and  $Var(\bullet)$  stand for covariance and variance, respectively. The confidence interval for  $\rho(\omega)$  can be computed in the conventional way.

The frequency-specific coefficient of correlation is normalised to take values in the [-1,1] interval. Unlike cospectral density  $c_{xy}$ , the FSCC  $\rho$  is free from data scale, hence a true measure of the strength of frequency- $\omega$  correlation between x and y. Comparing to coherence  $coh_{xy}$ , the FSCC  $\rho$  signs the direction of correlation existing in the data. The FSCC estimate is a clear improvement upon cospectrum and coherence estimates, and we use it as the main indicator of strength of bivariate correlation for Phillips relations. When the distribution theories for the cospectral density  $c_{xy}$  and the coherence  $coh_{xy}$  are complicated, p-values and confidence intervals for the FSCC estimates  $\hat{\rho}$  is can be provided in the usual way. In fact, these are often supplied automatically in an econometric or statistical software package.

#### A.3. Frequency-specific spectral regression

Consider a simple model for two time series  $y = [y_1, y_2, ..., y_T]^T$  and  $x = [x_1, x_2, ..., x_T]^T$  $y = \beta x + \varepsilon$ 

where  $\varepsilon \sim iid(0, \sigma^2 I)$  and x is uncorrelated with  $\varepsilon$ . The **periodogram** of x and the **cross-periodogram** between x and y at frequency  $\omega_s$  are, respectively

$$I_{x}(\omega_{s}) = |w_{s}x|^{2}$$
$$I_{xy}(\omega_{s}) = (w_{s}x)^{*}(w_{s}y)$$

where  $w_s$  is defined as before. The *s* -th frequency spectral regression is

$$A_s \widetilde{y} = \beta_s A_s \widetilde{x} + A_s \widetilde{\varepsilon}$$

where  $\tilde{q} = Wq$ , for any q, and W is defined as before. The s-th frequency spectral regression coefficient is

$$\widetilde{\beta}_{s} = (\widetilde{x}^{*}A_{s}\widetilde{x})^{-1}\widetilde{x}^{*}A_{s}\widetilde{\varepsilon} = I_{x}(\omega_{s})^{-1}I_{xy}(\omega_{s})$$

Since the unsmoothed periodogram  $I_x$  and the cross-periodogram  $I_{xy}$  are not consistent estimators of the spectral and cross-spectral densities, and we are interested in frequency-specific regression coefficients that do not involve averaging over a frequency band to obtain consistent estimates of the sums of periodogram and cross-periodogram ordinates, we may instead use smoothed spectral estimates to estimate  $\beta_s$ :

$$\hat{\boldsymbol{\beta}}_{s} = \hat{f}_{x}(\boldsymbol{\omega}_{s})^{-1}\hat{f}_{xy}(\boldsymbol{\omega}_{s})$$

In general, the estimator  $\hat{\beta}_s$  will be complex-valued. To obtain a real-valued estimate, one can take the real part of  $\hat{\beta}_s$ , but typically, both the real and complex parts of  $\hat{\beta}_s$  matter. Or we may simply use the gain, ie the modulus  $|\hat{\beta}_s|$ , which has the drawback of not allowing us to discern the sign of spectral regressions. We take advantage of the proposed data extraction procedure and run OLS regressions with frequency-specific data. Since the Fourier transform and inverse Fourier transform are linear operations, conventional asymptotic theory continues to apply, and the confidence interval for  $\hat{\beta}_s$  can be computed in the usual way (except at the zero frequency). Write the inverse Fourier transform as

$$L_s y = \beta_s L_s x + L_s \varepsilon$$

Simple OLS spectral regressions lead to the *frequency-specific coefficient of regression* (FSCR)  $\hat{\beta}_s$  corresponding to frequency  $\omega_s$ 

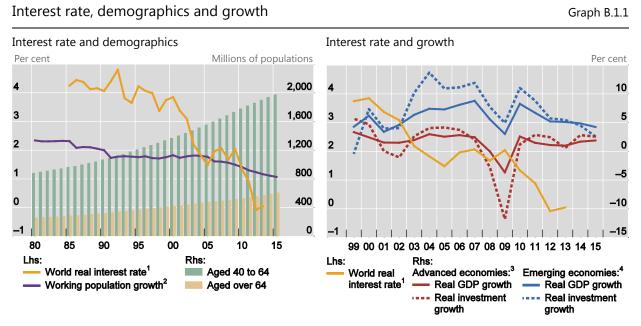
$$\hat{\boldsymbol{\beta}}_{s} = \left(\boldsymbol{x}^{T}\boldsymbol{L}_{s}^{T}\boldsymbol{L}_{s}\boldsymbol{x}\right)^{-1}\boldsymbol{x}^{T}\boldsymbol{L}_{s}^{T}\boldsymbol{L}_{s}\boldsymbol{y} = \left(\hat{\boldsymbol{x}}_{s}^{T}\hat{\boldsymbol{x}}_{s}\right)^{-1}\hat{\boldsymbol{x}}_{s}\hat{\boldsymbol{y}}_{s}$$

The great advantage of the data extraction procedure is that it is linear in nature; therefore all inferential apparatus in the conventional OLS regression theory can still be used as usual.

# Annex B: Graphs

This annex contains graphs of estimates of the equilibrium real interest rate and its relationship with macroeconomic, financial, demographic and global factors, based on the Hodrick-Prescott (1981) filter and on frequency domain indicators including Zhu's (2005) frequency-specific correlation and regression coefficients.

#### B.1. World real interest rate



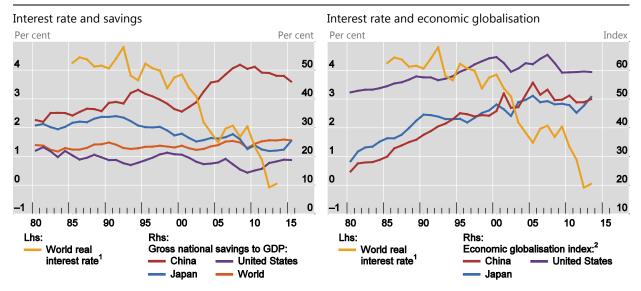
<sup>1</sup> Weighted world real interest rate estimated by King and Low (2014). <sup>2</sup> Working age population is defined as population aged between 15 and 64. <sup>3</sup> Advanced economies: Australia, Canada, Denmark, Japan, euro area, Japan, Sweden, Switzerland, the United Kingdom, and the United States. Weighted averages based on rolling GDP and PPP exchange rates. <sup>4</sup> Emerging economies: Brazil, Chile, China, Czech Republic, Hungary, India, Indonesia, Korea, Mexico, Poland, Russia, South Africa and Turkey. Weighted averages based on rolling GDP and PPP exchange rates.

Sources: King and Low (2014); IMF, World Economic Outlook; United Nations; BIS calculations.

#### Interest rate and global factors

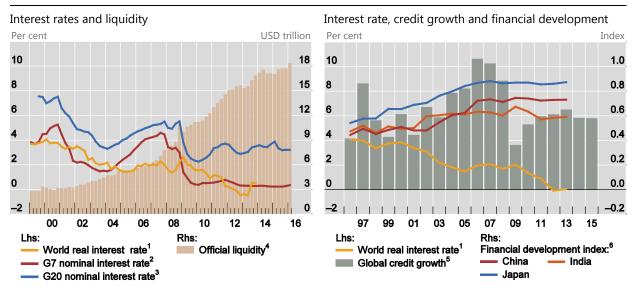
#### Graph B.1.2

Graph B.1.3



<sup>1</sup> Weighted world real interest rate estimated by King and Low (2014). <sup>2</sup> For details of the overall KOF Index of Globalisation and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008).

Sources: Dreher (2006) and Dreher, Gaston and Martens (2008); King and Low (2014); IMF, World Economic Outlook; United Nations; BIS calculations.



<sup>1</sup> Weighted world real interest rate estimated by King and Low (2014). <sup>2</sup> Weighted averages of nominal three-month interbank interest rates of G7 economies, based on rolling GDP and PPP exchange rates. <sup>3</sup> Weighted averages of nominal three-month interbank interest rates of G20 economies, excluding Argentina and Turkey, based on rolling GDP and PPP exchange rates. European Union is proxied by Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Portugal, Sweden and the United Kingdom. <sup>4</sup> Sum of central bank balance sheets of advanced economies (Australia, Canada, Denmark, euro area, Japan, Sweden, Switzerland, the United Kingdom and the United States) and foreign exchange reserves of emerging market economies (Brazil, Chile, China, Czech, Hungary, India, Indonesia, Korea, Mexico, Poland, Russia, South Africa and Turkey). <sup>5</sup> Weighted averages of real growth in total credit to private non-financial sector, using rolling GDP and PPP exchange rates and based on the BIS credit data for 38 economies. <sup>6</sup> Data on financial development index come from the data appendix of Sahay et al (2015).

Sources: King and Low (2014); Sahay et al (2015); IMF, International Financial Statistics; IMF, World Economic Outlook; Datastream; Global Financial Data; national data; BIS.

#### Interest rate and financial sector developments

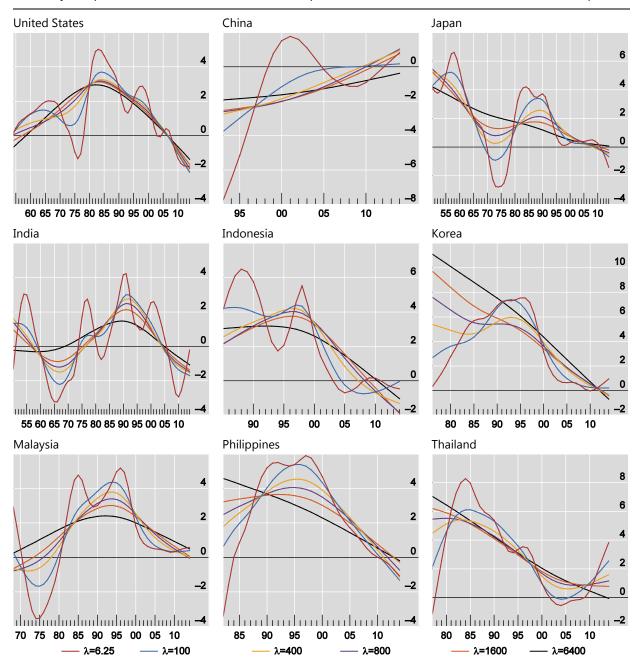
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#### B.2. Equilibrium real interest rate estimates

# Real overnight interest rate

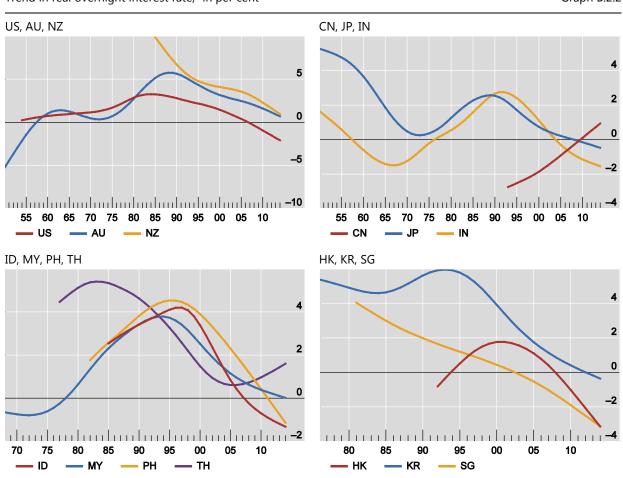
Sensitivity of equilibrium real interest rate estimates,<sup>1</sup> in per cent

Graph B.2.1



<sup>1</sup> Equilibrium real interest rate estimates are based on the Hodrick-Prescott (1981) filtering technique. The trend component is extracted with the smoothing parameter  $\lambda$  set at 6.25, 100, 400, 800, 1600 and 6400.

