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Programme

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Robert Z Aliber, University of Chicago

Reserve management

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11:30 ***Diversifying market and default risk in high grade sovereign bond portfolios***

Myles Brennan, Adam Kobor and Vidhya Rustaman, World Bank

Discussant: Fernando Monar, ECB

12:15 ***Sovereign credit scorecard***

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Discussant: Antonio Scalia, Bank of Italy

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15:30	***Explaining the returns of active currency managers*** Sam Nasypbek, World Bank, and Scheherazade S Rehman, George Washington University	***Fundamental allocation for government bond portfolios*** Cyril Caillault, Lombard Odier Darier Hentsch Investment Managers, Geneva

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Arjan B Berkelaar and Ali AlMansour, KAUST Investment Management Co, Arlington

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Reserve management

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Introduction[1]

This volume is a collection of papers presented at the Third Public Investors Conference, which was jointly organized by the Bank for International Settlements (BIS), the European Central Bank (ECB) and the World Bank (WB). This event, which took place on 2–3 November 2010 at the BIS's head office in Basel, brought together over 80 participants from more than 50 institutions comprising central banks, sovereign wealth funds and public pension funds.

The main aim of the current as well as previous Public Investor Conferences has been to create a forum where academics and private and public sector investment professionals can meet to discuss and ponder the issues of specific relevance to public sector investors. It is well recognized that public institutions differ markedly from their private sector peers in their investment activities. Investment rationales, preferences, eligible investments, governance structures and accountabilities as well as aspects relating to the availability of human and technical resources distinguish public investors. These idiosyncrasies have profound effects on how portfolio and risk management activities are organized and performed in public sector institutions.

Having discussed initial reactions to the financial crisis at the Second Public Investors Conference held at the World Bank in Washington DC, the 2010 Conference focused on how public investors are revising asset allocations and investment processes in response to the new financial market environment. Faced with high growth rates in foreign reserves and other pools of publicly managed funds, public investors are beginning again to discuss broader diversification of assets. Judging from the contributions to and discussions at the conference, central banks are concentrating their search for diversification opportunities on investment alternatives among sovereign obligations, including inflation-linked instruments and investments denominated in currencies other than those represented in the SDR basket. At the same time, public investors are becoming more aware of possible tension between what is optimal at the level of an individual investor and what might be required from the perspective of stability of financial markets. In terms of methodologies and techniques, similar to other institutional investors, public investors have accelerated efforts to develop and implement approaches for the management of market and credit risk that take on board lessons from the financial crisis. Also, further improved techniques for and oversight of active management of public funds received considerable attention at the conference.

In his keynote address, Professor Robert Z Aliber (International Economics and Finance, Booth School of Business, University of Chicago, emeritus) set the stage for the conference by describing four cycles of cross-border money flows since the early 1970s. These flows led to increases in the values of the currencies of the countries that experienced these money inflows, increases in their current account deficits, and increases in asset prices in these countries. These money inflows primarily financed increases in consumption spending. The countries that experienced these money inflows were in the "sweet spot" as long as the increase in indebtedness was larger than the interest payment on the indebtedness. These patterns of cash flows were not sustainable, and when they reversed, financial crises often followed.

Brief summaries of the papers that formed the main body of the conference are provided below. These contributions primarily focused on asset allocation from the specific

[1] This introduction was prepared by Joachim Coche (BIS), Ken Nyholm (ECB) and Gabriel Petre (World Bank). Comments by Robert N McCauley (BIS) are greatly appreciated.

perspective of public investors, aspects of active portfolio management, and credit risk modeling.

Asset allocation

Robert N McCauley and Jean-François Rigaudy of the BIS discuss in their contribution how the recent global financial crisis has impacted the asset allocation of central bank reserves. Using various data sources – inter alia, US authorities' annual surveys and data collected by the BIS – the authors analyze the extent to which reserve managers have reduced their exposures to bank debt and US agency debentures. They also report on sharp cutbacks in securities lending activities by central banks. Looking forward, the authors discuss the question of whether the crisis experience has halted the efforts of official reserve managers to diversify holdings more broadly. They argue that, given the high, and as a result of the crisis even increased, costs of holding reserves, the reversal in exposure to more credit-risky, less liquid instruments observed during the crisis may prove temporary. However, reserve managers will explore reserve diversification more cautiously than before the crisis. In particular, the limited size and liquidity of many alternatives to US Treasuries will pose ongoing challenges.

Myles Brennan, Adam Kobor and Vidhya Rustaman of the World Bank discuss the potential benefits of international diversification for high grade sovereign bond portfolios. To assess the potential diversification benefits, the authors decompose the returns on G7 sovereign bonds into global and local factors. They find that on average 75–80% of the bond returns are determined by global factors, whereas about 20–25% remains determined by local factors. Thus, while the sovereign bond market is integrated to a relatively high degree, there is still some room for diversification. The volatility reduction obtained by diversifying across the G7 issuers has in general been shrinking over the past decade, but local factors have gained in importance over the past two years for European issuers. Furthermore, in the light of the recent turbulence within the euro zone, the authors discuss diversification among sovereign obligations with a special focus on default risk. If an investor aims at enhancing expected return by going down the credit rating spectrum, diversification may mitigate the impacts of default risk to some degree.

José Luis Barros Fernandes and José Renato Haas Ornelas of the Central Bank of Brazil and Oscar Augusto Martínez Cusicanqui of the Central Bank of Bolivia propose a new methodology for portfolio construction that combines a Bayesian approach for formation of return expectations with a resampling approach for optimization. An application of this methodology to a sample of fixed income and equity markets in developed countries shows risk-return characteristics of the optimized allocations that are superior to those obtained with standard methods. The authors argue that the proposed approach is particularly suitable for long-term investors such as central banks and sovereign wealth funds, as it results in stable and well diversified portfolio allocations.

Carlos León of the Bank of the Republic (Colombia) and Alejandro Reveiz of the World Bank address portfolio choice for long-term investors. More specifically, they analyze how the presence of long-term serial dependence in time series of financial returns affects risk estimates for various horizons and consequently impacts results from portfolio optimization. For example, they show the extent to which optimal allocations are different for one- and 10-year investment horizons in the presence of long-term serial dependence. Technically, the authors employ a version of the so-called rescaled range analysis, a statistical technique to detect fractal structures in time series, to derive a scale-dependent covariance matrix. The techniques and results may be of particular interest to long-term investors such as central banks, pension funds and sovereign wealth managers, since they typically face a choice between asset classes that exhibit serial dependence to quite differing degrees – as is, for example, the case for emerging and developed market exposure.

Ricardo Selves and Marcin Stamirowski of the European Commission discuss the inclusion of inflation-linked instruments in a sovereign bond portfolio. Using linkers' market prices and euro sovereign securities, they derive both the real and nominal zero coupon bond price curves. More specifically, they use the Heath-Jarrow-Morton (HJM) framework to model the time-series evolutions of the inflation and the real and nominal zero coupon bond price curves. Finally, they use the estimated term structure parameters to validate the model via hedging analysis.

Marie Brière of the Université Libre de Bruxelles and Amundi and Ombretta Signori of Amundi focus on strategic asset allocation for investors seeking to hedge inflation risk. Using a vector autoregressive model, they investigate the optimal portfolio choice for an investor with a fixed real return target at different horizons and a shortfall probability constraint. They show that the strategic allocation differs sharply across regimes. In a volatile macroeconomic environment, inflation-linked bonds, equities, commodities and real estate play an essential role, while in a stable environment, nominal bonds are the most significant asset class alongside equities and commodities. This paper was first presented at the Second Public Investors Conference in 2009.

George Hoguet and Solomon Tadesse of State Street Global Advisors examine the role that securities denominated in special drawing rights (SDR) could play in the management of large institutional portfolios. They demonstrate that such securities could reduce portfolio variance and could provide a convenient method of diversification. However, despite favorable risk-return characteristics, a private market for SDR-denominated bonds remains has not developed. The authors point to the role that central banks and sovereign wealth funds could have in the further development of SDR markets by investing in SDR-denominated deposits and bonds, denominating their accounts in SDR and borrowing in SDR. This paper was first presented at the Second Public Investors Conference in 2009.

Active management

Roberto Violi of the Bank of Italy discusses a framework for the optimal choice between active risk and passive, benchmark risk. To that end, the author employs an extension of a model suggested by Treynor and Black where the optimal mix depends on the assumed capacity of active managers to predict excess returns and to avoid unsystematic risks. A particular hurdle in the practical application of this model has been the difficulty of forecasting active manager excess returns with sufficient confidence. The author clears this hurdle by using a unique dataset of US dollar government bond portfolios actively managed by the national central banks in the Eurosystem on behalf of the ECB. It turns out that an important source of fund managers' outperformance – in addition to skill in anticipation of returns of the benchmark portfolio – is the ability to predict the sign of a fixed set of active portfolios.

Sam Nasypbek of the World Bank and Scheherazade S Rehman of George Washington University explain and replicate returns of active currency managers by building an active currency replication index that optimally combines simple trading strategies defined in the literature. The study provides further evidence that the main trading strategies can explain a substantial portion of aggregate profits from active currency management. The results show that it is easy to replicate a diversified portfolio or a composite index of currency managers using simple currency trading strategies. Since public investors often rely on external currency managers, an active currency manager replication index can be a beneficial tool to evaluate the risk and performance of those managers, and thereby to contribute to good governance of public funds.

Credit risk modeling

Michael Jacobs, Jr, Office of the Comptroller of the Currency, develops a theoretical model for ultimate loss-given-default in the Merton (1974) structural credit risk model framework. He proposes an extension that allows for an independent recovery rate process, representing undiversifiable recovery risk, with stochastic drift. The comparative statics of this model are analyzed and compared with the baseline models having no independent recovery rate process. He validates the model in an out-of-sample bootstrap exercise using a large sample of losses by 800 defaulting firms in the period 1987–2008. He concludes that the model is worthy of consideration by risk managers, as well as supervisors concerned with advanced internal rating-based approaches under the Basel Capital Accords.

Daniel Rösch of the University of Hanover and Harald Scheule of the University of Melbourne provide an empirical study of the historical performance of credit ratings for securitisations. They find that credit rating agencies did not sufficiently address the systematic risk of the underlying collateral pools or the tranche structure. Furthermore, impairment risk, ie the risk that a securitization violates contractual payment obligations, is underestimated during origination years and years with high securitization volumes. Credit rating agencies also tend to measure a too low impairment risk level when fee revenue is high. Finally, securitization ratings are unable to predict impairment risk.

Financial turbulence and international investment

Robert Z Aliber[1]

Fifty years ago, an ongoing debate about international monetary reform was initiated by the publication of Robert Triffin's *Gold and the Dollar Crisis*[2]. Triffin had identified an apparent inconsistency in international financial arrangements; if the demand for international reserve assets of various foreign countries were to be satisfied, then the United States would incur payments deficits year after year, and the U.S Treasury's gold holdings eventually would be exhausted. But if the United States adopted measures to avoid balance of payments deficits, other countries as a group would not be able to satisfy their demand for international reserve assets. Competition among countries for international reserve assets would be deflationary and lead to declines in prices.

Three groups of proposals were directed at the Triffin dilemma – two would lead to more rapid increases in the supplies of international reserve assets, and the claim for the third was that it would reduce the demand for reserves. One generic approach – the dominant approach – was to produce "paper gold"; a new international reserve asset that would share the attribute of gold in that it would be an asset without being the liability of any institution or government. The motive for the paper gold proposals was the desire to enable the United States to maintain the U.S. dollar parity of $35 an ounce, then viewed as the centrepiece of international financial arrangements. The belief was that the annual or periodic increases in the supply of paper gold would satisfy the increases in the demand for international reserve assets.

The second approach toward increasing the supply of reserves was that the U.S. dollar price of gold be increased to $70 or perhaps to $100, with comparable percentage increases in the price of gold in terms of other most other currencies. (A few countries might use the occasion of the change in the U.S. dollar price of gold to change their parities in terms of the U.S. dollar.) The value of the gold owned by central banks immediately would increase in the same proportion as the increase in the U.S. dollar price of gold. Moreover gold production would be stimulated. Finally the higher price of gold would lead to a reduction in the private demand, and central banks would acquire a higher proportion of annual production.

The third approach to resolve the Triffin dilemma was to abandon the system of adjustable parities for currencies, which would then float, much as the Canadian dollar had from 1950 to 1962. A shock that would have led to a payments deficit if a currency had been pegged instead would lead to a decline in the price of that country's currency; similarly a shock that would have led to payments surplus if the currency had been pegged would have led to an increase in its price. Since central banks would no longer be committed to maintaining the value of their currencies, they would no longer acquire international reserve assets.

International monetary arrangements now incorporate each of the three major sets of proposals. The Special Drawing Rights arrangement embodied a paper gold proposal and was implemented in 1969 when the SDR equivalent of $3 billion of U.S. dollars was produced and attached to the International Monetary Fund; a member country could use its SDR to purchase the currencies of other IMF members. SDR outstanding now total $308 billion. A floating currency arrangement was adopted, initially in August 1971 when the

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[1] Professor of International Economics and Finance Emeritus, Booth School of Business, University of Chicago.

[2] Full disclosure: Triffin was my thesis advisor at Yale in the late 1950s.

U.S. Treasury formally closed its gold window and sought to achieve the revaluation of the Japanese yen and the French franc, and then again in February 1973 when the Smithsonian Agreement faltered. The private market for gold was segmented from the official market in the spring of 1968, and then the U.S. gold market window was formally closed in August 1971. Market forces led to an increase in the U.S. dollar price of gold to nearly $200 in 1973 and then to nearly $1,000 in January 1980. In the last few months gold has traded above $1,300; the gold component of central bank reserves is now five or six times larger than the SDR component. Moreover, the supply of international reserve assets has surged, despite the earlier argument that the demand for reserve assets would decline once currencies were no longer pegged.

One dominant feature of the last 40 years is that there have been four waves of financial crises; each wave has involved the failure of banks and other credit institutions in three, four or more countries. These financial crises often have occurred at the same time as currency crisis. The first of these waves of crises was in the early 1980s, when the governments, government-owned firms, and banks in Mexico, Brazil, and 10 other countries failed. Japan in the 1990s was the primary country in the second wave. The most recent wave began in 2007; banks in the United States, Britain, Ireland, Spain, and Iceland tumbled into bankruptcy. Each of these waves of crises was preceded by a wave of credit bubbles when the indebtedness of a group of borrowers increased by 20–30% a year; most of these credit bubbles led to rapid increases in the prices of real estate and stocks. The prices of these assets declined sharply when the credit bubbles were pricked, and financial crises followed. Most of these waves of credit bubbles followed from an increase in cross-border money flows to these countries, which led to the appreciation of their currencies and an increase in asset prices.

These cross-border money flows have been both much larger and much more variable than when currencies were attached to parities. The rates of return to the investors who undertook the cross-border money flows have been adversely impacted by the financial crises.

The first of the six sections of this paper is descriptive and summarizes the turbulence in international financial markets in the last 40 years. The second section is analytical, and highlights the sources of financial crisis. The third section identifies the impacts of structural shocks and monetary shocks on currency values. The fourth section highlights the role of carry trade investors and the impact of their transactions on currency values and asset prices. The fifth section examines the factors that lead to increases in cross-border money flows by carry trade investors. The sixth highlights the risks and the returns of cross-border investments in a world characterized by large movements in currency values. The concluding section summarizes the main features of the paper.

I. Monetary turbulence and financial crises

The striking development since the early 1970s has been the turbulence in the currency market and in national financial markets. The proponents of a floating currency arrangement had suggested that the changes in currency values would be reasonably small and would reflect changes in the differences in national inflation rates, and that the deviations between the market prices of currencies and the shadow prices computed from differences in inflation rates would be smaller than when currencies were pegged. But instead the range of movement in the currency prices has been much larger and the scope of overshooting and undershooting much much larger than when currencies were pegged.

There have been four waves of financial crises; each of these waves involved the failure of banks and other credit institutions in three, four, or more countries at about the same time. The first of the four waves of crises involved the failure of the governments and government-owned firms in Mexico, Brazil, Argentina, and 10 or so other developing countries in the early

1980s to pay the interest on their U.S. dollar indebtedness in a timely way; their currencies depreciated sharply. The domestic banks in these countries failed when many of the borrowers defaulted on their loans to the domestic banks after the currencies depreciated sharply, since the borrowers often had liabilities denominated in the U.S. dollar and their indebtedness surged when their currencies depreciated. The second wave was centred on the failures of banks and credit institutions in Japan in the early 1990s when property prices declined; at about the same time the banks in three of the Nordic countries tumbled in response to sharp declines in real estate prices. The Asian financial crisis was the third wave and involved the failures of banks in Thailand, Malaysia, Indonesia, and South Korea, although banks in Russia and Argentina also failed during this wave. Similarly the financial crisis in Mexico at the end of 1994 and the beginning of 1995 was the bellwether of events that would impact Thailand and Indonesia thirty months later. The fourth wave of failures of banks and credit institutions began in 2007 and 2008 and involved the United States, Britain, Ireland, Spain, and Iceland.

Each of these waves of financial crises followed a period of three, four, or more years when the indebtedness of a similarly placed group of borrowers in different countries increased at the rate of 20–30% a year. Most of the waves of indebtedness resulted from cross-border money flows. Thus bank loans to governments and government-owned firms in Mexico, Brazil, and the other developing countries increased by 30% a year for nearly a decade, and the total external indebtedness of these countries increased by 20% a year. Each of the next three waves led to bubbles in real estate prices Bank loans to buyers of real estate in Japan increased by 25–35% throughout the 1980s; the increases in the price of real estate led to comparably large increases in stock prices. The external indebtedness of most of the countries that were involved in the Asian Financial Crisis and of Mexico had increased sharply in the early 1990s; the money inflows resulted in part because the overhang of bank loans that were default was funded into long-term bonds. Moreover some investment banks had discovered that “emerging market equities were a new asset class”, which led pension funds and mutual funds to buy these securities. Some of these countries had privatized government owned firms, and some of the newly privatized firms were acquired by firms headquartered in the industrial countries. Banks headquartered in the emerging market countries sourced for money from the banks headquartered in the industrial countries because the interest rates were below those in the domestic money markets. The United States, Britain, Ireland, Spain, and Iceland experienced a large money inflows after 2002, and bank loans for real estate purchases in these countries increased rapidly.

Many of the banking crises have been associated with the abrupt depreciation of currencies; the principal exception was that most of the banks and many other financial institutions failed in Japan in the 1990s but there was no crisis in the yen. A second exception is that the financial crisis in Ireland in 2008 was not associated with a currency crisis, because Ireland did not have its own currency. The Greek currency crisis led to a significant depreciation of the euro.

The data on the changes in the prices of currencies belie one of assertions advanced by the proponents of floating exchange rates, that changes in prices of currencies would be systematically related to changes in national differences in inflation rates on a week to week and month to month basis, and in the short run – say intervals of up to four or five years. In the long run, purchasing power parity concept is validated, but at shorter intervals the deviations from the values are suggested by the differences in national inflation rates.

II. The causes of financial crises

Several features of these cross-border money flows should be noted. The first is that the increases in money flows to countries have two immediate impacts. One was that their currencies appreciated and the second was that asset prices in these countries increased in response to purchases by foreign buyers – who bought the currencies so they could buy securities. Household wealth increased as asset prices increased, which led to higher levels of consumption spending and more imports and a larger trade deficit. Increases in asset prices, household wealth, and imports were an integral part of the adjustment process, which required that the current account deficit increase by an amount that corresponded with the autonomous increase in the capital account surplus.

A second feature is that the indebtedness of many of those who had borrowed to buy real estate was increasing at two to three times the rate of growth of their incomes, which meant that that the ratio of their indebtedness to their incomes was increasing at a rapid rate – one which was too high to be sustained. Similarly the external indebtedness of these countries was increasing more rapidly than their GDPs. The third feature was that the rate of increase in the indebtedness of these borrowers was two to three times the interest rate on the loans, which meant that money available to the borrowers from new loans was several times larger than the interest payments on their outstanding loans. The borrowers were in a "sweet spot" because all the money they needed to pay the interest on their outstanding loans came from the lenders in the form of new loans.

This pattern of cash flows could not continue without limit, at some stage the lenders would reduce the rate at which they would extend more credit to the borrowers, who then would have to find a new source of money for the scheduled interest payments. When the flow of money from the lender to the borrower slowed, the borrower's currency would depreciate.

The implications of changes in cross-border money flows on the price of a currency and on the prices of assets in a country can be illustrated by reviewing the experience of Iceland between 2002 and 2008. Iceland had a modest current account surplus in 2002. Then the foreign demand for Icelandic securities increased sharply, more or less at the same time as the foreign demand for securities denominated in the U.S. dollar and the British pound increased. The Icelandic krona appreciated in response to the increase in the foreign demand for Icelandic securities; Iceland's capital account surplus and its current account deficit increased. Moreover the prices of the Icelandic securities increased in response to the purchases by foreign buyers.

The Icelandic sellers of the securities denominated in the krona then had to decide whether to use the money from the sale of these securities to buy other Icelandic securities from other Icelandic investors or to buy consumption goods – they could do both and they did both. To the extent that they purchased other Icelandic securities, the sellers had the same problem. In effect the initial purchases of Icelandic securities triggered a series of purchases by those who sold the securities, who used nearly all of their receipts to buy other Icelandic securities. The prices of these securities, and the financial wealth of Icelandic households increased. Their consumption spending increased, which stimulated an economic boom; Iceland's trade deficit increased sharply.

This series of transactions in Icelandic securities was an integral part of the adjustment process whereby the increase in the Icelandic imports and in the country's current account deficit matched the increase in the country's capital account surplus. The intermediate argument was that Icelandic household wealth increased as the prices of the securities owned by the borrowers increased which led to an increase in household consumption.

Iceland experienced two bubbles at the same time, one in the currency market and a second in its asset markets, both for stocks and for bonds. When the foreign demand for Icelandic krona securities slackened, it was inevitable that the krona would depreciate; at the same time, it was likely that the prices of Icelandic assets would decline in response to the increase in domestic interest rates, since some investors would become distress sellers.

The bubble in the U.S. housing market between 2002 and 2007 was similar to the events in Iceland, although on a much more massive scale. An increase in the foreign demand for U.S. dollar securities lead to an appreciation of the U.S. dollar (although it mostly dampened a depreciation that otherwise would have occurred). The U.S current account deficit increased, and the ratio of the U.S. current account deficit to U.S. GDP increased by 3–4 percentage points. U.S. real estate prices surged. And then real estate prices started to decline at the beginning of 2007, much as they did in Britain, Ireland, Iceland, and Spain.

Because the rate of increase of the indebtedness of the borrowers in these countries was so much greater than their incomes, it was inevitable that at some stage the lenders would become more cautious about increasing their loans. Similarly, because the rate of increase in the external indebtedness of these countries was so much higher than the increase in their GDPs, it was inevitable that lenders would become more cautious. When the flow of money to these countries slackened, it was inevitable that their currencies would depreciate. The initial depreciation by itself might induce other lenders to become more cautious. The combination of the decrease in the pace of money inflows and the depreciation of the currencies would lead to a decline in asset prices. Economic growth would slow, as households increased their saving in response to the decline in financial wealth.

Hence the increase in the values of the currencies and the increases in asset prices in the countries were not sustainable. And the increases in these prices can be considered bubbles because they were not sustainable.

III. Monetary shocks, structural shocks, and changes in currency values

Large changes in currency values relative to the values based on differences in national inflation rates may reflect more structural shocks such as sharp increases and declines in oil prices or more monetary shocks including changes in inflation and interest rates. Large variations in the prices of currencies can be related to an early literature on currency movements when currencies are not pegged. When Ragnar Nurkse, in his classic *International Currency Experience*, suggested that speculation in the currency market was destabilizing, he probably was referring to the French experience between 1924 and 1926. Milton Friedman responded in “The Case for Floating Exchange Rates” that if speculation had been destabilizing, it would have been unprofitable, and those speculators that had lost money would leave the market. Nurkse’s statement centred on the empirical properties of time series of changes in currency values and changes in national price levels. Friedman’s statement was derived from “first principles” and hence was not a direct refutation of Nurkse’s observation.

Changes in currency values in the first half of the 1920s were affected by two different factors. At the beginning of the First World War, most governments suspended the convertibility of their national currencies into gold; their currencies depreciated relative to the U.S. dollar in part because their money supplies had increased more rapidly than the U.S. price level. The view in the immediate post-war period was that currencies would again be attached to their prewar parities. Initially investors accumulated German marks in anticipation that the mark would appreciate toward its prewar parity, and then when they reversed their anticipations and sold their marks, the currency depreciated. When the mark collapsed in 1923, the cliché was that speculative pressure was deflected to the French franc. There were two “bear raids” on the French franc, one in 1924 and the second in 1926.

Two meanings can be attached to Nurkse’s use of the term “destabilizing speculation” – one is that speculators caused the amplitude of movements in the currency values attributable to shocks in the goods market to be larger than they would have been in their absence; in this sense the speculators are like “tape watchers” or “momentum traders” who followed the cliché that “the trend is your friend”. The second meaning attached to this term is that the transactions of speculators would induce changes in domestic income and employment by their impacts on the trade balance.

The logic is that if there are only two groups of participants – goods market traders and the speculators – in the currency market, the transactions of one group cause the prices of currencies to deviate from their long-run average prices, while the transactions of the second group will limit these deviations. Both the goods market traders and the speculators are responding to different shocks and different profit opportunities.

Both the goods market traders and the speculators will be impacted by various shocks. Shocks can be grouped as either structural or monetary; structural shocks include sharp changes in the prices of oil and other commodities, the loss of an export market, a domestic crop failure. Monetary shocks include changes in interest rates and changes in the anticipated inflation rate. If a shock in the form of an increase in the price of imports – say an oil price shock – might lead to a depreciation of the currency – and if speculators believe the shock is temporary, they may buy the currency and limit the depreciation. In contrast if foreign interest rates increase, speculators may move money to the foreign centre which will cause domestic currency to depreciate. Domestically produced goods will become more competitive in both the domestic market and in the foreign market, and the increase in the domestic trade surplus will limit the depreciation of the domestic currency.

Neither Nurkse nor Friedman identified who the speculators were – whether they were banks, trading firms, brokerage firms, insurance companies, or individual investors.

The debate between Nurkse and Friedman was never joined because they differed in the source of shocks. Nurkse implicitly suggested that the shocks originated in the money market, while Friedman believed that the shocks originated in the goods market. An extension of this distinction is whether the goods market shocks are more frequent than money market shocks, and the frequency and severity of each type of shock.

Money market shocks and goods market shocks have different impacts on the combination of changes in the trade balance and the value of the currency. For example, assume that there is a goods market shock in the form of an increase in the price of oil; the country's oil import bill increases and its currency depreciates so that exports will increase to match the increase in the imports. Speculators may buy the currency and limit the depreciation, which is the scenario envisioned by Friedman. In contrast, assume a money market shock in the form of an increase in interest rates in foreign country; investors move money to the foreign country; the foreign currency appreciates or what is the same thing, the domestic currency depreciates. The country's trade surplus increases, which provides goods market traders with the opportunities to arbitrage. The increase in the trade surplus leads to a higher level of domestic income and perhaps an increase in upward pressure on the price level. This is the type of shock envisioned by Nurkse.

The shortcoming of the Nurkse-Friedman debate is that it does not deal with the stylized fact that large changes in the prices of currencies have been associated with significant changes in the cross-border movements of money. Again, returning to Iceland, the sharp appreciation of the currency was associated with a massive flow of money to Iceland; Iceland's current account deficit increased as its capital account surplus increased. The money flow to Iceland might be viewed as consistent with a broad interpretation of Nurkse's view of destabilizing speculation, although Nurkse appears to have been concerned that money flows from a country might put upward pressure on the price level because of the increase in the trade surplus might lead to excess demand. The Iceland experience is one in which the money flows to a country led to increases in consumption spending and investment spending as result of the positive wealth effect.

IV. Carry trade investors and currency movements

One feature of the last 40 years has been large cross-border money movements and the variations in these flows, which is evident from the changes in the trade balances of individual countries. During the early 1990s, Mexico's current account deficit increased to 6% of its GDP; then the peso depreciated sharply at the beginning of 1995, and Mexico developed a current account surplus that was 4% of its GDP. Iceland went from a current account deficit that was more than 20% of its GDP to a current account balance. Similarly there were large changes in the current account balances of many other countries, although few were as dramatic as those for Mexico and Iceland.

The shocks that led to these changes in the cross-border money flows originated in the financial markets; these shocks included changes in interest rates and in anticipated inflation rates. These shocks have induced changes in cross-border money flows that led to changes in the values of currencies. (If the shocks that had led to an increase in the current and trade deficits had originated in the goods market, the currency would have depreciated as the trade deficit increased.)

These cross-border movements of money are initiated by "carry trade investors", who acquire foreign securities with the intent to own them for extended periods. Carry trade investors are like arbitragers in financial markets, they seek to profit from the difference in interest rates on comparable securities denominated in different currencies; they realize that they may incur losses from the depreciation of the foreign currencies – but obviously they believe that the values in the interest rate differential term is larger than the value in the currency term. Carry trade investors do not believe that "all the information is in the price"; instead they believe that the interest rate differential overstates the anticipated or likely change in the value of the currency during the term to maturity of their investments. The difference in the two streams of interest income can be considered the revenues for carry trade investors, and the anticipated change in the price of the currency is the cost.

At times the interest rate term and the currency term are additive. For example, assume that interest rates in a country increase, perhaps because its central bank has adopted a more contractionary monetary policy. The carry trade investors move money to the country, and its currency appreciates. The carry trade investors profit both from the additional interest income and the gain from the appreciation of the currency. In periods of two or three years, the additional income from the appreciation of the currency may be larger than from the difference in interest rates. In the long run, however, the interest rate differential and the currency term are offsetting, and the currencies of the countries identified with higher interest rates depreciate.

Carry trade transactions come in 57 varieties. Mrs. Watanabe took the money from one of her yen deposits in Tokyo to acquire a U.S. dollar annuity from AIG. Citibank used funds obtained from the sale of dollar deposits in London to fund its U.S. dollar loans to the Government of Mexico. Nomura acquired dollars in the offshore market to buy the IOUs of the Landsbanki of Iceland. Individuals in Reykjavik financed the purchase of autos by signing IOUs denominated in the Japanese yen and the Swiss franc because interest rates were lower than those on the Icelandic kronor. Similarly individuals in Poland have financed their purchases of homes by borrowing Swiss francs, and individuals in Australia have borrowed the yen to finance their home purchases.

Carry trade investors who bought Icelandic krona IOUs in 2002 and 2003 and 2004 profited from the appreciation of the krona as well as from the excess of interest rates krona securities over the interest rates on U.S. dollar securities. Similarly Icelandic borrowers who sold IOUs denominated in the U.S. dollar or the euro profited from the saving in interest costs.

The efficient market view is that the cross-border money flows surge whenever there is new "news"; the price of the currency changes immediately until there is no longer an excess return attached to the cross-border movement of money. However, the appreciation of the currency of a country that experience an inflow of money is slowed or dampened because of transactions in the goods market; as the currency appreciates, the opportunities for goods market arbitrage increase. Hence the anticipated excess return remains.

Carry trade investors can be distinguished from the Friedman's speculators and from some of the speculators noted by Nurkse. Friedman's speculators trade currencies for banks and other financial firms; they seek to profit from changes in the prices of currencies. These speculators hold their positions for a relatively short time – a few minutes, a few hours, a few days. A few of these traders are market makers, many are day traders, and a few are proprietary traders. The hallmark of these traders as a group is that their anticipated revenues are from changes in the prices of currencies, while their costs are the difference between domestic and foreign interest rates. The market makers provide both bid and offer for transactions of a standard size; while it may seem that they are providing a service, the information in the order flow is of high value. This group makes its money from the immense volume of transactions – and they make money regardless of whether their domestic currency appreciates or depreciates.

Consider the returns to the goods market traders, the carry trade investors, and speculators. The goods market traders profit from the arbitrage opportunities presented by the divergence in national price levels created by changes in the values of currencies, the greater the overshooting and undershooting, the larger their profit opportunities. Their trade transactions require that they buy and sell currencies as an intermediate transaction prior to the payment for the purchase of goods, and they may incur a cost for these transactions. Similarly, carry trade investors must buy foreign currencies before they can buy foreign securities; they incur a cost. The speculators profit from their market making activities, and from changes in currency values.

Casual empiricism suggests that the trading revenues of the major international banks have increased sharply since 1980 and perhaps from the early 1970s. Some of these revenues are from trading securities and some are from trading currencies and some from trading commodities. The volume of currency transactions is many times larger than the volume of trade or the volume of trade and investment. Most of the transactions of the speculators are with other speculators. Moreover developments in technology and competition have led to declines in the bid-ask spreads. The increase in the revenues seem larger than the amount that can be attributed to the bid-ask spreads; the implication is that a large share of these profits must have come from revaluation gains on their positions.

How can the currency traders and the carry market traders both profit at more or less the same time? Obviously they can't in terms of cash flows – the currency traders take money off the table, minute by minute and hour by hour, and stuff their profits in a sock. Some of that money may be placed on the table by the goods market traders; the costs of using the currency market are like transport costs. The carry trade investors are indifferent because they are continually re-valuing their positions on the basis of current prices. The carry trade investors earn money for an extended period – until the bubble is pricked, the currencies depreciate sharply, and firms and banks fail.

V. The sources of financial instability

The much greater variability in cross-border money flows since the early 1970s can be attributed to a larger number of shocks. Some of these shocks might be structural, including sharp changes in the price of oil, dramatic increases or decreases in the rates of return on a particular group of assets, and changes in financial regulations. Some of these shocks might be monetary, including significant changes in anticipated inflation rates, or in interest rates, or in currency values.

One of the principal arguments advanced by the proponents of a floating currency arrangement is that in the absence of parities, national central banks would be able to follow "independent" national monetary policies; they would no longer be constrained by the need to minimize their payments imbalance at their established parities. When currencies were pegged, national inflation rates were closely linked because countries could not finance large trade deficits. Because currencies are no longer attached to parities, national inflation rates are more likely to differ – and the larger possible difference in these rates means that the scope for changes in these differences is much larger. When interest rates change relative to the inflation rate, carry trade investors may recognize an exceptional profit opportunity.

The necessary condition for a significant increase in cross-border money flows is a shock that leads to an increase in the return on securities available in a particular country, or a shock that leads to a relaxation of restrictions that previously had restricted investor purchases of certain securities, or a change in controls on cross-border money movements. One of the two sufficient conditions for an increase in cross-border flows involves the willingness of carry trade investors to take on the risks associated with the cross-border movements of money, and the other is a pool of money that these investors can access. The large payments imbalances since the mid-1960s have led to a surge in international reserve assets which is an enormous pool of accessible money. (Central banks are more likely than others to hold funds in the offshore deposits.)

That there have been four waves of credit and asset price bubbles in 40 years suggests that there may be a connection between several of these waves – more precisely, between the implosion of one wave of bubbles, and the formation on another wave. That three, four, or more countries have been involved in each of the several waves of credit bubbles suggests a common cause. The shock that preceded the first wave of credit bubbles in the 1970s was a surge in the world inflation rate that led to significant increases in commodity prices and in the anticipated rates of growth of GDP in the countries that produce primary commodities. The shock that preceded the bubble in Japanese real estate and stocks was the decline in interest rates on US dollar securities, which lead to an increase in money flows toward Tokyo and a tendency toward the appreciation of the yen. The Japanese authorities relaxed restrictions on bank loans for real estate. The shock that led to the surge in money flows to the Nordic countries was the relaxation of restriction that limited the ability of banks headquartered in these countries to source for money in the offshore market. Several shocks contributed to the increase in money flows to the emerging market countries in the early 1990s, including the appreciation of the yen, and the liberalization of restrictions that had limited the ability of banks headquartered in these countries to source for money in foreign markets. The sharp depreciation of the Thai baht and the currencies of other emerging market countries in mid-1997 contributed significantly to the bubble in U.S. stocks. The shock that led to a rapid increase in the supply of credit available for real estate in the United States, Britain and other countries was the surge in China's trade surplus.

One feature of these shocks is that the adjustment process induced by the flow of carry trade money to these countries leads to increases in their rates of growth of GDP as a result of increases in consumption spending and investment spending in response to higher levels of household wealth. It is as if there is a feedback loop; an initial shock leads to increases in cross-border money flows, and then the increases in wealth induced by these flows lead to the more rapid growth of GDP – which induces carry trade investors to move more money abroad.

Although the cross-border money movement is induced by the increase in the rates of return, the primary impact of this movement is to finance higher levels of consumption spending – the story is that the increase in wealth induced by the money inflow leads to a decline in domestic saving as consumption spending increases.

Because currencies are not pegged, changes in national monetary policies lead to changes in anticipated values for currencies and induce changes in cross-border money flows; a move to more contractionary monetary policies may attract money because of the higher inflation-adjusted rate of return and the downward revision in the anticipated inflation rate. In this way the appreciation of the currency in response to the adoption of a more contractionary policy may be like a self-fulfilling prophecy.

Once a shock leads to an increase in the anticipated returns on securities denominated in a group of currencies or a reduction in the restrictions on the cross-border movement of money, the conditions are appropriate for the formation of a bubble. The money is there, and the initial movement of money across national border is likely to enhance the anticipated returns on the money market arbitrage. No one foresees the inevitable crunch because the rate of increase in indebtedness is not sustainable.

VI. Managing wealth in turbulent times

Keynes wrote several articles in the mid-1920s that questioned whether British foreign investment "paid" – whether British GDP was higher because London-based firms increased their investments abroad and because households bought foreign securities. His argument centred on the distinctions between private rates of return and social rates of return. One was that when the British owned the New York subway and the subway went bankrupt, the equity investors lost all their money and the American bondholders became the owners. (If the Americans had owned the equity and the British had owned the bonds, the conclusion would have differed.) His second point was that the private rate of return to the owners of the foreign investments was higher than the social rate of return to Britain as a country, and for two reasons – one is that the U.S. government rather than the British government would collect the taxes on the investment. His second was that the capital stock available to British workers was smaller because of the capital outflow, which involved a comparison between the decline in domestically produced GDP and the return to Britain on its foreign investment. His third argument was that anticipated rates of return by the first to invest abroad were lowered by others who followed them and increased their investments.

The textbook answer to the question whether foreign investment pays is an extension of the argument about the gains from trade; both the capital-exporting country and the capital-importing country gain, and a lot of country-specific factors determine the shares of the gain to each country. This textbook answer is in a "real economy" – one without money.

There are three primary characteristics of cross-border money flows when the currencies have been floating. One is that the risk of these flows has increased because of the much larger range of movement in the prices of currencies. A second is that the economic booms in the money-receiving countries are associated primarily with increases in consumption spending. The third is that there is a likelihood of financial failure when the bubble implodes.

The implicit assumption was that the increases in the flows of money to a country would be associated with an increase in the investment in the country. One of the unique features of cross-border money flows since the early 1970s is that they have been associated primarily with increases in household consumption or an increase in the fiscal deficits in the countries that have experienced increases in money inflows. For example, the bank loans to Mexico, Brazil, Argentina, and other developing countries in the 1970s primarily financed increases in the fiscal deficits of the governments in these countries and increases in the capital expenditures of government-owned firms. The surge in money flows to the United States after 2002 contributed to an increase in the supply of credit available for real estate; a significant part of this credit financed mortgage equity withdrawals. One feature of this period was that the household savings rate declined sharply and was not significantly different from zero; in effect the increase in the supply of foreign saving available to Americans induced a set of market developments that led to a decline in the household saving rate. The surge in money flows to Iceland enabled the domestic banks to finance large loans to firms that wanted to invest abroad; increases in household consumption accounted for 90% of the increase in the current account deficit.

The implication is that cross-border money flows have not been primarily associated with increases in investment and in the rate of economic growth in the countries that have received the money inflows. Still it may be that some countries have been able to achieve somewhat higher rates of economic growth, since they are no longer obliged to maintain parities for their currencies. Nevertheless there appears to have been a disproportionate increase in the risk relative to the increase in return from cross-border investments. Hence there has been a significant reduction in the “all-in” return available to the carry trade investors from the sum of the additional interest income and the currency losses and gains relative to the risk of revaluation losses and credit defaults.

These statements about increases in return and increases in risk follow from first principles. Obviously investors who get the timing right – who buy low and sell high and repatriate their money before the bubble implodes – will have a much higher rate of return. A few investors can pursue this strategy, however if many were to produce this strategy, the currency would depreciate and the bubbles would implode.

Consider the market in junk bonds otherwise known as high-yield bonds as a metaphor. Promises were made about a large supply of “free lunches” on junk bonds or high-yield bonds by Michael Milken in the 1980s, who convinced investors that there was market inefficiency because the rating agencies did not rank these bonds, and hence there was an excess return on these bonds. The excess return persisted until the market in these bonds collapsed, which occurred soon after the savings and loan associations and insurance companies that were managed by Milken’s buddies were no longer the “buyers of last resort” for these bonds. Subsequent studies have shown that the additional interest income was not sufficiently large relative to the credit losses that investors incurred.

The same point is made by considering the appropriate premium for selling flood insurance in New Orleans. How should the underwriters set the appropriate premium – high enough to cope with the losses due to the floods that occurs every 10th year? But then the premiums will not be large enough to reimburse the losses due to the exceptional flood that occurs every 50th year. If the premiums are set to cover the losses from the more frequent, less severe floods, they may be too low to cope with the more severe floods. The most severe flood may lie in the future.

The dominant implication of the increase in risk relative to return on cross-border carry trade investments is that market participants should devote more attention to determining the currency composition of their assets and liabilities that minimizes their exposure to loss from changes in currency values. The managers of the international reserves of central banks are in a position much like multinationals and other firms involved in international trade; they first need to determine the currency composition of their reserves that minimizes their exposure to gain and loss from changes in the price of currencies, and they then need to determine whether the anticipated interest income from maintaining a different composition is worthwhile in terms of the exposure to loss from changes in currency values.

VII. Summary and conclusion

During the 1960s the dominant concern of those involved in international finance was that the shortage of international reserve assets would lead to deflationary pressures on national economies. The waves of credit bubbles since the 1970s reflect in part the surge in the supply of international reserve assets. There have been four waves of financial crisis since the early 1980s; each of these waves has led to the failures of large number of banks in three, four, or more countries at the same time as many borrowers defaulted on their loans. The first wave occurred in the early 1980s and involved the governments of and government-owned firms in Mexico, Brazil, and 10 other developing countries. The inability or unwillingness of these borrowers to repay in a timely way led to a depreciation of their currencies, and domestic firms with liabilities denominated in the U.S. dollar failed because of the large revaluation losses on these loans, which contributed to the failure of the domestic banks. The second wave of financial crises occurred in the early 1990s, when banks and other financial institutions failed in Japan and three of the Nordic countries – Finland, Norway, and Sweden. The Asian Financial Crisis that began in mid-1997 was the third wave; the financial turmoil in Mexico at the end of 1994 was a prelude to the collapse in values in Asia. The fourth wave of crises that began in 2008 resulted from the sharp decline in real estate prices in the United States, Britain, Ireland, Iceland, and other countries; the decline in the value of mortgages and mortgage-related securities led to large losses by mortgage bankers, investment banks, and commercial banks, and other lenders.

Each of these waves of crises followed from increases in the indebtedness of a group of borrowers at rates of 20–30% a year for three, four, or more years. Each of these four waves of credit bubbles involved the cross-border movement of money; the principal exception was that the rapid increases in real estate prices in Japan followed from the rapid growth in the domestic supplies of money and credit. In contrast, the rapid growth in the credit in the Nordic countries in the late 1980s resulted from money inflows as domestic banks sourced money in the offshore market.

One central aspect of the period since the early 1970s has been that the range of movements in the prices of currencies has been much larger than the range that would have been forecast based on contemporary or lagged differences in national inflation rates. These very large changes in the value of currencies have resulted from changes in cross-border money flows; increases in the money flows to countries have led to extensive appreciations of their currencies.

The increases in cross-border money flows to countries have two immediate effects – their currencies appreciate and the asset prices in these countries increase in response to purchases by those who had moved money to these countries. The increases in asset prices were an integral part of the adjustment process; asset prices and household wealth increased until the increase in consumption spending and in imports led to an increase in the current account deficit that matched the autonomous increases in money inflows. The counterpart of the increase in the money flows to the country was that its savings rate declines as household consumption spending increased.

The increase in the indebtedness of the borrowers in these several waves was several times higher than the increase their incomes and GDPs; similarly the increase in the indebtedness was several times higher than the interest rate on the indebtedness. As long as the indebtedness of the borrowers was higher than the interest payments, the borrowers were in the sweet spot, since all the money needed to pay the interest on the indebtedness came from the lenders in the form of new loans. But it was inevitable that the lenders would reduce the rate of growth of new loans, which automatically would lead to a depreciation of the currencies of these countries.

The minimum requirement to generate a bubble is that the rate of the flow of money to a country is too high to be sustained; when the rate slackens, it is inevitable that the currency depreciates and interest rates increase, in part in response to the decline in the supply of credit.

Three factors have contributed to the four waves of bubbles. One is that since the early 1970s, there has been a large pool of "idle money" parked with the international banks, available to be tapped by those who have concluded that they enhance their own returns by taking on credit risk or currency risk. This pool has been inflated by the large payments imbalances since the late 1960s. The second is that there have been a series of shocks at national borders, which either have increased the anticipated returns available on securities in certain countries or increased the scope for cross-border investment by reducing restrictions at the border. The third is that the early stages of cross-border money flows enhance the returns in countries that receive the money, so that the flows are self-justifying – at least for a while.

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Managing foreign exchange reserves in the crisis and after

Robert N McCauley and Jean-François Rigaudy[1]

1. Introduction

The recent global financial crisis has posed a great challenge to official foreign exchange reserve managers. Events brutally reminded them of the original *raison d'être* of foreign exchange reserves, namely to deal with emergencies. Reserve managers faced the possibility of a need to mobilise rapidly funds in liquidity portfolios, and even investment portfolios, to meet the foreign currency needs of domestic banks (and in some places firms) or to support the foreign exchange value of the domestic currency. At the same time, the most common short-term placements, namely bank deposits, came into question as write-downs of asset-backed securities burned through bank equity and interbank funding liquidity dried up. And then the failure of Lehman Brothers exposed risks in repo and money market mutual funds, in which some central banks had invested. While central banks struggled with manifold challenges to their management of short-term funds, their losses on longer-term investments in private asset-backed securities – the securities that set the crisis in motion – sometimes showed up in surprising places, but appear to have been neither widespread nor large.

Accordingly, official reserve managers reacted most immediately in the management of their short-term portfolios. Judging from data on US dollar portfolios, they reversed the long-standing tendency to hold a greater share of short-term funds in riskier placements, especially with banks, and sought refuge in the quality of US Treasury bills and central bank liabilities. Many withdrew from or cut back on their participation in securities lending programmes, under which cash raised against securities of the highest quality turned out to have often been invested in lower-quality and less liquid securities.

Among longer-term holdings, official reserve managers' banks reversed their long-standing diversification into US agency securities, starting several months before the US authorities took over Fannie Mae and Freddie Mac in early September 2008. Central banks bought agencies on the supposition of official support, and then sold as the Treasury in fact provided support and the reputation of agencies suffered. Official holdings of agency mortgage-backed securities have held up much better than holdings of agency debentures. In contrast, among holdings of US corporate bonds, reserve managers now hold many fewer asset-backed bonds, while broadly maintaining their modest holdings of straight corporate bonds.

These developments raise the question of whether the searing experience of 2007–09 will prove to have stopped the trends towards greater acceptance of credit, market liquidity and duration risk by official reserve managers. It should be recalled that the need on the part of many reserve managers to mobilise resources was limited by the central bank swaps. These financed provision of foreign currency funding to commercial banks. Central banks must consider coping with such calls on their foreign exchange reserves in the absence of such

[1] Monetary and Economic Department and Banking Department of the Bank for International Settlements (BIS), respectively. The authors thank Gavin Browning, Michael Davies, Christine Kamil, John Nugée, Pat McGuire and Elizabeth Wrigley for discussions and Swapan Pradhan-Kumar and Michela Scatigna for research assistance. All mistakes remain those of the authors. Views expressed are those of the authors and not necessarily those of the BIS.

swaps. Nevertheless, the crisis may in retrospect represent more a temporary reversal or pause in reserve management trends than any enduring reversal of them.

This paper starts by sketching the investment by maturity and instrument of official portfolios as of mid-2007, focusing *faute de mieux* on the identified instrument composition of the US dollar portfolio of official reserve holders. The next section describes the challenges and responses from mid-2007 to mid-2008, and the following section those since the Lehman failure. The focus is on the data as of end-June because the US authorities' annual surveys drill down through layers of custodians once a year to produce better-quality data than captured in monthly reports. (This means that the most recent data on the period since mid-2010 can support only tentative generalisations.[2]) The following section reports the valuation gains and losses experienced by official reserve portfolios in US securities, which suggest that both US Treasury and agency securities served as rainy-day portfolios. The penultimate section poses five questions on the lessons reserve managers appear to be taking regarding their investments. The final section concludes.

2. Maturity and instrument composition in mid-2007

Official reserve management by mid-2007 had come a long way from the choice between Treasury bills and bank deposits faced by an earlier generation (Table 1, upper panel).[3] Identified US dollar reserves[4] were 70% invested in securities of over one year original maturity. US Treasury securities still represented the largest single holding, but had fallen to less than half of the identified portfolio overall. US agency paper had risen to half the level of Treasuries. In the short-term portion of the portfolio, bank deposits had long surpassed Treasury bills. Treasury bills stood at only 15% of short-term holdings, and less than 5% of overall holdings. Official reserve managers, like most households, kept their cash in the bank. This was not your grandfather's or even your father's reserve management.

It does not appear that the official dollar liquidity portfolio had come to be invested to any significant extent in the "shadow banking system". This term refers to non-bank financial institutions that funded themselves with open market paper or reverse repos and held securitised assets (Pozsar, et al (2010)). For instance, money market funds bought asset-backed commercial paper funding securities investment vehicles (SIVs) that held asset-backed securities (ABS). There were reports of some official investment in US dollar money market funds domiciled outside the United States (and thus not captured in Table 1). But reported official holdings of commercial paper, including asset-backed commercial paper, remained very modest. Reported official holdings of longer-term corporate asset-backed securities (mostly private label mortgage-backed securities, but also credit card- and auto loan-backed securities) amounted to $44 billion in mid-2007. Some of this sum might have come from official liquidity portfolios.[5] But unmeasured money fund holdings, measured commercial paper holdings and some asset-backed bonds together, perhaps aggregating

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2 See Warnock (2010), Figure 3, page 6 on the hazards of drawing inferences from the monthly Treasury International Capital data.

3 See Fung and McCauley (2003) for a longer view. De Kock (2010, p 19) analyses dollar holdings in the US.

4 While the survey data drill down with custodians to identify ultimate beneficial owners, official holdings are probably still understated owing to layers of holdings (Bertaut et al (2006)). However, the largest lacuna is central bank holdings of US dollar bonds issued, and held in custody, outside the United States by top-rated sovereigns like Sweden and Austria and agencies like KfW, CADES and export agencies. These are not captured in the Treasury survey, and their inclusion would tend to lower the share of US Treasuries.

5 These are included in long-term holdings of corporate bonds. See Table 7 below.

into the tens of billions of dollars, left the official liquidity portfolio overwhelmingly placed with banks at the outset of the crisis.[6]

Box A

Securities lending programmes

In order to raise returns in their investment portfolios, official reserve managers, like other institutional investors, contracted with agents to lend out their securities. As was argued by one of the largest custodians, such securities lending can contribute to the liquidity and efficiency of securities markets (State Street (2001)). Indeed, with a rising fraction of Treasury securities held by official reserve managers, the Federal Reserve Bank of New York encouraged official lending of securities and participation in repurchase markets in which cash is exchanged against securities.

The predominant model in securities lending as it developed in the first decade of the century is an integrated approach in which the custodian serves as agent for the securities lending and invests the cash raised. A client empowers the agent to lend to specified counterparties up to a specified fraction of the designated portfolio and to take specified securities as collateral or to invest cash in specified instruments. The agent typically provides indemnities against the failure of any counterparty to return the security.

Securities can be lent out against securities or cash collateral. When they are lent out against cash, the cash can be invested in reverse repos. In this case, a security lent is ultimately secured by a different security borrowed, and the return arises from the difference in the scarcity or the creditworthiness of the two securities. Alternatively, the agent can invest the cash received against borrowed securities in money market investments. In this case, the question arose of the quality, liquidity and maturity of the money market investments. As State Street (2001, p 19), noted:

> When cash is pledged as collateral, the general practice is to re-invest it in short-term, money-market instruments, because securities lenders have to price, purchase, sell and settle on a daily basis and holding any illiquid instrument in a short-term fund would be excessively risky.

The major securities lenders were said to differ in terms of how bespoke or pooled their cash investments were. Some agents worked with the lender on the parameters for a segregated account for all but the smallest lending programmes. This has the disadvantage of not getting the liquidity benefits of pooling. Other agents gave clients a choice of commingled pools, ranging in riskiness from low (government securities) to medium ("prime funds" invested in commercial paper and bank certificates of deposit with maturity limits similar those permitted by the SEC for so-called 2a-7 money market funds) to "enhanced" funds (investing outside such credit and maturity parameters). Even in the latter case, larger clients or those in special tax positions might have their cash investments segregated.

6 Thus, it appears that official reserve managers did not fall into the trap that caught UBS. According to UBS (2008), this bank turned a $25–$30 billion portion of its "liquidity buffer or reserve" portfolio into a profit centre in 2002–03. Ironically, the investment in AAA- and AA-rated asset-backed securities, mostly US originated, followed the bank's internal (credit risk control) downgrading of Japanese government bonds in which much of the portfolio had been invested. It was argued (ibid, p 16) at the time that the asset-backed securities qualified for a liquidity buffer because they were highly rated, repo-able and could be pledged at a major central bank. Small trading spreads, dollar denomination and, of course, positive carry were seen as pluses. Risk control's review (p 32) faulted putting the maintenance of the liquidity buffer in a profit centre accompanied by "no decision to forego some level of profit to ensure that the Group's liquidity reserve was *fully capable of liquidation in any event and at any time*" [emphasis added]. Also cited was "considerable reliance" on ratings and concentration limits that did not flag that 95% of the underlying assets were US assets, as well as a lack of granular data available to risk control regarding vintage, loan-to-value ratios and mortgage borrower credit scores. In the event, this portfolio of asset-backed securities not only lost value but also suffered impaired market liquidity precisely when UBS's own funding liquidity came under stress.

Box A (cont)

Securities lending programmes

What might not be apparent at first blush is that, to lend securities against cash and to invest the cash, is to leverage the portfolio. The yield of high-quality securities can be enhanced by lending those in short supply against similar, readily available securities. More incremental yield is available from lending cash taken in exchange for securities. Then, more than 100% of the portfolio is invested.

It appears that many institutional investors, including central banks, did not exercise the same care in specifying how the cash raised by securities lending might be invested as they would in managing their "own" cash.[①] Interviews suggest at least three different reasons for this. First, securities lending often started as an initiative of operations groups to harvest the return available by lending securities in short supply ("specials"). This gave the whole enterprise a frame of "free money" arising from securities markets' technical factors. Second, the notion of a guarantee by the securities lender may have reassured management that risks were more contained than they really were. As noted, the agent in the securities lending programmes often undertook to indemnify the security lender in the event of the failure of the security borrowers. But any indemnity against the failure of a counterparty to return the security (at the front end of the deal) did not extend to any guarantee on the securities in which cash might be invested (the back end). Third, there was apparently an incremental deterioration of the securities in which cash was invested, much as there was a progressive decline in the quality of securities accepted in the repo market (Gorton (2010, p 43)).

As a result, qualified investments for cash might be broadly characterised and the specific investments not even regularly communicated by the agent. For instance, qualifying securities might be any carrying an AAA or AA long-term rating or an A1/P1 short-term rating, or eligible for repo at the ECB. Moreover, average maturity might have been specified, allowing longer-term investments.

Or consent might be given to invest the funds in one of three or four available pooled investments, which itself might change in character over time. One interviewee reported that, when returns rose in late 2007 and the agent was asked to produce a list of current investments, the response was unreassuringly slow.

Thus, when cash was raised against the securities and then invested, holdings of very liquid securities inadvertently became investments in what could and did become very illiquid securities in stressed markets. To the extent that high-quality government securities were lent out and credit risk accepted in the portfolio, returns arose from so-called maturity and liquidity mismatches. While senior management might have signed off on a programme to exploit temporary supply shortages, yield (and risk) could end up arising from leverage, credit exposure ("credit arbitrage") and maturity and liquidity mismatches.

In retrospect, it is easy to see the risks inherent in not vetting the investments of cash collateral as much as investments in the liquidity portfolio. Even at the time, though, the Basel-based Committee on the Global Financial System (CGFS (2005)) warned:

> "[…] Despite the "value added" by the rating agencies, market participants need to be aware of the limitations of ratings. This applies, in particular, to structured finance and the fact that, due to tranching and the effects of default correlation, the one-dimensional nature of credit ratings based on expected loss or probability of default is not an adequate metric to fully gauge the riskiness of these instruments... As the unexpected loss properties of structured finance products tend to differ significantly from those of traditional credit portfolios or individual credit exposures, structured finance tranches can be significantly riskier than portfolios with identical weighted average ratings".

Such investments of cash collateral received in securities lending programmes appear not to have been well captured in the US Treasury/Federal Reserve or BIS data compiled in Table 1. As a result, it is very hard to put a number on the scale of official investment in cash collateral investments, whether bespoke or pooled.

Box A (cont)

Securities lending programmes

The Central Bank of Norway, which sets a standard for disclosure by official portfolios, provides a point of reference on magnitude. Of the total net assets of NOK 2.019 trillion at end-2007 (the first date for which data on security lending were made available), the Government Pension Fund had lent out NOK 516 billion (26%). Most of this was lent against cash (NOK 298 billion) and the rest directly against securities. Much of the cash, however, was lent out through reverse repo (NOK 201 billion), and so to this extent securities were lent indirectly against securities. In the end, only NOK 93 billion was held in fixed income instruments. Thus, cash had been raised against 15% of the overall portfolio, and cash collateral investments represented 5% of the portfolio.

If these proportions were representative of official reserve managers, then cash collateral investments by central banks would have amounted to $200–300 billion in June 2007. Of course, the Norwegian fund is a very unusual official investor, both in the securities it holds and in its risk appetite, and there is reason to believe that it is not representative of official investors. As a result, it could have been doing more than its share of securities lending (especially since the central bank carries the country's liquidity portfolio). However, its heavy reliance on securities collateral and cash investment in reverse repo might mean that other central banks ended up investing more of the proceeds of their securities lending in risky cash investments.

Market sources suggest that, after a sharp cut-back during the crisis, a number of central banks have returned to lending their securities.7 Central banks had reportedly lent securities to the extent of about $340 billion in August 2008. These were cut back in the following month by about a third, and amounts lent bottomed out at about $150 billion in the first half of 2009. Reportedly, amounts lent out recovered to as much as $200 billion by mid-2011.

Central banks that have returned to securities lending have tended to change their approach in the light of the crisis experience. Investments of cash are the concern not only of the back-office that deals with the custodians but also the front office that allocates funds to investment. Thus, such cash is invested in instruments meeting the usual credit, maturity and liquidity standards under limits set by risk control. Cash investments are more likely to be in bespoke rather than pooled investments. And reporting is more systematic than before the crisis.

[①] This case should not be confused with that of AIG, which raised cash with the securities of its insurance affiliates and itself invested in residential mortgage-backed securities, some of which ended up in the Federal Reserve Bank of New York portfolio, Maiden Lane II. See Kohn (2009).

If central banks avoided the trap of investing their own liquidity portfolios in untested securities and commercial paper backed by such securities, many faced similar challenges in outsourced liquidity management associated with securities lending programmes. While the short-term portfolio just reviewed reflected explicit choices by official reserve managers to invest their own portfolios, another set of money market positions arose more incidentally. And these more incidental cash holdings were apparently often left to agents to invest with a considerable reliance on ratings or other rules of thumb (see Box A). As a result, official reserve managers entered the shadow banking system as it were by the back door, with high quality securities that produced cash that was in some cases invested in surprisingly low quality securities.

[7] The underlying source for such market estimates are reports from custodians aggregated by Data Explorers in London. However, some central banks are said to be reluctant to have their securities lending identified as originating in central banks even in the absence of identification by name, so there may be a downward bias in such estimates.

Table 1

Instrument composition of official holdings of US dollars

In billion of US dollars

End-June 2007	**Short-term**	**Long-term**	**Total**
Treasury securities	159	1,452	1,611
Other assets	941	1,115	2,056
Repos and deposits in the United States	237		
Commercial paper and certificates of deposit in the United States	27		
Offshore deposits	597		
Agency securities	80	750	830
(ABS)		(236)	
(Other)		(515)	
Corporate bonds		99	
Equities		266	
Total	1,100 (30%)	2,567 (70%)	3,667 (100%)
Memo: Share of Treasury securities in identified assets of the given maturity	*15%*	*57%*	*43%*
Total IMF-reported US dollar reserves			*1,999*
End-June 2008	**Short-term**	**Long-term**	**Total**
Treasury securities	226	1,684	1,910
Other assets	871	1,435	2,306
Repos and deposits in the United States	199		
Commercial paper and certificates of deposit in the United States	23		
Offshore deposits	519		
Agency securities	130	967	1097
(ABS)		(435)	
(Other)		(532)	
Corporate bonds		105	
Equities		363	
Total	1,097 (28%)	3,119 (72%)	4,216 (100%)
Memo: Share of Treasury securities in identified assets of the given maturity	*21%*	*54%*	*45%*
Total IMF-reported *US dollar reserves*			*2,782*

Table 1 (cont)

Instrument composition of official holdings of US dollars

In billion of US dollars

End-June 2009	**Short-term**	**Long-term**	**Total**
Treasury securities	575	2,117	2,692
Other assets	573	1,212	1,785
Repos in the United States	102		
Deposits, brokerage balances and others in US	64		
Commercial paper and certificates of deposit in the United States	43		
Offshore deposits	330		
Agency securities	34	794	828
(ABS)		475	
(Other)		320	
Corporate bonds		107	
Equities		311	
Total	1,148 (25.6%)	3,329 (74.4%)	4,477 (100%)
Memo: Share of Treasury securities in identified assets of the given maturity	*50.1%*	*63.6%*	*60.1%*
Total IMF-reported US dollar reserves			*2,682*
End-June 2010	**Short-term**	**Long-term**	**Total**
Treasury securities	454	2,592	3,046
Other assets	457	1,223	1,680
Repos in the United States	90		
Deposits, brokerage balances and others in US	55		
Commercial paper and certificates of deposit in the United States	30		
Offshore deposits	255		
Agency securities	27	714	741
(ABS)		443	
(Other)		271	

Table 1 (cont)

Instrument composition of official holdings of US dollars

In billion of US dollars

End-June 20010	Short-term	Long-term	Total
Corporate bonds		83	
Equities		426	
Total	911 (19%)	3,815 (81%)	4,726 (100%)
Memo: Share of Treasury securities in identified assets of the given maturity	*50%*	*68%*	
Total IMF-reported US dollar reserves			*2,995*

Figures for US Treasury, agency and corporate bonds and equities are from US Treasury, Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, *Report on foreign portfolio holdings of U.S. securities as of June 30 2007* (April 2008), *June 30, 2008* (April 2009) and *June 30, 2009* (April 2010) and US Treasury, *Preliminary report on foreign holdings of U.S. securities at end-June 2010* (28 February 2011). Figures for deposits and money market paper in the United States are from BEA, International Transactions Table 5 (or the US *Treasury Bulletin*, Tables CM-I-1 and IFS-2). Figures for offshore US dollar deposits are estimated from the BIS *Quarterly Review*, Table 5C, and the Japanese SDDS. The US Treasury definition of official institutions, including "national government-sponsored investment funds", may be broader than those of the BIS and IMF. Long-term is defined by original maturity. IMF data from COFER.

In managing the predominant longer-term portfolio, reserve managers had reached for yield by substituting agency bonds for Treasuries. The agencies had accommodated central banks' demand for more tractable instruments with bullets and callable instruments ("other" in Table 1). Holdings of corporate bonds and equities were modest and concentrated.

3. Evolution of holdings from mid-2007 to mid-2008

Five developments stand out in this period, three concerning the short-term portfolio and the other two concerning the longer-term investment portfolio. The most notable development is that official reserve managers shifted cash away from unsecured deposits with commercial banks towards Treasury and agency bills.

3.1 Developments in the short-term portfolio

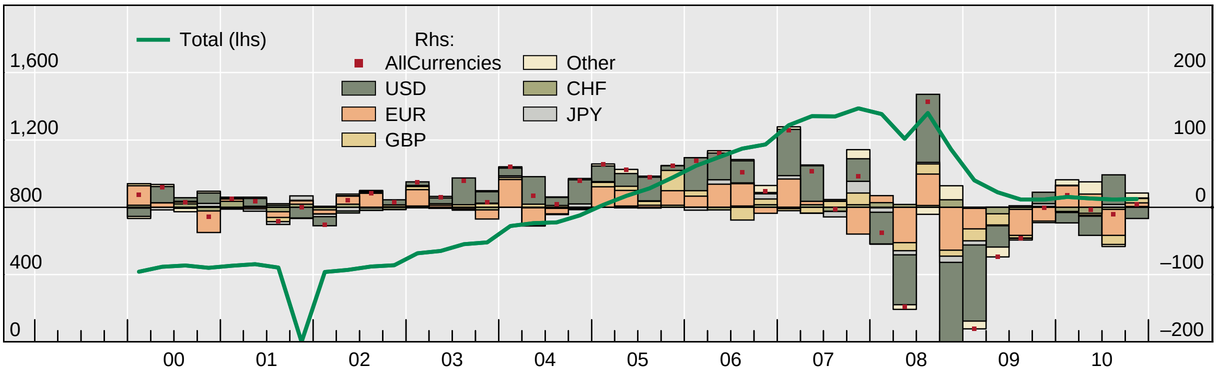
The three shifts in the short-term portfolio in this period all responded to the felt need to reduce the risk profile. Official reserve managers placed maturing bank deposits in sovereign paper, invested with central banks and cut back on often unappreciated risks in their security lending programmes.

Data reported by banks, aggregated nationally and collated by the BIS, show that reserve managers reacted in steps to the succession of bad news regarding bank losses. Overall official deposits in banks peaked in the last quarter of 2007 (Graph 1). Officials continued to reduce their deposits with banks well into 2009.

Graph 1

Bank liabilities to official monetary authorities by currency[1]

In billions of US dollars



[1] Liabilities booked by BIS reporting banks vis-à-vis official monetary authorities; expressed at constant Q4 2010 exchange rates. Green line relates to total amount outstanding; bars and dots relate to exchange rate and break adjusted changes in amount outstanding.

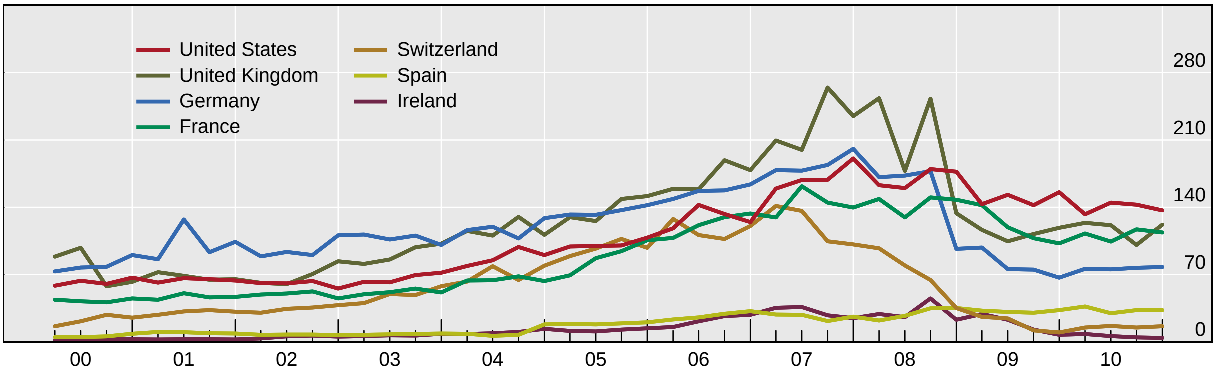
Sources: BIS locational banking by residence statistics; BIS calculations.

Thus, unlike US money market mutual funds, the largest non-bank providers of dollars to banks, official reserve managers ran down their deposits over quarters rather than weeks.[8] Reserve managers also drew distinctions as they backed away from banks. Already in the third quarter of 2007, official deposits in Swiss banks (in all currencies) started to decline and then they took a big step down in the fourth quarter (Graph 2). This backpedalling from bank risk spread in the following quarters to French, then German and US banks and finally to UK banks.

Graph 2

Liabilities to official monetary authorities by bank nationality[1]

Amounts outstanding, in billions of US dollars



[1] Liabilities booked by BIS reporting banks vis-à-vis official monetary authorities; expressed at constant end-Q4 2010 exchange rates.

Sources: BIS locational banking statistics by nationality; BIS calculations.

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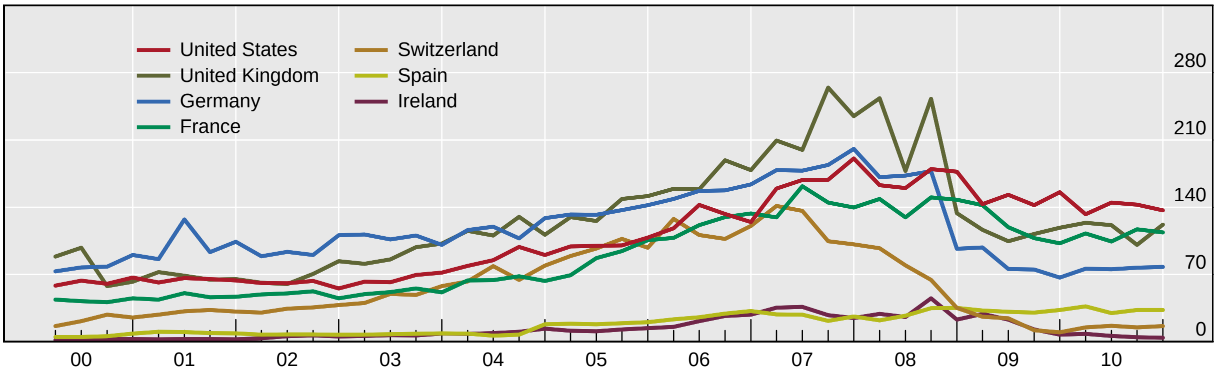
[8] US dollar money market funds kept ramping up the supply of funds to mostly European banks until the Lehman Brothers default, when they responded to a run by shareholders by pulling hundreds of billions of dollars out of banks in a matter of weeks. See Baba et al (2009) and Graph 5, below.

Focusing exclusively on US dollar deposits, the picture looks much the same, although the deposits in US banks now emerge as more clearly favoured (Graph 3). On balance, US and Spanish banks gained market share in official dollar deposits during the financial crisis.

Graph 3

US dollar-denominated liabilities outstanding to official monetary authorities

By bank nationality; amounts outstanding; in billions of US dollars



Source: BIS locational banking by nationality statistics.

As central banks cut their holdings of bank deposits, they returned to the safety of sovereign and semi-sovereign paper. Among dollar holdings of less than one year original maturity, the share of Treasury bills rose in the 12 months to June 2008 after years of decline. In particular, the Treasury bill share of measured short-term dollar holdings had fallen from about 40% in 1989 to about 15% in mid-2007, only to rise to 21% in mid-2008 (Table 1, middle panel). Reserve managers also took refuge in the presumed safety of agency bills.

The second development in official cash management was increased placements with official sector institutions (Graph 4). This sector includes the BIS and central banks. In Box B we discuss how the BIS did not increase its acceptance of funds – on the contrary, it reduced them. Some central banks of top-rated countries decided to step in and to play the credit intermediation role that private financial institutions were no longer able to fulfil. The central banks that received these flight-to-quality funds from official investors placed the proceeds to some extent in sovereign obligations and to some extent in banks, albeit reportedly on a secured basis.

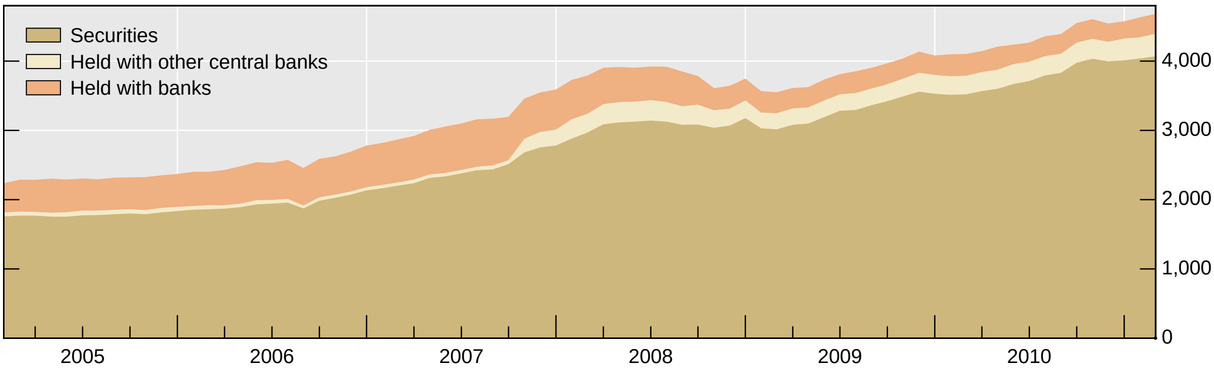
A third development in cash management was the early reduction by some official reserve managers of lending of their securities. For some, this reduction intended to make reserve holdings more readily mobilised, as they anticipated market strains. For others it was signs of strain in securities lending programmes themselves that led to their scaling back lending. Such programmes had often been marketed to operations departments as “free money”, producing an incremental yield by lending out securities against cash and investing the proceeds in pools of purportedly safe securities. With credit risk controls on invested cash often not as exacting as those applied to the own portfolio, official investors had inadvertently entered into the shadow banking system.

Setting a standard for transparency, Norway disclosed problems at end-2007. About half of the NOK 93 billion in cash collateral investments at that time was invested in asset-backed securities and an eighth in structured investment vehicles (NBIM (2008a, p 64). Write-downs of NOK 3 billion exceeded interest earnings of half that amount.

Graph 4

Foreign exchange reserves by broad instrument[1]

In billions of US dollars



[1] Holdings of foreign exchange reserves by 63 monetary authorities that report SDDS data to the IMF.

Source: IMF SDDS.

Unanticipated results in the cash collateral investment led some central banks to tighten their collateral and investment criteria and to re-examine their participation in securities lending programmes even before the Bear Stearns collapse. Part of the securities lending programme at the Norwegian Government Pension Fund was suspended in late 2007 (NBIM (2009, p 19)) and lending against cash peaked in the first quarter of 2008 (Box A, Table A-1). The Central Bank of Colombia exited its securities lending programme in March of 2008 (Banco de la República (2009, p 122)). A central bank could more easily exit from its securities lending programme at this stage than after the Lehman failure.

Box B

Developments at the BIS

The BIS deposit base had been increasing steadily over the two years ending March 2007 and March 2008 by some 15%, in line with the general increase in worldwide foreign exchange reserves. In the year to March 2009, confronted with a significant deterioration of financial markets, the BIS took actions to improve its balance sheet resilience, thereby fulfilling its fiduciary responsibilities towards its central bank shareholders and customers. Like most central bank reserve managers, but to a lesser extent owing to its credit intermediation function, the BIS reduced uncollateralised exposures to commercial banks and increased its collateralised exposures to them as well as its investments in sovereign and quasi-sovereign assets. The BIS also reduced the size of its liabilities by some 20% in the year to March 2009 through price actions and after the Lehman failure by quantitative measures as central banks focused primarily on the safety of their reserve investments ("flight to quality"). This cautious policy, coupled with a significant reduction of the main market risk drivers on its balance sheet (in particular a reduction in the duration of its investment portfolio) protected BIS credit quality and profitability to the benefit of the central bank community. Since then the BIS has continued to link the evolution of its investments and its liabilities to its capital adequacy with the prime objective of maintaining a very high credit quality.

3.2 Developments in the long-term portfolio

In the longer-term portfolio, the first salient development was the shift within agency holdings from debentures to mortgage-backed securities. In the year to mid-2008, official reserve managers did not reduce their holdings of debentures. But agency mortgage-backed securities received practically all the incremental investment. Recall that questions about the financial strength of the agencies built over the year to mid-2008. In July testimony before the Senate Banking Committee, US Treasury Secretary Paulson asked the Congress for a "bazooka", ie such heavy-duty authority to take over Fannie Mae and Freddie Mac that it would be unlikely to be needed. Official investors loaded up on two-name paper, ie mortgage-backed securities, suggesting an uncertain assessment of the agencies as going concerns.

The second noteworthy development in the long-term portfolio was continued flows into straight corporate bonds and even equities. This seems to be small in Table 1 until it is remembered that corporate bond spreads had widened and equity markets had sold off, imposing capital losses on holders. Among corporate bonds there was the opposite shift from that in holdings of agencies, that is, from asset-backed paper to straight bond holdings.[9] Foreign official holdings of US equities rose even in the face of price declines after the October 2007 peak in equity prices. China may have contributed to this growth,[10] but the Norwegian Government Pension Fund also adhered to the discipline of its target allocation and "rebalanced" by selling bonds and buying (worse performing) equities. (Many other official wealth managers suspended rebalancing in the face of losses.)

In conclusion, this period featured, rather than a flight, a walk to quality by reserve managers. Bank deposits looked less sure; Treasury and agency bills looked better. Some reserve managers reacted to previously unrecognised risks in securities lending, which may have been approved as a low-risk incremental return generator. In the long-term securities holdings, diversification by official reserve managers into agency mortgage-backed securities, straight corporate bonds and equities continued through mid-2008.

4. After the Lehman failure

Lehman's failure shocked official reserve management in manifold ways. It should be recalled that it followed, by a long week, the conservatorship of the major US agencies. This event may have relieved official investors in those agencies, but to the public it put a cloud over an entire asset class into which official reserve managers had diversified over the previous 10 years.

Again, our discussion first takes up liquidity management, starting with the flight to quality in the investment of liquidity portfolios and then the "de-risking" of securities lending programmes. Then, we turn to developments in the investment portfolio. Noteworthy developments here include the dumping of agency debentures (straight and callable) but no evidence of disinvestment from corporate bonds or equities.

4.1 Developments in the short-term portfolio

After Lehman's default, official reserve managers flew to sovereign quality in an unprecedented fashion. Nothing like late 2008 can be seen in the wake of Herstatt in 1974,

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9 Among corporate bonds, official monetary institutions are reported to have modestly reduced their private label asset-backed securities (from $44 billion in June 2007 to $40 billion in June 2008, including valuation losses). Reported holdings of other corporate bonds continued to increase in this stressed period, however, from $55 billion to $65 billion. The entry on Table 1 for corporate bonds represents the sum of asset-backed and other. See Table 3 below for loss estimates for corporate bonds and equities, and Table 7 for the breakdown of corporate bonds.

10 Setser and Pandey (2009). The Treasury/Federal Reserve survey data suggest that measured "equity" holdings are only to a minor extent shares in mutual funds or hedge funds.

Mexico in 1982, Continental Illinois in 1984, the stock market crash of 1987, Drexel in 1990, LTCM and Russia in 1998 or WorldCom in 2001.

The official walk to quality picked up its pace and official holdings of Treasury bills rose over the summer of 2008. Then, official holdings of US Treasury bills more than doubled from the end of August 2008 to the end of the year to over $500 billion.[11]

In retrospect, it was fortuitous that the supply of Treasury bills jumped in late 2008. The US Treasury accommodated the Federal Reserve's decision to sterilise dollars swapped with European central banks. The Treasury "overfunded" its cash flow needs by selling extra Treasury bills and depositing the proceeds in the Federal Reserve Bank of New York. As a result, central banks, as well as money market funds, found more bills in the market into which to flee than otherwise would have been available.

Reserve managers bought Treasury bills as they allowed riskier placements – agency paper and bank deposits – to mature without rolling them over. Whereas in the year to mid-2008 official reserve managers had found safety in agency bills, in the year to mid-2009 they let three quarters of their holdings mature without rolling them over.

With regard to bank deposits, the failure of Lehman led a large US dollar money market fund holding Lehman paper to announce that it could not redeem shares at $1.00 (ie, it had "broken the buck"). This led to a run on these funds (Baba et al (2009)), including by Asian central banks that reportedly withdrew funds from US dollar money market funds domiciled outside the United States in Dublin or Luxembourg. This run left European and other non-US banks scrambling for dollars and left other depositors in these banks hesitant to roll over maturating deposits.

Thus, it may be presumed that, among such depositors, official reserve managers reduced their bank deposits in the immediate aftermath of the Lehman Brothers bankruptcy. This is consistent with the aggregate of available disclosures by the authorities in Graph 4. But the BIS data on bank liabilities to officials rose in the fourth quarter of 2008 as the positive result of central bank liquidity provision overwhelmed withdrawals by reserve managers.[12] As a result, it is difficult to pin down the scale of the rundown of official deposits in banks in this crucial period. It does seem safe to say that the withdrawal of dollars by officials was not as acute as the withdrawal by the largest source of dollars for non-US banks, namely US dollar money market funds.

Stepping back to take a longer view, the facts are clearer (Table 1). While official holdings of Treasury bills rose by $350 billion in the year to June 2009, offshore dollar deposits and agency bills ran off in the amounts of about $200 billion and $100 billion, respectively.

Given that official deposits had long favoured non-US banks for their dollar deposits,[13] it is not surprising that deposits in mostly non-US banks placed outside the United States suffered the greatest decline (Table 2). These fell by more than half between mid-2007 and mid-2010, and most sharply in 2008-09. Even though the Federal Reserve swapped dollars freely with partner central banks to permit them to provide dollar funding to non-US banks, the market share gain of US banks in official deposits may reflect the perception that US banks are closer to the source of dollar liquidity.

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[11] Official depositors initially responded positively to the Irish blanket guarantee of deposits, stepping up their euro placements (Graph 2), only to reconsider as the underwriting of the relatively large bank liabilities visibly strained state finances.

[12] See discussion in Gadanecz et al (2009, p 20).

[13] See McCauley (2005), He and McCauley (2010).

Table 2

Official dollar deposits by location and nationality of banks

Amounts outstanding, in billions of US dollars

Nationality of banks	June 2007 Location of reporting banks			June 2008 Location of reporting banks			June 2009 Location of reporting banks			June 2010 Location of reporting banks		
	US	**Off-shore**[1]	**Total**	**US**	**Off-shore**[1]	**Total**	**US**	**Off-shore**[1]	**Total**	**US**	**Off-shore**[1]	**Total**
US	128.9	21.4	150.3	116.0	27.5	143.5	117.7	24.5	142.2	107.8	24.8	132.6
Others	96.9	462.1	558.9	75.5	404.7	480.1	47.1	258.8	306.0	37.0	209.9	246.9
Total	225.8	483.5	709.2	191.5	432.2	623.7	164.8	283.3	448.1	144.8	234.7	379.5

[1] Banks located in reporting countries other than the US.

Source: BIS locational by nationality statistics.

Elsewhere in cash management, Lehman's failure highlighted vulnerabilities at various points in the securities lending process. At the front-end repurchase transactions, in which securities are exchanged for cash, concerns heightened regarding counterparty credit and expectations of "fails", ie securities not being returned on time. In many securities lending contracts, however, the custodian had undertaken to indemnify the owner of the securities in the event that the security was not returned. However, the perception of risk also rose at the back end, where the cash collateral received in exchange for the security was invested. The news that a large US money market fund had "broken the buck" and the distress of AIG, which had actively invested cash raised against its insurance subsidiaries' securities, led to questions about the value and liquidity of other, less regulated pools of US dollar liquidity. These were known not only to have invested in regulated money market funds but also to have, in some cases, riskier investment profiles in terms of credit and maturity.

Under these circumstances, official reserve managers faced the following alternatives. They could exit from the securities programme. Alternatively, they could remain in the programme, possibly reducing its size, but invest any cash received as collateral only in reverse repos. Alternatively, they could stay in the programme, and tighten the range of permitted investments. Or they could continue with previous practice.

It is not possible to say with any precision how official reserve managers reacted. One well placed market participant suggested exits from securities lending programmes by half of his official clients. Out of 19 responses in late 2008 to a trade press survey, five reported exiting their securities lending programmes and another four to shifting the investment of cash collateral to reverse repo ("Survey answers" (2009)).[14]

Market participants report that some official reserve managers felt that they had a duty to maintain market liquidity by continuing to lend. Reportedly, this sense of duty was encouraged by the Federal Reserve Bank of New York and, in Europe, by the ECB.[15]

[14] Whether the exit rate by official reserve managers was closer to a quarter or a half, their propensity to exit was apparently higher than that of institutional investors in general. In a State Street fourth quarter 2008 earnings call, an executive put the rate of exit at 10% (45 clients; State Street (2009)). See also Carver and Pringle (2009).

[15] That said, the securities lending programme of the Federal Reserve (Fleming et al (2009)) can be seen in some respects as a substitute for securities lending by other institutional investors, including central banks.

Exits from securities lending would normally mean that the cash had to be returned, and this in turn would normally mean that the money market investments in which the cash had been placed had to be liquidated. The Colombian central bank (Banco de la República (2009, p 126)) reported a loss in March 2009 from its sale of an interest in a structured investment vehicle (Sigma Finance) in which funds raised from securities lending had been invested.

Some central banks found themselves stuck in investments of cash raised by securities lending. The Bank of New York, Northern Trust and State Street were reported in October 2008 to be restricting withdrawals from their riskier cash investment pools (in a process known as "gating"). Central banks were persuaded to leave their funds in cash investment pools on the premise that these would return to par value. In some cases, permitted redemptions were reported to have been transacted at a $1.00 net asset value, despite lower market values; in other cases, in kind redemptions were reported.[16] The upshot was that many central banks had to attend to thorny issues arising from transactions that had been entered into on the presumption of low risk and certain liquidity.

Central banks' securities lending bottomed out in 2009 at less than half of amounts reached before the Lehman failure, and have since recovered half of the decline. The risks arising from securities lending programmes are reportedly better understood and managed than before the crisis (see Box A).

4.2 Developments in the long-term portfolio

In the long-term portfolio, official reserve managers sold agency debentures and bought US Treasury notes. More than one governor at the meeting in Basel in September 2008 publicly welcomed the US government's taking the two agencies into conservatorship. Nevertheless, official reserve managers sold $200 billion (almost 40%) of agency debentures in the year to June 2009, and another $50 billion in the year to June 2010. Meanwhile, they added over $400 billion in US Treasury notes to their portfolio in each of the years to June 2009 and June 2010 (only in small part owing to valuation gains).

The Central Bank of the Russian Federation stood out among those officials selling agency paper. At the end of 2007 it held 37% of its R6.6 trillion portfolio of foreign securities in securities of Fannie Mae, Freddie Mac and the Federal Home Loan Banks, that is, a total of $100.8 billion. With the deterioration of the US agencies and considerable public discussion in Russia of the attendant risks, the central bank had reportedly sold 40% of its holdings by the time of the conservatorship of Fannie and Freddie in early September 2008 (Fabrichnaya and Bryanski (2008)). By the end of 2008, the holding had been slashed to 0.89% of the securities portfolio, and by the end of January to zero (Central Bank of the Russian Federation (2009, pp 75, 140)). Ironically, a successful bet on US taxpayer support for the agencies could not stand up against the Russian popular response. Official statements that the holdings of agency paper had produced good returns (see next section) did not avail and the officials liquidated the position.

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[16] See State Street 8-K 18 May 2009. Had these pools been subject to SEC rules, the $1.00 net asset value would have been abandoned (the "buck" broken). On 7 July 2010, State Street announced a $330 million charge in order to inject funds to return some of the security lending investment pools to a net asset value of $1.00 so that the gating could be lifted. Other pools received different treatment: "In December 2010, in order to increase participants' control over the degree of their participation in the lending program, we divided certain direct lending collateral pools into liquidity pools, from which clients can obtain cash redemptions, and duration pools, which are restricted and operate as liquidating accounts. Depending upon the direct lending collateral pool, the percentage of the collateral pool's assets that were represented by interests in the liquidity pool varied as of such division date from 58% to 84%" (State Street 8-K, 2 February 2011). See Annex Table A2 for information on the range of shortfall from the $1.00 net asset value of certain non-SEC registered cash collateral pools.

Again, there was no evidence of official reserve managers' turning away from other alternatives to US Treasury securities. Foreign official holdings of corporate bonds and equities held up, and their holdings of agency mortgage-backed securities even rose over the year to June 2009.

5. Returns on US securities

We have seen how reserve managers responded to the crisis. As a prelude to the next section's questions for the future, a review of the recent returns of official reserve managers on their US securities portfolios is useful. The conclusion is that US Treasury notes played their role well as a rainy day portfolio. While official holdings of corporate bonds and especially of equities produced losses from mid-2007 to mid-2009, their Treasury and agency holdings produced gains (Table 3).

Table 3

Capital gains/losses on official holdings of US securities

In billions of dollars

Year ending:	**Treasury**	**Agency**	**Corporate bonds**	**Equities**
June 2002	52			
June 2003	36 (6.4%)			
June 2004	–43 (–6.6%)			
June 2005	24 (2.6%)			
June 2006	–67 (–6.2%)			
June 2007	6 (0.5%)	2 (0.4%)	–1 (0.6%)	41 (18.8%)
June 2008	76 (5.2%)	20 (2.7%)	–4 (–3.6%)	–38 (–14.2%)
June 2009	40 (2.4%)	41 (4.2%)	–3 (–2.8%)	–103 (–28.3%)
June 2010	73 (3.6%)	23 (3.1%)	3 (2.7%)	37 (11.8%)

Valuation adjustment reported for official holdings of US Treasuries, but only for all holdings of agencies, corporate bonds and equities. Percentage adjustments for all holdings are applied to holdings of officials of agencies, corporate bonds and equities.

Sources: US Treasury et al (2008–11), authors' estimates.

These gains and losses, however, do not capture the reputational challenge that some official reserve managers faced in holding agency securities as the agencies racked up losses (borne by common and preferred shareholders), eventually entered an obscure process of conservatorship and required equity transfusions from the US Treasury. It is not easy to explain why the press harped on the exposure of foreign exchange reserves to agency securities in some countries and reported little on it in others.

6. Questions for the future

The question is whether the global financial crisis will be seen in retrospect as a watershed in reserve management, breaking trends, or whether it will look more like a pause. Pihlman and van der Hoorn (2010) argue that the lessons learned should result in a concentration of portfolios in the most creditworthy government bonds rather than in credit portfolios. Others wonder whether the concept of the credit risk-free government bond has survived the fiscal consequences of the crisis. Certainly, research into sovereign risk, for instance Hohensee and Prasad (2010) and Brennan et al (2010), does not take it as given that the best-rated sovereigns pose no credit risk. We take up five questions.

6.1 More emphasis on worst-case liquidity and bigger liquidity tranches?

We have seen that, while there was a flight to quality in the official short-term portfolio, official reserve managers did not in aggregate actually increase the proportion of short-term assets since mid-2007. To the contrary, the proportion of short-term holdings fell from 30% in mid-2007 to only 19% in mid-2010 (Table 4).

Table 4

Foreign official holdings of US dollars by instrument and maturity[1]

In billions of US dollars

	June 2007		June 2008		June 2009		June 2010	
	Short-term	**Long-term**	**Short-term**	**Long-term**	**Short-term**	**Long-term**	**Short-term**	**Long-term**
Treasuries	159	1,452	226	1,684	575	2,117	454	2,592
Agencies	80	750	130	967	34	794	27	714
Other	861	365	741	468	539	418	430	509
Total by maturity	1,100	2,567	1,097	3,119	1,148	3,329	911	3,815
Memo: maturity shares	*30%*	*70%*	*28%*	*72%*	*26%*	*74%*	*19%*	*81%*
Total official holdings	*3,667*		*4,216*		*4,477*		*4,726*	

[1] By original maturity.

Source: Table 1.

During the crisis period, a shortening of the portfolio to some extent reflected the drawing down of reserves, less in Europe or Japan than in emerging market economies like Brazil, India, Indonesia, Korea, Malaysia, the Philippines, Russia and Singapore. In many cases, the first reserves to be mobilised were the forward purchases of dollars, but then presumably the short-term portfolio was drawn upon. Foreign exchange reserves had in many cases returned to pre-crisis levels by mid-2010 (and in some cases well exceeded those levels), so the effect of the draw-down of reserves on the maturity composition should have passed.

One factor that may limit the shortening of the official dollar holdings in aggregate is the increasing concentration of holdings. Even if many reserve managers take the lesson that they need to hold bigger liquidity tranches, the largest reserve holders may not take this lesson and the aggregates may not show much portfolio shortening. Global reserves are increasingly concentrated (Table 5).

Indeed, the portfolios of the largest reserve holders seem to be invested only to a very limited extent in short-term instruments. Taking US Treasury securities held by China, including both the official sector per se and banks and institutional investors, holdings of US Treasury coupon securities were reported at $1.1 trillion in mid-2010, whereas holdings of bills amounted to only $4 billion. Less extreme was the position of the public and private sectors in Japan, with $737 billion of coupons and $63 billion of bills. However, since the Japanese foreign exchange reserves had only grown from March 2004 through mid-2010 with investment returns, it is the growing Chinese reserves and their evidently medium-term investment that account for the aggregate trend toward longer-term instruments in identified official holdings of US dollars. Indeed, it is striking that reserve managers as a whole are underweight US Treasury bills when measured against the outstanding portfolio (Graph 5).

Table 5

Concentration in the holdings of global foreign exchange reserves

In per cent

	Largest holder	Top 3 holders	Top 5 holders	Top 10 holders
1980	12.0	24.6	36.0	52.8
1985	10.2	22.5	32.9	50.0
1990	8.3	23.5	36.1	53.6
1995	12.4	24.5	34.7	50.2
2000	17.9	32.0	42.5	54.7
2005	19.2	44.0	52.9	66.7
2010 latest	29.7	46.6	55.7	70.3

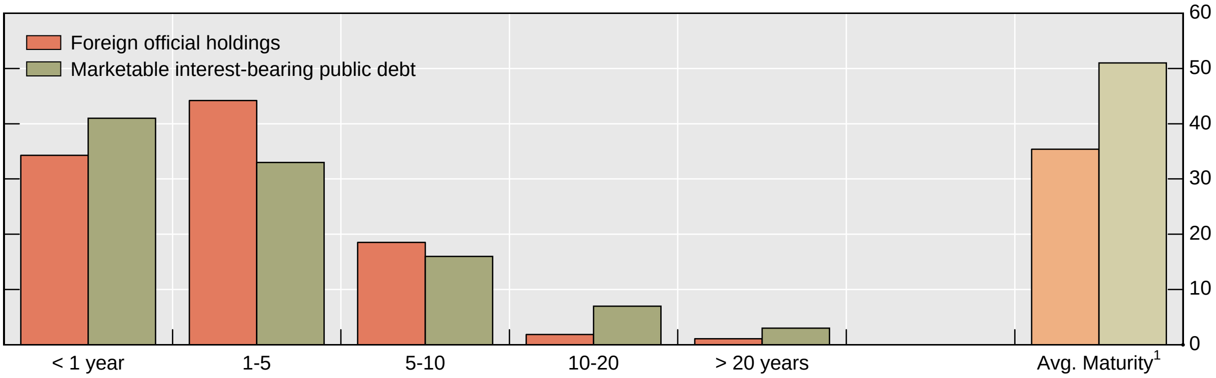
Sources: IMF; Central Bank of China.

Overall, it is not obvious that liquidity tranches have been increased. That said, there may be persistent changes to securities lending, including moving it in-house or lending out the cash more cautiously.

Graph 5

Maturity of foreign official holdings of Treasury securities and total outstanding, June 2009

In per cent



[1] Number of months.

Sources: Department of the Treasury, Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, *Survey of foreign portfolio holdings as of June 30, 2009*, April, 2010; US Treasury.

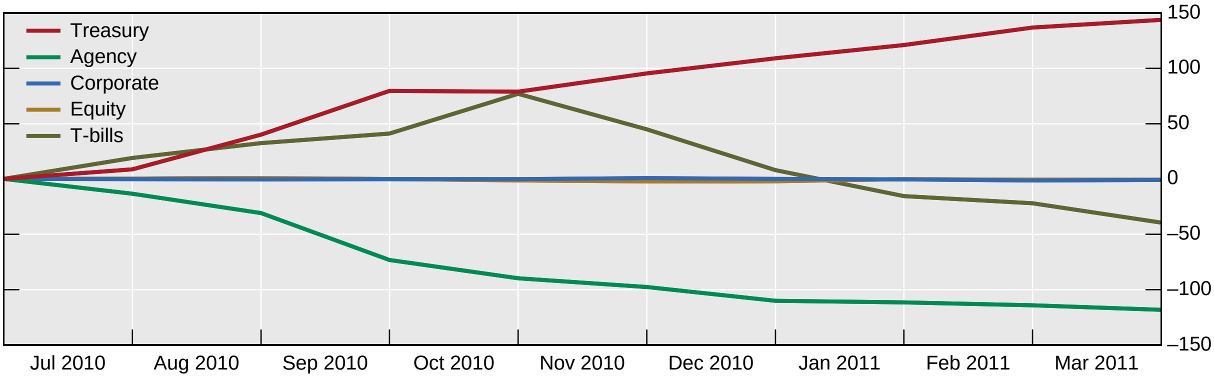
6.2 Will official investors restore the previous weight on bank deposits?

Going into the crisis, bank deposits bulked large in identified dollar holdings at over 20%, making up most of the short-term portfolio. These have now fallen to below 10% in substantially larger overall reserve holdings. These observations raise the question of whether official investors will return to holding more bank deposits.

Graph 6

Cumulative purchase of US securities by foreign officials

Since end-June 2010, in billions of US dollars



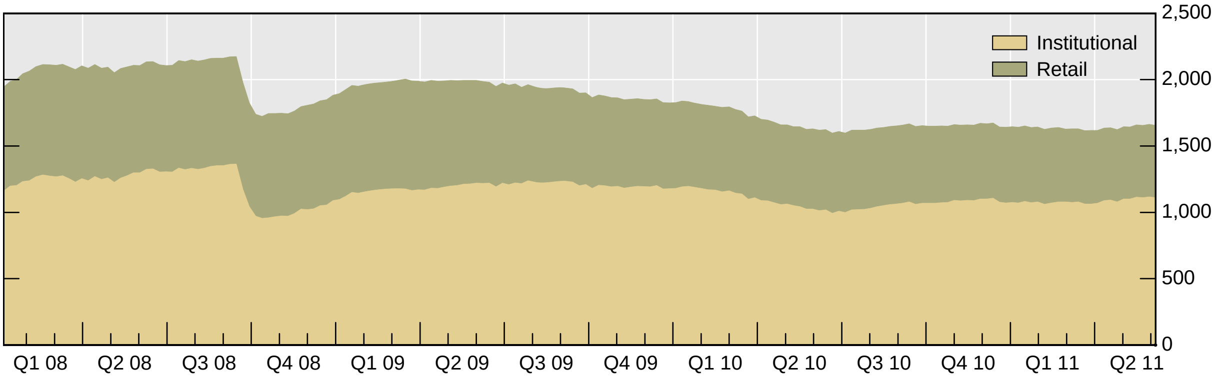
Source: Treasury International Capital.

Recent developments in the short-term portfolio do give a clear positive answer. While official reserve managers have held their deposits with central banks fairly steady (Graphs 1 and 4), they reduced their holdings of Treasury bills by over $100 billion in the year to June 2010 (Table 1) and on balance have not increased holdings since then (Graph 6). However, bank deposits have not benefited much from the reduced allocation to Treasury bills. Returning to Graph 1, the available 2010 data show hesitant signs of a recovery of official deposits with banks. Officials increased their euro deposits in banks in the first quarter of 2010 only to draw them down in the third quarter. After reducing US dollar deposits in the first two quarters of 2010, official investors increased their dollar deposits in the third quarter.

Graph 7

Total net assets of non-government money market mutual funds

In billions of US dollars



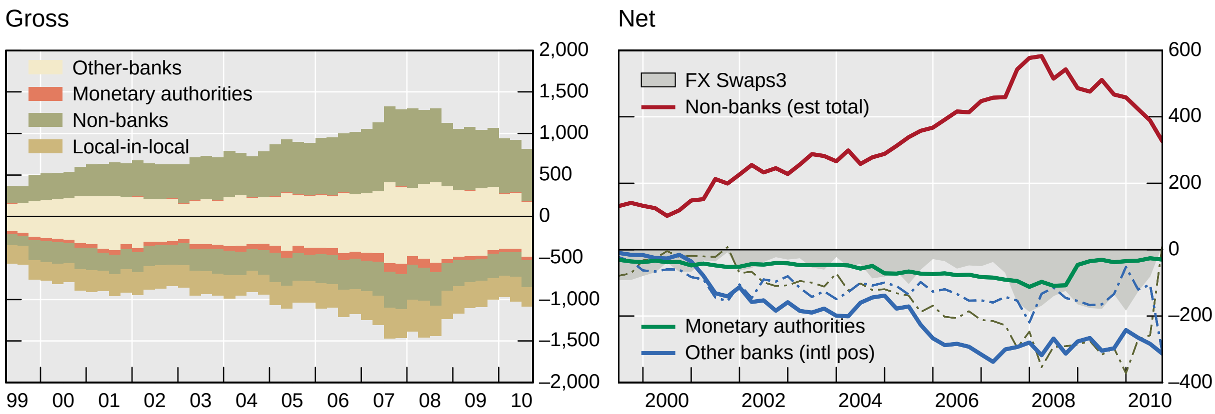
Source: Investment Company Institute.

Before the crisis, central banks were probably the second largest source of dollar funding for non-US banks, coming behind US dollar money market funds. US dollar money market funds have struggled in the low interest rate environment, with sponsors often waiving management fees to keep returns a whisker above 0%. Into 2010, the shrinkage of US money market funds, which place about half of their funds in non-US banks, put a squeeze on these banks' dollar funding (Graph 7).

Graph 8

German big banks' USD positions by sector

In billions of US dollars; dates indicate positions at end-Q1 of each year



Sources: BIS consolidated banking statistics (IB basis); BIS locational banking statistics by nationality; BIS calculations.

Lower official deposits and placements from money market funds have clashed with the still large needs of non-US banks for dollar funding. For instance, German banks still need, according to admittedly approximate estimates by our colleagues at the BIS,[17] more than $300 billion to fund dollar claims on non-banks (Graph 8). While they have reduced these dollar funding needs since the onset of the crisis, they still need to bid for sizeable amounts.

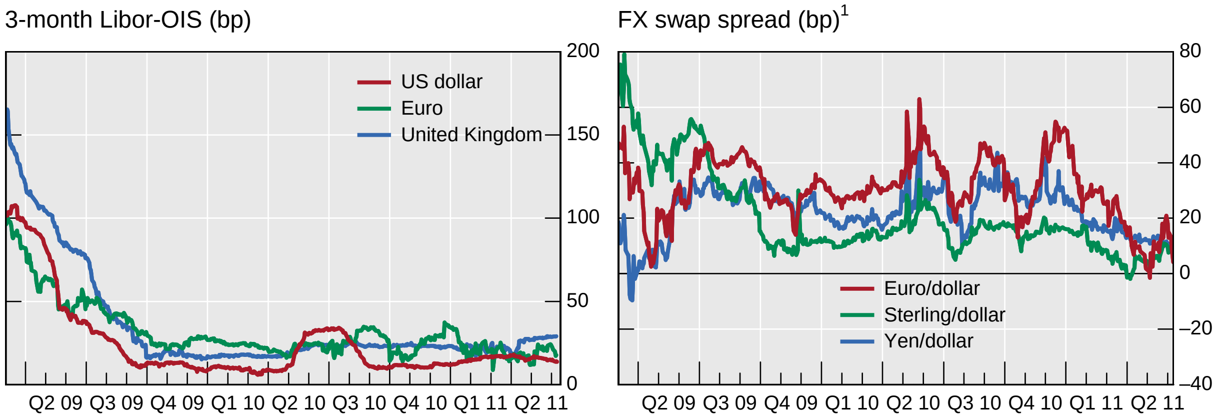
The result of the squeezed supply of dollar funding to foreign banks by central banks and money market funds and their ongoing demand for dollar funding is a persistent skew in the foreign exchange swap market (Graph 9). While Libor rates have to varying extents become more normal in relation to central bank policy rates (as captured by overnight interest rate swap yields in the left-hand panel), the skew in the swap markets has persisted at levels that were unimaginable before the crisis, especially in euro/dollar swaps (right-hand panel).

In conclusion, if official deposits with banks have bottomed out, it remains difficult to envisage a return to the former weight on banks. Among four large holders of bank deposits, only the Eurosystem central banks are showing a clear recovery of placements with banks (Graph 10). Many central banks are still put off by the financial risk and the lack of liquidity of bank deposits. The banks that emerged from the crisis in good shape do not bid for funds at very attractive rates at present because they are cash rich and unwilling to expand their balance sheets with interbank lending.

[17] See McGuire and von Peter (2009).

Graph 9

Domestic money market and foreign exchange swap spreads, 2009–10



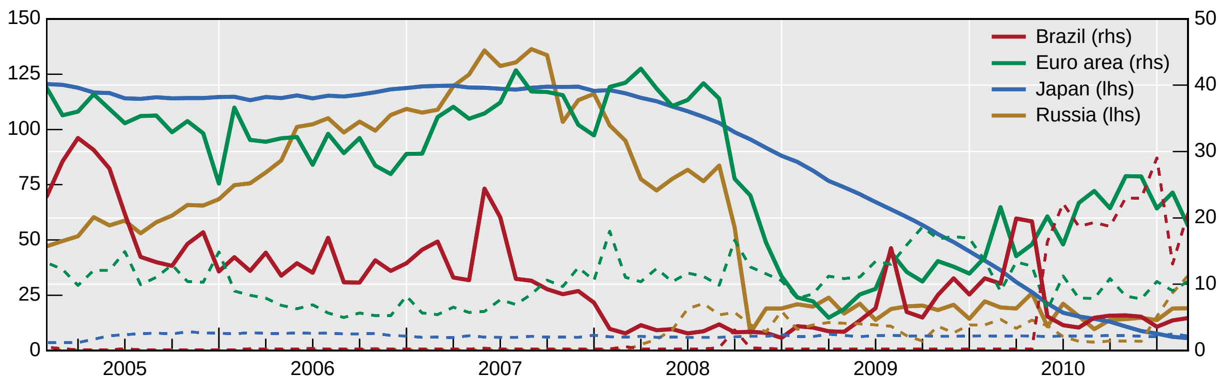
[1] Spread between three-month FX swap-implied dollar rate and the three-month USD Libor; the FX swap-implied rate is the implied cost of raising US dollars via FX swap using the funding currency.

Sources: Bloomberg; Datastream; Markit; national data.

Graph 10

Central bank deposits

In billions of US dollars



The solid lines refer to deposits in banks (domestic and foreign) and the dotted lines indicate deposits with other national central banks, the BIS and the IMF.

Source: IMF SDDS.

6.3 Do US agencies have a future as an asset class for official investors?

At writing, the US housing agencies' future as an asset class for reserve managers remains in doubt. We have seen how official holdings of agency obligations fell from $1.1 trillion in mid-2007 to $741 billion in mid-2010.[18] Monthly Treasury International Capital data suggest

[18] Thus, from the perspective of official portfolio managers, the Federal Reserve purchases were very timely. As the Fed bid for almost $200 billion in agency debentures and $1.25 trillion in mortgage-backed securities from late 2008 to March 2010, official portfolio managers sold.

another $100-plus billion in sales of long-term agency bonds (including prepayments on mortgage-backed securities) between June 2010 and March 2011 (Graph 6).

As we have seen, official reserve managers as a group shifted from the single-name debentures and bills of the agencies to their mortgage-backed securities. While official reserve managers originally sought to avoid the complexity of mortgage-backed securities in favour of the more predictable agency bullets and callables, they preferred the two-name mortgage-backed paper when the agencies got into trouble. Nevertheless, June 2010 holdings shows a decline even of the agency mortgage-backed securities holdings. On this showing, agency MBS just might remain a viable asset class for reserve managers, while the direct obligations of the agencies look to be in a run-off mode.

Table 6

Official holdings of US agency debt

	Long-term	**MBS**	**Deben-tures**	**Bills**	**Total**	***Memo: Fed + Treasury holdings***
March 2000	88					*0*
June 2002	134					*0*
June 2003	180					*0*
June 2004	211	23	194			*0*
June 2005	324	63	261	112	436	*0*
June 2006	473	118	355	110	583	*0*
June 2007	751	236	515	80	830	*0*
June 2008	967	435	532	130	1096	*0*
June 2009	795	475	320	34	828	*724*
June 2010	714	443	271	27	741	*1508**

Sources: US Treasury et al (2008–10); US Treasury (2011); Federal Reserve H.4.1 Release for 1 July 2010 and *Flow of Funds*.

6.4 Do corporate bonds have a future with official reserve managers?

Overall, official holdings of US corporate securities held up, albeit at fairly low dollar level, during the crisis (Table 7). To be sure, the turmoil in the commercial paper market after the Lehman Brothers failure led official reserve managers to back out of this money market in 2009. But selected official reserve managers have stayed with long-term corporate bonds.

The composition of official holdings of US corporate bonds, however, has shifted in the opposite direction from official holdings of agency bonds. As of June 2007, official reserve managers had bought almost equal amounts of corporate asset-backed securities and plain-vanilla corporate bonds. The experience of the crisis left them willing to continue to add to their holdings of straight corporate bonds – the increase in the year to mid-2009 came despite a significant spread widening that led to valuation losses (see above). The June 2010 holdings point to a small rise in straight corporate bonds – despite downgrades of AAA-rated issuers. On the asset-backed side, however, holdings of private-label mortgage-backed securities have fallen – through mid-2009 partly owing to valuation losses – resulting in a sharp overall decline in holdings.

On the available evidence, therefore, it appears that straight US corporate bonds remain in the sights of at least some official reserve managers. The future of private-label asset-backed securities is less clear, particularly that of private mortgage-backed securities.

Table 7

Official holdings of US corporate bonds and paper

	Asset-backed securities*	***Of which mortgage-backed***	**Other corporate bonds**	**Commercial paper**	**Total**
March 2000				na	12**
June 2002				na	18**
June 2003				na	21**
June 2004	18		29	na	47**
June 2005	17		44	7	68
June 2006	30		67	12	108
June 2007	44	*26*	55	17	116
June 2008	40	*18*	65	18	124
June 2009	35	*9*	72	9	116
June 2010	21	*7*	77	6	104

* Corporate ABS are backed by a wide variety of assets, such as car loans, credit card receivables, home and commercial mortgages, and student loans. ** Does not include commercial paper.

Sources: US Treasury et al (2008–11).

6.5 Is the diversification of official reserve portfolios continuing?

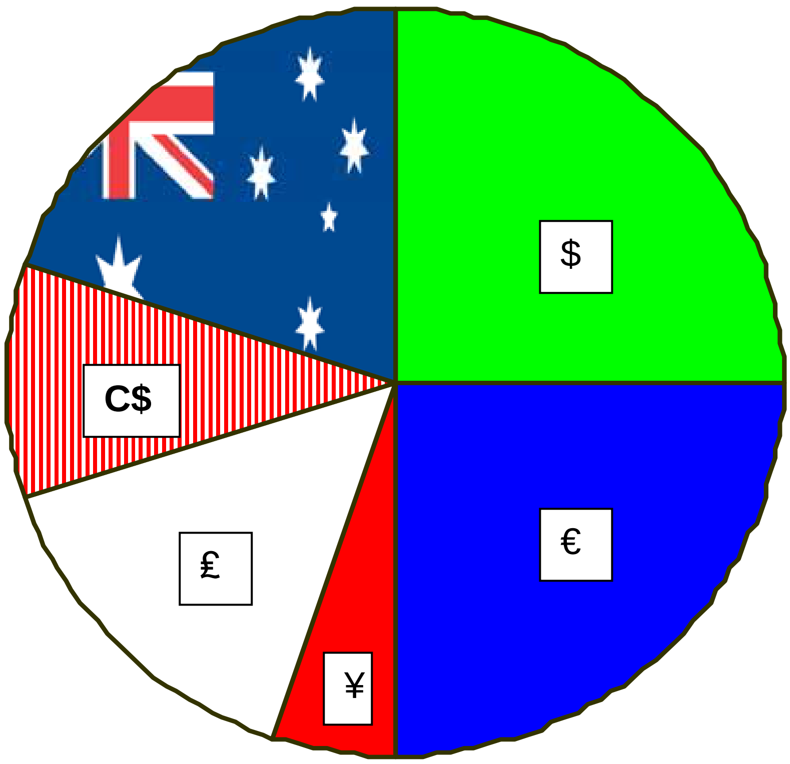
Our reading of the data and conversations with reserve managers suggest that a lively interest remains in diversification, particularly among large reserve holders. The attitude is probably best characterised as an amber light turning to green. Mistakes were made and lessons are being drawn. However, there are two forces putting diversification back on the agenda.

First, the monetary policy response to the financial crisis itself has lowered interest rates in the main reserve currencies to very low levels. True, many central banks finance (or "sterilise") reserve holdings with domestic currency instruments of systematically shorter duration than those instruments held in the reserve portfolio. Despite this yield curve boost, the relationship between funding costs and interest receipts is for many reserve portfolios very difficult. Even in places where interest rates are so low that the carry remains positive, there can be pressure to contribute more to the fiscal resources. On top of this "carry" problem is the experience of, or prospect of, valuation losses owing to the appreciation of the domestic currency against the major reserve currencies. Together, these make for a very challenging environment for official reserve managers.

In addition to the diversification of instruments that we have reviewed above, there is evidently interest in diversification by currency. An extreme example is New Zealand, but there is interest in commodity currencies by other commodity producing countries, and industrial countries alike. After a review of its currency allocation (Eckhold (2010)), the Reserve Bank of New Zealand has chosen to invest its reserves to a substantial extent in the commodity currencies of Australia and Canada (Graph 11). In addition, there are reports of central banks' buying Korean Treasury bonds.

Graph 11

Currency allocation of New Zealand's foreign exchange reserves



But it is important to recognise the limits of this kind of diversification. Sovereign debt outstanding of Australia, Canada, Korea and New Zealand amounts to only about $2 trillion. And in the emerging markets of Brazil, Colombia, Indonesia, Hungary, Malaysia, Mexico, the Philippines, Peru, Poland, Russia, South Africa, Thailand and Turkey, there is only $3 trillion in outstanding sovereign debt. Both of these are an order of magnitude smaller than the $38 trillion in the sovereign markets of the United States, the euro area, Japan, the United Kingdom and Switzerland.

In conclusion, US Treasury securities will for a time not only represent the plurality of US dollar reserves but also command a higher fraction of dollar holdings than before the global financial crisis. Reserve managers will continue to explore other investments, but with a more critical and cautious attitude than before the crisis. The limited size and liquidity of many alternatives to US Treasuries will pose ongoing challenges.

7. Conclusions

We have seen that the global financial crisis led official reserve managers to pare back their exposure to banks and to put their holdings of US agency debentures into a run-off mode. Harder to measure, but just as surely, official reserve managers have cut back sharply on the lending of their securities and especially the credit trade of raising cash against high-quality securities and investing the proceeds in lower-quality securities.

Going forward, central banks may increase their bank placements gradually and extend maturities. While the Basel III liquidity rules put pressure on banks to extend maturities, central bank placements remain by all accounts relatively short. In any case, banks need to deleverage and non-US banks need to work down their dollar funding requirements. Bank funding markets cannot be described as normal as long as the skew in the foreign exchange market persists.

From the standpoint of official reserve managers, there is good news and bad news on the sovereign debt front. Recession, fiscal stimulus and bank rescues have left lots of government paper for reserve managers to buy, although prospective returns may not be so attractive.

The bad news for official reserve managers is that the rise in debt resulting from recession, fiscal stimulus and bank rescues has undermined the notion of the risk-free placement. Can central banks assign a zero probability of default to any sovereign?

Thus, the job of the official reserve manager is very challenging. Investment yields are low, credit risks lurk and there is renewed pressure for returns.

Annex

Table A-1

Securities lending by Norwegian Government Pension Fund

In billions of Norwegian kroner

	Securities lent			**of which against cash**						***Memo: Gain/losses recognised***
	Total	**Equity**	**Bonds**	**Total**	**Reverse repo**	**Fixed income**				
						Total	**ABS**	**SIVs**	**Others**	
2007:Q4	516	181	334	298	201	93	46	11	37	*–3 (year)*
2008:Q1	541			327				8		*–1.5 (quarter)*
2008:Q2	515			249				5		*–.4 (quarter)*
2008:Q3	435			202						*–.1 (quarter)*
2008:Q4	374	183	191	186	114	63	39	2	22	*–6 (year)*
2009:Q1	335			160						*+1 (quarter)*

Source: Norges Bank Investment Management reports.

Table A-2

State Street's securities lending and unregistered cash collateral pools

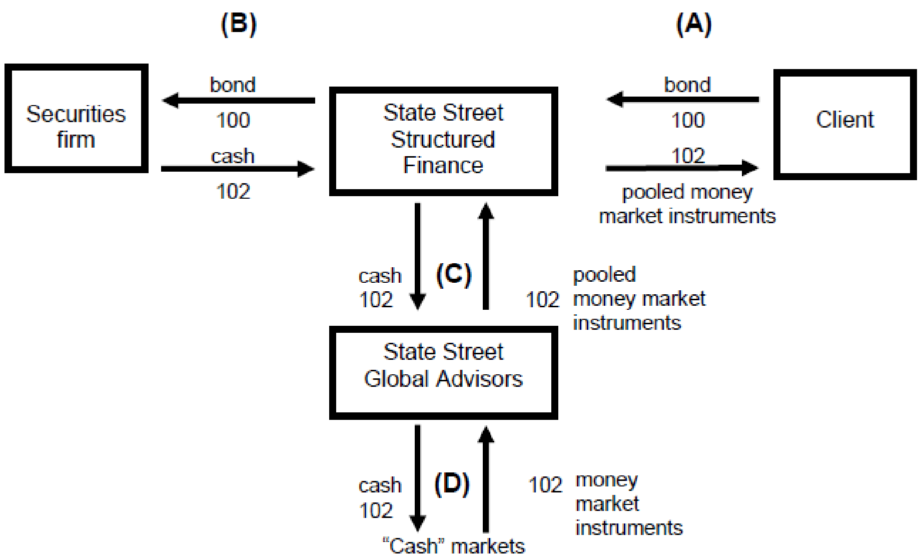
Date	Outstanding securities lending (USD billions)	Unregistered cash collateral pools (USD billions)	Net asset value range	Net asset value average
31 December 2007		194* (150)	.99-1.00	.993
30 September 2008		167		
31 December 2008	347	122* (85)	.908 (.92)-1.00	.939 (.941)
30 March 2009		122*	.904-1.00	.947
31 December 2009		85	.93-1.00	.986
30 June 2010				.989
31 December 2010		49	.91-1.00	.993

*. Figures in parentheses for end-years are from the 2 February 2011 8-K.

Source: State Street 8-Ks.

Graph A

Securities lending at State Street



(B) vs (D): Maturity mismatch
(B) vs (D): "Credit arbitrage" if "prime" or "enhanced" cash investment

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Diversifying market and default risk in high grade sovereign bond portfolios

Myles Brennan, Adam Kobor[1] and Vidhya Rustaman[2]

Introduction

Diversification is a keystone in modern finance. In this paper we discuss the potential benefits of diversification for high grade sovereign bond portfolios. A government bond investor may be motivated to diversify internationally for a number of reasons. The first and most classic reason would be to achieve overall volatility reduction in the portfolio. If the economic and business cycles of different countries are showing lags relative to each other, it is realistic to assume that lags across the expectations, monetary policy actions and other factors that influence interest rates would lead to less-than-perfect correlations. Beyond volatility reduction, investing in multiple countries could also be driven by return enhancement: the investor may find an attractive credit spread from a country with lower credit quality within the same currency zone, or the investor may expect positive carry from relative yield curve differences across different currencies. In conjunction, with an increasing focus on sovereign default risk, mitigating the impacts of possible financial distress may also be a motivation to diversify. In this paper we discuss both rate volatility reduction and the tail risk reduction aspects of diversification.

With globalization and free flow of capital, developed markets became highly integrated into the world market, and correlations across different countries have increased. The average correlation across the Bank of America/Merrill Lynch government indices of the G7 countries has been 0.66 over the past decade, but if we exclude Japan from the sample, in fact it would have been 0.78. These correlations suggest some diversification power, but clearly show a strong cross-border connection. We note that correlations across the MSCI equity indices of the G7 countries were similarly high over the past 10 years, on the order of 0.7. To get a more detailed and fundamental understanding of what drives government bond returns of a specific country, we apply a CAPM-based model to the G7 countries. This model attributes expected return to both global and local risk factors. If a market is fully integrated, its expected return depends solely on global risk factors and the market's exposure to them. If markets were fully integrated, assets with the same risk should have equal expected returns. However, if a market is fully segmented, its expected return should be derived from local factors only. We find that G7 government bond markets are partially integrated to the global market, with an average of 75-80% of their expected excess return coming from global risk factors. However, the impact of local factors is still on the order of 20-25%, meaning that these markets are not fully integrated, and still there is some room for diversification.

While volatility reduction is a typical result of diversification, investors may be more concerned about mitigating the impact of tail risk due to financial distress. Recent developments in the Eurozone government bond markets serve to highlight this concern. The Eurozone shares a

[1] Corresponding author; e-mail: akobor@worldbank.org.

[2] Myles Brennan is Director; Adam Kobor is Principal Portfolio Manager; and Vidhya Rustaman is Senior Portfolio Manager at the Investment Management Department of the World Bank Treasury.

The findings, interpretations and conclusions expressed herein are those of the authors and do not necessarily represent the views of the World Bank.

common monetary policy, so diversifying across issuers would have the impact of sharing exposure across different credit risk factors, and potentially picking up yields from lower credit quality borrowers. Recently, downside risk has been the dominant story for peripheral countries, driven by concerns about sovereign defaults, or the break-up of the Eurozone. Pure historical analysis is not sufficient for the assessment of diversification benefits against default losses, so we supplement our arguments with hypothetical simulation studies. Based on a default simulation exercise we find that diversification may mitigate severe credit losses, but the diversification may be limited due to the relatively low number of sovereign issuers.

This paper consists of two fairly distinct parts. The first part covers "business as usual" diversification, focusing on the reduction of volatility, and ignoring, or only implicitly considering, credit risk. This first part follows mainstream financial economic research, and historical time series can be used for reliable estimation. While in the second part we also apply historical estimations, we ultimately try to quantify the extremes, ie default loss. History in this context is less reliable, and the results are more dependent on qualitative assumptions and hypothetical scenarios. In 2010, we consider both aspects of diversification to be relevant.

The paper is organized as follows: first, we review the literature that deals with international government bond diversification and portfolio construction from both the academic and practitioner standpoint. In Section 2, we discuss diversification across G7 governments both in the context of asset pricing as well as from an empirical perspective. Then we turn our attention to sovereign credit risk considerations in Section 3. We review the recent history of risk and return within the Eurozone, applying a Markov switching model, and present credit risk simulation results based on hypothetical assumptions. Finally, we draw conclusions.

1. Literature review

Researchers discuss international diversification and global government bond portfolio construction from many different directions. International CAPM-based models discuss the degree to which national markets are integrated in the world market and explain the sources of risk and return. These models assume that the expected return comes from two systematic sources: the world market and the local market. Investing in a country that is fully integrated with the world market will only provide compensation against global market risk. In contrast, a fully segmented market will only provide compensation against local market risk. As fully integrated markets are likely to see the same expected returns for assets with the same risk, the case for diversification is more apparent if markets appear to be segmented. Several studies follow the paper by Bekaert and Harvey (1995), who tested the extent of integration in international equity markets. Barr and Priestley (2004) studied monthly returns in US, UK, German, Canadian and Japanese government markets between 1986 and 1996, and found that the average contribution of world factors to domestic returns was only 70%, implying that the full benefits of international diversification have not been realized in the international bond markets. In a more recent study discussing the integration of European government bond markets, Abad et al. (2009) found that based on a study of weekly returns over 1999 to 2008, euro-based countries are less sensitive to world risk factors, and are only partially integrated with the German market, suggesting room for diversification. Following an empirical statistical approach, Longstaff et al. (2008) find that excess return from sovereign credit is largely compensating for bearing global risk only, but find little country-specific credit risk premium after adjusting for global risk factors in general.

From the credit risk management perspective, the bulk of the literature discusses the risk of corporate bonds – see a broad review intended for risk practitioners by Ramaswamy (2004) among many others. Duffie and Singleton (2003) discuss credit risk from both the risk management and the asset pricing point of view, and they address both corporate as well as sovereign credit risk in their book. Wei (2003) discusses a multi-factor credit migration modeling approach that can be applied for both corporate and sovereign debts. Gray and

Malone (2008) extend Merton's contingent claim approach to the macro level, including sovereign balance sheet analysis. Remolona et al. (2007) discuss the factors explaining sovereign credit spreads, and relate potential loss based on historical data provided by rating agencies to the size of the credit spread. They also raise the point that diversifying sovereign default risk is more limited than, say, risk diversification in equities due to the relatively low number of issuers. Reinhart and Rogoff's (2009) book provides a historical synthesis of different kinds of crisis periods over several centuries, and we find it an essential read for understanding the nature of sovereign default risk.

Related to credit risk considerations, some practitioners suggest moving away from market weights as fixed income benchmarks given that these weights tend to overemphasize the more indebted countries. With respect to fixed income benchmarks, Laurence Siegel (2003) notes that market cap-weighted fixed income benchmarks bring rise to the "bums" problem, namely that the biggest debtors have the largest weights in the benchmarks and as a result, these benchmarks are unlikely to be mean-variance efficient. A number of practitioners have also increased focus on alternative weighting methodologies, including indices with weights linked to financial fundamental variables (Arnott et al. 2005 and Arnott el al. 2010), country weighting schemes related to GDP (see eg Barclays 2009 and PIMCO 2010), and indices with country level or regional caps (Dynkin and Ben Dor 2006).

2. Global diversification: searching for volatility reduction

In Figure 1 we compare the G7 diversified portfolio volatility to the market capitalization-weighted average of the volatilities of the seven countries on a 36-month rolling basis, based on Bank of America/Merrill Lynch government bond index data. As shown in Table 1, the volatilities of individual country indices are somewhat different; thus we found this approach to be the most neutral way of illustrating the volatility reduction. Figure 1 shows that volatility reducing power has actually been decreasing over the past decade. While between 1999 and 2001 the volatility of the diversified portfolio was only around 70% of the average individual country-level volatility, this ratio had climbed to 90% by 2008, suggesting that volatility reducing power had largely decreased. However, since the outbreak of the crisis, we can see a slight switch in the trend, and notice that global government diversification reduced volatility by around 15%. This suggests that some diversification benefits remain, and a trend of increasing correlations may change direction. In the following section we explore the factors behind country-level government bond returns in more depth.

Figure 1

Ratio of G7 volatility to the weighted average of volatilities

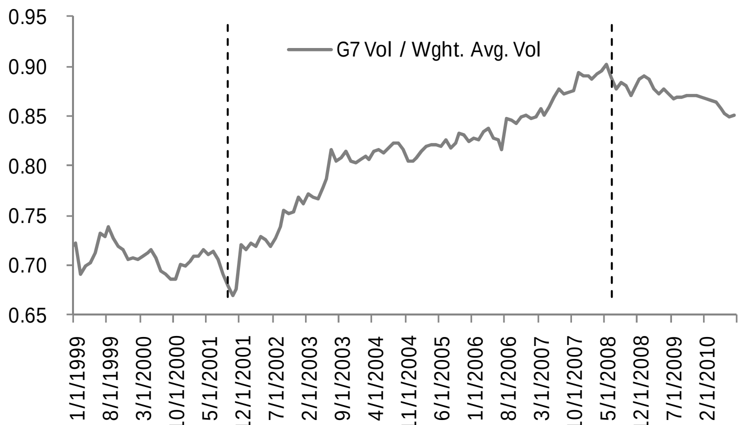


Table 1

Main statistics of local currency returns over cash

1999–2010	Canada	France	Germany	Italy	Japan	UK	US
Average (p.a.)	3.06%	2.56%	2.54%	2.18%	1.99%	1.40%	3.24%
Std. deviation (p.a.)	4.18%	3.94%	3.80%	4.05%	2.65%	5.72%	4.77%
Correlation	**Canada**	**France**	**Germany**	**Italy**	**Japan**	**UK**	**US**
Canada	1						
France	0.70	1					
Germany	0.71	0.98	1				
Italy	0.58	0.86	0.80	1			
Japan	0.26	0.29	0.31	0.23	1		
UK	0.66	0.81	0.81	0.68	0.25	1	
US	0.85	0.74	0.76	0.62	0.27	0.68	1

2.1 The impact of global and local risk factors in the pricing of sovereign bonds

We estimate and discuss a CAPM-based asset pricing model as presented by Barr and Priestley (2004) for global bond markets and by Abad et al. (2009) for European bond markets. These papers follow Bekaert and Harvey (1995), who assume that the expected excess return of an asset class can come from two sources: the global market price of risk, to the extent the specific market is integrated into the global markets; and the local market price of risk, which can be significant if the market is segmented from the global market. As fully integrated markets are likely to see the same expected returns for assets with the same risk, the case for diversification is more apparent if markets are segmented to some degree.

In our analyses we worked with Merrill Lynch/Bank of America Government index returns over the domestic 3-month government bill return as a proxy for the excess return in local currency terms. Our historical sample covers monthly observations between January 1999 and September 2010. We relate bond index returns to some selected global worldwide (*W*) and local (*L*) fundamental variables, namely the (1) yield curve slope (bond index yield over 3-month T-Bill); (2) lagged bond index excess return; (3) lagged equity excess return; and (4) 10-year swap spread as a proxy for liquidity. In the selection of the first three variables we are following the referenced literature. Others have also used dividend yield or earnings yield. For equity returns, we use the specific MSCI country index and the global developed markets index returns, and swap spread data was sourced from Bloomberg. The selection of the underlying variables is ultimately the analyst's choice; the theory does not have a closely defined set of variables.

As a first step, we test the predicting power of the selected variables in the following linear regression form, using only the local, only the global, or both sets of variables:

$$r_{i,t} = a_i + b_i^W Z_{t-1}^W + b_i^L Z_{i,t-1}^L + \varepsilon_{i,t} \qquad (1)$$

where $r_{i,t}$ is the government bond excess return in country i¸ and Z^W and Z^L are the vectors of the global and local information variables. We present our summary statistics in Table 2A. The R^2s on the order of 12-18% suggest that bond returns have some predictability. F-statistics suggest that we can reject the null hypothesis that both global and local variables can be excluded for all countries. However, partial F-tests show that, conditional on global variables, except for Japan, the omission of local variables cannot be rejected. Similarly, conditional on local variables, the omission of global variables cannot be rejected in the cases of Canada, Italy, and the US. We also present R^2s and F-statistics for regressions based on local and global variables only; based on these, except for global variables in Japan, we cannot reject the null hypothesis of the omission of the selected factors. We have to mention that the selected global and local information variables are correlated – see the correlations between the local bond returns, and also our observation on equities in the introduction. Thus, it may be understandable that the local-only or global-only regressions show significance, but we cannot reject the null hypothesis of omitting one set of the variables in several cases of the joint regressions. These correlations across global and local variables may impose some limitations on the analyses, but we still prefer using market-based variables as they should immediately reflect market perception, and unlike some macroeconomic variables they are not subject to reporting time lags, or revisions and methodological differences across countries. Nevertheless, had we chosen to work with, say, inflation and industrial production variables, they would also be correlated.

The actual conditional international CAPM takes the following form:

$$r_{i,t} = \theta_i \cdot \lambda_{W,t-1} \cdot \mathrm{cov}_{t-1}\left(r_{W,t}, r_{i,t}\right) + \left(1-\theta_i\right) \cdot \lambda_{i,t-1} \cdot \mathrm{var}_{t-1}\left(r_{i,t}\right) + \varepsilon_{i,t} \tag{2}$$

The first component of the equation represents the compensation for the global market risk: $\lambda_{W,t-1}$ is the time-varying global market price of risk; $\mathrm{cov}_{t-1}\left(r_{W,t}, r_{i,t}\right)$ is the covariance of market *i* with the global bond market and measures the sensitivity to the global market price of risk, and θ_i is the degree of integration into the global market. The second part of the formula represents the compensation for local risk, and $\lambda_{i,t-1}$ is the local price of risk. The dynamics of the global bond market are given by:

$$r_{W,t} = \lambda_{W,t-1} \cdot \mathrm{var}_{t-1}\left(r_{W,t}\right) + \varepsilon_{W,t} \tag{3}$$

and the error terms from equations (2) and (3) are assumed to be normally distributed with GARCH (1,1) covariances, ie $\varepsilon_t = [\varepsilon_{i,t} \quad \varepsilon_{W,t}] \sim N\left(0, H_t\right)$ with:

$$H_t = C'C + A'^{\varepsilon_{t-1}\varepsilon'_{t-1}}A + B'H_{t-1}B \tag{4}$$

The main significance of working with GARCH variances and covariances is that they allow for time-varying risk exposure in our model. In addition, the time-varying market price of risk is assumed to take non-negative values; thus, similarly to the literature, we express market price of risk in the form of exponential functions:

$$\lambda_{w,t-1} = \exp\left(K_w^{'} Z_{t-1}^{w}\right) \tag{5}$$

$$\lambda_{i,t-1} = \exp\left(\delta_i^{'} Z_{i,t-1}^{L}\right) \tag{6}$$

We estimate the parameters by maximum-likelihood method in two steps. First, we estimate the parameters of equation (3). Then, in step two we estimate equation (2) for each country, using the world market price of risk as input obtained in step one, and we fix the univariate GARCH parameter estimates a_{22} and b_{22} for the global world market to ensure consistency. For the sake of numerical tractability, we also assume *A* and *B* to be diagonal. The parameter estimates are found in the second panel of Table 2. Based on their θ_i estimates, Canada and the US appear to be the most integrated countries, whereas local factors and segmentation matter the most in Japan. European countries, except Italy, also show a relatively high degree of integration. We note that the asset pricing estimates and previous linear regression results seem to show similar messages. In the case of Japan, the global variables-only regression suggested that global variables may be less relevant, and the joint regression kept local variables significant conditionally on global variables. In the asset pricing context, Japan is estimated to be the least integrated country over our sample time period.

In terms of the local risk premium, yield curve slope seems to consistently play a significant role based on our estimations, whereas there is more variation in the significance of lagged bond and equity returns as well as in the role of swap spreads.

Our θ_i estimates are comparable to those reported by Barr and Priestley (2004). In our case, the average degree of integration is 0.8 and 0.75 on an equally weighted and market weighted basis, respectively. Barr and Priestley report an average of 0.7 for the five countries that they analyzed. The sample period in their case, however, was 1986 to 1996, so it is intuitive to see higher values in the case of a more recent period. Abad et al (2009) report lower estimated figures, but they worked with weekly observations. We similarly found lower figures in our experiments on a weekly basis; however, we consider the monthly horizon to be more relevant in the assessment of diversification benefits for institutional investors.

In Figure 2 we show the expected excess returns for the global bond market. The global premium took its highest levels at the most stressful periods of the crisis. In Figure 3 we present expected returns for all the seven countries, and we separately show the global and local return components.

Figure 2

World bond market expected excess return

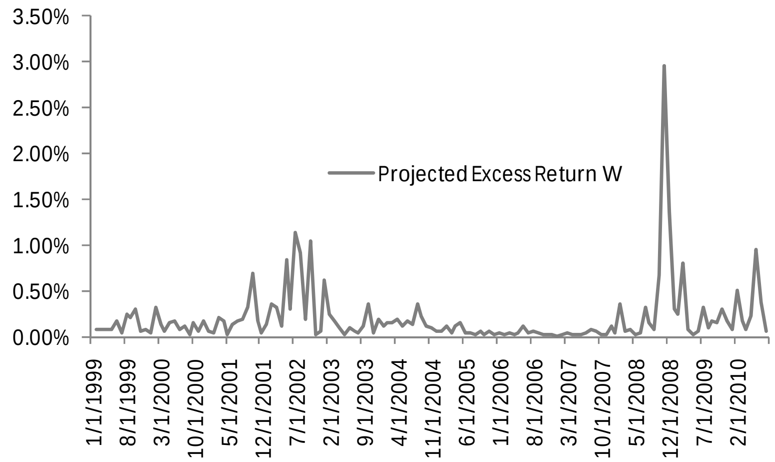


Figure 3 provides insights into the dynamics of country-level expected returns, and shows the breakdown of excess returns by country. While the degree of integration is assumed to be constant over the period, the charts highlight the time-varying contribution by local factors due to time-varying risk premium and time-varying sensitivities:

- The US has been a very highly integrated country, and only seemed to provide extra local premium on top of the global premium when the global premium was already high. Such periods include the tech bubble burst period around 2002 and the collapse of Lehman Brothers in 2008. US Treasuries did not seem to diversify against global governments, but rather intensified compensation in turbulent times. This is understandable if we consider US Treasuries the safe haven in a global flight to quality period;
- Canada, while estimated to be very highly integrated, seemed to provide a smoother and more balanced compensation over time;

- Germany and France are largely integrated, although they seemed to add some Eurozone-specific compensation over the past two years;
- Italy is integrated to a lesser degree than Germany and France, and the local risk premium picked up in Italy over the past two years, since sovereign credit became a concern;
- The UK is largely integrated, but also showed extra compensation recently, in line with signs of weakness in its banking system;
- Finally, Japan is estimated to be the least integrated out of the seven countries. The data shows that the local premium in the earlier years seemed to be more important than recently.

The results presented in Figure 3 and Table 1 suggest that Japan was a main contributor to diversification in the G7 over the past decade. Japan had the lowest volatility, as well as the lowest pairwise correlations with the other countries over the past decade. With a weight around 35% in the G7 universe, Japan was clearly the main diversifier for non-yen investors (see market weights in Appendix 1.) However, in Figure 4 we can see that the local return component played a more important role during the earlier years of historical sample. At the same time, the local component seemed to have gained importance for European countries over the past two years or so, and this coincides with the recently improving volatility reduction trend highlighted in Figure 1.

The practical conclusion of this analysis is that sovereign government bonds are relatively highly integrated into the global market, but local factors still play a role. In the asset pricing context this means that the expected return is not solely dependent on global factors. Bonds with the same global risk factor sensitivities may offer somewhat different expected returns even if their global risk sensitivity is the same. Given that local factors are not negligible, the case for diversification still holds.

Table 2

Asset pricing model estimation

A. Predicting equations (OLS)

		World	Canada	France	Germany	Italy	Japan	UK	US
Global and local variables	R^2 (%)	-	14.88	18.17	16.07	15.39	12.47	16.29	12.33
	F-stat exclude local	-	1.79	1.76	1.18	1.50	2.77*	1.64	0.77
	F-stat exclude global	-	0.42	2.64*	2.03*	1.92	2.19*	2.14*	0.53
	F-stat exclude both	-	2.84*	3.61*	3.11*	2.96*	2.32*	3.16*	2.29*
Local variables only	R^2 (%)	-	13.77	11.52	11.45	10.40	6.56	10.77	10.91
	F-stat exclude local	-	5.35*	4.36*	4.33*	3.89*	2.35*	4.05*	4.10*
Global variables only	R^2 (%)	13.97	10.20	13.73	13.03	11.50	5.01	11.95	10.26
	F-stat exclude global	5.44*	3.80*	5.33*	5.02*	4.35*	1.77	4.55*	3.83*

*significant at least at 90% confidence level

B. Asset pricing model parameters (ML)

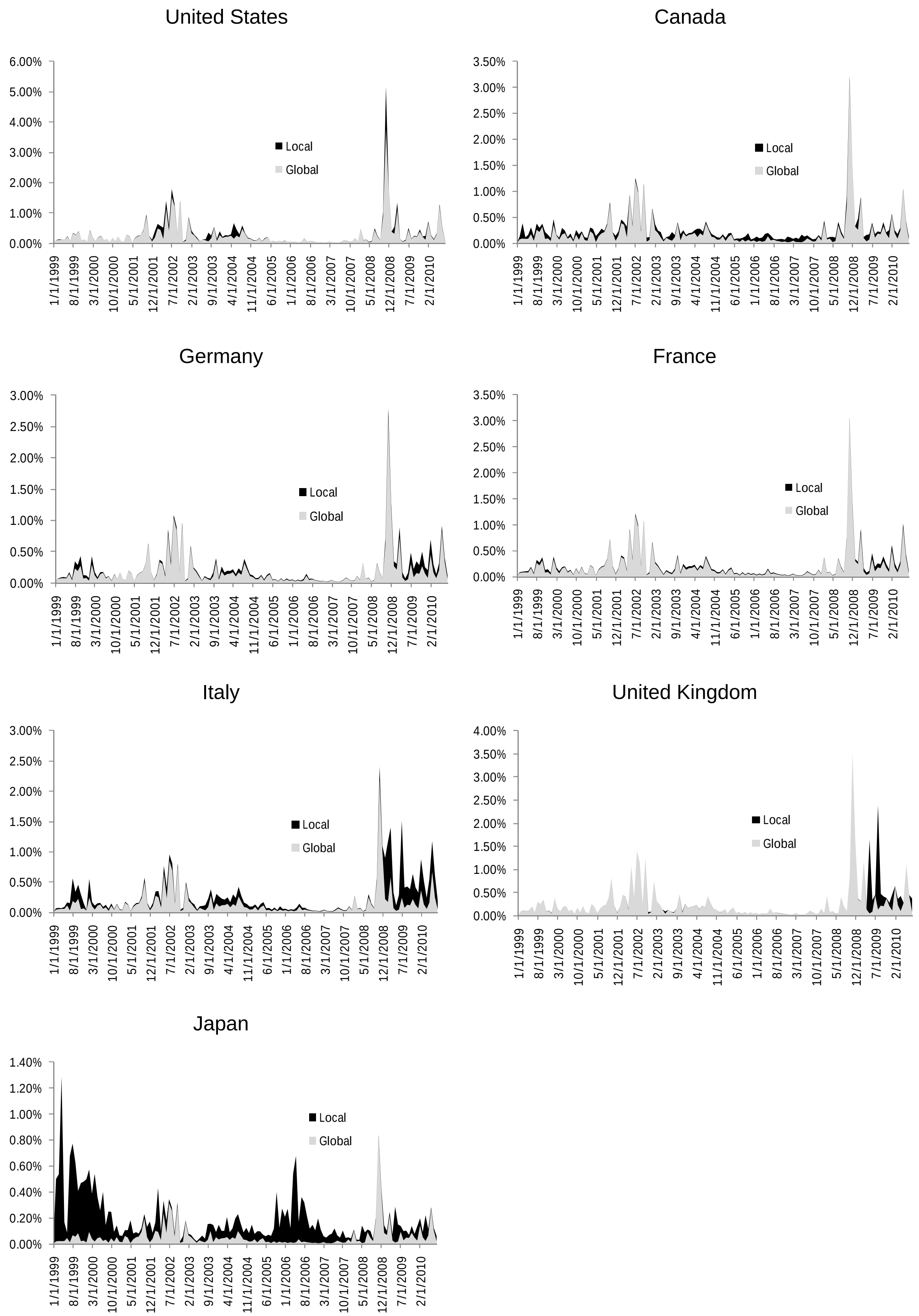
	κ_0	κ_1	κ_2	κ_3	κ_4	c	a	b			
World	1.29*	128.04*	44.66	-16.18*	-40.94*	0.0000*	0.0050*	0.7721*			
	θ_i	δ_0	δ_1	δ_2	δ_3	δ_4	c_{11}	c_{12}	c_{22}	a_{11}	b_{11}
Canada	0.96*	4.84*	0.61	-55.91*	-2.85	-2.57	0.0017*	0.0022*	0.0029*	0.0193*	0.8780*
France	0.86*	1.58*	100.91*	19.19*	-2.57	-5.91	0.0026*	0.0026*	0.0026*	0.2255*	0.9033*
Germany	0.78*	0.06	202.62*	8.89	-5.88	-28.31*	0.0031*	0.0032*	0.0019*	0.2388*	0.8780*
Italy	0.63*	-0.51	196.38*	30.84*	-5.27	0.51	0.0041*	0.0035*	0.0010*	0.2347*	0.7746*
Japan	0.58*	0.27	409.50*	14.62	11.43	-543.12*	0.0000*	0.0031*	0.0019*	0.0000*	0.8830*
UK	0.88*	-1.51*	142.48*	-19.02*	40.32*	-5.91	0.0036*	0.0035*	0.0012*	0.2597*	0.8907*
US	0.92*	-0.51	193.21*	-30.88*	-16.35*	-4.32	0.0032*	0.0032*	0.0017*	0.1797*	0.9165*

*significant at least at 90% confidence level.

κ and δ coefficients belong to the following factors: (1) yield curve slope; (2) lagged fixed income return; (3) lagged equity return; (4) swap spread.

Figure 3

Expected excess returns; global and local impact



2.2 An empirical look at the G7 portfolio

We complete our analysis by taking another empirical look at the G7 index volatility and performance. Table 3 summarizes a standard PCA analysis. We find that 76.4% of the variation in local currency excess returns of the seven markets can be explained by one factor. This first factor can be interpreted as the global bond market return, whereas the second factor appears to be a North America versus continental Europe factor, the third factor is UK-specific, and the fourth factor is a Japan-specific factor. Interestingly, we obtained a similar order of magnitude for global risk with PCA analysis as we did in the asset pricing analysis.

Table 3

Principal Component Analysis: variance explained and factor coefficients

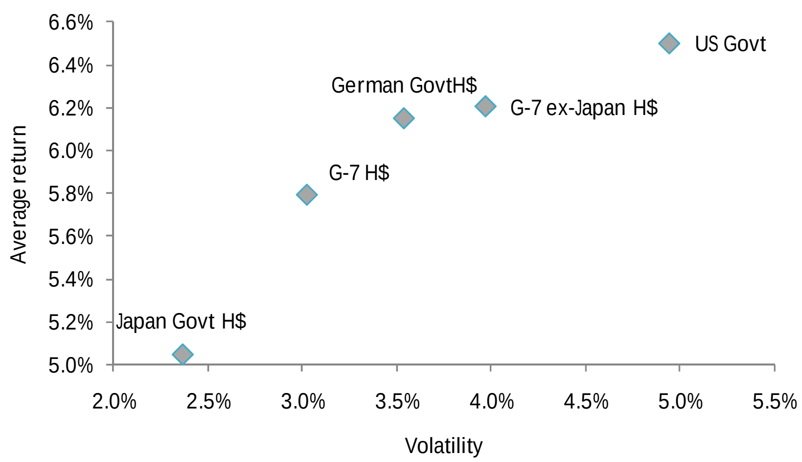
Factor	F1	F2	F3	F4
%ge explained	76.4%	7.4%	6.5%	5.5%
Canada	**0.38**	**0.44**	**0.22**	-0.06
France	**0.37**	***-0.44***	0.05	-0.06
Germany	**0.36**	***-0.37***	0.05	-0.03
Italy	**0.30**	***-0.53***	0.12	-0.16
Japan	0.09	-0.04	**0.45**	**0.88**
UK	**0.51**	0.18	***-0.76***	0.32
US	**0.48**	**0.41**	**0.39**	-0.30

Following the base currency neutral view of the G7 universe, illustratively we compare the G7 portfolio to a specific single currency alternative, namely US Treasuries. As Figure 4 shows, the volatility of the US Treasuries index was around 4.9% between January 2000 and August 2010, whereas the market cap weighted G7 index had a volatility of 3.0% after being hedged back to USD. In addition, the average return was not much behind: US Treasuries earned 6.5%, whereas the hedged index made 5.8%, thus showing a higher Sharpe ratio of 1.1 versus the UST-only alternative's Sharpe ratio of 0.8, using 3-month T-Bills as the risk-free asset.

Figure 4

Historical risk-return space from a USD-based investor's point of view

2000–2010

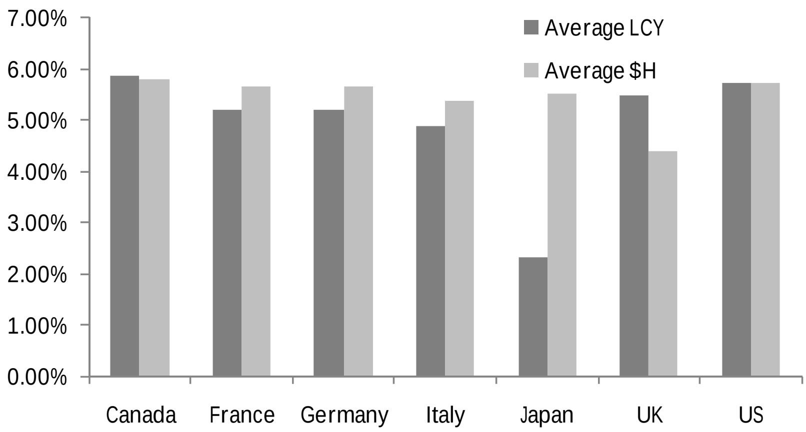


As previously shown in Table 1, US Treasuries had one of the highest excess return volatilities measured in local currency terms. Excess return volatility differences across the countries can be explained mainly by different levels of rate volatilities, but the durations of certain country indices were also somewhat different. We did not adjust durations to be equal in this paper, but even if we had, we would arrive at similar conclusions. That being said, given that US Treasuries exhibited the second highest volatility out of the seven countries, a dollar-based investor would have experienced a significant volatility reduction by simply switching from a US Treasuries-only portfolio to a G7 portfolio. The reduction in volatility as a result of a similar switch would seem to be smaller for a euro-based investor, and a yen-based investor would actually experience higher volatility. However, a G7 portfolio can still play an attractive role even for these investors in a portfolio optimization context.

Besides the differences in bond returns measured in local currency terms, currency hedging also has an impact on the ultimate performance of the diversified portfolio. Winkelmann (2003) finds that currency hedging is essential for bond portfolios as currency volatility exceeds government bond return volatility. Actually, applying our analysis to excess returns is consistent with analyzing hedged returns: the hedged return of a global bond portfolio is actually the same as the individual excess returns over cash, plus the cash return of the base currency. This is evident from covered interest rate parity. Figure 5 compares the bond index returns in local currency terms as well as in USD terms on a hedged basis between 1999 and 2010. Japanese cash rates were much lower than in the US during most of the period; thus, investing in Japanese bonds was very attractive on a currency hedged basis. We can consider this aspect similar to a typical carry trade. This also explains why the average return of the G7 hedged index did not fall much behind the US Treasury bond portfolio. Although this carry consideration goes beyond the scope of our paper, we note that the carry advantage from the perspective of a dollar-based investor has been diminished as cash rates are on a very similar scale at the moment of writing this paper. Ultimately, the optimization results for currency hedged bond portfolios depend on the expected excess returns, the excess return volatilities, and the correlations across the selected bond markets.

Figure 5

The impact of currency hedging



3. International diversification in the presence of credit risk

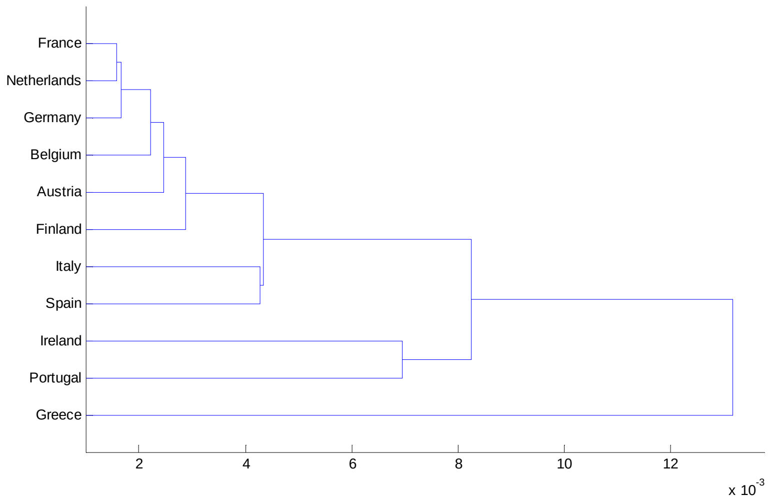
3.1 Switch in the Eurozone landscape

While behavior in G7 bond markets overall was relatively stable over the past decade, the markets in the periphery of the Eurozone became turbulent, as market participants became concerned at the path of increasing indebtedness in a number of Eurozone economies. For our analysis, we examine 11 Eurozone government bond markets, but we exclude Cyprus, Luxembourg, Malta, Slovakia and Slovenia due to their shorter histories within the Eurozone, or because of the lack of bond index data. We use bond index data from Bank of America/Merrill Lynch and Barclays Capital, and we cover the monthly history of index returns between July 31, 2001 and June 2010. Appendix 1 shows the latest market cap shares of the 11 countries in the Eurozone proxy, as well as the historical dynamics of the relative weights. We note that Barclays Capital stopped reporting index data for Greece after May 31, 2010, as Greece lost its eligibility as an investment grade country because of its S&P rating.

Figure 6 shows the dendrogram output of a cluster analysis based on monthly returns of the bond indices, using correlation-based distance measures. The clusters fairly closely reflect the commonly used "core" versus "periphery" separation. The core countries were closely correlated over the past nine years, whereas Ireland and some of the Mediterranean countries ("PIGS") showed more idiosyncratic behavior.

Figure 6

Dendrogram of Eurozone bond index monthly returns



Eurozone bond markets clearly show a distinct picture before and after the crisis. In order to capture this distinct behavior across Eurozone countries in the most refined way, we estimate a Markov switching model on the total returns of the 11 countries. Markov switching models with conditional normal distributions produce a better fit of the historical return distribution than a single unconditional normal distribution. To keep the model specification simple, we assume the presence of two regimes and assume that bond returns r_t are normally distributed with expected values μ_i and covariance matrices Σ_i, conditional on regime i. The return distribution is given by

$$\int\left(r_{it}\middle|s_t=i\right)=\frac{1}{\left(2\pi\right)^{m/2}\det\left(\Sigma_i\right)^{1/2}}\exp\left\{-\frac{1}{2}\left(y_t-\mu_i\right)'\sum\nolimits_i^{-1}\left(y_t-\mu_i\right)\right\}, \tag{7}$$

where s_t denotes the regime or state in period t. The regimes are assumed to evolve as a Markov chain: the probability of regime s_t=j (j=1...N) only depends on the previous observation:

$$P\left\{s_{t=j|s_{t-1}=i,s_{t-2}=k,...}\right\}=P\left\{s_t=j\middle|s_{t-1}=i\right\}P_{ij}, \tag{8}$$

where P_{ij} is usually referred to as transition probability from regime i to regime j. Regime switching models can be estimated using the so-called expectation maximization algorithm, as described by Kim and Nelson (1999) and Hamilton (1990, 1994). The estimation results are the following. Figure 7 shows the regime-dependent average returns versus Germany, as well as the regime-dependent volatilities. In Appendix 2 we show the same statistics, together with the regime-dependent correlations. Based on the results, regime 1 came across as a regime that can be labeled as a "high volatility" regime with very different average returns, whereas regime 2 shows lower volatilities and relatively uniform average returns Based on the likelihood maximizing estimation results, the transition probability from a lower volatility month to another lower volatility month (P_{22}) was 94.4%, and the transition probability from a hectic month to another hectic month (P_{11}) is estimated to be 87.8%. These would suggest reasonably persistent regimes.

Figure 7

Annualized conditional average returns over Germany and volatilities

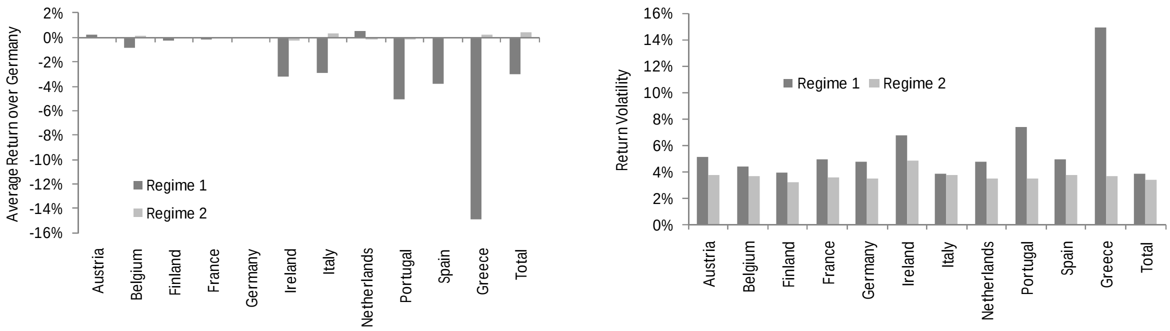


Figure 8 shows the estimated state probabilities for being in regime 1. Not surprisingly, the months belonging to the volatile regime were estimated very much in line with what we could have expected by intuition. Just for illustration, we also show the CDS for Greek bonds, although no spread history or any other data than the index returns themselves were used in the estimation.

By looking at the estimation output, we can make the following observations:

1. Considering the return pick-up in "normal times", ie during months belonging to the quieter regime 2, the average excess return of the Eurozone index was 9 bps over Germany. Some countries could pick up 20-30 bps extra return over Germany.

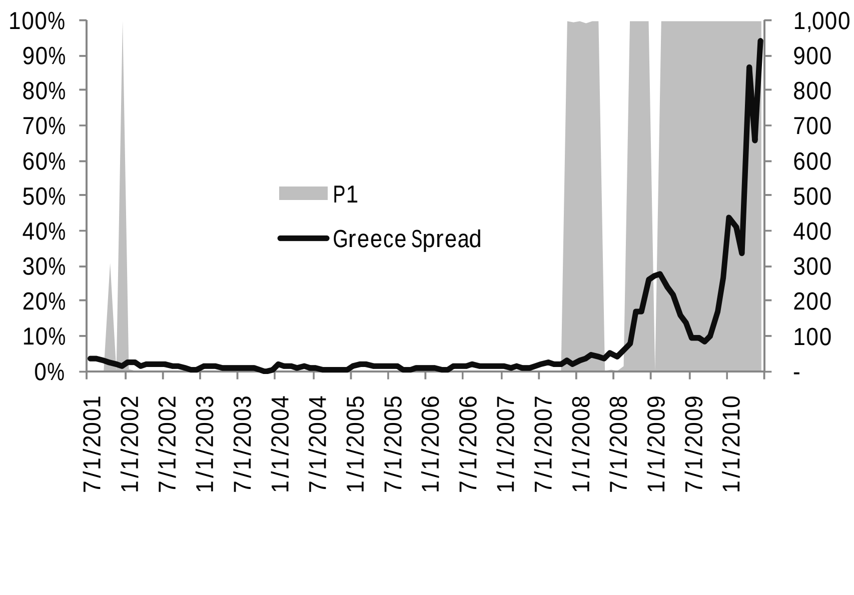
2. In stressful periods, however, the relative performance became gloomy. We have to point out the caveat that the history has not come to an end as of 2010, so some figures may show more comforting pictures once economic recovery takes place, but as of the summer of 2010, we can say that the diversified Eurozone index actually underperformed Germany by 42 bps over the full nine-year history in our sample, with Greece contributing -348 bps average annual relative performance. In

the more hectic regime 1, the average Eurozone return over Germany was -189 bps on an annual basis – with Greece showing -1488 bps; Portugal -504 bps, Spain -384 bps, and Ireland -324 bps relative annualized return individually. Again, these figures can become fairly different after some consolidation, but we note that right now the upside and downside pictures show a very asymmetric shape. Furthermore, if an investor's guidelines permit investment grade bonds only, or the investor simply replicates investment grade indices, the end of May was actually the end of the story for the Greek bond holdings. From the end of May, Greece was no longer considered investment grade, so only those who are willing to hold below-investment grade countries investors may benefit from a potential recovery.

3. Turning to risk diversification, in regime 2, the composite Eurozone volatility was 3.63%, 10 bps higher than that of Germany. In the more hectic regime 1, however, the aggregate volatility was 4.3%, 50 bps below the volatility of Germany. The conditional correlation matrices are worth a look as well: in regime 2, except for Ireland, Italy, and Spain, all pairwise correlations are in the range of 0.97-0.99. Even Ireland, Italy, and Spain show correlations between 0.84 and 0.96 with all other countries. In the hectic regime 1, however, correlations fall apart. Actually, the correlations across "core" countries remain high, on the order of 0.90-0.98 (this latter figure can be found between Germany and France, or the Netherlands and France), but the correlations between "core" and "peripheral" countries can be anywhere down to 0.11 (this figure is the correlation between Greece and Germany). If we want to summarize, we can say that "core" Europe diversified the poorer performance generated by bonds of the "periphery".

Figure 8

Probability of "high volatility" regime

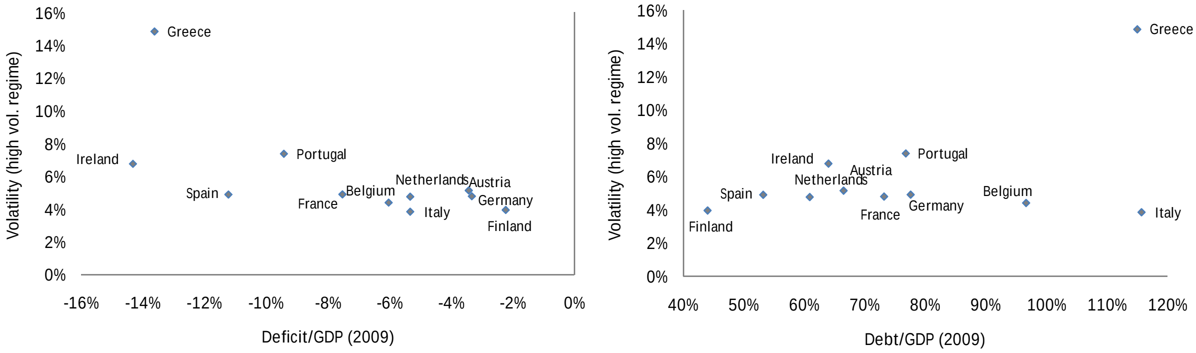


3.2 Credit risk and fundamentals

Within the Eurozone, the bulk of the country-specific risk is credit risk. We can assume that the market risk, ie the impact of the dynamics of the credit risk-free yield curve risk, is common across the Eurozone. In this section we connect bond market volatilities of the stressful regime to commonly used sovereign debt statistics and market capitalization. As Figure 9 suggests, countries with poorer public debt and budget deficit were associated with higher volatility in the stressful regime.

Figure 9

Debt statistics and volatility



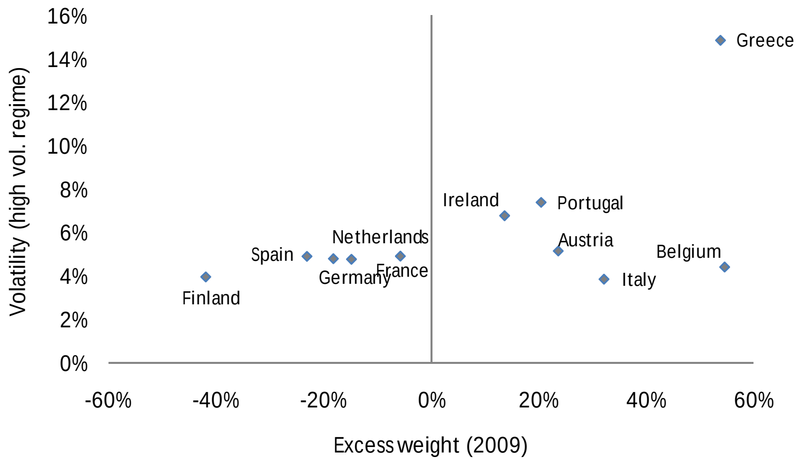
Source of debt and deficit data: Eurostat

It is worth noting some practical implications of these results. When determining a benchmark, investors commonly assign market weights to the benchmark constituents. It is of course reasonable to assume that larger components in the benchmark represent countries that issue liquid and easily accessible sovereign bonds. At the same time, as a country becomes more and more indebted, its market weight grows relative to others, but its increasing share in the market capitalization now potentially reflects increasing credit risk. This concern may well lead investors to deviate from market weights and consider alternative weighting schemes. Generally, determining neutral allocation weights can depend on institutions' individual characteristics, the time horizon of the benchmark selection, and the risk budgeting divided between strategic asset allocation and active portfolio management.

In Figure 10 we illustrate this with a comparison. We calculated the relative weights of the 11 selected Eurozone countries based on their market caps. We also calculated their relative weights based on their nominal GDPs as a first order approximation of their relative economic sizes. Finally, we calculated the ratio of these two weights – these ratios are presented in the horizontal axis. Countries on the left-hand side are those that have relatively lower market weights compared to their relative GDPs, whereas on the right-hand side we can find countries that have a relative market weight beyond their relative GDP ratio. According to the chart, the volatility had a tendency to be higher for the relatively more indebted countries. Still, this is a pure illustration, and does not substitute for a more thorough credit risk analysis. Debt/GDP can be one important indicator to be taken into consideration, but there are several other factors that will determine whether a given country is able or not to meet its financial obligations without any problems. For the sake of pure illustration, a GDP-weighted Eurozone composite would have earned 5.39% average annual return compared to 5.25% of the market weighted index over the 2001-2010 period. Also, the volatility of the GDP-weighted composite would have been 3.51% versus 3.53%. But historical backtesting can be misleading in many cases in this context; we would argue that the decision has to be made based on qualitative considerations, and we are not sure if a simple silver bullet formula exists to come up with an ideal allocation that will work in any period of time.

Figure 10

“Excess” weights and volatility



3.3 Diversification to mitigate default losses

Finally, we push the scope of risk quantification to the edge, and discuss the impact of default risk and diversification on portfolio return in a very simplified and naïve fashion. The Eurozone turbulence reminded investors that sovereign bond investment, even if high grade, is not free of default risk. Reinhart and Rogoff (2009) synthesized crisis periods over several centuries, and Figure 11 is made based on their account of sovereign defaults. Defaults on external and domestic debt have indeed happened, and they did not uniformly distribute over time. There were relatively quieter periods, as well as more default-intensive periods, and as the graph may suggest, defaults seemed to be concentrated by geographical regions.

Figure 11

History of sovereign defaults

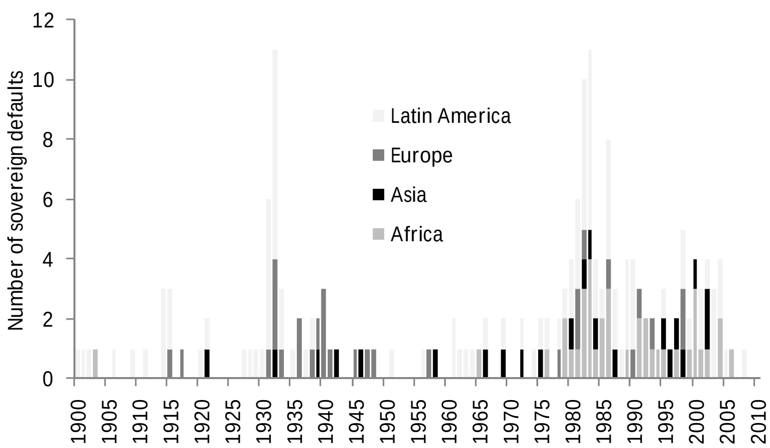


Table 4 summarizes historical sovereign default frequencies, reported by Standard and Poor’s (2010) and Moody’s (2010a). We also show corporate bond defaults based on significantly longer time series, again collected by Moody’s (2010b). In order to illustrate the possible magnitude of credit losses due to default, we run some naïve credit loss simulations, simply counting with default events, but excluding other factors associated with credit loss, such as credit spread movements due either to changing assessment of credit quality or changing risk tolerance of investors. Similarly to Remolona et al. (2007), we use five-year empirical default frequencies, and interpolate default frequencies for shorter horizons assuming that default probabilities are homogeneous over time. Our assumed default frequencies are shown in the two rightmost columns of the table. For AAA-A rated bonds, we work with the longer-history corporate default rates to show a more conservative picture, although the reader can argue that corporate and sovereign defaults are not directly comparable. We agree with that, but again, we simply consider this section as an illustration.

Table 4

Default frequencies over 5-year horizon

	S&P Fgn Svgn; 1975-2009	**Moody's Svgn; 1983-2009**	**Moody's Corp.; 1920-2009**	**Assumed 5-year freq.**	**Implied 1-year freq.**
AAA/Aaa	**0.00%**	**0.00%**	0.16%	0.16%	0.03%
AA/Aa	**0.00%**	**0.00%**	0.72%	0.72%	0.14%
A/A	**0.00%**	**0.00%**	1.26%	1.26%	0.25%
BBB/Baa	4.35%	2.44%	3.14%	4.35%	0.89%
BB/Ba	7.65%	8.08%	9.90%	7.65%	1.67%
B/B	13.74%	10.57%	22.42%	13.74%	2.91%
CCC/Caa	71.43%	32.46%	41.18%	71.43%	22.16%

In our illustrative analysis we assume a recovery rate of 50%, which is comparable to the range of the recovery rates reported by Moody's (2010a and 2010b). Our simulation approach follows the description by Duffie and Singleton (2003): we simulate the time to default, and impose dependence structure across issuers by working with Gaussian copula. The time to default τ can be determined by calculating the survival probability function $p_i(0,\tau)$ by the five-year default rates, and by simulating U_i uniform standard random number for issuer i such that:

$$p_i(0,\tau) = U_i \tag{9}$$

The Gaussian copula with a correlation matrix of ρ is expressed as:

$$C_p = N_p\left(N^{-1}(U_1), N^{-1}\left(U_2, \ldots N^{-1}(Un)\right)\right) \tag{10}$$

We refer to Embrechts et al. (1999) as one of the first published applications of copula functions in finance, and to Chen et al. (2009), who discuss tail dependence across sovereign bond CDSs by using different copula functions. In the following, we report simulation results over one-year horizon, based on 200,000 simulations. As our main interest is diversification, we present the results as a function of the number of issuers, ie countries, in a hypothetical portfolio, assuming equal weights across issuers. For a specific rating we consider a number of issuers in the range of 1 to 10. There is a simple practical consideration behind this: the total number of sovereign issuers rated by Moody's, S&P or Fitch are on the order of 100 to 120. As of 2009, we found the number of AAA rated issuers to be around 17-20 countries, AA rated countries on the order of 11 to 14, A rated around 17-24, and BBB around 18-20 countries. In other words, unlike in the case of equities, we can reach the limits of diversification very soon due to the limited number of issuers of sovereign bonds. In Appendix 3 we provide a detailed simulation report, but here we focus only on conditional value-at-risk (CVaR) as the relevant risk measure. CVaR with confidence level *α*, also known as expected shortfall (ES), quantifies the expected magnitude of loss if we exceed the highest expected loss, ie VaR, estimated at a confidence level *α* over the selected measurement horizon:

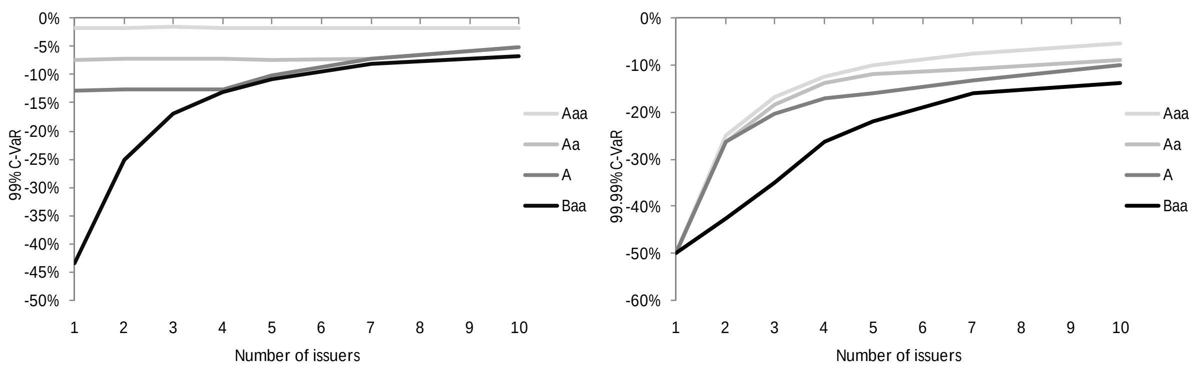
$$CVaT_a\left(w\right) = -E\left(r'w \middle| r'w \leq VaR_a\right) \tag{11}$$

We use CVaR because, unlike shortfall probability or VaR, it is a coherent measure of risk, as shown, for example, by Acerbi and Tasche (2001). Practically, this means that the risk of a portfolio measured by a coherent measure cannot exceed the weighted arithmetic average of risks of the individual assets, or in other words, coherent measures of risk recognize diversification. Shortfall probabilities, on the other hand, are not coherent, and credit risk is perhaps the best field to demonstrate it. The reader can also find the simulated probabilities of suffering *any* default loss as a function of the number of issuers in Appendix 4. As can be seen, the probability of *any* credit loss increases with the number of issuers – intuitively because if there are many issuers in the portfolio, chances are that a default will be seen sooner than when dealing with a single issuer. While the magnitude of loss is more devastating in the case of a concentrated loss, shortfall probability in this case would clearly argue against diversification.

In Figure 10 we show estimated CVaR for Aaa, Aa, A, and Baa rated bond portfolios from one to 10 issuers at the 99% or 99.99% confidence level, assuming 0 correlation. In the case of the 99% CVaR, only the Baa rated portfolio shows any kind of response to the number of issuers. This is simply because the probability of default is much less than 1% for the higher rated bonds. If we push the frontier to the extreme 1:10,000 confidence level, even Aaa rated bonds show some response, even though their probability of default is a mere 3:10,000. That being said, if this very unlikely outcome materializes, the impact can be severe. Figure 12, in any case, shows that diversification can significantly reduce extreme losses if default events are independent.

Figure 12

CVaRs at different confidence levels



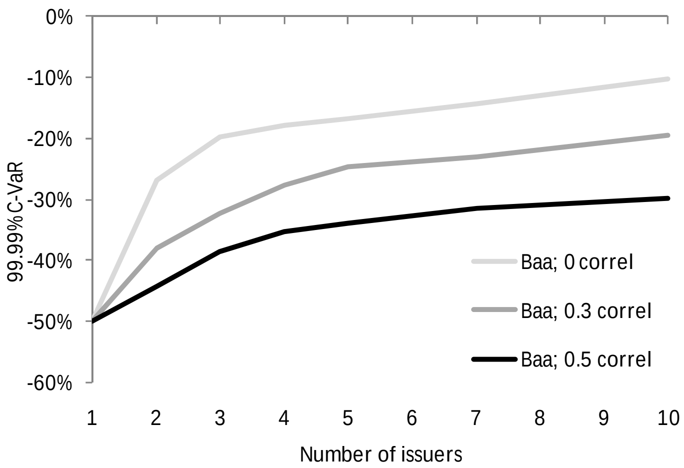
In Figure 13 we focus on 99.9% CVaRs of Baa rated bond portfolios only, but impose different degrees of dependence in the copula structure represented by 0.0, 0.3, and 0.5 correlations. CVaR still provides evidence for diversification, although at a decreasing rate with increasing correlations. In reality, we do not think that a uniform correlation would make sense across, say, 10 issuers. Instead, we could think of scenarios such that some countries become more correlated within a cluster, but less correlated with other clusters, as in the case of geographical regions, or the periphery versus the core within the Eurozone. We note that Chen et al. estimate Gaussian copula correlations on the order of 0.7 between selected Latin American countries. For clarification we also note that the correlation parameters underlying the copula function are not the same as default correlations. Default correlations can be extracted from the simulation results by using the formula

$$p = \frac{P\left(d_i, d_j\right) - P\left(d_i\right) \cdot P\left(d_j\right)}{\sqrt{P\left(d_i\right) \cdot \left(1 - P\left(d_i\right)\right) \cdot P\left(d_j\right) \cdot \left(1 - P\left(d_j\right)\right)}} \tag{12}$$

where $P(d_i)$ denotes the default probability for issuer i, as shown by Ramaswamy (2004), among others. For example, the default correlation for BBB rated bonds would be on the order of 0.07 and 0.13 for 0.3 and 0.5 Gaussian copula parameters.

Figure 13

99.9% CVaR of Baa rated portfolios with different correlations

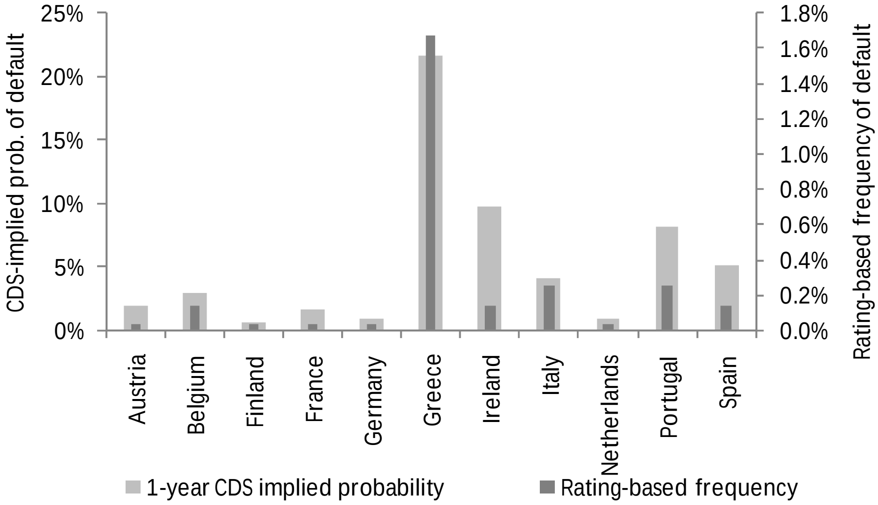


Readers might think that the reported 99.9% or 99.99% CVaR figures are completely irrelevant due to their very remote probabilities. However, when we contrast the historical statistical frequencies of default with the CDS implied risk neutral default probabilities over one year for selected Eurozone countries (spread levels taken in September 2010), we notice that credit default spreads “price in” default probabilities around 10 to 70 times higher than pure historical default frequencies. If risk neutral default probabilities had been equal to the actually expected default probabilities, our loss estimations would not seem to be so remote. Clearly, credit spreads contain a significant risk premium beyond the pure compensation against expected loss. Credit losses are hugely skewed. As an example, the default probability of a BBB bond is 0.9% over one year with a 50% recovery rate, so the expected credit loss due to default is 0.09*0.5=-0.45%. However, if default occurs, the investor will not lose 0.45% but 50% in our example. In addition, as Amato and Remolona (2003) point out, an investor would need a very high number of issuers to diversify the impact of credit losses away even in the case of corporate bonds.

We also note that sovereign defaults may take different forms. Hypothetically speaking, within the Eurozone a country would not need to formally declare a default. It would be enough for the country to leave the Eurozone in order to produce a similar impact. Leaving the Eurozone and returning to a national currency could potentially devalue the currency and thus reduce the value of the country’s debt, with a similar magnitude to that of a default.

Figure 14

CDS implied risk neutral probabilities of default versus rating-based frequencies



4. Conclusions

The aspects of diversification have become broader over the past years in the context of sovereign bond investments. In the traditional sense, diversification would primarily be considered in terms of volatility reduction. However, mitigating the impact of possible distress became part of the considerations.

In this paper we first discussed global governments from a broad international rate diversification perspective. By looking at the correlations and historical reduction in volatility, we noticed that global high grade sovereign bonds became largely integrated over the past decade. However, global governments did not become an entirely global asset class that is uniformly priced to the same factors. Local factors still have a considerable impact on returns, and the fact that local factors matter makes the fundamental case for diversification prevail. We found that local factors explain around 20-25% of total risk premia, and the volatility historically could have been reduced by 10-30% compared to the average volatility of the G7 countries based on historical observations. It is also important to note that while local factors in Japan became less significant recently, other countries in Europe showed the opposite dynamics.

Second, we discussed sovereign credit risk as this topic has become a key concern in Eurozone government bond markets. The Eurozone underwent a regime switch as recent volatilities and correlations vastly differ from their historical values. Considering diversification from the perspective of default risk, historical analysis does not provide too much insight. We rely on long-term historical default rate data, and ran a naïve hypothetical simulation with different correlation assumptions. Due to the limited number of issuers, diversification does not eliminate default impact, but at least mitigates losses. Also, sovereign default does not have to be a formal default; within the Eurozone context, leaving the euro and returning to a national currency would have a similar impact, as CDS levels show.

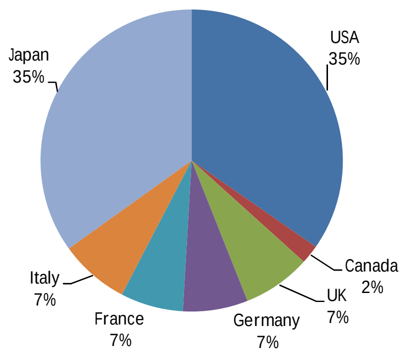
We can draw a couple of practical conclusions from the credit risk aspect. First, when an investor decides on the eligible issuers and constituents of the investment benchmark, sovereign credit risk cannot be ignored. The investor needs to own the decision and feel comfortable, whether the specific issuer is part of the benchmark or simply eligible for active management. Regarding the benchmark composition, the allocation weights to specific countries also matter. Market capitalization weights are typically used by investors; however, market weight increases with increasing indebtedness of an issuer. If credit risk is a concern, we understand considering weighting approaches that penalize the market weights of issuers with increasing credit risk based on qualitative considerations. Finally, the lower we go in the credit ratings, we believe that diversification becomes more and more valuable in mitigating potential losses due to financial distress.

We see several opportunities for further research. One is to bring the two parts presented in this paper to a more common ground, ie measuring interest rate and default risk diversification jointly. Also, some parts of the paper may be developed further as well. Bekaert and Harvey (1995) presented their CAPM-based approach in a regime switching context that would be a natural extension to the Eurozone countries. Finally, understanding sovereign credit risk and its implications for portfolio management remains a very timely issue; clearly many researchers are devoting their efforts to this topic.

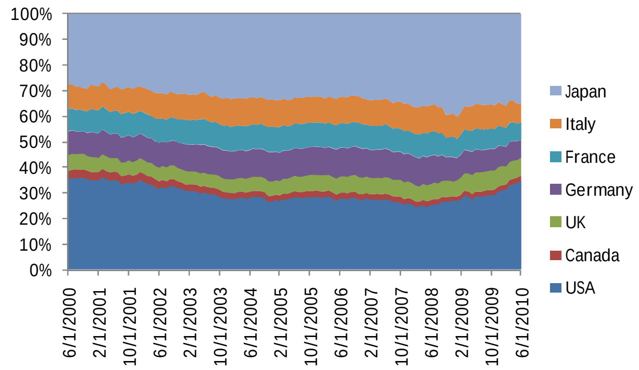
Appendix 1:
Structure of G7 and Eurozone government markets

Structure of G7 sovereign bond market

As of June 30, 2010

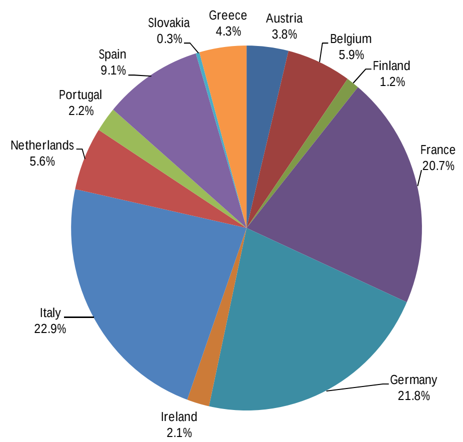


History

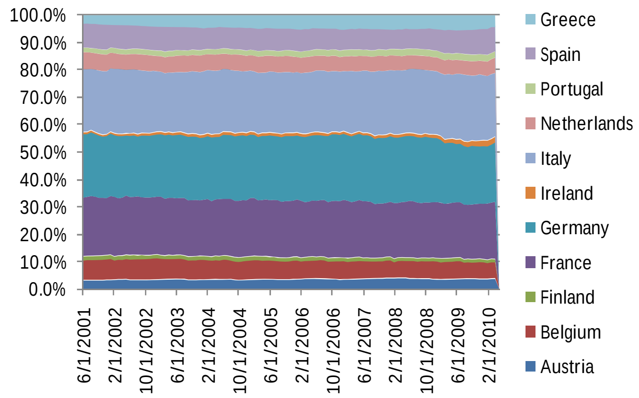


Structure of Eurozone sovereign bond market

As of May 31, 2010



History



Appendix 2:
Regime-dependent statistics

		Austria	Belgium	Finland	France	Germany	Ireland	Italy	Nether.	Portugal	Spain	Greece	Total
Regime 1	Average return	9.60%	8.52%	9.12%	9.24%	9.36%	6.12%	6.48%	9.84%	4.32%	5.52%	-5.52%	7.47%
	Ret. over Germany	0.24%	-0.84%	-0.24%	-0.12%	-	-3.24%	-2.88%	0.48%	-5.04%	-3.84%	-14.88%	-1.89%
	Volatility	5.15%	4.41%	3.96%	4.91%	4.80%	6.78%	3.85%	4.77%	7.40%	4.90%	14.88%	4.30%
Regime 2	Average return	4.44%	4.56%	4.44%	4.44%	4.44%	4.20%	4.80%	4.32%	4.32%	4.44%	4.68%	4.53%
	Ret. over Germany	0.00%	0.12%	0.00%	0.00%	-	-0.24%	0.36%	-0.12%	-0.12%	0.00%	0.24%	0.09%
	Volatility	3.74%	3.67%	3.22%	3.62%	3.53%	4.88%	3.79%	3.53%	3.53%	3.78%	3.64%	3.63%

Correlation matrices		Austria	Belgium	Finland	France	Germany	Ireland	Italy	Netherlands	Portugal	Spain	Greece
Regime 1	Austria	1										
	Belgium	0.95	1									
	Finland	0.92	0.92	1								
	France	0.94	0.96	0.95	1							
	Germany	0.90	0.90	0.97	0.98	1						
	Ireland	0.73	0.72	0.53	0.59	0.48	1					
	Italy	0.81	0.81	0.70	0.74	0.67	0.72	1				
	Netherlands	0.95	0.94	0.96	0.98	0.97	0.59	0.79	1			
	Portugal	0.60	0.65	0.48	0.54	0.45	0.88	0.57	0.51	1		
	Spain	0.71	0.79	0.63	0.72	0.65	0.71	0.90	0.71	0.67	1	
	Greece	0.30	0.33	0.15	0.21	0.11	0.75	0.26	0.16	0.90	0.40	1
Regime 2	Austria	1										
	Belgium	1.00	1									
	Finland	0.97	0.98	1								
	France	1.00	1.00	0.97	1							
	Germany	1.00	1.00	0.98	1.00	1						
	Ireland	0.92	0.92	0.86	0.92	0.90	1					
	Italy	0.98	0.98	0.96	0.98	0.98	0.84	1				
	Netherlands	1.00	1.00	0.98	1.00	1.00	0.93	0.97	1			
	Portugal	0.99	0.99	0.97	0.99	0.98	0.95	0.95	0.99	1		
	Spain	0.99	0.99	0.96	0.99	0.99	0.96	0.96	0.99	0.99	1	
	Greece	0.99	0.99	0.98	0.99	0.99	0.91	0.98	0.99	0.98	0.98	1

Appendix 3:
Credit simulation results

		Pairwise correl=0				Pairwise correl=0.3				Pairwise correl=0.5			
	# Issuers	1	3	5	10	1	3	5	10	1	3	5	10
Expected Loss	Aaa	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%	-0.02%
	Aa	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%	-0.07%
	A	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%	-0.13%
	Baa	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%	-0.45%
Prob. of credit loss	Aaa	0.03%	0.10%	0.17%	0.33%	0.03%	0.10%	0.17%	0.35%	0.03%	0.08%	0.13%	0.25%
	Aa	0.15%	0.42%	0.73%	1.43%	0.15%	0.44%	0.71%	1.37%	0.15%	0.40%	0.64%	1.13%
	A	0.26%	0.75%	1.27%	2.51%	0.26%	0.75%	1.24%	2.33%	0.26%	0.69%	1.07%	1.90%
	Baa	0.87%	2.59%	4.30%	8.42%	0.87%	2.52%	4.06%	7.40%	0.87%	2.18%	3.31%	5.61%
99% CVaR	Aaa	-1.7%	-1.6%	-1.6%	-1.6%	-1.7%	-1.7%	-1.7%	-1.7%	-1.7%	-1.5%	-1.5%	-1.5%
	Aa	-7.5%	-7.3%	-7.1%	-5.0%	-7.5%	-7.5%	-7.3%	-5.5%	-7.5%	-7.1%	-7.1%	-6.2%
	A	-12.8%	-12.6%	-10.1%	-5.1%	-12.8%	-12.7%	-10.6%	-6.2%	-12.8%	-12.2%	-11.5%	-7.4%
	Baa	-43.5%	-17.0%	-10.8%	-6.6%	-43.5%	-19.0%	-14.1%	-11.9%	-43.5%	-21.2%	-17.6%	-15.1%
99.9% CVaR	Aaa	-17.0%	-15.8%	-10.0%	-5.0%	-17.0%	-16.8%	-10.2%	-5.6%	-17.0%	-15.0%	-11.3%	-6.8%
	Aa	-50.0%	-16.8%	-10.2%	-5.4%	-50.0%	-18.4%	-12.4%	-9.6%	-50.0%	-20.1%	-17.0%	-13.7%
	A	-50.0%	-16.8%	-10.7%	-6.3%	-50.0%	-21.3%	-16.4%	-11.6%	-50.0%	-23.6%	-22.9%	-18.2%
	Baa	-50.0%	-19.9%	-16.5%	-10.4%	-50.0%	-32.3%	-24.8%	-19.4%	-50.0%	-38.6%	-34.0%	-29.8%
99.99% CVaR	Aaa	-50.0%	-16.7%	-10.0%	-5.3%	-50.0%	-18.3%	-12.0%	-10.3%	-50.0%	-25.8%	-23.0%	-16.3%
	Aa	-50.0%	-18.3%	-12.0%	-9.0%	-50.0%	-33.3%	-20.5%	-15.0%	-50.0%	-36.7%	-32.0%	-27.8%
	A	-50.0%	-20.3%	-16.0%	-10.0%	-50.0%	-34.2%	-23.0%	-17.8%	-50.0%	-40.0%	-35.0%	-32.5%
	Baa	-50.0%	-35.0%	-22.0%	-13.8%	-50.0%	-43.3%	-35.5%	-27.0%	-50.0%	-50.0%	-46.5%	-43.3%

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Combining equilibrium, resampling, and analysts' views in portfolio optimization

José Luis Barros Fernandes,[1] José Renato Haas Ornelas[2]
and Oscar Augusto Martínez Cusicanqui[3]

1. Introduction

Portfolio optimization methodologies play a central role in strategic asset allocation (SAA), where it is desirable to have portfolios that are efficient, diversified, and stable. Since the development of the traditional mean-variance approach of Markowitz (1952), many improvements have been made to overcome problems such as lack of diversification and strong sensitivity of optimal portfolio weights to expected returns.

The Black and Litterman (1992) model (BL) is among the most used approaches. The idea behind this model is that expected returns are the result of two important sources of information: the first is market information in the form of equilibrium returns (implicit returns that clear out the outstanding market allocation), and the second is analysts' views, which tilt the market portfolio to another diversified portfolio compatible with investor beliefs. In this fashion, portfolio managers get an intuitive but formal model to generate optimal allocation.

However, while the BL model offers a very useful and intuitive approach to deal with asset allocation, the inputs considered for the calculation of equilibrium returns are subject to estimation error, and thus expected returns will also contain estimation error. Michaud (1998) proposed the use of a statistical tool known as resampling to deal with estimation error, which is an important source of lack of diversification in mean-variance portfolios. This technique considers that data come from a stochastic process instead of being a deterministic input as in Markowitz (1952).

This paper proposes the use of a portfolio optimization methodology which combines features of both the BL and resampling methodologies. This novel methodology allows the combination of equilibrium and investor's views as in BL, and at same time deals with estimation risk as in Michaud (1998). Thus, it generates robust and diversified optimal allocations which are desirable properties for long-term investors such as central banks and sovereign wealth funds. We empirically test the new methodology using a sample of fixed income and equity indices, achieving very supportive results. We find strong evidence supporting the use of resampling techniques to improve standard models like BL and Markowitz, and this result is more pronounced for medium levels of risk. In general, our proposed methodologies, both with and without views, generated very competitive portfolios compared to the other methodologies, considering the three evaluation dimensions: financial efficiency, diversification, and allocation stability. For medium levels of risk, our methodologies are markedly better than others.

[1] Central Bank of Brazil.

[2] Central Bank of Brazil.

[3] Central Bank of Bolivia.

The views expressed in this work are those of the authors and do not reflect those of the Banco Central do Brasil, Banco Central de Bolivia or its members.

The remainder of this paper is as follows. The next section offers a brief literature review of asset allocation methodologies. The third section describes the BL resampling combined methodology. The fourth section describes the empirical study, including data and implementation and presents the results. The fifth section concludes the paper by reviewing the main results.

2. Literature review

The seminal work of Markowitz (1952) provided the first model for asset allocation, arguing that once expected returns and their joint variance were defined, a set of efficient portfolios could be generated and investors would choose the allocation according to their needs. Basically, the approach could be summarized as follows:

$$\min \frac{1}{2} a^T V a$$

subject to

$$E(R_a) = a^T X$$

where X is the vector of expected excess returns, a is the vector of allocations, and V is the variance-covariance matrix of returns. Despite its mathematical simplicity, this model typically generates concentrated allocations which heavily depend on expected returns estimation. Resampling techniques (Michaud, 1998) were developed as a way to deal with estimation error. Markowitz recognized that resampling methods could be used to obtain better estimates for the inputs of the mean-variance optimization (Markowitz and Usmen, 2003).

Jorion (1991) used the Bayesian approach to overcome the weakness of expected returns estimated solely by sample information. He proposed an estimator obtained by "shrinking" the mean values toward a common value, chosen to be the expected return for the minimum variance portfolio. Kempf et al. (2002) pursued this Bayesian line and considered estimation risk as a second source of risk, determined by the heterogeneity of the market, which is represented by the standard deviation of the expected returns across risky assets. Both methods proved to generate better out-of-sample estimates for expected returns (as opposed to in-sample estimates), and produced more diversified portfolios.

Black and Litterman (1992) built a bridge between statistical methods and expert judgment by recognizing that the capital asset pricing model (CAPM) offers an excellent starting point for expected excess returns. Thus, combing CAPM with investors' views would produce intuitive and diversified allocations. For that, BL assumes that equilibrium returns (CAPM returns that clear out the market) are well described by the following anchor relationship:

$$X \sim N(\Pi, \tau\Sigma)$$

where X is the observed returns vector, which is just a realization of the multivariate normal process with mean Π (equilibrium returns), covariance matrix Σ, and an scale parameter τ which measures the degree of confidence the investor has with regard to equilibrium estimates (the nearer the parameter to zero, the higher the confidence in equilibrium estimates).

In addition to this, BL postulates that returns have another important source of information, coming from investor's views:

$$X \sim N(Q, \Omega)$$

where Q denotes the vector of expected return views (this could be absolute or relative) and Ω is the uncertainty in those views. Since Ω is not an easy-to-obtain parameter, we employ the Idzorek (2004) approach, which measures the uncertainty through a degree of confidence and implicitly calculates Ω. With both sources of information, the combined process is also a multivariate normal, as follows:

$$X \sim N\left(\left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega Q\right], \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\right)$$

where P denotes the portfolio view matrix whose dimension is a function of the number of views (rows) and the number of assets (columns). Needless to say, since market capitalization offers a well-diversified portfolio, the optimal allocation (in general) will have this property, with tilts reflecting investors' views introduced in the model.

Finally, Michaud (1998) adapted the resampling statistic technique to mean-variance optimization, recognizing that return history is just a realization of the stochastic process behind it. Also, only if stationarity holds and in a large sample environment, the point estimates could statistically resemble the true distribution parameters. Suppose that we have a vector of expected excess return X_0 and a variance-covariance matrix denoted by Σ_0 (both estimated with a sample of returns of length k), assuming that returns come from a multivariate normal distribution (with parameters (X_0, Σ_0)), the procedure resamples *n* times joint returns of length k and estimates different parameters ({(X_1, Σ_1), (X_2, Σ_2), …, (X_n, Σ_n)}), which allow us to obtain *n* efficient frontiers. For a given portfolio, the resampled weights are given by the average of portfolio weights of the *n* samples:

$$a_R = \frac{1}{n}\sum_{i=1}^{n} a_i$$

where a_R is the vector of the assets' weights in the resampled portfolio, and a_i s are the weights of each of the *n* realizations.

Several out-of-sample evaluations have shown results in favor of resampling methodology, using different sets of data (see, for instance, Markowitz and Usmen, 2003; Pawley, 2005; Wolf, 2006; Fernandes and Ornelas, 2009). However, these evaluations cannot give definitive conclusions in favor of using resampling, given sampling limitations. Nevertheless, Fernandes and Ornelas (2009) and Kohli (2005) point out that resampled portfolios have two desirable characteristics for long-term investors. First, they usually generate portfolios that have greater diversification as more assets enter into the solution than in classical mean-variance efficient portfolios. Second, the model exhibits smoother transitions and less sudden shifts in allocations as return expectations change, meaning that the transaction costs of rebalancing the portfolio are typically lower. On the other hand, the traditional resampling methodology, considered as an ad hoc methodology, has been criticized because of its lack of a theoretical basis.

3. Description of the BL resampling methodology

Since, as already mentioned, the source of estimation error comes at the first part of the BL model and spreads out until the end, we combine the BL model with the resampling technique. This could be summarized as follows:

1. Estimate the BL expected return vector X and the covariance matrix Σ from historical inputs and possibly also in combination with analysts' views.
2. Resample from the results obtained in Step 1, by taking *n* draws of length *L* from a multivariate normal distribution with return vector X and covariance matrix Σ.
3. For each draw *n*, calculate the new expected return and variance matrix. Because estimation error is present, these resampling estimates are different from the ones calculated in Step 1.

4. For each of the n sets of expected returns and covariance matrix calculated in Step 3, calculate the efficient frontier using traditional Markowitz optimization. The output of this step will be a set of n efficient frontiers.

5. For each risk level, calculate the average portfolio weights across the n efficient frontiers. These weights define the portfolios of the BL resampling frontier. The risk × return profile of the BLR can be calculated using the expected return vector x and the covariance matrix Σ from Step 1.

The BL resampling methodology overcomes the highly criticized weakness of the traditional resampling approach as only an ad hoc methodology. This combined methodology has the theoretical background of an equilibrium model as in Black and Litterman (1992). At the same time, the use of resampling enriches the Bayesian BL approach to optimization, by recognizing estimation error.

4. Empirical study

4.1 Data and implementation

Our tests are based on monthly data of 15 indices of bonds and stocks from six developed countries. For bonds, we use six developed countries' government bond indices from BofA Merrill Lynch, and one US corporate bond index, also from BofA Merrill Lynch. For equities, we use the Thomson Datastream market indices from five countries, and three US equity market indices, divided by market capitalization: S&P500 Composite, S&P 400 Midcap, and S&P600 Small Cap.

Table 1

Descriptive statistics

Name	Market	Instrument Type	Mean	Std	Skewness	Kurtosis
TBill 3M	USA	Bonds	4.56	0.66	-0.13	-0.60
US Govt	USA	Bonds	7.14	4.91	-0.05	0.57
Canada Govt	CAN	Bonds	9.80	9.10	-0.45	1.87
Australia Govt	AUS	Bonds	11.34	12.59	-0.69	1.83
Japan Govt	JAP	Bonds	8.39	13.01	0.51	1.63
Germany Govt	GER	Bonds	9.21	11.72	0.06	0.47
UK Govt	UKG	Bonds	9.62	11.93	0.14	0.73
Germany Equity	GER	Equity	10.32	21.00	-0.53	1.32
Australia Equity	AUS	Equity	15.18	23.11	-1.31	7.01
Japan Equity	JAP	Equity	6.68	23.16	0.37	0.58
Canada Equity	CAN	Equity	12.75	19.08	-0.88	3.65
UK Equity	UKG	Equity	12.00	17.94	-0.40	1.66
SP500	USA	Equity	11.66	15.89	-0.81	2.25
SP600 SMALL	USA	Equity	10.27	18.90	-0.98	3.25
SP400 MID	USA	Equity	11.35	16.93	-0.97	3.22
US Corporate	USA	Corporates	7.81	5.51	-0.74	4.04

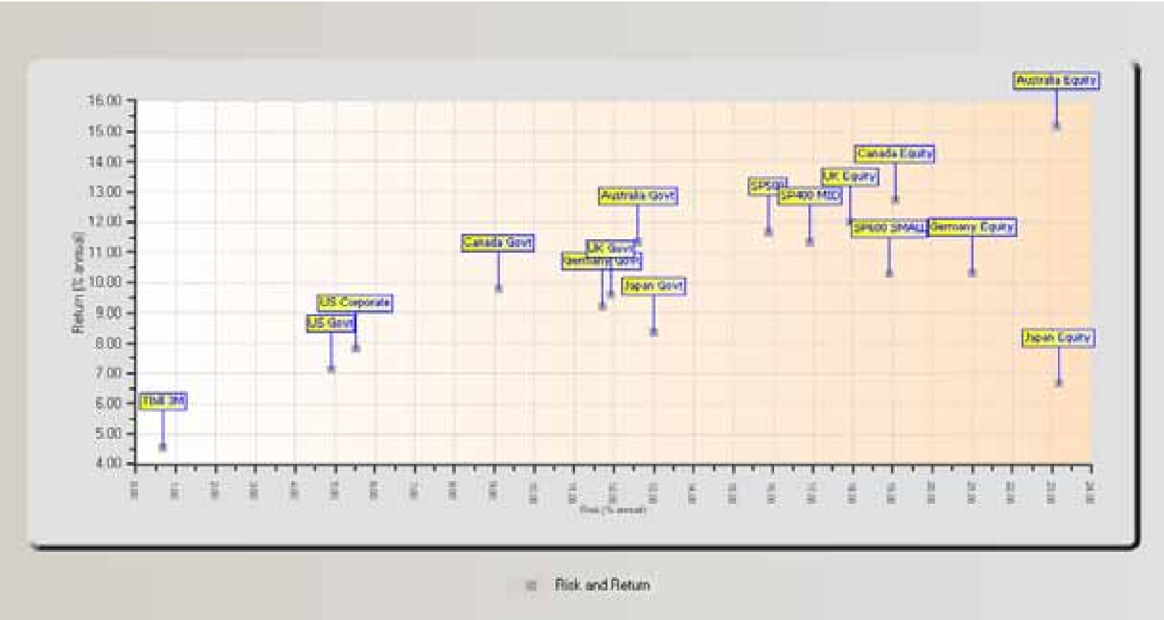
This table presents descriptive information of each asset class considered in the analysis. The presented values are annualized and calculated over the whole sample period.

The period of the sample is from January 1986 to December 2009, with a total of 288 months. The sample has data from market capitalization and total return index levels. All total return time series are calculated on a US-dollar basis, and we use the 3-month US T-Bill rate when calculating excess returns. We are aware that this assumption favors US assets. The US dollar is typically the *numeraire* considered by global investors, such as central banks, pension funds, and multinational institutions. Table 1 presents the descriptive statistics of each asset class considered.

The return time series used present the usual financial statistical characteristics, presenting a positive risk premium, negative skewness, and excess kurtosis. Figure 1 presents the risk return information for the considered asset classes. Note that, except for the Japan equity, there is an upward slope of the tendency line indicating the positive risk premium.

Figure 1

Risk-return estimates of asset classes



The list of methodologies that we address in this paper is shown in Table 2. The first one is the traditional Markowitz (Mark) approach (Markowitz, 1952), which uses quadratic optimization considering historical data to estimate risk and return. Resampling (Res) is the methodology proposed in Michaud (1998).

Two methodologies are based on the Black and Litterman (1992) article. The first one considers just equilibrium risk and return estimates (BL), while the second (BLView) applies the complete BL model, which combines equilibrium estimates with analysts' views. In this latter case, we consider the historical averages as the analysts' views with a confidence of 50% (we use the Idzorek (2004) framework to set the confidence level). It is important to highlight that we are not supporting any specific view methodology, but pointing out that our portfolio optimization technique may be used considering any analysts' views.

Finally, we have two methods that combine BL methodology with resampling, which were described in Section 3. The first one, BLRes, combines the BL model with the resampling methodology. The second, BLViewRes, combines the BL equilibrium model with the same analysts' views of the BLView model and also applies the resampling technique.

Table 2
Methodologies list

Mnemonic	Methodology
Mark	Markowitz Portfolio Selection Model
Res	Portfolio Resampling Model
BL	BL Equilibrium Model
BLView	BL Equilibrium Model with Analysts' Views
BLRes	BL Equilibrium Model with Resampling
BLViewRes	BL Equilibrium Model with Analysts' Views and Resampling

We analyze the performance of these six optimization strategies. We vary the estimation period (*p*) in an out-of-sample analysis. The parameters are estimated using monthly return observations of the past *p* months. Then we define the efficient risky portfolio and hold it for the next (*e*) months. Then we re-estimate the parameters and adjust the portfolio weights. We evaluate the methodology's performance with a no short-selling constraint as is usual in the industry. We consider three levels of investors' risk preference (low, medium, and high) in order to infer if the results are sensitive to investors' level of risk aversion. We use an estimation period of 60 months, and evaluation periods of 6 and 12 months.

To find out which methodology outperforms the others, we take into account three evaluation dimensions – financial performance, allocation stability, and diversification – as ideally an investor would prefer stable, diversified, and financially efficient portfolios. To judge the financial performance of the strategies, we compute their empirical Sharpe ratios, and to evaluate the stability of the allocation weights generated by the methodology, we calculate the average turnover of the portfolio in the re-estimation events. To infer the diversification, we use the mean Herfindahl index, given by the sum of the squared asset allocation weights.

4.2 Results

Table 3 presents the results of the Sharpe ratio, Herfindahl index, and turnover for each methodology considering three levels of risk for the optimal portfolio: low (panel A), medium (panel B), and high (panel C). The estimation period (*p*) considered was 60 months and the evaluation period (*e*) was 6 months. So, the optimizer considers the previous 5 years of data to estimate the frontier portfolios and holds the allocation for the next 6 months, for which the returns are calculated. After that, the frontier is re-estimated again considering the previous 5 years of data, and so on until the end of the sample period.

Comparing first the Markowitz (Mark) results against resampling (Res), we can see that for the three levels of risk, the Res portfolio outperformed the Mark one in all three evaluation dimensions considered. The Res portfolio turned out to be more financially efficient, diversified, and stable. This is a well-known result in the literature when we compare the traditional Markowitz approach with resampling (Markowitz and Usmen, 2003). As resampling techniques typically generate more diversified portfolios, they tend to reduce turnover and increase financial out-of-sample efficiency.

When we compare the BL equilibrium with the BL equilibrium with resampling (BLRes) results, we find out more supportive numbers related to the use of resampling. For every level of risk, the Sharpe ratio, the Herfindahl index, and the turnover are better for the BLRes.

Finally, in the results for the BL with analysts' views (BLView) and the resampled BL with analysts' views (BLViewRes) methodologies, the previous findings remain, with the BLViewRes outperforming in every dimension the BLView.

As a general first conclusion, we found strong evidence supporting the use of resampling techniques to improve standard models, and this result is more pronounced for medium levels of risk. The reason for this might be related to the fact that for intermediate levels of risk in the frontier, we will typically find more substitute asset classes (asset classes with similar risk-return characteristics). For low and high levels of risk in the frontier, we usually find few eligible asset classes and optimal portfolios tend to be more concentrated.

Another interesting result is that our proposed resampling BL methodologies (BLRes and BLViewRes) generated very competitive portfolios when compared to the other methodologies, considering the three evaluation dimensions and the three levels of risk. Specifically for the medium level of risk, our two proposed methodologies outperform all the others in terms of Sharpe ratio and Herfindahl index. For turnover, the equilibrium BL model is the only one that outperforms our proposed methodologies.

For the high-risk portfolios (panel C of Table 3), the low financial performance of the methodologies based on equilibrium (BL and BLRes) is notable. It may be caused by the outlier position of the Japan equity portfolio in Figure 1, since the equilibrium approach would increase the expected returns of this asset because of its high risk. Thus, Japan equities would enter into the optimization with a higher expected return, but realized returns would still be lower. In order to check whether this low performance is related to the presence of the Japan equity in the sample, we ran the optimization exercise without Japan equities. Results for the high-risk portfolios were considerably different, and now favor BL and BLRes against Mark and Res. This is evidence that these equilibrium methodologies are very sensitive to outliers' assets in terms of risk and return.

Table 3

Results *p*=60 and *e*=6

Panel A: 60 – 6 (Low)

	Mark	Res	BL	BLRes	BLView	BLViewRes
Sharpe Ratio	0.539	0.579	0.562	0.600	0.526	0.586
Herfindahl Index	0.446	0.403	0.377	0.354	0.337	0.335
Turnover	0.308	0.254	0.242	0.213	0.281	0.211

Panel B: 60 – 6 (Medium)

	Mark	Res	BL	BLRes	BLView	BLViewRes
Sharpe Ratio	0.367	0.423	0.402	0.518	0.388	0.512
Herfindahl Index	0.296	0.200	0.207	0.102	0.209	0.120
Turnover	0.577	0.375	0.191	0.208	0.494	0.259

Panel C: 60 – 6 (High)

	Mark	Res	BL	BLRes	BLView	BLViewRes
Sharpe Ratio	0.346	0.384	0.174	0.292	0.318	0.367
Herfindahl Index	0.344	0.179	0.339	0.113	0.330	0.126
Turnover	0.715	0.459	0.221	0.249	0.596	0.326

This table presents the results of the Sharpe ratio, Herfindahl index, and turnover for each methodology considering three levels of risk for the optimal portfolio: low (panel A), medium (panel B), and high (panel C). The estimation period (*p*) considered was 60 months and the evaluation period (*e*) was 6 months.

To evaluate the robustness of our findings so far, Table 4 presents the results with an evaluation period of 12 months, ie the optimizer considers the previous 5 years of data to estimate the frontier portfolios and holds the allocation for the next year, for which the returns are calculated. After that, the frontier is re-estimated again considering the previous 5 years of data, and so on until the end of the sample period.

The results are pretty much in line with the ones presented in Table 3, with resampling techniques typically improving results of the standard approaches. Again, this result is more pronounced for intermediate levels of risk. Our proposed methodologies (BLRes and BLViewRes) are still very competitive compared to the other methodologies, considering the three evaluation dimensions and the three levels of risk.