Source: Author's calculations.



#### Estimates of equilibrium real interest rate in Asia-Pacific

Trend in real overnight interest rate,<sup>1</sup> in per cent

Graph B.2.2

AU = Australia; CN = China; HK = Hong Kong; ID = Indonesia; IN = India; JP = Japan; KR = Korea; MY = Malaysia; NZ = New Zealand; PH = Philippines; SG = Singapore; TH = Thailand; US = United States.

<sup>1</sup> Equilibrium real interest rate estimates are based on the Hodrick-Prescott (1981) filtering technique. The trend component is extracted with the smoothing parameter  $\lambda$  set at 400.

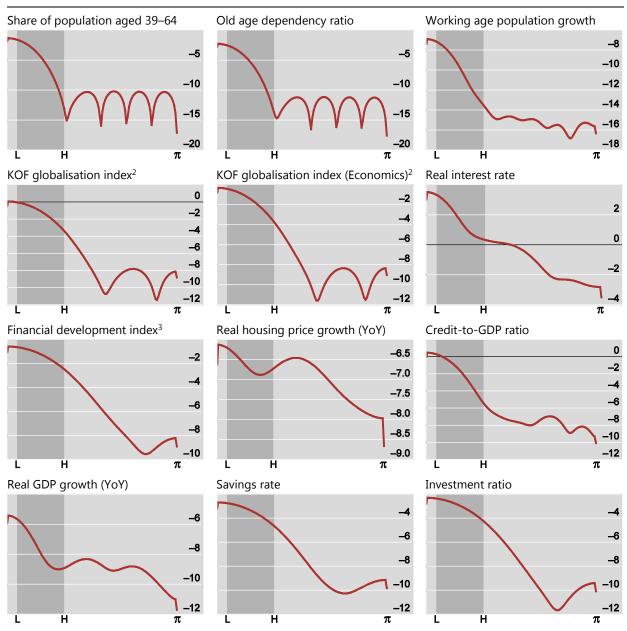
Source: Author's calculations.

#### B.3. Spectral analysis of real interest rates

#### B.3.1. Australia

#### Spectral density estimates: Australia<sup>1</sup>

In logarithm



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

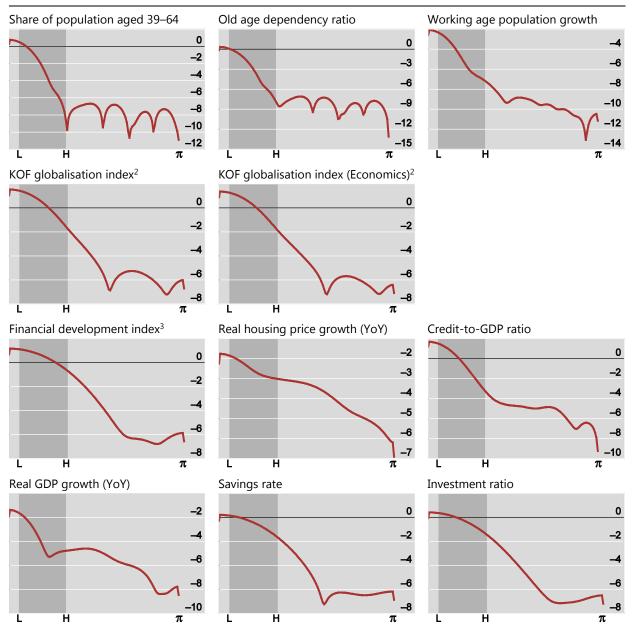
Source: Author's calculations.

Graph B.3.1.1

#### Cospectral density estimates: Australia<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.1.2



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

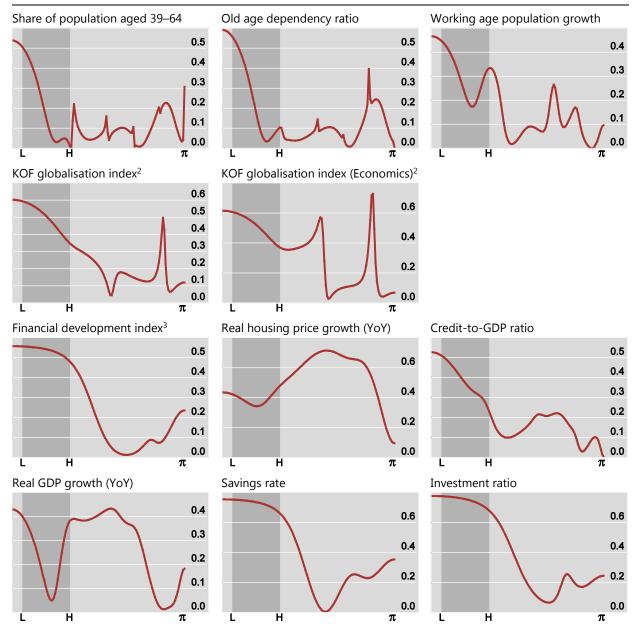
<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

Source: Author's calculations.

#### Squared coherence estimates: Australia<sup>1</sup>

With real interest rate

Graph B.3.1.3



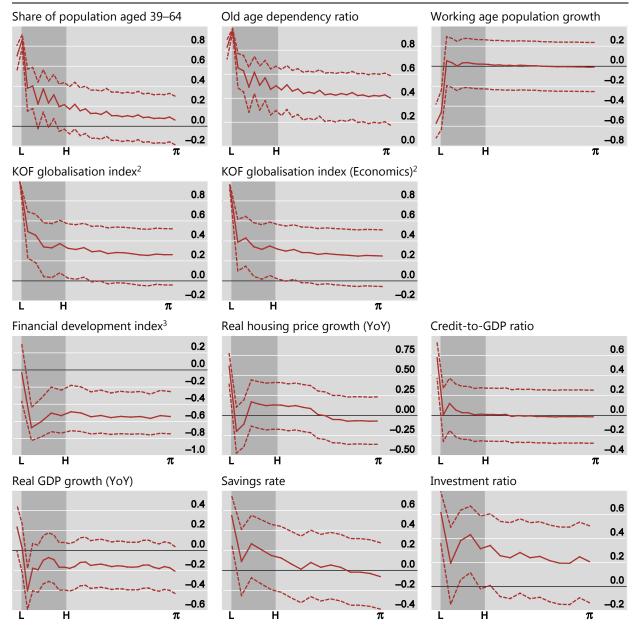
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

# Frequency-specific correlation coefficient estimates: Australia<sup>1</sup>

With real interest rate

Graph B.3.1.4



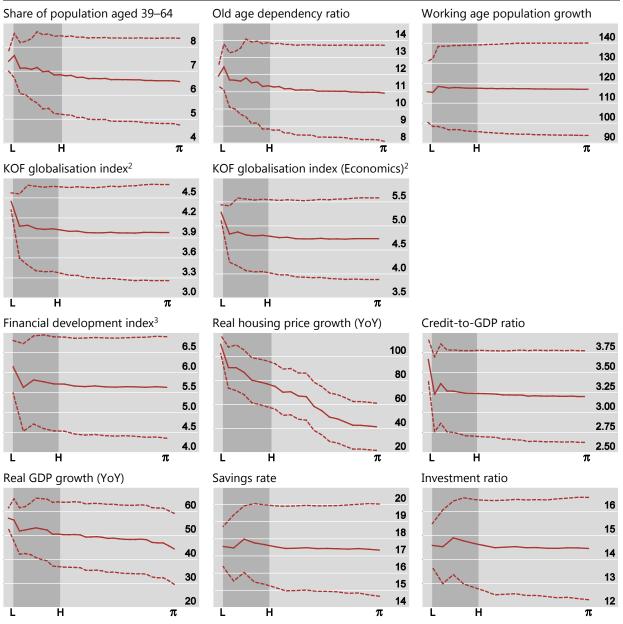
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Frequency-specific regression coefficient estimates: Australia<sup>1</sup>

Real interest rate as regressand

Graph B.3.1.5



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

Source: Author's calculations.

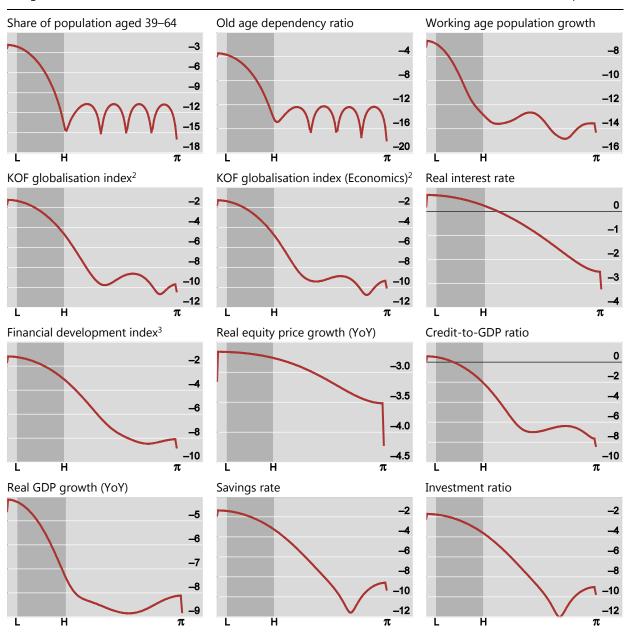
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## B.3.2. China

#### Spectral density estimates: China<sup>1</sup>

In logarithm

Graph B.3.2.1



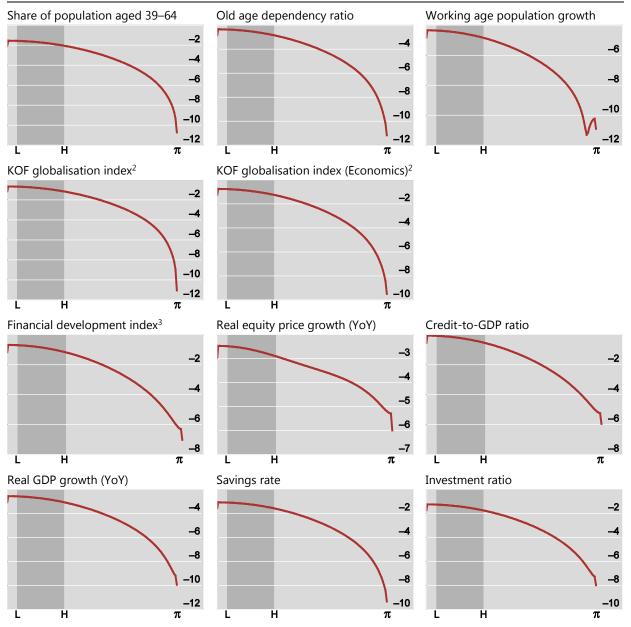
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

 $^{1}$  Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process.  $^{2}$  The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008).  $^{3}$  For details of the financial development index, see Sahay et al (2015).

## Cospectral density estimates: China<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.2.2



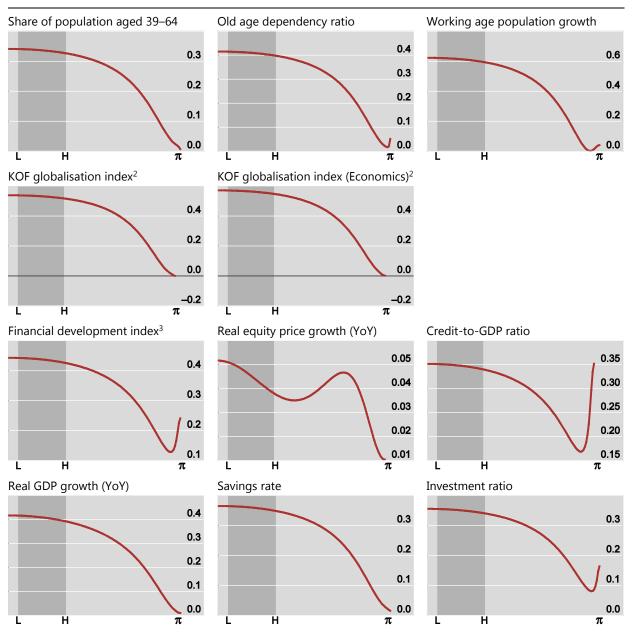
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: China<sup>1</sup>

With real interest rate

Graph B.3.2.3



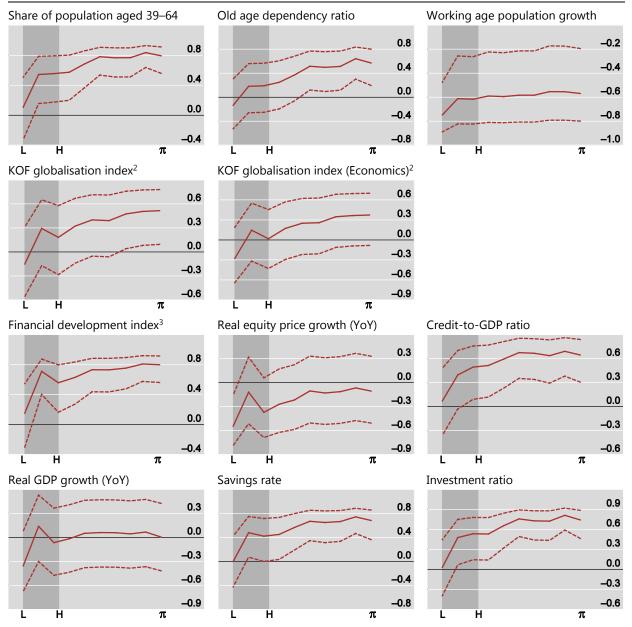
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Frequency-specific correlation coefficient estimates: China<sup>1</sup>

With real interest rate

Graph B.3.2.4



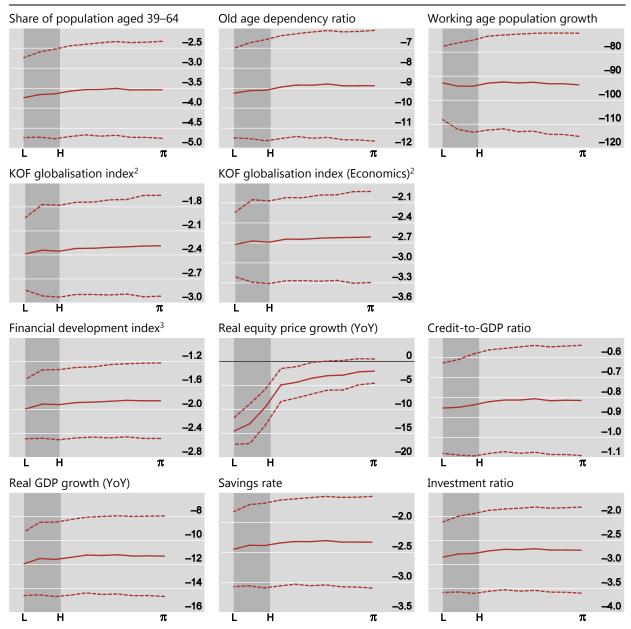
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

# Frequency-specific regression coefficient estimates: China<sup>1</sup>

Real interest rate as regressand

Graph B.3.2.5



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

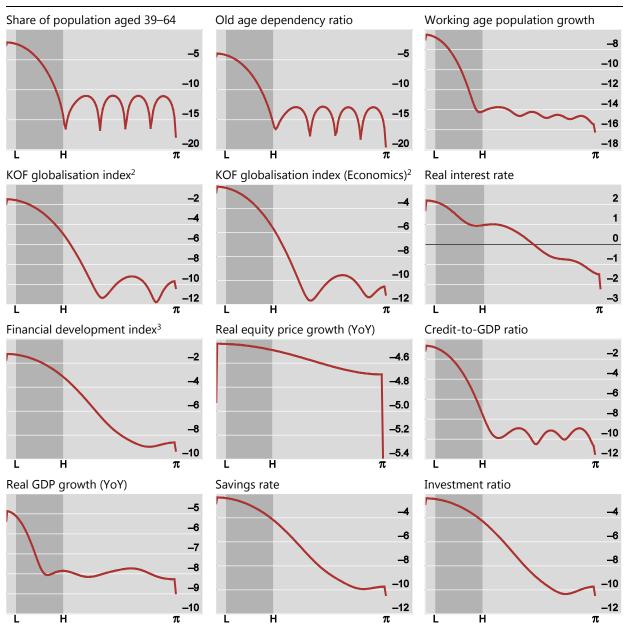
<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



#### Spectral density estimates: India<sup>1</sup>

In logarithm





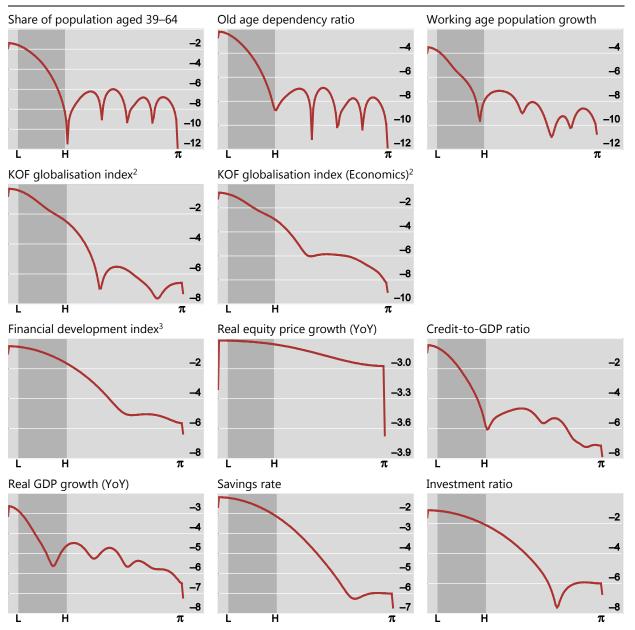
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Cospectral density estimates: India<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.3.2



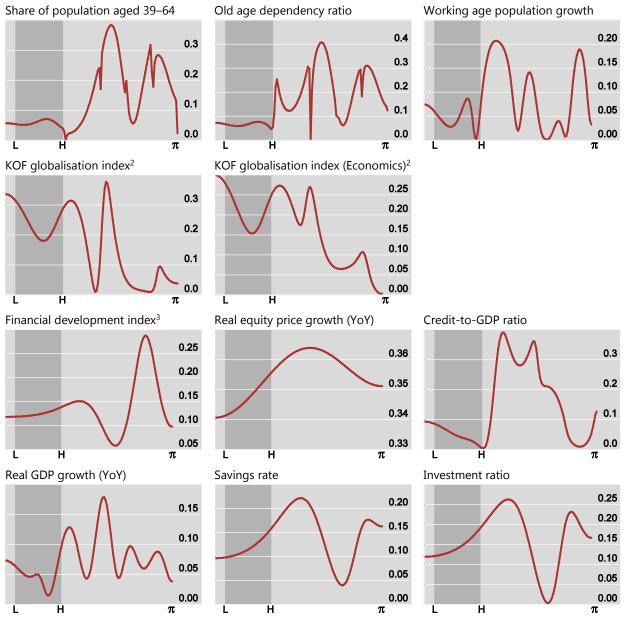
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: India<sup>1</sup>

#### With real interest rate

Graph B.3.3.3



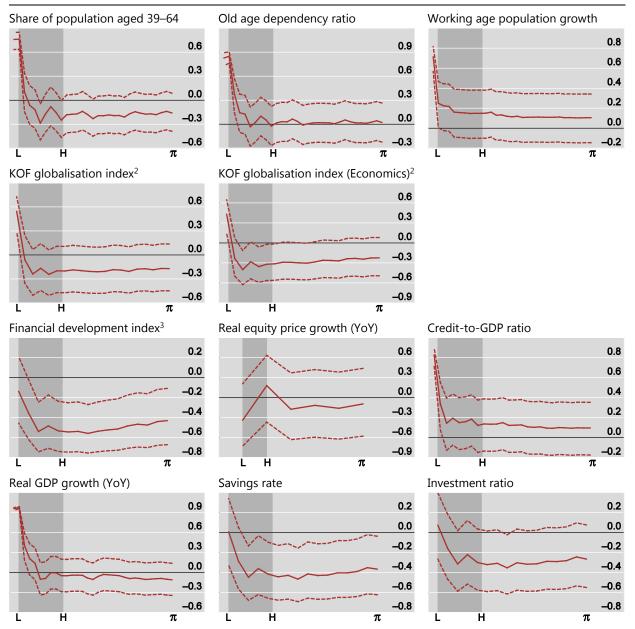
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Frequency-specific correlation coefficient estimates: India<sup>1</sup>

With real interest rate

Graph B.3.3.4



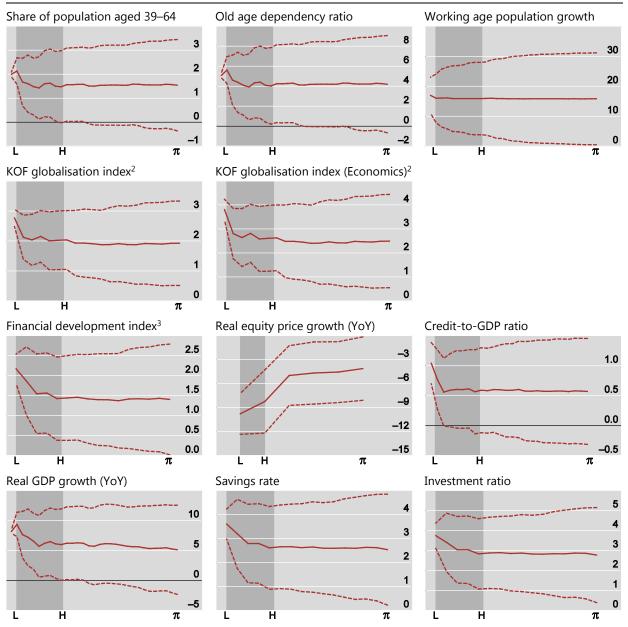
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Frequency-specific regression coefficient estimates: India<sup>1</sup>