In general, Sharpe ratios are smaller when compared with the results in Table 3. This is due to the fact that as we increase the evaluation period, the out-of-sample financial efficiency tends to diminish. In terms of diversification, the numbers are similar, so it seems that the methodologies are not affected in terms of diversification by the increase of the evaluation period. However, the turnover increased reasonably, indicating that in order to reduce transaction costs, the portfolios should be re-estimated more often.

Table 4

Results *p*=60 and *e*=12

Panel A: 60 – 12 (Low)

	Mark	Res	BL	BLRes	BLView	BLViewRes
Sharpe Ratio	0.439	0.480	0.493	0.534	0.413	0.504
Herfindahl Index	0.435	0.390	0.373	0.337	0.333	0.320
Turnover	0.485	0.428	0.456	0.384	0.439	0.363

Panel B: 60 – 12 (Medium)

	Mark	Res	BL	BLRes	BLView	BLViewRes
Sharpe Ratio	0.271	0.351	0.356	0.429	0.269	0.428
Herfindahl Index	0.297	0.202	0.228	0.101	0.201	0.114
Turnover	0.853	0.589	0.376	0.293	0.727	0.373

Panel C: 60 – 12 (High)

	Mark	Res	BL	BLRes	BLView	BLViewRes
Sharpe Ratio	0.270	0.337	0.110	0.272	0.216	0.336
Herfindahl Index	0.347	0.188	0.372	0.118	0.328	0.128
Turnover	0.999	0.670	0.458	0.379	0.781	0.456

This table presents the results of the Sharpe ratio, Herfindahl index, and turnover for each methodology considering three levels of risk for the optimal portfolio: low (panel A), medium (panel B), and high (panel C). The estimation period (*p*) considered was 60 months and the evaluation period (*e*) was 12 months.

5. Conclusion

This paper deals with a well-documented issue in mean-variance optimization, related to the fact that this methodology typically leads to unintuitive portfolios with extreme positions in asset classes. We proposed the use of an optimization approach that takes advantage of both BL and resampling techniques to incorporate the main positive aspects of both previous powerful techniques. It is a stochastic general equilibrium model, which can be used as a tool for both passive and active strategies. The main idea is to estimate the efficient frontier using the BL model but consider this frontier as just an input to the resampling method.

We empirically test our methodology using a comprehensive sample of bond and stock indices. Compared to traditional portfolio optimization methodologies, we have reached very supportive results. We found strong evidence supporting the use of resampling techniques to improve standard methodologies, and this result is more pronounced for medium levels of risk. The reason for this might be related to the fact that for intermediate levels of risk in the frontier, we will typically find more substitute asset classes (asset classes with similar risk-return characteristics). For low and high levels of risk in the frontier, we usually find few eligible asset classes and optimal portfolios tend to be more concentrated.

Generally speaking, our proposed methodologies, both with and without views, generated very competitive portfolios compared to the other methodologies, considering the three evaluation dimensions: financial efficiency, diversification, and allocation stability. For medium levels of risk, our methodologies are markedly better than others.

It is important to highlight that a recommendation of a specific methodology for deriving analysts' views is out of the scope of the present study. The view considered in this article is just a naive example to show that the proposed methodology may be adapted to the analysts' views. We argue that the proposal of views methodologies is still an open avenue for future research in portfolio management.

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Portfolio optimization and long-term dependence[1]

Carlos León[2] and Alejandro Reveiz[3]

Introduction

It is a widespread practice to use daily or monthly data to design portfolios with investment horizons equal or greater than a year. The computation of the annualized mean return is carried out via traditional interest rate compounding – an assumption free procedure –, while scaling volatility is commonly fulfilled by relying on the serial independence of returns' assumption, which results in the celebrated square-root-of-time rule.

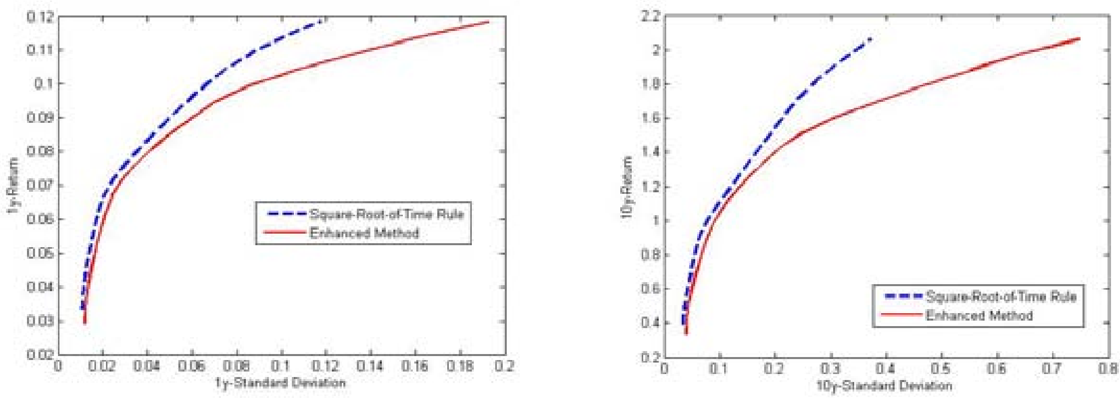
While it is a well-recognized fact that the serial independence assumption for asset returns is unrealistic at best, the convenience and robustness of the computation of the annual volatility for portfolio optimization based on the square-root-of-time rule remains largely uncontested.

As expected, the greater the departure from the serial independence assumption, the larger the error resulting from this volatility scaling procedure. Based on a global set of risk factors, we compare a standard mean-variance portfolio optimization (eg square-root-of-time rule reliant) with an enhanced mean-variance method for avoiding the serial independence assumption. Differences between the resulting efficient frontiers are remarkable, and seem to increase as the investment horizon widens (Figure 1).

Figure 1

Efficient frontiers for the standard and enhanced methods

1-year 10-year



Source: authors' calculations.

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[2] Research and Development Manager, Financial Infrastructure Oversight Department, Banco de la República (Central Bank of Colombia). Corresponding author: Tel. +57 1 343 0731; e-mail. cleonrin@banrep.gov.co / carlosleonr@hotmail.com. Involvement in this document occurred while working as Researcher for the Foreign Reserves Department, Banco de la República.

[3] Senior Investment Officer, World Bank, areveiz@worldbank.org.

Because this type of error lurks beneath customary asset allocation procedures, including the prominent Black-Litterman (1992) model, the main objective of this paper is to challenge the square-root-of-time rule as a proper volatility scaling method within the mean-variance framework, and to present a robust alternative.

In order to fulfill the stated objective, this paper estimates financial assets' long-run dynamic. The impact of long-term serial dependence on asset returns is assessed for a wide set of markets and instruments, with a sample which covers the most recent market turmoil. The estimation relies on a revised and adjusted version of the classic rescaled range analysis methodology (*R/S*) first introduced by Hurst (1951) and subsequently enhanced by Mandelbrot and Wallis (1969a and 1969b).

Similar to Hurst's results in geophysics and to financial literature (Malevergne and Sornette, 2006; Los, 2005; Daníelsson and Zigrand, 2005), the results confirm that numerous individual risk factors exhibit significant long-term dependence, thus invalidating the square-root-of-time rule. Interestingly, most previous findings related to long-term dependence in financial time series are still supported, even after the most recent period of market crisis.

The results also demonstrate that some major asset allocation issues could be explained to some extent by the inability of the square-root-of-time rule to properly scale up volatility in the presence of serial long-term dependence. Some of these issues are: (i) excessive risk taking in long-term portfolios (Valdés, 2010; Reveiz et al. 2010; Pastor and Stambaugh, 2009; Schotman et al. 2008); (ii) a tendency to hold a disproportionate level of investments within the domestic market –home bias– (Solnik, 2003; Winkelmann, 2003b); (iii) a reluctance to hold foreign currency-denominated assets (Lane and Shambaugh, 2007; Davis, 2005); and (iv) the presence of extreme portfolio weights or "corner solutions" (Zimmermann et al. 2003; He and Litterman, 1999).

This paper consists of six chapters; this introduction is the first one. The second chapter presents a brief examination of the square-root-of-time rule and its use for scaling high-frequency volatility (eg daily) to low-frequency volatility (eg annual). The third describes and develops the classic rescaled range analysis (*R/S*) methodology for detecting and assessing the presence of long-term serial dependence of returns. The fourth chapter exhibits the results of applying an adjusted version of *R/S* to selected risk factors. The fifth analyzes the consequences of the results for portfolio optimization. Finally, the last chapter highlights and discusses some relevant remarks.

1. The square-root-of-time rule

The square-root-of-time rule consists in multiplying the standard deviation calculated from a *d*-frequency (eg daily) time series by the square-root of *n*, where *n* is the number of *d* units to scale standard deviation up. For example, if σ_d is the standard deviation of a *d*-frequency time series, to scale volatility to an *a*-frequency, where $a = dn$, σ_d should be multiplied by the square-root of *n*, as follows:

$$\sigma_a = \sigma_{dn} = \sigma_d \sqrt[2]{n} = \sigma_d n^{0.5} \qquad \text{F1}$$

The value of this rule is evident for market practitioners: as acknowledged by Dowd et al. (2001), obtaining time series suitable –long enough– to make reliable volatility estimations for monthly or annual frequencies is rather difficult. Besides, even if such time series do exist, questions about the relevance of far-in-the-past data may arise.

Perhaps the most celebrated application of the square-root-of-time rule has to do with Value at Risk (VaR) estimation. According to the technical standards originally established by the Basel Committee on Banking Supervision (BIS, 1995), the VaR must be calculated for at least a ten-day holding period. VaR estimations could be based on shorter holding periods (eg using daily time series), but the ten-day holding period VaR should be attained by means of scaling up to ten days by the square-root-of-time.[4]

Discussing Bachelier's (1900) contribution to the construction of the random-walk or Brownian motion model, Mandelbrot (1963) described it as follows: if $Z(t)$ is the price of a stock at the end of time period t, successive differences of the form $Z(t+T)$ - $Z(t)$ are (i) independent, (ii) Gaussian or normally distributed, (iii) random variables (iv) with zero mean and (v) variance proportional to the differencing interval T.

These assumptions have been notably challenged by mere observation of financial markets, and rejected using traditional significance tests. Nevertheless, methodologies and practices based on the Brownian motion still endure; one of such lasting practices is volatility scaling *via* the square-root-of-time rule, which is the most important prediction of the Brownian motion model (Sornette, 2003).

The assumption underlying the square-root-of-time rule is independence. Under this assumption past behavior of the variable is irrelevant. This is also known as the weak form of the Efficient Market Hypothesis (EMH), and it is the core hypothesis of the martingale model for asset pricing, which states that the current price is the best forecast for future price (Campbell et al., 1997).

Under the independence assumption the probability distribution of changes in the same variable for two or more periods is the sum of the probability distribution; when two independent normal distributions are added, the result is a normal distribution in which the mean is the sum of means and the variance is the sum of variances (Hull, 2003).

Accordingly, if the probability distribution of changes of an independent variable (Ω) has an *A-B* range (Figure 2, left panel), the resulting range at the end of two periods will be proportional to twice *A-B*, and for three periods it will be proportional to three times *A-B*; it is irrelevant whether the probability distribution (Ω) is Gaussian or not.

If the distribution is Gaussian, the *A-B* range can be conveniently characterized by the variance. Hence, if the distribution of Ω can be defined as $\Omega \sim N(0,1)$, where N stands for normally distributed, zero is the mean and 1 the variance, after three periods the distribution of the possible values of the – independent – variable corresponds to $\Omega \sim N(0,1+1+1)$ or $\Omega \sim N(0,3)$.

Alternatively, the *A-B* range can be characterized by a different dispersion metric: standard deviation. However, because standard deviation corresponds to the square-root of variance, it is not additive; therefore, the three-period distribution of possible values of the – independent – variable corresponds to $\Omega \sim N\left(0,\sqrt[2]{1+1+1}\right)$ or $\Omega \sim N\left(0,\sqrt[2]{3}\right)$. This is the origin of the square-root-of-time-rule.

In the absence of independence this rule is no longer valid. As the right panel of Figure 2 reveals, if we let a return above the mean lead to a different (ϕ) more disperse distribution (*C-D*>*A-B*) – which is an example of dependence – then it is impossible to affirm neither that the resulting range at the end of two periods is going to be proportional to twice *A-B*, nor twice *C-D*. This impossibility applies even if the distributions (ϕ and Ω) are strictly Gaussian, and it would cause any standard rule to scale range, variance or standard deviation to falter.

Moreover, the presence of long-term dependence not only invalidates any use of the square-root-of-time rule, but helps explain the slow convergence of the distribution of financial assets' returns towards normality, even for low-frequency (eg monthly, quarterly) data (Malevergne and Sornette, 2006).

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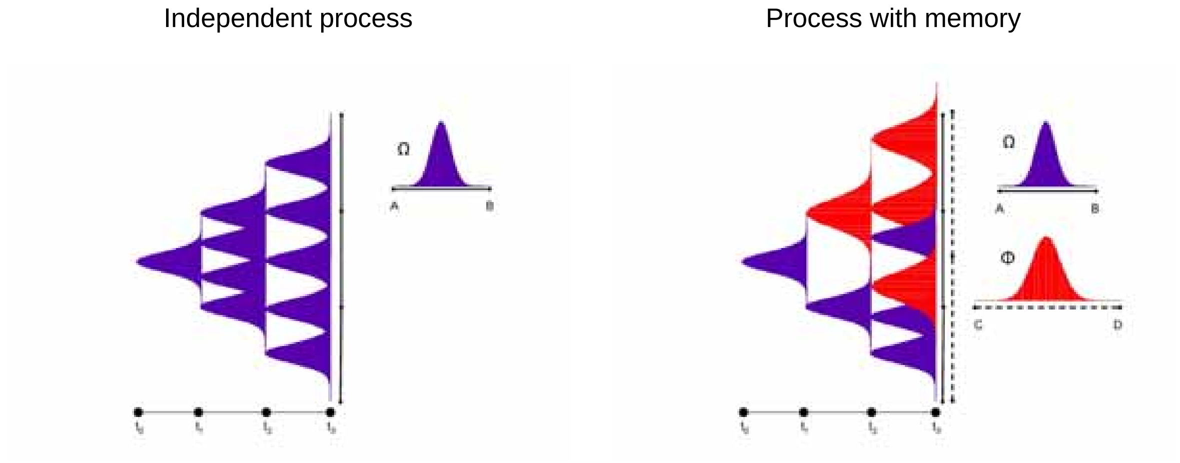
4 Technical caveats to the usage of the square-root-of-time rule were recently introduced (BIS, 2009).

Figure 2

Independence and the square-root-of-time rule



Source: authors' design.

While asset returns' independence has been accepted as one of the core foundations in economics and finance since Bachelier (1900), contradictory evidence also dates back to the dawn of the 20th century (Mitchell, 1927; Mills, 1927; Working, 1931; Cowles and Jones, 1937). However, it was complex natural phenomena which forced physicists to deal with the absence of independence. Geophysics, not economics or finance, was the source of methodologies to identify and measure long-term dependence.

2. Rescaled range analysis (*R/S*)

Long-term dependence detection and assessment for time series began with hydrology (Mandelbrot and Wallis, 1969a), when the British physicist H.E. Hurst (1880–1978) was appointed to design a water reservoir on the Nile River. The first problem Hurst had to deal with was to determine the optimal storage capacity of the reservoir; that is, restricted to a budgetary constraint, design a dam high enough to allow for fluctuations in the water supply whilst maintaining a constant flow of water below the dam.

Deciding on the optimal storage capacity depended on the inflows of the river, which were customarily assumed to be random and independent by hydraulic engineers at that time. Nonetheless, when checking the Nile's historical records (622 B.C.–1469 B.C.) Hurst discovered that flows could not be described as random and independent: data exhibited persistence, where years of high (low) discharges were followed by years of high (low) discharges, thus describing cycles but without an obvious periodicity.

Hurst concluded that (i) evidence contradicted the long-established independence assumption and (ii) that the absence of significant autocorrelation proved standard econometric tests to be ineffective (Peters, 1994). Thus, since the absence of independence vindicated caring about the size and sequence of flows, Hurst developed a methodology capable of capturing and assessing the type of dependence he had documented.

Hurst's methodological development was based on Einstein's (1905) work about particle movement, which Scottish botanist Robert Brown (1828 and 1829) had already depicted as inexplicable, irregular and independent. Einstein originally formulated that the distance or average displacement (R) covered by a particle suspended in a fluid per unit of time (n) followed $R = n^{0,5}$; this is analogous to the square-root-of-time rule.

Unlike Brown and Einstein, Hurst's primary objective was a broad formula, capable of describing the distance covered by any random variable with respect to time. Hurst found his observations of several time series were well represented by $R \sim cxn^{H}$, where H corresponds to the way that distance (R) behaves with respect to time.

Hurst defined that the metric for the distance covered per unit of time or sample (n) would be given by the range R_n [F2], where $x_1, x_2, x_3 .. x_n$ correspond to the change of the random variable within the sample, and $\overline{x_n}$ is the average of these changes. Range R_n is standardized by the standard deviation of the sample for that period (S_n), which results in the rescaled range for the n sample $(R/S)_n$ [F2].

$$\left(R/S\right)_n = R_n/S_n = \frac{\left[\max\limits_{1 \le k \le n}\left(\Sigma_{j=1}^{k}\left(x_j - \overline{x_n}\right)\right) - \min\limits_{1 \le k \le n}\left(\Sigma_{j=1}^{k}\left(x_j - \overline{x_n}\right)\right)\right]}{S_n} \qquad \text{F2}$$

Hurst found that the behavior of this rescaled range [F2] adequately fitted the dynamic of numerous time series from natural phenomena, where the adjustment could be represented as follows [F3]:

$$\left(R/S\right)_n \sim cxn^{H} \qquad \text{F3}$$

Paraphrasing Peters (1992), Hurst's novel methodology measures the cumulative deviation from the mean for various periods of time and examines how the range of this deviation scales over time. $\hat{H}$, the estimated exponent that measures the way distance (R) behaves with respect to time, takes values within the 0 and 1 interval ($0 < \hat{H} \le 1$), where $\hat{H}=0.5$ corresponds to Einstein's and Brown's independency case.

Mandelbrot and Wallis (1969a and 1969b) proposed to plot Hurst's function [F3] for several sample sizes (n) in a double logarithmic scale, which served to obtain $\hat{H}$ through a least squares regression. $\hat{H}$ would be the slope of the estimated equation [F4]; this procedure is known as the rescaled range analysis $\left(R/S\right)$.

$$Log\left(R/S\right)_n = Log(c) + HLog(n) \qquad \text{F4}$$

According to Mandelbrot (1965) the application of *R/S* to random series with stationary and independent increases, such as those characterized by Brown (1828 and 1829) and Einstein (1905), results in $\hat{H}=0.5$, even if the distribution of the stochastic process is not Gaussian, in which case $\hat{H}$ asymptotically converges to 0.5 ($\hat{H} \approx 0.5$).

As said by Sun et al. (2007), in the $\hat{H}=0.5$ and $\hat{H} \approx 0.5$ cases the process has no memory – is independent – hence the next period's expected result has the same probability of being lower or higher than the current result. Applied to financial time series this is akin to assuming that the process followed by asset returns is similar to coin tossing, where the probability of heads (rise in the price) or tails (fall in the price) is the same (½), and is independent of every other toss; this is precisely the theoretical base of the Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT), the Black & Scholes model and the Modern Portfolio Theory (MPT).

When $\hat{H}$ takes values between 0.5 and 1 ($0.5 < \hat{H} \le 1$) evidence suggests a persistent behavior; therefore, one should expect the result in the next period to be similar to the current one (Sun et al., 2007). According to Menkens (2007) this means that increments are positively correlated: if an increment is positive, succeeding increments are most likely to be positive than negative. In other words, each event has influence on future events; therefore there is dependence or memory in the process. Moreover, as $\hat{H}$ becomes closer to one (1) the range of possible future values of the variable will be wider than the range of purely random variables; Peters (1996) argues that the presence of persistency is a signal that today's behavior does not influence near future only, but distant future as well.

On the other hand, when $\hat{H}$ takes values below 0.5 ($0 \leq \hat{H} < 0.5$) there is a signal that suggests an antipersistent behavior of the variable. This means, as said by Sun et al. (2007), that a positive (negative) return is more likely followed by negative (positive) ones; hence, as stated by Mandelbrot and Wallis (1969a), this behavior causes the values of the variable to tend to compensate with each other, avoiding time series' overshooting. Applied to financial market series, Menkens (2007) affirms that this kind of continuously compensating behavior would suggest a constant overreaction of the market, one that would drive it to a permanent adjustment process. Similarly, Peters (1996) links this behavior to the well-known "mean-reversion" process.

Hurst's methodology and results[5] were gathered, corrected and reinterpreted by Mandelbrot (1972) and Mandelbrot and Wallis (1969a and 1969b). Based on random simulation models they verified that (i) Hurst's conclusions were correct, but his calculations were imprecise; (ii) their corrected version of *R/S* is robust to detect and measure dependence, even in the presence of significant excess skewness or kurtosis;[6] (iii) their corrected version of *R/S* is asymptotically robust to short-term dependency (eg autoregressive and moving average processes); (iv) asymptotically $\hat{H}=0.5$ for independent processes, even in the absence of Gaussian processes; and (v) in contrast to other methodologies, *R/S* can detect non-periodic cycles.

Shortcomings of Mandelbrot's (1972) and Mandelbrot and Wallis' (1969a and 1969b) developments regarding the presence of significant long-term dependence in financial time series were depicted by Lo (1991). He introduced modified rescaled range methodology (*mR/S*) as an effort to establish whether *R/S* results are due to the presence of genuine long-term dependence, or to some sort of short-term memory.

Despite considering comparative results of both *R/S* and *mR/S* as inconclusive, Los (2003) states that evidence documented by Peters (1994) shifts the balance of proof in the direction of the existence of the long-term dependence in financial assets' time series. Peters (1994) works on long-term dependence in capital markets discarded autoregressive processes (AR), moving average (MA) and autoregressive moving average (ARMA) as sources of the persistence effect or long-term memory that is captured by the *R/S*, while generalized autoregressive conditional heteroskedasticity (GARCH) processes showed a marginal persistence effect only.[7]

Although literature about short-term dependence in asset returns is abundant, that on long-term dependence is rather scarce, whereas *R/S* is a popular and robust methodology. As exhibited in Figure B1 (Annex B), evidence on *R/S* application to currencies, stock indexes, fixed income securities and commodities supports the long-term dependence hypothesis, as

5 Hurst (1956) studied 76 natural phenomena. $\hat{H}$ was significantly different from 0.5, and was close to 0.73 ($\sigma = 0.092$). Hurst found no evidence of significant autocorrelation in the first lags, which led him to reject short-term dependence as the source of this phenomenon; neither could he find a slow and gradual decay with increasing lags, which supported his rejection for long-term dependence in the traditional sense of Campbell et al. (1997).

6 Mandelbrot and Wallis (1969a) were the first to recognize *R/S* as non-parametric, even in the presence of extreme skewness or with infinite variance. León and Vivas (2010), Martin et al. (2003), Willinger et al. (1999) and Peters (1996 and 1994) verified this statement.

7 Moreover, since the purpose of this paper is not to establish the source of dependence, either short-term or long-term, but to detect and measure its impact on financial asset returns' long-run dynamic, Lo's (1991) criticism is rather irrelevant.

well as Peters' (1996) statement regarding the difficulty of finding antipersistent financial time series.

Evidence of significant antipersistence has been documented for energy prices; Weron and Przybylowicz (2000) explain such findings as resulting from energy's particularities (eg market regulation, storage problems, transmission, distribution). Reveiz (2002) documents similar findings for currencies floating within a band that introduces non-linear features to foreign exchange trading.

Peters (1996 and 1989) concluded that asset returns do not follow a pure random walk, but exhibit some degree of persistence ($0.5 < \hat{H} \leq 1$); Peters named this type of tainted random walk "biased random walk". When asset returns follow a biased random walk they trend in one direction until some exogenous event occurs to change their bias. The presence of persistency, according to Peters, is evidence that new events are not immediately reflected in prices, but are manifested as an enduring bias on returns; this contradicts the EMH.

Some explanations for financial assets' return persistence are found in human behavior, since the latter contradicts the rationality assumption in several ways; for example: (i) investors' choices are not independent, and they are characterized by non-linear and imitative behavior (LeBaron and Yamamoto, 2007; Sornette, 2003); (ii) investors resist changing their perception until a new credible trend is established (Singh and Dey, 2002; Peters, 1996); and (iii) investors do not react to new information in a continuous manner, but rather in a discrete and cumulative way (Singh and Dey, 2002).

Other explanations for financial assets' return persistence have to do with the importance of economic fundamentals (Nawrocki, 1995; Lo, 1991; Peters, 1989) and the use of privileged information (Menkens, 2007). Alternatively, some authors (Bouchaud et al., 2008; Lillo and Farmer, 2004), based on the persistence of the number and volume of buying and selling orders in transactional systems, conclude that markets' liquidity makes instantaneous trading impossible, leading to transaction splitting, and decision clustering, resulting in market prices which fully reflect information not immediately, but incrementally.

3. Estimated Hurst exponent ($\hat{H}$) for major risk factors

Estimating the Hurst exponent ($\hat{H}$) requires the implementation of the algorithm described in the Appendix, and the design of significance tests for evaluating the null hypothesis of independence.

Confidence intervals and significance tests

One of the main difficulties of *R/S* methodology is the selection of an ad-hoc optimal size of periods (n) to calculate $(R/S)_n$. In the literature there is consensus about *R/S* not being reliable for reduced periods because estimations may become unstable and biased (Cannon et al., 1997; Peters, 1994; Ambrose et al., 1993). However, there is no consensus about an optimal minimum size of periods (n_{min}).[8]

The same issue arises with the choice of optimal maximum period size (n_{max}). Cannon et al. (1997) and Peters (1996) recognize that the stability of $\hat{H}$ diminishes when using extended

8 Cannon et al. (1997) estimate optimal minimum size of periods to be $n_{min} \geq 2^8$ (≥ 256 observations) to achieve standard deviations below 0.05; Mandelbrot and Wallis (1969a) use 20 observations; Wallis and Matalas (1970) point out that the window must have at least 50 observations, unless series are of considerable length; Peters (1994) acknowledges that financial series are not long enough to discard reduced windows, and suggests at least 10 observations; Nawrocki (1995) argues that minimum number of observations should be large enough to minimize the effect of short-term dependence.

periods. Therefore, Cannon et al. advise dismissing the use of data windows where estimations are made on a few segments of the time series.

Given the absence of consensus on the optimal period size, all calculations were made using a minimum size of 32 observations ($n_{min} \geq 2^5$). This choice not only recognizes the intricacy of finding extended time series (Peters, 1994), but also results in reduced standard errors of the estimators in the sense of Cannon et al. (1997), and guarantees that the effect of conventional short-term serial dependence (eg autocorrelation) for a daily-frequency series is minimized (Nawrocki, 1995).

The maximum period size constraint (n_{max}) consists of restricting time series to be divided into at least ten contiguous non-overlapping segments; in this way, estimations based on a narrow number of samples and unstable estimators are avoided.

Concerning significance tests for $\hat{H}$, two well-documented issues have to be taken into account (León and Vivas, 2010; Ellis, 2007; Couillard and Davison, 2005; Peters, 1994). First, there is a positive bias in the estimation – overestimation – of H resulting from finite time series and a minimum size of periods below approximately 1,000 observations. Second, $\hat{H}$ distributes like a normal regardless of the empirical distribution of the random variables.

Regarding the first issue, the estimation bias resulting in the overestimation of $\hat{H}$ can be conveniently assessed. Several assessment methods for estimating such bias have been documented, but this work focuses on the single most well-known. First proposed by Anis and Lloyd (1976), subsequently revised by Peters (1994), and recently verified and applied by León and Vivas (2010), Ellis (2007) and Couillard and Davison (2005), the chosen method consists of a functional approximation for estimating the expected value of $(R/S)_n$ when the random variable is independent and of finite length. This method yields the expected Hurst exponent corresponding to an independent random variable, which will be noted as $\dot{H}$, and is based on the following calculation of the expected value of $(R/S)_n$:

$$E\left(R/S\right)_n = \frac{n-\frac{1}{2}}{n} \frac{1}{\sqrt{n\pi/2}} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}} \qquad \text{F5}$$

Any divergence of $\hat{H}$ from $\dot{H}$ would signal the presence of long-term memory in the time series. However, as customary in statistical inference, it is critical to develop appropriate statistical tests to distinguish between significant and non-significant deviations from the long-term independence null hypothesis.

The significance test used is similar to those proposed by Ellis (2007) and Couillard and Davison (2005). Because $\hat{H}$'s distribution is established to be normal, even for random variables that are not, a conventional *t*-statistic test may be implemented. Let $\hat{H}$ be the *R/S*'s estimated value of the Hurst exponent, $\hat{\mu}(\dot{H})$ and $\hat{\sigma}(\dot{H})$ the expected value and standard deviation of the expected Hurst exponent corresponding to an independent random variable ($\dot{H}$); the significance test would be as follows:[9]

$$t = \frac{\hat{H} - \hat{\mu}(\dot{H})}{\hat{\sigma}(\dot{H})} \qquad \text{F6}$$

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[9] Let N be the length of time series, due to $\hat{H}$ distributing like a normal the ordinary choice for $\hat{\sigma}(\dot{H})$ is $\approx 1/N^{1/2}$ as in Peters (1994). According to Couillard and Davison (2005), this choice corresponds to an infinite length time series, and yields easy and frequent rejections of the independence null hypothesis. They propose $\approx 1/eN^{1/3}$, which is the authors' choice.

As usual, if t is higher than ±1.96 it is possible to reject the null hypothesis of long-term independence with a 95% confidence level. The sign of t reveals the type of dependence: if it is positive (negative) there is evidence of persistence (antipersistence).

For convenience, given that $\dot{H}$ is the estimated Hurst exponent for random, independent and finite time series of length N, the spread between $\dot{H}$ and 0.5 corresponds to the bias estimation resulting from using finite time series and the choice of the size of periods (n). Subtracting such spread from the Hurst exponent estimated using R/S, namely $\hat{H}$, results in an adjusted estimated Hurst exponent, which will be noted as $\breve{H}$:

$$\breve{H} = \hat{H} - (\dot{H} - 0.5) \qquad \text{F7}$$

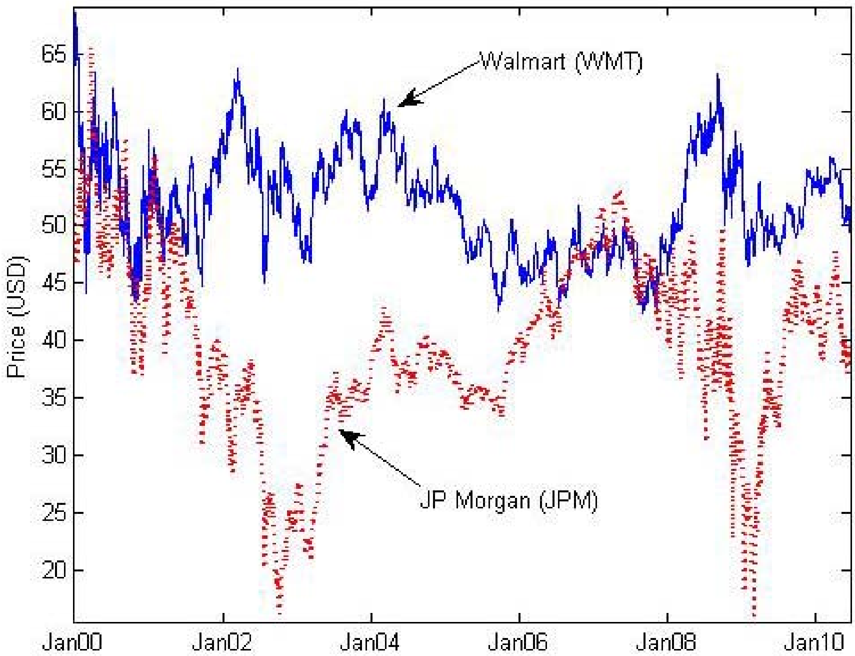
This adjusted estimated Hurst exponent ($\breve{H}$) is essential since it allows a practical and unbiased volatility scaling as will be presented in the following sections. Unlike prior literature on the estimation of the Hurst exponent in Finance and Economics, adjusting for estimation bias allows for practical applications such as portfolio theory and risk.

Estimated values of Hurst exponent ($\hat{H}$)

Figure 3 exhibits the Walmart and JP Morgan price-series from January 1st 2000 to June 25th 2010. Walmart's exhibits a narrower range in which prices fluctuate, where returns appear to compensate each other, while JP Morgan's appear to persist over time; since both share the same dollar scale, it is somewhat apparent that JP Morgan's time series are more persistent than Walmart's.

Figure 3

Daily prices for Walmart and JP Morgan



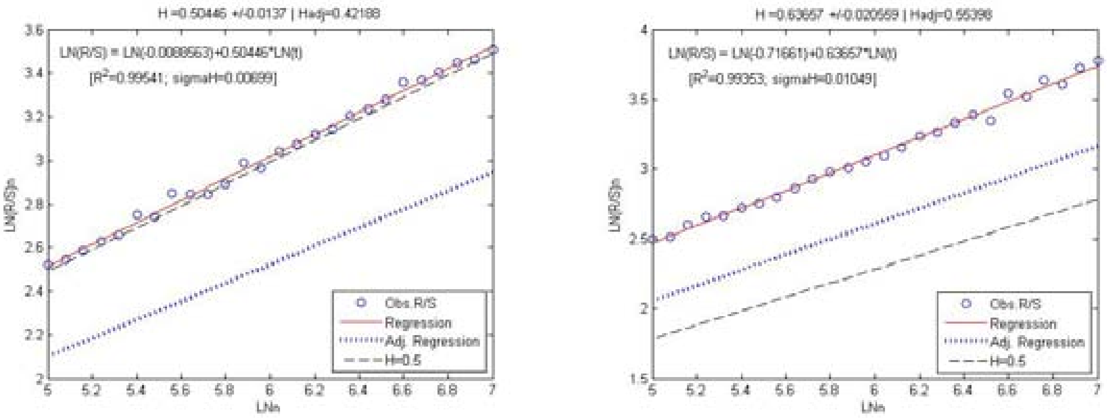
Source: authors' calculations.

Figure 4 exhibits the graphical result of applying R/S to both series' returns. Walmart exhibits an estimated Hurst exponent slightly above 0.5 ($\hat{H}_{WMT} = 0.504$), which would be a signal of non-significant persistence, while JP Morgan's $\hat{H}$ clearly diverges from 0.5 ($\hat{H}_{JPM} = 0.637$). Nevertheless, after acknowledging the positive estimation bias, the adjusted estimated Hurst exponent reveals that Walmart's time series is in fact significantly antipersistent ($\breve{H}_{WMT} = 0.422$; $t_{WMT} = -2.96$), whereas JP Morgan's remains significantly persistent ($\breve{H}_{JPM} = 0.554$; $t_{JPM} = 2.04$).

Figure 4

Walmart and JP Morgan (adjusted and unadjusted Hurst exponent)

Walmart (WMT) JP Morgan (JPM)



Source: authors' calculations.

Figure 5 exhibits the adjusted estimated Hurst exponent ($\breve{H}$) for individual risk factors (small dots) pertaining to different markets (eg developed and emerging) and instruments (fixed income, equity and commodities). As before, if the adjusted estimated Hurst exponent ($\breve{H}$) is greater (lower) than 0.5 there exists evidence of persistence (antipersistence), where the area between the vertical lines corresponds to the 95% confidence interval in which the independence hull hypothesis cannot be rejected.

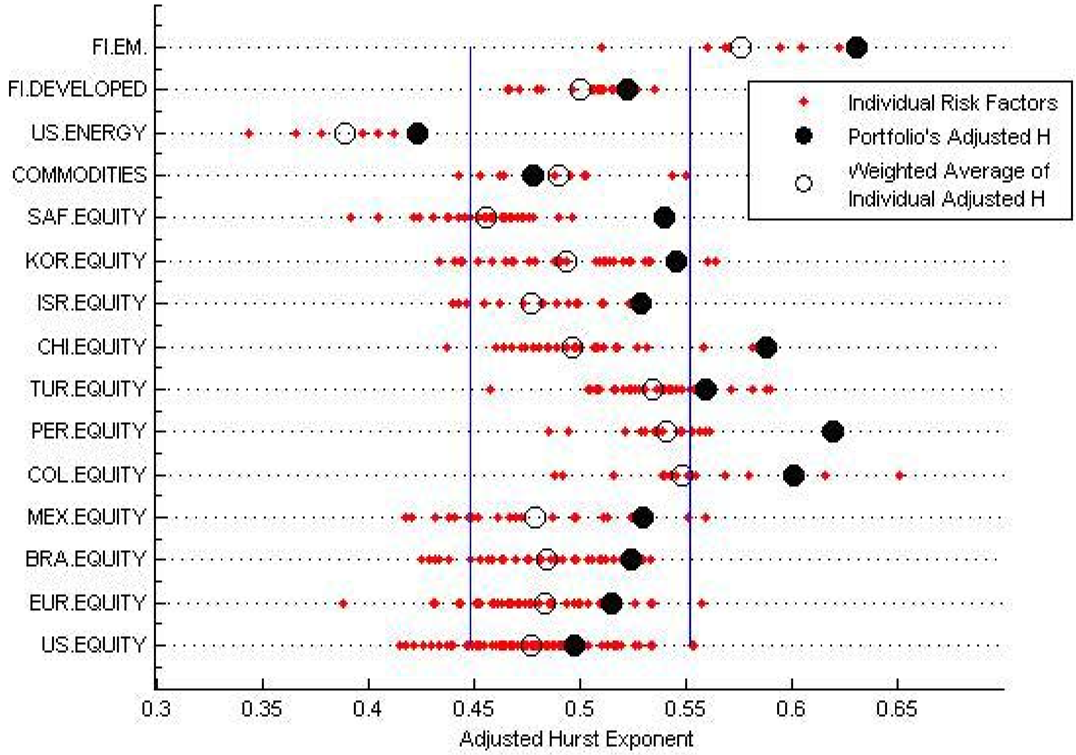
Individual risk factors across markets and instruments display different degrees of dependence, where persistence is typical of emerging markets' fixed income instruments (FI.EM) and of less-developed equity markets (eg Colombia, Turkey and Peru). Developed equity markets (eg US and EUR) and liquid emerging markets (eg Brazil, Mexico) show less incidence of persistent individual risk factors, even with several cases of antipersistence. These findings support Cajueiro and Tabak's (2008) comparison between developed and emerging markets. Results also correspond to the findings of Weron and Przybylowicz (2000) in relation to significant antipersistence of energy prices, but contradict Peters' (1996) affirmation about the difficulty of finding financial time series with antipersistent returns.

Regarding persistence at the portfolio level, Figure 5 displays the adjusted estimated Hurst exponent ($\breve{H}$) for an equally weighted portfolio of the individual risk factors (filled circles) and the equally weighted average of the individual risk factors' adjusted estimated Hurst exponent (empty circles). It is remarkable that the portfolios' adjusted estimated Hurst exponent tends to be higher than the weighted average of the individual exponents, which would indicate that the diversification effect does not apply to serial dependence as it does to variance or standard deviation.

It is also noteworthy that for emerging fixed income and equity markets the portfolios' adjusted estimated Hurst exponent ($\breve{H}$) is significantly higher than the weighted average of the individual exponents. Because aggregating risk factors should result in specific or idiosyncratic risk diversification, this could indicate that the remaining systemic risk is relatively more important for emerging than for developed markets; this could be the result of poor diversification opportunities within a small and illiquid market, or of the generalized impact of systemic shocks and the corresponding changes in risk appetite and liquidity in those markets.

Figure 5

Adjusted estimated Hurst exponent ($\breve{H}$)



Source: authors' calculations.

The markets included are: FI.EM. = Emerging Markets' Fixed Income (EMBI Global of Brazil, Mexico, Colombia, Peru, South Africa, Turkey and Chile); FI.DEVELOPED = Developed Markets' Fixed Income (as in Table 2); US.ENERGY = Off-peak day ahead electricity for several U.S. regions; COMMODITIES = oil, gold, copper, wheat, corn, cotton, aluminum, sugar, coffee, cocoa, rice, soy; and a market-capitalization representative set of securities from the equity markets of the United States (U.S.), Europe (EUR), Brazil (BRA), Mexico (MEX), Colombia (COL), Peru (PER), Turkey (TUR), Chile (CHI), Israel (ISR), Korea (KOR) and South Africa (SAF). All estimations were based on January 1st 2000-June 25th 2010 time series, except US.ENERGY (January 1st 2002–June 25th 2010).

4. Portfolio optimization under long-term dependence

The most far-reaching consequences of long-term dependence or memory in financial asset returns were pointed out by Lo (1991). He recognized that the long-term dependence conveys the invalidity of modern finance's milestones, where the most hard-hit would be the optimal consumption/savings and portfolio decisions, as well as the pricing of derivatives based on martingale methods.

Volatility scaling, investment decisions and portfolio optimization

Conventional portfolio optimization uses high-frequency data and customary procedures for return and volatility scaling in order to obtain allocations for low-frequency horizons. Let $\hat{\mu}_d$ and $\hat{\sigma}_d$ be the estimated high-frequency (eg daily) continuously compounded expected return and standard deviation, $\hat{\mu}_a$ and $\hat{\sigma}_a$ the estimated low-frequency (eg annual) continuously compounded expected return and standard deviation, and p the number of days-in-a-year convention. The standard procedure for asset allocation typically involves the following expected return [F8] and volatility escalation [F9]:

$$\hat{\mu}_a \sum_{t=1}^{p} \hat{\mu}_{d(t)} \qquad \text{F8}$$

$$\hat{\sigma}_a = \hat{\sigma}_d p^{0.5} \qquad \text{F9}$$

The standard procedure to scale returns up (eg from daily to annual) is assumption-free, and consists of interest compounding calculations. However, conventional volatility scaling inexorably involves the serial independence assumption.

If asset returns exhibit no serial dependence, using the square-root-of-time rule is adequate. Nevertheless, in the absence of independence some assets' volatility may increase with the time horizon, while others' may decrease; even if all assets' volatility increases, it may not increase at the same pace. Thus, Holton (1992) highlights the importance of considering volatility and the investment horizon as risk's first and second dimensions.

In the presence of long-term dependence, scaling returns up as in [F8] remains unchanged. However, for estimating volatility the scaling procedure should be generalized as follows:

$$\hat{\sigma}_a = \hat{\sigma}_d p^{\breve{H}} \qquad \text{F10}$$

Additionally, because mean-variance portfolio optimization involves working with the covariance matrix, the latter should be scaled up properly. Under the random-walk assumption, low-frequency covariance between two assets, *i* and *j*, corresponds to the arithmetic sum of high-frequency covariances (Winkelmann, 2003a); thus the relative variance between assets remains unrelated to the investment horizon.

Nevertheless, in the presence of dependence, either $\breve{H}_i \neq 0.5$ or $\breve{H}_j \neq 0.5$, as an extension to the volatility scaling procedure [F10], the *d*-frequency covariance between assets *i* and *j* ($\hat{\sigma}^2_{\{(i,j),d\}}$) should be scaled up to the *a*-frequency covariance ($\hat{\sigma}^2_{\{(i,j),a\}}$) as in Greene and Fielitz (1979) [F11]; this recognizes that memory in financial time series causes relative variance between assets to vary with the investment horizon.

$$\hat{\sigma}^2_{\{(i,j),a\}} = \left(p^{\breve{H}_i + \breve{H}_j}\right)\left(\hat{\sigma}^2_{\{(i,j),d\}}\right) \qquad \text{F11}$$

Long-term dependence inclusive portfolio optimization

In order to illustrate the impact of including long-term dependence adjustments to the covariance matrix scaling for asset allocation, a long-term portfolio optimization exercise is implemented based on the two methods for scaling volatility: (i) the square-root-of-time rule [F9] conventional method, and (ii) the method proposed by the authors [F10 and F11].

The square-root-of-time rule-based method begins by estimating the first two moments of the distribution of the risk factors and the covariance matrix from daily data. Afterwards, a traditional mean-variance optimization is employed, and the expected return and standard deviation of the resulting portfolios are customarily scaled up; since the square-root-of-time rule assumes volatilities' time-consistency, the portfolio weights remain the same regardless of the investment horizon.

The second method also begins by estimating the first two moments of the distribution and the covariance matrix from daily data. Next, because risk factors' dependence causes portfolio weights to vary according to the investment horizon, the standard deviation and covariance matrix scaling for long-term dependence effects [F10 and F11] takes place before optimizing.

Table 1 presents the set of risk factors to be considered in the portfolio optimization procedure. Consistent with the literature on strategic asset allocation, which points out that currency risk hedging is inappropriate for long-term portfolios (Solnik et al., 2003; Froot, 1993), all risk factors were included in their original currency.

According to Table 1, long-term dependence is significant for the two emerging market risk factors considered, namely equity and fixed income indexes, which – again – validates the findings of Cajueiro and Tabak (2008). Regarding commodities, divergence between $\breve{H}$ and 0.5 is rather low, with minor signals of antipersistence for metals and crude oil; agriculture and livestock commodities' $\breve{H}$ matches the independence assumption.

Table 1

Adjusted Hurst exponent for selected risk factors

Market	**Description**		**Abbreviation**	**Mean return**	**Standard deviation**	**Adjusted $\hat{H}$**	***t*-stat**
Commodities	Precious metals		PREC.MET.	0.03%	1.07%	0.47	(1.15)
	Industrial metals		IND.MET.	0.02%	1.38%	0.48	(1.00)
	Agriculture & livestock		AGR.&L.S.	-0.01%	0.91%	0.50	(0.11)
	Crude oil		CRUDE.OIL	0.04%	2.24%	0.48	(0.92)
Equity	Developed markets		EQ.DEV.	0.01%	1.01%	0.51	0.51
	Emerging markets		EQ.EM	0.02%	1.26%	0.59	**3.70**
Fixed income	Emerging markets		EMBI	0.04%	0.74%	0.59	**3.86**
	U.S. Treasury	1-5Y	US.T 1-5Y	0.02%	0.15%	0.53	1.14
		5-10Y	US.T 5-10Y	0.03%	0.36%	0.52	0.77
		10+Y	US.T 10+Y	0.03%	0.60%	0.51	0.26
	U.S. corporate AAA-AA	1-5Y	US.CORP 1-5Y	0.02%	0.17%	0.52	0.72
		5-10Y	US.CORP 5-10Y	0.03%	0.37%	0.50	0.05
		10+Y	US.CORP 10+Y	0.03%	0.55%	0.48	(0.94)
	U.S. mortgages AAA		US.MRTG	0.03%	0.21%	0.50	(0.20)
	GER treasury	1-5Y	GER.T 1-5Y	0.02%	0.12%	0.54	1.69
		5-10Y	GER.T 5-10Y	0.03%	0.27%	0.52	0.84
		10+Y	GER.T 10+Y	0.03%	0.53%	0.48	(0.87)
	JAP treasury	1-5Y	JAP.T 1-5Y	0.01%	0.09%	0.51	0.42
		5-10Y	JAP.T 5-10Y	0.02%	0.24%	0.50	(0.21)
		10+Y	JAP.T 10+Y	0.02%	0.41%	0.50	(0.05)
	U.K. treasury	1-5Y	UK.T 1-5Y	0.02%	0.14%	0.55	**2.10**
		5-10Y	UK.T 5-10Y	0.03%	0.30%	0.54	1.55
		10+Y	UK.T 10+Y	0.03%	0.52%	0.49	(0.28)

Calculations based on daily time series (January 1st 1995–June 25th 2010. Significant (95%) *t*-stats are highlighted. All denominated in their original currency. Precious metals, industrial metals, agriculture & livestock and crude oil correspond to S&P indexes; equity corresponds to MSCI indexes; Emerging market fixed income index is JP Morgan's EMBI; all other fixed income indexes correspond to Merrill Lynch (Bank of America) indexes.

Sources: All indexes provided by Bloomberg. Calculations are the authors'.

Developed markets' fixed income risk factors show low levels of persistence, except for short-term treasuries from U.K. and Germany, and medium-term treasuries from the U.K.; it is noteworthy that long-term fixed instruments consistently tend to exhibit lower persistence than short-term ones. Concerning developed markets' equity, the findings of Cajueiro and Tabak (2008), Menkens (2007), Couillard and Davison (2004), Ambrose et al. (1993) and

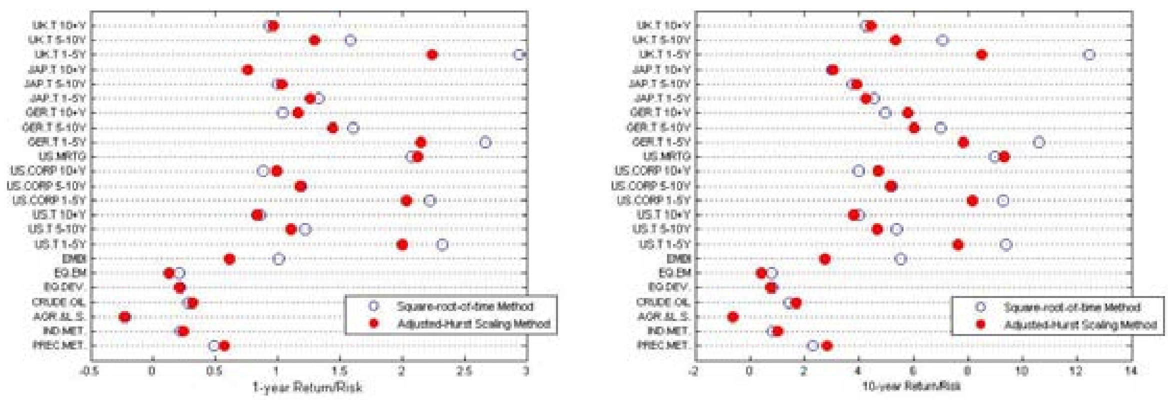
Lo (1991) are verified: there is no evidence of significant long-term dependence. Therefore, Peters' (1992) findings about long-term dependence in developed equity markets are contradicted; the reader should be aware that Peters and other authors in Table 1 do not adjust results for estimation bias.

Interestingly, contrary to conventional wisdom, fixed income instruments' mean returns significantly outperformed equity's for the time series under analysis; thus, it is likely that resulting efficient portfolios will disregard equity *vis-à-vis* academic basics. This supports recent concerns regarding the existence of a *natural hedge* from stocks in the long run and of a positive equity risk premium (Valdés, 2010; Arnott, 2009).

Figure 6

Return/risk ratio for the standard and enhanced methods

1-year 10-year

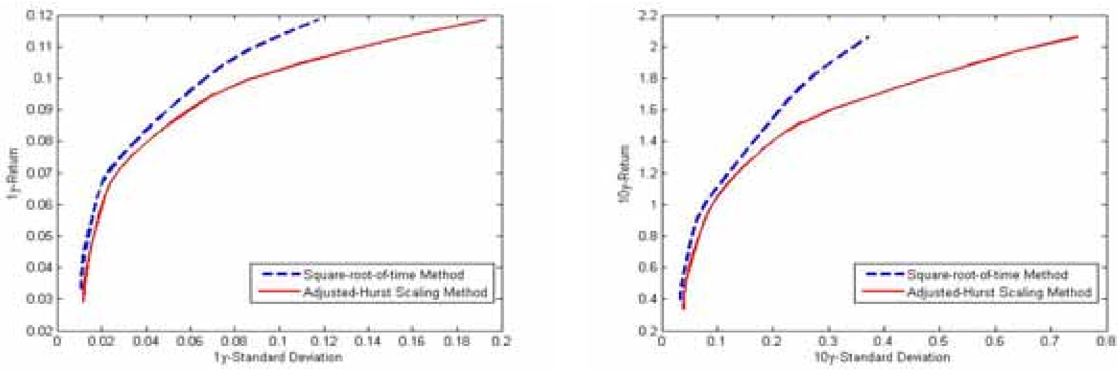


Source: authors' calculations.

Figure 7

Efficient frontiers for the standard and enhanced methods

1-year 10-year



Source: authors' calculations.

Using the adjusted estimated Hurst exponent (Table 2), Figure 6 exhibits the risk/return ratios for both scaling methods for 1-year and 10-year investment horizons. Relative return/risk ratios between methods clearly differ for almost all risk factors. Once dependence is taken into account, extreme differences between return/risk ratios due to concealed riskiness resulting from serial dependence are moderated; hence, it is plausible that adjusting for long-term persistence helps mitigate the well-known tendency of mean-variance optimization to provide extreme weights or corner solutions. The results shown in Figure 6 coincide with Greene and Fielitz's (1979) concern about how return/risk performance measures (eg Sharpe, Treynor and Jensen ratios) are affected by the differencing interval assumption in presence of long-term dependence.

Figure 7 exhibits the efficient frontiers for both scaling methods for 1-year and 10-year investment horizons. As expected, the standard method obtains a strictly dominating frontier with higher levels of return for each level of risk.

Table 2

1-year horizon efficient frontier weights

Panel a. – Square-root-of-time method

Port. #	**Return / risk**	**Commodities** [0.482; 0.19]	**Emerging markets**		**Developed markets**						
			Equity [0.585; 0.21]	**EMBI** [0.589; 1.00]	**Equity** [0.512; 0.22]	**U.S. Treas.** [0.517; 1.47]	**U.S. corp.** [0.499; 1.43]	**U.S. mortg.** [0.495; 2.07]	**GER treas.** [0.513; 1.77]	**JAP treas.** [0.501; 1.03]	**U.K. treas.** [0.526; 1.82]
1	3.0	0.9%	0.9%	0.0%	0.9%	11.0%	0.0%	0.0%	18.7%	58.4%	9.1%
2	3.5	0.7%	1.0%	0.3%	1.2%	15.0%	0.0%	0.0%	19.5%	39.3%	22.9%
3	3.6	1.0%	0.8%	1.3%	1.3%	11.8%	4.9%	1.4%	20.2%	21.4%	35.9%
4	3.5	1.2%	0.8%	2.0%	1.4%	6.9%	9.5%	4.4%	20.5%	6.8%	46.5%
5	3.3	2.1%	0.0%	6.2%	0.0%	0.0%	3.1%	19.3%	0.0%	1.8%	67.4%
6	2.5	3.5%	0.0%	16.8%	0.0%	0.0%	0.0%	26.3%	8.0%	0.0%	45.4%
7	2.0	5.0%	0.0%	25.1%	0.0%	0.0%	0.0%	31.9%	11.7%	0.0%	26.3%
8	1.6	6.5%	0.0%	37.8%	0.0%	2.5%	0.0%	7.4%	24.4%	0.0%	21.4%
9	1.4	6.4%	0.0%	53.5%	0.0%	2.1%	0.0%	0.0%	38.0%	0.0%	0.0%
10	1.0	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Panel b. – Adjusted Hurst scaling method

Port. #	**Return / risk**	**Commodities** [0.482; 0.22]	**Emerging markets**		**Developed markets**						
			Equity [0.585; 0.13]	**EMBI** [0.589; 0.62]	**Equity** [0.512; 0.21]	**U.S. Treas.** [0.517; 1.32]	**U.S. corp.** [0.499; 1.40]	**U.S. mortg.** [0.495; 2.13]	**GER treas.** [0.513; 1.59]	**JAP treas.** [0.501; 1.02]	**U.K. treas.** [0.526; 1.50]
1	2.4	1.2%	0.5%	0.0%	1.1%	11.9%	0.0%	0.0%	14.8%	66.0%	4.6%
2	2.9	1.2%	0.5%	0.0%	1.3%	5.8%	4.4%	7.5%	16.4%	48.2%	14.9%
3	3.0	1.9%	0.4%	0.0%	1.4%	0.0%	8.6%	16.0%	17.3%	29.5%	24.7%
4	3.0	2.4%	0.6%	0.0%	1.7%	0.0%	10.6%	20.4%	20.1%	13.2%	31.1%
5	2.8	4.1%	0.0%	2.1%	0.4%	0.0%	0.0%	41.7%	0.0%	3.7%	47.9%
6	2.2	7.1%	0.0%	6.6%	0.0%	0.0%	0.0%	49.6%	19.1%	0.0%	17.6%
7	1.7	10.7%	0.0%	12.9%	0.0%	0.0%	0.0%	34.1%	42.3%	0.0%	0.0%
8	1.3	13.5%	0.0%	21.9%	0.0%	2.4%	0.0%	0.0%	62.2%	0.0%	0.0%
9	1.0	8.1%	0.0%	49.9%	0.0%	0.0%	0.0%	0.0%	41.9%	0.0%	0.0%
10	0.6	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

The figures in brackets indicate the average adjusted Hurst exponent and the average return/risk ratio, respectively, for each risk factor.

Source: authors' calculations.

Strict dominance of the traditional method's efficient frontier occurs because relative return/risk ratios do not change with the time horizon; adjusting for long-term dependence causes efficient portfolio weights associated with high (low) persistence risk factors to decrease (increase) as the horizon increases. This becomes evident when observing portfolio weights obtained by each method along the 1-year horizon frontier (Table 2). Each frontier consists of ten portfolios, from the lowest risk to the highest return; the average adjusted exponent $(\breve{H})$ and average return/risk ratio for each category of risk factors are also reported in brackets.

Table 3

10-year horizon efficient frontier weights

Panel a. – Square-root-of-time method

Port. #	Return / risk	Commodities	Emerging markets		Developed markets						
		[0.482; 0.99]	Equity [0.585; 0.79]	EMBI [0.589; 5.54]	Equity [0.512; 0.82]	U.S. Treas. [0.517; 6.25]	U.S. corp. [0.499; 6.16]	U.S. mortg. [0.495; 9.00]	GER treas. [0.513; 7.52]	JAP treas. [0.501; 3.79]	U.K. treas. [0.526; 7.91]
1	10.9	0.9%	0.9%	0.0%	0.9%	11.0%	0.0%	0.0%	18.7%	58.4%	9.1%
2	13.4	0.7%	1.0%	0.3%	1.2%	15.0%	0.0%	0.0%	19.5%	39.3%	22.9%
3	14.3	1.0%	0.8%	1.3%	1.3%	11.8%	4.9%	1.4%	20.2%	21.4%	35.9%
4	14.6	1.2%	0.8%	2.0%	1.4%	6.9%	9.5%	4.4%	20.5%	6.8%	46.5%
5	14.1	2.1%	0.0%	6.2%	0.0%	0.0%	3.1%	19.3%	0.0%	1.8%	67.4%
6	11.3	3.5%	0.0%	16.8%	0.0%	0.0%	0.0%	26.3%	8.0%	0.0%	45.4%
7	9.3	5.0%	0.0%	25.1%	0.0%	0.0%	0.0%	31.9%	11.7%	0.0%	26.3%
8	8.0	6.5%	0.0%	37.8%	0.0%	2.5%	0.0%	7.4%	24.4%	0.0%	21.4%
9	7.2	6.4%	0.0%	53.5%	0.0%	2.1%	0.0%	0.0%	38.0%	0.0%	0.0%
10	5.5	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Panel b. – Adjusted Hurst scaling method

Port. #	Return / risk	Commodities	Emerging markets		Developed markets						
		[0.482; 1.23]	Equity [0.585; 0.41]	EMBI [0.589; 2.76]	Equity [0.512; 0.75]	U.S. Treas. [0.517; 5.38]	U.S. corp. [0.499; 6.01]	U.S. mortg. [0.495; 9.33]	GER treas. [0.513; 6.54]	JAP treas. [0.501; 3.74]	U.K treas. [0.526; 6.09]
1	8.5	1.3%	0.3%	0.0%	1.1%	11.9%	0.0%	0.2%	13.2%	68.7%	3.2%
2	11.5	1.9%	0.3%	0.0%	1.2%	0.0%	5.2%	16.6%	12.5%	47.9%	14.5%
3	12.0	3.1%	0.2%	0.2%	1.4%	0.0%	4.9%	28.8%	11.7%	25.1%	24.5%
4	11.7	4.6%	0.0%	0.9%	1.3%	0.0%	2.1%	43.9%	2.4%	8.3%	36.5%
5	10.3	7.4%	0.0%	3.7%	0.0%	0.0%	0.0%	53.8%	17.3%	0.0%	18.0%
6	8.3	11.4%	0.0%	8.2%	0.0%	0.0%	0.0%	38.2%	42.3%	0.0%	0.0%
7	6.8	15.6%	0.0%	13.8%	0.0%	1.9%	0.0%	2.0%	66.7%	0.0%	0.0%
8	5.2	9.9%	0.0%	34.2%	0.0%	0.0%	0.0%	0.0%	55.9%	0.0%	0.0%
9	3.8	12.9%	0.0%	57.7%	0.0%	0.0%	0.0%	0.0%	29.4%	0.0%	0.0%
10	2.8	0.0%	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

The figures in brackets indicate the average adjusted Hurst exponent and the average return/risk ratio, respectively, for each risk factor.

Source: authors' calculations.

Relative overweighting of persistent risk factors (eg emerging markets' fixed income – EMBI) is evident for the conventional method. When dependence is taken into account, this overweighting diminishes in favor of near-independent or antipersistent risk factors, such as Japanese and German treasuries, U.S. mortgages or commodities. The main difference between risk factors across panels consists in lower divergences in the return/risk ratios, which are conveniently obtained from proper volatility scaling. This difference explains persistent risk factors' relative overweighting when using traditional covariance scaling. This analysis is validated for the ten-year horizon too (Table 3).

Table 3 confirms the adjustment of extreme divergences between risk factor's return/risk ratios when using the proposed procedure, and the persistent risk factors' relative overweighting due to traditional covariance scaling. Moreover, against basic financial principles or intuition, but as an obvious consequence of the square-root-of-time rule, traditional 1-year and 10-year efficient frontiers do not differ from each other; this emphasizes that customarily use of mean-variance optimization disregards the investment horizon as a meaningful factor of the asset allocation process.

Table 4 presents a summary of the weights allocated according to each investment horizon. Three cases are depicted: (i) because using the square-root-of-time rule makes the allocation independent from the investment horizon, X-year corresponds to the weight at any horizon; (ii) the 1-year horizon; (iii) the 10-year horizon. Figure 8 presents a graphical summary of the weights assigned to the efficient frontier for the three cases.

Table 4

X-year, 1-year and 10-year horizon efficient frontier weights

Summary (mean and maximum)

	Investment horizon	Commodities [0.482; 0.99]	Emerging markets		Developed markets						
			Equity [0.585]	EMBI [0.589]	Equity [0.512]	U.S. Treas. [0.517]	U.S. corp. [0.499]	U.S. mortg. [0.495]	GER treas. [0.513]	JAP treas. [0.501]	U.K. treas. [0.526]
Mean	X-y	2.9%	0.3%	25.4%	0.5%	4.6%	1.9%	8.9%	16.8%	11.2%	27.4%
	1-y	5.6%	0.2%	20.2%	0.6%	1.6%	2.5%	17.3%	24.0%	14.3%	13.8%
	10-y	7.9%	0.1%	22.9%	0.4%	0.9%	1.3%	18.8%	25.4%	13.2%	9.3%
Max	X-y	7.1%	1.0%	100.0%	1.4%	15.0%	12.1%	31.9%	38.0%	58.4%	67.4%
	1-y	13.5%	0.6%	100.0%	1.7%	11.9%	10.6%	51.8%	62.2%	66.0%	47.9%
	10-y	15.9%	0.3%	100.0%	1.6%	11.9%	5.5%	55.4%	66.7%	68.7%	36.5%

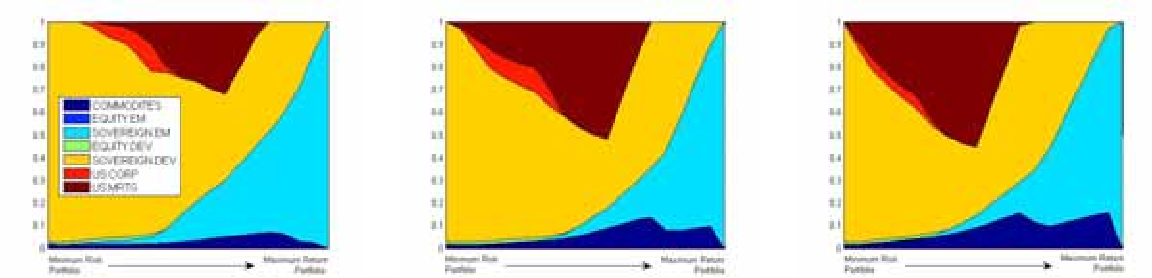
The figures in brackets indicate the average adjusted Hurst exponent for each risk factor.

Source: authors' calculations.

Figure 8

Square-root-of-time and adjusted Hurst methods for 1-year and 10-year weights

H=0.5 (*n*-year) H=$\breve{H}$ (1-year) H=$\breve{H}$ (10-year)



Source: authors' calculations.

5. Final remarks

Much attention has been given to financial asset returns' short-term dependence. In this sense many models are readily available to improve the estimation of the variance and, to a lesser degree, covariance inputs for short-term portfolio construction.

Less emphasis has been given to long-term dependence of returns. Akin to the financial literature, this paper shows that (i) significant long-term dependence is common in asset return time series; (ii) significant persistence is prevalent for emerging fixed income markets, and fairly frequent for emerging equity markets – mainly the less liquid ones; (iii) independence is representative of developed fixed income and equity markets, and somewhat recurrent for liquid emerging equity markets; (iv) U.S. energy markets exhibit significant antipersistence.

Interestingly, this document's support for prior evidence includes data from the most recent and severe episode of widespread financial disruption. Divergence with documented literature is circumscribed to our findings of recurrent antipersistence for developed equity markets, as well as a few liquid emerging markets.

This paper's long-term dependence assessment relies on rescaled range analysis (*R/S*), a popular and robust methodology designed for geophysics but extensively used in financial literature. Well-known issues of *R/S* such as the optimal minimum and maximum size of periods were surmounted vis-à-vis some previous studies, resulting in reduced estimators' standard errors and minimal interference of short-term serial dependence in the results.

Ahead of *R/S* financial literature, we used the spread between the estimated Hurst exponent ($\hat{H}$) and the expected Hurst exponent for independent and finite time series ($\dot{H}$) to estimate an adjusted Hurst exponent ($\breve{H}$). Under a generalized version of the conventional volatility and covariance scaling procedure, we suggest using this adjusted measure of long-term dependence for practical purposes, where long-term mean-variance portfolio optimization is a natural choice to begin with.

Comparing the efficient portfolio weights resulting from customary mean-variance optimization (eg independency assumption reliant) and the suggested enhanced procedure shows that the former tends to overweight persistent risk factors. Once long-term dependence is considered via the proposed covariance scaling procedure, the return per unit of risk of persistent (antipersistent) risk factors is adjusted downwards (upwards), decreasing (increasing) the weight of high (low) persistence risk factors as the investment horizon increases. Our results provide evidence of the significance of weight differences for 1-year and 10-year investment horizons and of how these differences reveal that adjusted efficient frontiers may be less optimistic than conventional ones.

Long-term dependence recognition conveys various practical advantages, especially for long-term institutional investors, such as central banks, pension funds and sovereign wealth managers. First, because the proposed scaling procedure exposes concealed riskiness resulting from persistence, extreme relative return/risk ratio differences due to inappropriate risk scaling are moderated, avoiding to some extent excessive risk taking in long-term portfolios and mitigating the presence of extreme portfolio weights.

Second, evidence of significant persistence in small and illiquid capital markets provides proof of masked risks within their securities. Such underestimation of local instruments' long-term risk could explain two well-known facts of those capital markets: (i) the tendency to hold a disproportionate level of investments within the domestic market or "home bias"; and (ii) the reluctance to hold foreign currency-denominated assets. Recognizing long-term dependence would make local – persistent – instruments from small and illiquid markets less attractive within the mean-variance asset allocation framework, and developed markets' – independent or antipersistent – instruments more attractive.

Given these insights, we are currently considering three research topics: firstly, to study the contribution of individual risk factors to portfolio's persistence. The initial results presented here show that persistence at the portfolio level can be significantly higher than the weighted persistence of individual assets, especially for small and illiquid markets, thereby reinforcing the case for international diversification.

Secondly, akin to upside and downside risk concepts, we also envision a methodology capable of differentiating upside from downside persistence. This is a key issue because persistence may be an asset's desirable (undesirable) feature if its price is expected to rise (fall) in the future (eg a persistent bond may be attractive on the verge of monetary expansion). In the meantime, we suggest considering the market environment and investors' views in order to decide on the convenience of underweighting persistent risk factors. Alternatively, including optimization constraints such as a threshold for maximum drawdown (Reveiz and León, 2010) may capture investors' natural inclination (reluctance) to hold upside (downside) persistent risk factors.

Finally, because Black-Litterman portfolio optimization is heavily reliant on the serial long-term independence assumption via traditional volatility scaling and the starting global CAPM equilibrium, our agenda also includes designing long-term dependence adjustments to this celebrated approach. A forthcoming paper by one of the authors will present how Colombia's foreign reserve management approach already incorporates the adjustment suggested here.

Appendix A

- For a time series of N returns, having k independent (non overlapping[10]) windows or samples of size n, divide the original series in such way that $n \times k = N$.
- Estimate the arithmetic mean of each k-segment ($\hat{\mu}_k$) of size n.
- Obtain the difference between each i-return and the mean of each k segment ($\hat{\mu}_k$).

$$Y_{i,k} = x_{i,k} - \hat{\mu}_k$$

- Calculate accumulative differences for each k segment.

$$D_{i,k} = \sum_{i=1}^{n} Y_{i,k}$$

- Calculate range ($R_{n,k}$) of the $D_{i,k}$ series.

$$R_{n,k} = \max\left(D_{1,k},\ldots,D_{i,k},\ldots D_{n,k}\right) - \min\left(D_{1,k},\ldots,D_{i,k},\ldots D_{n,k}\right)$$

- Estimate standard deviation for each k segment ($S_{n,k}$).

$$S_{n,k} = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} \left(x_{i,k} - \hat{\mu}_k\right)^2}$$

- Calculate rescaled range for each segment k.

$$\left(R/S\right)_{n,k} = R_{n,k}/S_{n,k}$$

- Calculate average rescaled range for k segments of size n.

$$\left(R/S\right)_n = \frac{1}{k} \sum_{i=1}^{k} \left(R/S\right)_{n,k}$$

$(R/S)_n$ corresponds to the average standardized distance covered per unit of time n.

The previous procedure must be done for different values of k, where $k_j = n_{min} \ldots n_{max}$, and where n_{min} y n_{max} corresponds to the minimum and maximum of the chosen window to calculate the rescaled range. Thus, we have j values of $(R/S)_n$, where $n_j = \frac{N}{k_j}$.

Finally, using n and $(R/S)_n$ values we estimate the ordinary least squares regression proposed by Mandelbrot and Wallis (1969a y 1969b), where H corresponds to the estimated Hurst exponent:

$$Log\left(R/S\right)_n = Log(c) + HLog(n)$$

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[10] For a discussion regarding the use of overlapping and non-overlapping segments, please refer to Nawrocki (1995) and Ellis (2007).

Appendix B

Table B1

Literature on R/S-estimated Hurst exponent

Author	Time series	Period (frequency)	H	Fit	
Peters (1992)	S&P500 – USA	01/1950 – 06/1988 (M)	0.780	N/A	N/A
Ambrose et al. (1993)	S&P500 – USA	07/1962 – 12/1988 (D)	0.531	1.380	‡
	S&P500 – USA	01/1950 – 07/1988 (M)	0.622	1.490	‡
Sierra (2007)	IPC – MEX.	01/1999 – 02/2006 (D)	0.557	0.990	§
	DJIA – USA	06/1999 – 05/2006 (D)	0.504	0.988	§
Palomas (2002)	IPC – MEX.	01/1988 – 09/2001 (D)	0.584	0.995	§
	IPC – MEX.	01/1983 – 05/2001 (M)	0.713	0.976	§
	DJIA – USA	01/1950 – 08/2001 (M)	0.658	0.994	§
	S&P500 – USA		0.686	0.993	§
Qian and Rasheed (2004)	DJIA – USA	11/1969 – 12/1973 (D)	0.650	N/A	N/A
Bilel and Nadhem (2009)	S&P500 – USA	03/1990 – 09/2008 (M)	0.525	1.400	‡
	S&PTSX – CAN.		0.541	1.465	‡
	CAC40 – FR.		0.537	2.088	‡
	DAX100 – GER.		0.541	1.644	‡
	MIB – ITALY		0.505	1.644	‡
	NIKKEI225 – JAP.		0.551	2.635	‡
	FTSE 100 – U.K.		0.511	2.420	‡
Cajueiro and Tabak (2008)	NIKKEI225 – JAP.	01/1999 – 12/2005 (D)	0.547	0.038	†
	MERVAL – ARG.		0.584	0.040	†
	BOVESPA – BRA.		0.612	0.040	†
	SENSEX – INDIA		0.591	0.040	†
	KOSPY – S.KOR.		0.551	0.039	†
	IPSA – CHILE		0.594	0.040	†
	IPC – MEX.		0.557	0.039	†
	IGBVL – PERU		0.656	0.042	†
	ISE – TURKEY		0.538	0.036	†
	TA-100 – ISRAEL		0.584	0.041	†
	FTSE100 – U.K.		0.521	0.039	†
	S&P500 – USA		0.519	0.037	†
Jagric et al. (2005)	PX50 – CZ.REP.	09/1993 – 07/2004 (D)	0.645	0.018	†
	BUX – HUNG.	01/1991 – 06/2004 (D)	0.626	0.015	†
	WSE – POLAND	03/1994 – 08/2004 (D)	0.569	0.018	†
	RTS – RUSSIA	09/1995 – 08/2004 (D)	0.648	0.020	†
	SAX – SLKIA	07/1995 – 07/2004 (D)	0.525	0.020	†
	SBI – SLVNIA	01/1993 – 07/2004 (D)	0.656	0.017	†
McKenzie (2001)	Australian Stock Exch.	04/1876 – 03/1996 (M)	0.571	2.027	¤
			0.622	1.850	¤

Table B1 (cont)

Literature on R/S-estimated Hurst exponent

Author	Time series	Period (frequency)	H	Fit	
Alptekin (2008)	Gold – Istambul Gold Exchange	01/2003 – 03/2008 (D)	0.600	2.100	‡
Corazza et al. (1997)	Corn Futures – CBOT	01/1981 – 10/1991 (D)	0.760	N/A	N/A
	Oats Futures – CBOT		0.700	N/A	N/A
	Soybean Futures – CBOT		0.740	N/A	N/A
	Soybean oil futures – CBOT		0.800	N/A	N/A
	Wheat futures – CBOT		0.650	N/A	N/A
Erzgraber et al.(2008)	Energy (NordPool) – Norway	01/1999 – 01/2007 (D)	0.270	N/A	N/A
Weron and Przandbandlowicz (2000)	Energy (CalPX) California	03/1998 – 01/2000 (H)	0.439	N/A	N/A
	Energy (SWEP) – Switzerland	03/1998 – 03/2000 (D)	0.529	N/A	N/A
Batten et al. (1999)	DMK/USD	01/1976 – 09/1998 (D)	0.623	2.248	¤
	CHF/USD		0.610	2.053	¤
	JPY/USD		0.609	1.954	¤
	GBP/USD		0.590	1.487	¤
Sierra (2007)	MXN/USD	01/1995 – 02/2006 (D)	0.526	0.994	§
	USD/EUR	06/1999 – 05/2006 (D)	0.559	0.995	§
Da Silva et al. (2007)	BRL/USD	01/1995 – 08/2006 (D)	0.630	3.260	¤
Souza et al. (2008)	DEM/USD	05/1986 – 12/1998 (D)	0.580	0.026	†
	3M future DEM/USD		0.571	0.026	†
	FRF/USD		0.576	0.026	†
	GBP/USD		0.567	0.026	†
	ITL/USD		0.598	0.026	†
Peters (1992)	30Y Treas. – USA	01/1950 – 06/1988 (M)	0.670	N/A	N/A

Frequencies correspond to the following convention: hourly (H), daily (D), monthly (M).

§ Corresponds to the R^2 of the regression [F4].

† Corresponds to the standard error of estimated H.

‡ Corresponds to Lo's Vq statistic (1991).

¤ Corresponds to the *t*-statistic by Couillard and Davison (2005).

Source: authors'design.

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Including linkers in a sovereign bond portfolio: an HJM approach

Ricardo Selves and Marcin Stamirowski[1]

1. Introduction

An inflation-linked bond (ILB) is a debt security which generates cash-flows linked to the evolution of a given price index. The aim of the indexation is to protect the “real” value of the investment. Contrary to conventional sovereign[2] fixed or floating rate securities, which offer investors certain nominal rates of return, inflation-linked bonds tie part of their economic result to the evolution of a price index, assuring in this sense a real rate of return. By so doing, the risk/return characteristics of these instruments differ from those of conventional bonds, while still offering the same credit exposure. The question naturally arises whether there are any advantages, from a risk/return perspective, on including this kind of instruments in a bond portfolio made up of conventional fixed/floating rate bonds and money market instruments. In other words, do ILBs constitute a different asset class able to enhance the efficient frontier if included in an otherwise conventional bond portfolio?

Inflation-linked securities have a long history, with the State of Massachusetts having issued a first bond linked to a basket of commodities as long ago as 1780. The modern development of the market is widely regarded to have started in 1981, the year in which the index-linked gilts were first issued by the UK Treasury.

Today, the global (government) market is worth well above EUR 1,000 billion and the main global issuers are the US, UK, France and Italy. Euro-denominated inflation-indexed bonds are issued mainly by the French Treasury (AFT-Agence France Tresor), and by the Italian and German Treasuries as well. The market is dominated by sovereign issuers. However, corporate issuance has also seen growth in recent years. It has been facilitated by the rapid development of inflation-indexed derivatives (such as inflation swaps), which enable greater flexibility in terms of determining the desired cash flows. Mainly due to its relative size versus the other euro-denominated markets, the French market for the inflation-linked bonds seems the most appropriate for an analysis of the impact of including linkers in a bond portfolio. For this reason, all references are made primarily to the (French) HICP-linked bonds. We start by a quick review of the main elements characterizing ILBs, then we address the issue of their inclusion in a bond portfolio so that we can later develop a model for pricing linkers and derivatives.

2. Inflation-linked bonds (ILBs)

The fundamental feature of inflation-indexed securities is that they offer investors the promise of a certain “real” yield or rate of return (r) on their investments as compared to

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[1] Disclaimer: the views expressed here are those of the authors and do not necessarily correspond to those of the European Commission.

[2] Our references to sovereign bonds include agency and supranational securities.

conventional bonds (either zero-coupon or coupon-paying bonds, fixed or floating) which offer investors the promise of a certain "nominal" rate of return (i).[3]

The classic Fisher equation suggests that the expected annual rate of inflation over the life of the two bonds would on average amount to:

$$\pi^e = (i - r)$$

If the actual average annual inflation proves later to be higher, ie $\pi = \pi^e + \varepsilon$, with $\varepsilon > 0$, then the ex-post real annual yield (rate of return) on the nominal bond will turn out to be just:

$$(i - \pi^e - \varepsilon < r).$$

On the other hand, the ILB will still have yielded its promised annual real rate (r).

Naturally, the ex-post real yield on the nominal bond could end up being higher than that on the inflation-linker, should the average inflation rate prove lower than π^e per annum.

The key feature of linkers is that they provide a mean to guarantee *ex-ante* a certain real rate of return, whereas real return on conventional bonds is only known ex-post, depending on the actual inflation rate realized over the investment period.

The actual mechanism inflation linkers use to ensure protection against inflation varies in details across the different countries. In general, however, most issuers, including France, have chosen a relatively simple framework, first introduced by Canada. Specifically, bonds are quoted in real terms, and both principal and coupons are adjusted for changes in the relevant consumer price index between issue date and cash-flow payment date, subject to a certain indexation lag. Such a cash-flow structure is commonly referred to as *capital*-indexed.

We will concentrate on French government linkers as they are the most liquid in the Euro-denominated ILB market.[4]

The following table presents the situation of the euro-denominated sovereign inflation-linked debt in the largest European markets, as of the end of November 2009, and its relative importance in the corresponding total government debt market.

	France	**Germany**	**Italy**	**Greece**	**Total**
Nom.	133.90	22.70	78.50	13.40	248.50
% of LT debt	20.9	3.9	8.6	N/A	11.0
% of EUR debt	13.2	2.3	6.1	5.1	7.0

Source: Periodic bulletins of the respective debt agencies and Barclays Capital.

The prices of inflation-linked bonds are quoted in real terms. Settlement values and cash-flows then adjust for accrued inflation. This mechanism makes linkers entirely equivalent to a conventional bond denominated in a foreign currency: Everything is traded, computed and negotiated in the foreign currency (in real terms in our case) and then the resulting

3 This promise of a real return is just a promise, on the same ground as the promise of a certain nominal yield offered by conventional bonds is subject to a series of assumptions such as reinvestment conditions etc.

4 French linkers account for more than 50% of the euro-linker government market, followed by Italy, representing about 30%, and then Germany and Greece both accounting for less than 10% each.

magnitudes so calculated (accrued interests, principal, coupons) are just multiplied by the "exchange rate" (the index ratio).

This means that for settlement amounts, real accrued interests are calculated as for ordinary OATs. Clean prices and accrued interests are then each multiplied by the index ratio to arrive at a cash settlement amount. As for the coupons paid, the (real) annual coupon rate is multiplied by the index ratio, and likewise for the par redemption amount (with the cash value subject to a par floor).

3. Including government ILBs in a bond portfolio

Now we come to the question if are there any advantages from a risk/return perspective in including ILBs on a bond portfolio made up of conventional fixed/floating rate bonds and possibly some money market instruments?[5] In other words, do ILBs have the potential to enhance the efficient frontier of such a portfolio?

The answer from a theoretical perspective is clearly yes. From an efficient frontier point of view, linkers can significantly enhance the risk/return characteristics of an otherwise classical portfolio. This argument effectively relies on the beta relationship between real and nominal yields. Recalling the Fisher equation that was introduced earlier, which relates nominal rates to real rates, inflation and risk; the offered yield on a nominal bond (i) can be decomposed into the required real return (r), a necessary compensation for inflation (π^e) and a certain risk premium (ρ), as previously stated.

In its loose version the Fisher equation states that

$$i = r + \pi^e + \rho$$

Based on the definition of breakeven inflation, $bei = \pi^e + \rho$, we can write:

$$i = r + bei$$

The variance of the nominal yield can then be written as:

$$Var(i) \equiv Cov(i,i) = Cov(i,r + bei) = Cov(i,r) + Cov(i,bei)$$

Based on this expression, dividing both sides by $Var(i)$ we can get:

$$1 = \frac{Cov(i,r)}{Var(i)} + \frac{Cov(i,bei)}{Var(i)}$$

Again from a theoretical perspective, we should expect some positive covariation between nominal yields and expected inflation, or more precisely between nominal yields and breakeven inflation, ie $Cov(i,bei) > 0$, which means that:

$$\frac{Cov(i,r)}{Var(i)} \equiv \beta(i,r) < 1$$

In other words, this means that part of the variability in nominal yields is accounted for by the variability in breakevens, which leaves the real yield relatively more stable, which in turn translates into additional stability in real prices and real returns on linkers. This also means

[5] References to risk are made to market risk and leave aside credit risk, which is completely similar to that already existing on conventional bonds.

that the sensitivity of linkers to changes in nominal yields will usually (but not necessarily) be less than 1.[6]

The attractiveness of an asset to a portfolio is usually measured in terms of the risk and return trade-off; so if the theory holds in reality, linkers should stand a very good chance of being included in a fixed income portfolio.

Several empirical studies have shown that the efficient frontier of portfolios including linkers as an asset class moves upward, meaning that better rewards are achieved for the same levels of risk.

Barclays ([3]) has tested empirically this assertion for several markets, but we concentrate on the euro-linkers. By the end of 2007 the size of the euro-linker market had surpassed that of the UK, making it the second-largest in the world. In their empirical analysis (with data covering 1998–2007) Barclays found that adding linkers to a portfolio of MM, conventional bonds and equities significantly improved the efficient frontier. Barclays also ran the exercise restricting the weight in the portfolio to 20%, to reflect the fact that linkers share a portion in the market that is lower than 20%, and the improvement remained significant (see Barclays([3])).

Société Générale (SG) also discusses the case of a portfolio investing in European securities (MM, conventional bonds and equities[7]). The study shows that the portfolio becomes more efficient when including linkers from a historical perspective.

In the following section we develop a 3-factor HJM to characterize the economy, with time-dependent (non-stochastic) volatilities. If validated, the fixed income market can be characterized as Gaussian and so the inclusion of linkers in a bond portfolio can be analyzed in the context of the classical portfolio analysis, ie building an efficient frontier just based in the variance-covariance matrix of returns.

4. An HJM approach to pricing bonds

We start from the modeling of the market itself by applying an HJM model to consistently price both ILB and conventional euro-denominated (French) sovereign bonds. As explained in the description of linkers, a foreign currency analogy is naturally suited to implementing this methodology. In the vein of Jarrow and Turnbull [9] and Jarrow and Yildirim [10] we consider a hypothetical world under the no-arbitrage assumption where nominal euros correspond to the domestic currency, real euros correspond to the foreign currency, and the HICP corresponds to the spot exchange rate. In this setup, the fluctuations of the real and nominal interest rates and the inflation rate will be correlated.

Following the foreign bonds analogy, nominal bonds will play the role of "national" bonds (upper-scripted N), the role of "foreign bonds" will be played by the real bonds (upper-scripted R) and the HICP will play the role of the "exchange rate". The following notation will be used:

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6 If this beta were always a stable number, it would be easy to calculate the equivalent nominal duration for an inflation bond. Equally, though, if it were that easy, then there would be no additional value to inflation-linked bonds as a diversifying asset class (Barclays Capital).

7 Total return for nominal bonds and linkers computed from total return Barclays Capital Euro Indices (France), money market returns based on one-month Euribor rates and equity returns derived from the total return MSCI Equity index for France.

1. $f_{t,T}^{h}$ stands for country's h forward rate (with $h \in \{N, R\}$), set at t ,for borrowing over $[T, T+dt], T > t$.[8]

2. $P_{t,T}^{h}$ stands for the price at t of country's h zero-coupon bond, maturing at $T > t$.

3. I_t stands for the HICP, ie the "exchange rate" for a unit of "foreign currency" expressed in terms of the local currency.[9]

4. r_t^h stands for country's h instantaneous risk-free interest rate.

5. $B_t^h = e^{\int_0^t r_u^h du}$ stands for country's h money market account.

In a general HJM-world, $f_{t,T}^{h}$ evolves according to:

$$df_{t,T}^{h} = \alpha_{t,T}^{h} dt + \underline{\sigma}_{t,T}^{h} \cdot \underline{dW}_t$$

with $\underline{\sigma}_{t,T}^{h} = \left(\sigma_{t,T}^{h1}, \cdots, \sigma_{t,T}^{hk}\right)$

and $\underline{W}_t = \left(W_t^1, \cdots, W_t^k\right)$
a k-dimensional Brownian Motion.[10]

In the spirit of Jarrow and Yildirim [10], we will assume a three-factor model, where nominal bonds depend on W_N, real bonds depend on W_R and the HICP depends on W_I, with:

$$dW_N \cdot dW_R = \rho_{N,R} dt$$

$$dW_N \cdot dW_I = \rho_{N,I} dt$$

$$dW_R \cdot dW_I = \rho_{R,I} dt$$

The price of a zero-coupon bond, $P_{t,T}^{h}, h \in \{N, R\}$, may be expressed as a function of these forward rates as:

$$P_{t,T}^{h} = e^{-\int_t^T f_{t,u}^h du} . \tag{1}$$

Letting $f_{0,T}^{h}$ be the forward rate curve at time 0, it is possible to express $f_{t,T}^{h}$ as:

$$f_{t,T}^{h} = f_{o,T}^{h} + \int_0^t \alpha_{s,T}^{h} ds + \int_0^t \sigma_{s,T}^{h} dW_s^h$$

As a particular case, the short rate $r_t^h = f_{t,t}^h$ results:

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8 The dynamic is with respect to calendar time, t , whereas the maturity, T , acts as a parameter.

9 Each ILB has associated a particular initial HICP value, I_0, which depends on its issuance date, and which constitutes the basis to calculate the applicable "exchange rate" at any particular time, so $\left(I_t / I_o\right)$ and not I_t should be used. For easiness of exposition however, we will assume that both, the HICP's basis and the initial ILB's index coincide, unless otherwise required by the context, which will be made clear in the text.

10 $\alpha_{t,T}$ and $\underline{\sigma}_{t,T}$ are adapted with respect to the σ-algebra generated by $W_s^j, 1 \le j \le k, s \le t$ (the filtration $F_t^{\underline{W}}$) and satisfy the boundary conditions $\mathrm{E}\left(\int_0^T \left|\alpha_{t,T}\right| dt\right) < \infty$ and $\int_0^T \mathrm{E}\left(\left|\underline{\sigma}_{t,T}\right|^2\right) dt < \infty$.

$$r_t^h = f_{t,t}^h = f_{0,t}^h + \int_0^t \alpha_{s,t}^h ds + \int_0^t \sigma_{s,t}^h dW_s^h$$

with dynamics:

$$dr_t^h = \left(\frac{\partial f_{0,t}^h}{\partial t} + \alpha_{t,t}^h + \int_0^t \frac{\partial \alpha_{s,t}^h}{\partial t} ds \right) dt + \sigma_{t,t}^h dW_t^h + \int_0^t \frac{\partial \sigma_{s,t}^h}{\partial t} dW_s^h dt$$

(assuming that $\alpha_{t,T}^h$ and $\sigma_{t,T}^h$ are differentiable with respect to maturity).[11]

Bond prices as given by (1) satisfy a SDE:

$$d_t P_{t,T}^h = P_{t,T}^h \left(\left(r_t^h - \alpha_{t,T}^{h*} + \frac{1}{2} \left| \sigma_{t,T}^{h*} \right|^2 \right) dt - \sigma_{t,T}^{h*} \cdot dW_t^h \right) \tag{2}$$

where we have put:

$$\alpha_{t,T}^{h*} = \int_t^T \alpha_{t,U}^h dU \,,$$

$$\sigma_{t,T}^{h*} = \int_t^T \alpha_{t,U}^h dU$$

for, respectively, the integrated drift and the integrated volatility with respect to maturity.

The HICP (or "exchange rate") I_t satisfies a SDE as well:

$$dI_t = I_t \left(\mu_t dt + \sigma_t^I dW_t^I \right) \tag{3}$$

Real ("foreign") bonds and the real current account are non-tradeable assets in the domestic economy, ie it is precise to express them (price them) in nominal terms (the domestic currency), in order for them to be tradeable:

- Let $P_{t,T}^T = I_t \times P_{t,T}^R$ be the price in "domestic currency" at t, of the real zero-coupon bond, maturing at $T > t$, ie $P_{t,T}^T$ is the price of a zero-coupon linker.
- Similarly, for the "foreign" money market account, let us define $P_t^T = I_t \times B_t^R$, the value at t, in the domestic currency, of the foreign money market holdings.

These two assets are governed by the following stochastic processes (as a simple application of Ito's rules):

$$\begin{pmatrix} d_t P_{t,T}^T \\ d_t P_t^T \end{pmatrix} = \begin{pmatrix} P_{t,T}^T \left[\left(r_t^R - \alpha_{t,T}^{R*} + \frac{1}{2} \left| \sigma_{t,T}^{R*} \right|^2 + \mu_t - \rho_{R,I} \sigma_t^I \sigma_{t,T}^{R*} \right) dt + \sigma_t^I dW_t^I - \sigma_{t,T}^{R*} dW_t^R \right] \\ P_t^T \left[\left(r_t^R + \mu_t \right) dt + \sigma_t^I dW_t^I \right] \end{pmatrix} \tag{4}$$

In order to price claims in this economy, we need:

1. A replicating *self-financing trading strategy* (SFTS).
2. An *equivalent martingale measure* (EMM) for discounted bond prices.

As there are three sources of uncertainty in our model, we need three securities and a savings account to build a SFTS. We choose a (any) nominal zero-coupon bond $\left(P_{t,T_1}^T \right)$, a

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[11] This is in general not a SDE, due to the final integral. In fact, r_t^h is in general not a Markov process.

(tradeable) inflation-linked zero-coupon bond $\left(P_{t,T_2}^T\right)$, the (tradeable) real saving account $\left(P_t^T\right)$ and the nominal saving account $\left(B_t^N\right)$.

The SFTS will be a vector of adapted processes $\left(\underline{\varphi}_t, \psi_t\right) = \left(\varphi_t^1, \varphi_t^2, \varphi_t^3, \psi_t\right)$ on this set of securities, such that, if $V_t\left(\underline{\varphi}_t, \psi_t\right)$ is the portfolio's value at t,

$$V_t\left(\underline{\varphi}_t, \psi_t\right) = \varphi_t^1 P_{t,T_1}^N + \varphi_t^2 P_{t,T_2}^T + \varphi_t^3 P_t^T + \psi_t B_t,$$

then, for $t < \min(T_1, T_2)$:

$$dV_t\left(\underline{\varphi}_t, \psi_t\right) = \varphi_t^1 P_{t,T_1}^N + \varphi_t^2 P_{t,T_2}^T + \varphi_t^3 dP_t^T + \psi_t dB_t,$$

(where we have put $B_t = B_t^N$).

If (φ_t, ψ_t) is self-financing for $\left(P_{t,T_1}^N, P_{t,T_2}^T, P_t^T, B_t\right)$, then it is also self-financing for the discounted bond prices $\left(Z_{t,T_1}^N, Z_{t,T_2}^T, Z_{t,t}^T, 1\right)$ with:

$$Z^h := \frac{P^h}{B_t}, h \in \{N, T\}$$

From equations (2) and (4), the definition of Bt and the rules of Ito's calculus, it results that:

$$\begin{pmatrix} \dfrac{dZ_{t,T_1}^N}{Z_{t,T_1}^N} \\ \dfrac{dZ_{t,T_2}^R}{Z_{t,T_2}^T} \\ \dfrac{dZ_t^T}{Z_t^T} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{2}\left(\sigma_{t,T_1}^{N*}\right)^2 - \alpha_{t,T_1}^{N*}\right)dt - \sigma_{t,T_1}^{N*} dW_t^N \\ \left(\frac{1}{2}\left(\sigma_{t,T_2}^{R*}\right)^2 - \alpha_{t,T_2}^{R*} + r_t^R - r_t^N + \mu_t - \rho_{R,I}\sigma_t^I \sigma_{t,T_2}^R\right)dt + \sigma_t^I dW_t^I - \sigma_{t,T_2}^{R*} dW_t^R \\ \left(r_t^R - r_t^N + \mu_t\right)dt + \sigma_t^I dW_t^I \end{pmatrix}$$

We now turn into the issue of the EMM for Z^h. Let us define:

$$\underline{W}_t = \underline{\hat{W}}_t - \int_0^t \underline{\gamma}_s ds$$

$$\underline{dW}_t = \underline{d\hat{W}}_t - \underline{\gamma}_t dt,$$

where $\underline{\hat{W}}_t$ is a three-dimensional Brownian motion with respect to a new probability measure $\Theta = Q^\gamma$, given by Girsanov's theorem, with Girsanov density $\exp\left(-\int_0^{T^*} \underline{\gamma}_t \underline{dW}_t - \frac{1}{2}\int_0^{T^*} \left|\underline{\gamma}_t\right|^2 dt\right)$,

With respect to this new probability measure, dZ/Z becomes:

$$\begin{pmatrix} \dfrac{dZ_{t,T_1}^N}{Z_{t,T_1}^N} \\ \dfrac{dZ_{t,T_2}^R}{Z_{t,T_2}^T} \\ \dfrac{dZ_t^T}{Z_t^T} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{2}\left(\sigma_{t,T_1}^{N*}\right)^2 - \alpha_{t,T_1}^{N*} + \sigma_{t,T_1}^{N*}\gamma_t^N\right)dt - \sigma_{t,T_1}^{N*} d\hat{W}_t^N \\ \left(\frac{1}{2}\left(\sigma_{t,T_2}^{R*}\right)^2 - \alpha_{t,T_2}^{R*} + r_t^R - r_t^N + \mu_t - \rho_{R,I}\sigma_{t,T_2}^{R*} + \sigma_{t,T_2}^{R*}\gamma_t^R - \sigma_t^I\gamma_t^I\right)dt + \sigma_t^I d\hat{W}_t^I - \sigma_{t,T_2}^{R*} d\hat{W}_t^R \\ \left(r_t^R - r_t^N + \mu_t - \sigma_t^I\gamma_t^I\right)dt + \sigma_t^I d\hat{W}_t^I \end{pmatrix} \quad (5)$$

and to have Z_*^h driftless, we need that:

$$\begin{pmatrix} \alpha_{t,T_1}^{N*} \\ \alpha_{t,T_2}^{R*} \\ \mu_t \\ \alpha_{t,T_2}^{R*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(\sigma_{t,T_1}^{N*}\right)^2 + \sigma_{t,T_1}^{N*}\gamma_t^N \\ \frac{1}{2}\left(\sigma_{t,T_2}^{R*}\right)^2 + \sigma_{t,T_2}^{R*}\gamma_t^R - \sigma_t^I\gamma_t^I + r_t^R - r_t^N + \mu_t - \rho_{RI}\sigma_t^I\sigma_{t,T_2}^{*R} \\ r_t^R - r_t^R + \sigma_t^I\gamma_t^I \\ \frac{1}{2}\left(\sigma_{t,T_2}^{R*}\right)^2 + \sigma_{t,T_2}^{R*}\gamma_t^R - \rho_{RI}\sigma_{t,T_2}^{*R} \end{pmatrix} \tag{6}$$

(the last equation for α_{t,T_2}^{R*} in the fourth line above, results from substituting μ_t for its expression in the third line.)

In order for $Q = Q^{\gamma}$ to be an EMM simultaneously for bond prices of all maturities, $\underline{\gamma}_t$ needs to be T-independent and this in turn means that, given the (integrated) volatilities $\underline{\sigma}_{t,T}^{h*}$, equation (6) is a condition on the (integrated) drifts, $\int_t^T \alpha_{t,U}^h dU$.

Differentiating both sides with respect to T we obtain a condition on the α's themselves:

$$\begin{pmatrix} \alpha_{t,T}^{N} \\ \alpha_{t,T}^{R} \\ \mu_t \end{pmatrix} = \begin{pmatrix} \sigma_{t,T}^{N}\sigma_{t,T}^{N*} + \sigma_{t,T}^{N}\gamma_t^N \\ \sigma_{t,T}^{R}\left(\sigma_{t,T}^{R*} - \rho_{RI}\sigma_t^I\right) + \sigma_{t,T}^{R}\gamma_t^R \\ r_t^N - r_t^R + \sigma_t^I\gamma_t^I \end{pmatrix}$$

Our HJM-model is therefore determined by:

- specifying the volatilities $\left\{\sigma_{t,T}^N, \sigma_{t,T}^R, \sigma_t^I\right\}$ with respect of the three risk-factors $\left\{W_{t,T}^N, W_{t,T}^R, W_t^I\right\}$ and
- specifying the corresponding market prices of risk $\left\{\gamma_{t,T}^N, \gamma_{t,T}^R, \gamma_t^I\right\}$.

These solutions for the drifts, $\{\alpha, \alpha^{*h}$ and $\mu\}$, allow us to write the following system of equations which is the basis for our estimation:

$$\begin{pmatrix} \frac{d_t P_{t,T}^N}{P_{t,T}^N} \\ \frac{d_t P_{t,T}^R}{P_{t,T}^R} \\ \frac{dP_{t,T}^T}{P_{t,T}^T} \\ \frac{dP_t^T}{P_t^T} \\ \frac{dI^t}{I^t} \end{pmatrix} = \begin{pmatrix} r_t^N dt - \sigma_{t,T}^{N*} d\hat{W}_t^N \\ \left(r_t^R + \rho_{RI}\sigma_t^I\sigma_{t,T}^{*R}\right)dt - \sigma_{t,T}^{R*} d\hat{W}_t^R \\ r_t^N dt + \sigma_t^I d\hat{W}_t^I - \sigma_{t,T}^{R*} d\hat{W}_t^R \\ r_t^N dt + \sigma_t^I d\hat{W}_t^I \\ \left(r_t^N - r_t^R\right)dt + \sigma_t^I d\hat{W}_t^I \end{pmatrix} \tag{7}$$

4.1 Pricing contingent claims

Let X be $F\frac{W}{T}$ - measurable.

- The martingale representation theorem allows to write any discounted claim's price as:

$$\widetilde{X} := \frac{X}{B_T} = X_0 + \int_0^T \left(\delta_t^N d\hat{W}_t^N + \delta_t^R d\hat{W}_t^R + \delta_t^I d\hat{W}_t^I \right) \tag{8}$$

for some $X_0 \in \mathrm{P}$ and $F\frac{\hat{W}}{t}$ adapted processes $\underline{\delta}_t = \left(\delta_t^N, \delta_t^R, \delta_t^I\right)$.

- Using equations (5) and (6), we get:

$$\begin{pmatrix} \frac{dZ_{t,T_1}^N}{Z_{t,T_1}^N} \\ \frac{dZ_{t,T_2}^R}{Z_{t,T_2}^R} \\ \frac{dZ_t^T}{Z_t^T} \end{pmatrix} = \begin{pmatrix} -\sigma_{t,T1}^{N*} d\hat{W}_t^N \\ \sigma_t^I d\hat{W}_t^I - \sigma_{t,T_2}^{R*} d\hat{W}_t^R \\ \sigma_t^I d\hat{W}_t^I \end{pmatrix} \tag{9}$$

and, assuming that the 3×3 matrix Σ

$$\sum = \begin{pmatrix} -\sigma_{t,T_1}^{N*} & 0 & 0 \\ 0 & -\sigma_{t,T_2}^{R*} & \sigma_t^I \\ 0 & 0 & \sigma_t^I \end{pmatrix}$$

is invertible, for all t, it is possible to invert the set of linear relations (9), and with

$$d\hat{W} = \Sigma^{-1} \frac{dZ}{Z}$$

substituting this back into (8), it results:

$$\widetilde{X} = X_0 + \int_0^T \left(\varphi_t^N dZ_{t,T_1}^N + \varphi_t^R dZ_{t,T2} T + \varphi_t^I dZ_t^T \right),$$

with:

$$\begin{pmatrix} \varphi_t^N \\ \varphi_t^R \\ \varphi_t^I \end{pmatrix} = \begin{pmatrix} \frac{-\delta_t^N}{\sigma_{t,T_1}^{N*} Z_{t,T_1}^N} \\ \frac{-\delta_t^R}{\sigma_{t,T_2}^{R*} Z_{t,T_2}^T} \\ \frac{\delta_t^R}{\sigma_{t,T_2}^{R*} Z_{t,T_2}^T} + \frac{\delta_t^I}{\sigma_t^I Z_t^T} \end{pmatrix}$$

$\underline{\varphi}_{t=} \left(\varphi_t^N, \varphi_t^R, \varphi_t^I\right)$, invested in the three assets together with

$$\psi_t = -\left(\varphi_t^N Z_{t,T_1}^N + \varphi_t^R Z_{t,T_2}^T + \varphi_t^I Z_t^T\right) + \int_0^t \left(\varphi_t^N dZ_{t,T_1}^N + \varphi_t^R dZ_{t,T_2}^T + \varphi_t^I dZ_t^T\right)$$

in a saving account, constitute a self-financing trading strategy replicating $\widetilde{X}$ at T.

- The price of our contingent claim, X_0, is obtained by taking expectations with respect to E_Θ, as $\mathrm{E}_{\Theta,t}\left(dZ_{t,*}^h\right) = 0$, by construction.

5. Data treatment/generation

5.1 Data description

The data set includes daily closings of selected euro-benchmark government nominal bonds[12] and French government inflation-linked bonds for the period 09/03/2007 to 26/02/2010, as well as monthly data on the harmonized consumer price index (HICP), covering the same period but with a monthly frequency. Consequently, the data set comprises around 800 daily observations for each tenor corresponding to nominal and real bonds and 36 observations corresponding to the HICP.

5.2 Nominal and real interest rates

Data for nominal bonds (spot rates) was available on 15 different maturities: three and six months, one to 10, 15, 20 and 30 years. Data on linkers (daily prices) corresponded to the five benchmark French HICP-linked bonds.

Zero-coupon (spot) nominal rates were available directly from Bloomberg information service.[13] They are estimated by Bloomberg itself, on the basis of the data on traded nominal bonds, issued by euro area-based sovereign issuers. The data as published by Bloomberg are constant maturity rates.

Data on zero-coupon real rates were not readily available from Bloomberg. Therefore, in order to extract the rates at the desired maturities, we estimated the relevant daily term structures on the basis of the five benchmark euro-denominated French sovereign bonds, linked to the euro area HICP inflation index (excluding tobacco) and published monthly by Eurostat.

The estimation procedure involved cross-sectional fitting of the zero-coupon, Nelson-Siegel (1987) term structure to all daily price observations, available from 09/03/2007 until 26/02/2010.[14] It is a fairly accurate approximation of the current term structure of zero-coupon rates, provided that its shape is not too irregular. The model is still widely used by the market participants (see eg BIS (2005 ([7])).

The starting point is the description of the forward rate curve. Its shape is given by the time to maturity, $T-t$, as well as four parameters: $\beta_0, \beta_1, \beta_2, \kappa,$ according to the following formula:

$$f_{t,T}^{R} = \beta_0 + \left(\beta_1 + \beta_2 (T-t)\right) e^{-\kappa(T-t)}$$

where $f_{t,T}^{R}$ stands for the rate, set at t, for borrowing over $[T, T+dt], T > t$ in the “foreign country”.