Real interest rate as regressand

Graph B.3.3.5



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

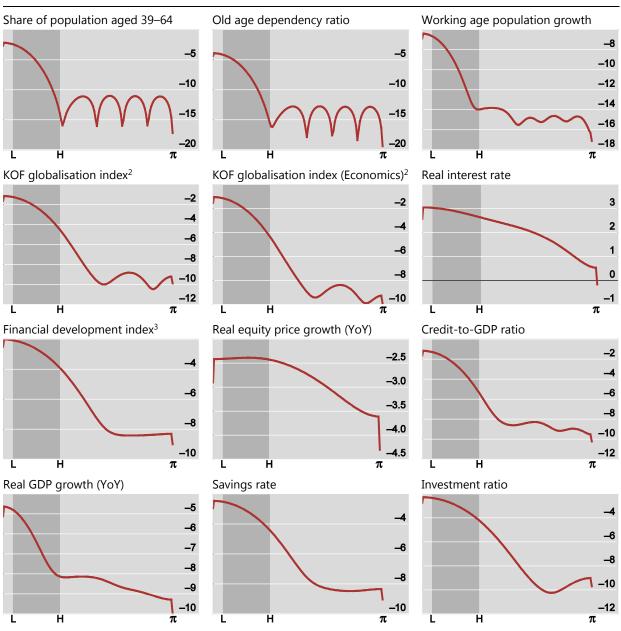
<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

### B.3.4. Indonesia

### Spectral density estimates: Indonesia<sup>1</sup>

In logarithm

Graph B.3.4.1



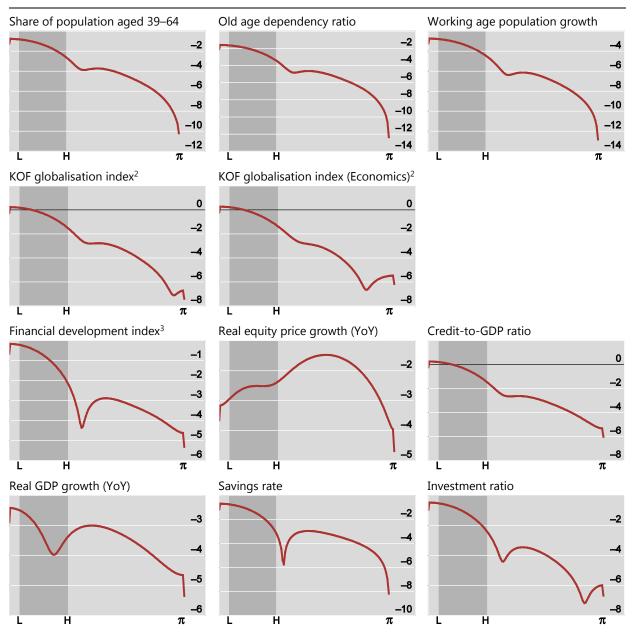
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

 $^{1}$  Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process.  $^{2}$  The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008).  $^{3}$  For details of the financial development index, see Sahay et al (2015).

#### Cospectral density estimates: Indonesia<sup>1</sup>

In logarithm, with real interest rate





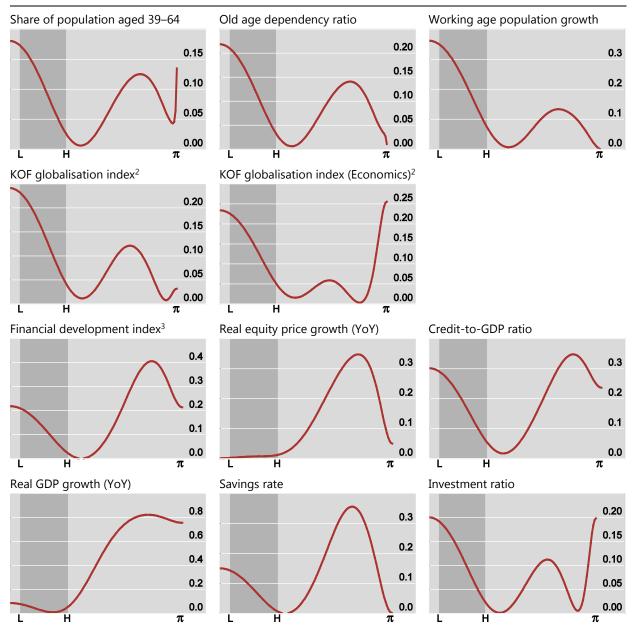
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: Indonesia<sup>1</sup>

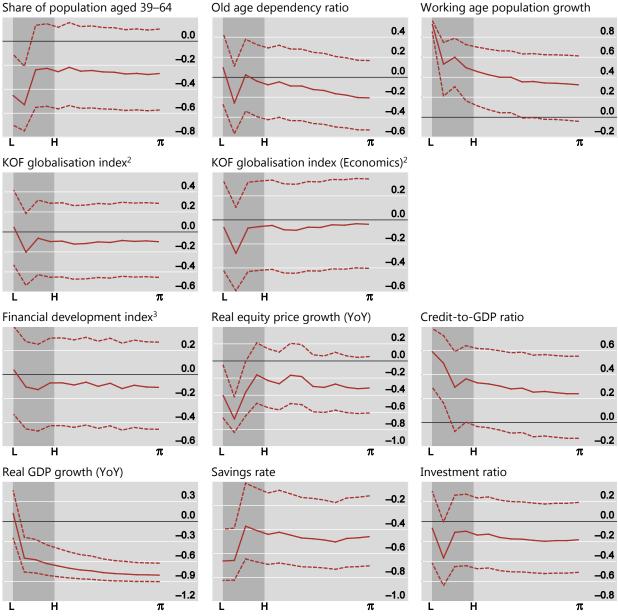
With real interest rate

Graph B.3.4.3



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



Frequency-specific correlation coefficient estimates: Indonesia<sup>1</sup>

With real interest rate

Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005).<sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

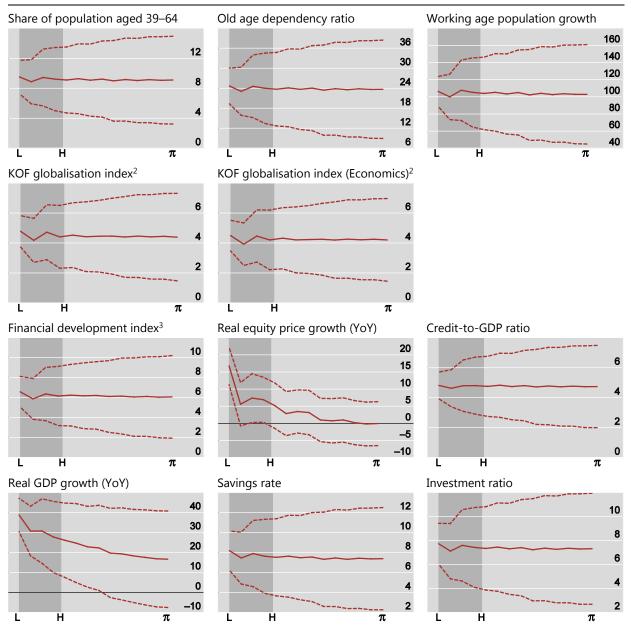
Source: Author's calculations.

Graph B.3.4.4

# Frequency-specific regression coefficient estimates: Indonesia<sup>1</sup>

Real interest rate as regressand

Graph B.3.4.5



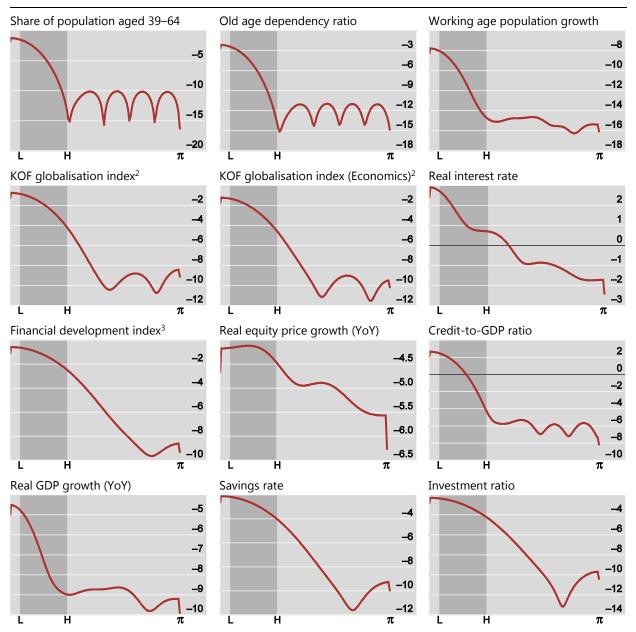
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Spectral density estimates: Japan<sup>1</sup>

In logarithm





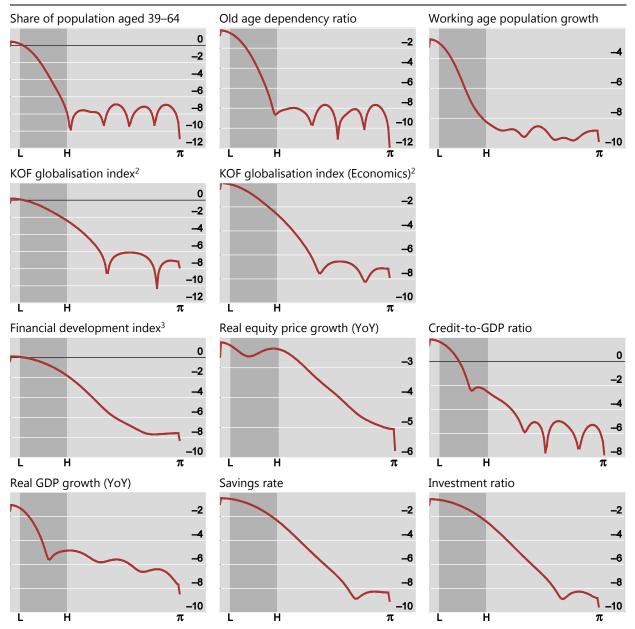
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Cospectral density estimates: Japan<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.5.2



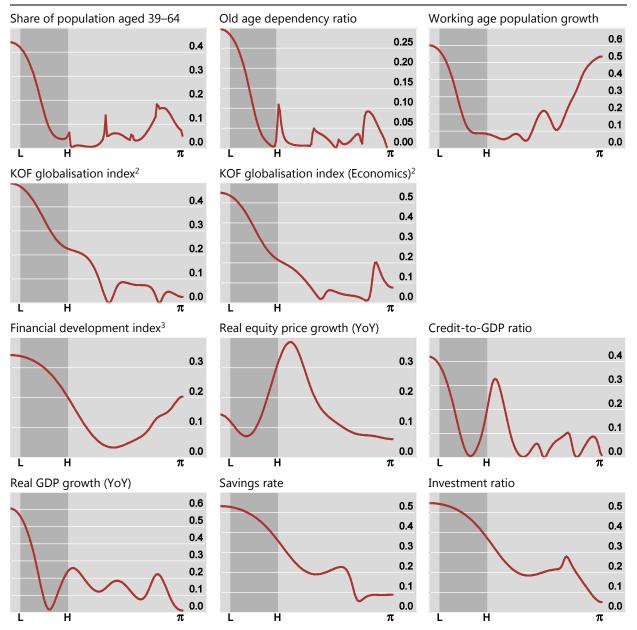
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: Japan<sup>1</sup>