The corresponding spot rate term structure takes the form:

$$r_{t,T}^{R} = \beta_0 + \left(\beta 1 + \frac{\beta_2}{\kappa}\right)\left(\frac{1-e^{-\kappa(T-t)}}{\kappa(T-t)}\right) - \frac{\beta_2}{\kappa} e^{-\kappa(T-t)}$$

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12 French government treasury bills for maturities up to one year and German government bonds for maturities beyond one year.

13 More precisely, the indices can be found using the Fair Market Curve (FMC) function in Bloomberg, and then choosing curve number F960.

14 Although the Nelson-Siegel model family is known to violate the no-arbitrage assumptions when considered in the time dimension, it must be noticed that the estimations were carried out for a series of cross-sectional observations. In other words, the inter-temporal dynamics of the Nelson-Siegel model did not play any role in the analysis.

By making T approach t the instantaneous short rate results as $r^R(t) = \beta_0 + \beta_1$ and allowing for T to grow unbounded the long-term rate $r^R_{t,\infty}$ becomes equal to $r^R_{t,\infty} = \beta_0$. The remaining parameters govern the location and size of the hump.

These spot rates can be easily transformed into the discount factors:

$$P^R_{t,T} = e^{-r^R_{t,T}(T-t)}$$

and these, in turn, can be used to price financial assets traded on the market, including bonds. However, the specific problem encountered in the present project necessitated the application of a reverse engineering, whereby the observed market prices of the five French HICP inflation-linked bonds served to estimate the unknown parameters. More specifically, the observed bond prices were compared to the theoretical prices given by the formula:

$$B^R_{t,T} = \sum c^R_j \times P^R_{t,j}$$

where c^R_j denote the (real) cash-flows, and $B^R_{t,T}$ stands for the sum of the discounted real cash-flows – ie the "foreign-currency' price of the bond. The estimation of the parameters was conducted by way of minimizing the sum of the squared errors between the prices of the five French inflation-linkers and their model counterparts. Obviously, given that the Nelson-Siegel model in its original form is static (ie it describes the term structure at a given moment, and not its evolution over time), the estimation procedure needed to be carried out separately for each day in the sample – ie around 800 daily observations.[15]

Then, the (theoretical) real zero-coupon rates were also calculated for the 15 selected maturities, as outlined above. From this set of zero-rates, the (daily) returns are derived as follows:

$$R_{t,T} = (r_{t-1,T} - r_{t,T})(T-t) + r_{t,T}\frac{1}{252} \tag{10}$$

Both the nominal zero-coupon rates published by Bloomberg and the model-derived real zero-coupon rates have in effect constant maturities. Thus, in order to compute the return (either nominal or real) for holding a Z-bond over a one day period, we need the corresponding ($(t-T)-1$day) maturity rate, which is not directly available from the data. For this purpose, we assume that the interest rate of a given maturity $(t-T)$ is also valid for maturity ($t-T-1$day).[16]

Zero-coupon forward bonds are martingales under the forward measure. To preserve consistency in the empirical part, parameter estimation was conducted using forward rates. For example, the three-month spot and six-month spot nominal rates for a given day were used to calculate the implied 3x3M forward rate for that same day. The same transformation was conducted for the remaining maturities. The resulting dataset comprised the forward nominal and real interest rates, with the following fourteen maturities: (3M, 9M, 1.75Y, 2.75Y, 9.75Y, 14.75Y, 19.75Y, 29.75Y).

[15] One of the French inflation-linkers, FRTR –3:15% maturing in July 2032, was deliberately left out of the estimation sample, in order to evaluate the out-of-sample performance of the model. The table at the end of this section presents the results of the estimation.

[16] Although several "fine-tuned" alternatives are possible, the impact of this assumption on the final result is negligible.

5.3 Returns on inflation-linked zero-coupon bonds

As explained in Section 2, inflation-linked bonds are traded on a nominal basis, after adjusting for the inflation accrued over a given period:

$$B_{t,T}^{T} = B_{t,T}^{R} \times IR_t = \sum c_j^R \times P_{t,j}^T \tag{11}$$

where $P_{t,j}^T = P_{t,j}^R \times IR_t$ denotes the price, at time t, of an index-linked zero-coupon bond, $P_{t,j}^R$ stands for the t-time price of the underlying real zero-coupon bond, and IR_t is the corresponding index ratio for day t.

The price of the synthetic 3M-forward zero-ILB was calculated according to the following formula:

$$P_{t,t+3M,j}^T = \frac{P_{t,j}^T}{P_{t,3M}^N} \tag{12}$$

with $P_{t,t+3M,j}^T$ denoting the price, agreed at t, of a 3M-forward index linked zero-coupon bond delivered at $t+3M$ and maturing at j. $P_{t,3M}^N$, in turn, is a price of a three-month nominal bill. The return on such synthetic forward zero-coupon bonds was calculated in line with the procedure as outlined in the equation (10).

5.4 Smoothing algorithm

The final step before estimating the variance was to apply the smoothing algorithm, similar to that implemented by Jarrow and Yildirim (2003). The aim of the procedure was to ensure that the obvious outliers (eg resulting from the poor market quotes), which generate noise in the data, are excluded from the analysis. The smoothing algorithm was based on the following formula:

$$\left| \frac{yield - Mean(yield)}{\sigma_{yield}} \right| \le k \tag{13}$$

where k varies from 3.25 to 2.50, depending on the maturity. The purpose of varying the parameter k was to ensure that the overall data sample is broadly balanced (ie the number of observations is approximately equal) across the maturities.

5.5 The rate of inflation

The monthly rate of inflation was calculated from the euro area HICP inflation index (excluding tobacco), published monthly by Eurostat.[17] In line with Jarrow and Yildirim (2003), the raw index data was transformed into the rate of inflation using the following formula:

$$\frac{dI_t}{I_t} = \ln\left(\frac{\Delta I_t}{I_t}\right) \tag{14}$$

[17] The data can be found eg in Bloomberg using the following mnemonic: CPTFEMU < Index >.

5.6 Estimation procedure

The aim of the procedure was to estimate the following parameters:

Parameter	Definition
α^N	Time decay factor of the nominal return's volatility: $\sigma^N e^{-\alpha^N(T-t)}$.
α^R	Time decay factor of the real return's volatility: $\sigma^R e^{-\alpha^R(T-t)}$.
σ^N	Scale factor for the nominal return's volatility.
σ^R	Scale factor for the real return's volatility.
σ^I	Constant HICP's volatility.
$\rho_{N,R}$	Correlation between Nominal and Real return risk drivers.
$\rho_{N,I}$	Correlation between Nominal return and Inflation risk drivers.
$\rho_{R,I}$	Correlation between Real return and Inflation risk drivers.

The estimation proceeded in several steps. In each case, it involved fitting of the variance/covariance function to the cross section of the observed variances/covariances of the returns on the forward real bonds, forward inflation-linked bonds, forward nominal bonds, as well as inflation. Fitting was performed using nonlinear least squares.

As shown in the table above, volatilities were assumed to be time dependent, but deterministic $\left(\sigma^h_{t,T} = \sigma^h e^{-\alpha^h(T-t)}, h \in \{N,R\}\right)$ and as a consequence the rates of price changes are Gaussian. The parameters of the nominal return process were then estimated using the following equation:

$$Var\left(\frac{dP^N_{\tau,T}}{P^N_{\tau,T}}\right) = \left[\int_\tau^T \sigma^N e^{-\alpha^N(U-\tau)} dU\right]^2 dt = \left\{\sigma^N\left(\frac{1-e^{-\alpha^N}(T,\tau)}{\alpha^N}\right)\right\}^2 dt \tag{15}$$

with τ denoting the (forward) maturity and $dt = 1/252$ representing the time step. As usual, the variable to be explained (the variance of the forward returns on the nominal bonds) is on the left-hand side, and the only explanatory variable on the right-hand side is the forward maturity. Likewise, the parameters of the real return process were evaluated based on the equation:

$$Var\left(\frac{dP^R_{\tau,T}}{P^R_{\tau,T}}\right) = \left[\int_\tau^T \sigma^R e^{-\alpha^R(U-\tau)} dU\right]^2 dt = \left\{\sigma^R\left(\frac{1-e^{-\alpha^R}(T,\tau)}{\alpha^R}\right)\right\}^2 dt \tag{16}$$

The four parameters $\alpha^N, \sigma^N, \alpha^R, \sigma^R$ served immediately to evaluate the correlation between the nominal and the real returns, $\rho_{R,N}$. To this end, use was made of the following equation:

$$Cov\left(\frac{dP^R_\tau}{P^R_\tau}, \frac{dP^N_\tau}{P^N_\tau}\right) = \rho_{R,N} \times \sigma^R \times \sigma^N \times \left(\frac{1-e^{-\alpha^R(T-\tau)}}{\alpha^R}\right)\left(\frac{1-e^{-\alpha^N(T-\tau)}}{\alpha^N}\right) dt$$

The next step required the evaluation of the volatility of inflation, σ^I. It was approximated by the sample standard deviation of the rate of inflation (Eurozone HICP) over the period starting in March 2007 and ending in February 2010 (36 observations). With use of this additional parameter, it was possible to estimate the correlation between the nominal returns and the inflation, based on the equation:

$$Cov\left(\frac{dI_\tau}{I_\tau},\frac{dP_\tau^N}{P_\tau^N}\right)=-\rho_{I,N}\times\sigma^I\times\sigma^N\times\left(\frac{1-e^{-\alpha^N(T-\tau)}}{\alpha^N}\right)$$

Another equation, analogous to the previous one, albeit involving the real rate and the inflation, was used to estimate $\rho_{R,I}$, the correlation between these two processes:

$$Cov\left(\frac{dI_\tau}{I_\tau},\frac{dP_\tau^R}{P_\tau^R}\right)=-\rho_{I,R}\times\sigma^I\times\sigma^R\times\left(\frac{1-e^{-\alpha^R(T-\tau)}}{\alpha^R}\right)$$

5.7 Calibration based on ZCIIS

An alternative way to calibrate the real part of our HJM model is to recur to the market for inflation derivatives, in particular to the Zero-Coupon Inflation-Indexed Swaps (ZCIIS).

ZCIIS are actively traded in the European, UK and US markets and are the most liquid inflation derivatives. As their prices are model-independent, the term structure of real rates can be easily derived from the nominal term-structure and market inflation swap rates.

On a ZCIIS one party pays inflation on a notional amount N, whereas the other party pays fixed on the same notional. The contract is for settlement at maturity (T) and its value is zero at inception (t). The fixed rate (k) is chosen so as to make the value of the fixed leg equal to that of the inflation leg, when the swap is initially traded at t. Formally:

$$B_t^N \mathrm{E}_\Theta\left\{B_T^{N-1}\left(\frac{I_T}{I_t}-(1+k)^{T-t}\right)\middle|F_t\right\}=0 \tag{17}$$

from where it results that:

$$P_{t,T}^R=P_{t,T}^N(1+k)^{T-t} \tag{18}$$

where k is the quoted ZCIIS, and the corresponding data series is available from Bloomberg.

5.8 Results of estimation

The following tables present the estimated coefficients, together with their standard errors and significance levels for both, the Nelson-Siegel derived real z-bonds and the ZCIIS-derived bonds:

Estimation using Nelson-Siegel-derived zero-coupon real bonds.		
Parameter	**Value**	**St. Error**
α^N	1.9713E – 03	(1.575E – 03)
α^R	6.0379E – 03***	(8.65E – 04)
σ^{N^2}	4.5289E – 05***	(1.94E – 06)
σ^{R^2}	4.4214E – 05***	(1.01E – 06)
σ^{I^2}	2.1052E – 04	*
$\rho_{N,R}$	0.7434***	(7.384E – 03)
$\rho_{N,I}$	0.3780***	(1.566E – 02)
$\rho_{R,I}$	0.2468***	(8.559E – 03)

*** significance at $\alpha = 1\%$

* estimate based on sample variance of inflation

Estimation using ZCIIS-derived zero-coupon real bonds.		
Parameter	**Value**	**St. Error**
α^N	1.9713E – 03	(1.575E – 03)
α^R	11.4361E – 03***	(13.14E – 04)
σ^{N^2}	4.5289E – 05***	(1.94E – 06)
σ^{R^2}	7.0432E – 05***	(2.37E – 06)
σ^{I^2}	2.1052E – 04	*
$\rho_{N,R}$	0.7995***	(8.29E – 03)
$\rho_{N,I}$	0.3780***	(1.566E – 02)
$\rho_{R,I}$	0.1809***	(8.563E – 03)

*** significance at $\alpha = 1\%$

* estimate based on sample variance of inflation

6. Hedging analysis

The three-factor HJM model we have fitted to the market needs now to be validated.

We will do this via a hedging analysis, ie we will price traded inflation-linked bonds and nominal bonds out of the model and compare these model-derived prices with those actually traded in the market. We will do this for the whole time range considered in this study

(09/03/2007–26/02/2010). Traded are coupon-bearing bonds, not zeroes, so we need to do it with actual traded bonds. The procedure to hedge the linker is as follows:[18]

- First build two portfolios: portfolio A, including the linker whose price we are trying to validate and portfolio B, including (in principle) three bonds (two linkers and one nominal bond). Portfolio B requires three different bonds in order to control for the three risk factors in the economy.

- Then calculate the required amounts of each bond in portfolio B ($n_{1,t}$, $n_{2,t}$ and $n_{3,t}$) so that the total investment required to build it at time t matches exactly the cost of buying the linker in portfolio A.

- Then calculate the daily return of each portfolio $\left(R_t^A, R_t^B\right)$ and compute the difference $\left(\varepsilon_t = R_t^A - R_t^B\right)$. If the model is correct, the difference should be indistinguishable from 0.

- Finally validate the model via analysis of residuals.

Due to the fact that we have specified the volatilities as deterministic functions of time, all zero-coupon bonds (nominal and real) are Markov in three state variables: The instantaneous nominal and real rates, r_r^N, r_r^R and the inflation index, I_t.

In particular, the specification of volatilities in the model – $\sigma_{t,T}^h = \sigma^h e^{-\alpha^h (T-t)}$ – translates into r^N and r^R following Ornstein–Uhlenbeck stochastic processes:

$$\begin{pmatrix} dr_t^N \\ dr_t^R \end{pmatrix} = \begin{pmatrix} \alpha^N\left(\theta_t^N - r_t^N\right)dt + \sigma^N d\hat{W}_t^N \\ \alpha^R\left(\theta_t^R - r_t^R\right)dt + \sigma^R d\hat{W}_t^R \end{pmatrix} \tag{19}$$

which in turn determines specific forms for the corresponding zero-coupon bond prices:

$$\begin{pmatrix} P_{t,T}^N \\ P_{t,T}^R \end{pmatrix} = \begin{pmatrix} e^{A_{t,T}^N - b_{t,T}^N r_t^N} \\ e^{A_{t,T}^R - b_{t,T}^R r_t^R} \end{pmatrix} \tag{20}$$

where $A_{t,T}^h, h \in (N, R)$ are functions of time that turn out not to matter for the hedging exercise, and

$$b_{t,T}^h = \left(\frac{1 - e^{-\alpha^h (T-t)}}{\alpha^h}\right)$$

We first build portfolio B so that it is worth the same as the price of the ILB we are trying to hedge:

$$n_1 B_{t,1}^T + n_2 B_{t,2}^T + n_3 B_{t,3}^N = B_{t,0}^T$$

where $B_{t,k}^f$, $f \in (T, N)$, $k \in$ (0, 1, 1, 3) stand for the price at t of the corresponding coupon bearing bond k.

[18] As stated in the section describing the index-linked bonds, linkers usually include a par floor, granting that the capital received will at least be equal to 100%. The value of this option is usually considered to be zero, and we treat them similarly, given that it is highly unlikely that it would ever need to be executed.

Prices of traded linkers are actually the product of the "real" bond prices, $B_{t,k}^{R}$ and the corresponding "exchange rate", I_t / I_k , with I_k standing for the associated base index.

$$B_{t,k}^{T} = \frac{I_t}{I_k} B_{t,k}^{R}\text{, with } B_{t,k}^{R} = \sum_{j=1}^{n_k} c_j^k P_{t,j}^{R}$$

where

$$\begin{pmatrix} \\ c_j^k \\ \\ \end{pmatrix} = \begin{pmatrix} c^k \quad \forall j < n_k \\ (c^k + 1) \;\; for \quad j = n_k \end{pmatrix}$$

In order for portfolio B to hedge portfolio A (the linker) we need:

$$\begin{array}{ccc} B_{t,0}^{N} & = & n_1 B_{t,1}^{T} + n_2 B_{t,2}^{T} + n_3 B_{t,3}^{N} \\ dB_{t,0}^{T} & = & n_1 dB_{t,1}^{T} + n_2 dB_{t,2}^{T} + n_3 dB_{t,3}^{N} \\ \frac{\partial \mathrm{B}_{\mathrm{t},0}^{\mathrm{T}}}{\partial \mathrm{r}_{\mathrm{t}}^{\mathrm{R}}} dr_t^R + \frac{\partial B_{t,0}^{T}}{\partial I_t} dI_t + \cdots & = & \sum_{i=1}^{2} n_i \frac{\partial B_{t,i}^{T}}{\partial r_t^R} dr_t^R + n_i \frac{\partial B_{t,i}^{T}}{\partial I_t} dI_t + n_3 \frac{\partial B_{t,3}^{N}}{\partial r_t^R} dr_t^R + n_3 \frac{\partial B_{t,3}^{N}}{\partial I_t} dI_t + \cdots \end{array}$$

where the second equation follows from the strategy being self-financing. The dots in the formulae involve other terms multiplied by dt , which cancel out. The system is solved by gathering all terms associated with each of the two random magnitudes (dI_t and dr_t^R) and making their coefficients equal to zero for each t :

$$\begin{pmatrix} n_{1,t} \\ \\ n_{2,t} \\ \\ n_{3,t} \end{pmatrix} = \begin{pmatrix} \dfrac{B_{t,0}^{T}}{B_{t,1}^{T}} \times \dfrac{\sum_{j=1}^{n_2} \omega_{t,j}^{(2)} b_{t,j}^{R} - \sum_{j=1}^{n_0} \omega_{t,j}^{(0)} b_{t,j}^{R}}{\sum_{j=1}^{n_2} \omega_{t,j}^{(2)} b_{t,j}^{R} - \sum_{j=1}^{n_1} \omega_{t,j}^{(1)} b_{t,j}^{R}} \\ \dfrac{B_{t,0}^{T}}{B_{t,2}^{T}} \times \dfrac{\sum_{j=1}^{n_0} \omega_{t,j}^{(0)} b_{t,j}^{R} - \sum_{j=1}^{n_1} \omega_{t,j}^{(1)} b_{t,j}^{R}}{\sum_{j=1}^{n_2} \omega_{t,j}^{(2)} b_{t,j}^{R} - \sum_{j=1}^{n_1} \omega_{t,j}^{(1)} b_{t,j}^{R}} \\ \\ 0 \end{pmatrix} \qquad (21)$$

where the $\omega_{t,j}^{k}$'s are:

$$\omega_{t,j}^{k} = \frac{c_j^k P_{t,j}^{R}}{B_{t,j}^{R}}, \;\; \sum_j \omega_j^k = 1$$

There are no nominal bonds in the hedging strategy $(n_3 = 0)$, which results from the fact that nominal bonds don't depend directly neither on r_t^R nor on I_t. However, they are correlated with them.

The procedure to hedge the nominal bond is as follows:

- First build two portfolios: portfolio A, including the nominal bond whose price we are trying to validate and portfolio B, including two other bonds (just nominal bonds, as their prices only depend directly on r_t^N , not on r_t^R or I_t). Portfolio B requires just one bond in order to control for the single risk factor r_t^N , but a second bond is required in order to ensure that the total value of portfolio B exactly matches that of portfolio A.

- Calculate the required amounts of each bond in portfolio B ($n_{1,t}$ and $n_{2,t}$) so that the total investment required to build it at time t matches exactly the cost of buying the bond in portfolio A.
- Then calculate the daily return of each portfolio (R_t^A, R_t^B) and compute the difference $\left(\varepsilon_t = R_t^A - R_t^B\right)$ in the same way we did for linkers. If the model is correct, the difference should be undistinguishable from 0.
- Finally validate the model via analysis of residuals.

In order for portfolio B to hedge portfolio A we need:

$$
\begin{aligned}
B_{t,0}^N &= n_1 B_{t,1}^N + n_2 B_{t,2}^N \\
dB_{t,0}^N &= n_1 dB_{t,1}^N + n_2 dB_{t,2}^N \\
\frac{\partial B_{t,0}^N}{\partial r_t^N} dr_t^N + \cdots &= \sum_{i=1}^{2} n_i \frac{\partial B_{t,i}^N}{\partial r_t^N} dr_t^N + \cdots
\end{aligned}
$$

where the second equation follows from the strategy being self-financing. The dots in the formulae involve other terms multiplied by dt, which cancel-out. Solving the system for each t in a similar way as before, the result is:

$$
\begin{pmatrix} n_{1,t} \\ \\ n_{2,t} \end{pmatrix} = \begin{pmatrix} \dfrac{B_{t,0}^N}{B_{t,1}^N} \times \dfrac{\sum_{j=1}^{n_0} \omega_{t,j}^{(0)} b_{t,j}^N - \sum_{j=1}^{n_2} \omega_{t,j}^{(2)} b_{t,j}^N}{\sum_{j=1}^{n_1} \omega_{t,j}^{(1)} b_{t,j}^N - \sum_{j=1}^{n_2} \omega_{t,j}^{(2)} b_{t,j}^N} \\ \dfrac{B_{t,0}^N}{B_{t,2}^N} \times \dfrac{\sum_{j=1}^{n_1} \omega_{t,j}^{(1)} b_{t,j}^N - \sum_{j=1}^{n_0} \omega_{t,j}^{(0)} b_{t,j}^N}{\sum_{j=1}^{n_1} \omega_{t,j}^{(1)} b_{t,j}^N - \sum_{j=1}^{n_2} \omega_{t,j}^{(2)} b_{t,j}^N} \end{pmatrix} \tag{22}
$$

6.1 Hedging results

The hedging exercise was run first run on the five existing benchmark linker bonds employed to estimate the HJM parameters: there are 10 possible combinations of three bonds each out of these five bonds, which are shown in the table below numbered from one to 10. We also included as portfolio 11 the hedging of the single bond left out of the parameter estimation, the OAT *i* 3, 15%2032, to check the model performance on an out-of-sample bond.

We performed this exercise both, for the NS and for the ZCIIS estimated parameters. All in all $11 \times 2 = 22$ portfolios were created.

These hedging portfolios generated 22 series of errors, which constitute the basis for the model-validation analysis. The error analysis was performed on the original series (as generated from the hedging exercise) and also on the same number of "filtered" series, ie series where the errors were filtered in line with the three sigma rule, according to which all the outlier observations exceeding three sample standard deviations were iteratively excluded from the series, until the sample moments (mean and variance) converged to a stable level.[19]

[19] This allowed for smoothing the series and decreasing the dispersion of the sample distribution.

Hedged bond	hedging bonds	
1) BTANi 1,25% 2010	OATi 3% 2012	OATi 1,6% 2015
2) BTANi 1,25% 2010	OATi 3% 2012	OATi 2,25% 2020
3) BTANi 1,25% 2010	OATi 3% 2012	OATi 1,8% 2040
4) BTANi 1,25% 2010	OATi 1,6% 2015	OATi 2,25% 2020
5) BTANi 1,25% 2010	OATi 1,6% 2015	OATi 1,8% 2040
6) BTANi 1,25% 2010	OATi 2,25% 2020	OATi 1,8% 2040
7) OATi 3% 2012	OATi 1,6% 2015	OATi 2,25% 2020
8) OATi 3% 2012	OATi 1,6% 2015	OATi 1,8% 2040
9) OATi 3% 2012	OATi 2,25% 2020	OATi 1,8% 2040
10) OATi 1,6% 2015	OATi 2,25% 2020	OATi 1,8% 2040
11) OATi 3,15% 2032	OATi 2,25% 2020	OATi 1,8% 2040

The following chart presents the result of the analysis:

The results clearly show that in all cases but the NS-filtered series from portfolio 11 (the 2020/2040 portfolio hedging the 2032 bond) there is no reason to reject the null hypothesis of zero mean error.[20] This constitutes a strong argument for validating the model: In other words, none of the strategies considered allows for making consistent profits (ie arbitrage opportunities).

[20] Regarding the filtered series, as it is apparent from the above results, the smoothing algorithm did not affect the inference regarding the zero mean error.

Figure 1

ILBs – Null hypothesis: mean error= 0

Portfolio	Estimate	Sample size	Mean (bps)	stdev	Test t-statistic	Probability	Reject Null?
P.1	NS	774	-0.3	7.4	-1.04	30%	no
	NS-filt	734	-0.2	5.6	-0.99	32%	no
	ZC	774	-0.3	23.5	-0.37	72%	no
	ZC-filt	748	-0.2	18.1	-0.28	78%	no
P.2	NS	774	-0.4	10.6	-1.10	27%	no
	NS-filt	739	-0.4	8.1	-1.33	18%	no
	ZC	774	-0.4	21.8	-0.50	62%	no
	ZC-filt	747	-0.3	16.6	-0.51	61%	no
P.3	NS	774	-0.5	53.9	-0.28	78%	no
	NS-filt	752	0.0	44.2	0.01	99%	no
	ZC	774	-0.8	65.6	-0.34	73%	no
	ZC-filt	754	-1.1	54.3	-0.57	57%	no
P.4	NS	774	-1.1	25.7	-1.15	25%	no
	NS-filt	736	-0.7	18.9	-1.06	29%	no
	ZC	774	-0.9	59.3	-0.41	69%	no
	ZC-filt	743	-1.0	41.0	-0.65	52%	no
P.5	NS	774	-0.5	23.8	-0.61	54%	no
	NS-filt	742	-0.4	18.6	-0.56	58%	no
	ZC	774	-0.4	38.4	-0.32	75%	no
	ZC-filt	752	-1.1	29.7	-0.99	32%	no
P.6	NS	774	0.0	43.0	0.03	98%	no
	NS-filt	731	-0.1	29.8	-0.14	89%	no
	ZC	774	-0.1	54.6	-0.04	97%	no
	ZC-filt	754	-1.0	45.3	-0.61	54%	no
P.7	NS	774	-0.4	10.1	-1.11	27%	no
	NS-filt	738	-0.4	7.3	-1.60	11%	no (marginal)
	ZC	774	-0.2	35.5	-0.18	86%	no
	ZC-filt	753	-0.7	26.0	-0.73	47%	no
P.8	NS	774	-0.2	13.6	-0.31	76%	no
	NS-filt	737	0.0	10.0	-0.13	90%	no
	ZC	774	-0.1	27.2	-0.08	93%	no
	ZC-filt	756	-0.4	21.3	-0.51	61%	no
P.9	NS	774	0.4	35.1	0.29	77%	no
	NS-filt	733	0.9	24.3	1.01	31%	no
	ZC	774	0.2	45.4	0.15	88%	no
	ZC-filt	750	-0.2	35.8	-0.13	89%	no
P.10	NS	774	0.4	18.0	0.58	56%	no
	NS-filt	728	0.4	12.4	0.79	43%	no
	ZC	774	0.2	30.2	0.15	88%	no
	ZC-filt	737	-0.5	20.8	-0.68	50%	no
P.11	NS	774	-0.4	11.2	-1.04	30%	no
	NS-filt	710	-0.5	6.4	-2.27	2%	reject at 5%
	ZC	774	-0.6	25.8	-0.63	53%	no
	ZC-filt	741	-0.7	17.4	-1.07	29%	no

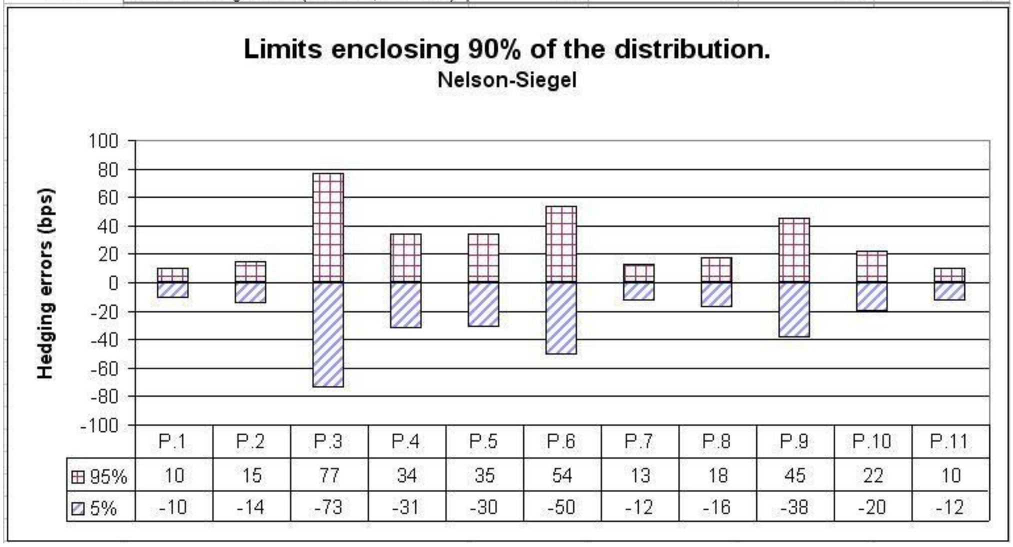
Figure 2 presents a table showing the mean error and corresponding standard deviation per portfolio (both, for the NS and the ZCIIS filtered series), and a chart showing the error range covering 90% of the distribution for the NS-filtered series.[21] The chart permits to have a second assessment on the quality of the HJM-model to represent the economy: six out of 11 hedging portfolios produced errors which stayed inside ±20 bps 90% of the time and two portfolios produced errors which stayed inside ±35 bps 90% of the time. Remarkably, portfolio 11, which hedges the bond 2032 (which wasn't included in the set of bonds to estimate the HJM parameters), produced very small errors.

[21] The table shows that the NS parameter estimation produced much more accurate results than the ZCIIS approach.

Figure 2

ILBs – NS-filtered selection

#	Hedging Portfolios	Errors NS		Errors ZCIIS	
		AVG (bps)	St.Dev.(bps)	AVG (bps)	St.Dev.(bps)
P.1	ILB 2010 hedged with (ILB 2012; ILB 2015)	-0.2	6	-0.2	18
P.2	ILB 2010 hedged with (ILB 2012; ILB 2020)	-0.4	8	-0.3	17
P.3	ILB 2010 hedged with (ILB 2012; ILB 2040)	0.0	44	-1.1	54
P.4	ILB 2010 hedged with (ILB 2015; ILB 2020)	-0.7	19	-1.0	41
P.5	ILB 2010 hedged with (ILB 2015; ILB 2040)	-0.4	19	-1.1	30
P.6	ILB 2010 hedged with (ILB 2020; ILB 2040)	-0.1	30	-1.0	45
P.7	ILB 2012 hedged with (ILB 2015; ILB 2020)	-0.4	7	-0.7	26
P.8	ILB 2012 hedged with (ILB 2015; ILB 2040)	0.0	10	-0.4	21
P.9	ILB 2012 hedged with (ILB 2020; ILB 2040)	0.9	24	-0.2	36
P.10	ILB 2015 hedged with (ILB 2020; ILB 2040)	0.4	12	-0.5	21
P.11	ILB 2032 hedged with (ILB 2020; ILB 2040)	-0.5	6	-0.7	17



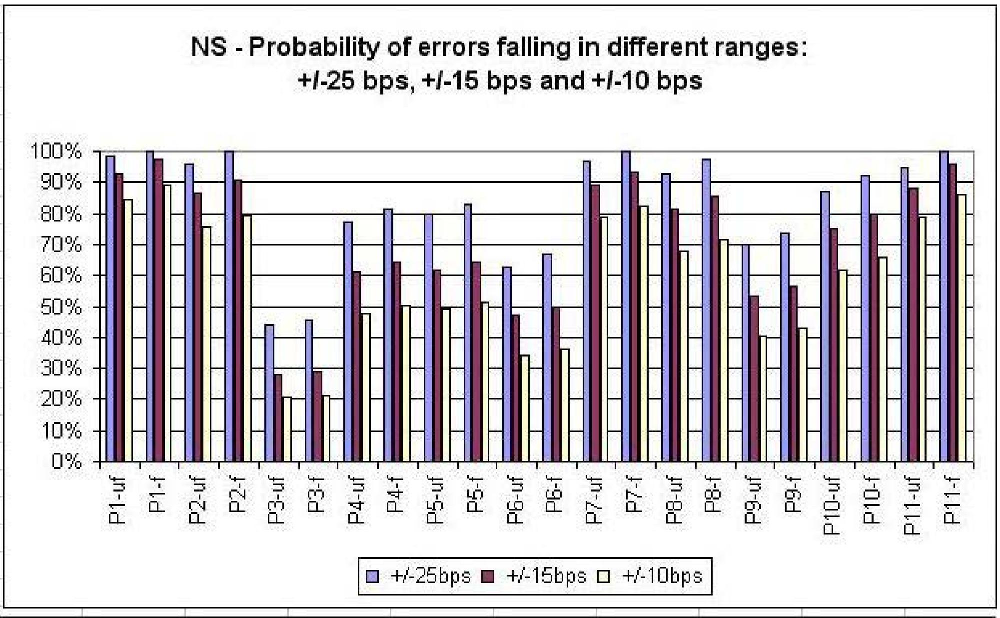
Finally, Figure 3 shows from another angle that the NS approach to build the zero-coupon bonds produced superior results compared to the ZCIIS approach. The probability mass of hedging errors enclosed in specific ranges is larger in the NS approach than in the ZCIIS. The results are presented both, for filtered and unfiltered errors.

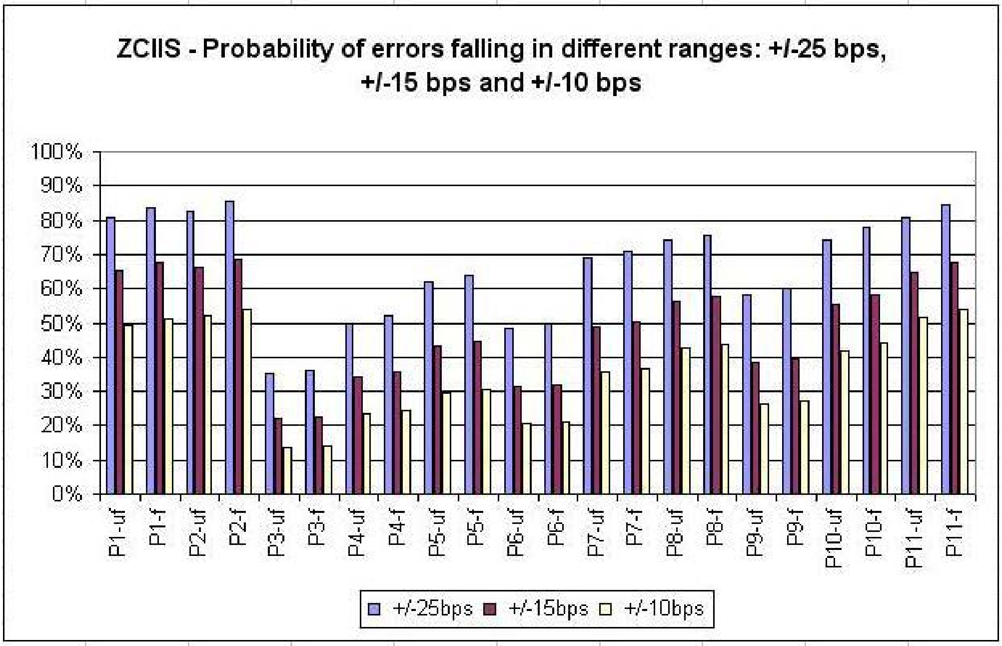
There is still a need to perform a similar hedging analysis on nominal bonds in order to validate the model, which is done below. The hedging analysis was based on 10 portfolios made out of five different synthetic bonds, built to exactly match the maturities of the five benchmark linker bonds and the same methodology used to hedge the ILBs was used to analyze the nominal bonds.

The following charts present the results of the analysis.

Figure 3

ILBs–probability mass





The results show that in all cases there is no reason to reject the null hypothesis of zero mean error.[22] This completes the argument for validating the model: In other words, none of the strategies considered allows for making consistent profits (ie arbitrage opportunities).

22 Regarding the filtered series, as was the case also for ILBs, the smoothing algorithm did not affect the inference regarding the zero mean error.

Figure 4

Nominal bonds-null hypothesis: mean error= 0

Portfolio	Estimate	Sample size	Mean (bps)	stdev	Test t-statistic	Probability	Reject Null?
P.1	Unfiltered	774	-0.2	18.4	-0.28	78%	no
	Filtered	718	0.3	7.9	0.95	34%	no
P.2	Unfiltered	774	-0.3	33.6	-0.28	78%	no
	Filtered	738	0.3	22.3	0.37	71%	no
P.3	Unfiltered	774	-0.1	34.2	-0.12	91%	no
	Filtered	733	0.5	22.7	0.60	55%	no
P.4	Unfiltered	774	0.3	44.7	0.21	83%	no
	Filtered	741	0.2	33.2	0.15	88%	no
P.5	Unfiltered	774	-0.2	16.9	-0.37	71%	no
	Filtered	719	0.2	8.1	0.71	48%	no
P.6	Unfiltered	774	-0.2	16.9	-0.34	74%	no
	Filtered	718	0.3	8.3	0.83	41%	no
P.7	Unfiltered	774	0.0	14.6	0.01	99%	no
	Filtered	743	0.0	9.8	-0.03	97%	no
P.8	Unfiltered	774	0.1	10.9	0.17	86%	no
	Filtered	737	0.1	6.7	0.47	64%	no
P.9	Unfiltered	774	0.2	11.3	0.53	60%	no
	Filtered	756	0.0	9.8	0.08	94%	no
P.10	Unfiltered	774	-0.1	8.5	-0.30	77%	no
	Filtered	745	0.0	6.7	-0.05	96%	no

7. Portfolio selection

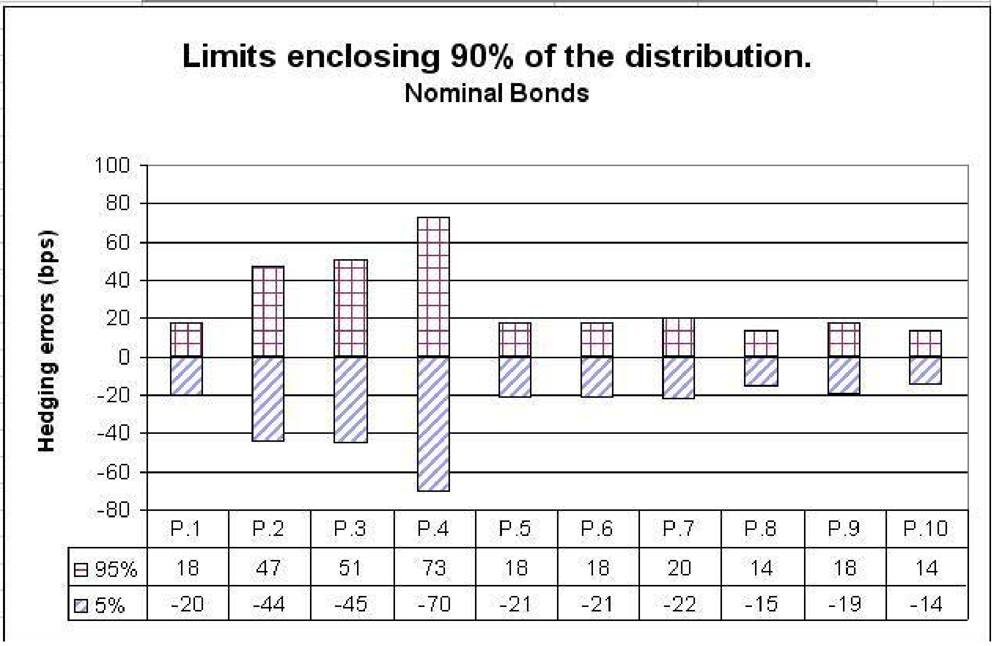
We have built a model to characterize the bond market (including nominal and inflation-linked sovereign bonds) and we are now capable of building a sovereign bond portfolio including both asset classes. As the model is Gaussian, the variance-covariance matrix characterizing all securities in the portfolio is required, as is the corresponding vector of expected returns.

As there are just three risk factors driving the market, all we need is the risk-free rate and three bonds (two linkers and one nominal bond) spanning the whole maturity range.

Figure 5

Nominal bonds–filtered selection

#	Hedging Portfolios	Errors	
		AVG (bps)	St.Dev.(bps)
P.1	2040 hedged with 2020 & 2032	-0.2	18
P.2	2040 hedged with 2020 & 2015	-0.3	34
P.3	2040 hedged with 2020 & 2012	0.1	34
P.4	2040 hedged with 2015 & 2012	0.3	45
P.5	2040 hedged with 2015 & 2032	-0.2	17
P.6	2040 hedged with 2012 & 2032	-0.2	17
P.7	2020 hedged with 2012 & 2032	0.0	15
P.8	2020 hedged with 2015 & 2032	0.1	11
P.9	2020 hedged with 2015 & 2012	0.2	11
P.10	2015 hedged with 2012 & 2032	-0.1	9



Let $R^N_{t,T}dt$ be the return on a nominal bond:

$$R^N_{t,T}dt = r^N_t dt - \sigma^N \left(\frac{1-e^{-\alpha^N (T-t)}}{\alpha^N} \right) dW^N_t = r^N_t dt - \sigma^N Q^N_{t,T} dW^N_t \tag{23}$$

where we have defined $Q^N_{t,T} \equiv \left(\frac{1-e^{-\alpha^N (T-t)}}{\alpha^N} \right)$

Let $R^T_{t,T}dt$ be the return on a ILB:

$$R^T_{t,H}dt = r^N_t dt - \sigma^R Q_{t,HR} dW^R_t + \sigma^I dW^I_t \tag{24}$$

with $Q^R_{t,H} \equiv \left(\frac{1-e^{-\alpha^R (H-t)}}{\alpha^R} \right)$

Let $B^h_{t,T} = \sum_{j=1}^{N} c^h_j P^h_{t,j}$ be the price of a traded bond $(h \in \{N,T\})$ where

$$c^h_j = \begin{cases} c^h \forall j < N \\ (c^h + 1) \;\; for \;\; j = N \end{cases}$$

The instantaneous rate of return on a bond (either nominal or real) is:

$$\frac{dB_{t,T}^h}{B_{t,T}^h} = \sum_{j=1}^{N} \frac{c_j^h P_{t,j}^h}{B_{t,T}^h} \frac{dP_{t,j}^h}{P_{t,j}^h} = \sum_{j=1}^{N} \omega_j^h R_{t,j}^h; \quad h \in \{N,T\}$$

7.1 Variance-covariance

In this subsection the required Variance-covariance matrix is derived. The covariances between the different returns of discount bonds are presented below:

$$Cov\left(R_{t,T_1}^N dt R_{t,T_2}^N dt\right) = \sigma^{N^2} Q_{t,T_1}^N Q_{t,T_2}^N dt$$

$$Cov\left(R_{t,H_1}^T dt R_{t,H_2}^T dt\right) = \sigma^{R^2} Q_{t,H_1}^R Q_{t,H_2}^R dt + \sigma^{I^2} dt - \sigma^R \sigma^I \rho_{R,I} \left(Q_{t,H_1}^R + Q_{t,H_2}^R\right) dt$$

$$Cov\left(R_{t,T}^N dt R_{t,H}^T dt\right) = \sigma^N \sigma^R \rho_{N,R} Q_{t,T}^N Q_{t,H}^R dt - \sigma^N \sigma^I \rho_{N,I} Q_{t,T}^N dt$$

Now, using these expressions together with those for the returns of traded bonds, the covariances between returns of the different traded (coupon-paying bonds) are derived:

$$V\left(\frac{dB_{t,T}^N}{B_{t,T}^N}\right) = \sigma^{N^2} D_n^2 dt$$

$$V\left(\frac{dB_{t,H}^T}{B_{t,H}^T}\right) = \left(\sigma^{R^2} D_T^2 + \sigma^{I^2} - 2\sigma^R \sigma^I \rho_{R,I} D_T\right) dt$$

$$Cov\left(\frac{dB_{t,T}^N}{B_{t,T}^N} \frac{dB_{t,H}^T}{B_{t,H}^T}\right) = \left(\sigma^N \sigma^R \rho_{N,R} D_N D_T - \sigma^N \sigma^I \rho_{N,I} D_N\right) dt$$

$$Cov\left(\frac{dB_{t,H_1}^T}{B_{t,H_1}^T} \frac{dB_{t,H_2}^T}{B_{t,H_2}^T}\right) = \left(\sigma^{R^2} D_{T_1} D_{T_2} + \sigma^{I^2} - \sigma^R \sigma^I \rho_{R,I} \left(D_{T_1} + D_{T_2}\right)\right) dt$$

Based on the sample period of our database (09/03/2007–26/02/2010) these formulae produced the following variance-covariance matrix for the set of the three selected bonds:

Nominal bond coupon 3.85% maturity July/2040

ILB bond coupon 1.25% maturity July/2010

ILB bond coupon 2% maturity July/2040

$$\Omega = \begin{pmatrix} 1.32\% & 0.10\% & 1.19\% \\ 0.10\% & 0.03\% & 0.18\% \\ 1.19\% & 0.18\% & 1.96\% \end{pmatrix}$$

Using this variance-covariance matrix, the portfolio allocation between nominal bonds and linkers results from an optimization exercise between three securities ($B_{t,T}^N$, B_{t,H_1}^T, B_{t,H_2}^T).

$$\min \lambda' \Omega \lambda \quad subject \quad to:$$

$$\lambda' 1 = 1$$
$$\lambda' R = r$$
$$\lambda \geq 0$$

where λ stands for the asset's optimal weights in the portfolio, R for the vector of expected returns and r for the portfolio's expected return.

8. Conclusions

The financial crisis changed the appreciation of different asset classes among public investors leading to a fundamental reassessing of their risks, which in turn reduced the investment universe. As a result, the quest for diversification became even more critical and the case for including inflation linkers in a fixed income portfolio grew stronger.

We first discussed the general case for including linkers in an otherwise traditional fixed income portfolio, to later develop a specific model to characterize the market.

Using French ILB's market prices and zero-coupon inflation indexed swaps, we derived corresponding real zero-coupon bond price curves. Zero (real) coupon prices were derived as it is typically done in the industry, ie by recourse to traded ZCIIS, but also by fitting Nelson-Siegel curves to the daily data. Both methodologies resulted in different parameter estimates, which were later tested in the hedging analysis to validate the model.

We then fitted a three-factor HJM model to characterize the economy, with time-dependent (non-stochastic) volatilities, which consequently resulted on a Gaussian economy.

Some 21 hedging portfolios were built and the statistical characteristics of their errors permitted to validate the model.[23] The validation of the model provided a coherent theoretical background to build a portfolio of bonds which includes linkers as well as nominal bonds.

In the context of this model, the asset returns are normally distributed, so the case for including linkers in a bond portfolio is reduced to the classical CAPM analysis, as assets are characterized by their expected returns and their variance-covariance matrix.

This is the first, to the authors' knowledge, attempt to calibrate the HJM framework using data on European inflation-linked bonds.

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Inflation hedging portfolios in different regimes

Marie Brière[1] and Ombretta Signori[2]

1. Introduction

Having weathered the worst crisis in terms of length and amplitude since the Second World War, investors may have to cope with one of the potential outcomes of the subprime meltdown: the threat of a surge in the cost of living. The accumulation of multiple factors raises the question as to whether a globally low and stable inflation environment can continue to exist (Barnett and Chauvet (2008), Cochrane (2009), Walsh (2009)), thereby raising the question of inflation hedging, a key concern for many investors. To support weak economies almost all developed countries applied unconventional monetary policies with significant stimulus packages and injections of liquidity into money markets. The resulting exceptional rise in government deficits and huge debt levels are a looming problem for the US and many European countries, while the recent oil price spike, dollar weakness and macroeconomic volatility are adding further pressures to the ongoing debate. These renewed concerns about inflation naturally raise the question of reconsidering how to build the ideal portfolio that will shield investors effectively from inflation risk and, where possible, generate excess returns. This applies both to long-term institutional investors (particularly pension funds, which operate under inflation-linked liability constraints) and to individual investors, for whom real-term capital preservation is a minimal objective.

Consider an investor having a target real return and facing inflation risk. Her portfolio is made of Treasury bills, government nominal and inflation-linked (IL) bonds, stocks, real estate and commodities. Three questions are to be solved. (1) What is the inflation hedging potential of each asset class? (2) What is the optimal allocation for a given target return and investment horizon? (3) What is the impact of changing economic environment on this allocation? To address those questions, we consider a two-regime approach: macroeconomic volatility is either high, as it was during the 1970s and 1980s, or low, as in the 1990s and 2000s marked by the “Great Moderation”.

We use a vector-autoregressive (VAR) specification to model the inter-temporal dependency across variables, and then simulate long-term holding portfolio returns up to 30 years. Recent research has pointed to instability and regime shifts in the stochastic process generating asset returns. Guidolin and Timmerman (2005), and Goetzmann and Valaitis (2006) stress that a full-sample VAR model can be mis-specified as correlations vary over time. Asset returns exhibit multiple regimes (Garcia and Perron (1996), Ang and Bekaert (2002), Connolly et al (2005)).The changing economic conditions, especially the strong

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[1] Université Libre de Bruxelles, Solvay Brussels School of Economics and Management, Centre Emile Bernheim, Av. F.D. Roosevelt, 50, CP 145/1, 1050 Brussels, Belgium and Amundi (Asset Management of Credit Agricole SA and Société Générale), 90 bd Pasteur, 75015 Paris, France.

[2] AXA Investment Managers, 100 Esplanade du G_en_eral de Gaulle, 92932 Paris, France.

Comments can be sent to *marie.briere@amundi.com*, or *ombretta.signori@axa-im.com*.

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decrease in macroeconomic volatility (the "Great Moderation", Blanchard and Simon (2001), Bernanke (2004), Summers (2005)) and the changing nature of inflation shocks – from countercyclical to procyclical – have been stressed as the two main factors affecting the level of stocks and bond prices (Lettau et al (2008), Kizys and Spencer (2008)), and also partially explaining the change of correlation sign between stocks and bond returns, from strongly positive to slightly negative (Baele et al (2009), Campbell (2009), Campbell et al (2009)). Using the Goetzmann et al (2005) breakpoint test for structural change in correlation, we split the sampling period into two sub-periods exhibiting the most stable correlations. The simulated returns based on our two estimated VAR models are thus used, on the one hand, to measure the inflation hedging properties of each asset class in each regime, and on the other hand to carry out a portfolio optimisation in a mean-shortfall probability framework. We determine the allocation that maximises above-target returns (inflation + x%) with the constraint that the probability of a shortfall remains lower than a threshold set by the investor.

We show that the optimal asset allocation differs strongly across regimes. In the periods of highly volatile economic environment, an investor having a pure inflation target should be mainly invested in cash when her investment horizon is short, and increase her allocation to IL bonds, equities, commodities and real estate when her horizon increases. In contrast, in a more stable economic environment, cash plays an essential role in hedging a portfolio against inflation in the short run, but in the longer run it should be replaced by nominal bonds, and to a lesser extent by commodities and equities. With a more ambitious real return target (from 1% to 4%), a larger weight should be dedicated to risky assets (mainly equities and commodities). These results confirm the value of alternative asset classes in shielding the portfolio against inflation, especially for ambitious investors with long investment horizons.

Our paper tries to complement the existing literature in three directions: inflation hedging properties of assets, strategic asset allocation, and alternative asset classes. The question of hedging assets against inflation has been widely studied (see Attié and Roache (2009) for a detailed literature review). Most studies have focused on measuring the relationship between historical asset returns and inflation, either by measuring the correlation between these variables or by adopting a factor approach such as the one used by Fama and Schwert (1977). These approaches present a number of difficulties, especially with regard to the lack of historical data available to study long-horizon returns, the problem of non-serially independent data, non-stationary variables, and instability over time of the assets' relationships to inflation.

The literature on strategic asset allocation has shed new light on this question. Continuing the pioneering work of Brennan et al (1997) and Campbell and Viceira (2002), many researchers have sought to show that long-term allocation is very different from short-term allocation when returns are partially predictable (Barberis (2000), Brennan and Xia (2002), Campbell et al (2003), Guidolin and Timmermann (2005), Fugazza et al (2007)). The approach developed in an assets-only framework was extended to asset and liability management (ALM) using traditional classes (van Binsbergen and Brandt (2007)) but also alternative assets (Goetzmann and Valaitis (2006), Hoevenaars et al (2008), Amenc et al (2009)). One common characteristic of these studies is their focus on the situation of investors, such as pension funds, with liabilities which are subject to the risk of both fluctuating inflation and real interest rates. In this article, we adopt a different point of view. Not all investors who seek to hedge against inflation necessarily have such liabilities. They may only wish to hedge their assets against the risk of real-term depreciation, and thus have a purely nominal objective that consists of the inflation rate plus a real expected return target, which is assumed to be fixed.

Thus far, most of the research into inflation hedging for diversified portfolios has been done within a mean-variance framework. The studies of inflation hedging properties in an ALM framework with a liability constraint generally focus on a "surplus optimisation" (Leibowitz (1987), Sharpe and Tint (1990), Hoevenaars et al (2008)). In our context, however, this risk measure is not the one that corresponds best to investors' objectives. Our portfolio's excess returns above target may be only slightly volatile but still significantly lower than the objective, presenting a major risk to the investor. The notion of "safety first" (Roy (1952)) is therefore more appropriate. We focus on the shortfall probability, ie the likelihood of not achieving the target return at maturity. In an ALM framework, Amenc et al (2009) measure the shortfall probability of ad hoc portfolios. We expand that work and determine optimal portfolio allocations in a mean-shortfall probability framework.

The properties of alternative asset classes have been studied in a strategic asset allocation context (Agarwal and Naik (2004), Fugazza et al (2007), Brière et al (2010)). In an ALM context, Hoevenaars et al (2008) and Amenc et al (2009) also find significant appeal in these asset classes, which are interesting sources of diversification and inflation hedging in a portfolio. To the best of our knowledge, however, these asset classes have not yet been studied in an asset-only context with an inflation target. Our research tries to fill the gap.

Our paper is organised as follows. Section 2 presents our data and methodology. Section 3 presents our results: the correlation structure of our assets with inflation at different horizons, and the optimal composition of inflation hedging portfolios. Section 4 concludes.

2. Data and methodology

2.1 Data

We consider the case of a US investor able to invest in six liquid and publicly traded asset classes: cash, stocks, nominal bonds, IL bonds, real estate and commodities. (1) Cash is the 3-month T-bill rate. (2) Stocks are represented by the Morgan Stanley Capital International (MSCI) US Equity index. (3) Nominal bonds are represented by the Morgan Stanley 7-10 year index. (4) IL bonds are represented by the Barclays Global Inflation index from 1997.[3] Before that date, to recover price and total return history before IL bonds were first issued in the US, we reconstruct a time series of real rates according to the methodology of Kothari and Shanken (2004). Real rates are thus approximated by 10-year nominal bond rates minus an inflation expectation based on a 5-year historical average of a seasonally adjusted consumer price index (CPI) (Amenc et al (2009)). The inflation risk premium is assumed equal to zero, a realistic assumption considering the recent history of US TIPS (Berardi (2005), D'Amico et al (2008), Brière and Signori (2009)). (5) Real estate investments are proxied by the FTSE NAREIT Composite Index representing listed real estate in the US (publicly traded property companies of the NYSE, Nasdaq, AMEX and Toronto Stock Exchange). (6) Commodities are represented by the Goldman Sachs Commodity Index (GSCI). We also add a set of exogenous variables: inflation (measured by CPI), dividend yield obtained from the Shiller database (Campbell and Shiller (1988)) and the term spread measured as the difference between the 10-year Treasury Constant Maturity Rate and the 3-month Treasury bill rate provided by the US Federal Reserve Economic Database. We consider monthly returns for the time period January 1973–June 2009.

Table 1 in Appendix 1 presents the descriptive statistics of monthly returns. The hierarchy of returns is the following: cash has the smallest return on the total period, followed by IL bonds, nominal bonds, real estate, equities and commodities. Adjusted for risk, the results show a slightly different picture: cash appears particularly attractive compared to other asset classes, nominal bonds are much more appealing than real estate (risk-adjusted return of 1 vs 0.4), and equities are more attractive than commodities (0.5 vs 0.4). Extreme risks are

[3] Note that the durations of the IL bond and nominal bond indices are comparable.

also different: negative skewness and strong kurtosis are strongly pronounced for real estate and, to a lesser extent, for equities and commodities.

2.2 Econometric model of asset return dynamics

VAR models are widely used in financial economics to model the intertemporal behaviour of asset returns. Campbell and Viceira (2002) provide a complete overview of the applications of VAR specification to solve intertemporal portfolio decision problems. The VAR structure can also be used to simulate returns in the presence of macroeconomic factors. Following Barberis (2000), Campbell et al (2003), Campbell and Viceira (2005) and Fugazza et al (2007), among others, we adopt a VAR(1) representation of the returns but expand it to alternative asset classes, as did Hoevenaars et al (2008).[4] Empirical literature has relied on a predetermined choice of predictive variables. Kandel and Stambaugh (1996), Balduzzi and Lynch (1999) and Barberis (2000) use the dividend yield; Lynch (2001) uses the dividend yield and term spread; Brennan et al (1997) use the dividend yield, bond yield and Treasury bill yield; and Hoevenaars et al (2008) use the dividend yield, term spread, credit spread and Treasury bill yield. We select the most significant variables in our case: dividend yield and term spread. As we are modelling nominal logarithmic returns, we also enter inflation explicitly as a state variable, which enables us to measure the link between inflation and asset class returns.[5]

The compacted form of the VAR(1) can be written as:

$$z_t = \phi_0 + \phi_1 z_{t-1} + u_t \qquad (1)$$

where ϕ_0 is the vector of intercepts; ϕ_1 is the coefficient matrix; z_t is a column vector whose elements are the log returns on the six asset classes and the values of the three state variables; and u_t is the vector of a zero mean innovations process.

Finally, to overcome the problem of correlated innovations of the VAR(1) model and to take into consideration the contemporaneous relationship between returns and the economic variables, we follow the procedure described in Amisano and Giannini (1997) to obtain structural innovations characterised by an iid process. The structural innovations ε_t, may be written as $Au_t = B\varepsilon_t$ where the parameters of A and B matrices are identified imposing a set of restrictions. The structure of ε_t is used to perform Monte Carlo simulations on the estimated VAR for the portfolio analysis. Imposing the restrictions we assume that inflation, as well as cash, impact on the returns of all the asset classes, and that commodities are not affected instantaneously by the returns of the other asset classes.

Meaningful forecasts from a VAR model rely on the assumption that the underlying sample correlation structure is constant. However, regime shifts in the relationship between financial and economic variables have already been widely discussed in the literature. Guidolin and Timmermann (2005) and Goetzmann and Valaitis (2006) find evidence of multiple regimes in the dynamics of asset returns. This suggests that a full-sample VAR model might be potentially mis-specified, as the correlation structure may not be constant. Changing macroeconomic volatility has been identified as one of the main causes of the changing correlation structure between assets (Li (2002), Ilmanen (2003), Baele et al (2009)). This has

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[4] The differences with the model lie in the fact that we include IL bonds but not corporate bonds and hedge funds in our investment set. As our investor is an asset-only investor, there are no liabilities in our model.

[5] As in the models of Brennan et al (1997), Campbell and Viceira (2002) and Campbell et al (2003), we do not adjust VAR estimates for possible small sample biases related to near non-stationarity of some series (Campbell and Yogo (2006)).

been accompanied by a change in the nature of inflation. During the 1970s and 1980s (marked by supply shocks and poor central bank credibility), inflation was mainly countercyclical, whereas in the most recent period (with demand shocks and credible monetary policy), inflation was more procyclical. This change has been stressed as an important driver of the decreasing correlation between stocks and bonds (Campbell (2009), Campbell et al (2009)).

Using the Goetzmann et al (2005) test[6] for structural change in correlations between asset returns and state variables, we determine the breakpoint that best separates the sample data, ensuring the most stable correlation structure within each sub-period.[7] The first period (January 1973–December 1990) corresponds to a volatile economic environment (major oil shocks, huge government deficits, large swings in GDP growth), the second (January 1991–June 2009) to a much more stable one.

Tables 2 to 5 in Appendix 1 present the results of our VAR model in the two identified sub-periods. Looking at the significance of the coefficients of the lagged state variables, inflation is mainly helpful in predicting nominal bond returns. Dividend yield has better explanatory power for equity returns in the second period than in the first. The high positive correlation coefficient of the residuals between nominal bonds and IL bonds (84% and 76% in the two sub-sample periods) confirms the strong interdependency between the contemporaneous returns of the two asset classes dominated by the common component of real rates. Real estate and equities have the second largest positive innovation correlation coefficient (61% and 55%, respectively), implying that a positive shock in real estate has a positive contemporaneous effect on stock returns and vice versa. Other results are in line with the common findings of positive contemporaneous correlation between inflation and commodities, and the intuition that inflation and monetary policy shocks have a negative impact on bond returns through the inflation expectations component.

2.3 Simulations

We use the iid structural innovation process of the two VAR models estimated on the two sub-samples to perform a Monte Carlo analysis based on the fitted model. We draw iid random variables from a multivariate normal distribution for the structural innovations and we obtain simulated returns for 5,000 simulated paths of length *T* (*T* varying from 1 month to 30 years). The simulated returns are thus used, on the one hand, to measure the inflation hedging properties of each asset class in each regime, and on the other hand in a portfolio construction context to generate expected returns and covariance matrices at different horizons (2, 5, 10 and 30 years).

2.4 Portfolio choice

The bulk of the research into inflation hedging for a diversified portfolio has used a mean-variance framework. And research into inflation hedging properties in an ALM framework with a liability constraint is usually based on surplus optimisation, in which the surplus is maximised under the constraint that its volatility be lower than a target value (Leibowitz (1987), Sharpe and Tint (1990), Hoevenaars et al (2008)). But for our purposes, this risk measure is not the one best suited to investors' objectives. Since the portfolio's excess returns above target may be only slightly volatile but still significantly lower than the

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6 Null hypothesis of stationary bivariate historical correlations between assets.

7 We have not presented the Goetzmann et al (2005) test results so as not to clutter the presentation of the results.

objective, the investor faces a serious risk. In this case, the notion of safety first (Roy (1952)) is more appropriate. Roy argues that investors think in terms of a minimum acceptable outcome, which he calls the “disaster level”. The safety first strategy is to choose the investment with the smallest probability of falling below that disaster level. A less risk-averse investor may be willing to achieve a higher return, but with a greater probability of going below the threshold. Roy defined the shortfall constraint such that the probability of the portfolio's value falling below a specified disaster level is limited to a specified disaster probability. Portfolio optimisations with a shortfall probability risk measure have been conducted before (Leibowitz and Henriksson (1989), Leibowitz and Kogelman (1991), Lucas and Klaassen (1998), Billio (2007), Smith and Gould (2007)), but as far as we know not in the context of an inflation hedging portfolio.

We determine optimal allocations that maximise above-target returns (the target being inflation + x%) with the constraint that the probability of a shortfall remains lower than a threshold set by the investor.

$$Min_w \ P(\sum_{i=1}^{n} w_i R_{iT} < \pi_T + \overline{R}) \tag{2}$$

$$E\left[\sum_{i=1}^{n} w_i R_{iT} - (\pi_T + \overline{R})\right] > 0 \tag{3}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{4}$$

$$w_i \geq 0 \tag{5}$$

Where $R_T = (R_{1T}, R_{2T}, ..., R_{nT})$ is the annualised return of the n assets in the portfolio over the investment horizon T, $w = (w_1, w_2, ..., w_n)$ the fraction of capital invested in the asset i, π_T the annual inflation rate during that horizon T, $\overline{R}$ the target real return in excess of inflation, and α the target shortfall probability. E is the expectation operator with respect to the probability distribution P of the asset returns.

We work in a mean-shortfall probability world and derive the corresponding efficient frontier (Harlow (1991)). For a portfolio with normally distributed returns $N(\mu, \sigma)$, the probability of portfolio shortfall is written:

$$p(w' R_T < \pi_T + \overline{R}) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\pi_T + \overline{R}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

For each investment horizon T (T = 1 year, 3 years, 5 years, 10 years, 20 years, 30 years), we draw all the efficient portfolios in the mean-shortfall probability universe for the two identified regimes.

3. Results

3.1 Inflation hedging properties of individual assets

Figures 1 and 2 in Appendix 1 display correlation coefficients between asset returns and inflation based on our VAR model, depending on the investment horizon, from 1 month to 30 years. We consider two sample periods: from January 1973 to December 1990 and from January 1991 to June 2009. The inflation hedging properties of the different assets vary strongly depending on the investment horizon. Most of the assets (the only exception being commodities and nominal bonds in the first period) display an upward-sloping correlation curve, meaning that inflation hedging properties improve as the investment horizon widens.

In the first sample period (1973–1990), marked by a volatile macroeconomic environment, cash and commodities have a positive correlation with inflation on short-term horizons, whereas nominal bonds, equities and real estate are negatively correlated. The correlation of IL bonds with inflation lies in the middle and is close to zero. In the longer run (30 years), cash shows the best correlation with inflation (around 0.6), followed by IL bonds and real estate (all showing a positive correlation), then equities, commodities, and finally nominal bonds (the latter with negative correlation).

The very strong negative correlation of nominal bonds with inflation both in the short run and in the long run is intuitive since changes in expected inflation and bond risk premiums are traditionally the main source of variation in nominal yields (Campbell and Ammer (1993)). IL bonds and inflation are positively correlated for an obvious reason: the impact of a strongly rising inflation rate has a direct positive impact on performance through the coupon indexation mechanism. Negative correlation between equities and inflation is a characteristic of countercyclical inflation periods when the economy is affected by supply shocks or changing inflation expectations, which shift the Phillips curve upwards or downwards (Campbell (2009)). This has been documented by many authors, with three different interpretations. The first is that inflation hurts the real economy, so the dividend growth rate should fall, leading to a fall in equity prices (an alternative explanation is that poor economic conditions lead the central bank to lower interest rates, which has a positive influence on inflation (Geske and Roll (1983)). The second interpretation argues that high expected inflation has tended to coincide with periods of higher uncertainty about real economic growth, raising the equity risk premium (Brandt and Wang (2003), Bekaert and Engstrom (2009)). The final explanation is that stock market investors are subject to inflation illusion and fail to adjust the dividend growth rate to the inflation rate, even though they correctly adjust the nominal bond rate (Modigliani and Cohn (1979), Ritter and Warr (2002), Campbell and Vuolteenaho (2004)). Commodities exhibit more contrasted behaviour, ie the correlation with inflation is positive in the short run but negative in the long run. This result is consistent with the fact that commodities have a tendency to overreact to money surprises (and therefore inflation) in the short run (Browne and Cronin (2007)), whereas the long-term link with inflation has been weak since the 1980s, when the commodity-consumer price connection seems to have broken down. This reflects the diminished role of traditional commodities in US production and the sterilisation of some inflation signals by offsetting monetary policy actions (Blomberg and Harris (1995), Hooker (2002)).

The correlation picture is very different if we now consider the second sample period (1991–2009), marked by a stable macroeconomic environment. The hierarchy of the different assets in terms of inflation hedging properties is very different, both in the long run and in the short run. In the short run, commodities have the strongest correlation with inflation, followed by cash, real estate, nominal bonds, IL bonds and equities. In the long run, the best inflation hedger is now cash, followed by equities, real estate, nominal bonds, IL bonds and commodities. The main differences compared to the first period are that nominal bonds and equities now have a positive correlation with inflation in the long run, and better inflation hedging properties than IL bonds. The moderation in economic risk, especially inflation volatility, has reduced correlations in absolute terms. IL bond returns have a much smaller positive correlation with inflation, whereas nominal bonds lose their negative correlation and become moderately positively correlated. Moreover, as inflation is now procyclical (the macroeconomy is moving along a stable Phillips curve), positive inflation shocks happen during periods of improving macroeconomic environment, leading to positive correlation between equities and inflation (Campbell (2009)). This changing behaviour is strongly linked to the much stronger credibility and transparency of central banks in fighting inflation during the last two decades, leading to more stable and lower interest rates, only slightly impacted by inflation changes (Kim and Wright (2005), Eijffinger et al (2006)).

Another way to look at the inflation hedging properties of individual assets is to measure the probability of having below-inflation returns at the investment horizon (shortfall probabilities). Tables 6 and 7 in Appendix 1 display the shortfall probabilities of the different asset classes for horizons of 2, 5, 10 and 30 years. A first observation is that shortfall probabilities decrease strongly with the investment horizon. This is true for all asset classes, but particularly for the most risky ones. Commodities, for example, have a probability of not achieving the inflation target of more than 35% at a 2-year horizon. At 30 years, this falls below 8% for both periods. An asset can be strongly correlated with inflation but also have a significant shortfall probability if its return is always lower than inflation. Looking at shortfall probabilities, the best inflation hedger in the short run appears to be cash on both inflation regimes. In the long run, the best hedgers are cash, equities and commodities in the volatile regime (IL bonds are well correlated with inflation during that period but with a strong shortfall probability, 25% for a 30-year horizon), and nominal bonds and commodities in the stable regime.

3.2 Inflation hedging portfolios

We now turn to the construction of inflation hedging portfolios. We examine the case of an investor wishing to hedge inflation on her investment horizon. This investor has a target real return ranging from 0% to 4%. For each of the investor targets, we show the optimal portfolio composition depending on the inflation regime.

How to attain a pure inflation target

We first consider the case of an investor simply wishing to hedge inflation, ie having a target real return of 0%. Table 8 and Table 13 in Appendix 2 show the optimal portfolio composition and the descriptive statistics of minimum shortfall probability portfolios for each horizon.

The first observation, common to both periods, is that the higher the required return, the greater the shortfall probability in the portfolio. The minimum shortfall probability (corresponding to Roy's (1952) "safety first" portfolio) generally decreases with the investment horizon, the only exception being for the 2-year horizon in the first period, where the minimum shortfall probability is lower than for the 5-year horizon.

In the first period, characterised by high macroeconomic volatility, the optimal portfolio composition of a "safety first" investor with a 2-year horizon is 88% cash, 6% IL bonds, 1% equities and 5% commodities. This very conservative portfolio has a 1.6% annualised return over inflation, 1.9% volatility of real returns and 11% shortfall probability. Diversifying the portfolio makes it possible to sharply diminish the achievable shortfall probability compared to individual assets: whereas the minimum shortfall probability over all assets in that period is 18% (for cash), it is 7% lower with a diversified portfolio. When the horizon is increased, the weight assigned to cash decreases and the weights of riskier assets (IL bonds, equities, real estate, commodities) rise. For a 30-year horizon, the optimal portfolio composition is 64% cash, 17% IL bonds, 8% equities, 5% real estate and 6% commodities. This portfolio generates an annualised excess return of 2.2% over inflation with stronger volatility (5.4%) but with a very low probability (1.4%) of falling below the inflation target at the investment horizon. Again, portfolio diversification makes it possible to decrease strongly the shortfall probability at the investment horizon.

In the second period, characterised by much lower macroeconomic volatility, the optimal portfolio composition is quite different. With a 2-year horizon, the optimal composition for a "safety first" investor is still very conservative: 81% cash, but the rest of the portfolio consists mainly of nominal bonds (17%), real estate (1%) and commodities (2%). Compared to the first period, nominal bonds now replace IL bonds and equities. This result is consistent with our previous findings on individual assets: the inflation hedging properties of nominal bonds increase strongly in the second period, with inflation correlation becoming even greater than for IL bonds and shortfall probabilities becoming much smaller. Increasing the investment horizon, the share of the portfolio dedicated to cash decreases, progressively replaced by nominal bonds, whereas the weights of commodities and equities increase slightly. With a 30-year horizon, the optimal portfolio of a "safety first" investor is composed of 73% nominal bonds, 10% equities and 17% commodities. This portfolio has slightly higher annualised real return than in the first period (3.2% vs 2.2%), with a smaller shortfall probability (0.02% vs 1.4%). Contrary to the first period, IL bonds no longer appear in the optimal composition of safety first portfolios.

To sum up, when macroeconomic volatility is high, a "safety first" investor having a pure inflation target should be mainly invested in cash when her investment horizon is short, and should increase her allocation to IL bonds, equities, commodities and real estate when her horizon increases. When economic volatility is much lower, the optimal investment set changes radically. Mainly invested in cash when the investment horizon is short, an investor should increase her holdings of nominal bonds, commodities and equities when her investment horizon increases.

Raising the level of required real return

We now consider the consequences for an investor of having a more ambitious target real return, ranging from 1% to 4%. Tables 9 to 12 and 14 to 17 in Appendix 2 present the optimal portfolio composition as well as the descriptive statistics of the minimum shortfall probability portfolios, for the first and second sample periods.

Consistent with intuition, when the required real return is increased, the shortfall probability increases strongly in both sub-periods. In the first period, for a 2-year horizon investor, the minimum shortfall probability is 10.8% for a target real return of 0%. It is 28.9%, 36.7%, 40.9% and 44.0% for a 1%, 2%, 3% and 4% real target return, respectively. The results are similar for the second period: shortfall probabilities rise from 4.7% to 44.9% for a 0% to 4% real return target.

Another intuitive result is that the more the investor increases her required real return, the more the optimal portfolio composition is biased towards risky assets. Considering the first period, for a 30-year horizon, the optimal weight of cash decreases from 64% (with a real return target of 0%) to 0% (1% to 4% target). The IL bond weight also decreases, from 17% to 0%. The explanation is intuitive: these assets provide a good inflation hedge but are not sufficient to achieve high real returns. On the contrary, the weights of risky assets (equities, and especially commodities) increase. A long-term portfolio seeking to achieve inflation +1% should comprise 63% equities and 37% commodities. With a 4% target, the investor should hold 32% equities and 68% commodities. Of course, if the investment horizon is shorter, a more substantial part of the portfolio should be dedicated to cash.

In the second sample period, the results are comparable. Increasing the real return target leads to a decrease in the cash investment and an increase in the more risky assets. The difference lies in the "risky" assets retained by the optimisation. A substantial portion of nominal bonds should now be added to the optimal mix of equities and commodities than in the first period. For a 30-year investor with a 1% real return target, the optimal portfolio composition is 69% nominal bonds, 10% equities and 21% commodities. It is 60% bonds, 9% equities and 31% commodities for a 2% target, and 100% commodities for a 3% or 4% target. As in the first period, commodities are the most rewarding asset class. This explains why, with a very ambitious real return target, the portfolio should be fully invested in commodities.

To sum up, a more ambitious real return target leads to a greater shortfall probability and a different optimal portfolio composition, with a larger weight in risky assets. In an unstable and volatile economic regime, an ambitious investor should abandon IL bonds and real estate and concentrate on equities and commodities. In a more stable economic environment, she should reduce her portfolio weight in nominal bonds and equities and invest a higher share in commodities.

4. Conclusion

A key challenge for many institutional investors is the preservation of capital in real terms, while for individual investors it is building a portfolio that keeps up with the cost of living. In this paper we address the investment problem of an investor seeking to hedge inflation risk and achieve a fixed target real rate of return. The key question is thus to determine the optimal asset allocation that will preserve the investor's capital from inflation with an acceptable probability of shortfall.

Following Campbell et al (2003) and Campbell and Viceira (2005), we used a vector-autoregressive (VAR) specification to model the joint dynamics of asset classes and state variables, and then simulated long-term holding portfolio returns for a range of different assets and inflation. The strong change in macroeconomic volatility and the varying nature of inflation shocks (leading to a change of correlation sign between inflation and the real economy) have been identified as the two main causes of the changing correlation structure between assets (Li (2002), Ilmanen (2003), Baele et al (2009), Campbell (2009), Campbell et al (2009)). Relying on the Goetzmann et al (2006) test for structural change in correlation, we determined the breakpoint that best separates the sample data, ensuring the most stable correlation structure within each sub-period. We estimated a VAR model for each period and performed a simulation-based analysis. We were thus able to measure the inflation hedging properties of each asset class in each regime and determine the allocation that maximises above-target returns (inflation + x%) with the constraint that the shortfall probability remains below a threshold set by the investor.

Our results confirm that the presence of macroeconomic regimes radically alters the investor's optimal allocation. In a volatile regime marked by countercyclical inflation, a "safety first" investor having a pure inflation target should be mainly invested in cash when her investment horizon is short and should increase her allocation to IL bonds, equities, commodities and real estate when horizon increases. In a more stable economic environment with procyclical inflation shocks, the optimal investment set changes radically. Mainly invested in cash when investment horizon is short, an investor should increase her investment in nominal bonds, but also, to a lesser extent, in commodities and equities when her horizon increases. Our results confirm the value of alternative asset classes in protecting the portfolio against inflation.

Having a more ambitious real return target (from 1% to 4%) leads automatically to a greater shortfall probability, but also to a different optimal portfolio composition. A larger weight should be dedicated to risky assets, which make it possible to achieve higher returns (with a greater shortfall probability). In the first period, an ambitious investor should gradually abandon IL bonds and real estate and concentrate on equities and particularly commodities. In the second period, she should reduce her portfolio weight in nominal bonds and equities and invest a higher share in commodities.

Our work could be extended in several ways. Different methodologies have been developed that move away from the standard mean-variance approach, by changing the risk measure of the portfolio. One branch of the literature considers portfolio selection with value at risk (Agarwal and Naik (2004), Martellini and Ziemann (2007)), or conditional VaR (Rockafellar and Uryasev (2000)); the other branch with shortfall probability (Leibowitz and Henriksson (1989), Leibowitz and Kogelman (1991), Lucas and Klaassen (1998), Billio and Casarin (2007), Smith and Gould (2007)). A useful development of our work would be to reconcile the two approaches and examine shortfall probabilities in the context of non-normal returns. We have considered only a static allocation on the whole investment horizon. A very interesting development would be to compare these results with a dynamic asset allocation, rebalancing the portfolio depending on active views on the different asset classes. Finally, we examined a fairly simple objective function. In the real world, many investors (especially pension funds) do not have a single well-defined goal but rather have to cope with multiple and sometimes contradictory objectives, with long-term return shortfall probability constraints and short-term performance objectives. An interesting development of this work would be to take these different constraints into account.

Appendix 1

Table 1

Summary statistics of monthly returns

January 1973–June 2009

	Cash	Nom Bonds	IL bonds	Equities	Real Estate	Commodities
Ann. Ret.	5.8%	7.8%	6.5%	8.6%	7.8%	8.3%
Max Monthly	1.3%	11.3%	13.9%	16.4%	26.9%	22.9%
Min Monthly	0.0%	-9.0%	-13.8%	-23.9%	-36.4%	-33.1%
Ann. Vol.	0.9%	7.6%	9.9%	15.9%	18.5%	20.6%
Risk/Adjusted Ret.*	6.6	1.0	0.6	0.5	0.4	0.4
Skewness	0.7	0.3	0.1	-0.7	-1.2	-0.3
Kurtosis	3.9	5.9	6.8	5.7	12.4	6.1

* Annualised return divided by annualised volatility.

Table 2

Results of VAR model, parameter estimates

January 1973–December 1990

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash(-1)	0.96	1.13	-0.96	-1.75	-3.52	-0.22	0.09	1.80	-1.26
	(-48.71)	(-1.11)	(-0.86)	(-0.92)	(-1.87)	(-0.09)	(-0.53)	(-1.39)	(-0.10)
Nom Bonds(-1)	-0.01	0.17	1.02	-0.01	0.41	-0.43	-0.04	-0.18	-5.96
	(-6.29)	(-1.66)	(-9.42)	(-0.03)	(-2.20)	(-1.91)	(-2.14)	(-1.39)	(-4.98)
IL Bonds(-1)	0.00	-0.09	-0.17	0.22	0.08	0.41	0.01	0.01	4.59
	(-0.46)	(-1.18)	(-2.14)	(-1.54)	(-0.57)	(-2.46)	(-1.16)	(-0.08)	(4.33)
Equities(-1)	0.00	-0.03	-0.07	-0.14	0.01	-0.07	0.01	-0.35	-0.59
	(-1.69)	(-0.66)	(-1.41)	(-1.58)	(-0.08)	(-0.69)	(-0.72)	(-5.91)	(-1.38)
Real Estate(-1)	0.00	-0.06	-0.07	0.15	-0.07	-0.11	-0.01	-0.08	0.55
	(-1.56)	(-1.24)	(-1.33)	(-1.76)	(-0.77)	(-1.02)	(-1.18)	(-1.39)	(1.64)
Commodities(-1)	0.00	-0.07	-0.05	-0.12	-0.19	0.13	0.02	0.07	-0.09
	(-1.91)	(-2.19)	(-1.59)	(-2.04)	(-3.46)	(-1.86)	(-3.54)	(-1.89)	(-0.35)
Inflation(-1)	0.00	-0.19	0.10	-0.22	-0.19	-0.08	1.00	0.14	0.52
	(-0.89)	(-2.83)	(-1.38)	(-1.78)	(-1.50)	(-0.52)	(-90.79)	(-1.62)	(0.32)
Div. Yield(-1)	0.00	0.02	0.02	0.05	0.09	-0.02	0.00	0.96	0.02
	(-0.23)	(-2.07)	(-1.24)	(-2.61)	(-4.26)	(-0.77)	(-2.41)	(-67.43)	(-0.36)
TermSpread(-1)	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.36
	(-3.57)	(-1.21)	(-0.46)	(-0.13)	(-0.20)	(-0.91)	(-1.35)	(-1.15)	(4.81)
Adj. R^2/F.stat	0.95	0.07	0.39	0.08	0.18	0.04	0.98	0.98	0.15
	(447.67)	(2.90)	(16.47)	(3.15)	(6.25)	(1.94)	(1522.93)	(958.73)	(5.29)

t-stat are given in parentheses. The last row reports the adjusted-R^2 and the F-statistics of joint significance.

Table 3

VAR residuals, correlation coefficients

January 1973–December 1990

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash	1.00								
Nom Bonds	-0.37	1.00							
IL Bonds	-0.47	0.84	1.00						
Equities	-0.14	0.25	0.21	1.00					
Real Estate	-0.25	0.17	0.14	0.61	1.00				
Commodi-ties	-0.06	-0.12	-0.06	-0.05	0.02	1.00			
Inflation	0.02	-0.07	-0.02	-0.12	-0.04	0.13	1.00		
Div. Yield	0.12	-0.20	-0.24	-0.80	-0.54	0.08	0.17	1.00	
Term Spread	-0.85	-0.09	-0.05	0.01	0.18	0.11	0.02	0.03	1.00

Table 4

Results of VAR model, parameter estimates

January 1991–June 2009

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash(-1)	0.99	1.83	1.47	7.06	1.24	3.75	0.20	-3.86	-1.26
	(119.42)	(1.99)	(1.41)	(3.09)	(0.44)	(1.18)	(1.21)	(-2.50)	(-0.10)
Nom Bonds(-1)	0.00	0.15	0.70	0.01	0.49	-0.44	-0.03	-0.23	-5.96
	(-3.81)	(1.64)	(6.69)	(0.02)	(1.74)	(-1.40)	(-2.07)	(-1.51)	(-4.98)
IL Bonds(-1)	0.00	-0.07	-0.28	0.16	0.31	-0.20	0.02	-0.18	4.59
	(-2.90)	(-0.81)	(-3.01)	(0.78)	(1.25)	(-0.72)	(1.16)	(-1.31)	(4.33)
Equities(-1)	0.00	-0.07	-0.01	-0.01	0.32	-0.06	0.00	-0.49	-0.59
	(2.06)	(-2.28)	(-0.16)	(-0.09)	(3.22)	(-0.55)	(-0.35)	(-9.00)	(-1.38)
Real Estate(-1)	0.00	-0.06	-0.04	0.07	-0.03	0.23	0.00	-0.03	0.55
	(0.25)	(-2.23)	(-1.36)	(1.10)	(-0.39)	(2.64)	(0.88)	(-0.63)	(1.64)
Commodities(-1)	0.00	-0.01	0.02	-0.01	0.17	0.17	0.03	0.06	-0.09
	(0.53)	(-0.41)	(1.12)	(-0.20)	(2.88)	(2.61)	(9.99)	(1.78)	(-0.35)
Inflation(-1)	0.00	0.07	-0.04	-0.84	-0.01	-1.04	0.95	0.77	0.52
	(-1.27)	(-0.61)	(-0.32)	(-2.78)	(-0.03)	(-2.46)	(-42.78)	(-3.74)	(0.32)
Div. Yield(-1)	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.99	0.02
	(-0.06)	(-0.51)	(-0.29)	(-2.23)	(-0.14)	(-0.28)	(-1.12)	(-153.36)	(-0.36)
TermSpread(-1)	0.00	0.00	-0.01	0.02	0.03	-0.01	0.00	-0.03	0.36
	(-5.90)	(0.66)	(-1.07)	(1.07)	(1.92)	(-0.59)	(-0.59)	(-2.71)	(4.81)
Adj. R^2/F.stat	0.99	0.10	0.20	0.04	0.12	0.10	0.91	0.99	0.18
	(1928.97)	(3.86)	(7.10)	(2.06)	(4.32)	(3.74)	(262.82)	(2860.87)	(6.41)

t-stat are given in parentheses. The last row reports the adjusted-R2 and the F-statistics of joint significance.

Table 5

VAR residuals, correlation coefficients

January 1991–June 2009

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash	1.00								
Nom Bonds	-0.18	1.00							
IL Bonds	-0.20	0.76	1.00						
Equities	0.08	-0.04	0.05	1.00					
Real Estate	0.11	0.10	0.16	0.55	1.00				
Commodities	0.10	0.09	0.20	0.16	0.21	1.00			
Inflation	0.09	-0.10	-0.01	0.05	-0.06	0.22	1.00		
Div. Yield	-0.22	0.10	-0.04	-0.73	-0.44	-0.24	-0.06	1.00	
Term Spread	-0.63	-0.49	-0.47	-0.07	-0.24	-0.11	-0.06	0.14	1.00

Table 6

Probabilities of not achieving the inflation target for individual assets

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Cash	18%	17%	14%	4%
Nom Bonds	39%	35%	29%	17%
IL Bonds	45%	42%	36%	25%
Equities	38%	29%	20%	6%
Real Estate	44%	40%	32%	18%
Commodities	35%	26%	19%	8%

Table 7

Probabilities of not achieving the inflation target for individual assets

January 1991–December 2009

Horizon	2 years	5 years	10 years	30 years
Cash	13%	19%	22%	21%
Nom Bonds	17%	8%	4%	1%
IL Bonds	30%	23%	19%	12%
Equities	32%	29%	26%	13%
Real Estate	36%	31%	27%	19%
Commodities	39%	29%	18%	4%

Figure 1

Correlations between asset returns and inflation depending on the investment horizon

January 1973–December 1990

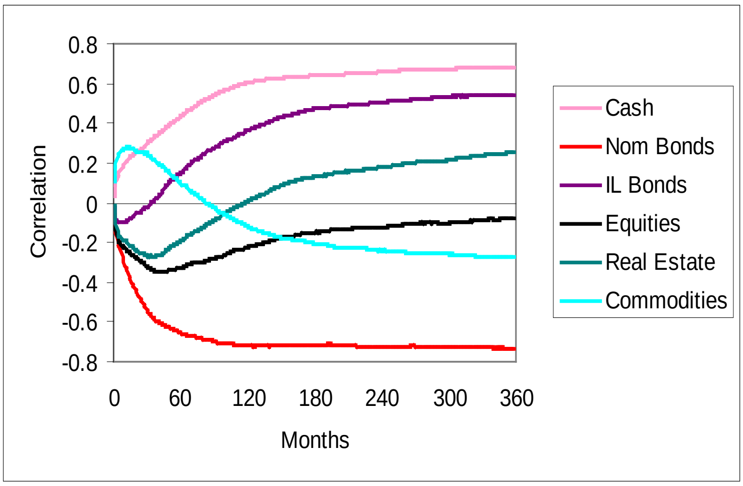
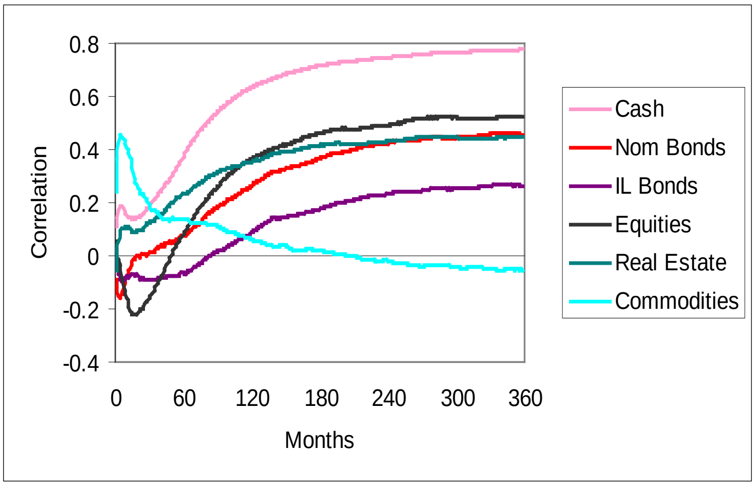


Figure 2

Correlations between asset returns and inflation depending on the investment horizon

December 1990–June 2009



Appendix 2

Table 8

Minimum shortfall probability portfolio, real return target 0%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	10.8%	11.5%	9.0%	1.4%
Ann. Excess Return Volatility	1.9%	3.6%	5.1%	5.4%
Ann. Excess Return*	1.6%	1.9%	2.2%	2.2%
Cumulated Excess Return	3.2%	9.7%	21.8%	65.2%
Weights				
Cash	88%	81%	72%	64%
Nom Bonds	0%	0%	0%	0%
IL Bonds	6%	7%	11%	17%
Equities	1%	3%	7%	8%
Real Estate	0%	0%	0%	5%
Commodities	5%	9%	10%	6%

* Excess returns are measured over target.

Table 9

Minimum shortfall probability portfolio, real return target 1%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	28.9%	23.7%	17.6%	5.8%
Ann. Excess Return Volatility	2.8%	7.1%	14.4%	14.9%
Ann. Excess Return	1.1%	2.3%	4.2%	4.3%
Cumulated Excess Return	2.2%	11.4%	42.4%	127.8%
Weights				
Cash	80%	50%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	1%	0%	0%	0%
Equities	9%	23%	55%	63%
Real Estate	0%	0%	0%	0%
Commodities	11%	27%	45%	37%

Table 10

Minimum shortfall probability portfolio, real return target 2%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	36.7%	30.0%	36.7%	11.4%
Ann. Excess Return Volatility	12.2%	13.1%	14.6%	15.1%
Ann. Excess Return	2.9%	3.1%	3.3%	3.3%
Cumulated Excess Return	5.9%	15.4%	33.0%	99.8%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	45%	47%	51%	59%
Real Estate	0%	0%	0%	0%
Commodities	55%	53%	49%	41%

Table 11

Minimum shortfall probability portfolio, real return target 3%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	40.9%	35.9%	30.7%	19.7%
Ann. Excess Return Volatility	14.1%	13.8%	15.3%	15.7%
Ann. Excess Return	2.3%	2.2%	2.4%	2.4%
Cumulated Excess Return	4.6%	11.2%	24.3%	73.4%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	33%	40%	44%	52%
Real Estate	0%	0%	0%	0%
Commodities	67%	60%	56%	48%

Table 12

Minimum shortfall probability portfolio, real return target 4%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	44.0%	41.5%	37.8%	29.9%
Ann. Excess Return Volatility	21.3%	18.1%	18.1%	18.4%
Ann. Excess Return	2.3%	1.7%	1.8%	1.8%
Cumulated Excess Return	4.5%	8.6%	17.7%	53.1%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	0%	14%	23%	32%
Real Estate	0%	0%	0%	0%
Commodities	100%	86%	77%	68%

Table 13

Minimum shortfall probability portfolio, real return target 0%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	4.7%	3.2%	1.3%	0.0%
Ann. Excess Return Volatility	1.3%	3.0%	4.8%	5.1%
Ann. Excess Return	1.5%	2.4%	3.4%	3.2%
Cumulated Excess Return	3.0%	12.2%	33.8%	96.7%
Weights				
Cash	80%	41%	0%	0%
Nom Bonds	17%	48%	77%	73%
IL Bonds	0%	0%	0%	0%
Equities	0%	5%	10%	10%
Real Estate	1%	0%	0%	0%
Commodities	2%	6%	13%	17%

Table 14

Minimum shortfall probability portfolio, real return target 1%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	16.0%	9.1%	5.8%	0.8%
Ann. Excess Return Volatility	4.5%	4.5%	4.8%	5.3%
Ann. Excess Return	3.2%	2.7%	2.4%	2.3%
Cumulated Excess Return	6.3%	13.3%	24.1%	70.2%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	76%	78%	76%	69%
IL Bonds	0%	0%	0%	0%
Equities	17%	13%	10%	10%
Real Estate	0%	0%	0%	0%
Commodities	7%	10%	14%	21%

Table 15

Minimum shortfall probability portfolio, real return target 2%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	24.7%	20.6%	17.4%	7.5%
Ann. Excess Return Volatility	4.5%	4.6%	5.1%	6.4%
Ann. Excess Return	2.2%	1.7%	1.5%	1.7%
Cumulated Excess Return	4.4%	8.4%	15.1%	50.4%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	75%	76%	72%	60%
IL Bonds	0%	0%	0%	0%
Equities	18%	13%	9%	9%
Real Estate	0%	0%	0%	0%
Commodities	7%	11%	19%	30%

Table 16

Minimum shortfall probability portfolio, real return target 3%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	35.4%	36.4%	34.1%	18.8%
Ann. Excess Return Volatility	4.7%	5.1%	9.9%	18.9%
Ann. Excess Return	1.2%	0.8%	1.3%	3.0%
Cumulated Excess Return	2.5%	3.9%	12.8%	91.5%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	73%	69%	42%	0%
IL Bonds	0%	0%	0%	0%
Equities	20%	11%	0%	0%
Real Estate	1%	5%	7%	0%
Commodities	5%	15%	51%	100%

Table 17

Minimum shortfall probability portfolio, real return target 4%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	44.9%	45.9%	41.3%	27.6%
Ann. Excess Return Volatility	12.5%	16.3%	18.3%	18.9%
Ann. Excess Return	1.1%	0.7%	1.3%	2.1%
Cumulated Excess Return	2.3%	3.7%	12.8%	61.6%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	26%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	33%	0%	0%	0%
Real Estate	40%	54%	0%	0%
Commodities	0%	46%	100%	100%

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decrease in macroeconomic volatility (the "Great Moderation", Blanchard and Simon (2001), Bernanke (2004), Summers (2005)) and the changing nature of inflation shocks – from countercyclical to procyclical – have been stressed as the two main factors affecting the level of stocks and bond prices (Lettau et al (2008), Kizys and Spencer (2008)), and also partially explaining the change of correlation sign between stocks and bond returns, from strongly positive to slightly negative (Baele et al (2009), Campbell (2009), Campbell et al (2009)). Using the Goetzmann et al (2005) breakpoint test for structural change in correlation, we split the sampling period into two sub-periods exhibiting the most stable correlations. The simulated returns based on our two estimated VAR models are thus used, on the one hand, to measure the inflation hedging properties of each asset class in each regime, and on the other hand to carry out a portfolio optimisation in a mean-shortfall probability framework. We determine the allocation that maximises above-target returns (inflation + x%) with the constraint that the probability of a shortfall remains lower than a threshold set by the investor.

We show that the optimal asset allocation differs strongly across regimes. In the periods of highly volatile economic environment, an investor having a pure inflation target should be mainly invested in cash when her investment horizon is short, and increase her allocation to IL bonds, equities, commodities and real estate when her horizon increases. In contrast, in a more stable economic environment, cash plays an essential role in hedging a portfolio against inflation in the short run, but in the longer run it should be replaced by nominal bonds, and to a lesser extent by commodities and equities. With a more ambitious real return target (from 1% to 4%), a larger weight should be dedicated to risky assets (mainly equities and commodities). These results confirm the value of alternative asset classes in shielding the portfolio against inflation, especially for ambitious investors with long investment horizons.

Our paper tries to complement the existing literature in three directions: inflation hedging properties of assets, strategic asset allocation, and alternative asset classes. The question of hedging assets against inflation has been widely studied (see Attié and Roache (2009) for a detailed literature review). Most studies have focused on measuring the relationship between historical asset returns and inflation, either by measuring the correlation between these variables or by adopting a factor approach such as the one used by Fama and Schwert (1977). These approaches present a number of difficulties, especially with regard to the lack of historical data available to study long-horizon returns, the problem of non-serially independent data, non-stationary variables, and instability over time of the assets' relationships to inflation.

The literature on strategic asset allocation has shed new light on this question. Continuing the pioneering work of Brennan et al (1997) and Campbell and Viceira (2002), many researchers have sought to show that long-term allocation is very different from short-term allocation when returns are partially predictable (Barberis (2000), Brennan and Xia (2002), Campbell et al (2003), Guidolin and Timmermann (2005), Fugazza et al (2007)). The approach developed in an assets-only framework was extended to asset and liability management (ALM) using traditional classes (van Binsbergen and Brandt (2007)) but also alternative assets (Goetzmann and Valaitis (2006), Hoevenaars et al (2008), Amenc et al (2009)). One common characteristic of these studies is their focus on the situation of investors, such as pension funds, with liabilities which are subject to the risk of both fluctuating inflation and real interest rates. In this article, we adopt a different point of view. Not all investors who seek to hedge against inflation necessarily have such liabilities. They may only wish to hedge their assets against the risk of real-term depreciation, and thus have a purely nominal objective that consists of the inflation rate plus a real expected return target, which is assumed to be fixed.

Thus far, most of the research into inflation hedging for diversified portfolios has been done within a mean-variance framework. The studies of inflation hedging properties in an ALM framework with a liability constraint generally focus on a "surplus optimisation" (Leibowitz (1987), Sharpe and Tint (1990), Hoevenaars et al (2008)). In our context, however, this risk measure is not the one that corresponds best to investors' objectives. Our portfolio's excess returns above target may be only slightly volatile but still significantly lower than the objective, presenting a major risk to the investor. The notion of "safety first" (Roy (1952)) is therefore more appropriate. We focus on the shortfall probability, ie the likelihood of not achieving the target return at maturity. In an ALM framework, Amenc et al (2009) measure the shortfall probability of ad hoc portfolios. We expand that work and determine optimal portfolio allocations in a mean-shortfall probability framework.

The properties of alternative asset classes have been studied in a strategic asset allocation context (Agarwal and Naik (2004), Fugazza et al (2007), Brière et al (2010)). In an ALM context, Hoevenaars et al (2008) and Amenc et al (2009) also find significant appeal in these asset classes, which are interesting sources of diversification and inflation hedging in a portfolio. To the best of our knowledge, however, these asset classes have not yet been studied in an asset-only context with an inflation target. Our research tries to fill the gap.

Our paper is organised as follows. Section 2 presents our data and methodology. Section 3 presents our results: the correlation structure of our assets with inflation at different horizons, and the optimal composition of inflation hedging portfolios. Section 4 concludes.

2. Data and methodology

2.1 Data

We consider the case of a US investor able to invest in six liquid and publicly traded asset classes: cash, stocks, nominal bonds, IL bonds, real estate and commodities. (1) Cash is the 3-month T-bill rate. (2) Stocks are represented by the Morgan Stanley Capital International (MSCI) US Equity index. (3) Nominal bonds are represented by the Morgan Stanley 7-10 year index. (4) IL bonds are represented by the Barclays Global Inflation index from 1997.[3] Before that date, to recover price and total return history before IL bonds were first issued in the US, we reconstruct a time series of real rates according to the methodology of Kothari and Shanken (2004). Real rates are thus approximated by 10-year nominal bond rates minus an inflation expectation based on a 5-year historical average of a seasonally adjusted consumer price index (CPI) (Amenc et al (2009)). The inflation risk premium is assumed equal to zero, a realistic assumption considering the recent history of US TIPS (Berardi (2005), D'Amico et al (2008), Brière and Signori (2009)). (5) Real estate investments are proxied by the FTSE NAREIT Composite Index representing listed real estate in the US (publicly traded property companies of the NYSE, Nasdaq, AMEX and Toronto Stock Exchange). (6) Commodities are represented by the Goldman Sachs Commodity Index (GSCI). We also add a set of exogenous variables: inflation (measured by CPI), dividend yield obtained from the Shiller database (Campbell and Shiller (1988)) and the term spread measured as the difference between the 10-year Treasury Constant Maturity Rate and the 3-month Treasury bill rate provided by the US Federal Reserve Economic Database. We consider monthly returns for the time period January 1973–June 2009.

Table 1 in Appendix 1 presents the descriptive statistics of monthly returns. The hierarchy of returns is the following: cash has the smallest return on the total period, followed by IL bonds, nominal bonds, real estate, equities and commodities. Adjusted for risk, the results show a slightly different picture: cash appears particularly attractive compared to other asset classes, nominal bonds are much more appealing than real estate (risk-adjusted return of 1 vs 0.4), and equities are more attractive than commodities (0.5 vs 0.4). Extreme risks are

[3] Note that the durations of the IL bond and nominal bond indices are comparable.

also different: negative skewness and strong kurtosis are strongly pronounced for real estate and, to a lesser extent, for equities and commodities.

2.2 Econometric model of asset return dynamics

VAR models are widely used in financial economics to model the intertemporal behaviour of asset returns. Campbell and Viceira (2002) provide a complete overview of the applications of VAR specification to solve intertemporal portfolio decision problems. The VAR structure can also be used to simulate returns in the presence of macroeconomic factors. Following Barberis (2000), Campbell et al (2003), Campbell and Viceira (2005) and Fugazza et al (2007), among others, we adopt a VAR(1) representation of the returns but expand it to alternative asset classes, as did Hoevenaars et al (2008).[4] Empirical literature has relied on a predetermined choice of predictive variables. Kandel and Stambaugh (1996), Balduzzi and Lynch (1999) and Barberis (2000) use the dividend yield; Lynch (2001) uses the dividend yield and term spread; Brennan et al (1997) use the dividend yield, bond yield and Treasury bill yield; and Hoevenaars et al (2008) use the dividend yield, term spread, credit spread and Treasury bill yield. We select the most significant variables in our case: dividend yield and term spread. As we are modelling nominal logarithmic returns, we also enter inflation explicitly as a state variable, which enables us to measure the link between inflation and asset class returns.[5]

The compacted form of the VAR(1) can be written as:

$$z_t = \phi_0 + \phi_1 z_{t-1} + u_t \tag{1}$$

where ϕ_0 is the vector of intercepts; ϕ_1 is the coefficient matrix; z_t is a column vector whose elements are the log returns on the six asset classes and the values of the three state variables; and u_t is the vector of a zero mean innovations process.

Finally, to overcome the problem of correlated innovations of the VAR(1) model and to take into consideration the contemporaneous relationship between returns and the economic variables, we follow the procedure described in Amisano and Giannini (1997) to obtain structural innovations characterised by an iid process. The structural innovations ε_t, may be written as $Au_t = B\varepsilon_t$ where the parameters of A and B matrices are identified imposing a set of restrictions. The structure of ε_t is used to perform Monte Carlo simulations on the estimated VAR for the portfolio analysis. Imposing the restrictions we assume that inflation, as well as cash, impact on the returns of all the asset classes, and that commodities are not affected instantaneously by the returns of the other asset classes.

Meaningful forecasts from a VAR model rely on the assumption that the underlying sample correlation structure is constant. However, regime shifts in the relationship between financial and economic variables have already been widely discussed in the literature. Guidolin and Timmermann (2005) and Goetzmann and Valaitis (2006) find evidence of multiple regimes in the dynamics of asset returns. This suggests that a full-sample VAR model might be potentially mis-specified, as the correlation structure may not be constant. Changing macroeconomic volatility has been identified as one of the main causes of the changing correlation structure between assets (Li (2002), Ilmanen (2003), Baele et al (2009)). This has

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[4] The differences with the model lie in the fact that we include IL bonds but not corporate bonds and hedge funds in our investment set. As our investor is an asset-only investor, there are no liabilities in our model.

[5] As in the models of Brennan et al (1997), Campbell and Viceira (2002) and Campbell et al (2003), we do not adjust VAR estimates for possible small sample biases related to near non-stationarity of some series (Campbell and Yogo (2006)).

been accompanied by a change in the nature of inflation. During the 1970s and 1980s (marked by supply shocks and poor central bank credibility), inflation was mainly countercyclical, whereas in the most recent period (with demand shocks and credible monetary policy), inflation was more procyclical. This change has been stressed as an important driver of the decreasing correlation between stocks and bonds (Campbell (2009), Campbell et al (2009)).

Using the Goetzmann et al (2005) test[6] for structural change in correlations between asset returns and state variables, we determine the breakpoint that best separates the sample data, ensuring the most stable correlation structure within each sub-period.[7] The first period (January 1973–December 1990) corresponds to a volatile economic environment (major oil shocks, huge government deficits, large swings in GDP growth), the second (January 1991–June 2009) to a much more stable one.

Tables 2 to 5 in Appendix 1 present the results of our VAR model in the two identified sub-periods. Looking at the significance of the coefficients of the lagged state variables, inflation is mainly helpful in predicting nominal bond returns. Dividend yield has better explanatory power for equity returns in the second period than in the first. The high positive correlation coefficient of the residuals between nominal bonds and IL bonds (84% and 76% in the two sub-sample periods) confirms the strong interdependency between the contemporaneous returns of the two asset classes dominated by the common component of real rates. Real estate and equities have the second largest positive innovation correlation coefficient (61% and 55%, respectively), implying that a positive shock in real estate has a positive contemporaneous effect on stock returns and vice versa. Other results are in line with the common findings of positive contemporaneous correlation between inflation and commodities, and the intuition that inflation and monetary policy shocks have a negative impact on bond returns through the inflation expectations component.

2.3 Simulations

We use the iid structural innovation process of the two VAR models estimated on the two sub-samples to perform a Monte Carlo analysis based on the fitted model. We draw iid random variables from a multivariate normal distribution for the structural innovations and we obtain simulated returns for 5,000 simulated paths of length T (T varying from 1 month to 30 years). The simulated returns are thus used, on the one hand, to measure the inflation hedging properties of each asset class in each regime, and on the other hand in a portfolio construction context to generate expected returns and covariance matrices at different horizons (2, 5, 10 and 30 years).

2.4 Portfolio choice

The bulk of the research into inflation hedging for a diversified portfolio has used a mean-variance framework. And research into inflation hedging properties in an ALM framework with a liability constraint is usually based on surplus optimisation, in which the surplus is maximised under the constraint that its volatility be lower than a target value (Leibowitz (1987), Sharpe and Tint (1990), Hoevenaars et al (2008)). But for our purposes, this risk measure is not the one best suited to investors' objectives. Since the portfolio's excess returns above target may be only slightly volatile but still significantly lower than the

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6 Null hypothesis of stationary bivariate historical correlations between assets.

7 We have not presented the Goetzmann et al (2005) test results so as not to clutter the presentation of the results.

objective, the investor faces a serious risk. In this case, the notion of safety first (Roy (1952)) is more appropriate. Roy argues that investors think in terms of a minimum acceptable outcome, which he calls the "disaster level". The safety first strategy is to choose the investment with the smallest probability of falling below that disaster level. A less risk-averse investor may be willing to achieve a higher return, but with a greater probability of going below the threshold. Roy defined the shortfall constraint such that the probability of the portfolio's value falling below a specified disaster level is limited to a specified disaster probability. Portfolio optimisations with a shortfall probability risk measure have been conducted before (Leibowitz and Henriksson (1989), Leibowitz and Kogelman (1991), Lucas and Klaassen (1998), Billio (2007), Smith and Gould (2007)), but as far as we know not in the context of an inflation hedging portfolio.

We determine optimal allocations that maximise above-target returns (the target being inflation + x%) with the constraint that the probability of a shortfall remains lower than a threshold set by the investor.

$$Min_w \; P(\sum_{i=1}^{n} w_i R_{iT} < \pi_T + \overline{R}) \tag{2}$$

$$E\left[\sum_{i=1}^{n} w_i R_{iT} - (\pi_T + \overline{R})\right] > 0 \tag{3}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{4}$$

$$w_i \geq 0 \tag{5}$$

Where $R_T = (R_{1T}, R_{2T}, ..., R_{nT})$ is the annualised return of the n assets in the portfolio over the investment horizon T, $w = (w_1, w_2, ..., w_n)$ the fraction of capital invested in the asset i, π_T the annual inflation rate during that horizon T, $\overline{R}$ the target real return in excess of inflation, and α the target shortfall probability. E is the expectation operator with respect to the probability distribution P of the asset returns.

We work in a mean-shortfall probability world and derive the corresponding efficient frontier (Harlow (1991)). For a portfolio with normally distributed returns $N(\mu, \sigma)$, the probability of portfolio shortfall is written:

$$p(w' R_T < \pi_T + \overline{R}) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\pi_T + \overline{R}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

For each investment horizon T (T = 1 year, 3 years, 5 years, 10 years, 20 years, 30 years), we draw all the efficient portfolios in the mean-shortfall probability universe for the two identified regimes.

3. Results

3.1 Inflation hedging properties of individual assets

Figures 1 and 2 in Appendix 1 display correlation coefficients between asset returns and inflation based on our VAR model, depending on the investment horizon, from 1 month to 30 years. We consider two sample periods: from January 1973 to December 1990 and from January 1991 to June 2009. The inflation hedging properties of the different assets vary strongly depending on the investment horizon. Most of the assets (the only exception being commodities and nominal bonds in the first period) display an upward-sloping correlation curve, meaning that inflation hedging properties improve as the investment horizon widens.

In the first sample period (1973–1990), marked by a volatile macroeconomic environment, cash and commodities have a positive correlation with inflation on short-term horizons, whereas nominal bonds, equities and real estate are negatively correlated. The correlation of IL bonds with inflation lies in the middle and is close to zero. In the longer run (30 years), cash shows the best correlation with inflation (around 0.6), followed by IL bonds and real estate (all showing a positive correlation), then equities, commodities, and finally nominal bonds (the latter with negative correlation).

The very strong negative correlation of nominal bonds with inflation both in the short run and in the long run is intuitive since changes in expected inflation and bond risk premiums are traditionally the main source of variation in nominal yields (Campbell and Ammer (1993)). IL bonds and inflation are positively correlated for an obvious reason: the impact of a strongly rising inflation rate has a direct positive impact on performance through the coupon indexation mechanism. Negative correlation between equities and inflation is a characteristic of countercyclical inflation periods when the economy is affected by supply shocks or changing inflation expectations, which shift the Phillips curve upwards or downwards (Campbell (2009)). This has been documented by many authors, with three different interpretations. The first is that inflation hurts the real economy, so the dividend growth rate should fall, leading to a fall in equity prices (an alternative explanation is that poor economic conditions lead the central bank to lower interest rates, which has a positive influence on inflation (Geske and Roll (1983)). The second interpretation argues that high expected inflation has tended to coincide with periods of higher uncertainty about real economic growth, raising the equity risk premium (Brandt and Wang (2003), Bekaert and Engstrom (2009)). The final explanation is that stock market investors are subject to inflation illusion and fail to adjust the dividend growth rate to the inflation rate, even though they correctly adjust the nominal bond rate (Modigliani and Cohn (1979), Ritter and Warr (2002), Campbell and Vuolteenaho (2004)). Commodities exhibit more contrasted behaviour, ie the correlation with inflation is positive in the short run but negative in the long run. This result is consistent with the fact that commodities have a tendency to overreact to money surprises (and therefore inflation) in the short run (Browne and Cronin (2007)), whereas the long-term link with inflation has been weak since the 1980s, when the commodity-consumer price connection seems to have broken down. This reflects the diminished role of traditional commodities in US production and the sterilisation of some inflation signals by offsetting monetary policy actions (Blomberg and Harris (1995), Hooker (2002)).

The correlation picture is very different if we now consider the second sample period (1991–2009), marked by a stable macroeconomic environment. The hierarchy of the different assets in terms of inflation hedging properties is very different, both in the long run and in the short run. In the short run, commodities have the strongest correlation with inflation, followed by cash, real estate, nominal bonds, IL bonds and equities. In the long run, the best inflation hedger is now cash, followed by equities, real estate, nominal bonds, IL bonds and commodities. The main differences compared to the first period are that nominal bonds and equities now have a positive correlation with inflation in the long run, and better inflation hedging properties than IL bonds. The moderation in economic risk, especially inflation volatility, has reduced correlations in absolute terms. IL bond returns have a much smaller positive correlation with inflation, whereas nominal bonds lose their negative correlation and become moderately positively correlated. Moreover, as inflation is now procyclical (the macroeconomy is moving along a stable Phillips curve), positive inflation shocks happen during periods of improving macroeconomic environment, leading to positive correlation between equities and inflation (Campbell (2009)). This changing behaviour is strongly linked to the much stronger credibility and transparency of central banks in fighting inflation during the last two decades, leading to more stable and lower interest rates, only slightly impacted by inflation changes (Kim and Wright (2005), Eijffinger et al (2006)).

Another way to look at the inflation hedging properties of individual assets is to measure the probability of having below-inflation returns at the investment horizon (shortfall probabilities). Tables 6 and 7 in Appendix 1 display the shortfall probabilities of the different asset classes for horizons of 2, 5, 10 and 30 years. A first observation is that shortfall probabilities decrease strongly with the investment horizon. This is true for all asset classes, but particularly for the most risky ones. Commodities, for example, have a probability of not achieving the inflation target of more than 35% at a 2-year horizon. At 30 years, this falls below 8% for both periods. An asset can be strongly correlated with inflation but also have a significant shortfall probability if its return is always lower than inflation. Looking at shortfall probabilities, the best inflation hedger in the short run appears to be cash on both inflation regimes. In the long run, the best hedgers are cash, equities and commodities in the volatile regime (IL bonds are well correlated with inflation during that period but with a strong shortfall probability, 25% for a 30-year horizon), and nominal bonds and commodities in the stable regime.

3.2 Inflation hedging portfolios

We now turn to the construction of inflation hedging portfolios. We examine the case of an investor wishing to hedge inflation on her investment horizon. This investor has a target real return ranging from 0% to 4%. For each of the investor targets, we show the optimal portfolio composition depending on the inflation regime.

How to attain a pure inflation target

We first consider the case of an investor simply wishing to hedge inflation, ie having a target real return of 0%. Table 8 and Table 13 in Appendix 2 show the optimal portfolio composition and the descriptive statistics of minimum shortfall probability portfolios for each horizon.

The first observation, common to both periods, is that the higher the required return, the greater the shortfall probability in the portfolio. The minimum shortfall probability (corresponding to Roy's (1952) "safety first" portfolio) generally decreases with the investment horizon, the only exception being for the 2-year horizon in the first period, where the minimum shortfall probability is lower than for the 5-year horizon.

In the first period, characterised by high macroeconomic volatility, the optimal portfolio composition of a "safety first" investor with a 2-year horizon is 88% cash, 6% IL bonds, 1% equities and 5% commodities. This very conservative portfolio has a 1.6% annualised return over inflation, 1.9% volatility of real returns and 11% shortfall probability. Diversifying the portfolio makes it possible to sharply diminish the achievable shortfall probability compared to individual assets: whereas the minimum shortfall probability over all assets in that period is 18% (for cash), it is 7% lower with a diversified portfolio. When the horizon is increased, the weight assigned to cash decreases and the weights of riskier assets (IL bonds, equities, real estate, commodities) rise. For a 30-year horizon, the optimal portfolio composition is 64% cash, 17% IL bonds, 8% equities, 5% real estate and 6% commodities. This portfolio generates an annualised excess return of 2.2% over inflation with stronger volatility (5.4%) but with a very low probability (1.4%) of falling below the inflation target at the investment horizon. Again, portfolio diversification makes it possible to decrease strongly the shortfall probability at the investment horizon.

In the second period, characterised by much lower macroeconomic volatility, the optimal portfolio composition is quite different. With a 2-year horizon, the optimal composition for a "safety first" investor is still very conservative: 81% cash, but the rest of the portfolio consists mainly of nominal bonds (17%), real estate (1%) and commodities (2%). Compared to the first period, nominal bonds now replace IL bonds and equities. This result is consistent with our previous findings on individual assets: the inflation hedging properties of nominal bonds increase strongly in the second period, with inflation correlation becoming even greater than for IL bonds and shortfall probabilities becoming much smaller. Increasing the investment horizon, the share of the portfolio dedicated to cash decreases, progressively replaced by nominal bonds, whereas the weights of commodities and equities increase slightly. With a 30-year horizon, the optimal portfolio of a “safety first” investor is composed of 73% nominal bonds, 10% equities and 17% commodities. This portfolio has slightly higher annualised real return than in the first period (3.2% vs 2.2%), with a smaller shortfall probability (0.02% vs 1.4%). Contrary to the first period, IL bonds no longer appear in the optimal composition of safety first portfolios.

To sum up, when macroeconomic volatility is high, a “safety first” investor having a pure inflation target should be mainly invested in cash when her investment horizon is short, and should increase her allocation to IL bonds, equities, commodities and real estate when her horizon increases. When economic volatility is much lower, the optimal investment set changes radically. Mainly invested in cash when the investment horizon is short, an investor should increase her holdings of nominal bonds, commodities and equities when her investment horizon increases.

Raising the level of required real return

We now consider the consequences for an investor of having a more ambitious target real return, ranging from 1% to 4%. Tables 9 to 12 and 14 to 17 in Appendix 2 present the optimal portfolio composition as well as the descriptive statistics of the minimum shortfall probability portfolios, for the first and second sample periods.

Consistent with intuition, when the required real return is increased, the shortfall probability increases strongly in both sub-periods. In the first period, for a 2-year horizon investor, the minimum shortfall probability is 10.8% for a target real return of 0%. It is 28.9%, 36.7%, 40.9% and 44.0% for a 1%, 2%, 3% and 4% real target return, respectively. The results are similar for the second period: shortfall probabilities rise from 4.7% to 44.9% for a 0% to 4% real return target.

Another intuitive result is that the more the investor increases her required real return, the more the optimal portfolio composition is biased towards risky assets. Considering the first period, for a 30-year horizon, the optimal weight of cash decreases from 64% (with a real return target of 0%) to 0% (1% to 4% target). The IL bond weight also decreases, from 17% to 0%. The explanation is intuitive: these assets provide a good inflation hedge but are not sufficient to achieve high real returns. On the contrary, the weights of risky assets (equities, and especially commodities) increase. A long-term portfolio seeking to achieve inflation +1% should comprise 63% equities and 37% commodities. With a 4% target, the investor should hold 32% equities and 68% commodities. Of course, if the investment horizon is shorter, a more substantial part of the portfolio should be dedicated to cash.

In the second sample period, the results are comparable. Increasing the real return target leads to a decrease in the cash investment and an increase in the more risky assets. The difference lies in the “risky” assets retained by the optimisation. A substantial portion of nominal bonds should now be added to the optimal mix of equities and commodities than in the first period. For a 30-year investor with a 1% real return target, the optimal portfolio composition is 69% nominal bonds, 10% equities and 21% commodities. It is 60% bonds, 9% equities and 31% commodities for a 2% target, and 100% commodities for a 3% or 4% target. As in the first period, commodities are the most rewarding asset class. This explains why, with a very ambitious real return target, the portfolio should be fully invested in commodities.

To sum up, a more ambitious real return target leads to a greater shortfall probability and a different optimal portfolio composition, with a larger weight in risky assets. In an unstable and volatile economic regime, an ambitious investor should abandon IL bonds and real estate and concentrate on equities and commodities. In a more stable economic environment, she should reduce her portfolio weight in nominal bonds and equities and invest a higher share in commodities.

4. Conclusion

A key challenge for many institutional investors is the preservation of capital in real terms, while for individual investors it is building a portfolio that keeps up with the cost of living. In this paper we address the investment problem of an investor seeking to hedge inflation risk and achieve a fixed target real rate of return. The key question is thus to determine the optimal asset allocation that will preserve the investor's capital from inflation with an acceptable probability of shortfall.

Following Campbell et al (2003) and Campbell and Viceira (2005), we used a vector-autoregressive (VAR) specification to model the joint dynamics of asset classes and state variables, and then simulated long-term holding portfolio returns for a range of different assets and inflation. The strong change in macroeconomic volatility and the varying nature of inflation shocks (leading to a change of correlation sign between inflation and the real economy) have been identified as the two main causes of the changing correlation structure between assets (Li (2002), Ilmanen (2003), Baele et al (2009), Campbell (2009), Campbell et al (2009)). Relying on the Goetzmann et al (2006) test for structural change in correlation, we determined the breakpoint that best separates the sample data, ensuring the most stable correlation structure within each sub-period. We estimated a VAR model for each period and performed a simulation-based analysis. We were thus able to measure the inflation hedging properties of each asset class in each regime and determine the allocation that maximises above-target returns (inflation + x%) with the constraint that the shortfall probability remains below a threshold set by the investor.

Our results confirm that the presence of macroeconomic regimes radically alters the investor's optimal allocation. In a volatile regime marked by countercyclical inflation, a "safety first" investor having a pure inflation target should be mainly invested in cash when her investment horizon is short and should increase her allocation to IL bonds, equities, commodities and real estate when horizon increases. In a more stable economic environment with procyclical inflation shocks, the optimal investment set changes radically. Mainly invested in cash when investment horizon is short, an investor should increase her investment in nominal bonds, but also, to a lesser extent, in commodities and equities when her horizon increases. Our results confirm the value of alternative asset classes in protecting the portfolio against inflation.

Having a more ambitious real return target (from 1% to 4%) leads automatically to a greater shortfall probability, but also to a different optimal portfolio composition. A larger weight should be dedicated to risky assets, which make it possible to achieve higher returns (with a greater shortfall probability). In the first period, an ambitious investor should gradually abandon IL bonds and real estate and concentrate on equities and particularly commodities. In the second period, she should reduce her portfolio weight in nominal bonds and equities and invest a higher share in commodities.

Our work could be extended in several ways. Different methodologies have been developed that move away from the standard mean-variance approach, by changing the risk measure of the portfolio. One branch of the literature considers portfolio selection with value at risk (Agarwal and Naik (2004), Martellini and Ziemann (2007)), or conditional VaR (Rockafellar and Uryasev (2000)); the other branch with shortfall probability (Leibowitz and Henriksson (1989), Leibowitz and Kogelman (1991), Lucas and Klaassen (1998), Billio and Casarin (2007), Smith and Gould (2007)). A useful development of our work would be to reconcile the two approaches and examine shortfall probabilities in the context of non-normal returns. We have considered only a static allocation on the whole investment horizon. A very interesting development would be to compare these results with a dynamic asset allocation, rebalancing the portfolio depending on active views on the different asset classes. Finally, we examined a fairly simple objective function. In the real world, many investors (especially pension funds) do not have a single well-defined goal but rather have to cope with multiple and sometimes contradictory objectives, with long-term return shortfall probability constraints and short-term performance objectives. An interesting development of this work would be to take these different constraints into account.

Appendix 1

Table 1

Summary statistics of monthly returns

January 1973–June 2009

	Cash	Nom Bonds	IL bonds	Equities	Real Estate	Commodities
Ann. Ret.	5.8%	7.8%	6.5%	8.6%	7.8%	8.3%
Max Monthly	1.3%	11.3%	13.9%	16.4%	26.9%	22.9%
Min Monthly	0.0%	-9.0%	-13.8%	-23.9%	-36.4%	-33.1%
Ann. Vol.	0.9%	7.6%	9.9%	15.9%	18.5%	20.6%
Risk/Adjusted Ret.*	6.6	1.0	0.6	0.5	0.4	0.4
Skewness	0.7	0.3	0.1	-0.7	-1.2	-0.3
Kurtosis	3.9	5.9	6.8	5.7	12.4	6.1

* Annualised return divided by annualised volatility.

Table 2

Results of VAR model, parameter estimates

January 1973–December 1990

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash(-1)	0.96	1.13	-0.96	-1.75	-3.52	-0.22	0.09	1.80	-1.26
	(-48.71)	(-1.11)	(-0.86)	(-0.92)	(-1.87)	(-0.09)	(-0.53)	(-1.39)	(-0.10)
Nom Bonds(-1)	-0.01	0.17	1.02	-0.01	0.41	-0.43	-0.04	-0.18	-5.96
	(-6.29)	(-1.66)	(-9.42)	(-0.03)	(-2.20)	(-1.91)	(-2.14)	(-1.39)	(-4.98)
IL Bonds(-1)	0.00	-0.09	-0.17	0.22	0.08	0.41	0.01	0.01	4.59
	(-0.46)	(-1.18)	(-2.14)	(-1.54)	(-0.57)	(-2.46)	(-1.16)	(-0.08)	(4.33)
Equities(-1)	0.00	-0.03	-0.07	-0.14	0.01	-0.07	0.01	-0.35	-0.59
	(-1.69)	(-0.66)	(-1.41)	(-1.58)	(-0.08)	(-0.69)	(-0.72)	(-5.91)	(-1.38)
Real Estate(-1)	0.00	-0.06	-0.07	0.15	-0.07	-0.11	-0.01	-0.08	0.55
	(-1.56)	(-1.24)	(-1.33)	(-1.76)	(-0.77)	(-1.02)	(-1.18)	(-1.39)	(1.64)
Commodities(-1)	0.00	-0.07	-0.05	-0.12	-0.19	0.13	0.02	0.07	-0.09
	(-1.91)	(-2.19)	(-1.59)	(-2.04)	(-3.46)	(-1.86)	(-3.54)	(-1.89)	(-0.35)
Inflation(-1)	0.00	-0.19	0.10	-0.22	-0.19	-0.08	1.00	0.14	0.52
	(-0.89)	(-2.83)	(-1.38)	(-1.78)	(-1.50)	(-0.52)	(-90.79)	(-1.62)	(0.32)
Div. Yield(-1)	0.00	0.02	0.02	0.05	0.09	-0.02	0.00	0.96	0.02
	(-0.23)	(-2.07)	(-1.24)	(-2.61)	(-4.26)	(-0.77)	(-2.41)	(-67.43)	(-0.36)
TermSpread(-1)	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.36
	(-3.57)	(-1.21)	(-0.46)	(-0.13)	(-0.20)	(-0.91)	(-1.35)	(-1.15)	(4.81)
Adj. R^2/F.stat	0.95	0.07	0.39	0.08	0.18	0.04	0.98	0.98	0.15
	(447.67)	(2.90)	(16.47)	(3.15)	(6.25)	(1.94)	(1522.93)	(958.73)	(5.29)

t-stat are given in parentheses. The last row reports the adjusted-R^2 and the F-statistics of joint significance.

Table 3

VAR residuals, correlation coefficients

January 1973–December 1990

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash	1.00								
Nom Bonds	-0.37	1.00							
IL Bonds	-0.47	0.84	1.00						
Equities	-0.14	0.25	0.21	1.00					
Real Estate	-0.25	0.17	0.14	0.61	1.00				
Commodi-ties	-0.06	-0.12	-0.06	-0.05	0.02	1.00			
Inflation	0.02	-0.07	-0.02	-0.12	-0.04	0.13	1.00		
Div. Yield	0.12	-0.20	-0.24	-0.80	-0.54	0.08	0.17	1.00	
Term Spread	-0.85	-0.09	-0.05	0.01	0.18	0.11	0.02	0.03	1.00

Table 4

Results of VAR model, parameter estimates

January 1991–June 2009

	Cash	Nom Bonds	IL Bonds	Equities	Real Estate	Com-modities	Inflation	Div. Yield	Term Spread
Cash(-1)	0.99	1.83	1.47	7.06	1.24	3.75	0.20	-3.86	-1.26
	(119.42)	(1.99)	(1.41)	(3.09)	(0.44)	(1.18)	(1.21)	(-2.50)	(-0.10)
Nom Bonds(-1)	0.00	0.15	0.70	0.01	0.49	-0.44	-0.03	-0.23	-5.96
	(-3.81)	(1.64)	(6.69)	(0.02)	(1.74)	(-1.40)	(-2.07)	(-1.51)	(-4.98)
IL Bonds(-1)	0.00	-0.07	-0.28	0.16	0.31	-0.20	0.02	-0.18	4.59
	(-2.90)	(-0.81)	(-3.01)	(0.78)	(1.25)	(-0.72)	(1.16)	(-1.31)	(4.33)
Equities(-1)	0.00	-0.07	-0.01	-0.01	0.32	-0.06	0.00	-0.49	-0.59
	(2.06)	(-2.28)	(-0.16)	(-0.09)	(3.22)	(-0.55)	(-0.35)	(-9.00)	(-1.38)
Real Estate(-1)	0.00	-0.06	-0.04	0.07	-0.03	0.23	0.00	-0.03	0.55
	(0.25)	(-2.23)	(-1.36)	(1.10)	(-0.39)	(2.64)	(0.88)	(-0.63)	(1.64)
Commodities(-1)	0.00	-0.01	0.02	-0.01	0.17	0.17	0.03	0.06	-0.09
	(0.53)	(-0.41)	(1.12)	(-0.20)	(2.88)	(2.61)	(9.99)	(1.78)	(-0.35)
Inflation(-1)	0.00	0.07	-0.04	-0.84	-0.01	-1.04	0.95	0.77	0.52
	(-1.27)	(-0.61)	(-0.32)	(-2.78)	(-0.03)	(-2.46)	(-42.78)	(-3.74)	(0.32)
Div. Yield(-1)	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.99	0.02
	(-0.06)	(-0.51)	(-0.29)	(-2.23)	(-0.14)	(-0.28)	(-1.12)	(-153.36)	(-0.36)
TermSpread(-1)	0.00	0.00	-0.01	0.02	0.03	-0.01	0.00	-0.03	0.36
	(-5.90)	(0.66)	(-1.07)	(1.07)	(1.92)	(-0.59)	(-0.59)	(-2.71)	(4.81)
Adj. R^2/F.stat	0.99	0.10	0.20	0.04	0.12	0.10	0.91	0.99	0.18
	(1928.97)	(3.86)	(7.10)	(2.06)	(4.32)	(3.74)	(262.82)	(2860.87)	(6.41)

t-stat are given in parentheses. The last row reports the adjusted-R2 and the F-statistics of joint significance.

Table 5

VAR residuals, correlation coefficients

January 1991–June 2009

	Cash	**Nom Bonds**	**IL Bonds**	**Equities**	**Real Estate**	**Com-modities**	**Inflation**	**Div. Yield**	**Term Spread**
Cash	1.00								
Nom Bonds	-0.18	1.00							
IL Bonds	-0.20	0.76	1.00						
Equities	0.08	-0.04	0.05	1.00					
Real Estate	0.11	0.10	0.16	0.55	1.00				
Commodities	0.10	0.09	0.20	0.16	0.21	1.00			
Inflation	0.09	-0.10	-0.01	0.05	-0.06	0.22	1.00		
Div. Yield	-0.22	0.10	-0.04	-0.73	-0.44	-0.24	-0.06	1.00	
Term Spread	-0.63	-0.49	-0.47	-0.07	-0.24	-0.11	-0.06	0.14	1.00

Table 6

Probabilities of not achieving the inflation target for individual assets

January 1973–December 1990

Horizon	**2 years**	**5 years**	**10 years**	**30 years**
Cash	18%	17%	14%	4%
Nom Bonds	39%	35%	29%	17%
IL Bonds	45%	42%	36%	25%
Equities	38%	29%	20%	6%
Real Estate	44%	40%	32%	18%
Commodities	35%	26%	19%	8%

Table 7

Probabilities of not achieving the inflation target for individual assets

January 1991–December 2009

Horizon	**2 years**	**5 years**	**10 years**	**30 years**
Cash	13%	19%	22%	21%
Nom Bonds	17%	8%	4%	1%
IL Bonds	30%	23%	19%	12%
Equities	32%	29%	26%	13%
Real Estate	36%	31%	27%	19%
Commodities	39%	29%	18%	4%

Figure 1

Correlations between asset returns and inflation depending on the investment horizon

January 1973–December 1990

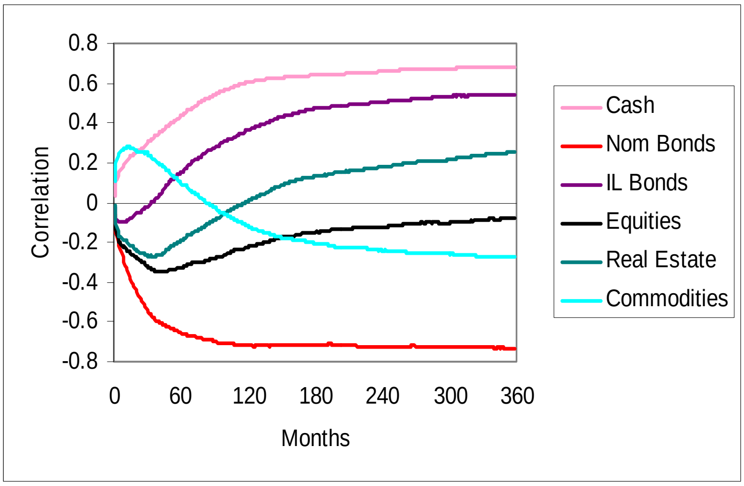
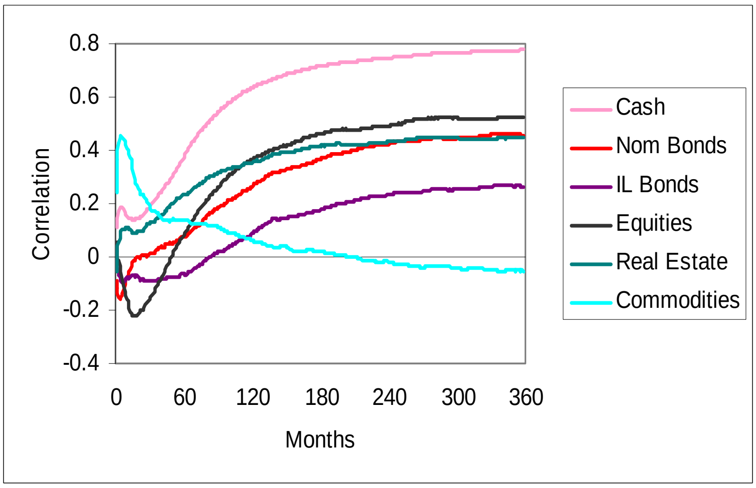


Figure 2

Correlations between asset returns and inflation depending on the investment horizon

December 1990–June 2009



Appendix 2

Table 8

Minimum shortfall probability portfolio, real return target 0%

January 1973–December 1990

Horizon	**2 years**	**5 years**	**10 years**	**30 years**
Min Shortfall Probability	10.8%	11.5%	9.0%	1.4%
Ann. Excess Return Volatility	1.9%	3.6%	5.1%	5.4%
Ann. Excess Return*	1.6%	1.9%	2.2%	2.2%
Cumulated Excess Return	3.2%	9.7%	21.8%	65.2%
Weights				
Cash	88%	81%	72%	64%
Nom Bonds	0%	0%	0%	0%
IL Bonds	6%	7%	11%	17%
Equities	1%	3%	7%	8%
Real Estate	0%	0%	0%	5%
Commodities	5%	9%	10%	6%

* Excess returns are measured over target.

Table 9

Minimum shortfall probability portfolio, real return target 1%

January 1973–December 1990

Horizon	**2 years**	**5 years**	**10 years**	**30 years**
Min Shortfall Probability	28.9%	23.7%	17.6%	5.8%
Ann. Excess Return Volatility	2.8%	7.1%	14.4%	14.9%
Ann. Excess Return	1.1%	2.3%	4.2%	4.3%
Cumulated Excess Return	2.2%	11.4%	42.4%	127.8%
Weights				
Cash	80%	50%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	1%	0%	0%	0%
Equities	9%	23%	55%	63%
Real Estate	0%	0%	0%	0%
Commodities	11%	27%	45%	37%

Table 10

Minimum shortfall probability portfolio, real return target 2%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	36.7%	30.0%	36.7%	11.4%
Ann. Excess Return Volatility	12.2%	13.1%	14.6%	15.1%
Ann. Excess Return	2.9%	3.1%	3.3%	3.3%
Cumulated Excess Return	5.9%	15.4%	33.0%	99.8%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	45%	47%	51%	59%
Real Estate	0%	0%	0%	0%
Commodities	55%	53%	49%	41%

Table 11

Minimum shortfall probability portfolio, real return target 3%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	40.9%	35.9%	30.7%	19.7%
Ann. Excess Return Volatility	14.1%	13.8%	15.3%	15.7%
Ann. Excess Return	2.3%	2.2%	2.4%	2.4%
Cumulated Excess Return	4.6%	11.2%	24.3%	73.4%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	33%	40%	44%	52%
Real Estate	0%	0%	0%	0%
Commodities	67%	60%	56%	48%

Table 12

Minimum shortfall probability portfolio, real return target 4%

January 1973–December 1990

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	44.0%	41.5%	37.8%	29.9%
Ann. Excess Return Volatility	21.3%	18.1%	18.1%	18.4%
Ann. Excess Return	2.3%	1.7%	1.8%	1.8%
Cumulated Excess Return	4.5%	8.6%	17.7%	53.1%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	0%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	0%	14%	23%	32%
Real Estate	0%	0%	0%	0%
Commodities	100%	86%	77%	68%

Table 13

Minimum shortfall probability portfolio, real return target 0%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	4.7%	3.2%	1.3%	0.0%
Ann. Excess Return Volatility	1.3%	3.0%	4.8%	5.1%
Ann. Excess Return	1.5%	2.4%	3.4%	3.2%
Cumulated Excess Return	3.0%	12.2%	33.8%	96.7%
Weights				
Cash	80%	41%	0%	0%
Nom Bonds	17%	48%	77%	73%
IL Bonds	0%	0%	0%	0%
Equities	0%	5%	10%	10%
Real Estate	1%	0%	0%	0%
Commodities	2%	6%	13%	17%

Table 14

Minimum shortfall probability portfolio, real return target 1%

December 1990–June 2009

Horizon	**2 years**	**5 years**	**10 years**	**30 years**
Min Shortfall Probability	16.0%	9.1%	5.8%	0.8%
Ann. Excess Return Volatility	4.5%	4.5%	4.8%	5.3%
Ann. Excess Return	3.2%	2.7%	2.4%	2.3%
Cumulated Excess Return	6.3%	13.3%	24.1%	70.2%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	76%	78%	76%	69%
IL Bonds	0%	0%	0%	0%
Equities	17%	13%	10%	10%
Real Estate	0%	0%	0%	0%
Commodities	7%	10%	14%	21%

Table 15

Minimum shortfall probability portfolio, real return target 2%

December 1990–June 2009

Horizon	**2 years**	**5 years**	**10 years**	**30 years**
Min Shortfall Probability	24.7%	20.6%	17.4%	7.5%
Ann. Excess Return Volatility	4.5%	4.6%	5.1%	6.4%
Ann. Excess Return	2.2%	1.7%	1.5%	1.7%
Cumulated Excess Return	4.4%	8.4%	15.1%	50.4%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	75%	76%	72%	60%
IL Bonds	0%	0%	0%	0%
Equities	18%	13%	9%	9%
Real Estate	0%	0%	0%	0%
Commodities	7%	11%	19%	30%

Table 16

Minimum shortfall probability portfolio, real return target 3%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	35.4%	36.4%	34.1%	18.8%
Ann. Excess Return Volatility	4.7%	5.1%	9.9%	18.9%
Ann. Excess Return	1.2%	0.8%	1.3%	3.0%
Cumulated Excess Return	2.5%	3.9%	12.8%	91.5%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	73%	69%	42%	0%
IL Bonds	0%	0%	0%	0%
Equities	20%	11%	0%	0%
Real Estate	1%	5%	7%	0%
Commodities	5%	15%	51%	100%

Table 17

Minimum shortfall probability portfolio, real return target 4%

December 1990–June 2009

Horizon	2 years	5 years	10 years	30 years
Min Shortfall Probability	44.9%	45.9%	41.3%	27.6%
Ann. Excess Return Volatility	12.5%	16.3%	18.3%	18.9%
Ann. Excess Return	1.1%	0.7%	1.3%	2.1%
Cumulated Excess Return	2.3%	3.7%	12.8%	61.6%
Weights				
Cash	0%	0%	0%	0%
Nom Bonds	26%	0%	0%	0%
IL Bonds	0%	0%	0%	0%
Equities	33%	0%	0%	0%
Real Estate	40%	54%	0%	0%
Commodities	0%	46%	100%	100%

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The role of SDR-denominated securities in official and private portfolios

George Hoguet and Solomon Tadesse[1]

1. Introduction

This paper examines the role that securities denominated in special drawing rights (SDRs) could play in the management of large institutional portfolios. We find such securities could reduce portfolio variance and could provide a convenient method of diversification. While a nascent market for SDR-denominated bonds began in the early 1980s, it did not develop and remained very small relative to global bond issuance. Yet China's recent call to expand the use of the SDR could provide the impetus for a renewed effort to use the SDR as a unit of account for short-term deposits and fixed income obligations. China's initiative has been supported by Russia and Brazil, among others.

In light of the comments by some of their sponsor governments, sovereign wealth funds (SWFs) and highly diversified monetary authorities (central banks that acquire risk assets) are well-positioned to promote the development of a market for certificates of deposit and bonds denominated in SDRs. While several technical issues like liquidity provision remain to be resolved, the evolution of the European Currency Unit (ECU) bond market provides some evidence that a market in SDR-denominated bonds could develop. Although ECU-denominated bonds at their peak never accounted for more than 10 percent of the issuance of all international bonds (Dammers and McCauley 2006), as with SDRs, both a private and official market for ECUs existed.

An investor can synthetically replicate the weights of an SDR-denominated bond, but a security denominated in SDRs is self-rebalancing and is likely to minimize rebalancing costs. Additional research, particularly on the coordination problem (which limits liquidity) and operational issues, including settlement, can facilitate the development of an SDR-denominated bond market. Williamson (2009a) suggests that greater private use of the SDR could possibly facilitate greater official use, including the pegging of currencies to the SDR rather than to a basket of currencies or to some bilateral exchange rate.

It is important for investors to understand the distinction between "private" SDRs, in which the SDR serves as a unit of account, and "official" SDRs, which are official reserve assets. In this paper, we focus on the use of the private SDR as a unit of account. However, as the private SDR relies on the portfolio composition and value of the official SDR, we will first briefly discuss the official SDR.

[1] Hoguet is Managing Director, State Street Global Advisors, One Lincoln Street, Boston, MA 02110, tel: +1-617-664-2487, fax: +1-617-664-5960, e-mail: George_Hoguet@SSgA.com. Tadesse is Vice President, State Street Global Advisors, One Lincoln Street, Boston, MA 02110, tel: +1-617-664-1539, fax: +1-617-664-5960, e-mail: Solomon_Tadesse@SSgA.com. The authors thank Warren Coats and Mark Hooker for their helpful comments. The views expressed in this paper are those of the authors only, and should not be interpreted as reflecting the views of State Street Global Advisors.

2. Background

The most severe financial crisis since the Great Depression has led to massive global policy interventions and renewed calls to re-examine the global financial architecture. In addition, some holders of U.S. Treasury bills and bonds have become concerned about potential losses on their holdings. For example, in March 2009 Chinese Premier Wen Jiabao stated that: "We have lent a huge amount of money to the U.S. Of course we are concerned about the safety of our assets. To be honest, I am definitely a little bit worried."[2] He reiterated his concerns the following month at the Boao Forum for Asia. Further, in March 2009, the People's Bank of China (PBOC) posted a speech by Governor Zhou Xiaochuan entitled "Reform the International Monetary System." Among other initiatives, Zhou emphasized:

1. Reforming the international monetary system and creating an international reserve currency that "is disconnected from individual nations and is able to remain stable in the long run, thus removing the inherent deficiencies caused by using credit-based national currencies" (p.1, Sec II).
2. Entrusting part of member countries' reserves to the centralized management of the International Monetary Fund (IMF).
3. Expanding the use of the IMF's SDRs, including as a means of payment, currency of denomination of securities, commodity denomination and reserve currency.
4. Expanding the basket of currencies forming the basis for SDR valuation to include currencies of all major economies, and including GDP as a factor in currency selection for the SDR.

China currently holds approximately $2.2 trillion in gold and foreign exchange reserves and about $800 billion in U.S. Treasuries, as well as an estimated $500 billion in U.S. Agency debt. The rapid build-up in reserves, however, is not limited to China. Williamson (2009b) points out that global foreign exchange reserves from 1975 to 2008 grew roughly 2.2 times as fast as global nominal GDP, and 1.10 times as fast as world trade. According to the June 2009 IMF COFER data, the dollar's share of allocated foreign exchange reserves now stands at about 63 percent, versus 66 percent in 2002–2003. As of the same date, for emerging and developed countries the dollar comprises about 48 percent of allocated foreign exchange reserves, versus 54 percent at the end of 2003 (IMF 2009a).

In April 2009, the G-20 countries agreed to a roughly $250 billion allocation of official SDRs, or newly created reserves. On August 28, 2009, members of the IMF that were participants in the Special Drawing Rights Department (currently all 186 members) duly received their official SDR allocations. In addition, the Fourth Amendment to the IMF Articles of Agreement provided a one-time allocation of SDRs equal to approximately $33 billion. This allocation had been delayed, but was finally acted upon in 2009. After the special and general allocations the cumulative total of SDR allocations totals roughly SDR 204 billion, or about $316 billion. SDR assets will thus represent roughly 4 percent of global foreign exchange as of the 2009:Q3. The United States, with an allocation of about 17 percent, is currently the largest official holder of SDRs followed by Japan at 6 percent (IMF 2009b).

Furthermore, on July 1, 2009, the IMF Executive Board approved a framework for the issuance of notes to member countries and their central banks. The principal of the notes will be denominated in SDRs, the Fund's unit of account. The notes will be tradable in the official sector, which includes all IMF members, their central banks, and 15 "prescribed holders" of

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[2] Wines, Michael, Keith Bradsher, and Mark Landler, "China's Leader Says He is 'Worried' Over U.S. Treasuries," New York Times, March 14, 2009, A1,www.nytimes.com/2009/03/14/world/asia/14china.html?_r=1&scp=1&sq=china's%20leader%20says&st=cse.

SDRs, which include four regional central banks, three intergovernmental monetary agencies, and eight development institutions. A permitted purchaser of the notes could also include a member's "fiscal agency" (2009b). Because in some cases the demarcation between a monetary authority and a sovereign wealth fund is not explicit, it is possible that a sovereign wealth fund will in fact purchase some of these notes. For example, some highly diversified monetary agencies, such as the Saudi Arabian Monetary Authority (SAMA) and the Hong Kong Monetary Authority (HKMA), while not explicitly SWFs, may possibly hold a portion of their portfolios in risk assets. The notes will have an initial maturity of three months, extendable to a maximum maturity of five years, and interest payments will be made quarterly. China has executed a note purchase agreement for up to $50 billion, and Brazil, India and Russia have indicated their intention to purchase up to $10 billion each. Per the IMF's website, as of October 23, 2009, the Fund has entered into SDR-denominated borrowings from several countries and placed some SDR-denominated notes with China.

In the past 18 months, several prominent economists, including Joseph Stiglitz and Robert Mundell, have suggested that the SDR should play a greater role in the international system. Stiglitz has argued that the proposal for the SDR as a new global reserve currency is "a good idea for many reasons." He has also suggested that replacing the dollar with a new global currency is "very much in the long-term interest of the [United States]" (Stiglitz 2009: p. 1). Mundell (2009) has advocated a large official SDR issuance, while Bergsten (2009) has proposed annual official SDR issues. Eichengreen (2009a) points out that "attempts in the 1980s to promote a private SDR bond market were not particularly successful, in part because "[t]he coordination problem – that many prospective issuers were reluctant to issue SDR-denominated claims in the absence of evidence that others were prepared to likewise – was substantial" (p. 11). He suggests that the IMF could possibly serve as a market-maker for SDR-denominated bonds.

Because the private SDR relies on the portfolio constituency and the value of the official SDR, in the next section we briefly examine the history and properties of the official SDR.

3. The definition and role of Special Drawing Rights

Public SDRs

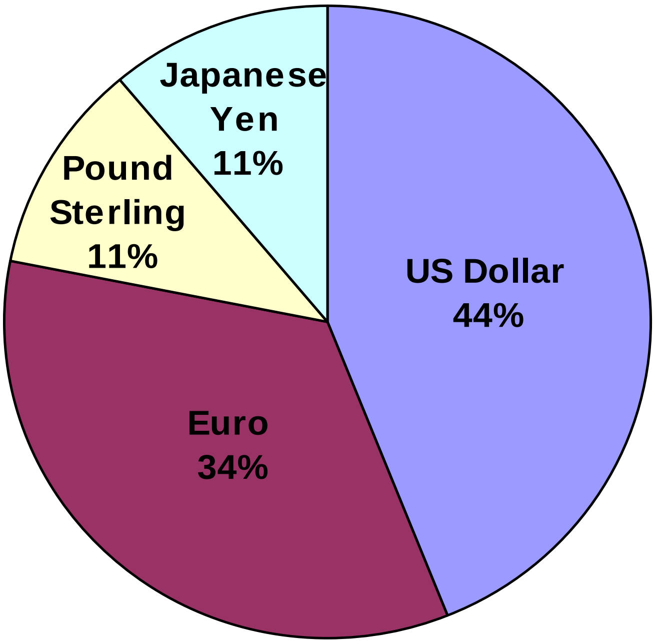
SDRs are an arcane and complex topic.[3] According to the IMF's website, "the SDR is an international reserve asset, created by the IMF in 1969 to supplement the existing official reserves of member countries" (IMF 2009c: p. 1). With the agreement of the Executive Board, the Fund periodically allocates official SDRs to member countries in proportion to their IMF quotas. A basket of currencies comprise the SDR. The Fund and the Board review the currencies included in the SDR every five years. Presently, the currencies included in the SDR are those currencies issued by Fund members whose exports of goods and services during the five-year period ending 12 months before the effective review date had the largest value and that are "*freely useable*" (IMF 2005: p. 6). SDR weights are currently based on the value of exports and the amount of reserves denominated in the respective currencies.

At the latest review in 2005, the Fund established the weights for the SDR as illustrated in Figure 1 below.

[3] For a fuller discussion, see the IMF's Articles of Agreement (IMF 1990) as well as Clark and Polak (2002). Margaret Garritson de Vries also wrote about SDRs for the IMF in 1976 and 1985, and in 1987 the IMF issued an occasional paper devoted to the topic, "The Role of the SDR in the International Monetary System."

Figure 1

Composition of the Special Drawing Right (SDR)



Five-year review period beginning January 1, 2006.

Source: International Monetary Fund.

At each five-year review, the IMF Board establishes the initial weights of the currencies in the IMF basket, but *during* the five-year period the weights change on a daily basis as a function of movements in exchange rates in the constituent currencies. For example, appreciating currencies gain a larger share of the basket and depreciating currencies a smaller share. Since 1969 the constituent currencies in the SDR and their weights have varied from time to time during sequential five-year periods. The next official reconstitution of the SDR will be in 2010 and will take effect in 2011.

SDRs are an official reserve asset and bear interest which is based on the weighted average interest rate of the representative short-term money market rates of the SDR basket currencies. As of October 23, 2009 the interest rate on SDRs is 0.27 percent. Coats (1990) suggests that the "[t]he official SDR can be thought of as an interest-bearing security that has the special quality that it can be transferred like a currency to settle obligations" (p. 979). Under XXII in the IMF's Articles of Agreement, "each participant undertakes to participate with the Fund with the objective of making the SDR the principal reserve asset in the international monetary system" (IMF 1990). In this context, Governor Zhou's statement may be seen as advocating that the international community implement already agreed upon – if not yet implemented – objectives.

The official SDR is neither a currency unit nor a claim on the IMF. Rather, it is a potential claim on the freely useable currencies of IMF members. The Fund and the Board can agree to create official SDRs, but only with 85 percent of the shareholders approving this creation. Therefore, the United States, with its roughly 16.8 percent vote in the IMF, exercises a de facto veto on the creation of official SDRs.

Up until 2009 and the current allocation, the issuance of official SDRs has been modest, with the IMF having issued only SDR 21.4 billion in official SDRs. This total represented around

0.3 percent of current world reserves. The recent allocation of SDRs will raise this percentage to roughly 4 percent but, as Williamson (2009a) points out, at the end of 1972 SDRs accounted for 9.5 percent of the world's stock of non-gold reserve assets.

Currently the Fund has voluntary arrangements to buy and sell SDRs with 13 IMF member participants and one prescribed holder (market-maker). However, the IMF (2009d) reports that as of May 2009 the buying and selling capacity of the market-maker, a prescribed institution, was less than SDR 3 billion.

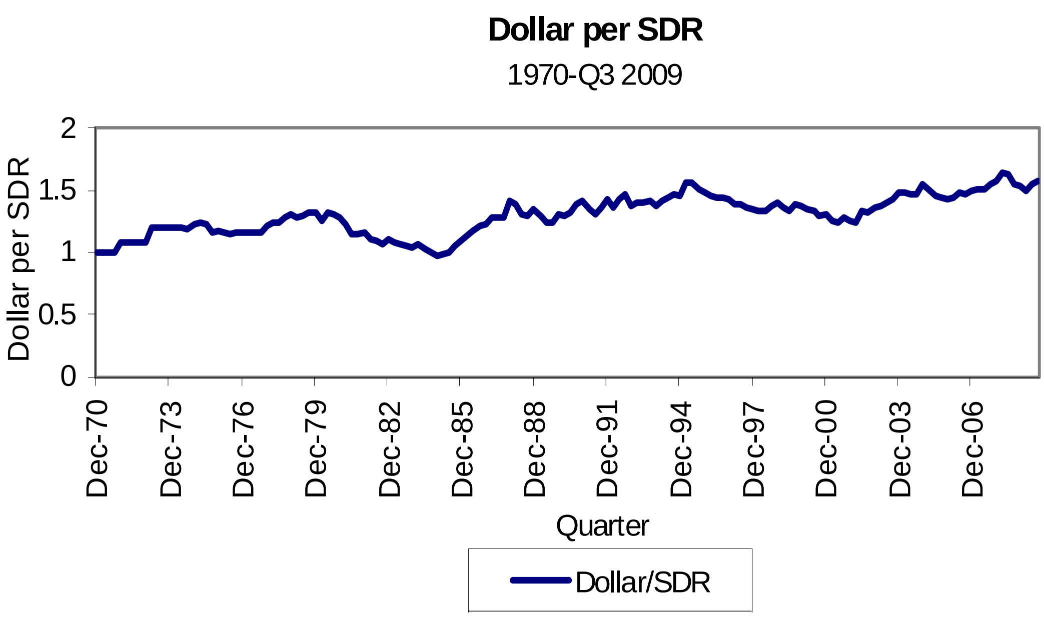
Private SDRs

The IMF created SDRs in 1969 as an international reserve asset meant to support the Bretton Woods fixed exchange rate system by supplementing the existing reserves of member countries. Since 1972 the Fund has also used the SDR as its basic unit of account. Over time, the SDR has found a number of applications outside the IMF official framework. Current accounting uses of the SDR include:

- Transit fees in the Suez Canal are denominated in SDRs.
- Some airlines now designate charges for overweight baggage in SDRs.
- A number of international organizations maintain their accounts in SDRs or accounting units linked to the SDR. The Arab Monetary Fund, for instance, maintains its accounts in Arab Accounting Dinars (AAD), which are linked to the SDR.
- In the past, some countries, such as Latvia, have pegged their currency to the SDR.

Coats (1990) suggests that the SDR's attractiveness as a unit of account for private sector use derives from the stability of its value relative to values of alternative units. By virtue of their currency composition, SDR-denominated securities can serve as a diversification vehicle and as a partial hedge against currency risk. For example, at the time the first SDR was created in 1969, 1 SDR was equivalent to 0.888 grams of fine gold and/or $1.00. However, as of Sept 30, 2009, 1 SDR is equivalent to roughly $1.58. Figure 2 outlines the dollar/SDR parity since 1970.

Figure 2



Source: Bloomberg.

Although its uses outside the official circle have been modest so far, information about the SDR is available on some commercial platforms. For example, the SDR has been assigned the ISO 4217 currency code XDR, and it is quoted on Bloomberg regularly (SDR Currency <GO>). The IMF determines the SDR's interest rate on a weekly basis. The IMF also updates the SDR every 20 minutes, although it does not quote an SDR rate on Saturday or Sunday, EST.

Sobol (1981) notes that private markets in SDR-denominated instruments first emerged in 1975 and included commercial bank deposits, syndicated credits, certificates of deposit (CDs), floating rate CDs, Eurobonds and floating rate notes. Interest grew in the SDR as a unit of account as the dollar weakened in 1977-1978. Sweden issued an SDR-denominated credit in 1981, and several other borrowers followed its lead. Sobol notes that JP Morgan offered demand deposits in SDRs and the ability to debit and credit accounts directly in SDRs without having first to convert the SDR into its component parts. The SDR thus served as limited means of payment. Banks also developed a secondary market in SDR-denominated CDs, and Euroclear and Cedel developed systems to accept assets in SDRs. Madura (1982) notes that banks in the United States, the United Kingdom and Japan issued SDR-denominated CDs.

No comprehensive data base of SDR-denominated bonds outstanding appears to exist, but Mingqi (2006) reports that, according to Bank for International Settlements (BIS) data, roughly SDR 594 million in bonds were issued by 13 different issuers in the 1970s and the 1980s. The private use of the SDR has not grown, even though Medeiros and Nocera (1988) observed more than 20 years ago that "the variance of the SDR exchange rate will always be lower than the weighted average of the variances of the component currencies in the basket" (p. 9). They suggested that SDR-denominated securities had a role to play in the construction of efficient portfolios.

Over time, the SDR has evolved in the composition of its basket of currencies, and the methodologies for setting its exchange value and interest rates have been refined. Likewise, the last couple of decades have witnessed substantial structural changes in the global economy due to regional economic integration, the creation of the euro, the globalization of trade and finance, and the emergence of China as an economic power. In light of the tremendous changes the world has undergone, it would be instructive to reevaluate if the desirable qualities of the SDR are as relevant today as they were in the past, a consideration that we take up in the following section. Despite the changing times, the SDR's potential to enhance the stability of portfolios' investment returns appears to be an enduring attribute.

4. The risk-return properties of SDR-denominated investments

As a basket currency, the SDR derives its exchange value from the exchange rates of its constituent currencies; currently the SDR is comprised of the U.S. dollar, the euro, the pound sterling and the Japanese yen. In this section, we present an analysis of the investment properties of the SDR, emphasizing its stability, to illustrate the potential benefits of SDR-denominated fixed income instruments in comparison to instruments denominated in the individual currencies within the SDR basket.

The analysis covers the period from January 1999, when the euro was introduced, through June 2009. Over this period, we consider hypothetical investments in short-term fixed income instruments, from the perspective of investors that use each of the component securities and the SDR as the base currency. The analysis evaluates the alternatives of investing in short-term instruments denominated in the local base currency, in the three other component currencies, and in the SDR itself. The total nominal returns to an investment strategy would consist of interest income accrued in the investment currency in which the instrument is denominated and exchange rate gains or losses in converting between the base currencies and investing currency. For each investor of a given base currency, the streams of total returns of each of the investment strategies are simulated and contrasted against each other, in terms of average returns, volatility of returns and reward per unit of risk.

The bilateral exchange rates used are monthly closing rates as reported by Reuters, obtained from Datastream. The investment instruments are assumed to be short-term government securities that currently constitute the SDR interest rate basket as well as an instrument denominated in SDRs which pays the SDR market interest rate.[4] We use market yields on three-month U.S. Treasury bills, three-month U.K. Treasury bills, the three-month Eurepo rate and three-month Japanese Financing bills, all obtained from Datastream. SDR interest rates are determined weekly as a weighted average of the yields on the securities in the SDR basket. The data on market interest rates on SDR are available on a monthly frequency, reported at mid-month, in the IMF's International Finance Series via Datastream.

The principal attraction of an SDR-based financial instrument, as for any other basket instruments, lies in the superior stability of returns it is capable of providing due to the diversification of currency risk, as well as in its convenience and cost advantage. The SDR serves as a convenient risk diversifier, because as a basket its value is based on component instruments that are imperfectly correlated with each other. As the total returns from such investment include both the currency gains/losses and the interest income in the investment currency, the SDR risk-reduction property accrues from both currency returns and interest income.

From the exchange risk perspective, the SDR's stability primarily results from the fact that exchange rate shifts among the currencies in the SDR basket tend to offset one another, depending on the degree of correlations among the component currencies. To the extent that movements in the exchange rates of the currencies within the basket are not perfectly positively correlated, changes in the value of one currency could be partially offset by smaller changes, if the correlations are positive, or by opposite changes in the values of the remaining currencies. As movements in the exchange rates of currencies in the SDR basket are not perfectly correlated, the volatility of the SDR's value in terms of any one of the component currencies (e.g., the U.S dollar) would be less than the average of the volatilities of the values of all the other SDR-component currencies in terms of that particular currency. Similarly, the SDR's interest rate is a weighted average of the nominal interest rates of the constituent currencies.

Table 1 presents the pair-wise correlation coefficients of changes in the exchange rates, expressed in terms of the SDR and of the component currencies in the SDR basket, as well as the correlations in changes in the interest rates of the currencies.

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4 The SDR interest rate is set on a weekly basis as a weighted average of the market interest rates on short-term domestic financial instruments denominated in the four component currencies. The instruments are the three-month U.S. Treasury bill, the three-month U.K. Treasury bill, the three-month Eurepo rate and the three-month Japanese Discount bill.

Table 1

Correlation Coefficients

Panel A: Changes in Exchange Rates (in terms of the SDR)				
	U.S. Dollar	Euro	Pound Sterling	Japanese Yen
U.S. Dollar	1.0000			
Euro	-0.8255	1.0000		
Pound Sterling	-0.2233	0.0793	1.0000	
Japanese Yen	-0.0302	-0.3832	-0.3829	1.0000
Panel B: Interest Rate Changes				
	U.S. Dollar	Euro	Pound Sterling	Japanese Yen
U.S. Dollar	1.0000			
Euro	0.3706	1.0000		
Pound Sterling	0.3562	0.7047	1.0000	
Japanese Yen	0.1848	0.3419	0.2211	1.0000

The table exhibits correlation coefficients in exchange rates and interest rates over the period from January 1999 through June 2009. Panel A shows the pair-wise correlations among exchange rates where the exchange rates are stated as SDR per unit of the currency. Panel B displays pair-wise correlations among the nominal interest rates of short-term government securities in the respective currencies.

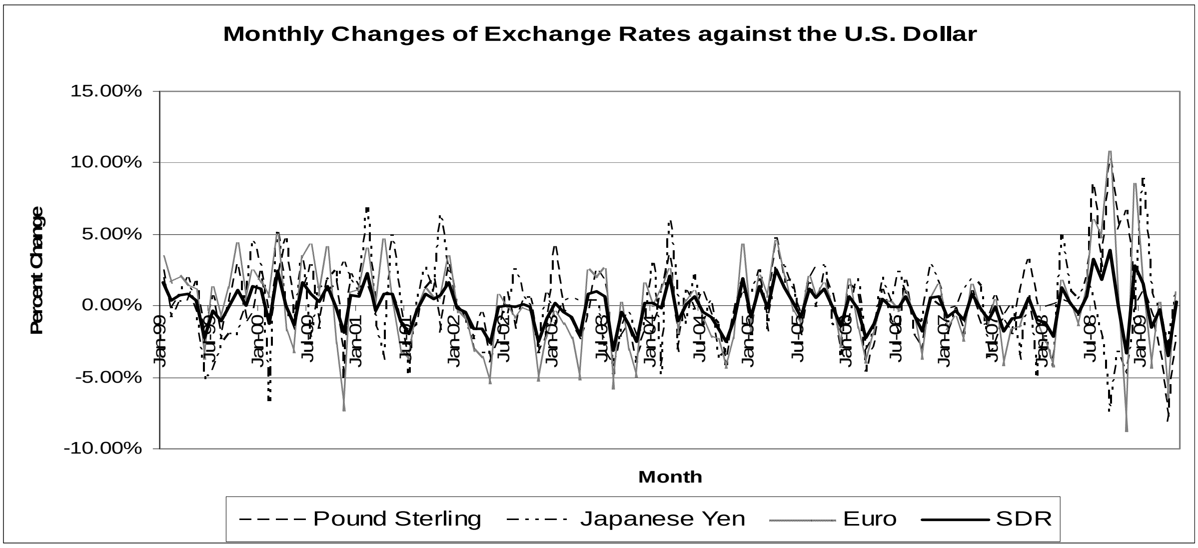
Source: Authors' calculations using Datastream.

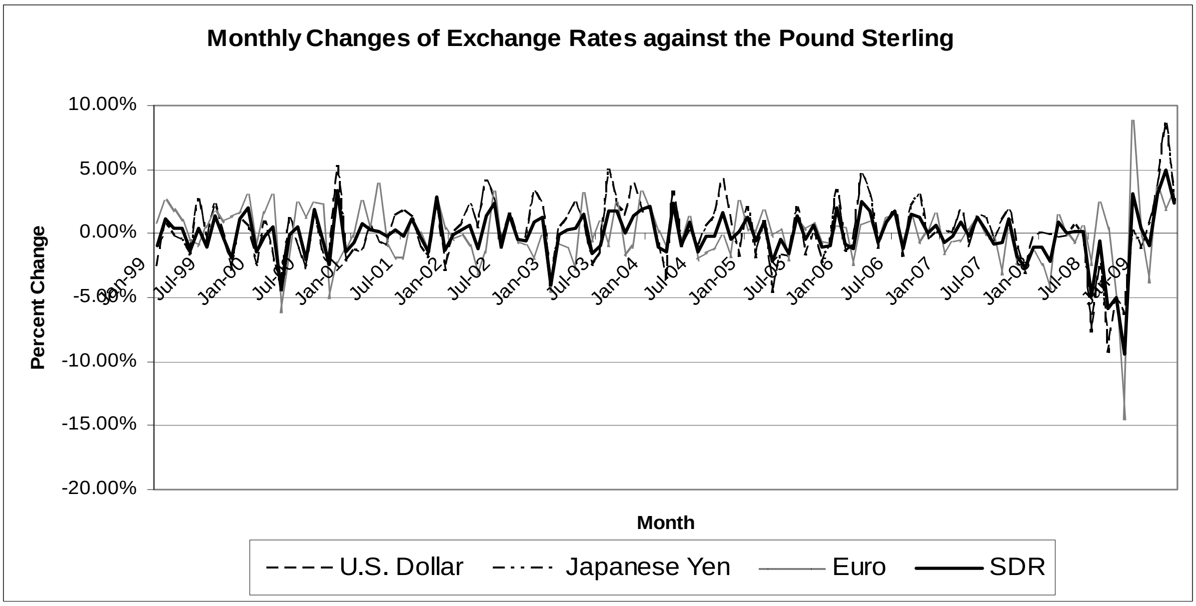
In Panel A, a large number of the pair-wise correlation coefficients of the shifts in the exchange rates are strongly negative, meaning that the exchange rates within the SDR basket are imperfectly and negatively correlated. For example, under the first column the U.S. dollar exhibits large negative correlations against each of the non-dollar currencies in the basket. As it is a dominant currency in the world economy, movements in the dollar are primarily determined by domestic events. The dollar's appreciation against the SDR would imply that the SDR would appreciate against at least one of the remaining currencies in the basket, resulting in a negative correlation. However, the degree of correlations varies from currency to currency, with the euro exhibiting the largest, and the yen the smallest. Given the imperfect and largely negative correlations in movements of exchange rates within the basket, changes in the value of one currency would be partially offset by opposite changes in the values of the remaining currencies, leaving the SDR as the least volatile in terms of its exchange rate value.

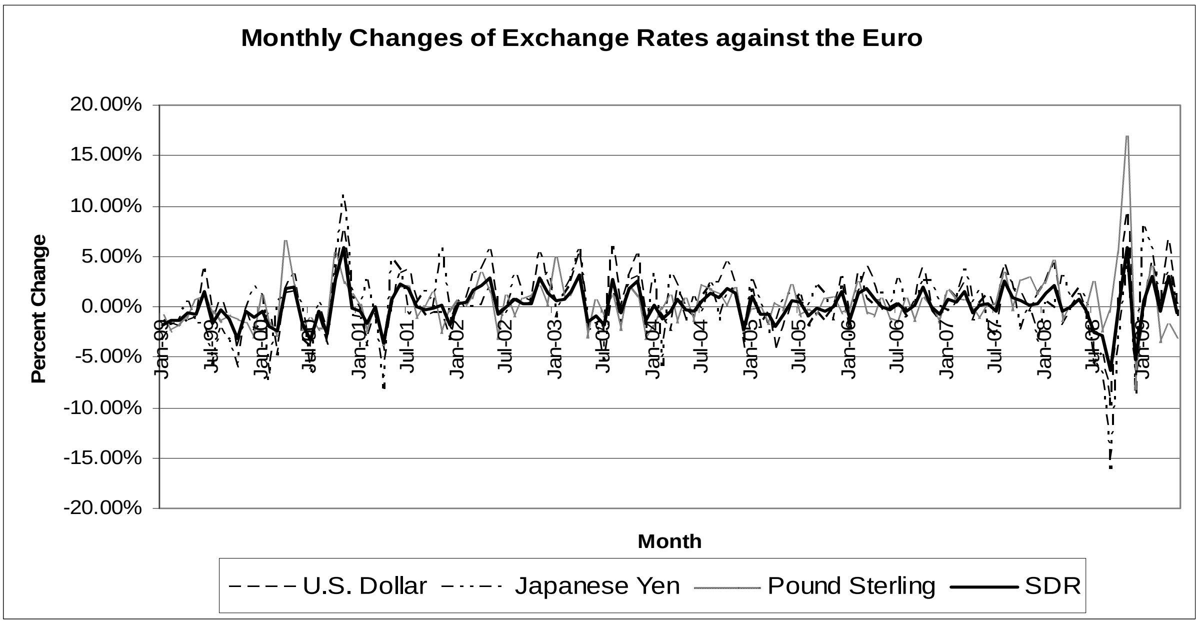
While exchange rate changes display largely negative correlations, panel B shows that changes in interest rates, while imperfectly correlated, tend to display positive co-movements. Though interest rates respond to a variety of unique domestic economic events, they also respond to a number of common factors. Thus, as Van Den Boogaerde (1984) argues, in terms of interest rates the risk-reduction property of the SDR primarily derives from the fact that its interest rate is an average of the constituent rates, and less so from offsetting opposite movements in interest rates within the SDR basket. The stability property of the SDR in terms of reducing exchange rate volatility is further illustrated in Figure 3, which plots the monthly changes in exchange rates of the basket's three remaining constituent currencies and the SDR in terms of the selected base currency.

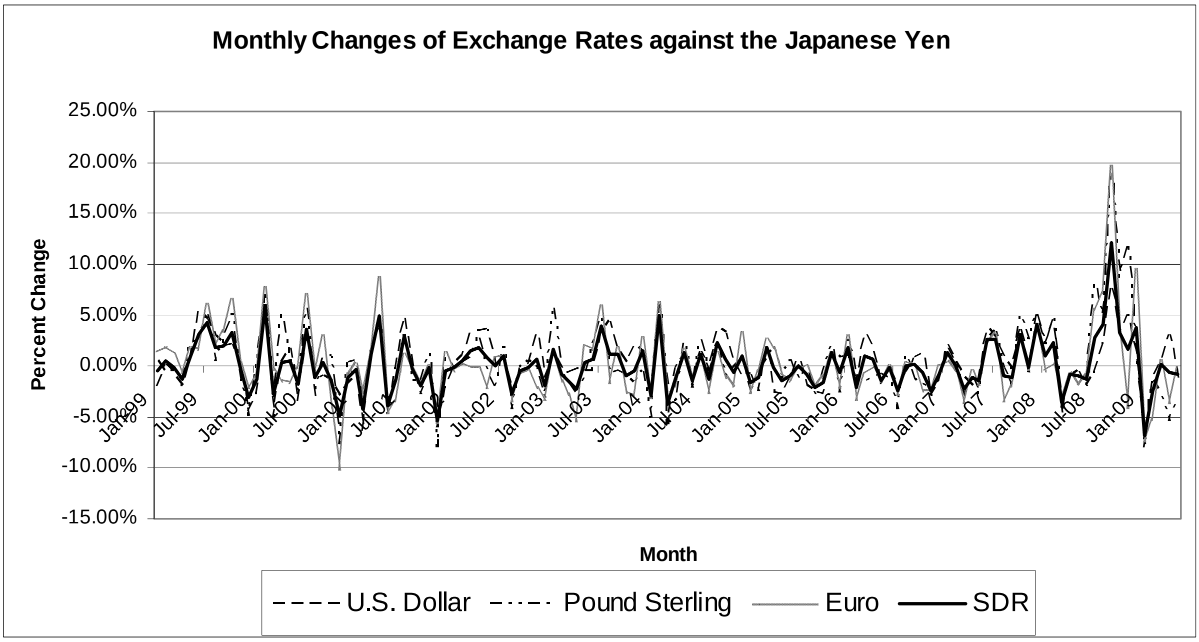
Figure 3

Monthly Changes in Bilateral Exchange Rates









Source: Authors' calculations using Datastream.

For example, taking the U.S. dollar as the base currency, the chart above shows that the SDR exchange rate, depicted by the thick line, displays the least variability over time compared to the constituent currencies. Similarly, expressing the exchange rates in terms of, respectively, the pound sterling, the Japanese yen and the euro as base currencies, the SDR's exchange rate is the least volatile.

4.1 Comparison of investments in single-currency denominations versus in the SDR

The total returns to an investor in a foreign-currency denominated instrument constitute the gain/loss in converting to the base currency from the investment currency and the interest income accruing from the investment currency. To further illustrate the relative performance of the SDR in terms of total return opportunities, the returns to an investor of a given base currency alternatively in the SDR and each of the remaining three currencies in the SDR basket were simulated and the results are presented in Table 2.

Table 2

Comparison of SDR and component currency denominated strategies

Panel A: Dollar-based Investor

	Mean	Standard Deviation
U.S Dollar	0.25%	0.15%
Pound Sterling	0.41%	2.53%
Japanese Yen	0.20%	2.80%
Euro	0.55%	2.99%
SDR	0.34%	1.34%

Correlation of Total Returns

	U.S Dollar	Pound Sterling	Japanese Yen	Euro	SDR
U.S Dollar	1.000				
Pound Sterling	-0.012	1.000			
Japanese Yen	-0.090	0.104	1.000		
Euro	-0.089	0.597	0.272	1.000	
SDR	-0.038	0.686	0.510	0.940	1.000

Panel B: Euro-based Investor

	Mean	Standard Deviation
Euro	0.26%	0.08%
U.S. Dollar	0.13%	3.00%
Pound Sterling	0.24%	2.44%
Japanese Yen	0.06%	3.57%
SDR	0.17%	1.78%

Correlation of Total Returns

	Euro	U.S. Dollar	Pound Sterling	Japanese Yen	SDR
Euro	1.000				
U.S. Dollar	0.141	1.000			
Pound Sterling	-0.058	0.577	1.000		
Japanese Yen	0.093	0.639	0.318	1.000	
SDR	0.125	0.967	0.648	0.774	1.000

Panel C: Pound Sterling-based Investor

	Mean	Standard Deviation
Pound Sterling	0.38%	0.10%
U.S. Dollar	0.28%	2.59%
Japanese Yen	0.23%	3.68%
Euro	0.51%	2.59%
SDR	0.34%	1.94%

Correlation of Total Returns

	Pound Sterling	U.S. Dollar	Japanese Yen	Euro	SDR
Pound Sterling	1.000				
U.S. Dollar	0.220	1.000			
Japanese Yen	0.121	0.640	1.000		
Euro	0.022	0.320	0.397	1.000	
SDR	0.172	0.859	0.788	0.719	1.000

Panel D: Japanese Yen-based Investor

	Mean	Standard Deviation
Japanese Yen	0.01%	0.02%
U.S. Dollar	0.14%	2.82%
Pound Sterling	0.29%	3.51%
Euro	0.37%	3.46%
SDR	0.21%	2.38%

Correlation of Total Returns

	Japanese Yen	U.S. Dollar	Pound Sterling	Euro	SDR
Japanese Yen	1.000				
U.S. Dollar	-0.020	1.000			
Pound Sterling	-0.151	0.710	1.000		
Euro	-0.027	0.573	0.751	1.000	
SDR	-0.051	0.881	0.872	0.881	1.000

The table shows monthly average returns, standard deviation and pair-wise correlation of total returns of instruments denominated in the SDR and component securities over the period from January 1999 through June 2009.

Source: Authors' calculations using Datastream.

As noted earlier, the investment instruments which constitute the SDR interest rate basket are short-term government fixed income securities. The holding period is assumed to be monthly, with the underlying instruments being rolled over at month's end. The total returns in currency *i*, in investing in an instrument denominated in currency *j*, are measured as the sum of the interest earnings accrued over the month's holding period and the exchange rate gain or loss at month's end. To simulate the return streams at the end of the month, the investments are rolled over by closing the old positions and taking equivalent positions at the current market yield.

Table 2 summarizes the average annualized total returns and the volatilities of returns, measured in terms of standard deviation, of the investment strategies for investors of different base currencies. As a general observation, across all the investment strategies for every investor type, investments denominated in the investor's base domestic currency tends to be by far the lowest risk investment strategy. The reason for this is that such strategies are immune to the substantial exchange rate risk exposure faced by the foreign-currency denominated strategies.

Excluding the strategies in the respective domestic base currencies, among the strategies that invest in foreign instruments the SDR provides the most stable return streams as reflected in lowest volatilities, regardless of the investor's currency base. If the desired investment objective is stable returns, as is the case in most long-term investment strategies, the SDR provides relatively stronger performance compared with investments in the single-currency individual instruments. In some cases, the SDR portfolio also generates the highest returns. Taking a dollar-based investor, for example, the SDR portfolio provides not only the lowest volatility, but also the highest yield, thus generating the highest rewards per unit of volatility among the foreign-denominated instruments.

Table 2's panel A summarizes the return and risk profile of the simulated strategies for a U.S. dollar-based investor. Between January 1999 and June 2009, the study period, such an investor would have had the highest returns investing in the euro, reflecting its strengthening, followed by the pound sterling, but both at a relatively higher risk. The lowest risk, as measured by the standard deviation, would have been realized by investing in SDR-denominated securities and assets. The U.S. dollar, accounting for the largest share in the SDR basket (as shown in Table 1 above), exhibits strongly negative correlations with the basket's non-dollar currencies, thus providing the SDR with a potentially natural hedge. The volatility of exchange rates from investing in any of the three non-dollar currencies would have been at least 1.6 times that of the SDR. In all, despite relatively lower average returns, the SDR instrument would have provided the highest return per unit of risk during this ten-year period. Of course, this period witnessed great volatility in financial markets, including the NASDAQ boom and bust, the worst financial crisis since the Great Depression, and the ensuing Great Recession.

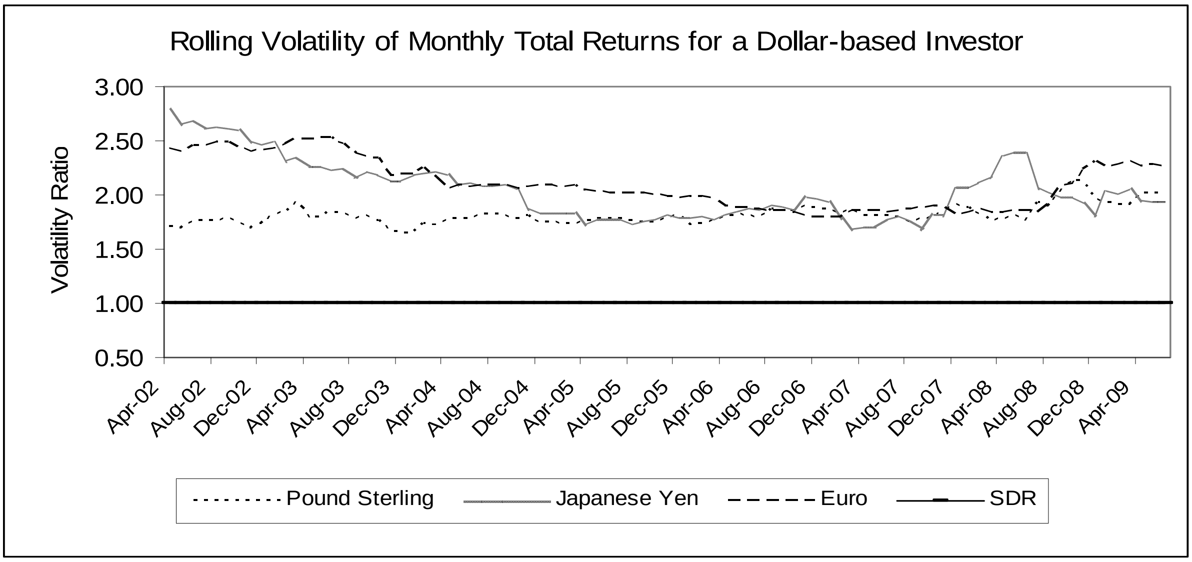
Panel B shows that, during this period, a euro holder investing in foreign-currency denominated instruments would have realized the highest returns by investing in the pound sterling (0.19 percent per month), closely followed by the SDR (0.17 percent per month). The investor, however, would have obtained the greatest stability in returns, as measured by standard deviations of total returns, from SDR-based instruments. For a euro-based investor, the yen instrument would have been both a low-yield and high-risk instrument.

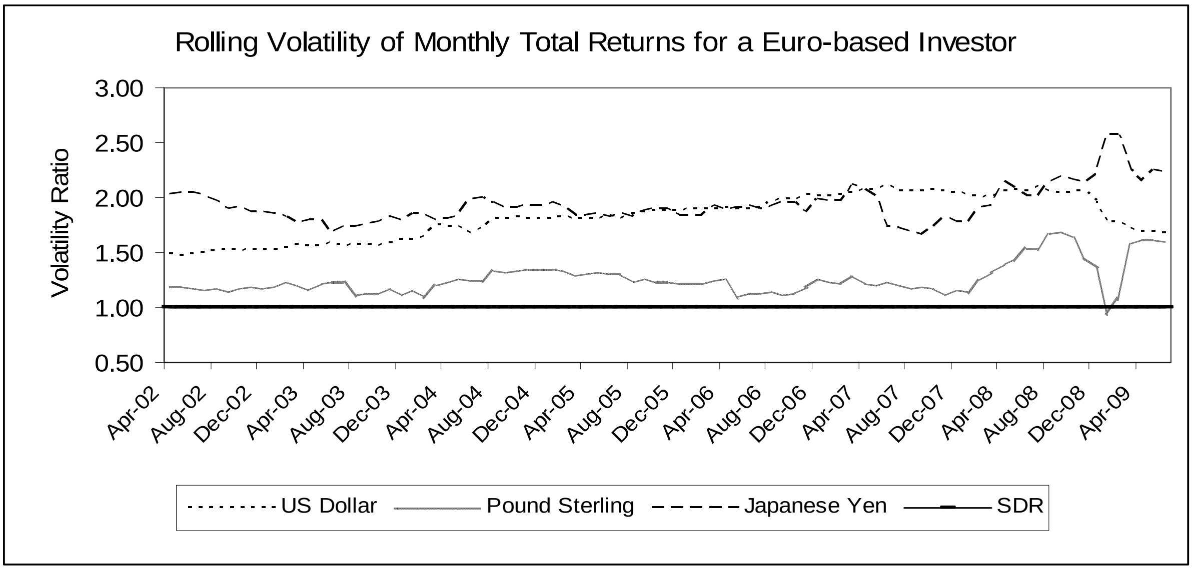
Panel C in Table 2 summarizes the risk-return profiles of the simulated investment alternatives from an investor using the pound sterling as her base currency. Again, a sterling holder investing in foreign-currency denominated instruments would achieve the highest level of stability by holding an SDR-based instrument. Furthermore, although not the highest-yielding, the SDR instrument would have generated above-average total returns. Reflecting the substantial appreciation of the euro during the study period, a euro-denominated instrument would provide the highest yield, but such an investment would have been more volatile. In all, despite lower yields, the risk-reduction advantage of the SDR instrument is such that it provides comparable yield per unit of risk as the high-yielding euro would have.

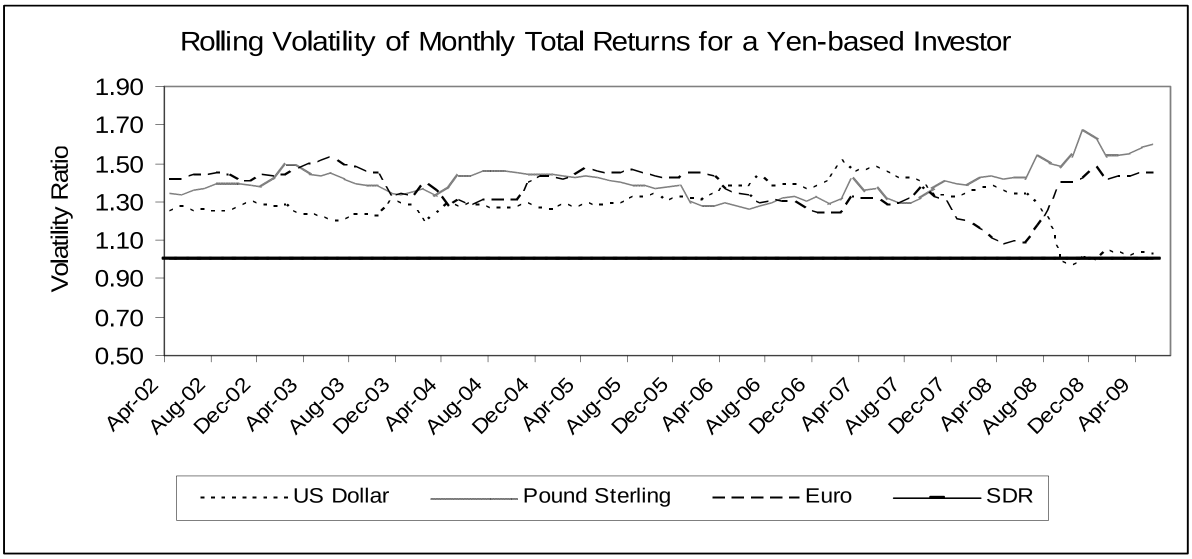
Panel D extends the comparison for a yen-based investor. The highest-yielding instrument would have been the euro and the pound sterling followed by the SDR and the dollar. In the risk category, however, the SDR instrument would have generated the least volatile total returns.

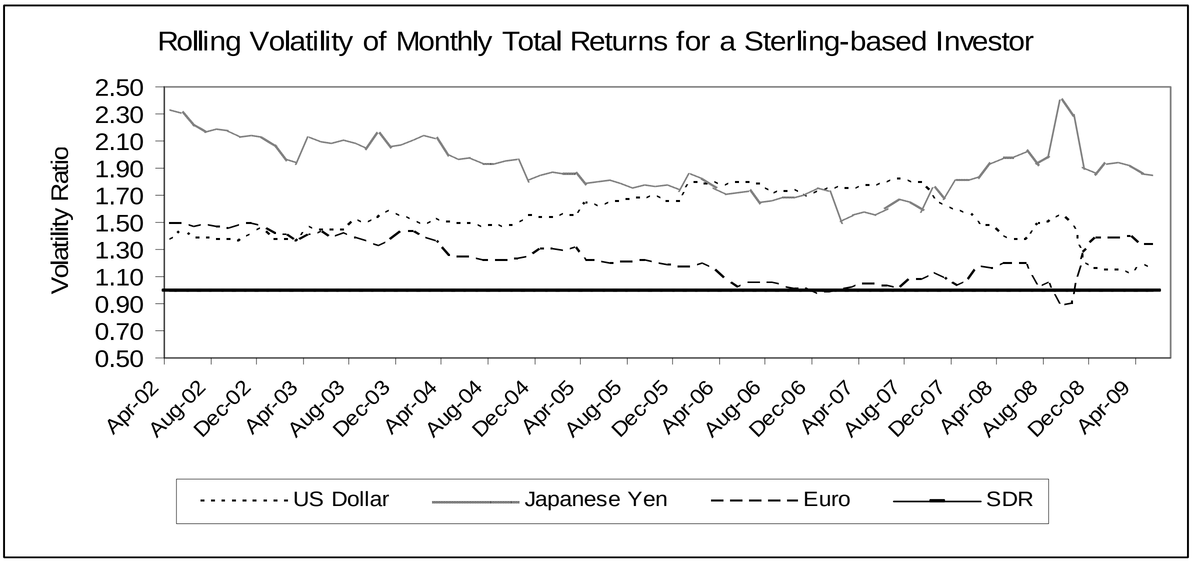
Figure 4 illustrates the risk-reduction potential of the SDR by plotting the total return volatilities of each of the three single-currency denominated instruments against that of the SDR. Volatilities, in standard deviation form, are computed over rolling 36-month windows and expressed as a ratio of the SDR's volatility. As an example, for a dollar-based investor, the volatility in total returns of the single-currency denominated instruments ranges between 1.6 (for the pound sterling) and 2.8 (for the Japanese yen) times the volatility of the SDR-denominated instrument, results that clearly indicate the SDR's potential risk-reduction properties. The results are qualitatively similar across all base currencies. From the euro-based investor's perspective, the SDR's volatility is the lowest, followed by that of the pound sterling, as it is for sterling and yen-based investors. For a sterling-based investor, however, while the SDR is the least volatile, during the study period the euro appears to provide comparable stability, perhaps reflecting the much closer economic integration within Europe as well as the shared co-movements of the euro and the pound sterling against the U.S. dollar, the basket's other major currency.

Figure 4









Source: Authors' calculations using Datastream.

In summary, irrespective of the base currency, the SDR has been the most stable among the foreign-currency denominated instruments and, by and large, has generated above-average total returns. If the objective is achieving stable investment returns, the SDR clearly wins as the strongest performer. However, the SDR may not generate the highest possible returns in some cases. During the period from January 1999 through June 2009, a yen-based investor would have realized substantially higher yields from the euro, despite the stability of returns an SDR instrument would have afforded.

An alternative convenient ranking that incorporates such tradeoffs between expected yield and low volatility might be to use the returns per unit of risk, as measured by the average returns of the investment divided by its volatility. Table 3 provides the average reward per unit of volatility for instruments based on the SDR and component currencies. The reward per risk is the highest along the diagonal for investments denominated in the respective base currencies. This finding is a reflection of the fact that such instruments are devoid of exchange rate risk exposure. For all investment alternatives other than the ones denominated in the domestic currency, investors would have substantially benefited from the SDR instrument. The table's last row shows that as an instrument the SDR ranked high, on reward per unit of risk basis (at least for dollar-based and euro-based investors), ranking in the top two across all strategies. From the yen- and sterling-based perspective, the SDR ranks second to the euro. Given that the SDR has been the lowest-risk instrument for sterling and yen-based investors, the euro's slight edge in those cases is attributable to the euro's higher yields due to its unprecedented appreciation in recent times. In sum, the foregoing descriptive statistics provide confirmation that the SDR could serve as a viable and superior vehicle, with a potential to generate above-average returns with the lowest variability.

Table 3

Reward-to-risk ratios

Base Currency	Dollar	Pound	Euro	Yen
Dollar	1.684	0.109	0.042	0.050
Pound	0.163	3.976	0.099	0.084
Euro	0.183	0.198	3.042	0.107
Yen	0.072	0.062	0.016	0.783
SDR	0.251	0.177	0.098	0.087

Source: Authors' calculations using Datastream.

4.2 Diversification: prepackaged versus customized portfolios

While some investors with the objective of improving expected returns may prefer to adopt a more active strategy of switching among their investments' currency denominations, the stated objectives of most investors oriented towards the long term might be a conservative strategy of preserving the stability of their investment returns. For such investors, including reserve managers and many institutional investors, the use of a prepackaged diversifying instrument such as an SDR would have an added advantage of convenience and low cost. As Dammers and McCauley (2006) note, the ready-made diversification through prepackaged portfolios could also prove advantageous to retail investors. Otherwise, for such investors, reasonable diversification would require both higher committed capital and buying a number of single-currency denominated instruments.

A counterargument against prepackaged portfolios as risk diversifiers is that in the set of potential portfolios of the constituent currencies that can be constructed, the ready-made portfolio (in this case, the SDR weights) may not represent the absolute minimum risk portfolio. It may as well be argued that if investors prefer to hold a portfolio of currencies as a hedge, they could customize one to their unique needs, reflecting their desired currency composition and unique constraints. With active currency markets, this can be accomplished through periodically rebalancing the hedge portfolio to tailor it to the desired investment objective.

It might be true that the SDR portfolio may not constitute the minimum variance portfolio even within the universe of current SDR constituent currencies, let alone within a larger set of currencies in which an investor may have interest. Nonetheless, despite the relative efficiency loss from the SDR basket, there are a number of practical advantages to a strategy of using the prepackaged SDR. First, customizing involves large transaction costs from the continuous rebalancing needed. Depending on the set of currencies in the customized portfolio and the degree of how active the investment strategy is, there could be substantial transaction costs incurred when moving away from a prepackaged portfolio denominated in the SDR to the customized one; such costs might not justify the potential efficiency gain. Moreover, the constituent currencies that denominate the SDR represent an overwhelmingly large amount of total global trade and investment transactions.

In addition, given that many other currencies are anchored through a peg and other mechanisms to these SDR basket currencies, any customized portfolio would more likely be a composition of the SDR currencies. Finally, within the set of the SDR currencies, the efficiency loss of using the SDR basket instead of a customized portfolio depends on the individual investor's base currency and constraints. Abstracting away from an investor's unique constraints, it can be shown that the SDR portfolio would be the closest to the efficient set compared to the constituent single-currency alternatives. In a similar study evaluating the SDR's role in reserve management, Medeiros and Nocera (1988) find that, depending on the base currency, the minimum variance portfolios, by and large, assign substantial weights to the SDR. It could, therefore, be argued that the efficiency loss of the prepackaged SDR portfolio relative to the customized version may not need to be as large in order to justify the complexity and higher costs of customizing it for many investors.

5. Pros and cons of SDR-denominated instruments

For the private SDR market to develop it must provide advantages to both issuers of and investors in the SDR. One of the SDR's advantages is that it is a prepackaged portfolio. The possible benefits from private SDR-denominated securities are delineated below.

5.1 Issuers

Exchange rate hedging: because the SDR is less volatile than its constituent currencies, it allows borrowers to hedge against mismatches of inflows and outflows. For example, a U.S.-based multinational corporation that sells in countries whose currencies help comprise the SDR basket could hedge its foreign receipts through the issuance of multicurrency liabilities.

Lower underwriting costs: the costs of underwriting an SDR-denominated bond may be lower than issuing four different currency-denominated bonds. In addition, privately placed SDR-denominated bonds could avoid the administrative and other costs associated with a public issue.

Potentially lower credit spreads: to the extent that a corporation's credit spreads differ by the currency market, the spread over an SDR-denominated instrument could possibly be lower than the weighted average of single-currency credit spreads.

Arbitrage opportunities: related to the above, an SDR bond, being a composite currency bond, could facilitate arbitrage opportunities from floating bonds denominated in the component currencies.

A framework to promote the SDR: if a holder of official SDRs would like to promote the development of the SDR as an alternative to other reserve currencies, it would be advantageous to promote private markets in SDR-denominated securities, perhaps as a primary issuer and as an investor, either at market rates or as an investor providing a modest subsidy.

Broadening the market: retail investors may find the SDR instruments a convenient way to diversify and could possibly open a new market for issues.

5.2 Investors

Prepackaged diversification: SDR-denominated securities provide official reserve holders and other investors exposed to particular currencies a convenient way to diversify the exchange risk exposure. Such securities may also incentivize a private investor with currency exposures in the SDR component currencies to reduce risk conveniently and at least cost.

Lower volatility: an investment's return and its realized Sharpe ratio can only be known after the fact. For private investors who may be subject to criticism by boards of directors and other parties for poor currency selection, SDR-denominated securities can reduce "currency regret."

Greater portfolio stability: as an overall portfolio diversifier, Chopra and Ziemba (1993) found that at the risk tolerance of a typical institutional investor, a mean-variance optimizer is most sensitive to estimates of means, then to variances and then to co-variances. In the estimation of optimizer inputs, errors in variance estimates are roughly twice as important as errors in co-variances. The lower variance of SDR securities can contribute to portfolio stability and facilitate the estimation of optimizer inputs.

An alternative reserve currency: the private use of the SDR may also provide opportunities to promote and brand the SDR as an alternative to other reserve currencies. A large official holder of reserves may wish to invest in private SDR-denominated bonds to facilitate greater official use of the SDR. In this sense, the IMF's recent issuance of SDR-denominated notes could possibly be viewed as a precursor to issuance by other official entities. While the notes are not publicly tradable (and are not newly issued public SDRs), because they can be traded among designated parties they appear to be more liquid than traditional private placements.

The potential challenges or "cons" of private SDR-denominated securities are delineated below. As a general observation, these issues are inevitable given the still-nascent state of a market for private securities denominated in SDRs, but will need to be confronted if a robust market is to develop.

Liquidity constraints: a public issuer of SDR-denominated bonds will have to pay a liquidity premium, and the first buyers of SDR instruments will enter an illiquid market. However, if several SWFs and/or other investors collectively agreed to purchase SDR instruments (say as a percentage of total cash and fixed income assets), then the liquidity of an SDR investment market could improve.

Uncertainty: the economic benefits the SDR will provide to issuers and investors are uncertain, at least at the outset of trying to create a more robust market in these securities. From an issuer's perspective, it is not clear that issuing SDR-denominated bonds will solve a pressing problem that cannot be dealt with otherwise.

The official sector's political will: it may take a subsidy from countries that wish to promote the SDR as a reserve currency to promote the beginning of a robust market in SDR-denominated securities, and relying on the political will of the official sector is an uncertain proposition from the standpoint of potential investors.

The absence of central bank participation: currently, central banks do not act as official issuers of SDRs and lenders of last resort for markets involving these instruments.

The current lack of a market: because the market for SDRs has yet to really develop, trade in these instruments lacks continuous pricing and market-making mechanisms. The complexity of the instrument and its reconstitution process make it an uncertain vehicle for many investors.

6. Expanding the market for SDR-denominated deposits and bonds

For a broad and deep market in publicly traded SDR-denominated securities to develop, at the very least the following need to be present:

- Multiple market-makers
- Continuous and transparent pricing

- Efficient financing mechanisms, including repurchase mechanisms
- Derivatives, such as swaps
- Hedging mechanisms
- Technology infrastructure, including Bloomberg/FactSet/Reuters
- An emergency lender, such as the IMF

Sobol (1981) points out the need for legal clarification on technical issues such as safeguard clauses (to cover, for example, recalculation of the SDR components or cessation of the SDR's use by the IMF; currencies to use in repaying interest and principal; and potential foreign exchange controls on SDR securities).

In short, several current impediments exist to the development of an SDR-denominated securities market, but historical precedent exists for such a market, and the emergence of large SWFs suggests that an additional class of investor is available to promote the use of SDR-denominated-instruments. Sovereign wealth funds and highly diversified monetary authorities could help develop a market by investing in SDR-denominated deposits and bonds, denominating their accounts in SDRs and potentially borrowing in SDRs. They could also make markets.

Conclusion

Dammers and McCauley (2006) point out that four basket currencies have been used since the second half of the twentieth century to denominate bonds: the European Unit of Account (EUA), the Eurco, the SDR and the ECU. They report that the first Eurobond was denominated in EUA, and observe that three out of the four basket currencies "turned out not to have much staying power" (p. 80).

The ECU was the most successful postwar basket currency, and preceded the euro by 20 years. The ECU served as a unit of account for target European food prices under the Common Agricultural Policy and served in 1979 as a unit of account for the currency area designated as the European Monetary System. As is the case currently with the SDR, there were at one point both private and public ECUs.

Allen (1986) observed that the ECU had many advantages over the SDR as a basket currency, among these being that:

1. the economy in the European Community (EC) was becoming increasingly integrated.
2. most EC currencies have been more stable against the ECU than against the SDR.
3. the SDR, with the dollar comprising about 40 percent of its value, is a relatively poor instrument for hedging or speculating against the dollar.

Eichengreen and Frankel (1996) refer to the SDR as the "Esperanto" of international currencies, contending that it lacks a natural constituency. Furthermore, Coats (2009) points out that most central bank transactions are not with other central banks but with the market. He argues that greater official use of the SDR will require greater linkages between the private and official SDR. Eichengreen (2009b) further points out that the SDR's limitations as an intervention and vehicle currency for foreign exchange transactions depends on the IMF's ability to issue SDRs rapidly in the case of a global liquidity shortage.

The IMF's recent increased issuance of SDR-denominated notes represents a new development in the SDR's evolution, and modestly blurs the distinction between private and official SDRs. Despite the recent global financial crisis and draw-down in some countries' reserves, many nations continue to accumulate reserves at a healthy pace. However, the IMF (Blanchard, Faruqee and Klyuev 2009) suggests that the current crisis may *raise* the precautionary demand for reserves. Thus currency selection will continue to be important for large official pools of capital. SDR-denominated securities can offer potential advantages to investors, but several structural impediments exist in developing a market. However, the desires expressed by some sponsors of large SWFs to diversify their currency exposures suggest that SWFs acting collectively could promote such a market.

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Optimal active portfolio management and relative performance drivers: theory and evidence

Roberto Violi[1]

This paper addresses the optimal active versus passive portfolio mix in a straightforward extension of the Treynor and Black (T-B) classic model. Such a model allows fund managers to select the mix of active and passive portfolio that maximizes the (active) Sharpe ratio performance indicator. The T-B model, here adapted and made operational as a tool for performance measurement, enables one to identify the sources of fund management performance (selectivity vs market-timing). In addition, the combination of active and passive risk exposures is estimated and fund manager choice is tested against the hypothesis of optimal (active) portfolio design.

The extended T-B model is applied to a sample of US dollar reserve management portfolios – owned by the ECB and managed by NCBs – invested in high-grade dollar denominated bonds. The best fund managers show statistically significant outperformance against the ECB-given benchmark. By far, market timing is the main driver. Positive (and statistically significant) selectivity appears to be very modest and relatively rare across fund managers. These results are not very surprising, in that low credit risk and highly liquid securities dominate portfolio selection, thus limiting the sources of profitable bond-picking activity. As far as the risk-return profile of the active portfolio is concerned, it appears that some of the best fund managers' outperformance is realised by shorting the active portfolio (with respect to the benchmark composition). Thus, portfolios that would be inefficient (eg negative excess return) if held long can be turned into positive-alpha yielding portfolios if shorted. The ability to select long-vs-short active portfolio can be seen as an additional source of fund manager's outperformance, beyond the skill in anticipating the return of the benchmark portfolio (market-timing contribution).

The estimated measure of fund managers' risk aversion turns out to be relatively high. This seems to be consistent with the fairly conservative risk-return profile of the benchmark portfolio. A relative measure of risk exposure (conditional Relative *VaR*) averaged across fund managers turns out to be in line with the actual risk budget limit assigned by the ECB. However, a fair amount of heterogeneity across fund managers is also found to be present. This is likely to signal a less-than-efficient use of their risk-budget by the fund managers – eg a deviation from the optimal level of relative risk accounted for by the model. At least in part, such variability might also be attributed to estimation errors. However, proper tests for *RVaR* statistics are sorely lacking in the risk management literature. Thus, the question remains open. This would warrant further investigation, which is left for future research.

1. Introduction

The performance of an investment portfolio that is diversified across multiple asset classes can be thought of as being driven by three distinct decisions that its manager makes:

(i) long-term (strategic or policy) asset allocations;

[1] Bank of Italy.

(ii) temporary adjustments (ie tactical) to these strategic allocations in response to current market conditions (market timing);

(iii) the choice of a particular set of holdings to implement the investment in each asset class (security selection).

The first of these performance components is commonly referred to as the passive portion of the portfolio, while the latter two collectively represent the active positions the manager adopts. Following the intuition of Treynor and Black (T-B,1973), I address the question of whether portfolio managers adopt an optimal active–passive risk allocation by seeing if they take full advantage of their alpha-generating capabilities or whether they “leave money on the table” by mixing their passive and active positions in a sub-optimal manner. I rely on a straightforward extension of the T-B model that allows to assess this issue in the context of the multi-asset class problem faced by fund managers who have the ability to make active decisions about broad market and sector exposures as well as for individual security positions.

The essential insight into the T-B analysis is that the optimal combination of the active portfolio – which results from the application of security analysis to identify a limited number of undervalued assets – and a passive benchmark portfolio is itself a straightforward portfolio optimization problem. That is, T-B treats the active and passive portions of an investment portfolio as two separate “assets” and then calculates the mix of those assets that maximizes the reward-to-variability (Sharpe) ratio. It is then demonstrated that the investment allocation assigned to the active portfolio strategy increases with the level of alpha it is expected to produce (ie the active “benefit”), but decreases with the degree of unsystematic risk it imposes on the investment process (ie the active “cost”). This is an important insight because it suggests that taking more active risk in a portfolio will not necessarily lead to an increase in total risk; if, for instance, the manager’s active investment is negatively correlated with the passive component (eg an effective short position in an industry or sector benchmark) the overall risk in the portfolio might actually decline.

Despite the importance of its insight, Kane *et al.* (2003) have noted that the T-B model has had a surprisingly low level of impact on the finance profession in the years since its publication. They attribute this neglect to the difficulty that investors have in forecasting active manager alphas with sufficient precision to use the T-B methodology as a means of establishing meaningful active and passive portfolio weights on an ex ante basis. However, this begs the question of whether the model offers a useful way of assessing ex post whether the proper active–passive allocation strategy was adopted by the portfolio manager. In other words, using the T-B model, did the fund manager construct an appropriate combination of active and passive exposures, given the alphas that were actually produced? In the subsequent sections, we explore the implications of the T-B model for designing optimal alpha-generating portfolio strategies and present an empirical analysis using a sample of US dollar reserve management portfolios owned by the European Central Bank (ECB) and managed on its behalf by national central banks (NCBs). These managed funds invest only in high grade government (or government-guaranteed) dollar denominated bonds.

In order to address the issue of whether fund managers deploy the various risks in their portfolio in an optimal manner, it is first necessary to split the returns they produce into their passive and active components. We follow a standard methodology and decompose the returns of a managed portfolio into their three fundamental components:

(i) strategic asset allocation policy (ie benchmark);

(ii) tactical allocation (ie market timing);

(iii) security selection.

Timing ability on the part of a fund manager is the ability to use superior information about the future realizations of common factors that affect bond market returns. Selectivity refers to the use of security-specific information. If common factors explain a significant part of the variance of bond returns, consistent with term structure studies such as Litterman and Scheinkman (1991), then a significant fraction of the potential (extra-) performance of bond funds might be attributed to timing. However, measuring the timing ability of bond funds is a subtle problem. Standard models of market timing ability rely on convexity in the relation between the fund's returns and the common factors. Unfortunately, the classical set-up does not control the non-linearity that are unrelated to bond fund managers' timing ability.[2]

Perhaps surprisingly, the amount of academic research on bond fund performance is small in comparison to the economic importance of bond markets and the size of managed funds invested in bonds. Large amounts of fixed-income assets are held in professionally managed portfolios, such as mutual funds, pension funds, trusts and insurance company accounts. Elton, Gruber and Blake (EGB, 1993, 1995) and Ferson, Henry and Kisgen (2006) study US bond mutual fund performance, concentrating on the funds' risk-adjusted returns. They find that the average performance is slightly negative after costs, and largely driven by funds' expenses. This might suggest that investors would be better off selecting low-cost passive funds, and EGB draw that conclusion. However, conceptually at least, performance may be decomposed into components, such as timing and selectivity. If investors place value on timing ability, for example a fund that can mitigate losses in down markets, they would be willing to pay for this insurance with lower average returns. Brown and Marshall (2001) develop an active style model and an attribution model for fixed income funds, isolating managers' bets on interest rates and spreads. Comer, Boney and Kelly (2009) study timing ability in a sample of 84 high-quality corporate bond funds, 1994-2003, using variations on Sharpe's (1992) style model. Aragon (2005) studies the timing ability of balanced funds for bond and stock indexes.

This paper is organised as follows. Section 2, I introduces a simple framework for identifying active vs passive asset allocation strategies; Section 3 illustrates the econometric implementation of performance decomposition, consistent with the active-passive asset allocation model; Section 4 discusses results obtained in the performance evaluation of NCBs' dollar reserve management.

2. Optimal active–passive asset allocation mix: a simple framework

Market-timing ability (timing) and security selection ability (selectivity) characterize active portfolio strategies. Like equity funds, bond funds engage in activities that may be viewed as selectivity or timing. Timing is closely related to asset allocation, where funds rebalance the portfolio among asset classes and cash. More specifically, managers may adjust the interest rate sensitivity (eg duration) of the portfolio to time changes in interest rates. They may vary the allocation to asset classes differing in credit risk or liquidity, and tune the portfolio's exposure to other economic factors. Since these activities relate to anticipating market-wide factors, they can be classified as market timing. Selectivity means picking good securities within the asset classes. Bond funds may attempt to predict issue-specific supply and demand or changes in credit risks associated with particular bond issues; funds can also attempt to exploit liquidity differences across individual bonds. We define the market timing component (tactical allocation) as the return that is achieved by over- or underweighting the benchmark asset in an attempt to increase returns or reduce risk. Security selection is the

[2] See Chen et al (2009) on the methodology that can be used to adjust for these potential biases.

excess return of the managed portfolio in a given asset class over the hypothetical return achievable by an investor who allocates resources in the benchmark according to the policy weights. Thus, portfolio's total return of fund manager, i, in any period t can be expressed as the sum of its passive and active components or:

$$R_{i,t}^{P} = R_{i,t}^{B} + R_{i,t}^{A} \tag{1}$$

To formalize the evaluation process, assume that the total actual return RP (subscripts are suppressed for convenience) contains a passive benchmark component RB and an active component RA representing, without loss of generality, the collective impact of the timing and selection decisions the manager makes. In this simplification, the active component can be written as

$$R_{i,t}^{A} = R_{i,t}^{P} - R_{i,t}^{B} \tag{2}$$

Notice that a fund can achieve exposure to its benchmark either by investing directly in the indices composing RB or indirectly through the formation of the active portfolio that generates RP. Consequently, we can always think of this active portfolio itself as being a (trivial) combination obtained by investing 100% in the assets that deliver RP and 0% in the assets that delivers RB. It is important to realize, though, that this "all active" portfolio will have only indirect exposure to the benchmark through RP. The crucial insight into this setting is that by rescaling the existing positions in RP and RB, the portfolio manager can construct an alternative portfolio that has the same monetary commitment to the benchmark assets but achieved with a different combination of asset class and security exposures. For example, consider the new return:

$$\underbrace{R_{i,t}^{P}\left[\lambda_{i,t}^{A}\right]}_{\substack{Total\\ \mathrm{Re}turn}} = \underbrace{\lambda_{i,t}^{A} R_{i,t}^{A}}_{\substack{Active\\ component}} + \underbrace{\left(1-\lambda_{i,t}^{A}\right) R_{i,t}^{B}}_{\substack{Benchmark\\ component}} \tag{3}$$

The rate of return of portfolio is obtained as a weighted average of active and benchmark portfolio. The portfolio implied by the weighted return in Eq. (3) is the de facto result of a swap transaction between the existing active portfolio and the benchmark allocation. If, for instance, the actual investment weights in the original portfolio and in the benchmark are identical (ie in the absence of market timing), the resulting swap portfolio has the same asset allocation weights, but its exposure to the benchmark is achieved through different securities than those contained in the active portfolio delivering R^P (see Appendix A1 for a detailed description). To implement such an exchange, a fund manager does not have to invest in asset classes with which it is not already familiar. To see this, consider the case of a portfolio comprising a single asset class, say European bonds.