With real interest rate

Graph B.3.5.3



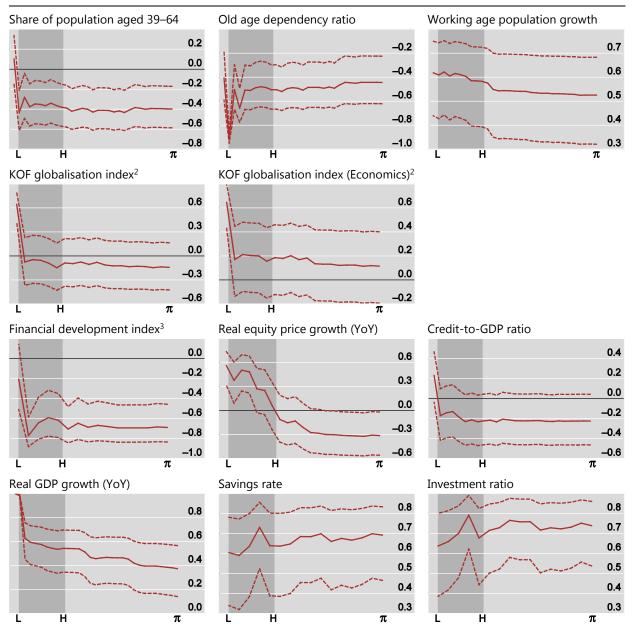
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Frequency-specific correlation coefficient estimates: Japan<sup>1</sup>

With real interest rate

Graph B.3.5.4



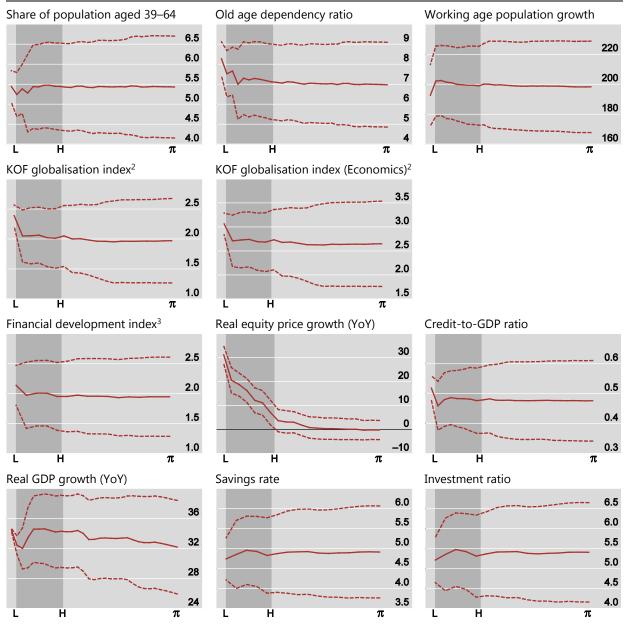
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Frequency-specific regression coefficient estimates: Japan<sup>1</sup>

Real interest rate as regressand

Graph B.3.5.5



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

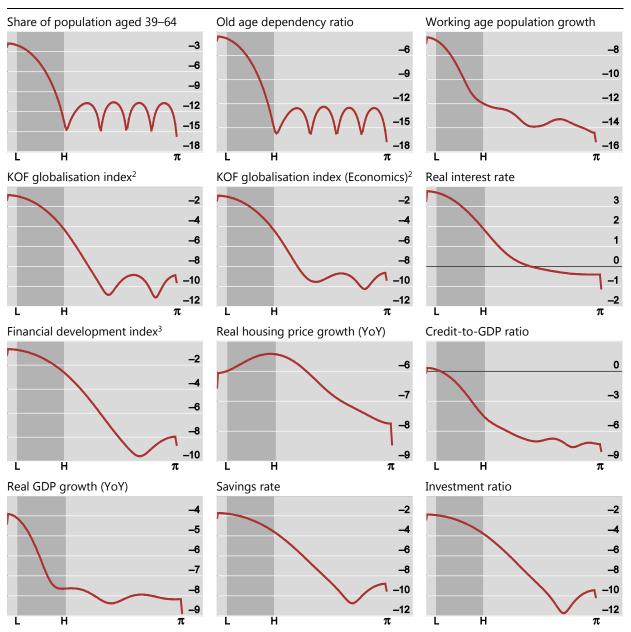
<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## B.3.6. Korea

## Spectral density estimates: Korea<sup>1</sup>

In logarithm

Graph B.3.6.1



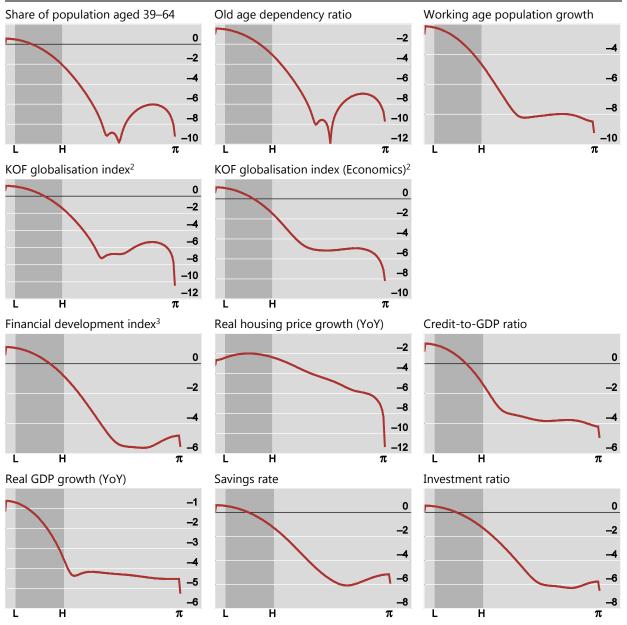
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

 $^{1}$  Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process.  $^{2}$  The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008).  $^{3}$  For details of the financial development index, see Sahay et al (2015).

#### Cospectral density estimates: Korea<sup>1</sup>

In logarithm, with real interest rate

#### Graph B.3.6.2



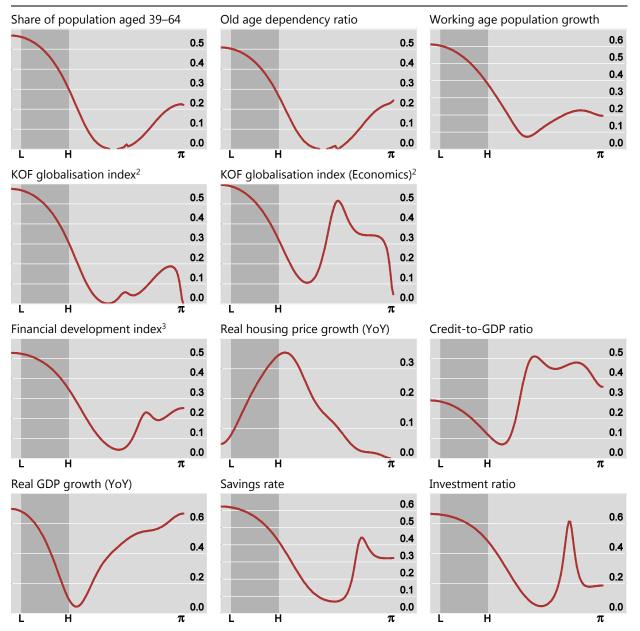
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

### Squared coherence estimates: Korea<sup>1</sup>

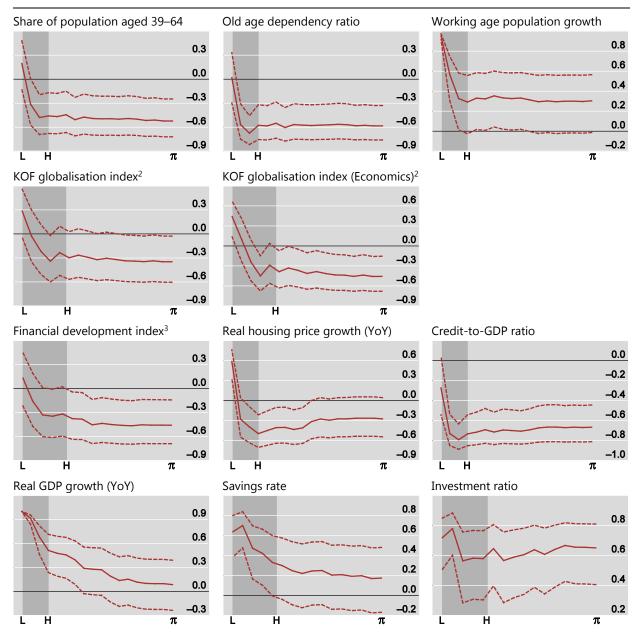
With real interest rate

Graph B.3.6.3



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



## Frequency-specific correlation coefficient estimates: Korea<sup>1</sup>

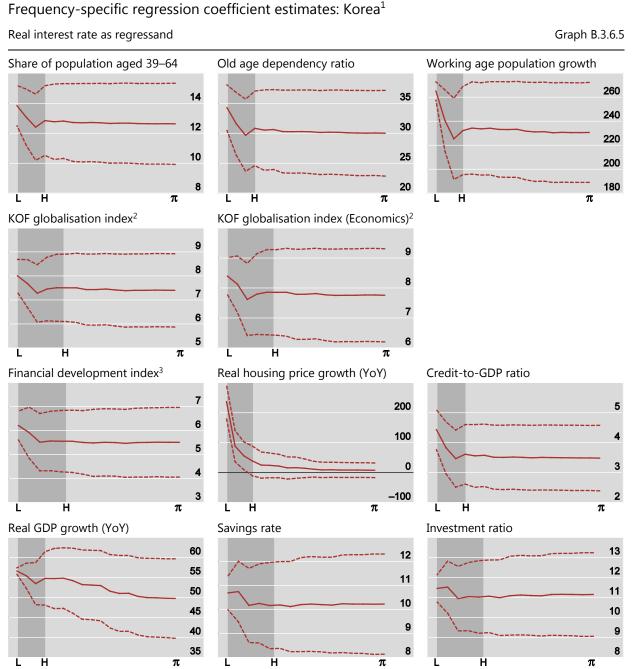
With real interest rate

Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

Source: Author's calculations.