Suppose that the manager initially holds 100 million euro of European bonds in portfolio R^P and then decides to revise his position by choosing an allocation of 110% in $R^P[\lambda^A = 1.1]$, while simultaneously shorting 10% of the benchmark. After this swap, the new portfolio will contain 110 million euro of this new bond position and will be short 10 million euro of the benchmark for European bonds (eg JPMorgan, GB EMU index). The net overall European bond position is still 100 million euro and so the exposure to this asset class is unaltered, although the actual securities held in the adjusted managed portfolio will be different. For the swap to be implementable in practice, it is important that it does not alter substantially the overall exposure to an asset class, since most (institutional) investors have policies limiting the variation of their actual portfolio weights around their benchmark weights (ie tactical ranges). Furthermore, the implementation of this swap does not necessarily require short selling the benchmark index; and it merely requires the sale of 10% of a combination of securities in a portfolio that is close to the index, while simultaneously investing proportionally the proceedings from this sale in those securities remaining in the portfolio. Hence, no new active management skills are required beyond those the manager already possesses. Since by construction (cf Eq. 1) total return R^P decomposes into the sum (R^B + R^A), Eq. (3) suggests that the ***passive component*** of the post-swap portfolio is

$$R^B(\lambda^A) = (1-\lambda^A)\, R^B \qquad (1')$$

while the active component is

$$R^A(\lambda^A) = \lambda^A\, R^A. \qquad (1'')$$

By extension, then, when $\lambda^A > 1$ the swap portfolio will have a higher emphasis on the active management component than did the manager's original portfolio. In other words, choosing how much emphasis is best placed on active risk is equivalent to choosing how to best rescale the existing managed portfolio R^P by swapping a fraction of it against the benchmark portfolio R^B. It is important to note that although the implementation of such a strategy requires the actual portfolio to be scalable, it does not require any additional alpha-generating abilities compared to the actual portfolio RP.

Once we know the fraction of wealth, $\lambda_{i,t}^A$, invested in the swap of asset *i* at time *t*, we could recover the implied return of active portfolio by inverting expression (3),

$$R_{i,t}^A = R_{i,t}^B + \frac{1}{\lambda_{i,t}^A}\left(R_{i,t}^P - R_{i,t}^B\right) \qquad (4)$$

With this background, the specific questions we would like to consider can be stated as follows:

(i) For any particular fund manager, is it possible to measure its commitment to active portfolio strategies implied by the portfolio currently held?

(ii) Can we say anything regarding the "optimality" of his/her commitment to active portfolio strategies?

(iii) To what extent are active management skills used in a way that add value through the market timing or security selection return components?

To answer questions concerning the optimality of the investment process, we need to identify fund manager's objectives (eg preferences) and then solve for the parameter λA that maximizes those preferences. We will assume that the investor is best served by the portfolio position – eg fraction of wealth, λ^A, invested in the active portfolio strategy – that maximizes risk-adjusted returns relative to the benchmark:

$$\underset{\lambda^A}{MAX}\ \mu_{P-B}\left[\lambda^A\right] - \frac{1}{2}\psi\sigma_{P-B}^2\left[\lambda^A\right] \qquad (5)$$

where

$$\mu_{P-B}\left[\lambda^A\right] \equiv E\left[R^P\left(\lambda^A\right) - R^B\right];\ \sigma_{P-B}^2\left[\lambda^A\right] \equiv VAR\left[R^P\left(\lambda^A\right) - R^B\right] \qquad (5')$$

and ψ represents the coefficient of investor's risk aversion, namely the marginal substitution rate between the return and the variance. Solving the first order condition of the optimization problem (5) yields the following expression for the optimal fraction of wealth invested in the active strategy

$$\lambda_*^A = \frac{1}{\psi}\frac{\mu_{A-B}}{\sigma_{A-B}^2} \qquad (6)$$

In solving the first order condition, it's useful to recall that Eq. (3) implies that the excess return over the benchmark is the product of the fraction of wealth, λA, invested in the active portfolio times the excess return over the benchmark earned by the active portfolio,

$$R_{i,t}^P\left[\lambda_{i,t}^A\right] - R_{i,t}^B = \lambda_{i,t}^A\left\{R_{i,t}^A - R_{i,t}^B\right\} \qquad (3')$$

Therefore, we can assert that first and second moment of the investor's excess return are related to those of the active portfolio strategy in the following way,

$$\begin{aligned}&\mu_{P-B}\left[\lambda^{\mathrm{A}}\right]=\lambda^{\mathrm{A}}\mu_{A-B}\equiv\lambda^{\mathrm{A}}E\left[R^{A}-R^{B}\right];\\&\sigma_{P-B}^{2}\left[\lambda^{\mathrm{A}}\right]=\left(\lambda^{\mathrm{A}}\right)^{2}\sigma_{A-B}^{2}\equiv\left(\lambda^{\mathrm{A}}\right)^{2}VAR\left[R^{A}-R^{B}\right]\end{aligned} \tag{6'}$$

The optimal fraction of wealth invested in the active portfolio in eq. (6) trades off relative risk (benchmark tracking error) and return (in excess of the benchmark), taking into account the tolerance for risk parameter, 1/ψ. With a risk tolerance parameter equal to 1, our representative fund manager would maximise the expected (log) excess return (deviation from the benchmark) of its active portfolio strategy. However, it is not clear from eq (6) whether we can measure the optimal choice of active portfolio share, as it is not based on observable variables. In Appendix A3 we show how to relate the optimal active portfolio choice to observable variables (eg benchmark and portfolio returns), as we obtain,

$$\begin{aligned}&\mu_{P-B}\left[\lambda^{\mathrm{A}}\right]=\lambda^{\mathrm{A}}\mu_{A-B}\equiv\lambda^{\mathrm{A}}E\left[R^{A}-R^{B}\right];\\&\sigma_{P-B}^{2}\left[\lambda^{\mathrm{A}}\right]=\left(\lambda^{\mathrm{A}}\right)^{2}\sigma_{A-B}^{2}\equiv\left(\lambda^{\mathrm{A}}\right)^{2}VAR\left[R^{A}-R^{B}\right]\end{aligned} \tag{6''}$$

where $\hat{\lambda}^{A}$ is the actual active portfolio share implied by the observed benchmark and portfolio return (see Eq. 12 below) and $\hat{\psi}$ is the estimated risk aversion parameter. Thus, the (estimated) optimal portfolio share would be obtained by adjusting the (estimated) implied active portfolio share with the risk-return profile of the excess returns over the benchmark – namely, estimated first and second moment taking risk aversion into account.

3. Decomposing fund manager relative performance

We can now provide some structure regarding the evolution over time of the active portfolio strategy. More specifically, we assume that the Active Portfolio return can be described by the following model:

$$R_t^{A}=\alpha_t+\beta_t\cdot R_t^{B}+\gamma_t\cdot X_t+\varepsilon_t \qquad \varepsilon_t\approx IID\left(0,\sigma_{\varepsilon,t}\right) \tag{7}$$

where (α_t, β_t, γ_t) are (time-varying) coefficients representing the security selection component of Active Portfolio return, its exposure to benchmark (return) risk and the market timing contribution to the active portfolio return, based on the market timing factor, X_t. The residual term, ε_t, captures return risk beyond those embedded in the benchmark return. As for the Benchmark Portfolio return, we assume that

$$\begin{aligned}&R_t^{B}\approx IID\left(\mu_{B,t},\sigma_{B,t}\right),\\&\text{with } Corr\left[R_t^{B};\varepsilon_t\right]=0; Corr\left[\left(R_t^{B}\right)^{2};\varepsilon_t\right]=0\end{aligned} \tag{7'}$$

The residual risk term of the active portfolio, ε_t, is supposed to be uncorrelated with the benchmark return. Other than that, we only assume that benchmark return has a well defined probability distribution, with finite mean and variance $\left(\mu_{B,t},\sigma_{B,t}\right)$. The stochastic rate of return of the active portfolio has a clear CAPM-like structure, augmented by a market-timing measure. If the benchmark portfolio were an efficient portfolio (in the mean variance sense), Eq. (7) would be consistent with a CAPM interpretation (Sharpe, 1994, market model). As we include a measure of market timing in our active return model, we rely on the Treynor and Mazuy (1966) and Hendriksson and Merton (1981) definition, in order to capture the fund manager's timing ability. More specifically we add to the standard CAPM bivariate regression the following extra market factor, X_t, with

$$X_t = MAX\left[-R_t^B, 0\right] \quad (8)$$

for Hendriksson and Merton (1981) and

$$X_t = \left[R_t^B\right]^2 \quad (8')$$

for Treynor and Mazuy (1966).

Both measures (8)-(8') are consistent with the ability of a fund manager to time the benchmark (market) returns. If fund managers are able to forecast benchmark portfolio returns, they will increase their proportion of the benchmark exposure when is high but will decrease it during a period of low returns on the benchmark portfolio.

The quadratic form of benchmark return in Eq. (8') can capture a manager's ability to forecast a market trend. For that reason, parameter γ_t represents not only a manager's timing ability but also the nonlinearity of benchmark realized returns. However, alpha performance measures can be misevaluated if realized returns are nonlinear at the benchmark. Although a significant timing coefficient can indicate the possibility of a misevaluated *alpha* measure, we cannot be totally sure about the source of the signal. In a recent paper, Goetzmann *et al.* (2007) revealed that a good timing measure can also be the outcome of performance manipulation (eg return smoothing) when assets are illiquid and subject to (for example) mark-to-model valuation. This paper only supposes that a significant timing coefficient from Treynor and Mazuy (1966)'s model indicates the presence of genuine timing ability, while neglecting performance manipulation issues.

In order to proceed with the implementation, it's useful to recall eq. (3) showing that the excess return over the benchmark is the product of the fraction of wealth invested in the active portfolio times the excess return earned by the Active portfolio over the benchmark. Replacing eq (3') in eq. (7) and subtracting the benchmark return, we get

$$R_t^P - R_t^B = \lambda_t^A \alpha_t + \lambda_t^A(\beta_t - 1)\cdot R_t^B + \lambda_t^A \gamma_t X_t + \lambda_t^A \varepsilon_t \quad (9)$$

It is clear from Eq. (9) that the excess return over the benchmark for the actual portfolio is controlled by same drivers determining the active portfolio return, scaled by the fraction of wealth , λ^A, invested in it:

1) security selection component: $\lambda_t^A \alpha_t$;

2) market-timing component: $\lambda_t^A \gamma_t X_t$;

3) exposure to benchmark (return) risk: $\lambda_t^A(\beta_t - 1)R_t^B$;

4) residual risk: $\lambda_t^A \varepsilon_t$.

3.1 Implementing fund manager performance measurement

For the sake of econometric implementation convenience, Eq. (9) is rewritten in the following reduced-form determination,

$$\begin{aligned} &R_t^P - R_t^B = a_t + b_t \cdot R_t^B + g_t\left(R_t^B\right)^2 + \varepsilon_t^P \quad \varepsilon_t^P \approx IID\left(0, \sigma_{\varepsilon,t}^P\right) \\ &a_t \equiv \lambda_t^A \alpha_t, \quad b_t \equiv \lambda_t^A(\beta_t - 1), \; g_t \equiv \lambda_t^A \gamma_t \\ &\varepsilon_t^P \equiv \lambda_t^A \varepsilon_t, \quad \sigma_{\varepsilon,t}^P \equiv \lambda_t^A \sigma_{\varepsilon,t} \end{aligned} \quad (10)$$

Parameters entering the set of Eqs. (10) ,

$$\left(a_t, b_t, g_t, \sigma_{\varepsilon,t}^P\right) \quad (10')$$

are easily amenable to standard econometric estimation technique, as we observe both benchmark and fund manager's portfolio returns. However we would still be in need of an identification procedure to measure the fraction of wealth, λ^A, invested in the active portfolio, in order to recover the (hidden active performance) parameters of interest,

$$\left(\lambda_t^A, \alpha_t, \beta_t, \gamma_t, \sigma_{\varepsilon,t}\right) \tag{10''}$$

Chen *et al.* (2009) derive an interesting generalisation of model (10) that incorporates the non-linear benchmark, replacing the market portfolio in the classical market-timing regression of Treynor and Mazuy (1966). Fund managers are assumed to time the market risk factors by anticipating their impact on the benchmark returns. Such impact may take a non-linear shape.[3]

Our identification strategy focuses on the level of risk determination for the active portfolio (eg its variance). The adopted key assumption relates the variance of the active portfolio to the variance of the benchmark portfolio by a coefficient, ϕ_t, assumed to be known in advance to the fund manager,

$$\sigma_{A,t}^2 = \phi_t^2 \sigma_{B,t}^2 \tag{11}$$

For the sake of simplicity, we set the value of ϕ_t equal 1 in equation (11), as if the fund manager would be choosing its active portfolio under the constraint of matching the risk of the benchmark portfolio. As a result, the variance of the active portfolio coincides with the variance of the benchmark return,

$$\sigma_{A,t}^2 = \sigma_{B,t}^2 \tag{11'}$$

We believe that there would not be much gain in relaxing risk constraint (11') by choosing different levels of (predetermined) deviation – albeit small – from the benchmark risk. In appendix A2 we prove that under the constraint (11'), the implied share of active portfolio share based on observable returns is given by

$$\hat{\lambda}^A = -\frac{1}{2}\frac{VAR\left(R_t^P - R_t^B\right)}{COV\left(R_t^P - R_t^B; R_t^B\right)} \tag{12}$$

Having estimated the unknown parameters (10') and (12), we can compute parameters (10''),

$$\gamma_t = \frac{g_t}{\lambda_t^A}; \quad \alpha_t = \frac{a_t}{\lambda_t^A}, \quad \beta_t = 1 + \frac{b_t}{\lambda_t^A}, \sigma_{\varepsilon,t}^2 = \frac{\left(\sigma_{\varepsilon,t}^A\right)^2}{\left(\lambda_t^A\right)^2} \tag{12'}$$

For the sake of simplicity, and in common with the classical market-timing models,[4] we maintain the hypothesis that returns can be represented by a static ordinary least squares (OLS) model with constant parameters,

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[3] One of the non linear forms considered in Chen et al. (2009) paper is a quadratic function, which has an interesting interpretation in terms of systematic coskewness. Asset-pricing models featuring systematic coskewness are developed, for example, by Kraus and Litzenberger (1976). Equation (10) would in fact be equivalent to the quadratic "characteristic line" used by Kraus and Litzenberger. Under their interpretation the coefficient on the squared factor changes does not measure market timing, but measures the systematic coskewness risk. Thus, a fund's return can bear a convex relation to a factor because it holds assets with coskewness risk.

[4] Cf Jensen (1968), Treynor and Mazuy (1966) and Henriksson and Merton (1981).

$$\left(a, b, g, \sigma_{\varepsilon}^{A}\right)_{OLS} \tag{13}$$

According to Jensen (1966), *a*>0 is a measure of (positive) abnormal performance, namely it captures the fund manager's ability to forecast extra-returns in excess of the exposure to market risk. Treynor and Mazuy (1966) argue that *g*>0 indicates market-timing ability. The logic is that when the market is up, the successful market-timing fund manager will be up by a disproportionate amount. When the market is down, it will be down by a lesser amount. All this makes sense from the perspective of the Capital Asset Pricing Model (CAPM, Sharpe, 1964). Under that model's assumptions there is two-fund separation and all investors hold the market portfolio and cash.

Can this approach still be valid for managed bond portfolios? Two-fund separation is generally limited to single-factor term structure models, and there is no central role for a "market portfolio" of bonds in most fixed income models. In practice, however, bond funds are managed to a "benchmark" portfolio that defines its investment style. Is it reasonable to assume that loadings (13) are really constant? After all, fund managers trade frequently in the hope of generating superior returns. This trading naturally generates time-varying loadings, as witnessed by the role played by a time-varying, λ^{A}_{t} in eq. (10). Moreover, expected market returns and fund managers' betas and gammas embedded in the active portfolio can change over time. If they are correlated, a constant coefficient (unconditional) model such as (13) would be misspecified.

Ferson and Schadt (1996), Christopherson *et al*. (1998), Mamaysky *et al.* (2007), Chen *et al*. (2009) propose a specific version of equation (10) to address such concerns. In essence, they introduce a conditional version of the market timing model of Treynor and Mazuy (1966) controlling for public information. These models generate time-varying loadings which can be forecasted by information signals observed by fund managers. While we would agree that a conditional model is likely to fit the data better than an unconditional model, in practice we retain our constant loadings assumption. As discussed in Section 4, we are going to apply eq. to a relatively short sample of daily data (one year). In this specific instance, the unconditional model may still provide a decent approximation of active portfolio strategy. However, in order to check the robustness of the unconditional (OLS) model estimates, we also test a GARCH(1,1) return model with heteroskedastic variance (see Appendix A4 for details)

$$\left(a, b, g, \sigma_{\varepsilon,t}^{A}\right)_{GARCH(1,1)} \tag{13'}$$

4. Euro-area NCBs fund managers: measuring US dollar reserves active performance

We test our active portfolio model (10) on the euro-area US dollar reserve fund managers. Nine national central banks (NCBs) are managing dollar-denominated bond funds on behalf of the ECB against a common benchmark. In the investment mandate, risk management and benchmark composition are strictly under the ECB's decision-making power. Fund managers can pursue active portfolio strategies only within narrow margins of discretion. The general guidelines of investment set strict risk limits – relatively short duration for the benchmark portfolio (below two years), a fairly tight tracking error volatility for benchmark deviations and a limited dose of credit risk are allowed.

Parameter estimates of model (10) – with parameters lists (13) and (13') according to the estimation method – for each fund manager are reported in Table 1. We do not report the NCB names for confidentiality reasons. The fund manager list is ordered according to the best performance (highest relative return; Table 2a). Daily (log) return data for each fund manager and benchmark (log) return for the year 2009 (1° January to 31 December;

365 observations) are used.[5] The same sample is again considered for the computation of statistical indicators and performance measurement.

Reported parameter values reflect three different estimation methods:

1) Standard OLS (homoskedastic residuals assumption);

2) Robust OLS, (non-gaussian returns; homoskedastic residuals assumption);

3) Maximum-Likelihood (ML), GARCH(1,1) model (heteroskedastic residuals assumption);

Method 1) and 2) assume constant residual variance, σ_{ε}^{2} . Method 3) include a GARCH(1,1) variance equation model to correct for residuals heteroskedasticity (see Appendix A4 for details). Parameters testing report t-Student statistics, standard as well as with Newey-West adjustment procedure. Coefficient of Determination, R^2, and autocorrelation of residuals test (Durbin-Watson; DW) are also reported.

The best fund managers – ranked according to table 2a by benchmark return out-performance – do show statistically significant parameters. In particular, the market timing performance parameter, *g*, is positive and significant for seven (out of nine) fund managers. The market portfolio parameter, *b*, is positive and significant for three fund managers; two of them are also the top performers in the ranking (the third one is found at the bottom). Moreover, there are three other fund managers that display statistically significant, *b*, with negative sign, however. In this case a negative exposure to market portfolio subtracts from fund performance, since the return of the benchmark turns out to be positive in the sample.

Selectivity appears to be very modest. The Jensen-*α* parameter, *a*, is statically significant (at 5% level) for one fund manager only (the third best performer in the ranking). The level of estimated residual autocorrelation (*DW* statistics) is confined to a range [1.73-2.35] consistent with absence (or modest level) of autocorrelation. The coefficient of determination, R^2, based on OLS estimates, is generally low, below 10% for eight out of nine fund managers. More often than not, this is strongly related to the heteroskedasticity of residuals: in seven out of nine cases R^2 coefficient jumps to well above 0.5 if the GARCH(1,1) estimates are considered. Changes in residuals volatility should capture risk factors dynamics beyond market portfolio risk (benchmark return) and market-timing risk (non linear or volatility risk implied by the benchmark portfolio). Such changes may reflect predictable, but unobserved (by the econometrician) adjustments in the active portfolio allocation selected by the fund manager (cf Eq. 12', normalised residuals). These portfolio adjustments may be driven by the dynamics of the risk-return trade-off (eg price-of-risk) faced by active fund managers. The implications of such important risk dynamics are not pursued further here, as they would require a more sophisticated identification strategy – this can be an interesting topic to investigate in future research. These considerations, as long as they are confined to residual risk, do not matter for performance decomposition, as we will see in a moment.

Table 2b reports the performance decomposition results based on the following ex-post identity derived from Eqs. (12-12')

$$\underbrace{T\hat{\mu}_{P-B}}_{\substack{Total\\ Holding\ \text{Period}\\ Excess\,\text{Re}turn}} \equiv \underbrace{T\hat{a}}_{\substack{ALPHA\\ Contribution}} + \underbrace{T\hat{b}\hat{\mu}_B}_{\substack{BETA\\ Contribution}} + \underbrace{T\hat{g}\left[\hat{\sigma}_B^2 + \left(\hat{\mu}_B\right)^2\right]}_{\substack{GAMMA\\ Contribution}} \qquad (14)$$

[5] Non-working day returns are linearly interpolated.

where T is the number of observations in the sample of return data.[6] Parameter estimates entering Eq. (14) are given by,

$$\begin{aligned}
&\hat{b} = COV\left(R_t^P - R_t^B; R_t^B\right) / \hat{\sigma}_B^2 \ ; \hat{g} = COV\left[R_t^P - R_t^B; \left(R_t^B\right)^2\right] / \mathrm{VAR}\left[\left(R_t^B\right)^2\right] ; \\
&\hat{a} = \hat{\mu}_{P-B} - \hat{b}\hat{\mu}_B - \hat{g}\left[\hat{\sigma}_B^2 + \left(\hat{\mu}_B\right)^2\right] \\
&\hat{\mu}_{P-B} = \frac{1}{T}\sum_{t=1}^{T}\left(R_t^P - R_t^B\right); \hat{\mu}_B = \frac{1}{T}\sum_{t=1}^{T} R_t^B; \\
&\hat{\sigma}_B^2 \equiv VAR\left(R_t^B\right) = \frac{1}{T}\sum_{t=1}^{T}\left(R_t^B - \hat{\mu}_B\right)^2; \mathrm{VAR}\left[\left(R_t^B\right)^2\right] = \frac{1}{T}\sum_{t=1}^{T}\left[\left(R_t^B\right)^2 - \sigma_B^2 - \left(\hat{\mu}_B\right)^2\right] \\
&COV\left(R_t^P - R_t^B; R_t^B\right) = \frac{1}{T}\sum_{t=1}^{T}\left(R_t^P - R_t^B - \hat{\mu}_{P-B}\right)\left(R_t^B - \hat{\mu}_B\right)
\end{aligned} \tag{14'}$$

Also, we can derive the decomposition of excess returns for the active portfolio by scaling the trading portfolio performance by the (estimated) share of wealth committed to active portfolio strategy,

$$\begin{aligned}
&T\hat{\mu}_{\text{A-B}} = \frac{T\hat{\mu}_{\text{P-B}}}{\hat{\lambda}^A} = T\hat{\alpha} + T\left(\hat{\beta} - 1\right)\hat{\mu}_{\text{B}} + T\hat{\gamma}\left[\hat{\sigma}_B^2 + \left(\hat{\mu}_B\right)^2\right] \\
&\hat{\alpha} = \hat{a} / \hat{\lambda}^A \ , \quad \hat{\beta} = 1 + \frac{\hat{b}}{\hat{\lambda}^A}, \hat{\gamma} = \frac{\hat{g}}{\hat{\lambda}^A}
\end{aligned} \tag{15}$$

Table 2b reports the computed performance decomposition obtained by plugging OLS (robust) estimates (see table, 1) in Eq. (14'). The OLS (robust) parameters, albeit with some relatively minor exceptions, do not differ much from the equivalent GARCH(1,1) estimates. Finally, table 2c reports the trading portfolio performance statistics for all fund managers – along with the benchmark return – separating actual vs active portfolio return statistics. Our main findings regarding performance decomposition can be summarised as follows:

1) Fund managers get most of their extra performance – about 60–80% of the total excess return – from market-timing ability ("Gamma" risk);

2) Selectivity (Jensen-*α* contribution) provides an, admittedly limited, boost to performance for few fund managers; such contribution is likely to be statistically insignificant (see my previous comments on the estimates of parameter *a*);

3) Exposure to "market-portfolio" (benchmark return) risk never materially contributes to extra-performance ("Beta" risk);

4) The implied (estimated) share of wealth allocated to active portfolio strategies varies substantially across fund managers, with no evident systematic pattern with regard to performance results.

It can be argued that conclusions 1)-2)-3) are broadly in line with the existing evidence about bond portfolio management performance. I therefore prefer to elaborate more on the fourth set of results.

Eq. (12) allows one to estimate the share of wealth allocated to active portfolio reported in table (2b; last column). These (estimated) values are plugged into Eq. (15) to simulate the active portfolio return statistics reported in table (2c; cf *Active* row). It is interesting to point

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6 Regression parameter estimates imply, $\sum_{s=1}^{T}\hat{\varepsilon}_s^P = 0$.

out that the two highest ranking fund performances take a short position in the active portfolio. In essence, these fund managers are investing all their money in the benchmark portfolio while selling their active portfolio (-0.51 and -0.36 dollar per dollar of invested wealth) to "buy with the proceeds" additional exposure to the benchmark portfolio. To be sure, their active portfolio strategy is expected to underperform the benchmark! Thus, fund managers can also profit from "inefficient" active strategies – with expected return lower than the benchmark (for given, identical "benchmark" and "active" portfolio risk) – if they are prepared to short them. On the other hand, there are fund managers – with supposedly more promising active portfolio strategies (ranking third, sixth and eight) – that are doing exactly the opposite: they are shorting the benchmark to finance with the proceeds additional exposure to their active strategy. In this case, their allocated share to their active strategy has to be larger than one (2.03, 1.50 and 1.31, respectively). Only three fund managers (out of nine) avoid leveraging their portfolio one way or the other; all of them end up in the middle of the performance ranking.

Computing the share of active portfolio based on the assumption of optimal portfolio mix (eq. 6") does not change the broad qualitative pattern of selected strategies (Table 3; cf column 4 vs 3). What is really changing drastically is the (absolute) level of wealth invested (or sold) in the active portfolio strategy. Only the worst performing fund manager happens to fully reverse its shorting strategy, eg from selling the active to selling the benchmark portfolio. As shown in Eq. (6"), the factor of proportionality to get the optimal allocation is given by the product of risk-tolerance parameter (the reciprocal of risk aversion) and the (estimated) risk-return ratio for the mixed (actual) portfolio,

$$\frac{1}{\hat{\psi}_B}\frac{\hat{\mu}_{P-B}}{\hat{\sigma}^2_{P-B}} \tag{16}$$

As reported in Table 3, the estimated (active) risk-return ratio is typically much larger than the (estimated) measure of risk aversion (cf col. 5 and 6), partly because the relative risk measure is by and large fairly small (only few basis points; cf standard deviation of excess return reported in cols 8–9). Hence, it should not come as a surprise that the estimated (uniform across fund managers) risk aversion parameter turns out to be relatively high (eg 11.71), as it is forced to reflect a fairly conservative benchmark portfolio risk-return profile.

As a robustness check, I also compute an implied measure of relative (conditional) VaR (RVaR). The average RVaR value is reckoned at around 0.50% annualised – with one notable exception (outlier). This is close to the actual risk budget limit assigned to NCB fund managers by the ECB risk management function. The (theoretical) optimal RVaR measure, computed under the assumption of a single value of risk aversion, $\hat{\psi}_B$, should be identical across fund managers. In practice, a fair amount of heterogeneity across fund managers seems to be present. The estimated RVaR has a range of variation between [0.24%–0.64%] (excluding the single outlier) across fund managers. Such variability is likely to be a signal of a less-than-efficient use of their risk budgets – eg an unexplained deviation from the optimal level of relative risk. At least in part, such variability might possibly be attributed to model estimation errors. To separate out the uncertainty due to sampling errors in estimating the RVaR measure, one would need to design a proper test for the RVaR statistics, so that a confidence interval for such a test is obtained. This is an area largely unexplored by the risk management literature and therefore further investigation is warranted. Since this statistical issues is well beyond the scope of this paper, it is left for future research.

5. Concluding remarks

The question of whether fund managers adopt an optimal active-passive risk allocation is addressed using in a straightforward extension of the T-B model. The essential insight into the T-B analysis is that the optimal combination of the active portfolio and a passive benchmark portfolio is itself a straightforward portfolio optimization problem. The T-B model allows fund managers to select the mix of active and passive portfolio that maximizes the (active) Sharpe-ratio performance indicator. The investment allocation assigned to the active portfolio strategy increases with the level of alpha (excess return over the benchmark portfolio) and decreases with the degree of unsystematic risk of the invested portfolio. The T-B model is here adapted and made operational as a tool for performance measurement. More specifically, the sources of fund management performance are isolated (selectivity vs market timing); the combination of active and passive risk exposures are estimated; individual fund manager portfolio choice (eg the active vs passive mix) and the related risk budget absorption are tested against the hypothesis of optimal design for the alpha-generating portfolio strategy.

The T-B model is applied to a sample of US dollar reserve management portfolios (owned by the ECB) invested in high grade dollar denominated bonds. Model parameters are estimated using standard OLS and GARCH(1,1) technique on daily portfolio returns for each fund managers. A performance decomposition, based on the well known selectivity and market-timing factors, is computed for each fund manager. The best fund managers show statistically significant outperformance against the benchmark. By far, market timing is its main driver. Selectivity appears to be very modest. These results are not very surprising after all, in that low credit risk and highly liquid securities dominate portfolio selection. Thus, very few opportunities are probably available to fund managers looking for (systematically) profitable bond-picking activity.

As far as the risk-return profile of the active portfolio is concerned, it appears that some of the best fund managers outperformance is realised by shorting the active portfolio with respect to the benchmark composition. Such portfolios are (rightly) shorted, because their equivalent long position would imply a negative (expected) excess return. Thus, long portfolios that are inefficient with respect to the their benchmark (negative excess return) can be turned into positive-alpha yielding portfolios provided that they are shorted. The long vs short choice of active portfolio requires a certain degree of fund manager ability in predicting the sign of excess returns. This ability can be seen as an additional source of fund manager’s outperformance, beyond the skill in anticipating the returns of the benchmark portfolio (market timing contribution).

Based on the model parameters estimates, I derive a measure of risk aversion (uniform across fund managers), consistent with the optimal active portfolio choice hypothesis. Such measure of risk turns out to be relatively high, as it is forced to reflect a fairly conservative benchmark portfolio risk-return profile. I also compute an implied measure of relative risk exposure, based on the concept of conditional Relative VaR (RVaR). The implied average level of RVaR (0.50% annualised) is close to the actual risk budget limit assigned to fund managers. However, a fair amount of heterogeneity across fund managers is found to be present, as the range of variation of (optimal implied) RVaR measures is material. Such variability across fund managers is a likely signal of inefficient use of their risk budget – eg a deviation from the optimal level of relative risk. At least in part, such variability could also be attributed to model estimation errors. To separate out the uncertainty due to sampling errors in estimating the RVaRs, one would need to design a proper test for the RVaR statistics, so that a confidence interval for such a test is obtained. This is an area largely unexplored by the risk management literature and therefore requires further investigation. For this reason it is left for future research.

Table 1

FUND MANAGER	Parameter Estimates: ERp = a + (b-1)*Rb + g*Rb^2 + other risks					
	a	b-1	g	Estimation Method	DW	R^2
I	**0.0004**	**0.0088**	**0.1013**	**OLS**	**2.3465**	**0.0432**
	1.1175	*1.6726*	*3.5401*	*(t-Stat)*		
	1.3841	*1.2045*	*3.039*	*(t-Stat-NW)*		
	0.0001	**0.0233**	**0.1121**	**Robust**		**0.0994**
	0.357	*6.7573*	*6.0147*	*(t-Stat)*		
	5.04E-04	**0.014187**	**0.096683**	**GARCH**		**0.7929**
	1.4431	*3.1267*	*3.9738*	*(t-Stat)*		
II	**-0.0003**	**0.0241**	**0.2351**	**OLS**	**2.2178**	**0.1311**
	-0.5563	*3.4662*	*6.2402*	*(t-Stat)*		
	-0.5966	*2.1021*	*3.0557*	*(t-Stat-NW)*		
	-0.0003	**0.0218**	**0.1467**	**Robust**		**0.0648**
	-1.0329	*5.4013*	*6.7126*	*(t-Stat)*		
	-1.58E-04	**0.020769**	**0.24891**	**GARCH**		**0.6842**
	-0.3298	*4.158*	*13.6675*	*(t-Stat)*		
III	**0.0007**	**-0.0022**	**0.0022**	**OLS**	**2.1455**	**5.64E-04**
	1.95	*-0.4496*	*0.0826*	*(t-Stat)*		
	2.1857	*-0.2679*	*0.0476*	*(t-Stat-NW)*		
	0.0005	**-0.0016**	**0.0435**	**Robust**		**0.0072**
	1.7309	*-0.4428*	*2.2694*	*(t-Stat)*		
	7.43E-04	**-0.00078**	**0.000208**	**GARCH**		**0.8524**
	1.9171	*-0.2025*	*0.0114*	*(t-Stat)*		
IV	**0.0001**	**-0.012**	**0.1068**	**OLS**	**1.9159**	**0.0536**
	0.2252	*-2.4278*	*3.9965*	*(t-Stat)*		
	0.2262	*-1.9979*	*3.2689*	*(t-Stat-NW)*		
	0.0001	**-0.0094**	**0.0974**	**Robust**		**0.0417**
	0.3632	*-3.0046*	*5.7386*	*(t-Stat)*		
	-9.84E-07	**-0.00964**	**0.1011**	**GARCH**		**0.3622**
	-0.003	*-2.7588*	*4.4604*	*(t-Stat)*		

FUND MANAGER	Parameter Estimates: ERp = a + (b-1)*Rb + g*Rb^2 + other risks					
	a	b-1	g	Estimation Method	DW	R^2
V	**0.0002**	**-0.0162**	**0.0424**	**OLS**	**2.0568**	**0.0335**
	0.5071	*-3.2806*	*1.5828*	*(t-Stat)*		
	0.6899	*-1.3434*	*1.1272*	*(t-Stat-NW)*		
	0.0001	**-0.0012**	**0.0604**	**Robust**		**0.0136**
	0.6405	*-0.4668*	*4.3686*	*(t-Stat)*		
	0.000301	**0.004349**	**0.015882**	**GARCH**		**0.7733**
	0.9332	*1.4734*	*1.0134*	*(t-Stat)*		
VI	**-0.0004**	**-0.0804**	**0.185**	**OLS**	**1.7275**	**0.0302**
	-0.2458	*-3.3007*	*1.4098*	*(t-Stat)*		
	-0.265	*-1.8918*	*1.0688*	*(t-Stat-NW)*		
	8.4E-07	**2.95E-05**	**-4.6E-05**	**Robust**		**0.0000**
	0.0033	*0.0081*	*-0.0024*	*(t-Stat)*		
	0.00129	**0.028739**	**-0.041485**	**GARCH**		**0.0569**
	3.8272	*8.7015*	*-2.9574*	*(t-Stat)*		
VII	**0.0001**	**-0.0089**	**0.0625**	**OLS**	**1.9707**	**0.0431**
	0.2385	*-2.5491*	*3.3204*	*(t-Stat)*		
	0.2351	*-1.1093*	*1.3569*	*(t-Stat-NW)*		
	0.0001	**-0.0078**	**0.052**	**Robust**		**0.0309**
	0.5812	*-3.2811*	*4.0617*	*(t-Stat)*		
	6.52E-05	**-0.008736**	**0.062266**	**GARCH**		**0.8217**
	0.2266	*-4.3444*	*6.7314*	*(t-Stat)*		
VIII	**0.0001**	**-0.01**	**0.0135**	**OLS**	**2.1627**	**0.004**
	0.0913	*-1.1899*	*0.2958*	*(t-Stat)*		
	0.1022	*-0.7111*	*0.1523*	*(t-Stat-NW)*		
	-1.8E-05	**-0.0155**	**0.0198**	**Robust**		**0.0095**
	-0.046	*-2.8549*	*0.6735*	*(t-Stat)*		
	5.11E-04	**-0.008439**	**0.015619**	**GARCH**		**0.8428**
	0.72	*-1.5506*	*0.649*	*(t-Stat)*		
IX	**-0.0002**	**0.033**	**0.0267**	**OLS**	**2.0432**	**0.3053**
	-1.0517	*12.3008*	*1.8364*	*(t-Stat)*		
	-0.989	*4.8458*	*0.5739*	*(t-Stat-NW)*		
	0.000001	**0.0371**	**-0.0251**	**Robust**		**0.3666**
	0.0113	*25.5755*	*-3.192*	*(t-Stat)*		
	-0.00018	**0.045263**	**0.022803**	**GARCH**		**0.5422**
	-1.2107	*40.7129*	*5.3379*	*(t-Stat)*		

ERp = Rp - Rb ; Rp = Absolute Return ; Rb = Benchmark Return

Table 2a (annual returns; in percent)

RANKING	FUND MANAGER	ABSOLUTE RETURN	RELATIVE RETURN
1.	I	0.8459	0.3882
2.	II	0.7959	0.3382
3.	III	0.7396	0.2819
4.	IV	0.7083	0.2506
5.	V	0.6198	0.1621
6.	VI	0.5930	0.1353
7.	VII	0.5672	0.1095
8.	VIII	0.5342	0.0765
9.	IX	0.4431	-0.0146
-	BNCHMRK	0.4577	-

Table 2b (annual relative returns; basis points)

RANKING	FUND MANAGER	Performance Decomposition				Active Portfolio Share
		Alfa	Beta	Gamma	Total	
1.	I	4.7	1.5	27.9	34.2	-0.511
2.	II	-23.1	2.1	53.1	32.1	-0.365
3.	III	18.2	-0.1	8.4	26.6	2.029
4.	IV	3.3	-0.5	18.4	21.2	0.443
5.	V	4.0	-0.1	9.6	13.5	0.291
6.	VI	12.2	0.0	0.0	12.2	1.500
7.	VII	2.8	-0.4	7.4	9.8	0.285
8.	VIII	-0.2	-1.4	7.6	6.0	1.305
9.	IX	0.0	0.9	-2.3	-1.4	-0.056

Table 2c (daily returns; basis points)

RANKING	FUND MANAGER	Trading Portfolio: Daily Performance Statistics (bps)				
		Mean	Median	St-dev	Skewness	Kurtosis
1.	I	0.2212	0.0673	7.0611	0.3156	5.7436
8.	*I (ACTIVE)*	*-0.0559*	*0.0677*	*6.9538*	*-0.1102*	*3.7592*
2.	II	0.2154	0.0708	7.2098	0.5244	6.7584
9.	*II (ACTIVE)*	*-0.1133*	*0.065*	*6.9538*	*-0.481*	*3.1649*
3.	III	0.2004	0.0698	6.9697	0.1704	5.5243
5.	*IIII (ACTIVE)*	*0.1634*	*0.0645*	*6.9538*	*0.1698*	*5.2977*
4.	IV	0.1857	0.0691	6.9132	0.3282	5.6375
1.	*IV (ACTIVE)*	*0.259*	*0.0678*	*6.9538*	*0.5277*	*6.368*
5.	V	0.1644	0.065	6.8765	0.2293	5.6593
2.	*V (ACTIVE)*	*0.2542*	*0.0674*	*6.9538*	*0.3517*	*6.8619*
6.	VI	0.147	0.0619	6.9489	0.0918	5.0259
6.	*VI (ACTIVE)*	*0.1363*	*0.0684*	*6.9538*	*0.2069*	*5.8908*
7.	VII	0.1627	0.0695	6.9141	0.2572	5.7319
3.	*VII (ACTIVE)*	*0.2508*	*0.0647*	*6.9538*	*0.4438*	*7.4773*
8.	VIII	0.1384	0.0683	6.9747	0.1806	6.2984
7.	*VIII (ACTIVE)*	*0.1359*	*0.0669*	*6.9538*	*0.1792*	*6.038*
9.	IX (ACTIVE)	0.1237	0.0672	7.195	0.206	5.6805
4.	*IX (ACTIVE)*	*0.1954*	*0.0717*	*6.9538*	*-0.2685*	*6.7898*
-	BNCHMRK	0.1275	0.0648	6.9538	0.1685	5.0573

Table 3

RANKING	FUND MANAGER	Active Portfolio Share: *Implied* (1)	Active Portfolio Share: *Optimal* (2)	Active Risk-Return Ratio (3)	Risk Aversion Parameter (benchmark-based Estimate)	Information-Ratio (daily rate)	Relative Return Standard deviation (daily; bps)	Relative Return Standard Deviation (annual rate; percentage points)	Implied RVaR Constraint (annual rate; percentage points) (4)
1.	I	-0.51	-80.48	1844.37	11.71	0.1315	0.7128	0.14%	0.43%
2.	II	-0.36	-28.30	908.08	11.71	0.0893	0.9839	0.19%	0.58%
3.	III	2.03	295.55	1705.68	11.71	0.1115	0.6538	0.12%	0.39%
4.	IV	0.44	49.15	1299.51	11.71	0.0870	0.6692	0.13%	0.39%
5.	V	0.29	20.87	838.59	11.71	0.0556	0.6633	0.13%	0.39%
6.	VI	1.50	0.98	7.64	11.71	0.0039	5.0509	0.96%	2.90%
7.	VII	0.29	38.98	1600.67	11.71	0.0751	0.4689	0.09%	0.28%
8.	VIII	1.31	9.78	87.76	11.71	0.0098	1.1145	0.21%	0.64%
9.	IX	-0.06	1.00	-210.46	11.71	-0.0089	0.4249	0.08%	0.24%

(1) 'Implied' equal 'Optimal' Share if Active Risk-Return Ratio (column 5) equal Risk Aversion Parameter

(2) Computed under the assumption of exact benchmark-based risk aversion estimate (cf column 6)

(3) Active Risk-Return Ratio equal Risk Aversion Parameter if 'Implied' equal 'Optimal' Share of Active Portfolio

(4) 'Implied' equal 'Optimal' Active Portfolio Share. Multiplier κ is set at 3 (time conversion factor √365)

Appendix

A1. Relative asset allocation: active vs benchmark portfolio

Let us consider a typical investment mandate. A fund manager is assigned the task to beat a benchmark portfolio over a specified time horizon. The benchmark portfolio, as specified in the mandate, should be attainable and investable. Depending on his expertise, the portfolio manager can overweight certain asset classes and/or securities and by the same token underweight others, thus building a zero-investment active portfolio. The composition of this active portfolio reflects the selection bets made by the portfolio manager.

Let us assume that there are {i=1,N } asset classes/securities to invest our portfolio. Its shares at time t can be represented as a vector of portfolio holdings adding up to 1:

$$\theta_t^P \equiv \left[\theta_{t,1}^P, \theta_{t,2}^P, ..., \theta_{t,N}^P\right], \text{ with } \sum_{i=1}^{N} \theta_{t,i}^P \equiv 1 \qquad \text{(a1.1)}$$

Our fund manager confronts a known ***Benchmark Portfolio, B***, with a given structure,

$$\theta_t^B \equiv \left[\theta_{t,1}^B, \theta_{t,2}^B, ..., \theta_{t,N}^B\right], \text{ with } \sum_{i=1}^{N} \theta_{t,i}^B \equiv 1 \qquad \text{(a1.2)}$$

In constructing her ***Managed Portfolio, P*** (eq, a1.1), our fund manager separates her active investment strategies in two related steps. In her first step she tries to construct an ***Active Portfolio, A***, which in her view differs from the benchmark in various desirable ways

$$\theta_t^A \equiv \left[\theta_{t,1}^A, \theta_{t,2}^A, ..., \theta_{t,N}^A\right], \text{ with } \sum_{i=1}^{N} \theta_{t,i}^A \equiv 1 \qquad \text{(a1.3)}$$

In her second step, she has to decide how much wealth she would commit to her Active Portfolio, A, in building her managed portfolio P. More specifically she has to ***set aside a fraction***, λ_t^A, of her total wealth (equal to 1) to be invested in portfolio A and the remaining fraction, $1-\lambda_t^A$, in the benchmark holdings. Thus, her Managed Portfolio has the following structure:

$$\underbrace{\theta_t^P}_{\substack{\textit{Managed}\\ \textit{Portfolio}}} = \underbrace{\lambda_t^A \theta_t^A}_{\substack{\textit{Active}\\ \textit{component}}} + \underbrace{\left(1-\lambda_t^A\right)\theta_t^B}_{\substack{\textit{Benchmark}\\ \textit{component}}} \qquad \text{(a1.4)}$$

Thus, her managed portfolio, P, turns out to be a combination of both active and passive portfolios, with exposure to the active component regulated by the amount, λ_t^A ,

We can rewrite Eq. (a1.4) highlighting the Managed Portfolio deviations from the benchmark holdings,

$$\underbrace{\theta_t^P - \theta_t^B}_{\substack{\textit{Managed Portfolio}\\ \textit{Deviation from}\\ \textit{the Benchmark}}} = \underbrace{\lambda_t^A}_{\substack{\textit{Active}\\ \textit{Portfolio}\\ \textit{Share}}} \underbrace{\left(\theta_t^A - \theta_t^B\right)}_{\substack{\textit{Difference between}\\ \textit{Active and Benchmark}\\ \textit{Portfolio holdings}}} \qquad \text{(a1.5)}$$

The left-hand-side holdings in Eq. (a1.5) can be observed directly by inspecting our fund manager's allocation, whereas the right-hand side decomposition is not known, unless we were to know in detail the two steps procedure highlighted above, namely the Active Portfolio holdings, θ_t^A, as well the associated fraction of wealth, λ_t^A, selection process. However, we can argue that if we happen to know the fraction of wealth invested in the Active Portfolio, we can easily recover the implied holdings of the Active Portfolio by inverting Eq. (a1.5) as follows

$$\theta_t^A = \theta_t^B + \frac{1}{\lambda_t^A}\left(\theta_t^P - \theta_t^B\right) \qquad \text{(a1.6)}$$

Our suggested two steps procedure may sound a bit contrived. Why bother paying attention to the decomposition suggested by the right-hand side of Eq. (a1.5) if what ultimately matters are only the bets (deviations from the benchmarks holdings) laid out in its left-hand side ? As investors, we are interested in the fund manager ability of selecting portfolio that can beat the benchmark. However, we can infer from decomposition (a1.5) that there perhaps be a wider range of active strategies than we have thought enabling us to achieve the extra-performance target. To appreciate such implication, let us transform decomposition (a1.5) in its return equivalent format (recall Eq. 3' in the main text)

$$R_t^P - R_t^B = \lambda_t^A\left(R_t^A - R_t^B\right) \qquad \text{(a1.7)}$$

Eq. (a1.7) suggests that, in principle, any active strategies (Portfolio A) can be used in order to beat the benchmark, provided that the share of wealth allocated to it (exposure) has the appropriate sign – long or short – depending upon the its expected performance relative to the benchmark. In brief, if the fund manager is convinced that her active Portfolio, A, can beat the benchmark, she would certainly want to be long portfolio A ($\lambda_t^A > 0$). Conversely, she may well come across an active portfolio, A, which (she believes) would very likely underperform the benchmark. Such underperforming (active) portfolio can equally provide a perfectly good foundation for a successful active strategy, if the appropriate short exposure ($\lambda_t^A < 0$) is chosen. Thus, the set of active strategies (portfolios A) seems much wider than we tend to believe. It all hinges upon the fund manager ability to assess the risk return profile of her selected Active Portfolio, A, vs the returns of the benchmark In the following paragraphs we illustrate several identification procedure for the share of wealth allocated to active strategies, λ_t^A ,based on a the risk of the active portfolio, A.

A2. Identifying the implied share of active portfolio return

To derive the implied share invested in the Active Portfolio one need to multiply both side of constraint (11) by the square of the fraction of wealth, $\left(\lambda^A\right)^2$, invested in the active portfolio,

$$VAR\left(\lambda^A R_t^A\right) = \left(\lambda^A\right)^2 VAR\left(R_t^B\right) \qquad \text{(a2.1)}$$

The left-hand side of eq. (a2.1) can be rewritten using the definition laid out in Eq. (3'):

$$VAR\left(R_t^P - R_t^B + \lambda^A R_t^B\right) = \left(\lambda^A\right)^2 VAR\left(R_t^B\right) \qquad \text{(a2.2)}$$

Eq. (a2.2) now depends entirely upon observable variables – benchmark and fund manager's returns – and the unknown value, λ_t^A. It is convenient to develop the variance on the left-hand in eq. (a2.2),

$$VAR\left(R_t^P - R_t^B + \lambda^A R_t^B\right) = VAR\left(R_t^P - R_t^B\right) + 2\lambda^A COV\left(R_t^P - R_t^B; R_t^B\right) + \left(\lambda^A\right)^2 VAR\left(R_t^B\right) \qquad \text{(a2.3)}$$

and equate the right-hand side of eq. (a2.2)- and (a2.3). As the term $\left(\lambda_t^A\right)^2 VAR\left(R_t^B\right)$ cancels out, we are left with the following equation,

$$VAR\left(R_t^P - R_t^B\right) + 2\lambda^A COV\left(R_t^P - R_t^B; R_t^B\right) = 0 \tag{a2.4}$$

which can be solved in the unknown share, λ^A, as,

$$\hat{\lambda}^A = -\frac{1}{2}\frac{VAR\left(R_t^P - R_t^B\right)}{COV\left(R_t^P - R_t^B; R_t^B\right)} \; , \; COV\left(R_t^P - R_t^B; R_t^B\right) \neq 0 \tag{a2.5}$$

QED

A3. Solving for the optimal active portfolio share

The first order condition of the optimization problem (5) is given by

$$\mu_{A-B} - \lambda^{A}\psi\sigma_{A-B}^2 = 0 \tag{a3.1}$$

which can be solved as

$$\lambda_*^A = \frac{1}{\psi}\frac{\mu_{A-B}}{\sigma_{A-B}^2} \tag{a3.2}$$

We can obtain an estimate of the optimal active portfolio share based on observable returns by manipulating the right-hand side of (a3.2) as follows (recall Eq. 6' of the main text)

$$\lambda_*^A = \frac{\left(\hat{\lambda}^A\right)^2}{\left(\hat{\lambda}^A\right)^2}\frac{1}{\psi}\frac{\mu_{A-B}}{\sigma_{A-B}^2} = \frac{\hat{\lambda}^A}{\psi}\frac{\mu_{P-B}}{\sigma_{P-B}^2} \tag{a3.3}$$

where $\hat{\lambda}^A$ is the implied value of the share of active investment according to the definition give in the main text (Eq. 3, with subscripts dropped)

$$R^P - R^B = \hat{\lambda}^A\left(R^A - R^B\right) \tag{a3.3'}$$

Notice that in Eq. (a3.3) the optimal share, λ_*^A ,coincides with the implied share, $\hat{\lambda}^A$, if (and only if) the level of risk aversion equates the risk-return ratio of the managed portfolio, P,

$$\psi_* = \frac{\mu_{P-B}}{\sigma_{P-B}^2} \tag{a3.4}$$

The level of risk aversion guiding fund manager risk control can be discussed in the standard portfolio management delegation framework, where the difference between the return on the managed portfolio (P) and the return on the benchmark portfolio (B) – eg tracking error – is subject to certain constraints. In order to control the active portfolio risk, investment mandates normally include a constraint on the Tracking Error Volatility (TEV), namely a limit on the maximum amount of risk borne by the investor in deviating from the benchmark. Typically, such risk constraint employs a Relative Value-at-Risk (RVaR) indicator as a *TEV* measure,

$$VaR_{P-B} \leq \upsilon \; , \; \upsilon > 0 \tag{a3.5}$$

where υ sets an upper bound on the *TEV*. Without a too great loss of generality, we assume that the relative *VaR* measure, VaR_{P-B}, is proportional to the standard deviation of the return differential, σ_{P-B}

$$VaR_{P-B} = \mu_{P-B} + \kappa\sigma_{P-B} = \lambda^A\mu_{A-B} + \kappa\left|\lambda^A\right|\sigma_{A-B} \quad \kappa > 0 \tag{a3.6}$$

where κ is a given multiplicative factor, depending upon the degree of confidence associated to the *VaR* measure as well as the shape of the return differential distribution. For a 99% confidence level and a Gaussian (daily) excess return distribution (with zero mean), κ would equal 2.3.

Combining (a3.5) and (a3.6), we get a measure of the maximum allowed size of the active portfolio share under a *TEV* constraint,

$$\lambda_{\nu}^{A} = \frac{\nu}{\mu_{A-B} + \kappa_{\pm}\sigma_{A-B}} \quad \kappa_{\pm} \equiv (\pm 1)\cdot\kappa \tag{a3.7}$$

In order to implement the allowed (maximum) size of active investment, λ_{υ}^{A}, in eq. (a3.7) as an optimal strategy, λ_{*}^{A} ,

$$\lambda_{\upsilon}^{A} = \lambda_{*}^{A} \tag{a3.7'}$$

The corresponding risk aversion parameter entering Eq. (a3.2) should be set as,

$$\psi_{\nu} = \frac{\mu_{A-B}}{\nu\sigma_{A-B}}\left[\frac{\mu_{A-B}}{\sigma_{A-B}} + \kappa_{\pm}\right] = \frac{1}{\nu} IR_{P-B}\left[IR_{P-B} + \kappa\right] \tag{a3.8}$$

where $IR_{P\text{-}B}$ is the managed portfolio excess return information ratio, which fulfils the following property,

$$IR_{P-B} \equiv \frac{\mu_{P-B}}{\sigma_{P-B}} = \frac{\lambda_A}{|\lambda_A|}\frac{\mu_{A-B}}{\sigma_{A-B}} = (\pm 1)\frac{\mu_{A-B}}{\sigma_{A-B}} \tag{a3.9}$$

Moreover, we can ask the question whether we can find the appropriate level of (maximum) *TEV*, so that implied and optimal (*TEV* constraint) active strategy would yield the same share of active investment, eg

$$\psi_{\upsilon} = \psi_{*} \tag{a3.10}$$

Recalling Eqs. (a3.4) and (a3.8), we can find the desired level of *TEV* fulfilling the assumption (a3.10),

$$\nu_{*} = \sigma_{P-B}\left[IR_{P-B} + \kappa\right] \tag{a3.11}$$

Under conditions (a3.4), (a3.10), (a.3.3) implies that the optimal (*RVaR* constrained) share of active portfolio allocation is equal to the implied value,

$$\lambda_{*,\upsilon}^{A} = \hat{\lambda}^{A}\frac{1}{\psi_{\upsilon}}\frac{\mu_{P-B}}{\sigma_{P-B}^{2}} = \hat{\lambda}^{A}\frac{1}{\psi_{*}}\frac{\mu_{P-B}}{\sigma_{P-B}^{2}} = \hat{\lambda}^{A} \tag{a3.12}$$

Eqs. (a3.12), (a3.11) and (a3.4) yield the "observationally equivalent" estimate of the optimal active portfolio share under *RVaR* constraint, with the associated risk aversion and (maximum) *TEV* estimates,

$$\left(\lambda_{*,\upsilon}^{A}, \psi_{*}, \nu_{*}\right) \tag{a3.13}$$

We also test a different identification strategy following a performance measurement methodology explained in Goetzmann *et al.* (2007). Their proposal is centred around the concept of Manipulation-Proof Performance Measures. (*MPPMs*). They show that if the benchmark portfolio return R_B has a (log)-normal distribution, then the coefficient of (relative) risk aversion entering the computation of *MPPMs* should be selected so that,

$$\psi_{B} = \frac{\mu_{B} - r_{f}}{\sigma_{B}^{2}} \tag{a3.14}$$

where r_f measures the risk-free rate of return. Since *MPPMs* ae typically associated with some benchmark portfolio, in the absence of any private information the *MPPM* should score the chosen benchmark highly.

Goetzmann *et al.* (2007) show that this would be the implication of eq. (a3.4) in computing their suggested *MPPMs*. What does it mean for a measure to be manipulation-free? Intuitively, if a manager has no private information and markets are efficient, then holding some benchmark portfolio, possibly levered, should maximize the measure's expected value. The benchmark portfolio might coincide with the market-portfolio, but in some contexts other benchmarks could be appropriate. Static manipulation is the tilting of the portfolio away from the (levered) benchmark even when there is no informational reason to do so. Dynamic manipulation is altering the portfolio over time based on past performance rather than on new information. A good performance measure penalises uninformed manipulation of both types in ranking fund managers' returns. Substituting the benchmark-based risk aversion measure (a3.14) into the optimal active portfolio share (eq. a3.3), we obtain,

$$\lambda_{*,B}^{A} = \frac{\hat{\lambda}^{A}}{\psi_B} \frac{\mu_{P-B}}{\sigma_{P-B}^{2}} \tag{a3.15}$$

A4. GARCH model for residual risk in the active portfolio

The error terms in the least-square model (10)-(13) are assumed to be homoskedastic (the same variance at any given data point). Sample data in which the variances of the error terms are not equal – the error terms may reasonably be expected to be larger for some points or ranges of the data than for others – are said to suffer from heteroskedasticity. The standard warning is that in the presence of heteroskedasticity, the regression coefficients for an ordinary least squares regression are still unbiased, but the standard errors and confidence intervals estimated by conventional procedures will be too narrow, giving a false sense of precision. Instead of considering this as a problem to be corrected, ARCH /GARCH models treat heteroskedasticity as a variance to be modelled. As a result, not only are the deficiencies of least squares corrected, but a prediction is computed for the variance of each error term. The ARCH/GARCH models, which stand for autoregressive conditional heteroskedasticity and generalized autoregressive conditional heteroskedasticity, are designed to deal with just this set of issues.

The GARCH(1,1) is probably the simplest and most robust of the family of volatility models. Since we are dealing with a relatively short sample (one year of daily data), higher order models – which would include additional lags – are unlikely to add much value. The GARCH model for variance looks like this (omitting superscript A):

$$\sigma_{\varepsilon,t}^{2} = \sigma_{\varepsilon,*}^{2} + \eta\sigma_{\varepsilon,t-1}^{2} + \phi\varepsilon_{t-1}^{2} \tag{a4.1}$$

where $\sigma_{\varepsilon,t}^{2}$ defines the variance of the residuals of model (10). I estimate the constants parameters $\left(\sigma_{\varepsilon,*}^{2};\eta;\phi\right)$. Updating Eq, (4.1) simply requires knowing the previous forecast, $\sigma_{\varepsilon,t-1}^{2}$, and (squared) residual term, $\varepsilon_{\varepsilon,t-1}^{2}$. The weights are $(1-\eta-\phi,\eta,\phi)$ and the long run average variance is given by $\sigma_{\varepsilon,*}^{2}/(1-\eta-\phi)$. This latter is just the unconditional variance. Thus, the GARCH(1,1) model is mean reverting and conditionally heteroskedastic, but have a constant unconditional variance. It should be noted that this only works if , and only really makes sense if the weights are positive, requiring $\left(\sigma_{\varepsilon,*}^{2};\eta;\phi\right)\geq 0$.

Parameters in eqs. (10) and (a4.1) are jointly estimated using Maximum Likelihood under the assumption of constant coefficients (eg using parameters' list, 13'). The GARCH(1,1) estimates are included in Table1. Reported standard errors are computed using the robust method of Bollerslev-Wooldridge. The coefficients in the variance equation are omitted here to save space and are available upon request from the author. The variance coefficients always sum up to a number less than one which is required in order to have a mean reverting variance process. In certain cases the sum is very close to one, therefore this process only mean reverts slowly. Standard Errors and p-values for parameters' list (13') are reported in Table 1.

The standardized residuals are examined for autocorrelation. In most cases, the autocorrelation is dramatically reduced from that observed in the portfolio returns themselves. Applying the same test for autocorrelation, we find the p-values are about 0.5 or more indicating that we can always accept the hypothesis of "no residual ARCH". As a result, we obtain a larger R^2 (coefficient of determination) statistics than standard OLS estimates, as the unanticipated residual variance component is drastically reduced by GARCH(1,1) variance prediction model.

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Explaining the returns of active currency managers[1]

Sam Nasypbek[2] and Scheherazade S Rehman[3]

1. Introduction

Currency markets have soared to have a trading volume of over $4 trillion a day. The $4 trillion is a 20% gain in the global foreign exchange markets from $3.3 trillion in 2007.[4] Over the years, the players in currency markets, the world's largest financial markets,[5] have changed. Traditionally, foreign exchange markets were mostly only a network of bank dealers and electronic trading systems used by (a) investors or corporations needing currency conversion to buy and sell financial instruments (i.e. stocks, bonds, etc.), repatriate profits home from abroad, and/or offset currency risks as part of their daily operations; (b) banks converting cash borrowed from foreign investors; (c) mutual-fund managers managing portfolios and using currency derivatives to offset the risk of currency swings; and (d) currency speculators (mostly interbank). Historically, the interbank market has accounted for the lion's share of daily volume; large banks not only have provided liquidity to multinational firms and global investors, but also have engaged in speculative activities through their proprietary trading desks.

With the rise of globalization and electronic trading, non-bank players such as hedge funds[6] have emerged as major players in the currency market with their share of daily volume matching the interbank as of 2007 (Gallardo and Heath, 2009). With hedge funds and other types of investors more active in currency markets, banks' traditional role as intermediaries in currency markets has diminished in terms of trading volume. Perhaps even more important is that all types of funds, from hedge funds to mutual funds, are increasingly now using currency markets as a distinct asset class (and not just a venue for an investment to be priced in another currency). "Non-interbank" (non-dealer) trading increased by 49% to $1.9 trillion a day, while trading in the interbank market (amongst dealers) grew by only 11% to $1.5 trillion a day (BIS, 2010). See Table 1. A large part of the rise in "non-dealer" trade and accompanied volatility is attributed to algorithms (trading models that are computer-driven).

In this fray, the number of small investors entering foreign exchange markets (i.e. investing in mutual funds whose core strategy is profits on currency fluctuations) has also dramatically

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[1] The findings, interpretations, and conclusions expressed in this study are entirely those of the authors and should not be taken to reflect those of the World Bank, its Executive Directors, or the countries they represent. All errors are our own.

[2] World Bank.

[3] George Washington University and EU Research Centre.

[4] Bank for International Settlements, "Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in April 2010 - Preliminary global results – Turnover", http://www.bis.org/publ/rpfx10.htm.

[5] By comparison, it dwarfs the stock markets, i.e. the U.S. stock trading averaged approximately $134 billion a day, and the U.S. Treasury markets which average about $456 billion a day (BIS, 2010).

[6] Hedge funds are unregulated private investment vehicles who have historically only been open to wealthy investors and institutions; they are less constrained in the use of trading strategies and instruments (eg, short selling and derivatives). Many hedge funds actively trade currencies, and until recently only a small segment has had an active "currency only" focus.

increased. Moreover, this demand has led exchange-traded mutual funds[7] to greatly enhance their products for small investors so as facilitate their participation in currency markets. There are approximately 44 currency exchange-traded funds (ETFs) currently in 2010, up from 16 ETFs in 2007, and 1 ETF in 2004.

A large portion of the increase amongst the non-bank players in currency markets has also come in the form of public institutional investors and sovereign wealth funds.[8] While currency markets have historically been deemed too risky for the investment fund managers who are the designated professional money managers administering pooled investments on behalf of local, regional, or central governments, it would seem there has been a shift and they are now more active players in foreign exchange markets. It should be noted that there is a maze of laws governing the agencies and persons (i.e. trustees) and, therefore, fiduciaries, authorized to make investment decisions on behalf of public agencies. They are subject to what are generally known as strict national prudent investor standards. In the United States, for example, the prudent investor standard is founded upon the presumption that a fiduciary will make the same decisions with respect to the use of public funds that a prudent person, seeking to maintain principal and meet the agency's cash needs, would make if provided with the same information. American courts have strictly interpreted the fact that fiduciaries must act in the same manner as a prudent person who is familiar with public investing. It should be noted that the large increase amongst the non-bank players in currency markets (especially public investors) has helped shift the notion of a "prudent investment" in the United States as (a) there is a surge of small investors in the currency markets and financial foreign exchange products; (b) currency funds are much more common today than 10 years ago; (c) foreign exchange markets are no longer viewed as the domain of large banks' treasury rooms; and (d) perhaps even more importantly, currency markets are increasingly being viewed as a distinct "asset class" of their own.

Table 1

Daily turnover in the foreign exchange markets

($ trillions)

Currency trading volumes		**1998**	**2004**	**2007**	**2010**
Total volume	All	$1.5	$1.9	$3.3	$4.0
By instrument	Spot	$0.6	$0.6	$1.0	$1.5
	Outright forwards	$0.1	$0.2	$0.4	$0.5
	Other: swaps, options	$0.8	$1.1	$2.0	$2.0
By source	Banks	$1.0	$1.0	$1.4	$1.5
	Funds, investors	$0.3	$0.6	$1.3	$1.9
	Non-financial customers	$0.3	$0.3	$0.6	$0.5

Source: BIS Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity, http://www.bis.org/publ/rpfx10.pdf.

[7] Exchange-traded mutual funds' shares trade similar to stocks.

[8] Worldwide domestic and foreign financial assets of all central banks and public wealth funds were estimated to be over $12 trillion in 2007.

As various types of hedge funds have increasingly marketed their currency investment products to outside investors (public fund managers, sovereign wealth funds, and private investors), their historical returns and stated philosophy and strategies have become more publicly available. This has begun to shed light on currency trading strategies. Previously, there was very little data available in this area as the primary participants were interbank (dealers) whose strategies or ROR are not publicly disclosed. Thus, perhaps for the first time in the field of currency trading, we are beginning to understand trading strategies and associated rates of return (ROR).

Given the rise in public sector (local and central government investors, and sovereign wealth funds) and small private investors' participation in the currency markets, we seek to explain and replicate the profits of active currency managers. It is important to clarify that *this study does not argue for or against investing in active currency managers*. We hope to develop a venue for enhancing our knowledge and evaluating the management and ROR of existing active currency funds using a currency beta composite index. We believe that such an active currency replication tool can be particularly beneficial to many public institutions facing large currency hedging decisions and considering employing external active currency managers to help manage the risk. An active currency replication index could serve as an alternative redundant risk evaluator or performance gauge that enhances informed choices with respect to currency risk management. Despite extensive literature on exchange rates, few academic studies have provided an in-depth analysis of active currency managers.

As such, in this study we see if it is possible to (a) explain returns of active currency managers[9] (the active currency managers used in this study include currency overlay managers, asset management units of large banks, and hedge funds) using simple trading strategies in the historical sample, and (b) replicate individual manager returns out-of-sample using an optimal combination of simple trading strategies. In addition, rolling regressions and Kalman filters are used to build an active currency replication index fund; its performance is then compared with the equal-weighted currency beta portfolio and optimized currency beta portfolios using classical Markowitz and Bayesian approaches.

In this study we specifically examine the profitability of active currency managers and apply a further definition to them as being from those asset management firms that offer strictly profit-oriented currency trading investments. The main purpose of this work is to explain the sources of their profits in-sample and replicate their returns out-of-sample using clearly defined currency trading strategies. We use a large database of 200 active currency managers for which monthly returns are available from 1993 to 2008. We contribute to an emerging literature on active currency managers by applying hedge fund replication methodology to active currency managers and extending previous studies with smaller datasets.

Since hedge funds started reporting their data to major databases, researchers have developed methodologies for replicating hedge fund returns using transparent investable trading rules. Given the less regulated and less transparent nature of hedge funds, the researchers have aimed to understand how these funds made money and whether it was possible to reverse-engineer their trading activities using statistical methods. Since the seminal article in 1997 by Fung and Hsieh, many systematic hedge fund trading strategies have been published (Fung and Hsieh, 1997, 2004, 2006; Schneeweis and Spurgin, 1998; Mitchell and Pulvino, 2001; Agarwal and Naik, 2004; Duarte, Longstaff, and Yu, 2006),

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[9] Although active currency managers (also sometimes called currency speculators) include market-making banks (interbank traders), investment firms, and individual investors, this study focuses only on investment firms and includes only currency overlay managers, asset management units of large banks, and hedge funds. From this point onwards in this study we will use the term "active currency managers" to refer only to currency overlay managers, asset management units of large banks, and hedge funds.

including active currency strategies (Lequeux and Acar, 1998; Binny, 2005; Middleton, 2005). These simple or beta strategies help explain the historical returns of different hedge fund sectors, including long/short equity and fixed income arbitrage. Several leading hedge fund researchers have suggested that given the success of historical return analysis and the expensive fees charged by hedge funds, a large portion of hedge fund returns could be replicated at less cost and used for multiple purposes, ranging from an investable index product (Hasanhodzic and Lo, 2007; Fung and Hsieh, 2004) to alternative risk redundancy mechanisms for currency risk management or investment. Thus, replication strategies could be used not only for analysis of past returns but also for index-like investment and risk management.

There is a gap between the academic literature and the practice of active currency management. Active currency trading involves making an informed bet on the direction and/or magnitude of future currency movements using historical and forecasted data. To date, economists have found it very difficult to forecast exchange rates out-of-sample and generally view currency speculation as a futile activity. Because of this skepticism, few academic studies have been conducted on active currency managers. Several investment consulting firms have reported that currency managers, on average, provide positive excess returns to their clients.[10] Given the lack of consensus on the optimal model of exchange rate determination, active currency managers use a variety of heuristic trading rules to capture the profit opportunities in the FX market. Researchers at investment banks have summarized the most popular trading strategies used by active currency managers as carry, value, momentum, and volatility, and several providers now offer investable indices based on these strategies (Binny, 2005; Middleton, 2005; Hafeez, 2006).[11]

Following the advances in hedge fund literature, Middleton (2005) conducted a pioneering analysis of historical returns of 40 currency managers which involved using five simple trading strategies. In another major study, Levich and Pojarliev (2007) used four simple strategies to analyze the historical returns of 34 currency managers. We contribute to this emerging literature by examining a large database of 200 active currency managers and investable currency beta strategies defined by Binny (2005) and Middleton (2005) and other researchers for the period from 1993 to 2008. The main focus of this paper is whether we can explain the returns of a large universe of active currency managers using simple beta trading strategies as an extension of previous work on historical return analysis. We also investigate whether we can replicate active currency managers' performance out-of-sample using a combination of currency beta strategies.

Our analysis shows that more than half of the profits from currency speculation can be explained by managers systematically taking advantage of forward rate bias, mean reversion to equilibrium fundamental value, trending of currency prices, and mean reversion of currency volatilities. This study, using a larger and more recent sample, confirms the findings from previous studies by Middleton (2005) and Levich and Pojarliev (2007).

We make further contributions to the literature on active currency managers by adapting the hedge fund methodology to replicate the manager returns out-of-sample. The results show that it is difficult to fully imitate the dynamic trading and allocation strategies of individual currency managers. The allocation rules proposed could fully replicate the total dollar return

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[10] Brian Strange, *Pensions and Investments*, September 1998; Watson Wyatt, *Global Pensions*, February 2000; Frank Russell, *2000 Russell Research Commentary*; and William Mercer, the Currency Overlay Supplement, *Global Pensions*, 2001.

[11] For example: *Financial Times*, "Investment banks up the ante for institutional FX allocations", 2007, retrieved March 10, 2009 from: http://www.ftmandate.com/news/fullstory.php/aid/1438/Investment_banks_up_the_ante_for_institutional_FX_allocations.html.

for only 30% of the 200 managers, while half of the total return could be replicated for up to 60% of the managers. In general, it is much easier to replicate a diversified portfolio or a composite index of currency managers using simple currency betas, as individual alphas tend to cancel out. Hence, investors, who do not have high confidence in their ability to select the best currency managers, may use this as a mechanism to learn, to inform themselves, and, finally, to use as a redundant risk management mechanism that could gauge ROR and strategies or, for that matter, perhaps even obtain roughly the same returns from buying a low-priced currency beta composite index product as they would from investing in much more expensive diversified currency manager fund of funds. Here we would like to restate that the main objective of this study is to develop an alternative redundant risk management tool for public investors.

The main limitations of this study are the choice of data and replication methodology. Hedge fund databases suffer from survivorship and backfill biases as only the best performing funds report their returns. We also assume it is possible to approximate the trading styles of currency managers using rolling regressions and Kalman filters with monthly data, but this methodology may not be appropriate for all managers. For example, managers may be using high frequency algorithmic trading models, or not using models at all and relying on fully discretionary analysis to trade around macroeconomic events. In either case it might be hard to capture the systematic trading style using regressions on monthly data.

In the remainder of this paper we explain in more detail. In Section 2, we review the related literature on exchange rate forecasting and active currency managers' performance studies. In Section 3, we describe the data and methodology used for the analysis. In Section 4, we present the results of this study, and finally, in Section 5, we present the main conclusions.

2. Literature review

FX market participants can be broadly classified into market-makers, passive hedgers, and active currency return-seeking traders. The large FX dealing banks primarily act as market-makers by providing liquidity to outside customers and by trading with each other to manage inventory. The passive hedging segment includes corporate treasuries and institutional investors that are mainly interested in insuring against FX fluctuations affecting their primary trade and investment activities. In contrast, active currency managers – including overlay firms, commodity trading advisors (CTAs),[12] and hedge funds – see FX as a speculative profit-generating opportunity.

Profits are generated in several ways, which change in response to the environment. In addition to bid-ask spread fees, FX desks of market-making banks also profit from proprietary trading activities. Initially, the primary responsibility of proprietary traders was to provide liquidity to their clients, but their mandate has evolved to taking active positions beyond liquidity needs.[13] Proprietary traders operated as hedge funds within banks. However, during the last decade many of these traders left the banks to open their own hedge funds, using their bank track records as evidence of past profitability. In addition, a recent proposal by the Obama administration is intended to limit risk-taking and proprietary trading activities of large banks (Weisman, 2010).

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[12] A Commodity Trading Advisor is an individual or firm which advises others about buying and selling futures and/or futures options and is licensed by the CFTC. See Spurgin, R., "Some Thoughts on the Sources of Returns to Managed Futures", *CISDM Working Paper Series*, 2005.

[13] For a brief history, please see http://en.wikipedia.org/wiki/Proprietary_trading.

FX traders outside major banks are usually part of currency overlay firms and hedge funds. Currency overlay managers traditionally serviced passive hedging clients, but eventually started shifting to active currency trading. Many hedge funds, especially in sectors such as global macro and managed futures, often employ active currency strategies. Although many hedge funds use some form of currency strategy in their trading, the focus of this analysis will be on those programs that have a dedicated active currency product. In particular, we focus on active currency managers investing in currency as an asset class (with the primary objective to generate profits from buying and selling currencies, independent of any other activity such as hedging, market-making, or trading equity and bond securities). In the next section we introduce the literature on exchange rate forecasting and provide background for the study.

2.1 Exchange rate forecasting: theory and practice

There is extensive academic literature on currency exchange rates, one of the central variables of open economy macroeconomics and international finance.[14] The main theoretical approaches to exchange rate determination are the macroeconomic and microstructure (micro) approaches. The empirical success of academic forecasting models remains limited. Although much of academic literature uses sophisticated forecasting methods to predict the level of the exchange rate, in practice most currency trading involves using simpler rules to forecast the direction of the move combined with strict risk management (Neely, Weller, and Ulrich, 2007).

The macroeconomic approach models the exchange rate as a function of differentials in money, interest rates, inflation, growth, productivity, fiscal balances, and balance of payments variables (Cheung, Chinn, and Pascual, 2005). The microstructure approach focuses on the currency market trading mechanisms and how the joint behavior of heterogeneous agents affects exchange rates (Lyons, 2001). The micro approach also attempts to explain technical patterns such as autocorrelation and mean reversion of currency prices and volatility (Osler, 2003).

The empirical success of exchange rate forecasting using macroeconomic models is poor, especially in the short run (Meese and Rogoff, 1983; and Cheung et al., 2005). Andersen et al. (2003) examined the US dollar spot rates around macroeconomic announcements and concluded that at high frequency exchange rate behavior is linked to fundamentals. However, researchers have had less success forecasting exchange rates using macroeconomic models at lower frequencies (daily, monthly, quarterly). In their seminal paper, Meese and Rogoff (1983) evaluated the out-of-sample fit of classical macroeconomic models of exchange rates, including the monetary and portfolio balance models of the 1970s. They found that none of the models could outperform the random walk;[15] therefore, they concluded that fundamental variables do not help predict future changes in exchange rates. Although a large number of studies have subsequently claimed to find success for various versions of fundamentals-based models using improved econometric methodology, their success has not proven to be robust across currencies and sample time periods (Sarno and Taylor, 2002) Cheung et al., 2005). There is evidence that forecasting performance improves for macroeconomic (macro) models incorporating non-linearities and using longer horizons (Mark, 1995; Killian and Taylor, 2003). Mark (1995) found that the fundamentals-based model outperforms the random walk when the forecast horizon is increased from 1 to

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[14] Recent comprehensive surveys include Sarno and Taylor (2002) and McDonald (2007).

[15] The best forecast for the next period's exchange rate is "no change" if it is a random walk. Exchange rates tend to exhibit daily and monthly serial correlations from time to time, allowing some predictability, but these dynamics are typically not related to fundamentals in the short run.

16 quarters. Killian and Taylor (2003) proposed a non-linear threshold autoregressive model: when exchange rates are near equilibrium they behave like random walks, but large deviations from fundamentals activate mean reversion tendencies.

One of the most persistent and important anomalies in exchange rate theory is the violation of the uncovered interest rate parity (UIP) principle. Also known as forward rate bias, UIP states that the forward exchange rate is an unbiased estimator of future spot rates. The validity of UIP would suggest that currency markets are efficient and that the long-term FX return is zero. There is now an accumulated body of evidence to claim that the forward exchange rate is a biased and inefficient predictor of the future spot rate (Bilson, 1981; Fama, 1984; Bekaert and Hodrick, 1993). In other words, the currency market is not efficient, possibly due to the existence of a risk premium or the failure of rational expectations (Sarno and Taylor, 2002). As a result, market participants might use forward premium information to predict the direction of exchange rates and earn carry profits during low volatility periods (Burnside et al., 2006; Jorda and Taylor, 2009).

In contrast to macro models focused on longer horizon forecasting, FX microstructure research has made progress in forecasting short-run exchange rate movements (Lyons, 2001). The microstructure approach focuses on heterogeneity of currency market participants and how information asymmetries influence exchange rates (Evans and Lyons, 2004). The FX market is large and highly liquid, but it is not as transparent as other financial markets; FX dealers can exclusively observe the order flows of both informed and less informed customers. Evans and Lyons (2005) examined forecasting over horizons from 1 day to 1 month and found that the microstructure-based order-flow model consistently outperformed both the random walk and the macro model. Overall, in the short run (less than 1 year), noise and microstructure effects are more prevalent, while in the longer run the fundamental factors become more important (Knott, 2002).

In practice, these theories are implemented using essentially four types of trading techniques or models: fundamental, carry, technical, and mixed. The fundamental model focuses on large deviations of the currency rates from macroeconomic fair value (Binny, 2005). The carry model takes advantage of the forward rate bias by collecting the interest rate differential between two currencies when exchange rate volatility is relatively low (Burnside et al., 2006, 2009). The technical model relies on the assumption that market data, such as charts of price, volume, and flow, can help predict future market trends (Le Baron, 1992, 2000; Lyons, 2001; Okunev and White, 2003; Irwin and Park, 2007). For carry and technical models the theoretical value of exchange rates is not considered. The mixed model, used most frequently by the marketplace, combines the other techniques.

Both macroeconomists and currency trading practitioners generally agree that the long-run fair value of the exchange rate should be based on economic fundamentals; however, since practitioners have capital at risk they cannot rely on fair value estimation alone. As the literature on limits to arbitrage suggests, asset prices can deviate from fair value longer than arbitrageurs can remain solvent (Shleifer and Vishny, 1995). Hence, currency traders often use carry, market sentiment and flow-based strategies in the shorter term. Allen and Taylor (1990, 1992) studied the London foreign exchange market and found that at short time horizons (up to 3 months) 90% of surveyed currency traders reported using the technical model, while at longer horizons (more than 6 months) 85% assumed fundamentals to be more important than charts. Researchers at investment banks have summarized the most popular trading strategies used by active currency managers as carry, value, momentum, and volatility, and the banks now offer investable indices based on these strategies (Binny, 2005; Middleton, 2005; Hafeez, 2006, etc.).

Because exchange rates are hard to forecast and currency risk can be substantial, currency management has historically focused on risk minimization or passive hedging. With the rise of alternative investments such as hedge funds, investors increasingly have come to view currency trading as a return-generating activity. The next section will introduce the emerging literature on active currency managers.

2.2 Active currency managers: performance analysis and replication

Over the past decade, starting with a pioneering study by Fung and Hsieh (1997), much research has been dedicated to defining and replicating[16] various hedge fund strategies and styles (Fung and Hsieh, 1997, 2004, 2006; Schneeweis and Spurgin, 1998; Mitchell and Pulvino, 2001; Agarwal and Naik, 2004; Duarte, Longstaff, and Yu, 2006, etc.). Extending the classic Sharpe (1992) style analysis of long-only mutual funds, successful replication of hedge funds involves defining a set of asset exposures or trading strategies capturing the systematic risk premium of the hedge fund. Schneeweis and Spurgin (1998) described the properties of managed futures hedge funds, Mitchell and Pulvino (2001) defined the trading strategy to replicate merge arbitrage. Agarwal and Naik (2004) covered equity hedge fund strategies, while Duarte, Longstaff, and Yu (2005) focused on explaining the returns of fixed income hedge funds. Fung and Hsieh (2004) introduced a seven-factor model for replicating diversified portfolios of hedge funds that incorporates the alternative betas from equity, fixed income, and managed futures hedge fund sectors. Using a 24-month rolling regression window, Fung and Hsieh (2004) replicated as much as 85% of the return variation of the average hedge fund, up to 64% of the global macro return, and 60% of the funds-of-funds hedge fund index returns.

For this study, we applied the approach used by Fung and Hsieh (2004) to the active currency sector, building on previous studies by Lequeux (2001), Middleton (2005) and Levich and Pojarliev (2007). Lequeux (2001) had analyzed 32 leveraged currency funds using an index of simple technical moving average trading rules (AFX) for the time period from October 1991 to September 1996 and demonstrated how to distinguish between trend following, systematic, and discretionary CTAs in the sample based on their rolling correlation to the AFX index. Middleton (2005) defined five naïve trading strategies (carry, forward curve, inflation differentials, momentum, and value) for developed currencies and analyzed the returns of 24 currency CTAs and 14 overlay managers (1996-2004) using principal component analysis and linear regressions. The author concluded that the CTA returns were primarily attributed to momentum strategies, while overlay managers were more likely to have diversified strategy sets including value, interest rate differentials, and momentum. The five factors defined by Middleton (2005) explained up to 75% of the returns of overlay managers and 44% of the CTAs. Levich and Pojarliev (2007) examined the returns of 34 individual currency managers from the Barclays Currency Trader Index (2001-2006) using four factors for studying developed currencies: carry, value, trend, and volatility. Similar to Middleton (2005), the authors found that the trend approach dominated the sample, followed by the carry approach. In addition, Levich and Pojarliev (2007) estimated the alpha to the benchmark of the naïve strategies (the intercept of the regression) and found that few managers in the sample generated statistically significant alphas. The authors also found significant factor timing ability by regressing the returns on squared factors.

In this study, we examine the historical returns of active currency managers using a larger sample over a longer time frame to determine how active currency managers make money: what trading strategies they typically use, how skilled they are at market timing, and whether they beat simple trading beta strategies on average. We use the currency beta indices by Binny (2005).

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[16] Hedge fund replication refers to observing the returns of a hedge fund and defining a set of simple investable trading strategies that allows the performance of given hedge fund to be imitated (replicated).

While several currency manager studies have focused on retrospective performance analysis, no attempt has been made to create an out-of-sample replicating portfolio using currency beta indices. With the rising demand from institutional investors, investment banks have constructed investable naïve trading strategies which track the main drivers of active currency returns (e.g. carry, value, momentum). This raises the possibility of creating passive replicating portfolios or “clones" using liquid investable instruments that provide similar risk exposures at lower cost and with greater transparency than that achievable through investing with the external managers. We examine how much of the active currency manager returns can be replicated using simple currency beta trading strategies developed by investment banks. Thus, if the simple currency beta index can achieve comparable returns to active currency managers, investors can create cost-efficient active currency portfolios consisting of less expensive currency betas.

We apply the hedge fund replication methodology (Fung and Hsieh, 1997, 2004; Hasanhodzic and Lo, 2007) to replicate active currency manager performance using pre-defined currency trading strategies. To build a replication we use rolling regressions (Fung and Hsieh, 2004; Hasanhodzic and Lo, 2007) and the Kalman filter (Roncalli and Teiletche, 2008). We also compare the replication's performance with the equal-weighted currency beta portfolio and optimized currency beta portfolio using the classical Markowitz mean-variance approach (Markowitz, 1952) and the Bayesian Black-Litterman approach (Black and Litterman, 1992). In other words, we assess whether an outsider can reproduce the active currency manager's process for generating returns based on simple building blocks using rules of thumb and Bayesian updating. This is the first study to apply the out-of-sample hedge fund replication methodology to active currency managers. It should be noted that the out-of-sample tests are performed using historical samples, and there is no guarantee that the past performance of active currency managers will be similar in the future. The following Section 3 describes the empirical analysis.

3. Empirical analysis

3.1 Data

We use three types of active currency returns data in this analysis: composite manager indices, the individual manager database, and currency beta strategies.

The manager indices published on Bloomberg[17] include Barclay, Stark, CISDM, and FXSelect. The Barclay, Stark, and CISDM indices have a long track record going back to the 1980s, while FXSelect has emerged in the last 10 years.

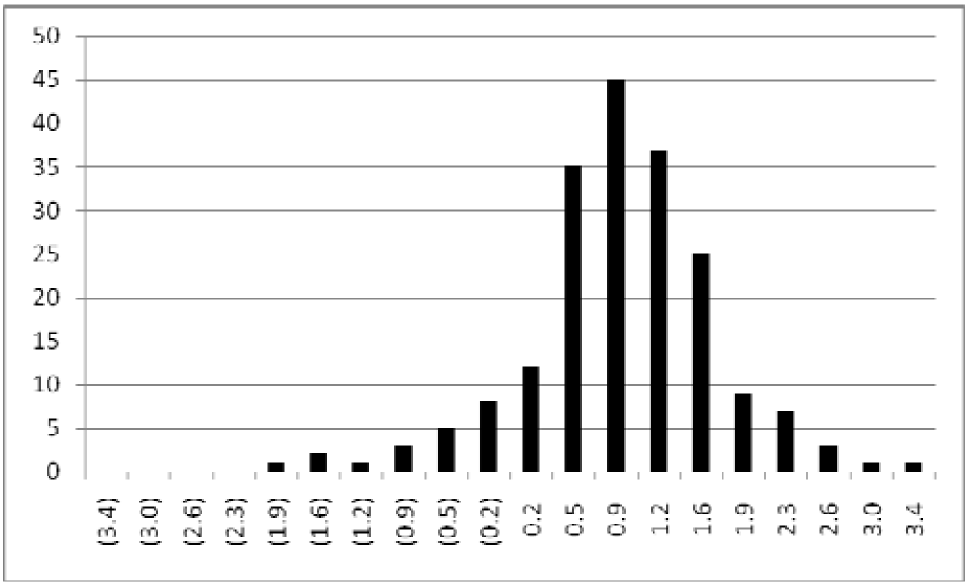
The main study dataset is the proprietary database that includes self-reported returns from 200 individual currency managers for the period of 1993-2008 (for a detailed description, see Ladekarl and Nasypbek, 2009). The dataset for this study does not contain any manager names, only style classifications and monthly returns; all returns are scaled to 10% annualized volatility. The sample used for this study is for the period from January 1993 to June 2008, with individual manager track records from 1.5 to 15 years. Since the database includes managers that stopped reporting, the number of active managers varied from 15 in 1993 to a high of 176 in 2005, and fell back to 120 in 2008. Two-thirds of the managers in the database follow mainly the technical price-driven approach, while 23% use the fundamental

[17] The choice of manager indices for this study does not indicate a judgment about the merits of individual index providers; the data are used strictly for research purposes. The Bloomberg codes are as follows: for Barclays and Stark Indices <FXTP>, for CISDM <CISDMACY Index> and for FXSelect <FXSTUNIU Index>.

macro and 9% employ a mixed approach. In terms of the investment process, over two-thirds of the managers in the database are model-driven or use a quantitative approach, while the other third use a discretionary/judgmental approach. Manager returns data used for this study are net of fees and net of Libor. For the entire period under review, 200 managers on average made 7.2% over 1M Libor per annum with a Sharpe ratio of 0.72. Figure 1 presents a distribution of the annualized Sharpe ratios, or risk-adjusted returns, of individual managers' live track records between January 1993 and June 2008.

Figure 1

Distribution of Sharpe ratios of individual managers



Source: Ladekarl and Nasypbek (2009).

Various investment banks have produced investable currency beta indices based on either stand-alone naïve trading strategies or a combination of the most successful substrategies. This study does not intend to compare the merits of individual currency beta providers, but rather focuses on the overall explanatory power of simple strategies grouped using Binny (2005) style definitions. Binny (2005) first formulated the now widely recognized carry, value, trend, and volatility currency trading styles.

Table 2

List of currency betas used in this study

Strategy	Rationale	Typical formulation
Carry	High-yielding currencies tend to outperform lower-yielding currencies in the short to medium term.	Long positions in high-yielding currencies and short positions in low-yielders; some of these strategies employ filters to close positions in high volatility periods.
Trend	Currency markets have exhibited a tendency to trend over time.	Moving average crossovers (MA), some with volatility filters. Buy if shorter MA outpaces long MA, sell otherwise.
Value	A currency with strong relative fundamental factors (PPP, high growth, BoP surplus, etc.) tends to outperform over long horizons. Buy the most undervalued currencies and sell the most overvalued currencies	Volatility tends to mean-revert. Sell in-the-money put and call options when volatility is high based on the expectation that volatility will mean-revert.
Volatility	Currency markets have exhibited a tendency to trend over time.	Moving average crossovers (MA), some with volatility filters. Buy if shorter MA outpaces long MA; sell otherwise.

Source: Binny (2005).

Table 3

List of currency beta indices

Provider	FX trading strategies	Data period	Source
Bank of America	Carry, forward curve, inflation differentials, momentum and PPP	01/01/85-06/30/08	Middleton (2005,2006)
Barclays	Carry, trend, value	01/30/00-06/30/08	Bloomberg <BFXIVTUS>
Bloomberg	Forward rate bias/carry	02/28/89-06/30/08	Bloomberg <FXFB>
Citibank	Carry, value, trend	01/01/97-06/30/08	CitiFX, Bloomberg <CACMUUSD Index>
Deutsche Bank	Carry, value, momentum	10/15/97-06/30/08	Hafeez (2006), Bloomberg <DBCRPLU Index>
RBS	Carry, value, trend, vol	12/31/74-06/30/08	Binny (2005), Bloomberg <FXTP>

For this analysis each individual beta is designated by its strategy type and the number of providers used (e.g. Carry1 to Carry4). As Table 4 shows, the currency beta indices achieved Sharpe ratios from 0.32 to 1.1 during the list's published history, but most indices came out only recently and are most likely back-tested (synthetically estimated) with only a few years of live trading history. However, these beta strategies are simple and transparent.

In total, we use as many as 20 individual currency beta strategies (stand-alone and components of the five composite indices above) for the analysis; we scale or standardize the volatility of all the active currency beta indices to 10% per annum (the same as for the active managers).

Table 4

Sample currency beta indices: performance and correlations (2002-2008)

Performance	**Beta Index1**	**Betal Index2**	**Betal Index3**	**Betal Index4**	**Betal Index5**
Ann Return	4%	3%	6%	4%	2%
Ann Volatility	5%	4%	6%	4%	7%
Sharpe	0.71	0.80	1.01	1.10	0.32
Correlations	Betal Index1	Betal Index2	Betal Index3	Betal Index4	Betal Index5
Beta Index1	1.00	0.72	0.17	0.53	0.52
Beta Index2	0.72	1.00	0.30	0.58	0.62
Beta Index3	0.17	0.30	1.00	0.54	0.58
Beta Index4	0.53	0.58	0.54	1.00	0.57
Beta Index5	0.52	0.62	0.58	0.57	1.00
Performance	Ann Return	Ann Vol	Sharpe		
Carry1	5%	8%	0.58		
Carry2	6%	8%	0.66		
Carry3	7%	7%	1.00		
Carry4	5%	6%	0.87		
Value1	4%	11%	0.35		
Value2	2%	5%	0.44		
Value3	1%	7%	0.19		
Value4	4%	5%	0.77		
Trend1	4%	12%	0.34		
Trend2	-6%	10%	(0.58)		
Trend3	4%	7%	0.60		
Trend4	4%	6%	0.64		
Volatility1	5%	4%	1.19		

Sources: Bloomberg, authors' calculations.

3.2 Methodology

The objective of the replication exercise is to reverse-engineer the target investment fund's returns, first by explaining its returns in-sample using various risk factors, and, second, by specifying a rule to produce similar returns out-of-sample using investable indices. In this study we focus on replicating the returns of active currency funds.

Traditionally, the relationship between risk factors and investment returns is described in classic multi-factor models such as the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT) approach (Treynor, 1961; Sharpe, 1964; Lintner, 1965; Ross, 1976). The main concept is that every systematic risk should be rewarded with a risk premium. For an investment fund, performance attribution and replication of the systematic part of the fund's return can be achieved if one can link returns of a fund to returns of rule-based trading strategies. For mutual funds, performance attribution turns out to be a comparatively straight-forward exercise. A typical mutual fund employs relatively static, long-only, strategies, and seldom uses leverage. Thus, the indices of standard asset class returns are suitable benchmarks for mutual funds, as shown by Sharpe (1992); however, these benchmarks can be inadequate for hedge funds. For hedge funds the replication process is more difficult since they take both long and short positions that create non-linear return series (Fung and Hsieh, 1997, 2007; Glosten and Jagannathan, 1994). Fung and Hsieh (1997) were the first to suggest replicating hedge funds with linear combinations of rule-based trading strategies; these trading strategies, in turn, could be non-linear positions in underlying instruments (Fung and Hsieh, 2007).

In this study we apply the Fung and Hsieh (1997) concept to currency funds. Hence, for a given active currency fund:

$$\textit{Return} = \textit{Alpha} + \textit{Beta}^{*}\ \textit{Currency Beta Factors} \qquad (1)$$

The beta is based on exposures to systematic risk factors (currency beta trading strategies), and the alpha represents the fund manager's skill in dynamic allocation and selection of individual strategies. Fung and Hsieh (2007) highlight the advantages of replication using low cost and liquid trading strategies: for investors seeking active returns it allows us to evaluate the performance of their hedge fund investment relative to costs, while for passive investors it offers a way to measure their factor exposure.

This study's research question is whether active currency funds' returns can be replicated using simple currency beta strategies. We use a two-part strategy to respond to this question. First, we will attribute the profits of active currency funds to a set of simple currency trading strategies in a historical sample. Second, we will replicate the active currency fund returns out-of-sample using pre-defined currency trading strategies.

Hypothesis 1 Historical profits of active currency funds can be attributed to the following trading strategies: carry, value, trend, and volatility. In particular, the profits are generated through:

a) direct exposure to these risk factors, and

b) market timing these risk factors.

We test this hypothesis using principal component analysis (PCA) as well as factor analysis (using pre-defined currency trading strategies). First, we use the first five principal components from the total variance of 200 currency funds and link them to the known currency beta strategies. We then follow Lequeux (2001), Middleton (2005) and Levich and Pojarliev (2007) and estimate the historical currency beta exposures in a full sample regression. This exercise evaluates how active currency managers historically have used currency trading strategies and determines the explanatory power of beta strategies in the total return of active managers.

To determine the explanatory power of common trading strategies for active currency managers, we perform a time-series regression for each of the 200 active currency managers in the sample by regressing the managers' monthly returns on the currency beta factors. The return of the active currency manager *i* at date *t* is denoted by the term R_{it}, in which R_{it} satisfies the following linear relationship:

$$R_{it} = \alpha_i + \beta_{i1} * F_{1t} + \beta_{i2} * F_{2t} + ... + \beta_{ik} * F_{kt} + \varepsilon_{it} \quad (2)$$

where the *F*s represent the returns of *k* currency beta strategies, and the βs represent the sensitivities of active manager R_i with respect to the different beta strategies.

The hedge fund approach differs in a few ways from classic factor models. First, as Hasanhodzic and Lo (2007) specify, the relationship in (2) is mainly statistical, unconstrained by any particular economic theory. In addition, the hedge fund risk factors, as opposed to academic risk factors, have to satisfy three criteria. They must be clearly defined and measurable, common and statistically significant to a set of hedge funds, and tradable using liquid instruments (Hasanhodzic and Lo, 2007).

We also use the methodology by Levich and Pojarliev (2007) to determine if the manager's alpha can be explained in terms of market timing ability. This methodology includes

- regressing the manager indices on currency betas with the intercept term to measure alpha in equation (2); the objective is to identify significant beta exposures and whether the alpha term αi is negative or positive and statistically significant.
- repeating the regressions but including the squared beta terms F^2 to measure market timing ability (manager's skill to forecast the sign of the factor), and noting whether the new intercept term A_i remains positive and significant.

$$R_{it} = A_i + \beta_{i1} * F_{1t} + \beta_{2t} * F_{2t} + ... + \beta_{ik} * F_{kt} + \quad (3)$$
$$+ \beta_{i1} * F^2_{1t} + \beta_{2t} * F^2_{2t} + ... + \beta_{ik} * F^2_{kt} + \varepsilon_{it}$$

If the squared beta terms β are positive and significant and new alpha A_i is reduced or becomes insignificant, it might suggest that the manager excess return over currency beta strategies can be partially explained by the market timing ability of the manager.

Hasanhodzic and Lo (2007) suggest using R^2, or the estimated fraction of the total variance attributable to the beta factors, as a measure of explanatory power. This is expressed as:

$$R^2 = Var[\sum(\beta_{ik} * F_{kt})] / Var[R_{it}] \quad (4)$$

For the majority of hedge funds in the TASS database, Hasanhodzic and Lo (2007) found that R^2s range from 25% to 75% for a three-factor risk model. They also demonstrated that where a hedge fund falls in this range depends on several characteristics: the hedge fund's investment style, the set of risk factors, and the time period. The seven-factor model developed by Fung and Hsieh (2004) achieves R^2 up to 85%.

We perform this analysis for the published active currency manager indices and 200 individual managers in the manager database. After the in-sample analysis is completed, we conduct the out-of-sample replication analysis.

The second objective in this study is to replicate active currency manager returns out-of-sample using a combination of currency beta strategies as a more cost effective (than formal hedge funds) alternative risk management redundancy mechanism for currency risk management or investment, or as an alternative less expensive investable index product. It should be noted that the former objective of developing an alternative redundant risk management tool is the primary focus and goal of our study. This objective will be tackled in two ways: (a) by optimally tracking the target manager's performance using beta strategies, and (b) by using an independent currency beta portfolio. The strategies for achieving the second objective lead to the second and third hypotheses.

Hypothesis 2 It is possible to replicate the performance of active currency funds, out-of-sample, by tracking their historical exposures to investable currency trading strategy indices.

To build the tracking portfolio, we use 24-month rolling regressions (Fung and Hsieh, 2004; Hasanhodzic and Lo, 2007) and the Kalman filter (Roncalli and Teiletche, 2008, Roncalli and Weisang, 2009).

For the replication exercise we use the managers with track records of at least 36 months (173 managers). Denoted by the symbol R_{rt}, the return of a replication portfolio consists of the currency beta indices corresponding to the *K* risk factors in (2). To replicate the target active currency manager *i*, we regress the target manager returns on the currency betas without intercepts and with weights summing to one:

$$R_{it} = \beta_{i1} * F_{1t} + \beta_{i2} * F_{2t} + ... + \beta_{ik} * F_{kt} + \varepsilon_{it} \tag{5}$$

subject to: $\beta_{i1} + \beta_{2t} + ... + \beta_{ik} = 1$.

To finalize the replication, we scale the volatility (or leverage in trading terms) of the estimated factor portfolio from (5) to match the target manager volatility:

$$R_{rt+1} = \sum \beta_{ik} F_{kt+1} * Var(R_{it}) / Var(\sum \beta_{ik} F_{kt}) \tag{6}$$

The ability of the factor model to map the market is not constant over time. A number of techniques can be employed to capture the dynamic portfolio adjustments, including time-varying-parameter regressions such as rolling regressions (Fung and Hsieh, 2004; Hasanhodzic and Lo, 2007). These techniques also include the Kalman filter and Bayesian dynamic models, which were recently applied to hedge fund replication by Roncalli and Teiletche (2008) and Roncalli and Weisang (2009).

For this study we use 24-month rolling-window regressions on currency betas (as in most hedge fund replication models, e.g. Fung and Hsieh, 2004; Hasanhodzic and Lo, 2007). The truncated regression approach has obvious limitations: A limited dataset is employed which does not use all available data, and the smaller samples of 24 monthly observations are relatively easily influenced by outliers (Fisher and Kamin, 1985).

We utilize the Kalman filter estimation to account for the dynamic nature of beta exposures, following the process developed by Roncalli and Teiletche (2008). To replicate the hedge fund exposures, Roncalli and Teiletche (2008) assumed smooth changes in weights using the following state-space model formulation:

$$R_t = F_t \beta_t + \varepsilon_t \tag{7}$$

$\beta_t = \beta_{t-1} + \acute{\eta}_t$ where

$\varepsilon_t \sim N(0,H_t)$ and $\acute{\eta}_{kt} \sim N(0,Q_t)$ are uncorrelated processes, and Qt=diag(σ_1,... σ_m).

The initial distribution of the state vector is $p(\beta_0) = \Phi(\beta_0, b_0, P_0)$, where $\Phi(\beta, b, P)$ is the Gaussian pdf with argument *β*, mean *b* and covariance matrix *P*.

Following Roncalli and Weisang (2009), the Bayes filter is then described by the following recursive equations: $p(\beta_t|R_{1:t-1}) = \Phi(\beta_{t-1}, b_{t|t-1}, P_{t|t-1})$ and $p(\beta_t|R_{1:t}) = \Phi(\beta_t, b_{t|t}, P_{t|t})$, estimated using a Kalman filter algorithm.

There is an important limitation of a look-ahead bias in using a full-sample Kalman filter for replication (Roncalli and Teiletche, 2008; Hasanhodzic and Lo, 2007). While the smoothed series using the full sample provide a useful benchmark, the clean out-of-sample replication would involve only using data available prior to the allocation decision at time t. We present results both for the forecasted $\beta t|t-1$ as well as for the full-sample smoothed $\beta t|t$.

The first evaluation is calculated using the manager's cumulative return, the rolling window and Kalman filter clones. These calculations produce simple clones that can be constructed using dynamic regression tools.

In addition to imitating active currency management, we also create a number of independent benchmarks using simple and optimized combinations of currency betas. The goal is to achieve the same performance as the active currency manager by creating an independent allocation strategy using simple currency betas.

Hypothesis 3 It is possible to replicate the performance of active currency funds, out-of-sample, using independent combinations of investable currency trading strategy indices including:

a) equally weighted currency betas,

b) mean-variance optimized currency betas, and

c) a Bayesian Black-Litterman portfolio of currency betas.

To create an independent beta portfolio we use a naïve equally weighted beta portfolio, a mean-variance optimized 24-month rolling currency beta portfolio, and a Bayesian blend of market-weighted and views-informed portfolios based on the Black-Litterman model (1992).

The equally-weighted beta portfolio return is estimated using the formula:

$R^{ave}_t = average\ (F_{1t} \ldots F_{mt})$ assuming m active currency beta factors and scaling the volatility to get the naïve clone:

$$R^{eq}_{rt+1} = R^{ave}_{t+1} * Var(R_{it}) / Var(R^{ave}_t) \tag{8}$$

The mean-variance optimized portfolio (Markowitz, 1952) is constructed as follows:

$\mathrm{Argmax}_w\ (w^T \mu_t - (0.5*\gamma)\ w^T\ \Sigma_t\ w^T$ st. $\sum w=1$,

μ is the vector of expected factor returns, and Σ equals the covariance of expected returns. Inputs μ_t and Σ_t are estimated using historical 24-month rolling window data. Volatility is scaled to find the mean-variance beta clone using the formula

$$R^{mv}_{rt+1} = w_t^{mv\ T}\ F_{t+1} * Var(R_{it}) / Var(w_t^{mv\ T}\ F_t) \tag{9}$$

The Black-Litterman optimized portfolio is constructed using the market equilibrium implied return as the ex-ante and subjective (forecasted) views (Black and Litterman, 1992). The Black-Litterman model uses a Bayesian approach to combine the subjective views of an investor with the market equilibrium vector of expected returns (the prior distribution) to form a new, mixed estimate of expected returns (Black and Litterman, 1992; He and Litterman, 1999; Walters, 2009). The resulting new vector of returns (based on the posterior distribution) can be used to develop intuitive portfolios with sensible portfolio weights.