Graph B.3.6.4



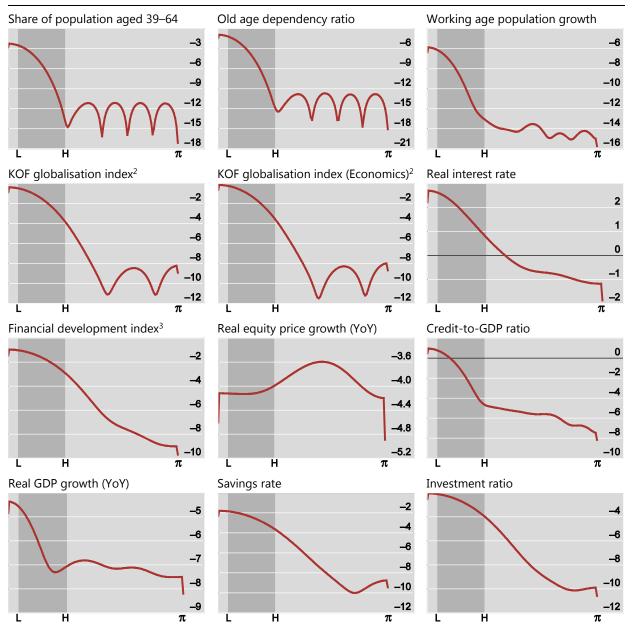
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Spectral density estimates: Malaysia<sup>1</sup>

In logarithm





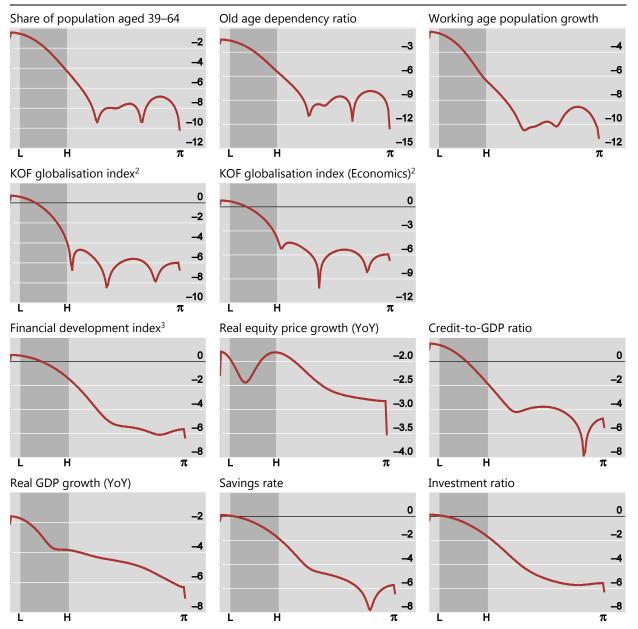
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Cospectral density estimates: Malaysia<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.7.2



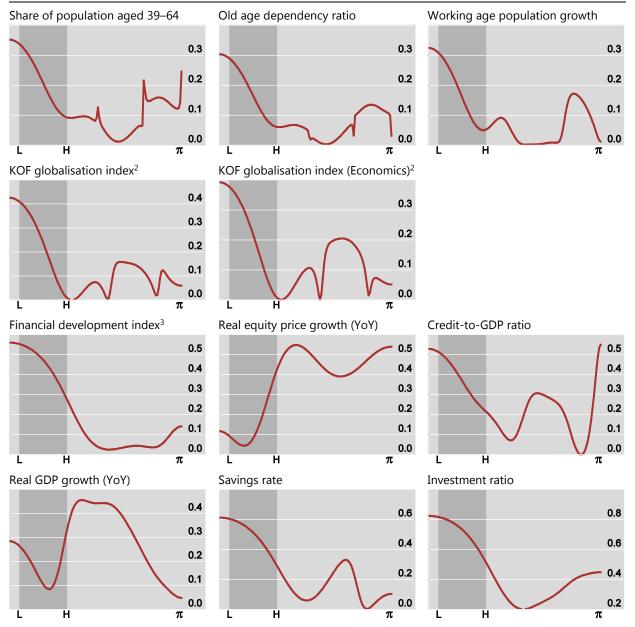
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

## Squared coherence estimates: Malaysia<sup>1</sup>

With real interest rate

Graph B.3.7.3



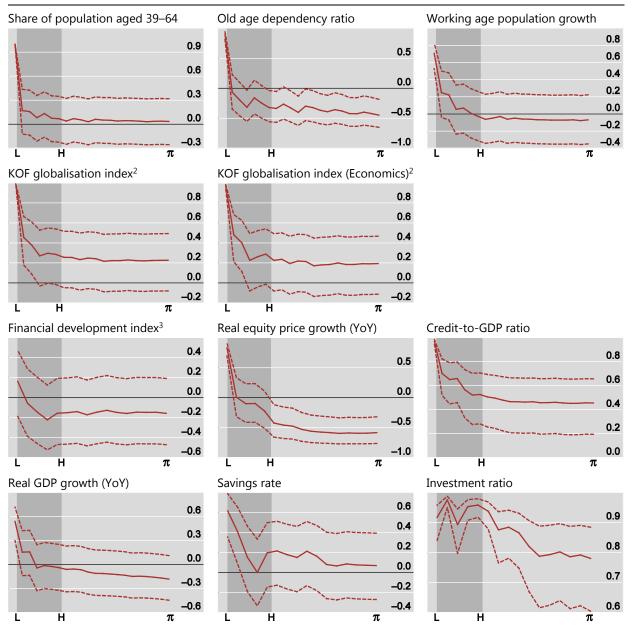
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

# Frequency-specific correlation coefficient estimates: Malaysia<sup>1</sup>

With real interest rate

Graph B.3.7.4



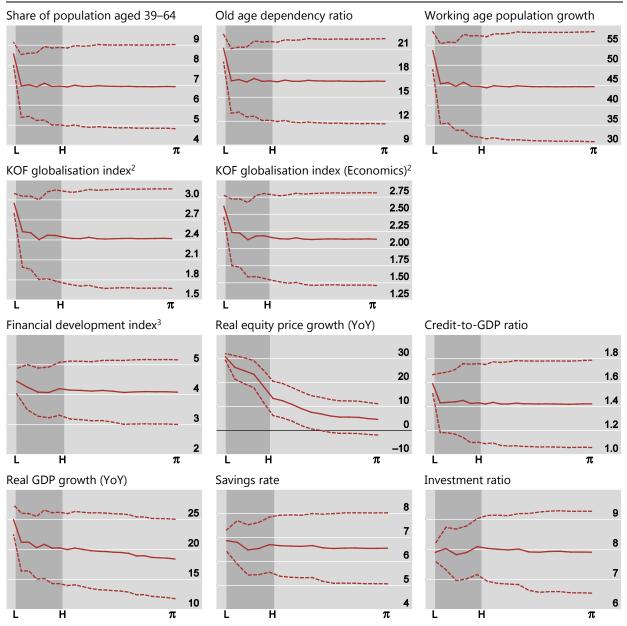
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

# Frequency-specific regression coefficient estimates: Malaysia<sup>1</sup>

Real interest rate as regressand

Graph B.3.7.5



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

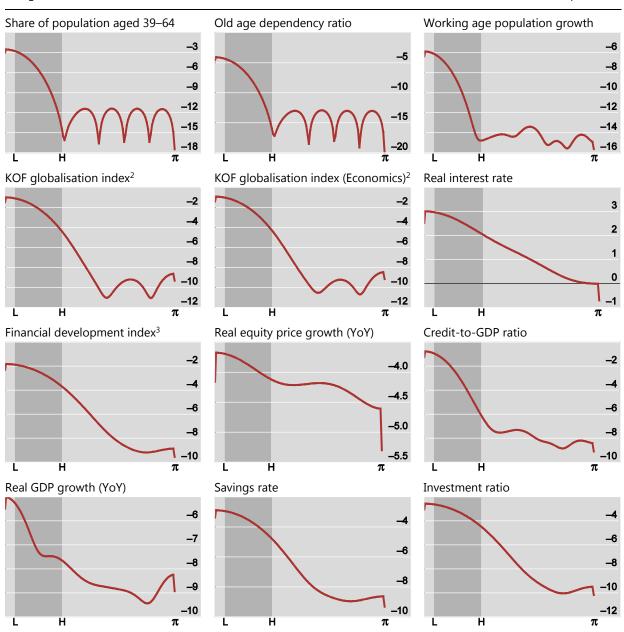
<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

### B.3.8. The Philippines

#### Spectral density estimates: Philippines<sup>1</sup>

In logarithm

Graph B.3.8.1



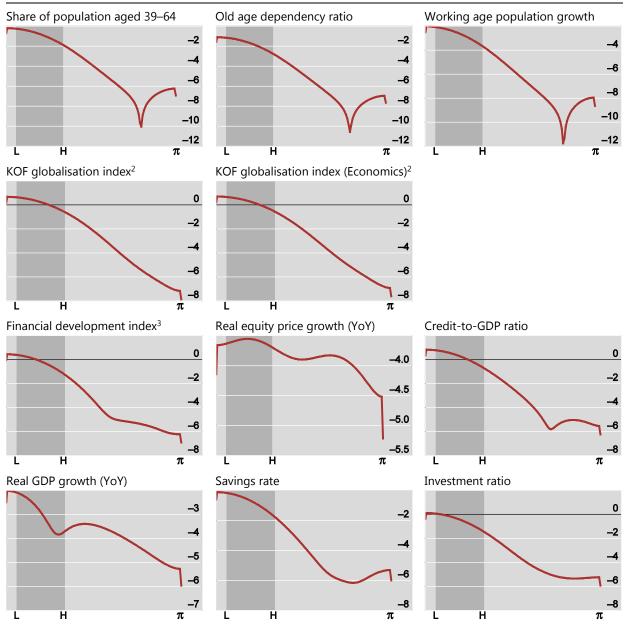
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

 $^{1}$  Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process.  $^{2}$  The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008).  $^{3}$  For details of the financial development index, see Sahay et al (2015).

### Cospectral density estimates: Philippines<sup>1</sup>

In logarithm, with real interest rate

#### Graph B.3.8.2



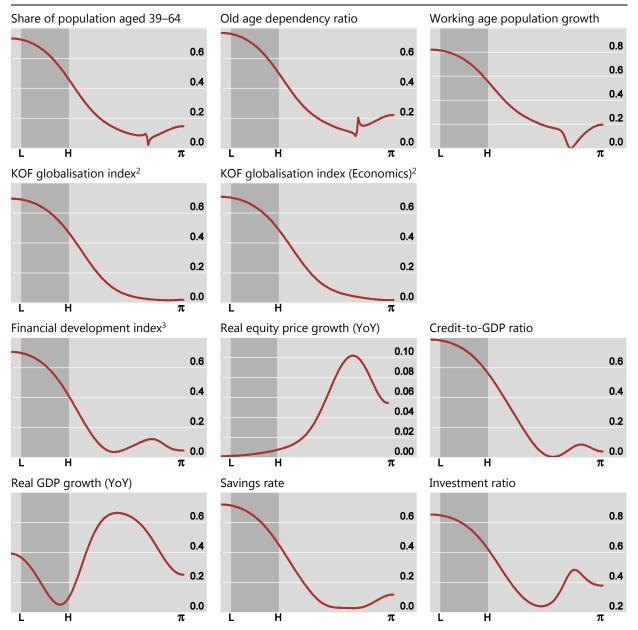
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: Philippines<sup>1</sup>

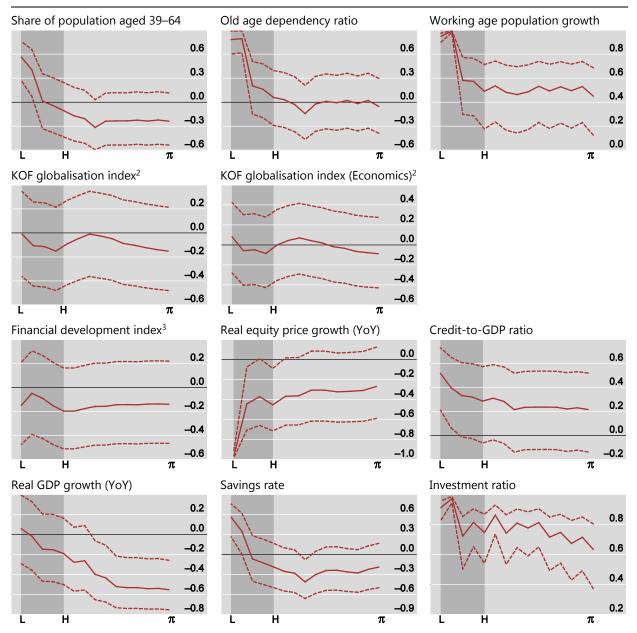
With real interest rate

Graph B.3.8.3



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



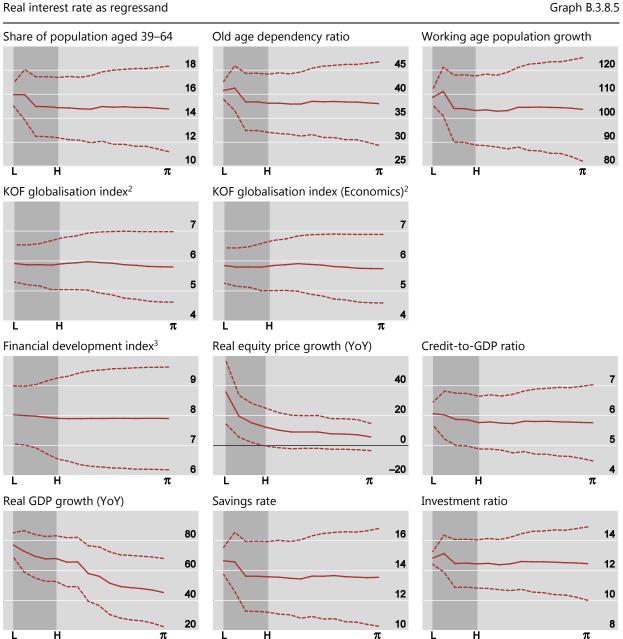
### Frequency-specific correlation coefficient estimates: Philippines<sup>1</sup>

With real interest rate

Graph B.3.8.4

Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



Frequency-specific regression coefficient estimates: Philippines<sup>1</sup>

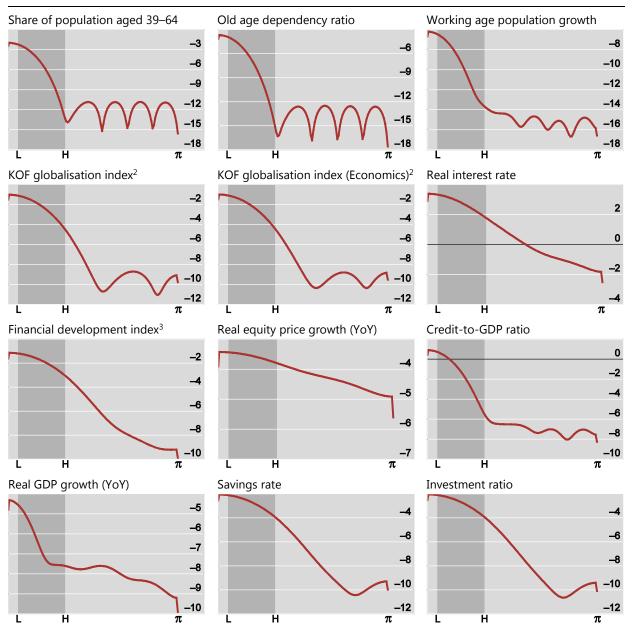
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005).<sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Spectral density estimates: Thailand<sup>1</sup>

In logarithm





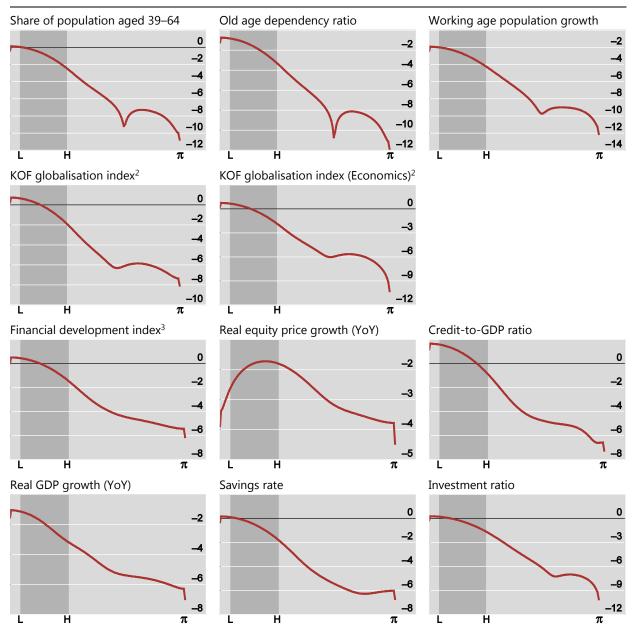
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Cospectral density estimates: Thailand<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.9.2



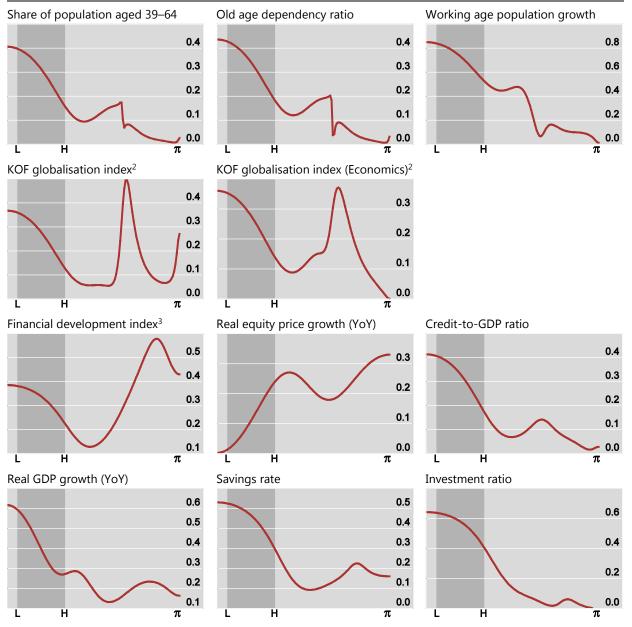
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: Thailand<sup>1</sup>

With real interest rate

Graph B.3.9.3



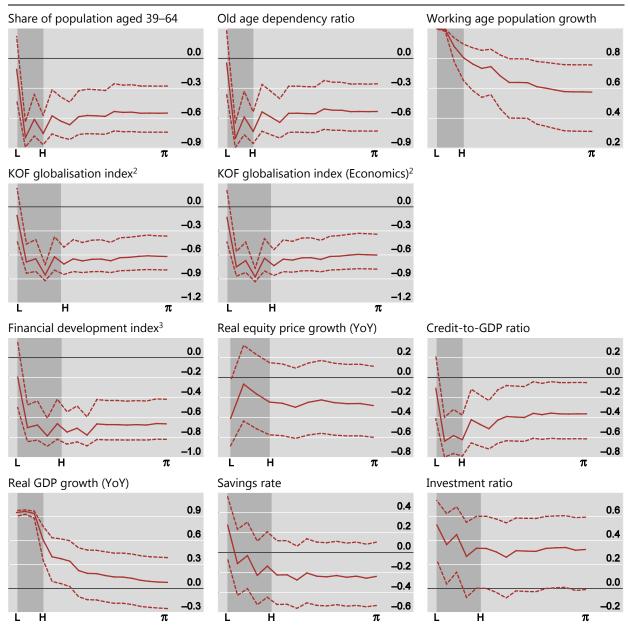
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

### Frequency-specific correlation coefficient estimates: Thailand<sup>1</sup>

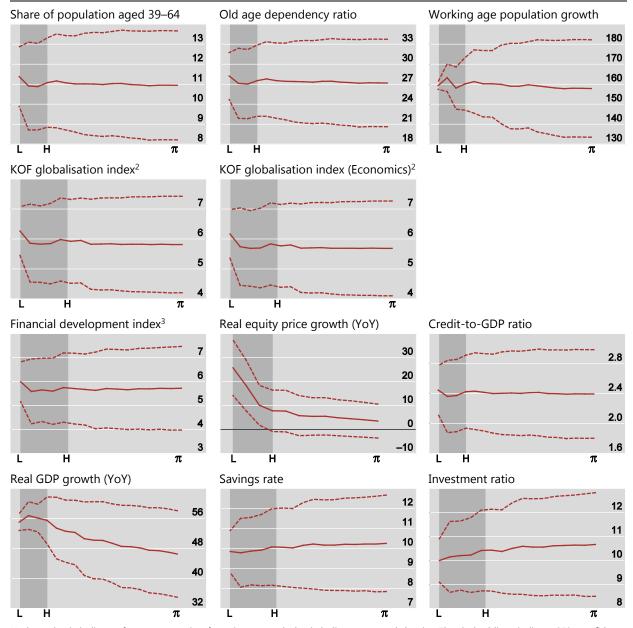
With real interest rate

Graph B.3.9.4



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



#### Frequency-specific regression coefficient estimates: Thailand<sup>1</sup>

Real interest rate as regressand

Graph B.3.9.5

Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

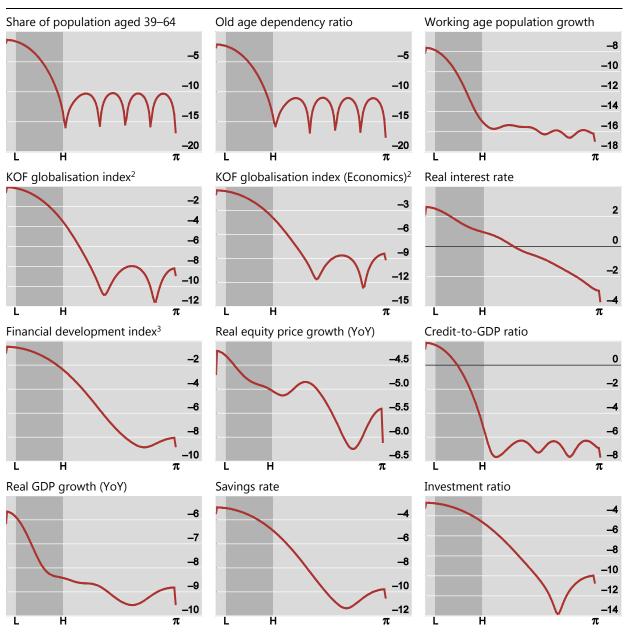
<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### B.3.10. United States

#### Spectral density estimates: United States<sup>1</sup>

In logarithm





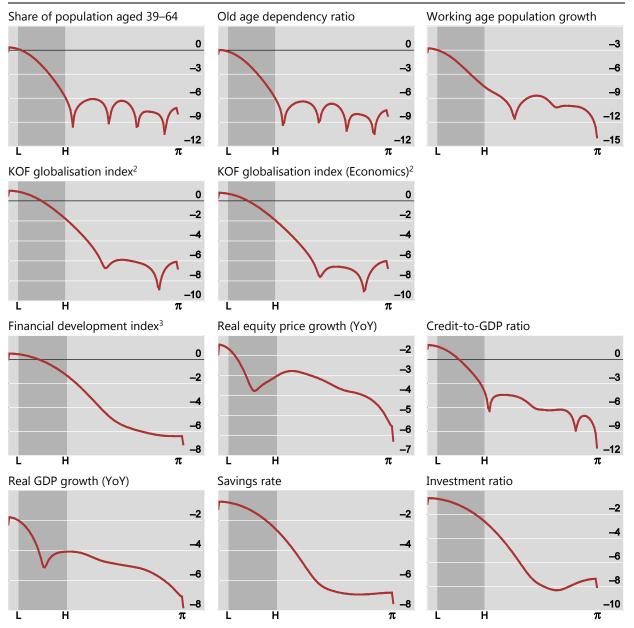
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

 $^{1}$  Spectral density, or power spectrum, is estimated using Welch's method. It records the contribution of the component(s) belonging to a specific frequency or frequency band to the total variance of a stochastic process.  $^{2}$  The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008).  $^{3}$  For details of the financial development index, see Sahay et al (2015).