In a slight departure from the Black-Litterman model, we assume the fixed-weight market equilibrium for active currency beta indices[18] and derive the implied return using $\mu^e_t = \gamma \Sigma_t w^e$. In this formula, γ is the risk-aversion parameter, w^e is the equilibrium portfolio weights, and Σ_t is the sample covariance matrix of factor returns. It is possible to stipulate that, $\mu = \mu^e + \varepsilon^e,\ \varepsilon^e \sim N(0, \tau\Sigma_t)$, where ε^e is the deviation of μ from μ^e that is normally distributed with zero mean and covariance matrix $\tau\Sigma_t$ and τ is a scalar indicating the degree of belief in how close μ is to the equilibrium value (in line with previous studies we use τ= 0.05; see Walters, 2009).

In the absence of any investor views on future stock returns, and in the special case of τ = 0, the investors' portfolio weights must equal w^e. However, an active portfolio manager is likely to have views on μ that are different from w^e in a substantial way. Black and Litterman (1992) illustrate that views on the relative performance of the stocks can be represented

[18] As the currency beta market develops it might be possible to obtain actual market value weights, but for this exercise we use fixed weights of 30% for each established factor (carry, value, and trend) and 10% for the lesser used volatility strategy. We understand that this does not fall under the exact Black-Litterman formulation, which derives the weights from established equity and bond indices with easily accessible capitalization weights.

mathematically by a single vector equation: $P\mu = \mu^v + \acute{\varepsilon}^v$, $\acute{\varepsilon}v \sim N(0, \Omega_t)$, where we set $P=I_m$ and μ^v is an m-vector summarizing the prior means of the view portfolios, and $\acute{\varepsilon}^v$ is the residual vector. The covariance matrix of the residuals, Ω_t, measures the degree of confidence the investor has in his or her views. For the views matrix, we use the weights from mean-variance optimization based on 24-month rolling window. Following He and Litterman (1999), we set $\Omega_t = [P\ (\tau\Sigma_t)\ P^T]$.

Applying the Bayesian rule to the equilibrium relationship and to the view equation, Black and Litterman (1992) show that Bayesian updated expected returns and risks may be expressed as

$$\mu^{BL} = [(\tau\Sigma_t)^{-1} + P^T\ \Omega_t^{-1}P\ ]^{-1}\ [(\tau\Sigma_t)^{-1}\ \mu^e + P^T\ \Omega_t^{-1}\ \mu^v] \tag{10}$$

$$\Sigma^{BL} = \Sigma_t + [(\tau\Sigma_t)^{-1} + P^T\ \Omega_t^{-1}P\ ]^{-1}$$

The Black-Litterman model tilts the investor's optimal portfolio away from the market portfolio according to the strength of her views. Because the market portfolio does not include any extreme positions, any suitably controlled tilt should also yield a portfolio without any extreme positions.

We then input the adjusted μ^{BL} and Σ^{BL} into mean-variance optimization and scale the volatility obtaining the Black-Litterman beta clone:

$$R^{BL}{}_{rt+1} = w_t^{BL\ T}\ F_{t+1} * Var(R_{it}) / Var(w_t^{BL\ T}\ F_t) \tag{11}$$

Hence, we create three new beta portfolios: naïve equally weighted, sample mean-variance optimized, and Black-Litterman optimized. We compare these new beta indices with the active manager and with the rolling window and Kalman filter clones from the first section using various performance measures including the cumulative return, Sharpe ratio, percentage of positive months, and maximum monthly loss. We also evaluate the statistical measures of forecasting success such as the mean absolute deviation (MAD) and the root mean-squared error (RMSE).

4. Study results

4.1 Explaining historical returns of currency manager indices

We examine the relationship between the currency beta indices developed by investment banks and actual currency manager index returns. Please note that this study does not compare the merits of individual currency managers and currency beta providers, but rather looks at the overall explanatory power of simple strategies grouped using Binny (2005) style definitions. The results show that the cumulative performance of the active currency manager indices can be mirrored quite well using the currency beta indices from multiple providers. In short, simple beta indices can achieve long-run returns which are similar to those generated by the target active manager indices. Table 5 shows that all manager indices have at least one currency beta index that is closely correlated.

Table 5

Correlations of currency manager and currency beta indices

(2000-2008)

Correlations	Barclays	Stark	CISDM	FXSelect
Barclay	1.00	0.75	0.61	0.80
Stark	0.75	1.00	0.73	0.61
CISDM	0.61	0.73	1.00	0.47
FXSelect	0.80	0.61	0.47	1.00
Correlations	**Barclays**	**Stark**	**CISDM**	**FXSelect**
BetaIndex1	0.53	0.35	0.25	0.63
BetaIndex2	0.38	0.24	0.20	0.50
BetaIndex3	0.20	0.27	0.46	0.14
BetaIndex4	0.40	0.44	0.48	0.38
BetaIndex5	0.63	0.61	0.67	0.47

Sources: Bloomberg; authors calculations.

Table 6

Style regression results: currency CTA indices on currency betas

(1993-2008)

	Barclays	Stark	CISDM
Adj R-squared	37%	47%	39%
Alpha	0.10%	0.09%	**0.29%**
t-stat	0.56	0.60	1.79
Carry	0.03	**0.10**	(0.04)
t-stat	0.51	1.80	(0.71)
Value	(0.03)	0.03	(0.08)
t-stat	(0.44)	0.56	(1.28)
Trend	**0.60**	**0.61**	**0.55**
t-stat	9.51	11.92	9.37
Volatility	**(0.21)**	**(0.15)**	**(0.27)**
t-stat	(3.44)	(2.91)	(4.62)

Sources: Bloomberg; authors' calculations.

We next conduct a factor analysis of FX manager indices to determine the predominant trading approach of the managers represented in each index. For the 1990-2008 data from the CTA indices (including Barclays, CISDM, and Stark) we use the currency beta factors and run a full sample regression as described in the methodology section under equation (2). The regression analysis results are presented in Table 6. In line with the previously discussed literature (Middleton, 2005), we find that the CTA indices primarily utilize trend following and long volatility trading strategies; this is evidenced by their statistically significant positive exposure to the trend factor and negative exposure to the volatility mean reversion factor. All of the beta factors explain from 37% to 47% of the variation in the overall history of CTA index returns from 1990 to 2008.

We also perform a similar factor analysis of the FXSelect index, which has a history beginning in 2000. This analysis shows that FXSelect is best explained by carry, value, and trend betas. As noted in the following table, the R-squared is 0.41 (Table 7).

Table 7

Style regression results: FXSelect index on currency betas

(2000-2008)

	Coefficient	**t-statistics**
Adj R-squared	41%	
Alpha	0.22%	2.51
Carry	1.09	3.72
Value	0.89	3.04
Trend	1.72	5.85

Sources: Bloomberg; authors' calculations.

Next we check for market timing ability and find that it is significant for all three strategies, with the adjusted R-squared improving to 0.52 and the alpha turning negative (Table 8).

Table 8

Style regression results: FXSelect with market timing

(2000-2008)

	Coefficient	**t-statistics**
Adj R-squared	52%	
Alpha	(0.11%)	(1.03)
Carry	**0.70**	2.49
Value	0.45	1.60
Trend	**1.12**	3.62
Carry_Sq	**126**	1.82
Value_Sq	**104**	2.06
Trend_Sq	**216**	2.58

Sources: Bloomberg; authors' calculations.

For the active currency indices above consisting of a large number of managers, the idiosyncratic approaches cancel each other out, and a large beta component can be identified as suggested in the hedge fund replication literature (e.g. Hasanhodzic and Lo, 2007).

The next step is to perform a style analysis of individual FX managers to determine their predominant trading approaches. Given the highly dynamic nature of active currency trading, we expect it might be harder to replicate the performance of an individual manager vs. the composite manager index. In this case, the alphas (excess returns) over simple currency beta strategies would be higher for the individual managers than for the aggregated index.

4.2 Explaining historical returns of individual managers

Following Fung and Hsieh (1997) and Middleton (2005), we use PCA to span the active currency manager returns space as represented in the manager database, and find that 52% of the variation in the returns can be explained by the first five principal components (Table 9). This is a relatively well-explained sample similar to the Fung and Hsieh (1997) finding that 45% of variance was due to the first five principal components in a total hedge fund sample.

Table 9

Percentage of total variance of 200 individual managers explained by first 5 principal components

	% of total variance explained	
	10-year data	5-year data
PCA1	22	25
PCA2	12	8
PCA3	8	6
PCA4	6	6
PCA5	5	4
Sum of 5	52	49

To gauge whether these principal components are related to currency beta factors, we conduct a regression analysis for each component. The results, as shown in Table 10, indicate that as much as 60% of the principal components can be explained by currency beta factor exposures, e.g. the first component consists of carry, trend, and volatility exposure, while the third component primarily consists of value and trend.

Table 10

Relationship between 5 principal components and currency beta factors

(1998-2008)

Regression results	PCA1	PCA2	PCA3	PCA4	PCA5
Adj R-squared	60%	26%	43%	12%	16%
t-statistics					
– Carry	5.89	4.25	(1.44)	1.36	(2.20)
– Value	1.42	(2.24)	(7.95)	0.74	3.22
– Trend	8.87	(3.37)	3.12	0.44	0.21
– Vol	(2.18)	(0.07)	1.80	3.35	1.13

These results suggest that the common factors driving the sample of 200 currency managers are indeed linear combinations of the currency beta factors.

The next step is to perform historical regression analysis for individual managers. This study does not intend to compare the merits of individual managers or currency beta providers, but rather to seek overall explanatory power of simple strategies grouped using Binny (2005) style definitions. For this analysis, each individual beta is designated using strategy type and number of providers used, e.g. Carry1 to Carry12.

Figure 2

Distribution of correlations: managers' returns to currency betas

(1993-2008)

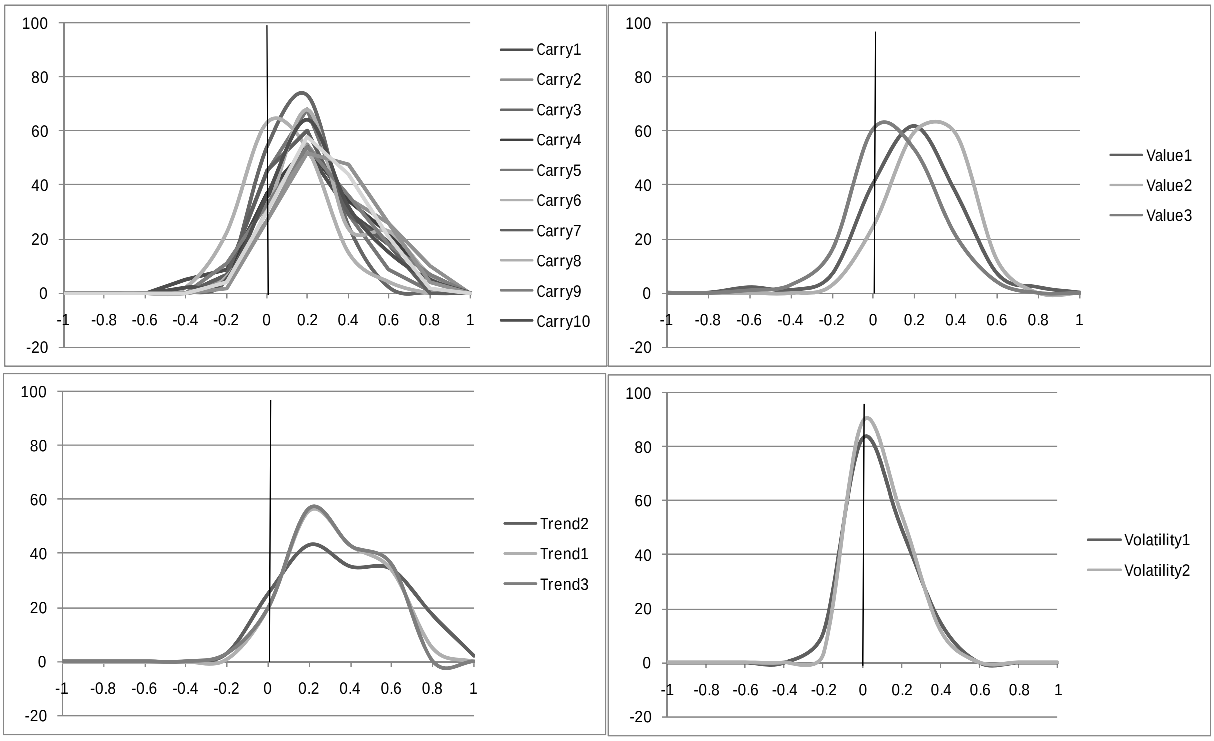


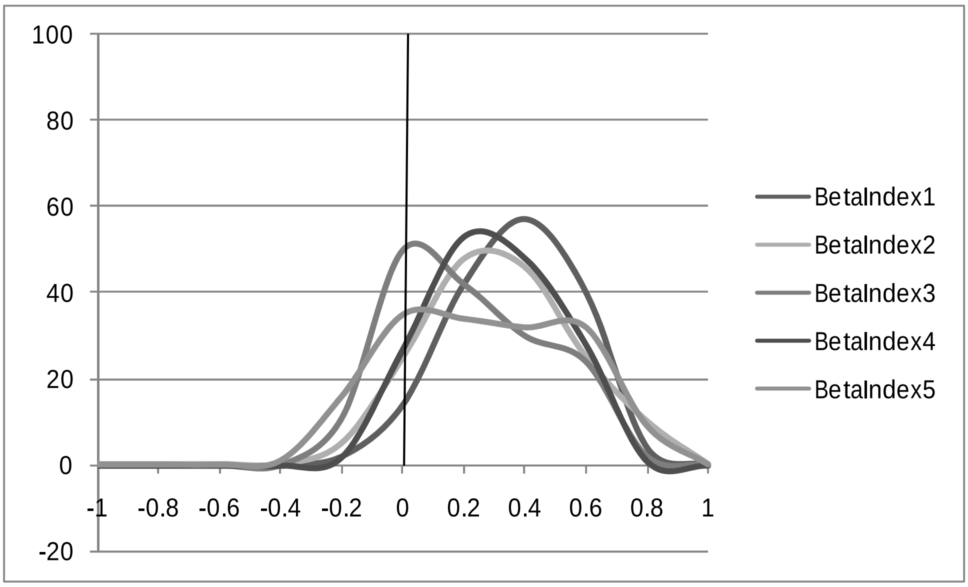
Figure 2 presents the correlation analysis for all betas by strategy type (i.e. carry, value, trend, and volatility). Various individual beta strategies within their style groups are broadly similar in the distribution of correlations to currency managers.

Figure 3 presents the distribution of currency managers' correlations to the composite currency beta indices. With the possible exception of BetaIndex3, all beta composites have similar correlation distributions to the manager universe.

Figure 3

Distribution of correlations: managers' returns to currency beta indices

(1993-2008)

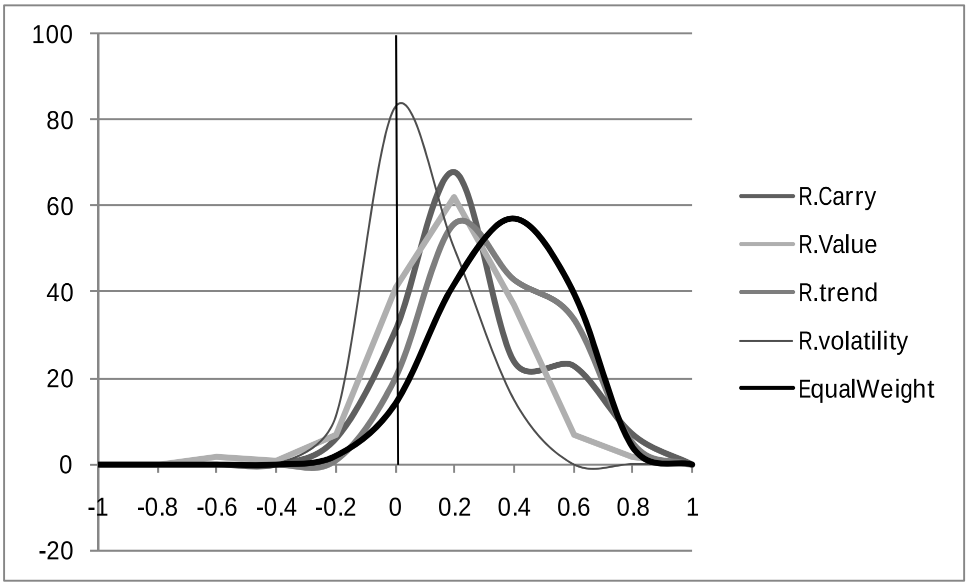


To conclude, the currency beta providers have broadly homogeneous categories of beta strategies that fall within the definitions established by Binny (2005). Hence, for the purpose of this study we could use any of currency beta bundles to address the research objective of judging the relationship between currency manager returns and naïve beta strategies. Without giving preference to any beta providers based on their individual merits, this study focuses on beta strategies formulated by Binny (2005).

The next step is to perform an analysis using the Binny (2005) currency beta strategies to see if they are correlated to individual manager returns. Figure 4 presents the distribution of the correlations. The majority of the managers have positive correlations to individual currency betas, and 90% of the managers have positive correlations to the equally weighted beta index.

Figure 4

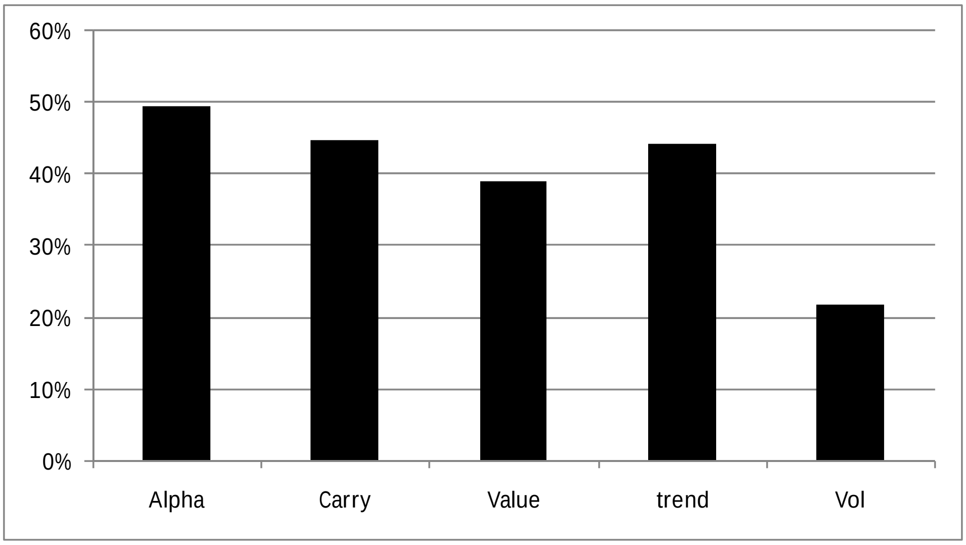
Distribution of correlations: managers' returns to Binny (2005) currency betas



For a more detailed analysis, we run full sample regressions and find that about half of the managers in the sample have statistically significant alphas over naïve trading strategies, as presented in Figure 5. All four strategies are used by managers; carry and trend are the most popular, followed by value and volatility.

Figure 5

Percentage of total managers with significant (at 95%) exposure to currency betas



On average, simple currency betas explain only 23% of the individual managers' return for this sample, with R-squared ranging from 1% to 83%. On average, the more profitable managers in the sample (information ratio or IR>0.5) tend to have significant alphas and significant exposures to all betas, while less profitable managers (IR<0.5) have no significant alphas and tend to be predominantly trend followers, as shown in Table 11.

Table 11

Summary of historical style analysis for currency managers using Binny (2005) beta factors

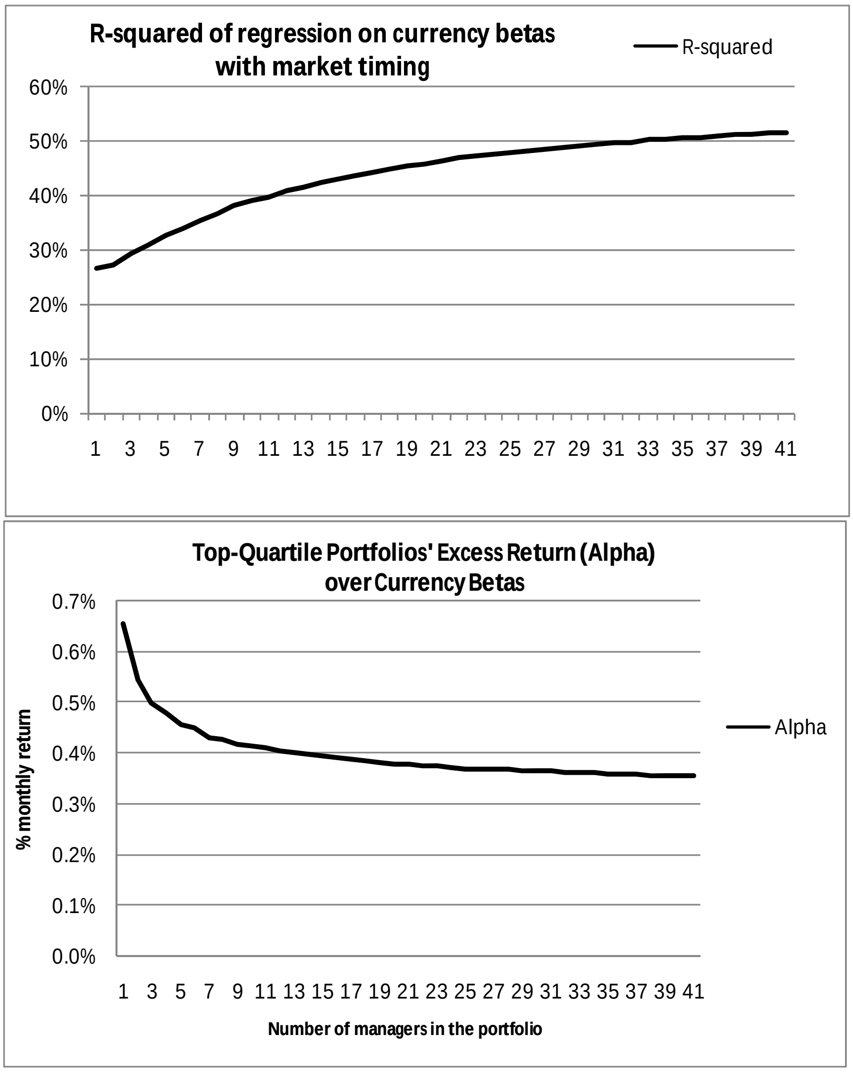
	Average	**IR>0.5**	**IR<0.5**	**Max**	**Min**	**StDev**
Adj R-Squared	23%	22%	25%	83%	1%	18%
Alpha	**0.51%**	**0.73%**	(0.03)	**3.15%**	**-1.81%**	0.66%
t-stat	1.87	2.60	0.01	6.53	(4.55)	1.83
Carry	**0.16**	**0.19**	0.09	**1.05**	**(2.15)**	0.36
t-stat	1.85	2.21	0.96	17.47	(4.28)	2.74
Value	0.06	0.07	0.06	**2.51**	**(3.55)**	0.55
t-stat	0.82	0.82	0.80	16.81	(6.58)	2.43
Trend	**0.20**	**0.17**	**0.26**	**0.66**	**(0.42)**	0.22
t-stat	2.69	2.37	3.55	10.42	(2.39)	2.72
Volatility	0.00	0.01	(0.01)	**0.37**	**(0.63)**	0.15
t-stat	(0.01)	0.02	(0.07)	3.91	(4.42)	1.33

Sources: Bloomberg; Binny (2005); authors' calculations.

Figure 6 illustrates how idiosyncratic components (unexplained by currency betas) of manager returns dissipate as we combine individual managers into indices or portfolios. We simulate the portfolios of managers using combinations of 1 to 40 and observe the average R-squared and excess return (alpha) for each combination: as the number of managers increases, the percentage explained by currency betas rises from 23% to over 50%, while monthly excess return for portfolios in the top quartile drops from 0.65% to 0.35%.

Figure 6

Percentage explained by currency beta strategies vs. number of managers



We next determine whether the currency managers in the sample possess market timing skills. A summary of the data is presented in Table 12 and Figure 7.

Table 12

Market timing skills analysis for sample currency managers

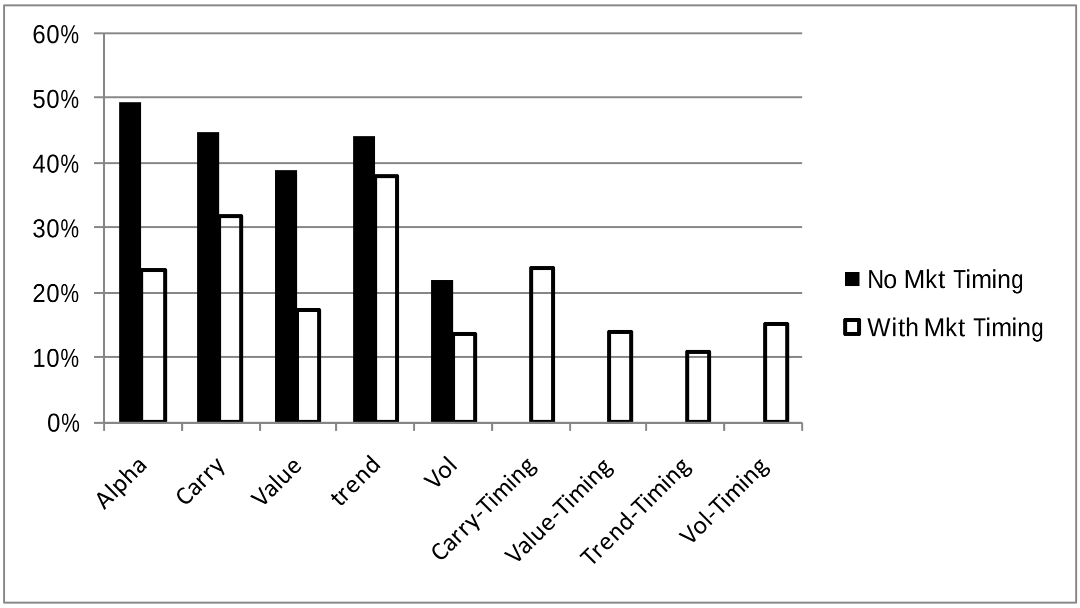
	Average	IR>0.5	IR<0.5	Max	Min	StDev
Adj R-Squared	29%	28%	33%	86%	1%	18%
Alpha	0.21%	0.47%	-(0.43)	**4.25%**	**-3.07%**	0.91%
t-stat	0.42	1.08	1.23	4.15	(3.64)	1.59
Carry	**0.15**	0.19	0.06	**1.11**	**(5.00)**	0.63
t-stat	1.80	2.17	0.94	17.53	(3.90)	2.74
Value	0.01	0.05	(0.08)	**1.99**	**(12.25)**	1.28
t-stat	0.51	0.62	0.26	16.43	(5.65)	2.30
Trend	**0.16**	**0.14**	**0.21**	**0.84**	**(0.71)**	0.24
t-stat	1.99	1.77	2.62	7.99	(2.74)	2.27
Volatility	0.03	0.03	0.02	**2.01**	**(0.54)**	0.22
t-stat	0.24	0.27	0.18	4.08	(3.59)	1.31
Carry-timing	2.33	2.19	2.76	**109.84**	**(62.51)**	14.89
t-stat	0.79	0.72	0.98	4.40	(3.13)	1.43
Value-timing	(1.02)	(0.55)	(2.22)	**67.01**	**(566.6)**	49.10
t-stat	0.39	0.29	0.65	4.25	(3.22)	1.27
Trend-timing	0.50	0.09	1.54	**27.89**	**(23.86)**	4.82
t-stat	0.26	0.07	0.77	3.60	(3.27)	1.23
Volatility-timing	0.99	1.15	0.63	**91.19**	**(24.27)**	7.30
t-stat	0.54	0.62	0.36	3.65	(2.19)	1.18

Sources: Bloomberg; Binny (2005); authors' calculations.

The percentage of managers with statistically significant alphas is halved if we consider skills in market timing.

Figure 7

Percentage of total managers with significant (at 95%) exposure to currency beta factors and market timing



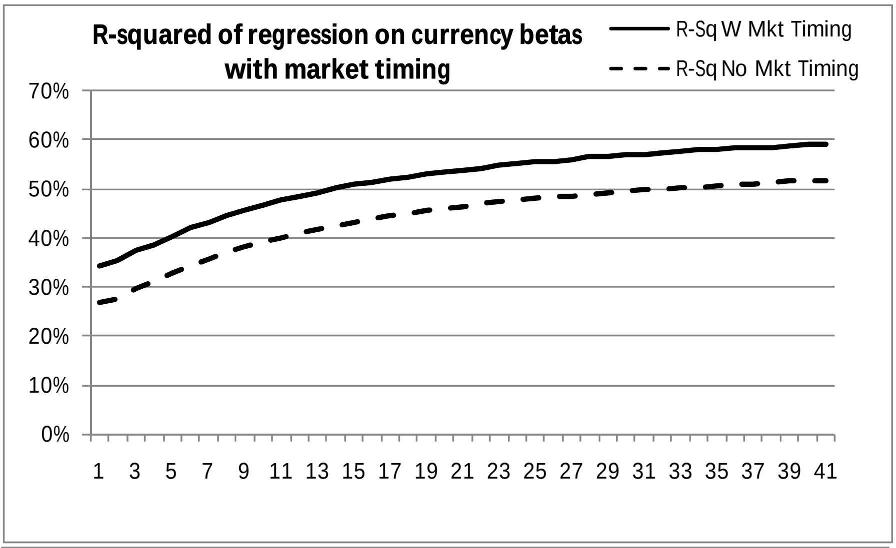
About a quarter of the managers in the carry category have statistically significant market timing skills. The figure is 15% for the value and volatility category and 10% for the trend-following category.

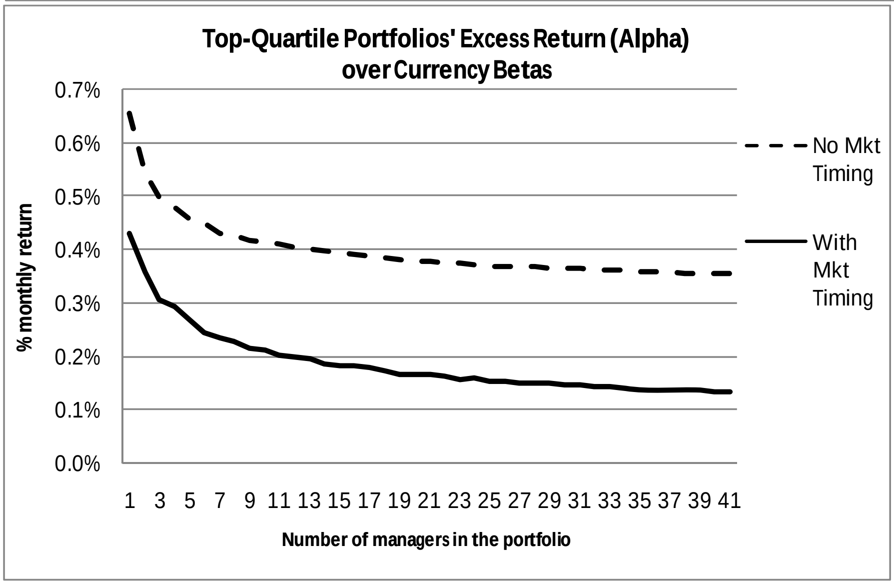
Figure 8 shows the results of the simulation of portfolios (as in Figure 6) for various combinations of managers (from 1 to 40) incorporating market timing skills. First, the percentage explained by currency betas increases and excess return (alpha) for top quartile portfolios decreases if we include market timing. Second, we can conclude that as the number of managers in the portfolio increases, the percentage explained by currency betas rises (from 30% to 60%), while monthly excess return for the top quartile portfolios drops (from 0.43% to 0.14%).

Hence, we can reconcile the findings from the analysis of currency manager indices (e.g. FXSelect) and individual managers. As we increase the number of managers in the portfolio, we find less alpha and a higher percentage of returns can be attributed to currency beta strategies, i.e. the portfolio starts looking like a manager index. Hence, historical in-sample analysis shows that simple currency betas can explain about half of the currency manager index/portfolio returns.

Figure 8

Percentage explained by currency beta strategies vs. number of managers: impact of market timing skills





We also examine whether the managers' beta exposures vary by self-reported trading style: fundamental vs. technical and model-based vs. discretionary. Fundamental managers use macroeconomic factors, while technical managers rely on currency prices for trading. Figure 9 illustrates the differences in exposures between technical and fundamental style managers: fundamental managers have a higher number of statistically significant exposures to carry and value beta strategies, while technical managers use trend and volatility strategies more often and are generally better at market timing. Model-based vs. discretionary style differences are based on whether the manager uses a systematic approach to trading. Figure 10 summarizes the differences in exposures for the model-based and discretionary styles. Model-based managers are much more likely to use carry and trend strategies; discretionary managers have more alpha and less beta exposures. The results are intuitive, as discretionary managers by definition should have more idiosyncrasies.

Figure 9

Percentage of total managers with significant (at 95%) exposure to currency beta factors by style: fundamental vs. technical

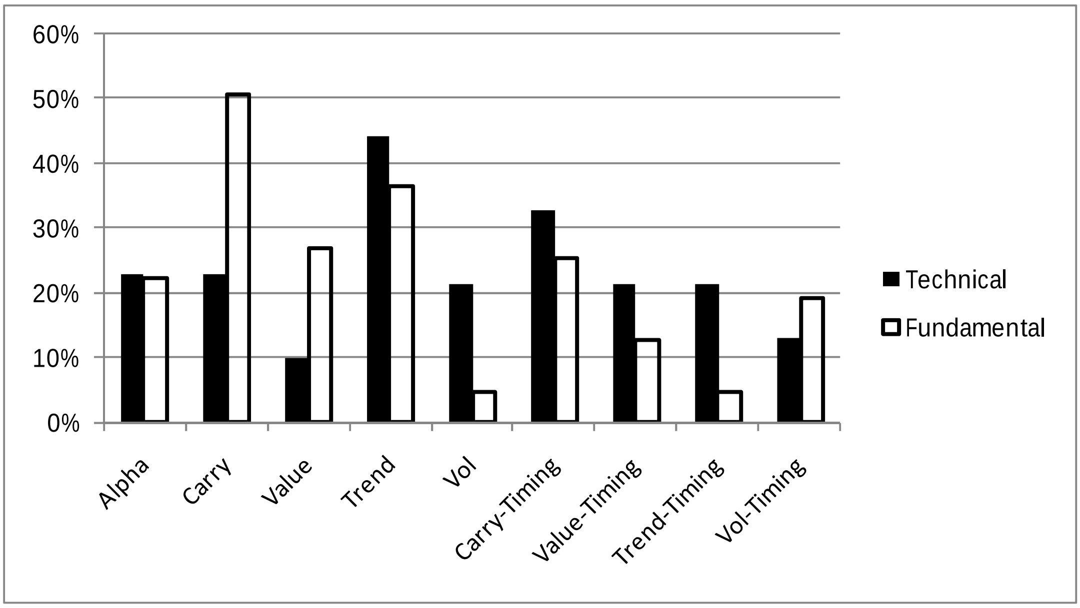
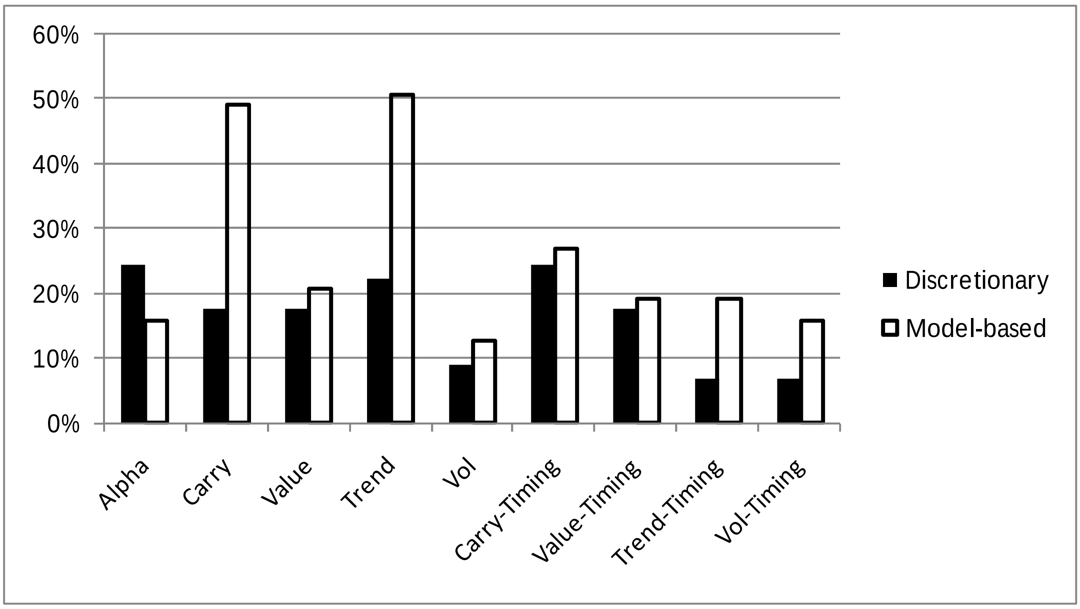


Figure 10

Percentage of total managers with significant (at 95%) exposure to currency beta factors by style: model-based vs. discretionary



The general findings from the historical style analysis based on the full sample regression can be summarized as follows:

- It is possible to explain up to 83% of the variation of currency manager returns using simple currency betas; the average R-squared for the sample is 23%.
- About half of the currency managers in the sample have statistically significant alphas over simple currency betas (with 95% confidence).
- Currency managers with higher profitability (IR>0.5) have statistically significant alpha to currency beta factors, while less profitable managers (IR<0.5) do not.
- As we increase the number of managers in the portfolio, alpha starts to decrease and a higher percentage of returns (50-60%) can be attributed to currency beta strategies, i.e. the portfolio starts looking like a manager index.
- Half of the statistically significant alphas can be attributed to market timing skills.
- The percentage explained by currency betas increases and excess return (alpha) for top quartile portfolios decreases if we include market timing.
- More than one-third of currency managers have statistically significant exposures to carry, trend, and value currency beta factors.
- Fundamental managers have a higher number of statistically significant exposures to carry and value beta strategies, while technical managers use trend and volatility strategies more often and are generally better at market timing.
- Model-based managers are much more likely to use carry and trend strategies, while discretionary managers have more alpha and fewer beta exposures.

Although a static full-sample analysis shows that half of the managers' return variation is not attributed to currency beta factors, some evidence indicates that a portion of the alpha can be explained by the dynamic nature of exposures (market timing). Of course, the static full sample regression is only an approximation and might not adequately represent the actual dynamic allocations by individual managers over time.

4.3 Out-of-sample replication of currency manager indices

We turn to forward-looking out-of-sample replication to measure how much of a manager's dollar return can be captured in practice by using simple currency beta trading strategies. Tracking replication portfolios are estimated for each manager using rolling regressions and Kalman filters. The equal-weighted, mean-variance optimized, and Black-Litterman currency beta portfolios are calculated independently and serve as benchmarks for all managers. The ultimate objective is to achieve a similar total dollar return to the target manager.

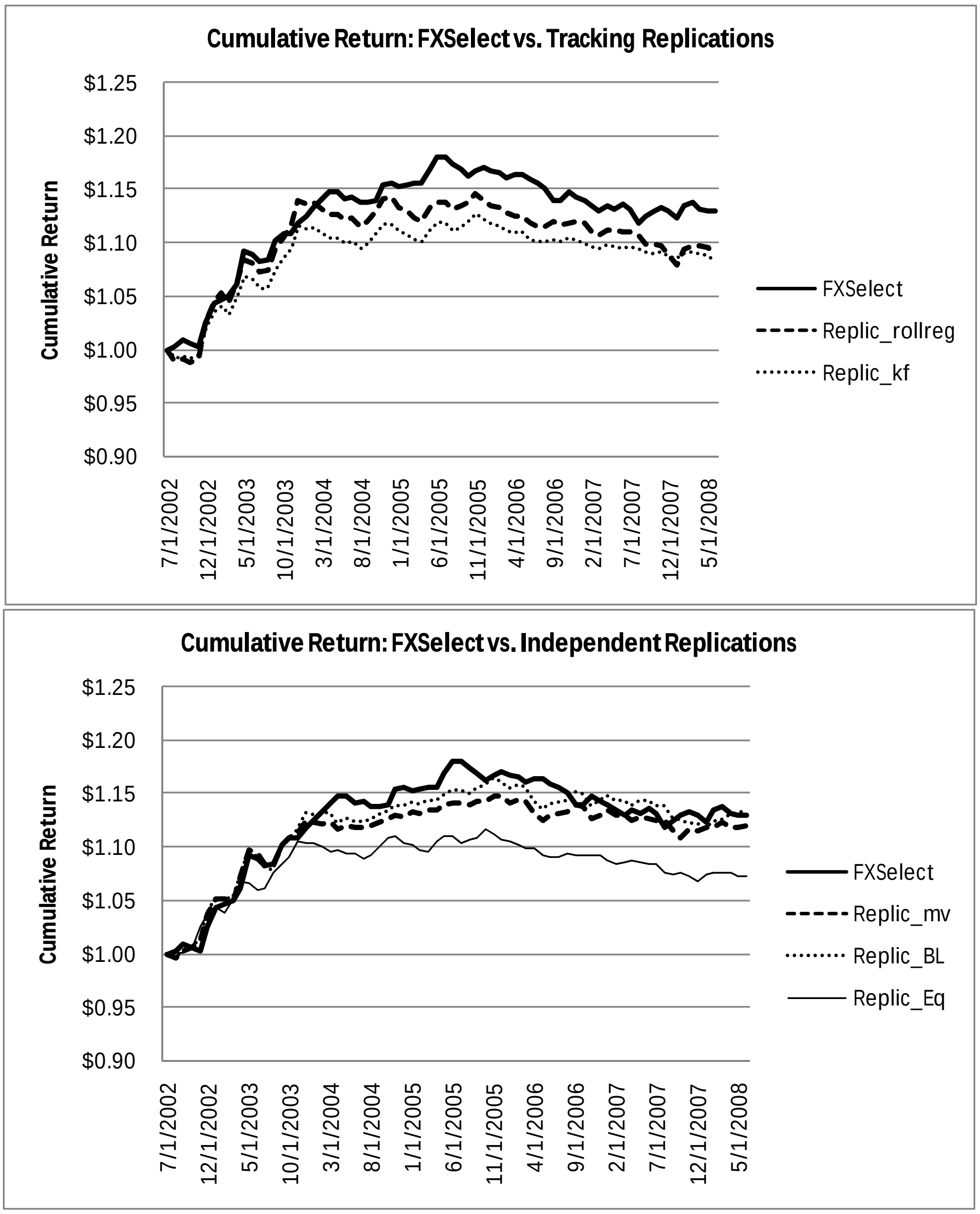
The first step is replicating the FXSelect index that has the best risk-adjusted performance among published active currency managers. We use the three most common currency betas—carry, value, and trend—that were statistically significant in the in-sample regression analysis with an R-squared of 0.41. We apply the weights from a 24-month rolling regression and the Kalman filter for the sample at time t to the portfolio at time t+1, and compare the cumulative return of $1.00 invested in the following alternatives:

1. FXSelect Manager Index
2. 24-month rolling window Regression Replic_rollreg
3. Kalman Filter Beta Replic_kf
4. Equal-weighted Beta Replic_eq
5. Mean-Variance Optimized Beta Replic_mv
6. Black-Litterman Optimized Beta Replic_BL

The results, which are presented in Figure 11 and Table 13, demonstrate that the FXSelect manager index delivers excess performance over dynamic tracking clones and the equal-weighted beta index. However, independently optimized portfolios of beta factors achieve comparable returns to the FXSelect manager index. Therefore, we conclude that the diversified active currency manager index returns can be roughly approximated using a mean-variance optimized portfolio of simple currency betas.

Figure 11

Out-of-sample replication of FXSelect index using currency beta factors



As Table 13 shows, the statistical forecasting accuracy measures do not necessarily correspond to the risk-adjusted performance measures; for example, the naïve equal-weighted portfolio has the lowest mean absolute deviation (MAD) and mean-squared error (MSE), but has a relatively lower Sharpe ratio and percentage of positive months.

Table 13

Performance comparison: FXSelect manager index vs. currency beta replications, 2000-2008

	Average	**Replic_ RollReg**	**Replic_ KalmanF**	**Replic_ MeanVar**	**Replic_ BlackLitt**	**Replic_ EqWeight**
Ann Ret	2.2%	1.5%	1.4%	2.0%	2.2%	1.1%
Ann Vol	2.7%	3.0%	2.5%	2.4%	2.6%	2.0%
Sharpe	0.81	0.49	0.56	0.82	0.88	0.54
Cumulative Return	13%	9%	8%	12%	13%	7%
%, positive	56%	47%	44%	54%	58%	44%
Skewness	1.2	1.4	1.4	1.2	0.9	1.3
Kurtosis	2.3	2.3	1.9	3.0	1.9	1.6
MaxLoss	-1%	-1%	-1%	-1%	-1%	-1%
Correlation	1.00	0.69	0.70	0.50	0.50	0.68
MAD		0.5%	0.4%	0.6%	0.6%	0.4%
MSE		0.004%	0.003%	0.005%	0.006%	0.003%

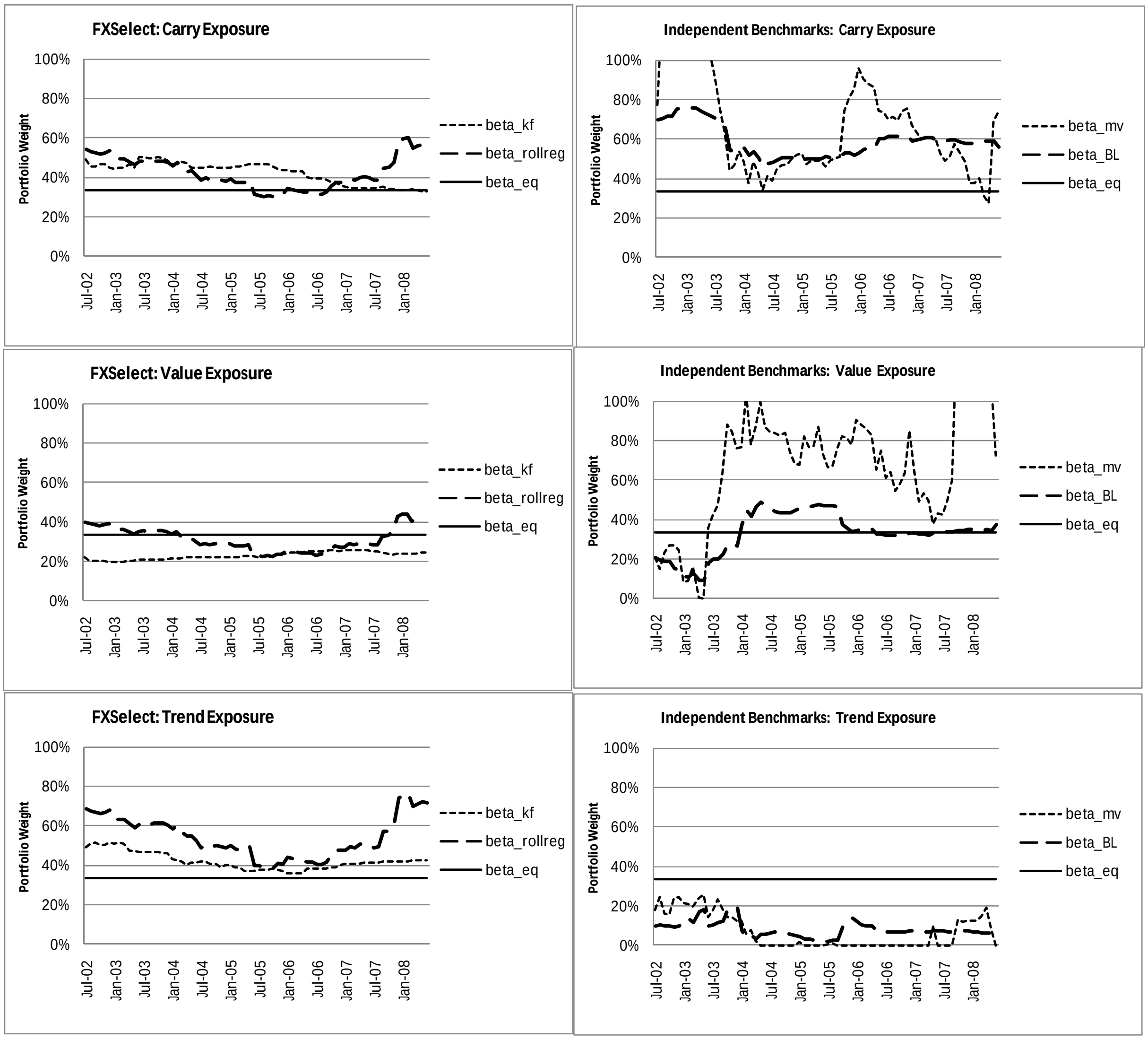
Sources: Bloomberg; authors' calculations.

In terms of portfolio weights, the rolling window replication requires frequent rebalancing of portfolio weights as Figure 12 shows, while the Kalman filter weights are slow-changing over time. Among independent replications, mean-variance optimized weights are prone to sudden and extreme changes, while the Black-Litterman portfolio is more stable. With estimation errors in mind, the FXSelect index seems to be diversified across three main styles. The relative exposures are stable over time, with larger allocations to carry and trend and smaller allocations of value strategies.

Based on these data, we can report that the FXSelect active currency manager index can be replicated out-of-sample using three currency beta strategies: carry, trend, and value. As Table 13 shows, out of cumulative excess return of 13% for the target index, the Black-Litterman beta portfolio replicated 13%, the mean-variance beta portfolio 12% and the rolling regression 9%.

Figure 12

FXSelect index vs. replications: portfolio weights for currency betas



We extend this analysis to the three currency manager indices dominated by the trend-following style as the in-sample analysis demonstrated. The out-of-sample replication results are shown in Figure 13 and Table 14. Barclays, Stark, and CISDM currency manager indices are relatively well replicated using the Kalman filter method (Replic_KF) and rolling regressions (Replic_rreg), while independent replication methods (Replic_EQ, Replic_MV, and Replic_BL) have quite different performances even after volatility is adjusted to the target manager index. The explanation is simple: these currency manager indices are primarily exposed to trend-following strategies, while independent beta indices are diversified across all four strategies.

Figure 13

Out-of-sample replications of currency CTA manager indices

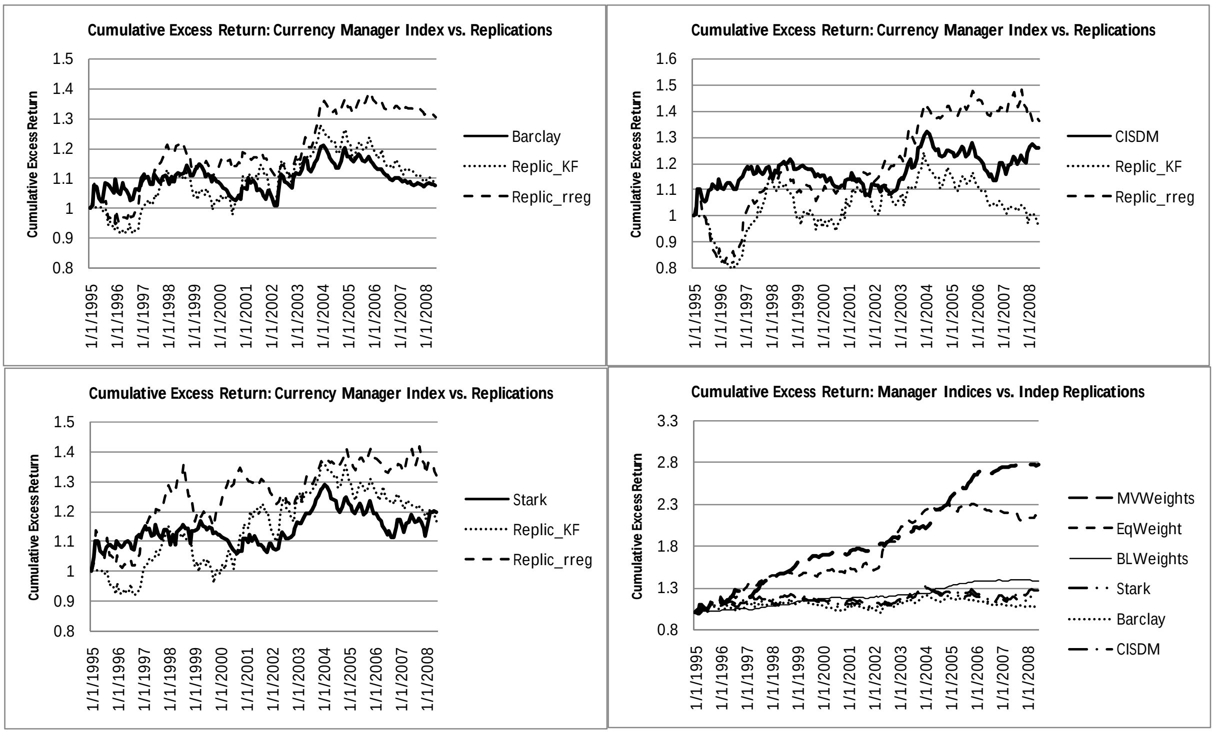


Table 14

Performance comparison: currency manager indices vs. replications

Index name		Manager index	In-Sample K-Filter	Replica_K-Filter	Replica_ rollreg
Barclays	RMSE	-	0%	2%	2%
	MAD	-	0%	2%	1%
	Total return	8%	24%	7%	30%
	Sharpe	0.09	0.31	0.06	0.35
Stark	RMSE	-	0%	2%	2%
	MAD	-	0%	2%	1%
	Total return	20%	28%	16%	32%
	Sharpe	0.22	0.50	0.14	0.29
CISDM	RMSE	-	1%	3%	2%
	MAD	-	1%	2%	2%
	Total return	26%	26%	-4%	36%
	Sharpe	0.29	0.39	(0.04)	0.33

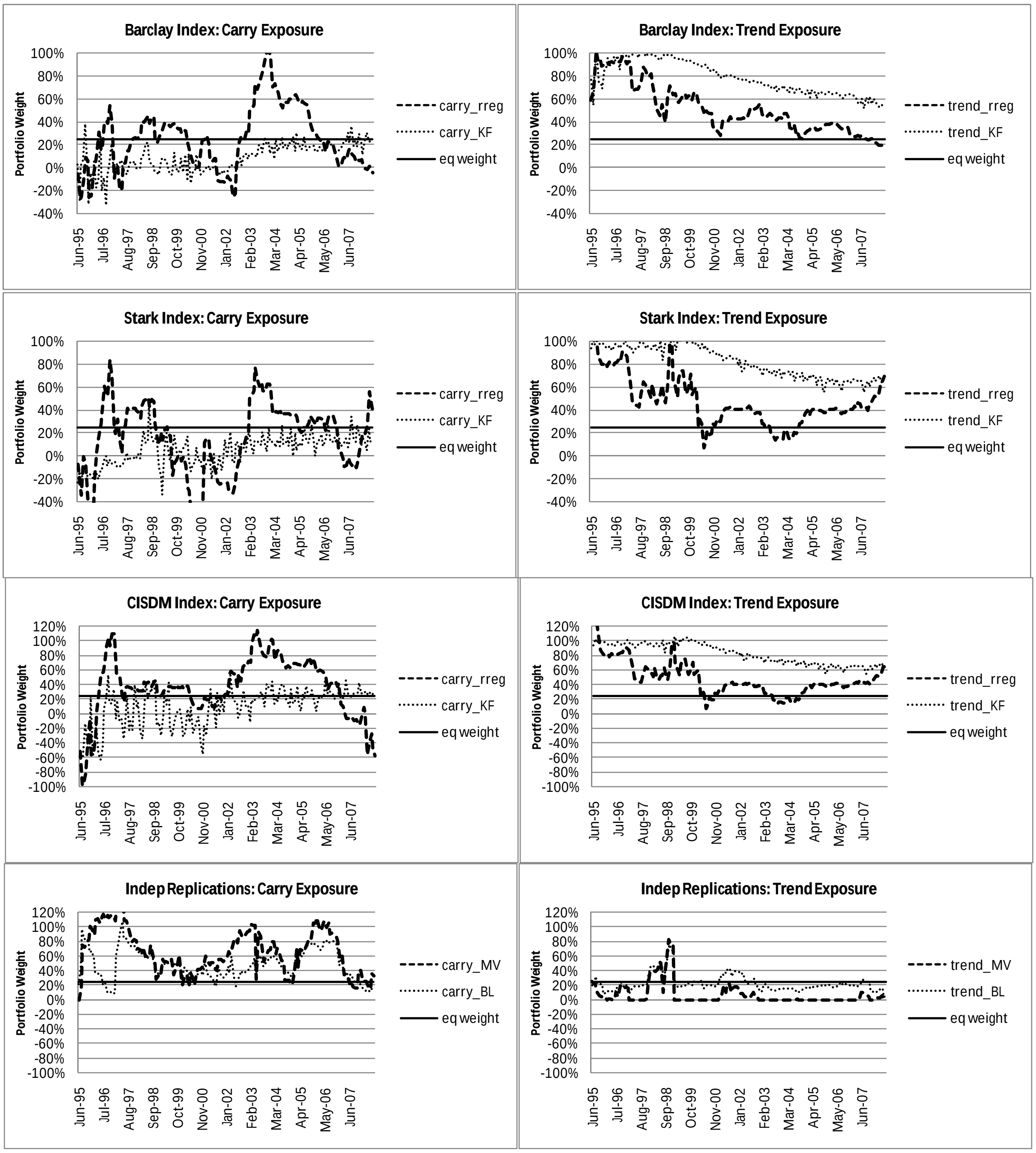
Sources: Bloomberg; authors' calculations.

The exposures of replication indices to the currency betas are shown in Figure 14. As Figure 14 demonstrates, these indices have historically been dominated by the trend strategy, while independent replications have allocated little weight to this strategy. This finding is in line with those of Middleton (2005). Since the year 2000, trend exposures have been declining and carry allocations increasing, probably reflecting relative profitability.

Based on these results, we can conclude that, similarly to managers represented in FXSelect index, constituents of the Barclays, CISDM, and Stark currency manager indices have adjusted their aggregate allocations to currency beta strategies over time to adapt to the market environment.

Figure 14

CTA indices vs. out-of-sample replications: portfolio weights for currency betas



4.4 Out-of-sample replication of individual currency managers

This section presents the results of the individual manager replications using the tracking methodology (i.e. rolling regressions and Kalman filters) as well the independent benchmarks. Two versions of the Kalman filter replication are presented. The following abbreviations are used in the figures below: equal-weighted (EW), mean-variance optimized (MV) and Black-Litterman (BL) beta indices. The abbreviations for the Kalman filter replications are: smoothed full sample beta index with look-ahead bias (kalmanSMO) and out-of-sample forecasted beta index (kalmanFOR). All of the beta indices (RollReg, kalmanFOR, EW, MV, BL) except kalmanSMO represent out-of-sample replications as described in the methodology chapter.

Figures 15-19 compare the performance characteristics of sample currency managers vs. their beta replicated return indices.[19] The figures provide data regarding average total return, Sharpe ratio, maximum monthly loss, skewness and kurtosis, and the forecasting errors in terms of MAD and MSE, as well as the correlations to the target manager.

In terms of total returns, the equal-weighted beta index achieves the highest return, followed by the BL and MV indices (Figure 15); the tracking beta indices such as kalmanFOR and RollReg achieve lower total return than independent indices. The relative underperformance of the tracking beta indices could be due to estimation errors.

Figure 15

Average total return: managers vs. beta replication indices

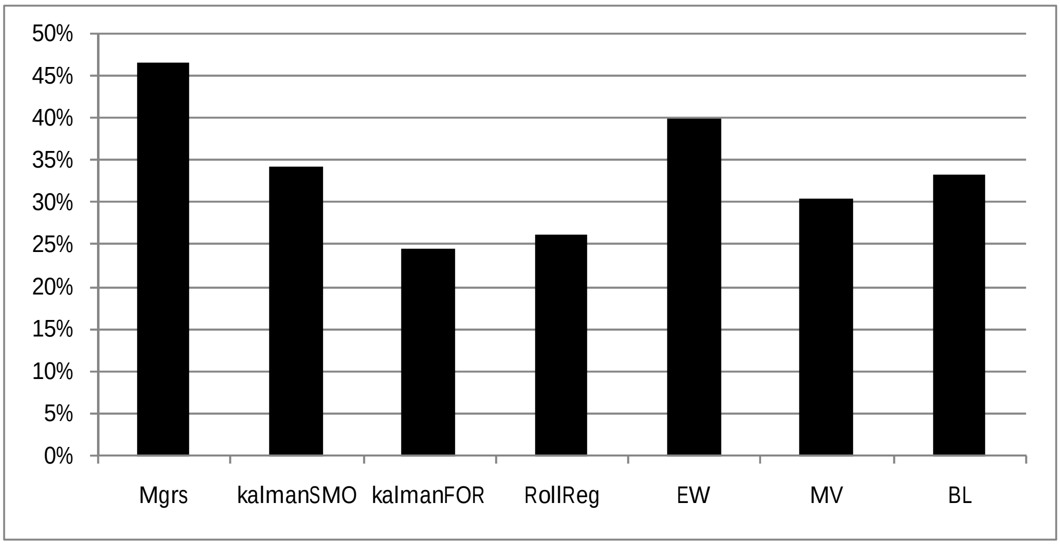


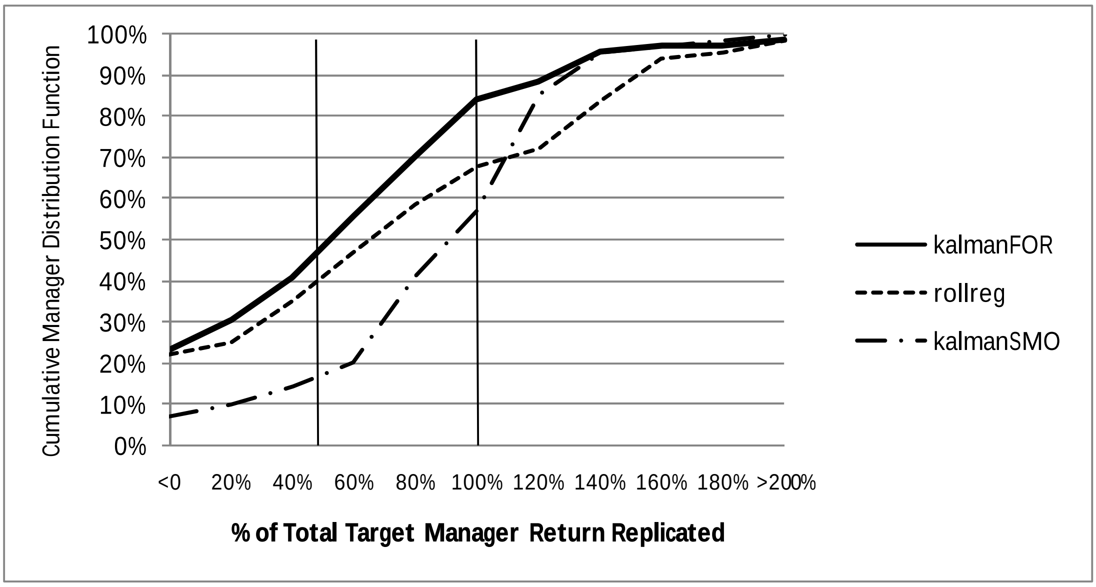
Figure 16 presents the cumulative distribution of sample managers and replication success (in terms of percent of total target return) of various beta indices. Based on these data, we can report that the rolling regression method succeeds at fully replicating total target manager return for only 30% of the sample managers and the Kalman filter only has a success rate of 15%, although the in-sample version (kalmanSMO) has a better success rate of 40%. The rolling regression succeeds in replicating at least half of the total target return for

[19] Please note that individual managers have differing lengths of reporting history and replication indices are estimated for each manager separately using currency betas for that time period.

60% of sample managers, and the Kalman filter succeeds for 50% of the managers, although the rate is 80% for the in-sample kalmanSMO.

Figure 16

Tracking replication: CDF function vs. % of total target return



All three independent currency beta combinations – equal-weighted, mean-variance, and Black-Litterman – can replicate the full total target return for 30% of managers (Figure 17). In terms of replicating at least half of the total target return, the mean-variance beta index succeeds for 60% of managers, while the equal-weighted and Black-Litterman beta indices do so for 50% of managers.

Figure 17

Independent replications: CDF function vs. % of total target return replicated

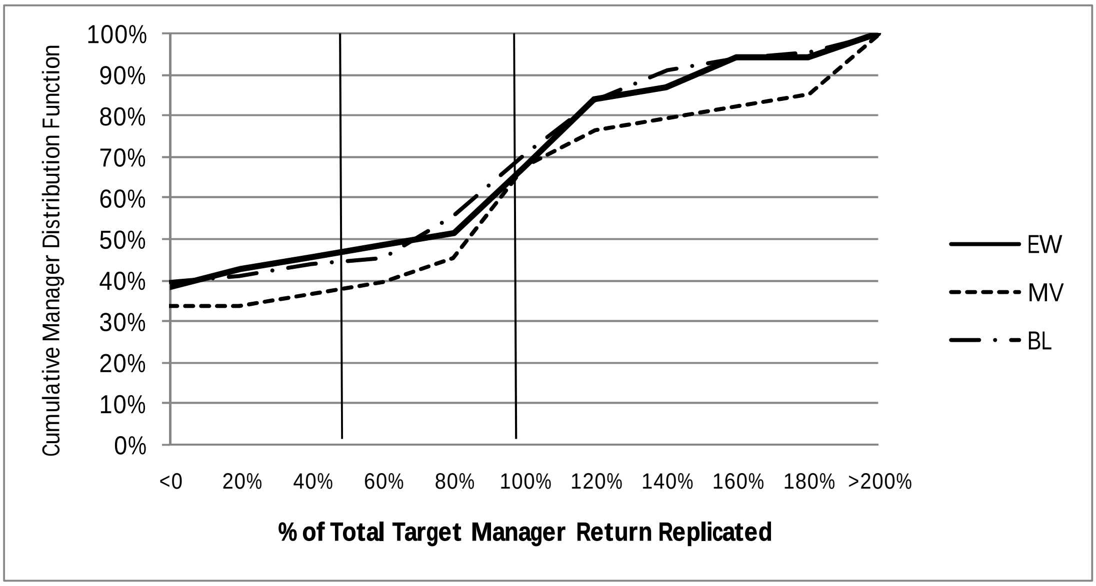
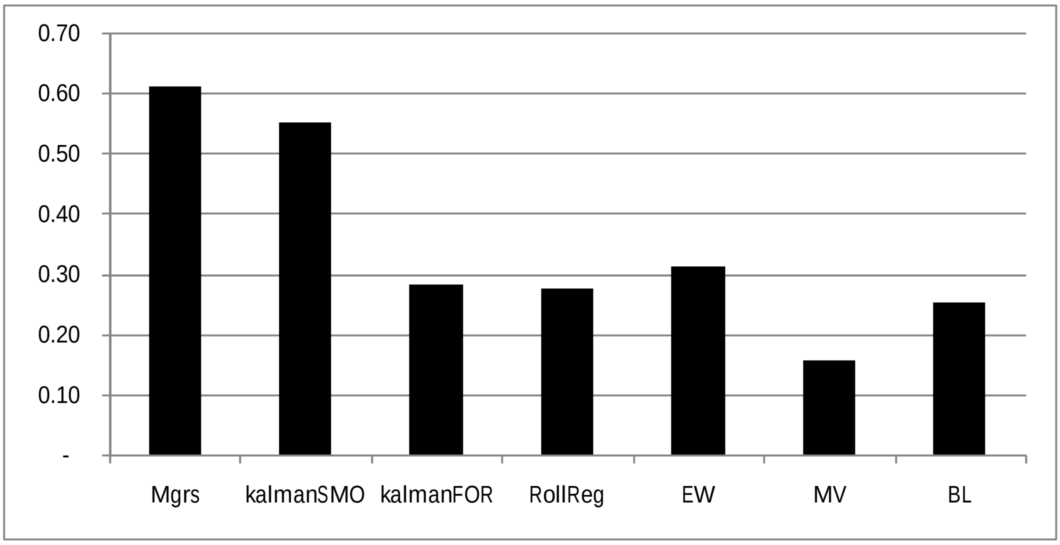


Figure 18 presents the average risk-adjusted returns of various replication methods. The in-sample Kalman smoothed replication achieves a very similar Sharpe ratio when compared to the target managers ex-post, but the out-of-sample Sharpe ratios are only half of the target managers' ratios on average in the sample.

Figure 18

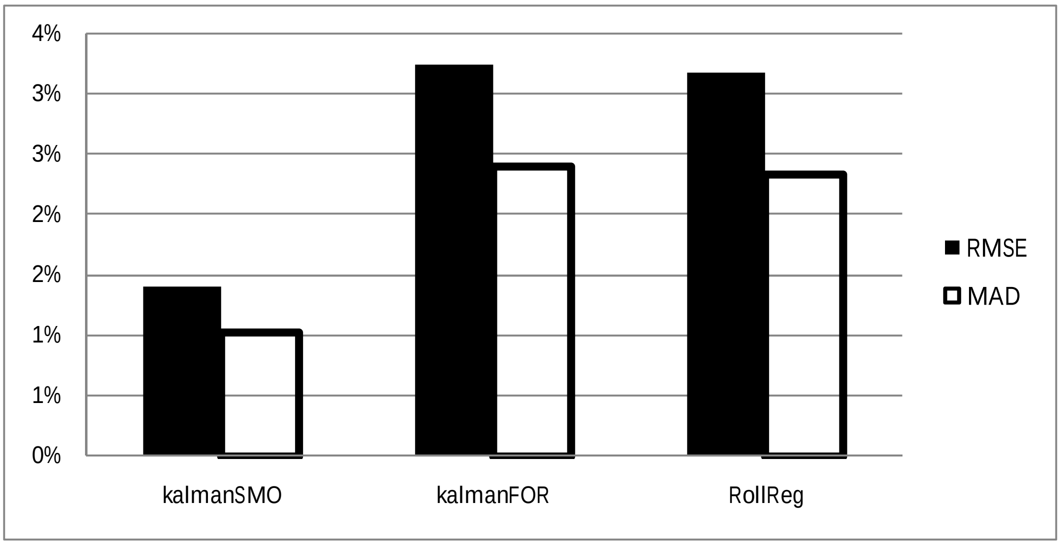
Average Sharpe ratio: managers vs. beta replication indices



In terms of statistical errors (MAD and RMSE), the in-sample Kalman filter naturally achieves the lowest errors, while the out-of-sample tracking beta indices, kalmanFOR and RollingReg, have broadly similar errors on average (Figure 19).

Figure 19

Forecasting errors for beta replication indices



To conclude, the independent beta indices have a better chance than tracking beta indices in terms of replicating both returns of the currency managers. In fact, a simple equally weighted beta index performs quite well in replicating currency manager returns across a number of criteria.

The results show it is quite hard to track the month-to-month performance of individual currency managers using simple beta currency strategies. It is possible to achieve comparable performance over time, but very hard to fully imitate individual currency managers' dynamic allocation between currency trading strategies. On the other hand, it is relatively easier to imitate the results of a diversified currency manager index which reduces the idiosyncratic approaches and delivers the average returns for the most popular and profitable currency strategies.

5. Study results

In this study we show that the profits of active currency managers can be attributed to the deviations from purchasing power parity (PPP), uncovered interest rate parity (UIP), overshooting, and the risk-averse behavior of currency market participants. In particular, half of the profits from currency speculation can be explained by managers systematically taking advantage of the forward rate bias, mean reversion to equilibrium fundamental value, trending of currency prices, and mean reversion of currency volatilities. In this study we use a larger and a more recent sample to confirm and expand upon the findings from previous studies, such as, Middleton (2005) and Levich and Pojarliev (2007).

Using full sample historical regressions, the results reveal that about half of the currency managers' returns can be attributed to the simple trading strategies. The alpha (excess return) over the simple trading strategies is positive and significant; however, half of this can be attributed to the market timing skills of simple trading strategies. The remainder of the returns reflects either unique trading strategies and/or other manager-related skills. Therefore, we can conclude that individual currency managers have substantial skill and, using their own trading strategies or market timing systems, they achieve better returns than those associated with naïve trading strategies. On the other hand, the idiosyncratic manager alpha components of composite currency manager indices and larger portfolios cancel each other out, leaving mostly currency beta returns, such as, carry, value, and trend. While we find that many individual managers have unique skills that result in outperformance over simple beta strategies, some of it could be due to luck. This is especially true for managers with smaller track records (1-2 years) – given the smaller sample size, investors typically exercise caution when evaluating performance potential based on return history alone. Yet if the manager track record spans several different market environments, there is more room for returns-based analysis.

Out-of-sample replication was conducted using tracking replications (rolling regressions and Kalman filters) as well as by using independent combinations of simple trading strategies, such as the naïve equally weighted, mean-variance optimized, and the Black-Litterman approaches. We can conclude that it is very hard to forecast the monthly returns of individual managers with simple trading strategies using rolling regressions and Kalman filters or independent combinations of currency betas. However, it is possible to achieve a comparable absolute total return over a reasonable period of time by using these simple strategies. Hence, it is very difficult to define a systematic rule for imitating individual currency managers' trading behavior. In fact, the naïve equal-weighted beta strategy achieves the best replication record among all alternatives judging across various criteria.

Composite manager indices, however, may be well replicated using a mean-variance optimized portfolio of currency betas. In other words, the aggregate behavior of active currency managers can be described as a mean-variance optimizing allocation between major trading strategies of carry, value, and trend. Overall, at least one beta replication index matches over 100% of total return for FXSelect, Barclays, Stark and CISDM composite currency manager indices.

The key contributions to the academic literature resulting from this study are that we can explain about half of the returns from currency speculation by capturing the forward rate bias (the carry factor), timing the deviations from PPP (the value factor), currency trending behavior (overshooting, interventions, etc.) and risk-aversion (volatility).

How then do we reconcile the random walk nature of exchange rates and the profitability of the active currency managers? In our results we find that about half of the active currency managers have statistically significant exposures to carry beta strategy. Forward rate bias is a UIP violation, the failure of future spot rates to offset the forward implied interest rate differential between two currencies. Although spot rates resemble the random walk at shorter horizons, the interest rate differentials embedded in the forward rates are rather slow-moving and allow some predictability (Burnside et al., 2006). The empirical properties of carry profits resemble insurance premiums; they tend to be erased during market crises and typically have a negative skew. A skilled currency manager captures the carry profits during periods of low volatility and avoids the strategy during volatile times.

Although the short-term spot exchange rate behavior is very close to a random walk, over longer horizons the exchange rates tend to revert to the fundamental equilibrium values (PPP, etc.). As highlighted in Killan and Taylor (2003), the likelihood of reversion to the mean is higher for significant deviations from the equilibrium, and currency managers are able to capture this tendency using customized trading rules. Value strategy typically has a positive skew. This strategy is less profitable because few currencies are substantially over- or undervalued at a given point in time, thus limiting the scope for profits. The study's results show that about one-third of the managers have statistically significant exposure to the value strategy.

The results also demonstrate that almost half of the currency managers maintain statistically significant exposures to the trend-following strategy. Trend-following is driven by the autocorrelation of currency prices over various horizons. Autocorrelation can be purely spurious or a result of central bank intervention, currency overshooting, continuous economic expansion, or a sell-off during a market crisis (Meese et al., 2002; Neely et al., 2007; Xin, 2003). In the very short term, the autocorrelation can be the result of investor positioning (for example, using stop-loss limits). Currency trend-following is based on moving average crossovers and filters. It does not reflect the underlying economic rationale; rather, it focuses on the empirical properties of short-term currency movements. Although it does not require extensive macroeconomic research, strict risk management is a must for getting out of losing trades—it is not a surprise that trend-following strategy returns typically exhibit a positive skew. Trend-following was the most profitable currency trading strategy before the year 2000, but it did not perform well again until the 2008 market crisis. There is some evidence of trend-following being profitable in high frequency (intra-day) trading and in emerging market currencies.

Finally, about one-fifth of the currency managers use the volatility strategy, taking advantage of the mean reversion of currency price volatilities. During sell-offs, market participants tend to favor certain currencies such as the Japanese yen, Swiss franc or U.S. dollar despite their macroeconomic fundamentals. Volatility also tends to spike around macroeconomic announcements or political events and then tends to return to normal. Currency managers can take advantage of the mean reversion properties of volatilities.

We make a further contribution to the academic literature about active currency managers by adapting the hedge fund methodology to replicate the manager returns out-of-sample. The findings show that while the total dollar return of composite manager indices and diversified manager portfolios can be replicated over time, it is not as easy to imitate the dynamic trading and allocation strategies of individual currency managers. The proposed allocation rules could fully replicate the total dollar return for only 30% of the managers, while half of the total return could be replicated for up to 60% of managers.

These results have several implications for both public and private investment practitioners. First, the results provide further evidence that the main trading strategies defined in the literature, such as carry, value, trend, and volatility, can explain the substantial portion of aggregate profits from active currency management. Second, the findings demonstrate that it is not easy to replicate the trading approach of any individual manager, although achieving a similar absolute dollar return over time is quite possible. Third, the results show that it is much easier to replicate a diversified portfolio or a composite index of currency managers using simple currency betas, as individual alphas tend to cancel out. Hence, private and, in particular, public investors can use the active currency replication index as an alternative mechanism or, more importantly, as a redundancy risk evaluator for currency risk management or investment. Although this vehicle also provides an alternative for investors as a less expensive currency beta composite index (as opposed to a more expensive diversified currency manager fund), we note a need for a very high degree of caution in that application. Instead, we encourage the use of a less expensive currency beta composite index as a venue for enhancing our knowledge and evaluating the management and ROR of existing active currency funds. This currency beta composite index may have multiple applications, ranging from education and tracking market strategies to ROR, either before investing public or private funds or in parallel to track and gauge the management of funds already invested with large currency fund managers. It would provide public investors with an additional redundant mechanism that serves as a companion risk evaluator or performance gauge to make more informed choices with respect to currency risk management. We believe this contributes to the best practices in the arena of public fund investments.

The caveat is, of course, that things change rapidly, especially in currency markets. Andrew Lo states in his adaptive markets hypothesis that simple trading strategies will become less profitable over time and will have to be modified to achieve competitive return (Lo, 2004). Therefore, it may be reasonable to conclude that individual managers will be more adaptive to market conditions and will revise their active currency trading models and strategies to remain profitable, while simple strategy formulations become obsolete over time.

Future research in this area could focus on identifying other trading strategies commonly used by active currency managers that could shed further light on how profits are generated from currency trading. In particular, defining the trading rules that capture the non-linear properties of currency manager returns, such as market timing, could help better replicate the currency manager returns out-of-sample. Alternatively, future research could also focus on better statistical methodologies to replicate the profits using existing simple strategies.

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An option theoretic model for ultimate loss-given-default with systematic recovery risk and stochastic returns on defaulted debt

Michael Jacobs, Jr[1]

1. Introduction and Summary

Loss-given-default (LGD),[2] the loss severity on defaulted obligations, is a critical component of risk management, pricing and portfolio models of credit. This is among the three primary determinants of credit risk, the other two being the *probability of default* (PD) and *exposure of default* (EAD). However, LGD has not been as extensively studied, and is considered a much more daunting modeling challenge than other components, such as PD. Starting with the seminal work by Altman (1968), and after many years of actuarial tabulation by rating agencies, predictive modeling of PD is currently in a mature stage. The focus on PD is understandable, as traditionally credit models have focused on systematic components of credit risk which attract risk premia, and unlike PD, determinants of LGD have been ascribed to idiosyncratic borrower specific factors. However, now there is an ongoing debate about whether the risk premium on defaulted debt should reflect systematic risk, in particular whether the intuition that LGDs should rise in worse states of the world is correct, and how this could be refuted empirically given limited and noisy data (Carey and Gordy, 2007).

The recent heightened focus on LGD is evidenced by the flurry of research into this relatively neglected area (Acharya et al [2007], Carey and Gordy [2007], Altman et al [2001, 2003, 2005], Altman [2006], Gupton et al [2000, 2005], Araten et al [2004], Frye [2000 a,b,c, 2003], Jarrow [2001]). This has been motivated by the large number of defaults and near simultaneous decline in recovery values observed at the trough of the last credit cycle circa 2000-2002, regulatory developments such as Basel II (BIS [2003, 2005, 2006], OCC et al [2007]) and the growth in credit markets. However, obstacles to better understanding and predicting LGD, including dearth of data and the lack of a coherent theoretical underpinning, have continued to challenge researchers. In this paper, we hope to contribute to this effort by synthesizing advances in financial theory to build a model of LGD that is consistent with a priori expectations and stylized facts, internally consistent and amenable to rigorous validation. In addition to answering the many questions that academics have, we further aim to provide a practical tool for risk managers, traders and regulators in the field of credit.

LGD may be defined variously depending upon the institutional setting or modeling context, or the type of instrument (traded bonds vs. bank loans) versus the credit risk model (pricing debt instruments subject to the risk of default vs. expected losses or credit risk capital). In the

1 Senior Financial Economist, Credit Risk Analysis Division, Department of Economic and International Affairs, Office of the Comptroller of the Currency, One Independence Square, Suite 3144, Washington, DC 20024, 202-874-4728, michael.jacobs@occ.treas.gov. The views expressed herein are those of the author and do not necessarily represent a position taken by of the Office of the Comptroller of the Currency or the U.S. Department of the Treasury.

2 This is equivalent to one minus the *recovery rate*, or dollar recovery as a proportion of par, or EAD assuming all debt becomes due at default. We will speak in terms of LGD as opposed to recoveries with a view toward credit risk management applications.

case of bonds, one may look at the price of traded debt at either the initial credit event,[3] the market values of instruments received at the resolution of distress[4] (Keisman et al, 2000; Altman et al, 1996) or the actual cash-flows incurred during a workout.[5] When looking at loans that may not be traded, the eventual loss per dollar of outstanding balance at default is relevant (Asarnow et al, 1995; Araten et al, 2004). There are two ways to measure the latter – the *accounting LGD* refers to nominal loss per dollar outstanding at default,[6] while the *economic LGD* refers to the discounted cash flows to the time of default taking into consideration when cash was received. The former is used in setting reserves or a loan loss allowance, while the latter is an input into a regulatory or economic credit capital model.

In this study we develop various theoretical models for ultimate loss-given-default in the Merton (1974) structural credit risk model framework. We consider an extension that allows for differential seniority within the capital structure, an independent recovery rate process, representing undiversifiable recovery risk, with stochastic drift. The comparative statics of this model are analyzed and compared to a baseline model, all of these in a framework that incorporates an optimal foreclosure threshold (Carey and Gordy, 2007). In the empirical exercise, we calibrate alternative models for ultimate LGD on bonds and loans having both trading prices at default and at resolution of default, utilizing an extensive sample of rated defaulted firms in the period 1987-2008 (Moody's Ultimate Recovery Database™ - URD™), 800 defaults (bankruptcies and out-of-court settlements of distress) that are largely representative of the US large corporate loss experience, for which we have the complete capital structures and can track the recoveries on all instruments to the time of default to the time of resolution.

We find that parameter estimates vary significantly across models and recovery segments. estimated volatilities of the recovery rate processes, as well as of their random drifts, are found to increasing in seniority, in particular for bank loans as compared to bonds. We interpret this as reflecting greater risk in the ultimate recovery for higher ranked instruments having lower expected loss severities (or ELGDs). Analyzing the implications of our model for the quantification of downturn LGD, we find the later to be *declining* in expected LGD, higher for worse ranked instruments, increasing in the correlation between the process driving firm default and recovery on collateral, and increasing in the volatility of the systematic factor specific to the recovery rate process or the volatility of the drift in such. Finally, we validate the leading model derived herein in an out-of-time and out-of-sample bootstrap exercise, comparing it to a high-dimensional regression model, and to a non-parametric benchmark based upon the same data, where we find our model to compare favorably. We conclude that our model is worthy of consideration to risk managers, as well as supervisors concerned with advanced IRB under the Basel II capital accord.

This paper is organized as follows. Section 2 reviews the literature, focusing on the treatment of LGD in theoretical credit models, both academic and practitioner. Section 3 presents the theoretical framework. Section 4 discusses comparative statics of the alternative models. Section 5 describes the econometric framework. Section 6 describes the data used in this study and presents the calibration analysis of structural model parameters. In Section 7 we

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[3] By default we mean either bankruptcy (Chapter 11) or other financial distress (payment default). In a banking context, this defined as synonymous with respect to non-accrual on a discretionary or non-discretionary basis. This is akin to the notion of default in Basel, but only proximate.

[4] Note that this may be either the value of pre-petition instruments received valued at emergence from bankruptcy, or the market values of new securities received in settlement of a bankruptcy proceeding or as the result of a distressed restructuring.

[5] Note that the former may viewed as a proxy to this, the pure economic notion.

[6] In the context of bank loans, this is the cumulative net charge-off as a percent of book balance at default (the *net charge-off rate)*.

discuss the implications of our modeling framework for downturn LGD. In Section 8 we perform an out-of-sample validation of our model and two alternative benchmarks. Finally, Section 9 concludes and discusses directions for future research.

2. Review of the literature

In this section we will examine the way in which different types of theoretical credit risk models have treated LGD – assumptions, implications for estimation and application. Credit risk modeling was revolutionized by the approach of Merton (1974), who built a theoretical model in the option pricing paradigm of Black and Scholes (1973), which has come known to be the *structural approach*. Equity is modeled as a call option on the value of the firm, with the face value of zero coupon debt serving as the strike price, which is equivalent to shareholders buying a put option on the firm from creditors with this strike price. Given this capital structure, log-normal dynamics of the firm value and the absence of arbitrage, closed form solutions for the default probability and the spread on debt subject to default risk can be derived. The LGD can be shown to depend upon the parameters of the firm value process as is the PD, and moreover is directly related to the latter, in that the expected residual value to claimants is increasing (decreasing) in firm value (asset volatility or the level of indebtedness). Therefore, LGD is not independently modeled in this framework; this was addressed in much more recent versions of the structural framework (Frye [2000 a,b], Dev and Pykhtin [2002], Pykhtin [2003]).

Extensions of Merton (1974) relaxed many of the simplifying assumptions of the initial structural approach. Complexity to the capital structure was added by Black and Cox (1976) and Geske (1977), with subordinated and interest-paying debt, respectively. The distinction between long- and short-term liabilities in Vasicek (1984) was the precursor to the KMV™ model. However, these models had limited practical applicability, the standard example being evidence of Jones, Mason and Rosenfeld (1984) that these models were unable to price investment-grade debt any better than a naïve model with no default risk. Further, empirical evidence in Franks and Touros (1989) showed that the adherence to absolute priority rules (APR) assumed by these models are often violated in practice, which implies that the mechanical negative relationship between expected asset value and LGD may not hold. Longstaff & Schwartz (1995) incorporate into this framework a stochastic term structure with a PD-interest rate correlation. Other extensions include Kim at al (1993) and Hull & White (2002), who examine the effect of coupons and the influence of options markets, respectively.

Partly in response to this, a series of extensions ensued, the so-called “second generation” of structural form credit risk models (Altman, 2003). The distinguishing characteristic of this class of models is the relaxation of the assumption that default can only occur at the maturity of debt – now default occurs at any point between debt issuance and maturity when the firm value process hits a threshold level. The implication is that LGD is exogenous relative to the asset value process, defined by a fixed (or exogenous stochastic) fraction of outstanding debt value. This approach can be traced to the barrier option framework as applied to risky debt of Black and Cox (1976).

All structural models suffer from several common deficiencies. First, reliance upon an unobservable asset value process makes calibration to market prices problematic, inviting model risk. Second, the limitation of assuming a continuous diffusion for the state process implies that the time of default is perfectly predictable (Duffie and Lando, 2001). Finally, the inability to model spread or downgrade risk distorts the measurement of credit risk. This gave rise to the *reduced form approach* to credit risk modeling (Duffie and Singleton, 1999), which instead of conditioning on the dynamics of the firm, posit exogenous stochastic processes for PD and LGD. These models include (to name a few) Litterman & Iben (1991), Madan & Unal

(1995), Jarrow & Turnbull (1995), Lando (1998) and Duffie (1998). The primitives determining the price of credit risk are the term structure of interest rates (or short rate), and a default intensity and an LGD process. The latter may be correlated with PD, but it is exogenously specified, with the link of either of these to the asset value (or latent state process) not formally specified. However, the available empirical evidence (Duffie and Singleton, 1999) has revealed these models deficient in generating realistic term structures of credit spreads for investment and speculative grade bonds simultaneously. A hybrid reduced – structural form approach of Zhou (2001), which models firm value as a jump diffusion process, has had more empirical success, especially in generating a realistic negative relationship between LGD and PD (Altman et al, 2006).

The fundamental difference between reduced and structural form models is the unpredictability of defaults: PD is non-zero over any finite time interval, and the default intensity is typically a jump process (eg Poisson), so that default cannot be foretold given information available the instant prior. However, these models can differ in how LGD is treated. The *recovery of treasury* assumption of Jarrow & Turnbull (1995) assumes that an exogenous fraction of an otherwise equivalent default-free bond is recovered at default. Duffie and Singleton (1999) introduce the *recovery of market value* assumption, which replaces the default-free bond by a defaultable bond of identical characteristics to the bond that defaulted, so that LGD is a stochastically varying fraction of market value of such bond the instant before default. This model yields closed form expressions for defaultable bond prices and can accommodate the correlation between PD and LGD; in particular, these stochastic parameters can be made to depend on common systematic or firm specific factors. Finally, the *recovery of face value* assumption (Duffie [1998], Jarrow et al [1997]) assumes that LGD is a fixed (or seniority specific) fraction of par, which allows the use of rating agency estimates of LGD and transition matrices to price risky bonds.

It is worth mentioning the treatment of LGD in credit models that attempt to quantify unexpected losses analogously to the *value-at-risk* (VaR) market risk models, so-called *credit VaR* models (Creditmetrics™ [Gupton et al, 1997], KMV CreditPortfolioManager™ [Vasicek, 1984], CreditRisk^{+}™ [Credit Suisse Financial Products, 1997], CreditPortfolioView™ [Wilson, 1998]). These models are widely employed by financial institutions to determine expected credit losses as well as economic capital (or unexpected losses) on credit portfolios. The main output of these models is a probability distribution function for future credit losses over some given horizon, typically generated by simulation of analytical approximations, as it is modeled as highly non-normal (asymmetrical and fat-tailed). Characteristics of the credit portfolio serving as inputs are LGDs, PDs, EADs, default correlations and rating transition probabilities. Such models can incorporate credit migrations (*mark-to-market mode* - MTM), or consider the binary default vs. survival scenario (*default mode* - DM), the principal difference being that in addition an estimated transition matrix needs to be supplied in the former case. Similarly to the reduced form models of single name default, LGD is exogenous, but potentially stochastic. While the marketed vendor models may treat LGD as stochastic (eg a draw from a beta distribution that is parameterized by expected moments of LGD), there are some more elaborate proprietary models that can allow LGD to be correlated with PD.

We conclude our discussion of theoretical credit risk models and the treatment of LGD by considering recent approaches, which are capable of capturing more realistic dynamics, sometimes called “hybrid models”. These include Frye (2000a, 2000b), Jarrow (2001), Bakshi et al (2001), Jarrow et al (2003), Pykhtin (2003) and Carey & Gordy (2007). Such models are motivated by the conditional approach to credit risk modeling, credited to Finger (1999) and Gordy (2000), in which a single systematic factor derives defaults. In this more general setting, they share in common the feature that dependence upon a set of systematic factors can induce an endogenous correlation between PD & LGD. In the model of Frye (2000a, 2000b), the mechanism that induces this dependence is the influence of systematic factors upon the value of loan collateral, leading to a lower recoveries (and higher loss severity) in periods where default rates rise (since asset values of obligors also depend upon the same factors). In a reduced form setting, Jarrow (2001) introduced a model of co-dependent LGD and PD implicit in debt and equity prices.[7]

3. Theoretical model

The model that we propose is an extension of Black and Cox (1976). The baseline mode features perpetual corporate debt, a continuous and a positive foreclosure boundary. The former assumption removes the time dependence of the value of debt, thereby simplifying the solution and comparative statics. The latter assumption allows us to study the endogenous determination of the foreclosure boundary by the bank, as in Carey and Gordy (2007). We extend the latter model by allowing the coupon on the loan to follow a stochastic process, accounting for the effect of illiquidity. Note that in this framework, we assume no restriction on asset sales, so that we do not consider strategic bankruptcy, as in Leland (1994) and Leland and Toft (1996).

Let us assume a firm financed by equity and debt, normalized such that the total value of perpetual debt is 1, divided such that there is a single loan with face value λ and a single class of bonds with a face value of $1-\lambda$. The loan is senior to that bond, and potentially has covenants which permit foreclosure. The loan is entitled to a continuous coupon at a rate c, which in the baseline model we take as a constant, but may evolve randomly. Equity receives a continuous dividend, having a constant and a variable component, which we denote as $\delta+\rho V_t$, where V_t is the value of the firm's assets at time t. We impose the restriction that $0 \leq \rho \leq r \leq c$, where r is the constant risk-free rate. The asset value of the firm, net of coupons and dividends, follows a geometric Brownian motion with constant volatility σ:

$$\frac{dV_t}{V_t}=\left(r-\rho-\frac{C}{V_t}\right)dt+\sigma dZ_t \tag{3.1}$$

Where in (3.1) we denote the fixed cash outflows per unit time as:

$$C=c\lambda+\gamma\left(1-\lambda\right)+\delta \tag{3.2}$$

Where in (3.2), γ and δ are the continuous coupon rate on the bond and dividend yield on equity, respectively. Default occurs at time t and is resolved after a fixed interval τ, at which point dividend payments cease, but the loan coupon continues to accrue through the settlement period. At the point of emergence, loan holders receive $\left(\lambda\exp\left(c\tau\right),V_{t+\tau}\right)^{-}$, or the minimum of the legal claim or the value of the firm at emergence. We can value the loan at resolution, under risk neutral measure, using the standard Merton (1974) formula. Denote the total legal claim at default by:

$$D=\lambda\exp\left(\tau c\right)+\left(1-\lambda\right). \tag{3.3}$$

This follows from the assumption that the coupon c on the loan with face value λ continues to accrue at the contractual rate throughout the resolution period τ, whereas the bond with face value $1-\lambda$ does not.

[7] Jarrow (2001) also has the advantage of isolating the liquidity premium embedded in defaultable bond spreads.

Thus far we have assumed that the senior bank creditors foreclose on the bank when the value of assets is V_t, where t is the time of default. However, this is not realistic, as firm value fluctuates throughout the bankruptcy or workout period, and we can think that there will be some *foreclosure boundary* (denoted κ) below which foreclosure is effectuated. Furthermore, in most cases there exists a *covenant boundary*, above which foreclosure cannot occur, but below which it may occur as the borrower is in violation of a contractual provision. For the time being, let us ignore the latter complication, and focus on the optimal choice of κ by the bank. In the general case of time dependency in the loan valuation equation $F\left(V_t \mid \lambda, \sigma, r, \tau\right)$, following Black and Cox (1976), we have to solve a second-order partial differential equation. Following Carey and Gordy (2007), we modify this such that the value of the loan at the threshold is not a constant, but simply equal to the recovery value of the loan at the default time. Second, we remove the time dependency in the value of the perpetual debt. It is shown in Carey and Gordy (2007) that under these assumptions, so long as there are positive and fixed cash flows to claimants other than the bank, $\gamma\left(1-\lambda\right)>0$ or $\delta>0$, then there exists a finite and positive solution κ^*, the optimal foreclosure boundary.

We model undiversifiable recovery risk by introducing a separate process for recovery on debt, R_t. This can be interpreted as the state of collateral underlying the loan or bond. R_t is a geometric Brownian process that depends upon the Brownian motion that drives the return on the firm's assets Z_t, an independent Brownian motion W_t and a random instantaneous mean α_t:

$$\frac{dR_t}{R_t} = \alpha_t dt + \beta dZ_t + \upsilon dW_t \tag{3.6}$$

$$d\alpha_t = \kappa_\alpha\left(\bar{\alpha} - \alpha_t\right)dt + \eta dB_t \tag{3.7}$$

Where the volatility parameter β represents the sensitivity of recovery to the source of uncertainty driving asset returns (or the "systematic factor"), implying that the instantaneous correlation between asset returns and recovery is given by $\frac{1}{dt}Corr_t\left(\frac{dA_t}{A_t}\times\frac{dR_t}{R_t}\right)=\sqrt{\beta\sigma}$. On the other hand, the volatility parameter υ represents the sensitivity of recovery to a source of uncertainty that is particular to the return on collateral, also considered a "systematic factor", but independent of the asset return process. The third source of recovery uncertainty is given by (3.7), where we model the instantaneous drift on the recovery by an Orhnstein-Uhlenbeck mean-reverting process, with κ_α the speed of mean-reversion, $\bar{\alpha}$ the long-run mean, η the constant diffusion term, and B_t is a standard Weiner process having instantaneous correlation with the source of randomness in the recovery process, given heuristically by $\varsigma = \frac{1}{dt}Corr_t\left(dB_t, dW_t\right)$. The motivation behind this specification is the overwhelming evidence that the mean LGD is stochastic.

Economic LGD on the loan is given by following expectation under physical measure:

$$LGD_\lambda^P\left(R_t, \alpha_t \mid \lambda, c, \beta, \upsilon, \kappa_\alpha, \eta, \varsigma, \tau\right) =$$

$$= 1 - \frac{\exp\left(-c\tau\right)}{\lambda}E_t\left(\min\left[\lambda\exp\left(c\tau\right), R_t\exp\left(\left(\alpha_t - \frac{\beta^2+\nu^2}{2}\right)\tau + \beta Z_{t+\tau} + \nu W_{t+\tau}\right)\right]\right)$$

$$=1-\frac{\exp\left(\left(\alpha_t-c\right)\tau\right)}{\lambda}B\left(R_t,\alpha_t\,|\,\lambda\exp\left(c\tau\right),\hat{\sigma}_\tau,\alpha_t,\tau\right) \tag{3.8}$$

Where the modified option theoretic function $B\left(\bullet\right)$ is given by:

$$B\left(R_t,\alpha_t\,|\,\lambda\exp\left(c\tau\right),\hat{\sigma}_\tau,\alpha_t,\tau\right)=R_t\Phi\left(-d_+^{'}\right)+\exp\left(\left(c-\alpha_t\right)\tau\right)\lambda\Phi\left(d_-^{'}\right) \tag{3.9}$$

having arguments to the Gaussian distribution function $\Phi\left(z\right)=\frac{1}{2\pi}\int_{u=-\infty}^{z}e^{-\frac{u^2}{2}}du$:

$$d_\pm^{'}=\frac{1}{\hat{\sigma}_\tau\sqrt{\tau}}\left(\log\left(\frac{R_t}{\lambda\exp\left(c\tau\right)}\right)+\tau\left(\alpha_t\pm\frac{1}{2}\hat{\sigma}_\tau^2\right)\right) \tag{3.10}$$

A well-known result (Bjerksund, 1991) is that the maturity-dependent volatility $\hat{\sigma}_\tau$ is given by:

$$\hat{\sigma}_\tau=\left(\beta^2+\upsilon^2-\frac{\eta}{\kappa_\alpha}\left(2\sqrt{\beta^2+\upsilon^2}\varsigma-\frac{\eta}{\kappa_\alpha}\right)\right)\tau+\frac{2\eta}{\kappa_\alpha^2}\left(\sqrt{\beta^2+\upsilon^2}\varsigma-\frac{\eta}{\kappa_\alpha}\right)\left(1-e^{-\kappa_\alpha\tau}\right)+\frac{1}{2}\left(\frac{\eta}{\kappa_\alpha}\right)^2\left(1-e^{-2\kappa_\alpha\tau}\right) \tag{3.11}$$

The recovery to the bondholders is the expectation of the minimum of the positive part of the difference in the recovery and face value of the loan $\left[R_{t+\tau}-\lambda\exp\left(c\tau\right)\right]^+$ and the face value of the bond B, which is structurally identical to a compound option valuation problem (Geske, 1977):

$$LGD_B^P\left(V_t,R_t,\alpha_t\,|\,\lambda,c,\gamma,\beta,\upsilon,\kappa_\alpha,\eta,\varsigma,\tau_\lambda\right)=$$

$$=1-\frac{\exp\left(-\gamma,\tau_\lambda\right)}{B}E_t\left(\min\left[B,\max\left[R_t\exp\left(\left(\alpha_t-\frac{\beta^2+\nu^2}{2}\right)\tau_\lambda+\beta Z_{t+\tau}+\nu W_{t+\tau}\right)-\lambda\exp\left(c,\tau_\lambda\right),0\right]\right]\right) \tag{3.12}$$

where $R_{t+\tau}=R_t\exp\left(\left(\alpha_t-\frac{\beta^2+\nu^2}{2}\right)\tau_\lambda+\beta Z_{t+\tau_\lambda}+\nu W_{t+,\tau_\lambda}\right)$ is the value of recovery on the collateral at the time of resolution. We can easily write down the closed-form solution for the LGD on the bond according to the well-known formula for a compound option, where here the “outer option” is a put, and the “inner option” is a call. Let R^* be the critical level of recovery such that the holder of the loan is just breaking even:

$$\lambda\exp\left(c\tau_\lambda\right)=1-LGD_\lambda^P\left(R^*,\alpha_t\,|\,\lambda,c,\beta,\upsilon,\kappa_\alpha,\eta,\varsigma,\tau_\lambda\right) \tag{3.13}$$

where τ_λ is the time-to-resolution for the loan, which we assume to be prior to that for the bond, $\tau_\lambda<\tau_B$. Then the solution is given by:

$$LGD_B^P\left(R_t,\alpha_t\,|\,\lambda,c,\gamma,\beta,\upsilon,\kappa_\alpha,\eta,\varsigma,\tau_\lambda,\tau_B\right)=1-\frac{\exp\left(-\gamma\tau_b\right)}{B}\mathrm{B}\left(R_t,\alpha_t\,|\,\lambda,c,\gamma,\beta,\upsilon,\kappa_\alpha,\bar{\alpha},\eta,\varsigma,\tau_B\right) \tag{3.14}$$

$$\mathrm{B}\left(R_t,\alpha_t\,|\,\lambda,c,\gamma,\beta,\upsilon,\kappa_\alpha,\eta,\varsigma,\tau_\lambda,\tau_B\right)=$$

$$= B\exp\left(\alpha_t - \gamma\right)\Phi_2\left(-a_-, b_-; -\sqrt{\frac{\tau_\lambda}{\tau_B}}\right) - R_t\Phi_2\left(-a_+, b_+; -\sqrt{\frac{\tau_\lambda}{\tau_B}}\right) + \lambda\exp\left(c\tau_\lambda\right)\Phi\left(-a_-\right) \tag{3.15}$$

$$a_\pm = \frac{1}{\hat{\sigma}_\tau\sqrt{\tau_\lambda}}\left(\log\left(\frac{R_t}{R^*}\right) + \tau_\lambda\left(\alpha_t \pm \frac{1}{2}\hat{\sigma}_\tau^2\right)\right) \tag{3.16}$$

$$b_\pm = \frac{1}{\hat{\sigma}_\tau\sqrt{\tau_B}}\left(\log\left(\frac{R_t}{B}\right) + \tau_B\left(\alpha_t \pm \frac{1}{2}\hat{\sigma}_\tau^2\right)\right) \tag{3.17}$$

Where $\Phi_2\left(X, Y; \rho_{XY}\right)$ is the bivariate normal distribution function for Brownian increments the correlation parameter is given by $\rho_{XY} = \sqrt{\frac{T_X}{T_Y}}$ for respective "expiry times" T_X and T_Y for X and Y, respectively. Note that this assumption, which is realistic in that we observe in the data that on average earlier default on the bond even if the emerges from bankruptcy or resolve a default at a single time (which in addition is random), is matter of necessity in the log-normal setting in that the bivariate normal distribution is not defined for $\rho_{XY} = \sqrt{\frac{\tau}{\tau}} = 1$ in the case that $T_X = T_Y = \tau$

We can extend this framework to arbitrary tranches of debt, such as for a subordinated issue, in which case we follow the same procedure in order to arrive at an expression that involves trivariate cumulative normal distributions. In general, a debt issue that is subordinated to the d^{th} degree results in a pricing formula that is a linear combination of d+1 variate Gaussian distributions. These formulae become cumbersome very quickly, so for the sake of brevity we refer the interested reader to Haug (2006) for further details.