#### Cospectral density estimates: United States<sup>1</sup>

In logarithm, with real interest rate

Graph B.3.10.2



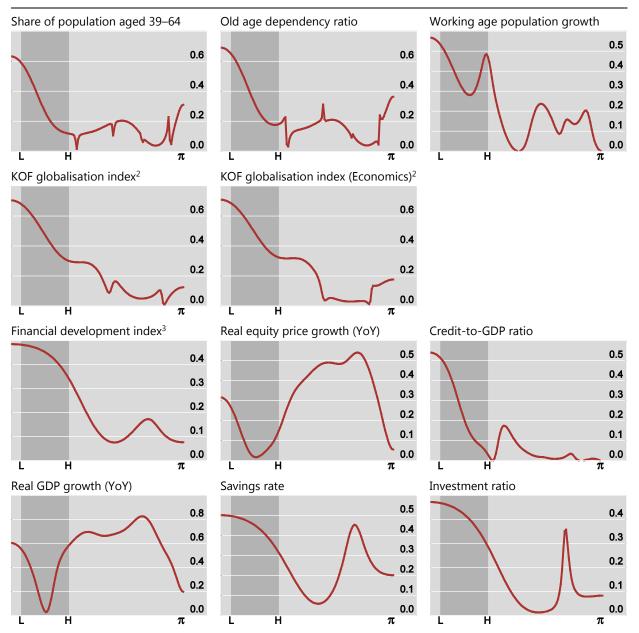
Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Cospectral density, also known as cross power spectral density or cospectrum, is estimated using Welch's averaged, modified periodogram method. It represents the covariance between the in-phase components of two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

#### Squared coherence estimates: United States<sup>1</sup>

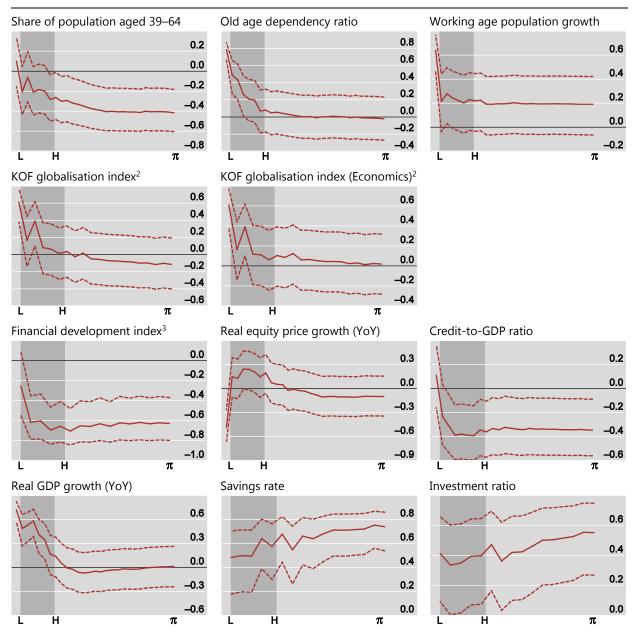
With real interest rate

Graph B.3.10.3



Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> Squared coherence, known as coherency, is estimated using Welch's averaged periodogram method. It is similar to the coefficient of determination  $R^2$ , and measures the strength of linear association between two stochastic processes at a specific frequency or frequency band. <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



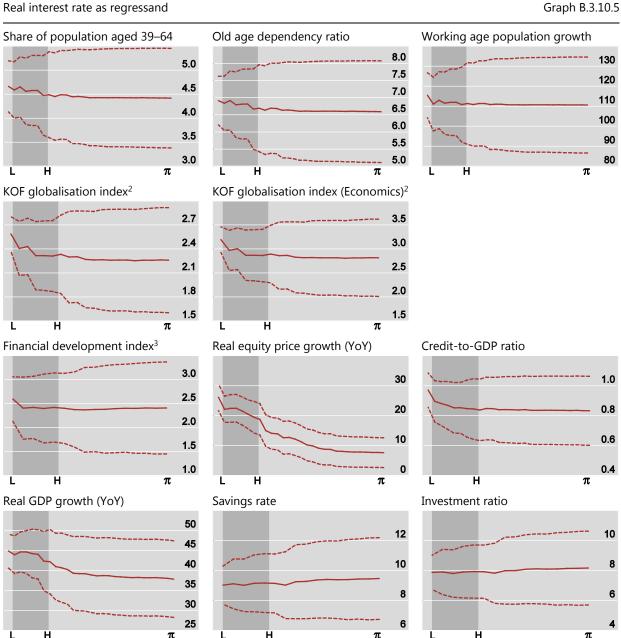
### Frequency-specific correlation coefficient estimates: United States<sup>1</sup>

With real interest rate

Graph B.3.10.4

Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of correlation measures the strength of correlation between two stochastic processes at a specific frequency or frequency band. See Zhu (2005). <sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).



Frequency-specific regression coefficient estimates: United States<sup>1</sup>

Horizontal axis indicates frequency ranging from 0 to  $\pi$ ; vertical axis indicates spectral density. The dashed lines indicate 95% confidence intervals. The shaded area indicates the business-cycle frequency, ranging from six quarters (marked by "H") to 32 quarters (marked by "L"). The area between 0 and L corresponds to the low-frequency range which contains the trend component.

<sup>1</sup> The frequency-specific coefficient of regression indicates the degree of linear association between two stochastic processes at a specific frequency or frequency band. See Zhu (2005).<sup>2</sup> The overall KOF Index of Globalisation covers the economic, social and political dimensions of globalisation. For details of the Index and the Economics sub-index, see Dreher (2006) and Dreher, Gaston and Martens (2008). <sup>3</sup> For details of the financial development index, see Sahay et al (2015).

# Annex C: Data descriptions

This annex provides a detailed description of data used in this paper.

Macro-economy Ta			
Variable	Description	Source	Notes
Real GDP	Level in billions of local currencies; year-on-year growth.	IMF IFS, OECD, GFD, national data, BIS.	Different base years in different countries
GDP deflator		IMF IFS, OECD, GFD, national data, BIS.	Rebased with 2005=100.
Consumer price index (CPI)		CEIC, GFD, Datastream, national data.	
Inflation forecasts	current and next year; 6-to- 10-year ahead	Consensus Economics ©; BIS calculations	
Savings	Gross national saving.	IMF WEO.	In billions of local currency.
Savings rate	Gross national savings as a percentage of GDP.	IMF WEO.	
Investment	Gross fixed capital formation.	IMF WEO.	In billions of local currency.
Investment ratio	Ratio of nominal investment to nominal GDP.	IMF WEO, BIS calculations.	
Unemployment rate		IMF WEO, OECD, CEIC, Datastream, GFD, national data.	Quarterly data not available for India and Indonesia; annual data available for India.
Employment		Eurostat, IMF IFS, IMF WEO.	in thousands; Quarterly data not available for China and India; annual data not available for India.
Oil price	Crude oil price for spot Brent.	Datastream.	In USD

Variable	Description	Source	Notes
Nominal total credit	Total credit to private non- financial sector.	National data, BIS.	In billions of local currency. Data not available for New Zealand and the Philippines.
Nominal bank credit	Bank credit to private non- financial sector.	National data, BIS.	In billions of local currency.
Domestic credit to GDP ratio	Domestic credit as a percentage of GDP.	World Bank.	
Equity price index		Bloomberg, CEIC, BIS calculations.	
Housing price index		National data.	Rebased with 2010=100
Interest rate	Overnight interest rates.	Bloomberg, Datastream, GFD.	
China's interest rate	Average of overnight, 1- week repo, three-month time deposit, six-month lending and one-year lending rates.	Datastream.	
Three-month government bill rate		Bloomberg, GFD.	Data are not available for Indonesia.
10-year government bond yield		Datastream, GFD.	

## Demographics

Demographics			Table C.3
Variable	Description	Source	Notes
Working age population	Population aged 15–64.	United Nations.	In thousands.
Total population		United Nations.	In thousands.
Ratio of population aged 39–64	Ratio of population aged 39–64 to total population.	United Nations, BIS calculations.	
Ratio of population aged over 64	Ratio of population aged over 64 to total population.	United Nations, BIS calculations.	
Total dependency ratio	Ratio of population aged below 20 and over 64 to population aged 20–64.	United Nations.	
Old-age dependency ratio	Ratio of population aged over 64 to population aged 20–64.	United Nations.	
Birth rate		World Bank.	Per 1000 people.
Life expectancy		World Bank.	In years.

# Global economy and external sector

Variable	Description	Source	Notes
Global official liquidity	Total assets of central banks in advanced economies and foreign reserves of emerging economies. <sup>1</sup>	IMF IFS, Datastream, national data, BIS calculations.	In trillions of USD
M3-to-GDP ratio	Weighted averages.	IMF IFS, IMF WEO, Datastream, national data.	Based on rolling GDP and PPP exchange rates.
World interest rate	Weighted (by real GDP) and unweighted world real interest rate.	King and Low (2014).	
Nominal effective exchange rate		BIS.	Index 2010=100.
Real effective exchange rate		BIS.	Index 2010=100.
Globalisation index (Overall)	KOF globalisation index (Overall).	ETH Zurich; Dreher (2006).	Data not available for Hong Kong.
Globalisation index (economics)	KOF Index of Globalisation (Sub-index of economics).	ETH Zurich; Dreher (2006).	
Financial development index		Sahay et al (2015)	

<sup>1</sup> Advanced economies include Canada, Denmark, euro area, Japan, Sweden, Switzerland, Australia and Denmark. Emerging market economies include Brazil, Chile, China, the Czech Republic, Hungary, India, Indonesia, Korea, Mexico, Poland, Russia, South Africa and Turkey.