4. Comparitive statics

In this section we discuss and analyze the sensitivity of ultimate LGD in to various key parameters. In Figures 1 through 5 we examine the sensitivity of the ultimate LGD in the two-factor model of Section 3, incorporating the optimal foreclosure boundary. In Figure 1, we look at the ultimate LGD on the bond and the loan for three different settings of the factor loading of the recovery rate process on the systematic factor in the firm value processes (β = 0.05, 0.45 and 0.9), while fixing other parameters at "reasonable" values motivated by prior literature (drift in recovery α = 0.08, face value of loan λ = 0.5, coupon rate on loan c = 0.06, LGD side volatility υ = 0.3, volatility of recovery return drift process η = 0.5, speed of mean reversion in LGD return κ = 0.5, correlation between LGD side systematic factor and random factor in recovery rate drift ξ =0.3, and time-to-resolution τ =1). We observe that ultimate LGD is monotonically decreasing at increasing rate in value of the firm at default, that this increasing in the correlation between the PD and LGD side systematic factors, and that this is also uniformly higher for bonds than for loans. In Figure 2 we show the ultimate LGD as a function of the volatility in the recovery rate process attributable to the LGD side systematic factor η, fixing firm value at default at V_t = 0.5. We observe that ultimate LGD increases at an increasing rate in this parameter, that for higher correlation between firm asset value and recovery value return the LGD is higher and increases at a faster rate, and that for bonds these curves lie above and increase at a faster rate. In Figure 3 we show the ultimate LGD as a function of the volatility β in the recovery rate process attributable to the

PD side systematic factor, fixing LGD side volatility $\upsilon = 0.5$, for different firm values at default at $V_t = (0.3, 0.5, 0.8)$. We observe that ultimate LGD increases at an increasing rate in this parameter, that for lower firm asset values the LGD is higher but increases at a slower rate, and that for bonds these curves lie above and increase at a lower rate. In Figure 4 we show the ultimate LGD as a function of the volatility η of the stochastic drift in the recovery rate process, for three different settings of the factor loading of the recovery rate process on the systematic factor in the firm value processes (β = 0.05, 0.45 and 0.9). We observe that ultimate LGD in this parameter decreases at a decreasing rate, although the sensitivity is not great (especially for loans), and that as expected the curves lie above for greater PD-LGD correlation and for bonds as compared to loans. Finally, in Figure 5 we fix $\eta = 0.3$ and vary κ, the coefficient of mean reversion in the drift process for the recovery rate, and observe that ultimate LGD is increasing in this parameter, at a decreasing rate and having a discontinuity for these parameter settings; and as expected, for higher levels of default and recovery correlation, or for bonds as compared to loans, the curves lie everywhere above.

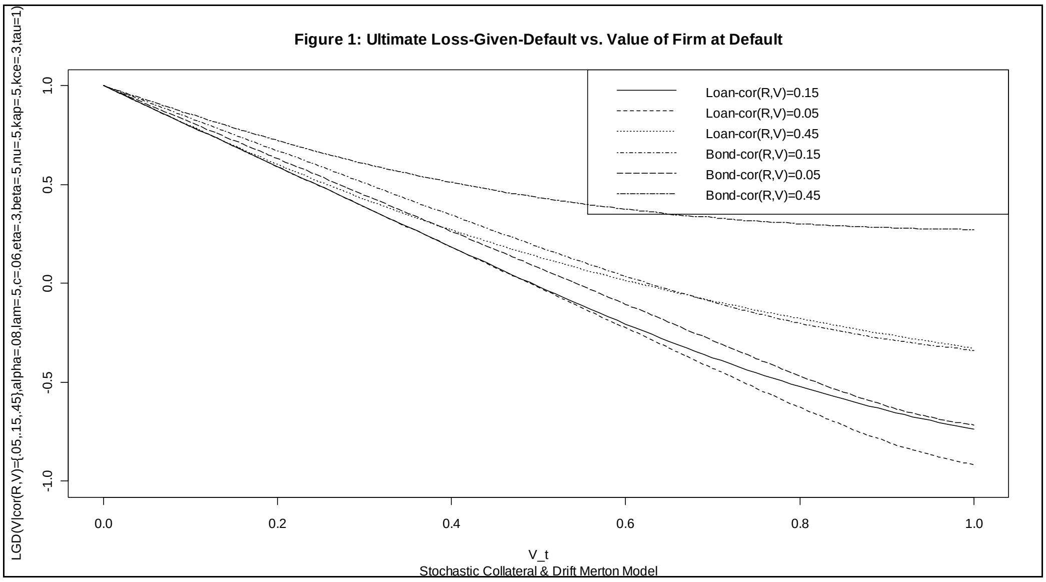


Figure 1: Ultimate Loss-Given-Default vs. Value of Firm at Default

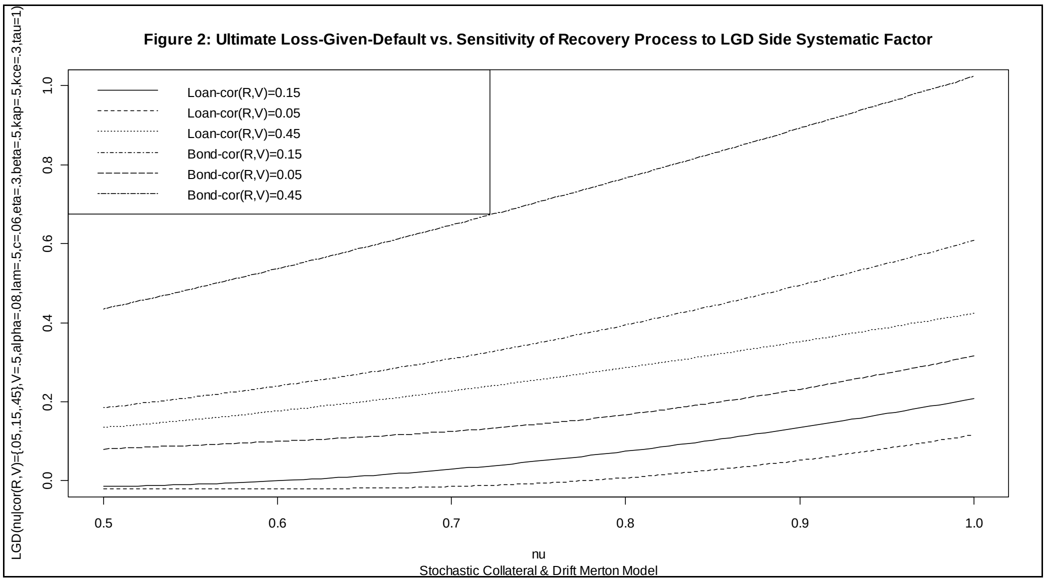


Figure 2: Ultimate Loss-Given-Default vs. Sensitivity of Recovery Process to LGD Side Systematic Factor

Figure 3: Ultimate Loss-Given-Default vs. Sensitivity of Recovery Process to PD Side Systematic Factor

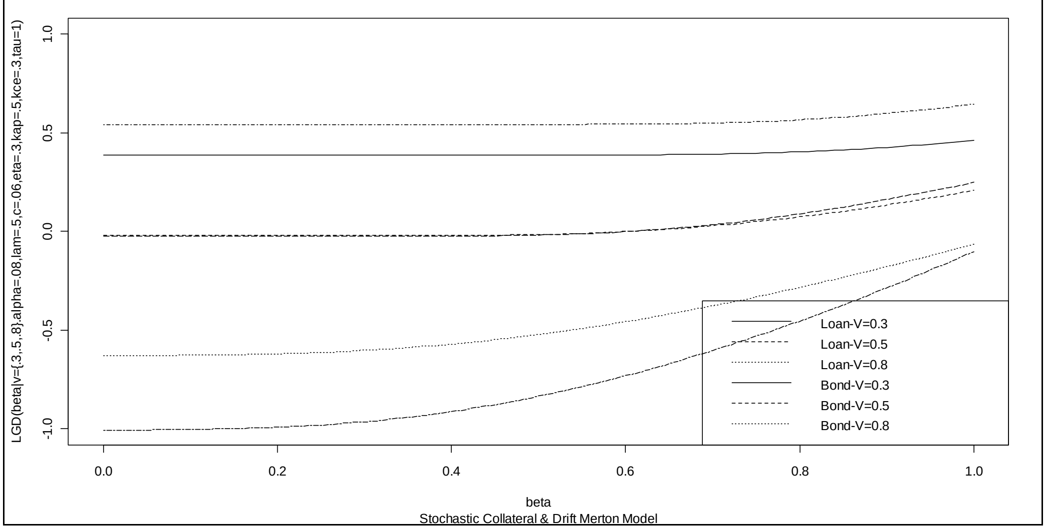


Figure 4: Ultimate Loss-Given-Default vs. Volatility in the Drift of the Recovery Rate Process

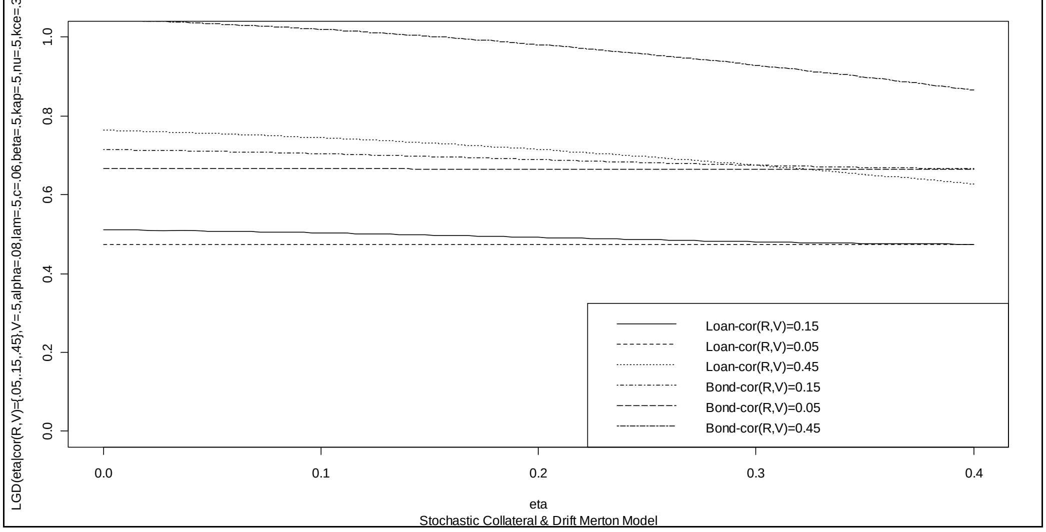
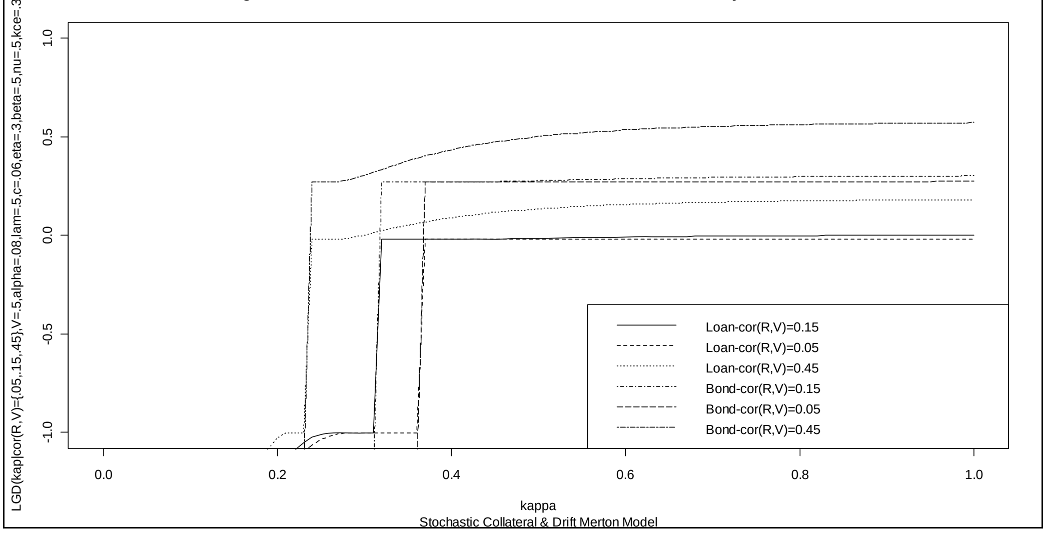


Figure 5: Ultimate Loss-Given-Default vs. Mean-Reversion Recovery Drift Process



5. Empirical analysis: calibration of models

In this section we describe our strategy for estimating parameters of the different models for LGD by *full-information maximum likelihood* (FIML.) This involves a consideration of the LGD implied in the market at time of default t_i^D for the i[th] instrument in recovery segment s, denoted LGD_{i,s,t_i^D}. This is the expected, discounted ultimate loss-given-default LGD_{i,s,t_i^E} at time of emergence t_i^E as given by any of our models m, $LGD_{s,m}^P\left(\boldsymbol{\theta}_{s,m}\right)$ over the resolution period $t_{i,s}^E - t_{i,s}^D$:

$$LGD_{i,s,t_i^D} = \frac{E_t^P\left[LGD_{i,s,t_i^E}\right]}{\left(1+r_{i,s}^D\right)^{t_{i,s}^E - t_{i,s}^D}} = LGD_{s,m}^P\left(\boldsymbol{\theta}_{s,m}\right) \tag{5.1}$$

where $\boldsymbol{\theta}_{s,m}$ is the parameter vector for segment s under model m, expectation is taken with respect to physical measure P, discounting is at risk adjusted rate appropriate to the instrument $r_{i,s}^D$ and it is assumed that the time-to-resolution $t_{i,s}^E - t_{i,s}^D$ is known.

In order to account for the fact that we cannot observe expected recovery prices ex ante, as only by coincidence would they coincide with expectations, we invoke market rationality to postulate that for a segment homogenous with respect to recovery risk the difference between expected and average realized recoveries should be small. We formulate this by defining the normalized forecast error as:

$$\tilde{\varepsilon}_{i,s} \equiv \frac{LGD_{s,m}^P\left(\boldsymbol{\theta}_{s,m}\right) - LGD_{i,s,t_i^E}}{LGD_{i,s,t_i^D} \times \sqrt{t_{i,s}^E - t_{i,s}^D}} \tag{5.2}$$

This is the forecast error as a proportion of the LGD implied by the market at default (a "unit-free" measure of recovery uncertainty) and the square root of the time-to-resolution. This is a mechanism to control for the likely increase in uncertainty with time-to-resolution, which effectively puts more weight on longer resolutions, increasing the estimate of the loss-severity. The idea behind this is that more information is revealed as the emergence point is approached, hence a decrease in risk. Alternatively, we can analyze $\varepsilon_{i,s} \equiv \frac{LGD_{s,m}^P\left(\boldsymbol{\theta}_{s,m}\right) - LGD_{i,s,t_i^E}}{LGD_{i,s,t_i^D}}$, the forecast error that is non-time adjusted, and argue that its standard error is proportional to $\sqrt{t_{i,s}^E - t_{i,s}^D}$, which is consistent with an economy in which information is revealed uniformly and independently through time (Miu and Ozdemir, 2005). Assuming that the errors $\tilde{\varepsilon}_{i,s}$ in (5.2) are standard normal,[8] we may use *full-information maximum likelihood* (FIML), by maximizing the *log-likelihood* (LL) function:

[8] If the errors are i.i.d and from symmetric distributions, then we can still obtain consistent estimates through ML, which has the interpretations as the quasi-ML estimator.

$$\widetilde{LGD}^{P}_{s,m} = \arg\max_{\boldsymbol{\theta}_{s,m}} LL = \arg\max_{\boldsymbol{\theta}_{s,m}} \sum_{i=1}^{N_s^D} \log\left[\phi\left(\tilde{\varepsilon}_{i,s}\left(\boldsymbol{\theta}_{s,m}\right)\right)\right]$$
$$= \arg\max_{\boldsymbol{\theta}_{s,m}} \sum_{i=1}^{N_s^D} \log\left[\phi\left(\frac{LGD^{P}_{s,m}\left(\boldsymbol{\theta}_{s,m}\right) - LGD_{i,s,t_i^E}}{LGD_{i,s,t_i^D} \times \sqrt{t^E_{i,s} - t^D_{i,s}}}\right)\right] \quad (5.3)$$

This turns out to be equivalent to minimizing the squared normalized forecast errors:

$$\widetilde{LGD}^{P}_{s,m} = \arg\min_{\boldsymbol{\theta}_{s,m}} \left\{\sum_{i=1}^{N_s} \frac{1}{t^E_{i,s} - t^D_{i,s}} \left(\frac{LGD^{P}_{s,m}\left(\boldsymbol{\theta}_{s,m}\right) - LGD_{i,s,t_i^E}}{LGD_{i,s,t_i^D}}\right)^2\right\} = \arg\min_{\boldsymbol{\theta}_{s,m}} \left\{\sum_{i=1}^{N_s} \tilde{\varepsilon}_{i,s,m}{}^2\right\} \quad (5.4)$$

We may derive a measure of uncertainty of our estimate by the ML standard errors from the Hessian matrix evaluated at the optimum:

$$\hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\theta}}_{s,m}} = \left[-\frac{\partial^2 LL}{\partial \boldsymbol{\theta}_{s,m} \partial \boldsymbol{\theta}^T_{s,m}}\right]^{-\frac{1}{2}} \Bigg|_{\boldsymbol{\theta}_{s,m} = \hat{\boldsymbol{\theta}}_{s,m}} \quad (5.5)$$

6. Data and estimation results

We summarize basic characteristics of our data-set in Tables 1 and 2, and the maximum likelihood estimates are shown in Table 3. These are based upon our analysis of defaulted bonds and loans in the Moody's Ultimate Recovery (MURD™) database release as of August, 2009. This contains the market values of defaulted instruments at or near the time of default,[9] as well as the values of such pre-petition instruments (or of instruments received in settlement) at the time of default resolution. This database is largely representative of the U.S. large-corporate loss experience, from the mid-1980's to the present, including most of the major corporate bankruptcies occurring in this period.

Table 1 shows summary statistics of various quantities of interest according to instrument type (bank loan, bond, term loan or revolver) and default type (bankruptcy under Chapter 11 or out-of-court renegotiation). First, we take the annualized return or yield on defaulted debt from the date of default (bankruptcy filing or distressed renegotiation date) to the date of resolution (settlement of renegotiation or emergence from Chapter 11), henceforth abbreviated as "RDD". Second, the trading price at default implied LGD ("TLGD"), or par minus the trading price of defaulted debt at the time of default (average 30-45 days after default) as a percent of par. Third, our measure of ultimate loss severity, the dollar loss-given-default on the debt instrument at emergence from bankruptcy or time of final settlement ("ULGD"), computed as par minus either values of pre-petition or settlement instruments at resolution. We also summarize two additional variables in Table 1, the total instrument outstanding at default, and the time in years from the instrument default date to the time of ultimate recovery.

9 This an average of trading prices from 30 to 45 days following the default event. A set of dealers is polled every day and the minimum /maximum quote is thrown out. This is done by experts at Moody's.

The preponderance of this sample is made up of bankruptcies as opposed to out-of-court settlements, 1,322 out of a total of 1,398 instruments. We note that out-of-court settlements have lower LGDs by either the trading or ultimate measures, 37.7% and 33.8%, as compared to Chapter 11's, 55.7% and 51.6%, respectively; and the heavy weight of bankruptcies are reflected in how close the latter are to the overall averages, 54.7% and 50.6% for TLGD and ULGD, respectively. Interestingly, not only do distressed renegotiations have lower loss severities, but such debt performs better over the default period than bankruptcies, RDD of 37.3% as compared to 28.1%, as compared to an overall RDD of 28.6%. We also note that the TLGD is higher than the ULGD by around 5% across default and instrument types, 55.7% (37.7%) as compared to 51.6% (33.8%) for bankruptcies (renegotiations). We also see that loans have better recoveries by both measures as well higher returns on defaulted debt, respective average TLGD, ULGD and RDD 52.5%, 49.3% and 32.2%. I

In Table 2 we summarize ULGD, TLGD and RDD by major collateral categories and seniority classes. We observe for this sample that either LGD measure appears to weakly exhibit the usual decreasing pattern observed in the literature with respect to higher seniority class, but this relationship is not consistent with respect to collateral categories. On the other hand, while also not monotonic, we a somewhat stronger relationship for RDD, as these tend to be higher for either better secured or more highly ranked instruments. We have average TLGD (ULGD) of 53.3% (49.3%), 51.6% (35.0%), 56.0% (38.0%), 58.5% (36.5%) and 65.8% (33.5%) for Revolving Credit/Term Loan, Senior Secured Bonds, Senior Unsecured Bonds, Senior Subordinated Bonds and Junior Subordinated Bonds, respectively. The corresponding averages of RDD in descending order of seniority class are 32.2%, 36.6%, 23.8%, 33.2% and 15.6% - an overall decreasing albeit non-monotonic pattern. On the other hand, for this particular sample and segmentation of collateral codes, we fail to see much of a rank ordering as we might have expected. We have average TLGD (ULGD) of 66.5% (65.0%), 41.6% (32.9%), 50.6% (47.6%), 61.6% (48.6%), 59.3% (59.4%) and 57.4% (51.46%) for Cash, Accounts Receivables & Guarantees, Inventory/Most Assets & Equipment, All Assets & Real Estate, Non-Current Assets & Capital Stock, PPE/Second Lien and Unsecured/Other Illiquid Collateral, respectively. Even just focusing upon the split between secured and unsecured, we fail to see much (any) of a difference in TLGD (ULGD), 57.58% vs. 53.40% (37.69% vs. 36.13%), respectively. The corresponding averages of RDD in descending order of collateral quality are: 22.6%, 33.2%, 33.8%, 46.2%, 29.0% and 24.1% - a humped shaped pattern. However, RDD is higher for secured as compared to unsecured, 34.5% vs. 3.6%, respectively.

Table 1

Characteristics of loss-given-default and return on defaulted debt observations by default and instrument type

(Moody's Ultimate Recovery Database 1987-2009)

		Bankruptcy			Out-of-Court			Total		
		Count	**Average**	**Standard Error of the Mean**	**Count**	**Average**	**Standard Error of the Mean**	**Count**	**Average**	**Standard Error of the Mean**
Bonds and Term Loans	Return on Defaulted Debt[1]	1072	28.32%	3.47%	59	45.11%	19.57%	1131	29.19%	3.44%
	LGD at Default[2]		55.97%	0.96%		38.98%	3.29%		55.08%	0.93%
	Discounted LGD[3]		51.43%	1.15%		33.89%	3.05%		50.52%	1.10%
	Time-to-Resolution[4]		1.7263	0.0433		0.0665	0.0333		1.6398	0.0425
	Principal at Default[5]		207'581	9'043		416'751	65'675		218'493	9'323
Bonds	Return on Defaulted Debt[1]	837	25.44%	3.75%	47	44.22%	21.90%	884	26.44%	3.74%
	LGD at Default[2]		57.03%	1.97%		37.02%	5.40%		55.96%	1.88%
	Discounted LGD[3]		52.44%	1.30%		30.96%	3.00%		51.30%	1.25%
	Time-to-Resolution[4]		1.8274	0.0486		0.0828	0.0415		1.7346	0.0424
	Principal at Default[5]		214'893	11'148		432'061	72'727		226'439	11'347
Revolvers	Return on Defaulted Debt[1]	250	26.93%	7.74%	17	10.32%	4.61%	267	25.88%	7.26%
	LGD at Default[2]		54.37%	1.96%		33.35%	8.10%		53.03%	1.93%
	Discounted LGD[3]		52.03%	2.31%		33.33%	7.63%		50.84%	2.23%
	Time-to-Resolution[4]		1.4089	0.0798		0.0027	0.0000		1.3194	0.0776
	Principal at Default[5]		205'028	19'378		246'163	78'208		207'647	18'786
Loans	Return on Defaulted Debt[1]	485	32.57%	5.71%	29	26.161 %	18.872 %	514	32.21%	5.49%
	LGD at Default[2]		53.31%	9.90%		38.86%	7.22%		52.50%	3.21%
	Discounted LGD[3]		50.00%	1.68%		38.31%	5.79%		49.34%	2.25%
	Time-to-Resolution[4]		1.3884	0.0605		0.0027	0.0000		1.3102	0.0816
	Principal at Default[5]		193'647	11'336		291'939	78'628		199'192	16'088

Table 1 (cont)

Characteristics of loss-given-default and return on defaulted debt observations by default and instrument type

(Moody's Ultimate Recovery Database 1987-2009)

		Bankruptcy			Out-of-Court			Total		
		Count	**Average**	**Standard Error of the Mean**	**Count**	**Average**	**Standard Error of the Mean**	**Count**	**Average**	**Standard Error of the Mean**
Total	Return on Defaulted Debt[1]	1322	28.05%	3.17%	76	37.33%	15.29%	1398	28.56%	3.11%
	LGD at Default[2]		55.66%	0.86%		37.72%	3.12%		54.69%	0.84%
	Discounted LGD[3]		51.55%	1.03%		33.76%	2.89%		50.58%	0.99%
	Time-to-Resolution[4]		1.6663	0.0384		0.0522	0.0260		1.5786	0.0376
	Principal at Default[5]		207'099	8'194		378'593	54'302		216'422	8'351

[1] Annualized return or yield on defaulted debt from the date of default (bankruptcy filing or distressed renegotiation date) to the date of resolution (settlement of renegotiation or emergence from Chapter 11). [2] Par minus the price of defaulted debt at the time of default (average 30-45 days after default) as a percent of par. [3] The ultimate dollar loss-given-default on the defaulted debt instrument = 1 - (total recovery at emergence from bankruptcy or time of final settlement)/(outstanding at default). Alternatively, this can be expressed as (outstanding at default - total ultimate loss)/(outstanding at default). [4] The total instrument outstanding at default. [5] The time in years from the instrument default date to the time of ultimate recovery.

Table 2

Loss-given-default by seniority ranks and collateral types

(Moody's Ultimate Recovery Database 1987-2009)

Collateral Type			Cash, Accounts Receivables & Guarantees	Inventory, Most Assets & Equipment	All Assets & Real Estate	Non-Current Assets & Capital Stock	PPE & Second Lien	Unsecured & Other Illiquid Collateral	Total Unsecured	Total Secured	Total Collateral
Revolving Credit / Term Loan	Count		39	8	367	38	29	33	32	482	514
	LGD at Default[1]	Average	66.81%	46.60%	51.95%	59.94%	55.02%	45.63%	46.25%	53.79%	53.31%
		Standard Error	4.44%	11.79%	1.70%	5.27%	6.08%	5.07%	5.20%	1.47%	1.42%
	Ultimate LGD[2]	Average	64.38%	56.03%	48.58%	50.62%	56.53%	30.70%	31.78%	50.51%	49.34%
		Standard Error	5.09%	13.85%	1.91%	6.10%	6.88%	6.17%	5.20%	1.47%	1.42%
	Return on Defaulted Debt[3]	Average	22.57%	-5.80%	33.49%	35.68%	46.07%	22.39%	19.77%	33.03%	32.21%
		Standard Error	18.20%	30.27%	6.89%	15.01%	27.64%	8.12%	7.93%	5.83%	5.49%
Senior Secured Bonds	Count		2	38	41	35	7	142	3	139	142
	LGD at Default[1]	Average	61.50%	40.19%	36.02%	62.99%	61.24%	51.67%	50.73%	51.59%	51.57%
		Standard Error	36.50%	5.50%	5.03%	4.71%	11.63%	2.48%	23.79%	2.76%	2.74%
	Ultimate LGD[2]	Average	76.81%	23.87%	36.67%	46.70%	60.32%	49.68%	50.15%	34.88%	35.04%
		Standard Error	19.39%	3.90%	5.61%	5.71%	12.68%	3.19%	28.95%	2.96%	2.94%
	Return on Defaulted Debt[3]	Average	23.86%	47.53%	35.03%	55.99%	14.33%	17.44%	-27.66%	38.02%	36.63%
		Standard Error	40.63%	7.18%	22.04%	20.10%	27.41%	6.34%	36.65%	9.05%	8.92%
Senior Unsecured Bonds	Count		0	0	1	0	1	459	452	9	461
	LGD at Default[1]	Average	0.00%	0.00%	85.00%	N/A	80.00%	55.83%	55.94%	56.63%	55.96%
		Standard Error	N/A	N/A	N/A	N/A	N/A	1.42%	1.43%	10.36%	1.42%
	Ultimate LGD[2]	Average	0.00%	0.00%	78.76%	N/A	74.25%	48.33%	38.14%	32.03%	38.00%
		Standard Error	N/A	N/A	N/A	N/A	N/A	1.78%	1.79%	10.68%	1.77%
	Return on Defaulted Debt[3]	Average	0.00%	0.00%	86.47%	n	119.64%	23.40%	23.71%	25.62%	23.75%
		Standard Error	N/A	N/A	N/A	N/A	N/A	4.80%	4.86%	22.61%	4.78%
Senior Subordinated Bonds	Count		0	0	1	0	1	159	158	3	161
	LGD at Default[1]	Average	0.00%	N/A	85.00%	N/A	90.50%	58.09%	57.98%	83.46%	58.48%
		Standard Error	N/A	N/A	N/A	N/A	N/A	2.48%	2.50%	4.58%	2.47%
	Ultimate LGD[2]	Average	N/A	N/A	74.72%	N/A	97.74%	54.51%	36.50%	40.47%	36.46%
		Standard Error	N/A	N/A	N/A	N/A	N/A	2.89%	2.90%	23.36%	2.87%
	Return on Defaulted Debt[3]	Average	0.00%	N/A	57.45%	N/A	-45.98%	33.57%	31.01%	150.30%	33.23%
		Standard Error	N/A	N/A	N/A	N/A	N/A	10.44%	10.18%	147.62%	10.32%

Table 2 (cont)

Loss-given-default by seniority ranks and collateral types

(Moody's Ultimate Recovery Database 1987-2009)

Collateral Type			Cash, Accounts Receivables & Guarantees	Inventory, Most Assets & Equipment	All Assets & Real Estate	Non-Current Assets & Capital Stock	PPE & Second Lien	Unsecured & Other Illiquid Collateral	Total Unsecured	Total Secured	Total Collateral
Junior Subordinated Bonds	Count		0	1	0	0	0	119	117	3	120
	LGD at Default[1]	Average	N/A	27.33%	0.00%	N/A	N/A	66.15%	66.58%	37.42%	65.81%
		Standard Error	N/A	N/A	N/A	N/A	N/A	2.50%	2.48%	22.25%	2.50%
	Ultimate LGD[2]	Average	N/A	20.15%	0.00%	N/A	N/A	65.36%	33.62%	32.77%	33.54%
		Standard Error	N/A	N/A	N/A	N/A	N/A	3.06%	3.11%	18.92%	3.06%
	Return on Defaulted Debt[3]	Average	N/A	72.13%	0.00%	N/A	N/A	15.11%	15.74%	9.49%	15.59%
		Standard Error	N/A	N/A	N/A	N/A	N/A	10.93%	11.11%	31.36%	10.85%
Total Instruments	Count		41	28	407	79	66	777	762	636	1398
	LGD at Default[1]	Average	66.53%	41.57%	50.55%	61.56%	59.31%	57.41%	57.58%	53.40%	55.66%
		Standard Error	4.41%	6.18%	1.63%	3.39%	3.86%	1.09%	1.10%	1.28%	0.84%
	Ultimate LGD[2]	Average	64.98%	32.93%	47.60%	48.58%	59.43%	51.46%	37.69%	36.13%	36.99%
		Standard Error	4.90%	5.99%	1.82%	3.99%	4.28%	1.35%	1.37%	1.43%	0.99%
	Return on Defaulted Debt[3]	Average	22.63%	33.17%	33.82%	46.22%	28.96%	24.12%	34.46%	23.63%	28.56%
		Standard Error	17.36%	11.74%	6.56%	12.02%	13.90%	3.94%	3.97%	4.89%	3.11%

[1] Par minus the price of defaulted debt at the time of default (average 30-45 days after default) as a percent of par. [2] The ultimate dollar loss-given-default on the defaulted debt instrument = 1 - (total recovery at emergence from bankruptcy or time of final settlement)/(outstanding at default). Alternatively, this can be expressed as (outstanding at default - total ultimate loss)/(outstanding at default). [3] Annualized return or yield on defaulted debt from the date of default (bankruptcy filing or distressed renegotiation date) to the date of resolution (settlement of renegotiation or emergence from Chapter 11).

In Table 3 we present the full-information maximum likelihood estimation (FIML) results of the leading model for ultimate LGD derived in this paper, the two-factor structural model of ultimate loss-given-default, with systematic recovery risk and random drift (2FSM-SR&RD) on the recovery process.[10] The model is estimated along with the optimal foreclosure boundary constraint.

We first discuss the MLE point estimates of the parameters governing the firm value process and default risk, or the "PD-side". Regarding the parameter σ, which is the volatility of the firm-value process governing default, we observe that estimates are decreasing in seniority class, ranging from 9.1% to 4.3% from subordinated bonds to senior loans. As standard errors range in 1% to 2%, increasing in seniority rank, these differences across seniority classes and models are generally statistically significant. Regarding the MLE point estimates of the parameter μ, which is the drift of the firm-value process governing default, we observe estimates are increasing in seniority class, ranging from 9.6% to 18.6% from subordinated bonds to loans, respectively.

[10] Estimates for the baseline Merton structural model (BMSM) and for the Merton structural model with stochastic drift (MSM-SD) are available upon request.

Table 3

Full information maximum likelihood estimation of option theoretic two-factor structural model of ultimate loss-given-default with optimal foreclosure boundary, systematic recovery risk and random drift in the recovery process

(Moody's Ultimate Recovery Database 1987-2009)

Recovery Segment		Parameter	σ[1]	μ[2]	β[3]	v[4]	σR[5]	πRβ[6]	πRv[7]	(βσ)0.5	κα[8]	α[9]	ηα[10]	ς[11]
Seniority Class	Revolving Credit / Term Loan	Est.	4.32%	18.63%	18.16%	36.83%	41.06%	19.55%	80.45%	12.82%	3.96%	37.08%	48.85%	20.88%
		Std. Err.	0.5474%	0.9177%	0.7310%	1.3719%				0.4190%	0.0755%	4.2546%	3.2125%	0.9215%
	Senior Secured Bonds	Est.	5.47%	16.99%	16.54%	30.41%	34.62%	22.83%	77.17%	11.64%	4.40%	33.66%	44.43%	18.99%
		Std. Err.	0.5314%	0.8613%	0.6008%	1.3104%				0.7448%	0.0602%	3.5085%	2.6903%	0.8297%
	Senior Unsecured Bonds	Est.	6.82%	14.16%	13.82%	24.38%	28.02%	24.30%	75.70%	9.71%	5.50%	28.07%	37.04%	15.83%
		Std. Err.	0.5993%	1.0813%	1.3913%	1.9947%				0.6165%	0.0281%	2.8868%	2.2441%	0.6504%
	Senior Subordinated Bonds	Est.	8.19%	11.33%	12.02%	17.35%	21.11%	32.43%	67.57%	7.76%	4.42%	22.45%	29.68%	12.69%
		Std. Err.	0.6216%	1.0087%	1.0482%	1.0389%				0.9775%	0.0181%	2.0056%	2.0132%	1.0016%
	Subordinated Bonds	Est.	9.05%	9.60%	10.24%	12.37%	16.06%	40.66%	59.34%	5.97%	3.34%	18.80%	18.69%	9.43%
		Std. Err.	0.6192%	1.0721%	1.0128%	1.0771%				0.9142%	0.0106%	2.0488%	2.0014%	1.0142%

Value Log-Likelihood Function		-371.09
Degrees of Freedon		1391
P-Value of Likelihood Ratio Statistic		4.69E-03
In-Sample / Time Diagnostic Statistics	Area Under ROC Curve	93.14%
	Komogorov-Smirnov Stat. (P-Val.)	2.14E-08
	McFadden Pseudo R-Squared	72.11%
	Hoshmer-Lemeshow Chi-Squared (P-Values)	0.63

[1] The volatility of the firm-value process governing default. [2] The drift of the firm-value process governing default. [3] The sensitivity of the recovery-rate process to the systematic governing default in (or the component of volatility in the recovery process due to PD-side systematic risk). [4] The sensitivity of the recovery-rate process to the systematic governing collateral value (or the component of volatility in the recovery process due to LGD-side systematic risk). [5] The total volatility of the recovery rate process: $\mathrm{sqrt}(\beta^2+v^2)$. [6] Component of total recovery variance attributable to PD-side (asset value) uncertainty: $\beta^2/(\beta^2+v^2)$. [7] Component of total recovery variance attributable to LGD-side (collateral value) uncertainty: $v^2/(\beta^2+v^2)$. [8] The speed of the mean-reversion in the random drift in the recovery rate process. [9] The long-run mean of the random drift in the recovery arte process. [10] The volatility of the random drift in the recovery rate process. [11] The correlation of the random processes in drift of and the level of the recovery rate process.

These too are statistically significant across seniorities. The fact that we are observing different estimates of a single firm value process across seniorities is evidence that models which attribute identical default risk across different instrument types are misspecified – in fact, we are measuring lower default risk (i.e., lower asset value volatility and greater drift in firm-value) in loans and senior secured bonds as compared to unsecured and subordinated bonds.

A key result regards the magnitudes and composition of the components of recovery volatility across maturities inferred from the model calibration. The MLE point estimates of the parameter β, the sensitivity of the recovery-rate process to the systematic factor governing default (or due to PD-side systematic risk), increases in seniority class, from 10.2% for subordinated bonds to 18.2% senior bank loans. On the other hand, estimates of the parameter ν, the sensitivity of the recovery-rate process to the systematic factor governing collateral value (or due to LGD-side systematic risk), are greater than β across seniorities, and similarly increases in from 12.4% for subordinated bonds to 36.8% for bank loans. This monotonic increase in both β and ν as we move up in the hierarchy of the capital structure from lower to higher ranked instruments has the interpretation of a greater sensitivity in the recovery rate process attributable to both systematic risks, implying that total recovery volatility $\sigma_R = \sqrt{\beta^2 + \upsilon^2}$ increases from higher to lower ELGD instruments, from 16.1% for subordinated bonds to 41.1% for senior loans. However, we see that the *proportion* of the total recovery volatility attributable to systematic risk in collateral (firm) value, or the LGD (PD) side, is increasing (decreasing) in seniority from 59.3% to 80.5% (40.7% to 19.6%) from subordinated bonds to senior bank loans. Therefore, more senior instruments not only exhibit greater recovery volatility than less senior instruments, but a larger component of this volatility is driven by the collateral rather than the asset value process.

The next set of results concern the random drift in the recovery rate process. The MLE point estimates of the parameter κ_α, the speed of the mean-reversion in, is hump-shaped in seniority class, ranging from 3.3% subordinated bonds, to 5.5% for senior unsecured bonds, to 4.0% for loans, respectively. Estimates of the parameter α, the long-run mean of the random drift in the recovery rate process, increase in seniority class from 18.8% for subordinated bonds to 37.1% for senior bank loans. This monotonic increase in α as we move from lower to higher ranked instruments has the interpretation of greater expected return of the recovery rate process inferred from lower ELGD (or greater expected recovery) instruments as we move up in the hierarchy of the capital structure. We see that the volatility of the random drift in the recovery rate process η_α, increases in seniority class, ranging from 18.7% to 48.9% from subordinated bonds to senior loans, respectively. The monotonic increase in η_α as we move from lower to higher ranked instruments has the interpretation of greater volatility in expected return of the recovery rate process inferred from lower ELGD (or greater expected recovery) instruments as we move up in the hierarchy of the capital structure. Finally, estimates of the parameter ζ, the correlation of the random processes in drift of and the level of the recovery rate process, increases in seniority class from 9.4% for subordinated bonds to 20.9% for senior bank loans.

Finally with respect to parameter estimates, regarding the MLE point estimates of the correlation between the default and recovery rate processes $\sqrt{\beta\sigma}$ in the 2FSM-SR&RD, we observe estimates are increasing in seniority class, ranging from 6.0% to 12.8% from subordinated bonds to loans, respectively.

We conclude this section by discussing the quality of the estimates and model performance measures. Across seniority classes, parameter estimates are all statistically significant, and the magnitudes of such estimates are in general distinguishable across segments at conventional significance levels. The likelihood ratio statistic indicates that we can reject the null hypothesis that all parameter estimates are equal to zero across all ELGD segments, a p-value of 4.7e-3. We also show various diagnostics that assess in-sample fit, which show that the model performs well-in sample. The *area under receiver operating characteristic curve* (AUROC) of 93.1% is high by commonly accepted standards, indicating a good ability of the model to discriminate between high and low LGD defaulted instruments. Another test of discriminatory ability of the models is the *Kolmogorov-Smirnov* (KS) statistic, the very small p-value 2.1e-8 indicating adequate separation in the distributions of the low and high LGD instruments in the model.[11] We also show two tests of predictive accuracy, which is the ability of the model to accurately quantify a level of LGD. The *McFadden psuedo r-squared* (MPR2) is high by commonly accepted standards, 72.1%, indicating a high rank-order correlation between model and realized LGDs of defaulted instruments. Another test of predictive accuracy of the models is the *Hoshmer-Lemeshow* (HL) statistic, high p-values of 0.63 indicating high accuracy of the model to forecast cardinal LGD.

7. Downturn LGD

In this section we explore the implications of our model with respect to downturn LGD in the 2FSM-SR&RD. This is a critical component of the quantification process in the Basel II advanced IRB framework for regulatory capital. The Final Rule (FR) in the U.S. (OCC et al, 2007) requires banks that either wish, or are required, to qualify for treatment under the advanced approach to estimate a *downturn LGD*. We paraphrase the FR, this is an LGD estimated during an historical reference period during which default rates are elevated within an institution's loan portfolio.

In Figures 6 through 8 we plot the ratios of the downturn LGD to the expected LGD. This is derived by conditioning on the 99.9th quantile of the PD side systematic factor in the 2FSM-SR&RD. We show this for loans and bonds, as well as for different settings of key parameters ($\sqrt{\beta\sigma}$, ν or η_α) in each plot, with other parameters set to the MLE estimates. We observe that the LGD mark-up for downturn is monotonically declining in ELGD, which is indicative of lower tail risk in recovery for lower ELGD instruments. It is also greater than unity in all cases, and approaches 1 as ELGD approaches 1. This multiple is higher for bonds than for loans, as well as for either higher PD-LGD correlation $\sqrt{\beta\sigma}$, collateral specific volatility ν or volatility in the drift of the recovery rate drift process η_α; although these differences narrow for higher ELGD. For example, in Figure 6, we see that for loans having ELGD of 15% and $\sqrt{\beta\sigma}$ = 10% (=20%), the ratio of downturn to ELGD is about 2 (2.5); but for ELGD of 50%, this is about 1.5 (1.6); and for ELGD of 80%, this about 1.2 (1.3). And for bonds having ELGD of 15% and $\sqrt{\beta\sigma}$ = 10% (=20%), the ratio of downturn to ELGD is about 2.5 (23); but for ELGD of 50%, this is about 2 (2.2) ; and for ELGD of 80%, this is about 1.6 (1.7).

[11] In these tests we take the median LGD to be the cut-off that distinguishes between a high and low realized LGD.

8. Model validation

In this final section we validate our preferred model, the 2FSM-SR&RD. In particular, we implement an out-of-sample and out-of-time analysis, on a rolling annual cohort basis for the final nine years of our sample. Furthermore, we augment this by resampling on both the training and prediction samples, a non-parametric bootstrap (Efron [1979], Efron and Tibshirani [1986], Davison and Hinkley [1997]). The procedure is as follows: the first training (or estimation) sample is established as the cohorts defaulting in the 10 years 1987-1996, and the first prediction (or validation) sample is established as the 1997 cohort. Then we resample 100,000 times with replacement from the training sample the 1987-1996 cohorts and for the prediction sample 1997 cohort, and then based upon the fitted model in the former we evaluate the model based upon the latter. Then we augment the training sample with the 1997 cohort, and establish the 1998 cohort as the prediction sample, and repeat this. This is continued until we have left the 2008 cohort as the holdout. Finally, to form our final holdout sample, we pool all of our out-of-sample resampled prediction cohorts, the 12 years running from 1997 to 2008. We then analyze the distributional properties (such as median, dispersion and shape) of the two key diagnostic statistics: the Spearman rank-order correlation for discriminatory (or classification) accuracy, and the Hoshmer-Lemeshow Chi-Squared (HLCQ) P-values for predictive accuracy, or calibration.

Figure 6: Ratio of Ultimate Downturn to Expected LGD vs. ELGD at 99.9th Percentile of PD-Side Systematic Factor Z

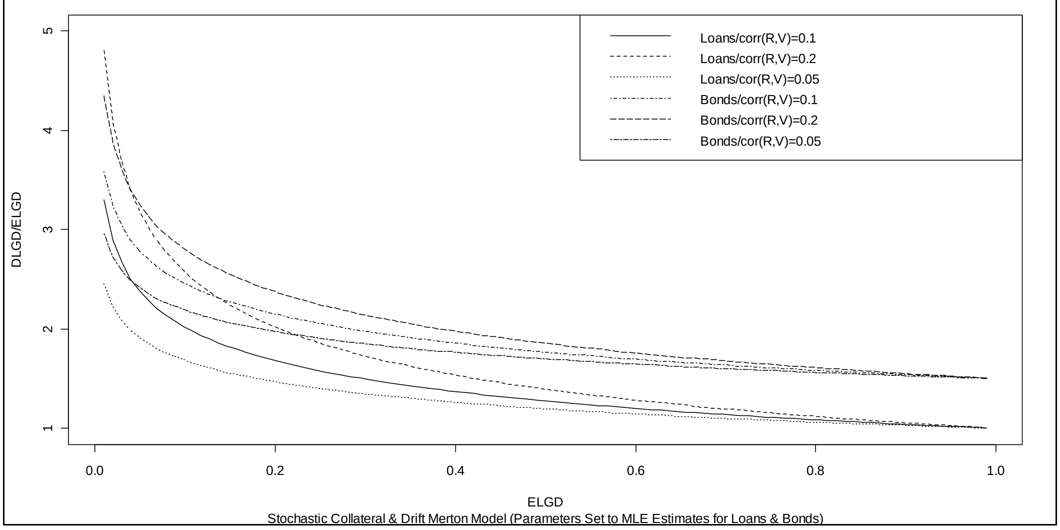


Figure 7: Ratio of Ultimate Downturn to Expected LGD vs. ELGD at 99.9th Percentile of PD-Side Systematic Factor Z

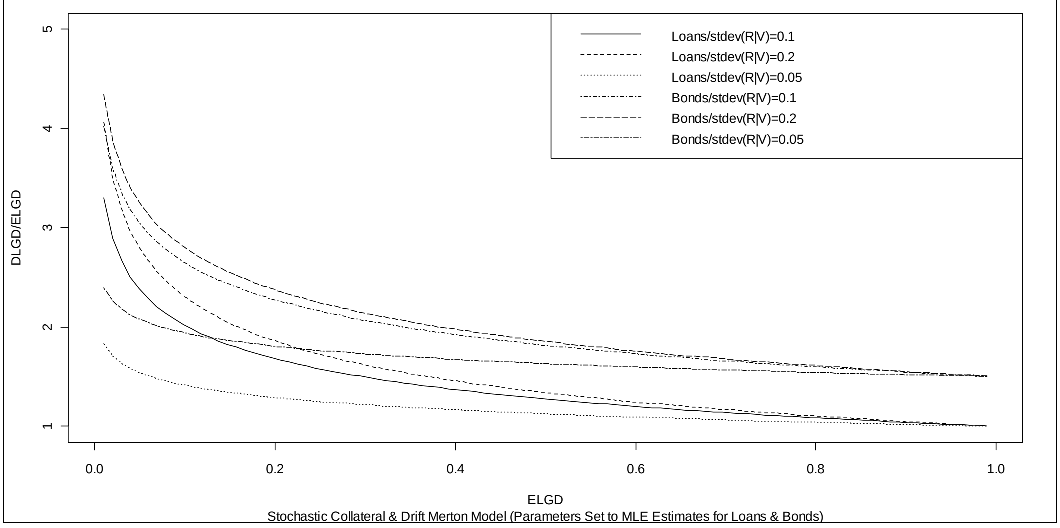
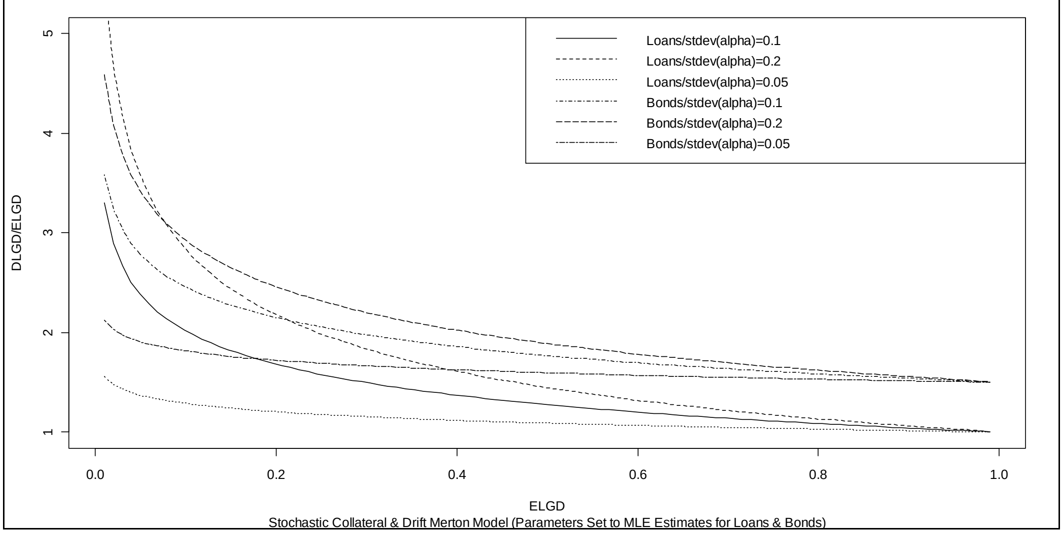


Figure 8: Ratio of Ultimate Downturn to Expected LGD vs. ELGD at 99.9th Percentile of PD-Side Systematic Factor Z



Before discussing the results, we briefly describe the two alternative frameworks for predicting ultimate LGD that are to be compared to the 2FSM-SR&RD developed in this paper. First, we implement a *full-information maximum likelihood simultaneous equation regression model* (FIMLE-SEM) for ultimate LGD, which is an econometric model built upon observations in URD at both the instrument and obligor level. FIMLE is used to model the endogeneity of the relationship between LGD at the firm and instrument levels in an internally consistent manner. This technique enables us to build a model that can help us understand some of the structural determinants of LGD, and potentially improve our forecasts of LGD. This model contains 199 observations from the URD™ with variables: long-term debt to market value of equity, book value of assets quantile, intangibles to book value of assets, interest coverage ratio, free cash flow to book value of assets, net income to net sales, number of major creditor classes, percent secured debt, Altman Z-Score, debt vintage (time since issued), Moody's 12-month trailing speculative grade default rate, industry dummy, filing district dummy and a pre-packaged bankruptcy dummy. Detailed discussion of the results can be found in Jacobs and Karagozoglu (2011). The second alternative model we consider addresses the problem of non-parametrically estimating a regression relationship, in which there are several independent variables and in which the dependent variable is bounded, as an application to the distribution of LGD. Standard non-parametric estimators of unknown probability distribution functions, whether or not conditional or not, utilize the Gaussian kernel (Silverman (1982), Hardle and Linton (1994) and Pagan and Ullah (1999)). It is well known that there exists a boundary bias with a Gaussian kernel, which assigns non-zero density outside the support on the dependent variable, when smoothing near the boundary. Chen (1999) has proposed a *beta kernel density estimator* (BKDE) defined on the unit interval [0,1], having the appealing properties of flexible functional form, a bounded support, simplicity of estimation, non-negativity and an optimal rate of convergence $n^{-4/5}$ in finite samples. Furthermore, even if the true density is unbounded at the boundaries, the BKDE remains consistent (Bouezmarni and Rolin, 2001), which is important in the context of LGD, as there are point masses (observation clustered at 0% and 100%) in empirical applications. We extend the BKDE (Renault and Scalliet, 2004) to a *generalized beta kernel conditional density estimator* (GBKDE), in which the density is a function of several independent variables, which affect the smoothing through the dependency of the beta distribution parameters upon these variables. Detailed derivation of this model can be found in Jacobs and Karagozoglu (2007), who also present a "horse-race" as herein between GBKDE the FIMLE-SEM.

Results of the model validation are shown in Table 4 and Figures 9-10. We see that while all models perform decently out-of-sample in terms of rank ordering capability, FIMLE-SEM performs the best (median = 83.2%), the GBKDE the worst (median = 72.0%), and our 2FSM-SR&RD in the middle (median = 79.1%). It is also evident from the table and figures that the better performing models are also less dispersed and exhibit less multi-modality. However, the structural model is closer in performance to the regression model by the distribution of the Pearson correlation, and indeed there is a lot of overlap in these. Unfortunately, the out-of-sample predictive accuracy is not as encouraging for any of the models, as in a sizable proportion of the runs we can reject adequacy of fit (ie p-values indicating rejection of the null of that the model fits the data it at conventional levels). The rank ordering of model performance is the same as for the Pearson statistics: FIMLE-SEM performs the best (median = 24.8%), the GBKDE the worst (median = 13.2%), and our 2FSM-SR&RD in the middle (median = 23.9%); and the structural model developed herein is comparable in out-of-sample predictive accuracy to the high-dimensional regression model. We conclude that while all models are challenged in predicting cardinal levels of ultimate LGD out-of-sample, it is remarkable that a relatively parsimonious structural model of ultimate LGD can perform so closely to a highly parameterized econometric model.

Table 4

Bootstrapped[1] out-of-sample and out-of-time classification and predictive accuracy model comparison analysis of alternative models for ultimate loss-given-default

(Moody's Ultimate Recovery Database 1987-2009)

	Test Statistic	Model	GBKDE[4]	2FSM-SR&RD[5]	FIMLE-SEM[6]
Out-of-Sample / Time 1 Year Ahead Prediction	Spearman Rank-Order Correlation[2]	Median	0.7198	0.7910	0.8316
		Standard Deviation	0.1995	0.1170	0.1054
		5th Percentile	0.4206	0.5136	0.5803
		95th Percentile	0.9095	0.9563	0.9987
	Hoshmer-Lemeshow Chi-Squared (P-Values)[3]	Median	0.1318	0.2385	0.2482
		Standard Deviation	0.0720	0.0428	0.0338
		5th Percentile	0.0159	0.0386	0.0408
		95th Percentile	0.2941	0.5547	0.5784

[1] In each run, observations are sampled randomly with replacement from the training and prediction samples, the model is estimated in the training sample and observations are classified in the prediction period, and this is repeated 100,000 times. [2] The correlation between the ranks of the predicted and realizations, a measure of the discriminatory accuracy of the model. [3] A normalized average deviation between empirical frequencies and average modelled probabilities across deciles of risk, ranked according to modelled probabilities, a measure of model fit or predictive accuracy of the model. [4] Generalized beta kernel conditional density estimator model. [5] Two-factor structural Merton systematic recovery and random drift model. [6] Full-information maximum likelihood simultaneous equation regression model. 199 observations with variables: long-term debt to market value of equity, book value of assets quantile, intangibles to book value of assets, interest coverage ratio, free cash flow to book value of assets, net income to net sales, number of major creditor classes, percent secured debt, Altman Z-Score, debt vintage (time since issued), Moody's 12-month trailing speculative grade default rate, industry dummy, filing district dummy and prepackaged bankruptcy dummy.

Fig. 9 - Densities of Pearson Correlations for LGD Prediction

100,000 Repetitions Out-of-Sample and Out-of-Time 1997-2008

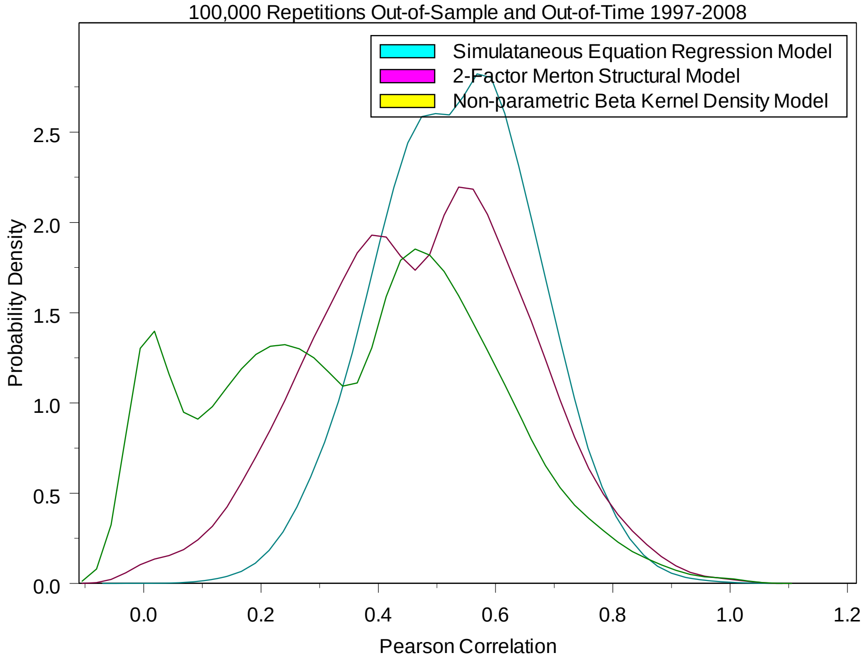
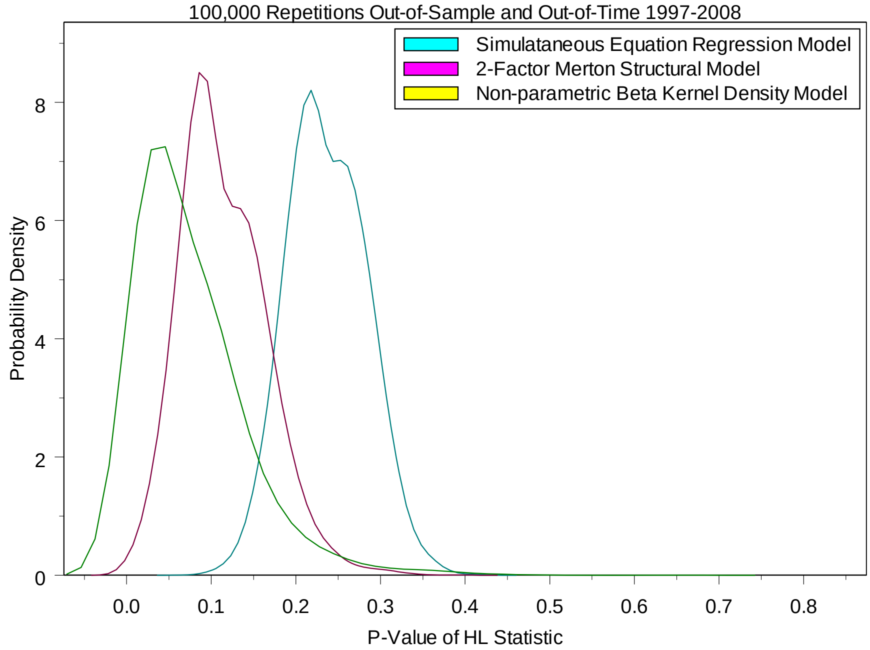


Fig.10 - Densities of Hoshmer-Lemeshow P-Values for LGD Prediction

100,000 Repetitions Out-of-Sample and Out-of-Time 1997-2008



9. Conclusions and directions for future research

In this study, we have developed a theoretical model for ultimate loss-given-default, having many intuitive and realistic features, in the structural credit risk modeling framework. Our extension admits differential seniority within the capital structure, an independent process representing a source of undiversifiable recovery risk with a stochastic drift, and an optimal foreclosure threshold. We have analyzed the comparative statics of this model and compared these to a baseline structural model. In the empirical analysis we calibrated alternative models for ultimate LGD on bonds and loans, having both trading prices at default and at resolution of default, utilizing an extensive sample of agency-rated defaulted firms in the Moody's URD™. These 800 defaults are largely representative of the US large corporate loss experience, for which we have the complete capital structures, and can track the recoveries on all instruments to the time of default to the time of resolution.

We demonstrated that parameter estimates vary significantly across models and recovery segments, finding that the estimated volatilities of the recovery rate processes and their random drifts are increasing in seniority; in particular, for first-lien bank loans as compared to senior secured or unsecured bonds. We argued that this as reflects the inherently greater risk in the ultimate recovery for higher ranked instruments having lower expected loss severities. In an exercise highly relevant to requirements for the quantification of a downturn LGD for advanced IRB under Basel II, we analyzed the implications of our model for this purpose, finding the later to be *declining* for higher expected LGD, higher for lower ranked instruments, and increasing in the correlation between the process driving firm default and recovery on collateral. Finally, we validated our leading model derived herein in an out-of-sample bootstrapping exercise, comparing it to two alternatives, a high-dimensional regression model and a non-parametric benchmark, both based upon the same URD data. We found our model to compare favorably in this exercise.

We conclude that our model is worthy of consideration to risk managers, as well as supervisors concerned with advanced IRB under the Basel II capital accord. It can be a valuable benchmark for internally developed models for ultimate LGD, as this model can be calibrated to LGD observed at default (either market prices or model forecasts, if defaulted instruments non-marketable) and to ultimate LGD measured from workout recoveries. Risk managers can use our model as an input into internal credit capital models.

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Securitization rating performance and agency incentives

Daniel Rösch[1] and Harald Scheule[2,3]

1. Introduction

This paper compares and analyzes cross-sectional and time-series characteristics of credit rating agency (CRA) ratings, implied impairment rate estimates and realized impairment rates of asset portfolio securitizations (also known as structured finance transactions). Three distinct hypotheses are analyzed, which provide empirical evidence on the role of ratings for securitizations during the global financial crisis (GFC).[4] This is of highest importance as shortcomings may have been instrumental to past, current and future loss rates of investors in relation to structured finance transactions, which are generally called securitizations. Structured finance ratings and associated fee revenue have experienced an unprecedented growth in past years. Until the GFC, such ratings were also the dominant rating category – both in terms of numbers of ratings issued as well as CRA fee revenue.[5]

The GFC led to an unprecedented and unexpected increase of impairment rates for securitizations. The disappointment of investors resulted in the criticism of models applied by credit rating agencies (CRAs). Examples are VECTOR from Fitch (see Fitch Ratings (2006)), CDOROM from Moody's (see Moody’s Investors Service (2006)) and CDO Evaluator from Standard and Poor's (see Standard and Poor’s (2005)). A similar critique was ventured after the Asian crisis of 1997 in relation to corporate bond issuer and bond issue credit ratings. For example, Leot et al (2008) find that ratings follow rather than predict the crisis as systematic downgrades occurred subsequent to the crisis.

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[1] Institute of Banking & Finance, Leibniz University of Hannover, Königsworther Platz 1, 30167 Hannover, Germany, Phone: +49-511-762-4668, Fax: +49-511-762-4670. Daniel.Roesch@finance.uni-hannover.de

[2] Department of Finance, Faculty of Economics and Commerce, University of Melbourne, Victoria 3010, Australia, Phone: +61-3-8344-9078, Fax: +61-3-8344-6914, hscheule@unimelb.edu.au

[3] We would like to thank Louis Ederington, Bruce Grundy, Marty Subrahmanyam and Hans Genberg for valuable suggestions. We would also like to thank the participants of financial seminars at the Deakin University, Deutsche Bundesbank, Global Association of Risk Professionals, University of Innsbruck, University of St. Gallen, Leibniz University Hannover, Macquarie University, Melbourne Centre for Financial Studies and The University of Melbourne as well as the discussants of various conferences for valuable comments. The paper was presented at C.R.E.D.I.T. 2009, the Asian Finance Conference 2009, the International Risk Management Conference 2009, the Australasian Finance and Banking Conference 2009, the 20th Asia Pacific Futures Research Symposium, the 59th Midwest Finance Conference, the 2010 Global Finance Conference, the 2010 FMA European Conference, the 2010 FMA Asian Conference, the 17th Annual Conference of the Multinational Finance Society, Finlawmetrics 2010, the Fourth Annual NUS Risk Management Conference and the Joint BIS/ECB/World Bank Public Investors' Conference 2010. The paper received the Australian Securities Exchange's Best Paper Award on Derivatives and Quantitative Finance in 2009. The support of the Melbourne Centre for Financial Studies and the Hong Kong Institute for Monetary Research is gratefully acknowledged.

[4] Namely, the impairment risk, agency incentive and prediction hypotheses, compare Section 2.

[5] Rating fee revenue peaked in 2007. According to Table 1, CRA Moody’s Investors Services has generated in 2007 a fee revenue of $873 million for structured finance ratings, $412 million for corporate issuer and issue ratings, $274 million for financial institution issuer and issue ratings and $221 million for public project and infrastructure ratings. The relative fee revenues in 2007 (1998) were 49% (32%) for structured finance ratings, 23% (33%) for corporate issuer and issue ratings, 15% (20%) for financial institution issuer and issue ratings and 12% (15%) for public project and infrastructure ratings.

Securitizations involve the sale of asset portfolios to bankruptcy-remote special purpose vehicles, which are funded by investors of different seniorities (tranches). Based on the nature of the securitized asset portfolios, important transaction types include asset-backed securities (ABSs), collateralized debt obligations (CDOs), home equity loan-backed securities (HELs) and mortgage-backed securities (MBSs). Despite their name, securitizations are generally over-the-counter instruments. Information is available to measure the risk of securitizations and includes credit ratings, impairment histories and proxies for the asset portfolio risk, such as asset value indices or cash flow indices. The evaluation of individual risks, their dependence structure and derivatives is complicated by the low liquidity of the underlying assets, the unavailability of secondary markets and the recent origination of such transactions.

Two main streams exist in literature on the measurement of financial risks of securitizations and – with regard to the risk exposure – similar credit derivatives. The first stream focuses on the pricing, where the central issue is to explain observed (market) prices such as credit spreads of credit default swap indices. The most prominent examples are the CDX North America and iTraxx Europe indices, which reference the default events in relation to bond portfolios. These indices were originated in 2003 and 2004. Credit spreads for the indices as well as tranches are generally available daily. Longstaff and Rajan (2008) and Hull and White (2004) apply a risk-neutral pricing framework to develop pricing techniques for these spreads. A central point of these risk models is the specification of the dependence structure for the portfolio assets.

The second stream is concerned with the modeling and estimation of risk characteristics of the underlying asset portfolio without relying on market prices. The focus is on the derivation of the distribution of future asset values (or losses) based on individual risk parameters. In the case of a loan portfolio, the relevant parameters are default probabilities, loss rates given default, exposures at default and dependence parameters such as correlations or more general copulas. Examples are as follows: Merton (1974), Leland (1994), Jarrow and Turnbull (1995), Longstaff and Schwartz (1995), Madan and Unal (1995), Leland and Toft (1996), Jarrow et al (1997), Duffie and Singleton (1999), Shumway (2001), Carey and Hrycay (2001), Crouhy et al (2001), Koopman et al (2005), McNeil and Wendin (2007) and Duffie et al (2007) address the default likelihood. Dietsch and Petey (2004) and McNeil and Wendin (2007) model the correlations between default events. Carey (1998), Acharya et al (2007), Pan and Singleton (2008), Qi and Yang (2009) and Grunert and Weber (2009) develop economically motivated empirical models for recoveries using explanatory co-variables. Altman et al (2005) model correlations between default events and loss rates given default.

Within this stream, credit ratings are often used to explain credit risk. Ratings aim to measure the credit risk of corporate bond issuers, corporate bond issues, sovereigns and structured finance issues. In the contemporary climate of the GFC, the role and importance of ratings to all market participants (eg issuers, investors and regulators), while controversial, is acknowledged. Previous research focuses on the degree to which corporate credit rating changes introduce new information. For example, Radelet and Sachs (1998) find that rating changes are pro-cyclical. This suggests that they provide only a limited amount of new information to the market. Ederington and Goh (1993), Dichev and Piotroski (2001) and Purda (2007) find that corporate credit rating downgrades provide news to the market. Loeffler (2004) finds that the default prediction power of ratings is low. Jorion et al (2005) show that after Regulation Fair Disclosure, the market impact of both downgrades and upgrades is significant and of greater magnitude compared to that observed in the pre-Regulation Fair Disclosure period. The relative roles of different CRAs have also been studied. For example, Miu and Ozdemir (2002) examine the effect of divergent Moody's and S&P's ratings of banks and Becker and Milbourn (2009) analyze the link between information efficiency of ratings and competition after the market entry of CRA Fitch.

With regard to the GFC, Rajan et al (2008) show that omission of soft information in ratings can lead to substantial model risk. Mayer et al (2008) find that the decline of housing prices was responsible for increasing sub-prime mortgage delinquency rates. Benmelech and Dlugosz (2008) analyze collateralized loan obligations (CLOs) rated by Standard and Poor's and find a mismatch between credit ratings and the quality of the underlying loan portfolios. Crouhy et al (2008) point out that CRAs' fee revenues depend on the number of ratings and may be linked to ratings quality. Similarly, Franke and Krahnen (2008) argue that incentive effects have played an important role in the GFC, particularly associated with the allocation of equity tranches of securitizations. Hull (2009) and Hellwig (2008) identify deficient CRA models as a cause of the GFC. Bolton et al (2008) show that the fraction of naive investors is higher, and the reputation risk for CRAs of getting caught understating credit risk is lower during economic booms, which gives CRAs the incentive to understate credit risk in booms.

Unfortunately, the literature has not yet empirically analyzed CRA ratings of securitizations and their accuracy in explaining impairment risk. This may have been due to the complexity of securitizations and the limited availability of data through traditional data sources. Impairment risk is the risk of a securitization to violate contractual payment obligations. Impairment events are a good proxy for the likelihood that an investor in a securitization may experience a loss.[6] To date, investors and prudential regulators assume the existence of such a link by acknowledging CRAs and assigning risk premia and risk weights to CRA rating categories. This paper addresses the accuracy of CRA securitizations. Based on the rating and impairment data of one CRA, cross-sectional and time-series characteristics of ratings, implied impairment rate estimates and realized impairment rates of asset portfolio securitizations are compared and analyzed.

The remainder of this paper is organized as follows. Section 2 develops the main hypotheses, consistent with the current literature in relation to the risk and uncertainty of CRA assessments. A framework to test the hypotheses is presented. Section 3 describes the data used in the study and analyzes three central hypotheses. Section 4 discusses the major ramifications of the empirical results for securitizations risk models and provides first suggestions in relation to a new stability framework for financial markets, institutions and instruments.

2. Hypotheses

The paper aims to answer empirically whether CRA structured finance ratings (from now on referenced as "ratings") are information efficient and may have been causal for the GFC. More specifically, information efficiency will be linked to i) the average impairment risk over time, ii) the impairment risk at and after origination and iii) the impairment risk given the economic cycle.

Rating agencies have been accused of the failure to measure impairment risk, ie the risk that investors may experience losses. Rating agencies address various elements of the asset (H1a) and liability side (H1b) of securitizations. *Impairment Risk Hypotheses* are as follows:

H1a: Ratings contain all information about the average asset quality of the asset portfolio relevant for impairment risk such as asset class, resecuritization status and transaction size.

H1b: Ratings contain all information about the characteristics of securitizations relevant for impairment risk, such as subordination level and tranche thickness.

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[6] Note that securitizations are generally structured as specific purpose companies which borrow from investors.

H1a addresses characteristics of the asset portfolio. Rajan et al (2008) find that securitization risk models omit “soft” information. This implies that CRA ratings, relying on such incomplete models omit important risk factors and hence misevaluate the average credit quality of the asset portfolio. Crouhy et al (2008) suggest that CRAs did not monitor raw data and were tardy in recognizing the implications of the declining state of the sub-prime market and support the argument by Rajan et al (2008) that other asset portfolio characteristics such as soft facts may be important drivers of asset portfolio risk.

H1b addresses the tranching structure of securitizations and the current discussion on the appropriate specification of the dependence structure of the asset portfolio, compare Hull (2009), Hellwig (2008). The probability distribution and hence the percentiles of losses associated with the pool are particularly sensitive to the correlations in the underlying asset pool. Thus, the level of subordination may be a key driver and should explain tranche impairments after controlling for credit ratings if correlations are mis-specified in the CRA model.

Furthermore, the rating agencies may have an incentive to bias the measures of impairment risk. Crouhy et al (2008) argue generally that CRA fees are paid by issuers and that CRA competition is limited by regulation. This may imply that the credit quality measured by a CRA and CRA fee revenue is positively correlated. However, CRAs publish default and rating migration tables, which are used to calibrate ratings to metric risk measures. Thus, a systematic “rating for fee” policy would be noticed and priced by investors when analyzing the financial risk in relation to ratings. H2 addresses two potential ways in which rating agencies may circumvent this rating performance mechanism. Our *Agency Incentive Hypotheses* are:

H2a: Rating-implied impairment risk and time since origination are positively correlated.

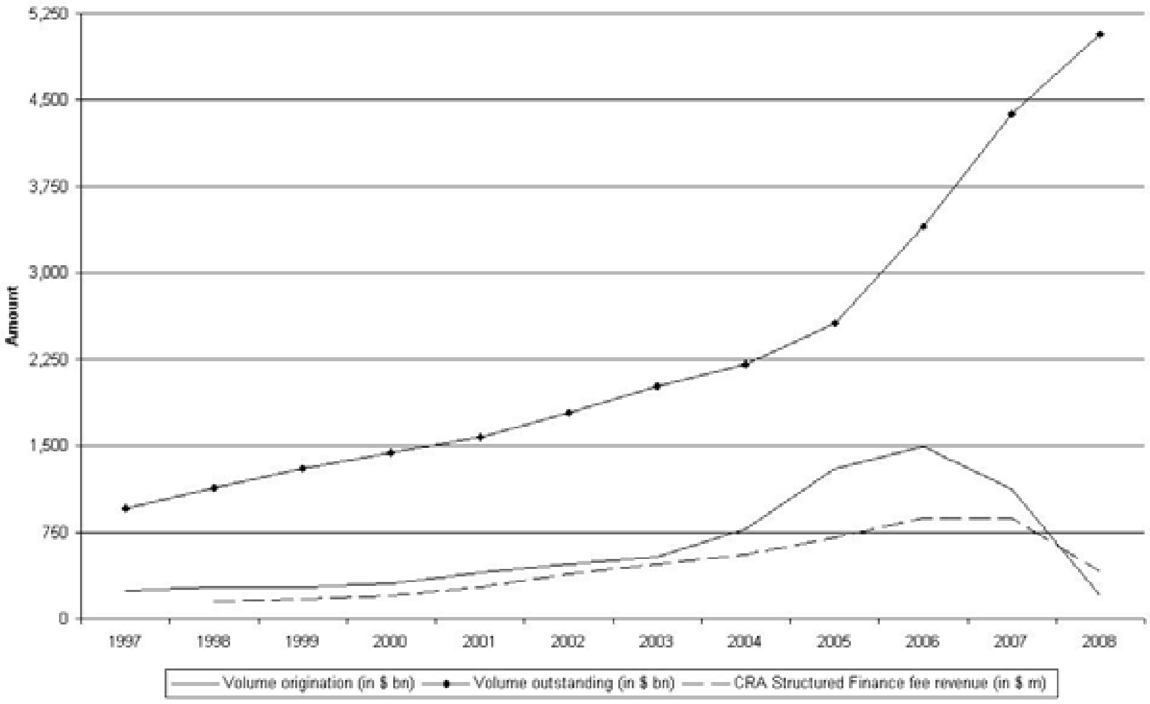
H2b: Rating-implied impairment risk and rating intensity at origination are negatively correlated.

The first incentive problem (H2a) relates to the assumption that investors do not price the risk with regard to origination and monitoring years. Rating performance measures are generally calculated as an average per rating class. The fee revenue of rating agencies is high when the first rating is generated (origination year) and low in later years when ratings are revisited (monitoring years). Figure 1 shows the origination volume and outstanding volume of the analyzed tranches as well as the CRA fee revenue.[7] It is apparent and insightful that despite the fact that CRAs provide origination and monitoring ratings, CRA fee revenue corresponds with the origination volume rather than the outstanding volume.

[7] Please note that outstanding volume as well as fee revenue relate to origination years and monitoring years while the origination volume relates to origination years only.

Figure 1

Origination volume, outstanding volume and CRA structured finance fee revenue



This chart shows the origination volume, outstanding volume and structured finance fee revenue of the CRA Moody's Investors Service. Origination volume relates to the year starting from the time that a rating was first assigned. Origination volume has increased prior to the GFC and decreased during the GFC. Outstanding numbers relate to issues which are rated at the beginning of the year and hence are originated in prior years. Outstanding volume has increased during the whole observation period. Origination volume and structured finance fee revenues have increased prior to the GFC and decreased during the GFC. Therefore, structured finance fee revenue coincides more with the origination volume which is in line with the recognition of the majority of fee revenue at or shortly after origination by the CRA.

The reason for this finding is that origination fees exceed the monitoring fees in absolute terms.[8] In addition, the fees in relation to origination and monitoring years are often paid upfront despite their lagged recognition as accounting income. As a result, CRAs may have an incentive to assign i) too low risk ratings in origination years to increase fee revenue and ii) too high risk ratings in monitoring years to maintain stable default and rating migration performance measures. The hypothesis tests whether the underestimation of risk decreases over time since origination.

The second incentive problem (H2b) relates to a critique by Bolton et al (2008) who suggest that the fraction of naive investors is higher, and the reputation risk for CRAs of getting caught understating credit risk is lower during economic booms, which gives CRAs the incentive to understate credit risk in economic booms. Figure 1 supports this argument

[8] In financial year 2007, CRA Moody's Investors Service generated 77% of fee revenue for origination of ratings and 23% for monitoring of ratings. The empirical data suggests that 37% of structured finance ratings relate to an origination year and 63% of structured finance ratings relate to a monitoring year. These numbers imply that an origination rating generates approximately 5.7 times more fee revenue than monitoring a rating for one year.

visually by showing that the origination volume and thus fee volume is high in economic booms.

Hence H2b tests whether impairment risk is underestimated during periods of high securitization activity at origination.

H3 addresses the information degree of credit ratings and their ability to forecast impairment risk. Hellwig (2008) argues that the omission of systematic factors related to real estate prices such as interest rates and the availability of housing finance may have led to an overoptimism of valuations and ratings. Such expectations may be adjusted in an economic downturn. Consequently, credit ratings which are overoptimistic and do not account for all relevant risk factors are poor predictors for impairment risk. Thus our *Prediction Hypothesis* is:

H3: Ratings predict impairment risk.

Please note that the Impairment Risk Hypotheses H1a and H1b relate to idiosyncratic risk. The Agency Incentive Hypotheses H2a and H2b relate to incentive mechanisms induced by the fee structure for securitization ratings. The Prediction Hypothesis H3 relates to the interaction between idiosyncratic and systematic risk characteristics of securitizations.

Following the models in Gordy (2000), Gordy (2003), McNeil and Wendin (2007), and Gupton et al (1997), the attachment probability (ie the propensity of being exposed to a loss in the underlying asset pool) for a tranche i of transaction (or asset pool) j in period t $\left(i=1,\ldots,I_j; j=1,\ldots,J, t=1,\ldots,T\right)$ is approximated by

$$\begin{aligned} P\left(D_{ijt}=1\right) &= 1-\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(AL_{ijt}\right)-\Phi^{-1}\left(\pi_{it}\right)}{\sqrt{\rho}}\right) \\ &= \Phi\left(\frac{-\sqrt{1-\rho}\,\Phi^{-1}\left(AL_{ijt}\right)+\Phi^{-1}\left(\pi_{it}\right)}{\sqrt{\rho}}\right) \\ &= \Phi\left(\eta_{ijt}\right) \end{aligned} \tag{1}$$

which implies that the tranche impairment probability is a function of the

- Average portfolio asset quality π_{it};
- Asset correlation ρ;
- Attachment level of a tranche relative to the total deal principal AL_{ijt}.

Please note that $\eta_{ijt} \equiv \dfrac{-\sqrt{1-\rho}\,\Phi^{-1}\left(AL_{ijt}\right)-\Phi^{-1}\left(\pi_{it}\right)}{\sqrt{\rho}}; i=1,\ldots,I_j; j=1,\ldots,J, t=1,\ldots,T.$

Reasonable assumptions in this body of literature are the modeling of credit risk of an individual borrower by a Gaussian factor model for the individual asset return based on Merton (1974) as well as a large number of assets in the pool.

All three hypotheses test whether CRAs capture impairment risk accurately. If credit ratings correctly assess the impairment risk of a tranche, then the tranche impairment probability should solely be explained by the ratings.

The impairment of tranche i $(i = 1,\ldots,I_j)$ of pool j $(j = 1,\ldots,J)$ in time t $(t = 1,\ldots,T)$ is linked with observable information by the probit regression[9]

$$P(D_{ijt} = 1) = \Phi(\beta' \chi_{ijt}) \tag{2}$$

where χ_{ijt} is a vector of tranche ratings at the beginning of an observation period. β is the respective vector of sensitivities and includes an intercept.

The models may be used for forecasting as the CRA ratings are measured at the beginning of the observation year. Note that the left hand side is the same probability as in equation (1). If ratings fully explain the impairment probability, then no other variable besides the ratings should be significant in the probit regression. In other words, if ratings reflect the tranche impairment probability accurately, they should include the information as specified in equation (1).

However, if a rating omits information, then additional information besides the rating may explain the tranche impairment probability. Examples may relate to the asset portfolio quality, the securitization structure as well as observable information about the business cycle. Consider an error in assigning one or more of the pool parameters resulting in $\tilde{\eta}_{ijt} \neq \eta_{ijt}$ which will lead to a bias in the estimated impairment probability. Then the impairment probability can be written as

$$P(D_{ijt} = 1) = \Phi(\tilde{\eta}_{ijt} + \Delta_{ijt}) \tag{3}$$

with $\Delta_{ijt} \equiv \eta_{ijt} - \tilde{\eta}_{ijt}$ denoting the measurement error in pool variables which may refer to characteristics of the pool, the tranche or time. Model (3) will provide the basis for the empirical tests in the following section.

Please note that this paper focuses on the ability of ratings and other risk factors to explain the (binary) impairment risk. Thus, the above probit analysis is appropriate to compare ratings and impairment events as it links the probability of impairment with explanatory variables. Krahnen and Weber (2001) argue that such a link is a necessity under generally accepted rating principles. These types of models have also been employed in other studies for analyzing corporate bond issue and issuer ratings or bank's loan credit ratings, compare eg Grunert et al (2005).[10]

3. Empirical analysis

3.1 Structured finance data

The paper analyzes a comprehensive panel data set of structured finance transactions rated by CRA Moody's Investors Service. The data covers characteristics of asset portfolios (which

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9 The models were also estimated for robustness using only one tranche per pool to analyze the dependence between multiple tranches in relation to a single asset portfolio. The results are qualitatively similar to the ones presented.

10 The research question is slightly different to the analysis of rating standard dynamics. One important study in this area is by Blume et al (1998) who analyze corporate rating standards and find that such rating standards have become more stringent from 1978 to 1995. Rating standard is defined in this study as the propensity to assign a certain rating category and thus an ordered probit models is estimated where the ratings grades are the dependent variables. Another example for such an approach is Becker and Milbourn (2008).

are also known as collateral portfolios), characteristics of tranches, ratings of tranches as well as occurrences of impairment events of tranches.

The focus of the present study is on the performance of CRA ratings, which involves a comparison of CRA ratings with the likelihood of occurrence of impairment events. An impairment event is defined as (compare Moody's Investors Service (2008)):

> ''[...] one of two categories, principal impairments and interest impairments. Principal impairments include securities that have suffered principal write-downs or principal losses at maturity and securities that have been downgraded to Ca/C, even if they have not yet experienced an interest shortfall or principal write-down. Interest impairments, or interest-impaired securities, include securities that are not principal impaired and have experienced only interest shortfalls."

Alternative measures for rating performance may exist. Firstly, ratings may be compared to the performance of the asset portfolios. The approach may be reasonable for asset portfolios such as mortgage-backed securities where information on the default rates of the underlying portfolios is available. We chose not to follow this approach for two reasons. Firstly, we focus on the securitization market rather than mortgage-backed securities only and find distinct differences between various asset portfolios. Secondly, credit ratings are issued for individual securities (tranches) and a key element in credit ratings is the credit enhancement (subordination) of these securities.

Secondly, ratings may be compared to the propensity of occurrence of rating downgrades. We chose not to follow this approach as our research question aims to analyze the accuracy of credit ratings. Analyzing rating downgrades limits the interpretation of results as the link between downgrades and losses to investors is less transparent.

Structured finance transactions are very heterogeneous by definition. The authors are aware of potential prudential policy implications of the research project and applied the seven filter rules to generate a homogeneous data set. Hence, the following observations are deleted:

(1) Transaction observations which can not be placed into the categories ABS, CDO, CMBS, HEL or RMBS. These are mainly asset-backed commercial paper, structured covered bonds, catastrophe bonds, and derivative product companies. 22.0% of the original number of observations are deleted;

(2) Transaction observations where the monetary volume and therefore relative credit enhancement and thickness of individual tranches could not be determined without setting additional assumptions due to i) multiple currency tranches and ii) missing senior unfunded tranche characteristics. 13.5% of the original number of observations are deleted after the application of filter rule (1);

(3) Transaction observations which are not based on the currency USD or transaction observations which are not originated in the USA. 5.0% of the original number of observations are deleted after the application of filter rule (1) and (2);

(4) The time horizon is 1997-2008. Tranche observations which relate to years prior to 1997 due to a limited number of impairment events. Impairment events are the focus of this paper and years prior to 1997 have experienced few impairment events. Years after 2008 are not yet available at the time of writing this paper. Some 7.3% of the original number of observations are deleted after the application of filter rule (1) to (3);

(5) Tranche observations which have experienced an impairment event in prior years. 0.2% of the original number of observations are deleted after the application of filter rule (1) to (4).

The resulting data comprise 325,443 annual tranche observations. The number of impaired tranche observations is 13,072.[11] The data set is one of the most comprehensive data sets on securitization collected to date.

Table 1 shows various proxies for origination[12] and outstanding volume of the data: number of tranches, number of deals and volume. In addition, rating fee revenues of the CRA Moody's Investors Service are shown. The outstanding number relates to issues which are rated at the beginning of the year and hence originated in prior years. Outstanding volume has increased during the whole observation period. Origination volume and structured finance fee revenues have increased prior to the GFC and decreased during the GFC. Therefore, structured finance fees coincide more with the origination volume which is in line with the recognition of the majority of fee revenue at or shortly after origination by the CRA.[13]

Table 1

Origination volume, outstanding volume and CRA structured finance fee revenue, various categories

	Origination volume			**Outstanding volume**			**CRA fee revenue (in $ m)**			
Year	**Tranches**	**Deals**	**Volume (in $ bn)**	**Tranches**	**Deals**	**Volume (in $ bn)**	**SF**	**Corporate**	**Financials**	**PPI**
1997	2,704	582	243	10,957	2,958	959				
1998	2,501	559	269	12,839	3,360	1,130	143	144	90	65
1999	2,665	574	271	13,855	3,702	1,298	172	166	105	60
2000	2,674	582	302	14,941	3,944	1,441	199	163	112	46
2001	4,533	761	402	16,309	4,193	1,579	274	226	131	64
2002	5,727	855	477	18,814	4,536	1,782	384	228	155	81
2003	6,783	1,014	537	21,416	4,888	2,012	475	267	181	87
2004	9,599	1,189	781	22,728	5,065	2,202	553	300	209	82
2005	16,597	1,617	1,301	28,302	5,438	2,565	709	277	214	185
2006	19,929	1,827	1,491	41,247	6,312	3,401	873	336	233	198
2007	12,958	1,405	1,126	57,661	7,511	4,380	873	412	274	221
2008	1,014	231	199	66,374	8,453	5,067	411	301	263	230
Total	87,684	11,196	7,399	325,443	60,360	27,816	5,066	2,817	1,967	1,319

This table shows the Origination volume, outstanding volume and structured finance fee revenue of the CRA Moody's Investors Service. Origination numbers relate to the year starting from the time that a rating was first assigned. Origination numbers have increased prior to the GFC and decreased during the GFC. Outstanding numbers relate to issues which are rated at the beginning of the year and hence originated in prior years. Outstanding numbers have increased during the whole observation period. SF stands for structured finance (securitization) rating revenues and PPI stand for Public, Project & Infrastructure rating revenues. SF rating fee revenues have increased prior to the GFC and decreased during the GFC.

[11] The original data set included 15,083 impairment events before the application of filtering rules.

[12] Origination volume relates to the year starting from the time that a rating was first assigned.

[13] Compare footnote 5.

From the resulting raw data, the following categorical variables were generated:

- Impairment (1: impairment, 0: no impairment) indicates that a tranche is impaired in the observation year;
- Rating at the origination of the transaction (Aaa, Aa, A, Baa, Ba, B, Caa) reflects the risk of a tranche and is measured at the beginning of an observation year;[14]
- Rating at the beginning of the respective year (Aaa, Aa, A, Baa, Ba, B, Caa) reflects the risk of a tranche and is measured at the beginning of an observation year;
- Deal category (ABS: asset backed security, CDO: collateralized debt obligation, CMBS: commercial mortgage-backed security, HEL: home equity loan security, RMBS: residential mortgage-backed security);[15]
- Resecuritization (1: resecuritization, 0: no resecuritization) indicates whether a transaction is a resecuritization of previous transactions. These transactions are often called 'squared' (e.g., CDO-squared). The database allows for the identification of resecuritizations for CDO and MBS transactions;
- Deal size: indicates the inflation-adjusted logarithm of the size of the underlying asset portfolio;
- Subordination indicates the relative size (in relation to the deal size) of the tranches that are subordinated to the respective tranche;
- Thickness indicates the relative size (in relation to the deal size) of the respective tranche;
- Origination year: year in which a tranche was first rated which coincides with the year in which transaction was closed;
- Time since origination (TSO) indicates the time in years since a tranche was first rated;
- Securitization volume at origination (SVO) indicates logarithm of the volume of rated tranches for a given year.[16]

Table 2 and Table 3 describe the number of observations over time. The overall number of rated securitizations has increased at an increasing rate over time.[17]

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[14] In the empirical analysis, the rating categories Aaa to A are aggregated to category Aaa-A due to the limited number of past impairment events in these categories.

[15] In the empirical analysis, the categories RMBS and CMBS are aggregated to category MBS due to the limited number of past impairment events in these categories.

[16] Alternative indicators of origination volumes such as the number of originated tranches or transactions were tested for robustness and resulted in similar results.

[17] All tables weight individual transactions equally and similar observations may be made for the value of securitizations.

Table 2

Total number of observations, relative frequencies of ratings at origination and at the beginning of the year

Panel A: Rating at Origination

Year	All	Aaa	Aa	A	Baa	Ba	B	Caa
1997	10,957	69.66%	16.72%	6.20%	5.04%	1.58%	0.80%	0.00%
1998	12,839	69.41%	15.02%	6.82%	5.97%	1.79%	0.97%	0.01%
1999	13,855	67.10%	13.95%	7.87%	7.28%	2.41%	1.34%	0.04%
2000	14,941	64.86%	12.76%	8.96%	8.49%	3.00%	1.84%	0.09%
2001	16,309	62.50%	12.17%	9.91%	9.67%	3.59%	2.06%	0.10%
2002	18,814	60.31%	11.45%	10.73%	11.04%	4.26%	2.10%	0.10%
2003	21,416	57.49%	11.26%	11.95%	12.16%	4.70%	2.32%	0.11%
2004	22,728	53.78%	11.39%	13.38%	13.89%	4.90%	2.55%	0.11%
2005	28,302	51.08%	12.06%	14.12%	15.21%	4.98%	2.47%	0.07%
2006	41,247	50.04%	13.48%	13.88%	15.43%	5.14%	1.99%	0.04%
2007	57,661	47.43%	15.07%	14.48%	15.86%	5.46%	1.66%	0.03%
2008	66,374	47.25%	16.18%	14.38%	14.89%	4.99%	2.02%	0.29%
Total	325,443	58.41%	13.46%	11.06%	11.25%	3.90%	1.84%	0.08%

Panel B: Rating at the beginning of a year

Year	All	Aaa	Aa	A	Baa	Ba	B	Caa
1997	10,957	72.09%	13.50%	6.74%	4.74%	1.93%	1.00%	0.00%
1998	12,839	72.57%	11.37%	7.24%	5.76%	1.94%	1.11%	0.01%
1999	13,855	70.70%	10.04%	8.05%	6.79%	2.79%	1.52%	0.10%
2000	14,941	68.04%	9.46%	9.02%	8.33%	2.94%	1.93%	0.28%
2001	16,309	65.95%	9.01%	9.97%	8.92%	3.78%	2.13%	0.25%
2002	18,814	63.03%	9.00%	10.76%	10.28%	4.44%	2.21%	0.27%
2003	21,416	58.92%	9.51%	11.88%	11.67%	4.89%	2.68%	0.44%
2004	22,728	53.96%	10.35%	13.20%	13.21%	5.31%	3.24%	0.74%
2005	28,302	51.24%	11.25%	13.86%	14.39%	5.34%	3.05%	0.87%
2006	41,247	50.70%	12.81%	13.56%	14.66%	5.31%	2.34%	0.62%
2007	57,661	48.61%	14.61%	14.00%	14.91%	5.51%	1.93%	0.44%
2008	66,374	48.23%	15.63%	12.12%	12.68%	6.16%	3.89%	1.29%
Total	325,443	60.34%	11.38%	10.87%	10.53%	4.19%	2.25%	0.44%

This table shows the total number of observations and the relative frequencies of ratings at origination and at the beginning of the year. The panel data is based on securitizations rated by CRA Moody's Investors Service. The following observations were excluded: i) transaction observations which can not be placed into the categories asset-backed security, collateralized debt obligation, commercial mortgage-backed security, residential mortgage-backed security or home equity loan security; ii) transaction observations where the monetary volume and therefore relative credit enhancement and thickness of individual tranches could not be determined without setting additional assumptions; iii) transaction observations which are not based on the currency USD or transaction observations which are not originated in the USA; iv) tranche observations which relate to years prior to 1997 due to a limited number of observations, v) tranche observations which have experienced an impairment event in prior years. The number of rated tranches has increased at an increasing rate. The rating quality of rated tranches has generally decreased over time as a smaller fraction of tranches are rated Aaa.

Table 3

Total number of observations, relative frequencies of asset portfolio and securitization characteristics

Panel A: Asset portfolio characteristics

Year	All	ABS	CDO	CMBS	HEL	RMBS	Sec.	Re-Sec.	Small	Medium	Big
1997	10,957	17.03%	0.77%	2.92%	14.88%	64.41%	93.01%	6.99%	79.55%	15.80%	4.65%
1998	12,839	20.05%	1.16%	4.15%	18.70%	55.94%	94.34%	5.66%	75.91%	18.40%	5.69%
1999	13,855	22.29%	2.36%	6.05%	21.52%	47.78%	95.51%	4.49%	72.39%	20.27%	7.34%
2000	14,941	23.97%	4.69%	8.28%	22.07%	40.99%	96.31%	3.69%	69.47%	22.46%	8.07%
2001	16,309	24.29%	6.97%	9.60%	21.94%	37.19%	96.87%	3.13%	68.61%	22.92%	8.47%
2002	18,814	21.95%	8.77%	11.43%	20.75%	37.11%	97.47%	2.53%	64.87%	25.76%	9.37%
2003	21,416	19.91%	9.96%	12.49%	20.83%	36.81%	97.87%	2.13%	61.16%	28.52%	10.32%
2004	22,728	18.73%	11.83%	13.24%	24.17%	32.03%	97.95%	2.05%	55.39%	31.34%	13.27%
2005	28,302	14.17%	12.14%	13.20%	28.26%	32.23%	98.32%	1.68%	49.68%	33.31%	17.02%
2006	41,247	9.53%	11.00%	11.35%	30.42%	37.69%	98.85%	1.15%	43.58%	35.66%	20.76%
2007	57,661	6.75%	11.40%	10.38%	31.80%	39.67%	98.97%	1.03%	39.99%	37.45%	22.56%
2008	66,374	6.11%	12.10%	10.70%	29.76%	41.33%	98.85%	1.15%	39.65%	37.29%	23.07%
Total	325,443	17.06%	7.76%	9.48%	23.76%	41.93%	97.03%	2.97%	60.02%	27.43%	12.55%

Panel B: Securitization characteristics

Year	All	Junior	Mezzanine	Senior	Thin	Thick	OY ≤ 2004	OY2005	OY2006	OY2007
1997	10,957	30.51%	38.49%	31.00%	35.43%	64.57%	100.00%	0.00%	0.00%	0.00%
1998	12,839	28.23%	39.82%	31.95%	34.88%	65.12%	100.00%	0.00%	0.00%	0.00%
1999	13,855	27.82%	42.24%	29.94%	35.22%	64.78%	100.00%	0.00%	0.00%	0.00%
2000	14,941	26.56%	44.85%	28.59%	36.51%	63.49%	100.00%	0.00%	0.00%	0.00%
2001	16,309	25.19%	47.05%	27.76%	38.18%	61.82%	100.00%	0.00%	0.00%	0.00%
2002	18,814	24.26%	48.86%	26.87%	42.18%	57.82%	100.00%	0.00%	0.00%	0.00%
2003	21,416	24.47%	49.61%	25.92%	45.60%	54.40%	100.00%	0.00%	0.00%	0.00%
2004	22,728	24.98%	49.50%	25.52%	46.44%	53.56%	100.00%	0.00%	0.00%	0.00%
2005	28,302	24.24%	50.58%	25.19%	51.09%	48.91%	100.00%	0.00%	0.00%	0.00%
2006	41,247	22.10%	51.01%	26.89%	57.52%	42.48%	59.76%	40.24%	0.00%	0.00%
2007	57,661	22.47%	51.28%	26.25%	61.73%	38.27%	37.35%	28.09%	34.56%	0.00%
2008	66,374	21.28%	52.27%	26.44%	62.16%	37.84%	29.14%	23.29%	28.04%	19.52%
Total	325,443	25.18%	47.13%	27.69%	45.58%	54.42%	85.52%	7.64%	5.22%	1.63%

This table shows the total number of observations and the relative frequencies of asset portfolio and securitization characteristics. Asset portfolio characteristics are the asset portfolio category, the resecuritization status and the asset portfolio size. The asset portfolio categories are asset backed security (ABS), collateralized debt obligation (CDO), commercial mortgage-backed security (CMBS), home equity loan security (HEL) and residential mortgage-backed security (RMBS). The resecuritization status indicates whether a transaction is a resecuritization of previous transactions or a primary securitization. The asset portfolio size is categorized into Small (inflation-adjusted asset portfolio size less than or equal to $500 million), Medium (asset portfolio size greater than $500 million and less than or equal to $1,000 million) and Big (asset portfolio size greater than $1,000 million). The number of rated tranches has increased at an increasing rate. The relative frequency of CDO and HEL has increased. The relative frequency of resecuritizations has generally decreased. The asset portfolio size has increased.

Securitization characteristics are the subordination level, the thickness and the origination year. The subordination level Junior indicates that a tranche attaches between 0 and 5%, Mezzanine indicates that a tranche attaches between 5% and 30% and Senior indicates that a tranche attaches between 30% and 100%. The relative frequency of mezzanine and thin tranches has increased.

Table 2 shows the relative frequency of rating categories at origination (Panel A) and at the beginning of the observation year (Panel B). In both panels, the average rating quality deteriorates over time as the relative frequency of the rating category Aaa declined. This may reflect i) a deterioration of the average asset portfolio quality, ii) a higher average risk level induced by the securitization structure (eg subordination, thickness or features such as embedded options, which are not addressed in this paper) or iii) a change of the CRA rating methodology.

Table 3 shows the relative frequency of asset portfolio (Panel A) and securitization characteristics (Panel B). Asset portfolio characteristics are the asset portfolio category, the resecuritization status and the asset portfolio size. The asset portfolio categories are asset-backed security (ABS), collateralized debt obligation (CDO), commercial mortgage-backed security (CMBS), home equity loan security (HEL) and residential mortgage-backed security (RMBS). The asset portfolio size is categorized into Small (asset portfolio size less than or equal to $500 million), Medium (asset portfolio size greater than $500 million and less than or equal to $1,000 million) and Big (asset portfolio size greater than $1,000 million).

The number of rated tranches has increased at an increasing rate. The relative frequency of CDO and HEL has increased. The relative frequency of resecuritizations has generally decreased. The inflation-adjusted asset portfolio size has increased.

Securitization characteristics are the subordination level, thickness and origination year. The subordination level Junior indicates that a tranche attaches between 0% and 5%, Mezzanine indicates that a tranche attaches between 5% and 30% and Senior indicates that a tranche attaches between 30% and 100%.

The relative frequency of mezzanine and thin tranches has increased while the relative frequency of the various origination years (OY) depends on the origination as well as the maturity and impairment of securitizations.

Generally speaking, the validation of credit ratings is complicated as the use of ratings involves two steps: firstly the ordinal assessments of the financial risk of issuers or issues by CRAs and secondly the calibration of these ordinal ratings to metric credit risk measures such as default rates, loss rates given default or unconditional loss rates. This calibration step is generally opaque and investors rely on impairment rate tables which are periodically published by CRAs. These tables aggregate the impairment events over dimensions such as rating class or observation year. The data set enables the estimation of impairment risk based on the most detailed information level, ie the individual transaction in a given observation year. Table 4 and Table 5 show the impairment rates over time for all tranches as well as per rating category, asset portfolio and securitization characteristics.

US securitizations have experienced two economic downturns during the observation period: the first one in 2002 subsequent to the US terrorist attacks (a period characterized by large bankruptcies such as Enron, WorldCom and various US airlines) and the Global Financial Crisis. With regard to the GFC, the impairment rate has increased by a factor of approximately 80 within two years between 2006 and 2008. Approximately 81% of all impairment events relate to 2008.[18]

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[18] While this number underlines the severity of the GFC and the importance of this study it raises the concern of imbalances in the data set. We address this issue for robustness by i) controlling for rating years, ii) analyzing the data for the period prior to the GFC and the GFC and iii) focusing on relative differences within these controlled environments.

Table 4

Impairment rates for all observations, per rating at origination and at the beginning of the year

Panel A: Rating at origination

Year	All	Aaa-A	Baa	Ba	B	Caa
1997	0.27%	0.00%	2.17%	4.62%	11.36%	0.00%
1998	0.19%	0.03%	1.83%	1.74%	2.40%	0.00%
1999	0.35%	0.15%	1.88%	2.40%	1.08%	0.00%
2000	0.31%	0.08%	0.95%	3.79%	2.55%	0.00%
2001	0.58%	0.07%	2.47%	2.74%	8.63%	5.88%
2002	1.08%	0.10%	4.77%	7.61%	7.09%	0.00%
2003	0.85%	0.19%	3.88%	2.88%	3.02%	20.83%
2004	0.94%	0.61%	1.55%	2.70%	3.11%	26.92%
2005	0.27%	0.07%	0.95%	0.43%	1.86%	5.00%
2006	0.20%	0.07%	0.41%	0.57%	2.68%	0.00%
2007	2.49%	0.48%	7.37%	16.80%	1.77%	0.00%
2008	16.02%	9.88%	38.05%	36.96%	28.07%	90.63%
Total	1.96%	0.17%	2.57%	4.21%	4.14%	5.33%

Panel B: Rating at the beginning of a year

Year	All	Aaa-A	Baa	Ba	B	Caa
1997	0.27%	0.00%	0.39%	6.64%	12.73%	0.00%
1998	0.19%	0.03%	1.08%	4.42%	2.10%	0.00%
1999	0.35%	0.06%	1.70%	2.84%	5.21%	21.43%
2000	0.31%	0.02%	0.56%	2.96%	3.13%	35.71%
2001	0.58%	0.06%	2.13%	3.57%	8.36%	12.50%
2002	1.08%	0.06%	2.43%	11.72%	8.89%	26.00%
2003	0.85%	0.05%	2.16%	4.96%	8.00%	23.16%
2004	0.94%	0.27%	1.37%	3.07%	5.30%	28.99%
2005	0.27%	0.00%	0.17%	0.79%	2.89%	13.06%
2006	0.20%	0.00%	0.12%	0.50%	2.07%	17.25%
2007	2.49%	0.44%	7.20%	16.49%	4.68%	16.73%
2008	16.02%	7.53%	34.11%	45.93%	55.16%	77.84%
Total	1.96%	0.09%	1.75%	5.27%	5.76%	17.71%

This table shows impairment rates for all observations, per rating at origination and at the beginning of the year. The impairment rate is the ratio between the number of impairment events and the total number of observations in a given category and observation year. Impairment events '[...]fall into one of two categories, principal impairments and interest impairments. Principal impairments include securities that have suffered principal write-downs or principal losses at maturity and securities that have been downgraded to Ca/C, even if they have not yet experienced an interest shortfall or principal write-down. Interest impairments, or interest-impaired securities, include securities that are not principal impaired and have experienced only interest shortfalls.' (compare Moody's Investors Service (2008)).

Impairment rates are high in 2002 and 2007/2008. Impairment rates increase from rating category Aaa to C and fluctuate over time. The rating categories Aaa, Aa and A are aggregated into one category Aaa-A due to the limited number of impairment events.

Table 5

Impairment rates for all observations as well as asset portfolio and securitization characteristics

Panel A: Asset portfolio characteristics

Year	All	ABS	CDO	HEL	MBS	Sec.	Re-Sec.	Small	Medium	Big
1997	10,957	0.00%	0.00%	1.41%	0.09%	0.29%	0.00%	0.34%	0.00%	0.00%
1998	12,839	0.16%	0.00%	0.79%	0.03%	0.20%	0.14%	0.26%	0.00%	0.00%
1999	13,855	0.36%	0.61%	0.97%	0.08%	0.36%	0.00%	0.47%	0.04%	0.00%
2000	14,941	0.42%	1.43%	0.49%	0.07%	0.30%	0.54%	0.41%	0.03%	0.17%
2001	16,309	0.73%	3.96%	0.34%	0.12%	0.60%	0.20%	0.71%	0.27%	0.43%
2002	18,814	2.15%	4.91%	0.36%	0.22%	1.11%	0.00%	1.36%	0.62%	0.45%
2003	21,416	2.18%	1.97%	0.58%	0.21%	0.87%	0.22%	0.94%	0.72%	0.72%
2004	22,728	3.27%	1.56%	0.20%	0.20%	0.95%	0.21%	1.28%	0.55%	0.43%
2005	28,302	0.45%	0.58%	0.21%	0.17%	0.28%	0.00%	0.37%	0.16%	0.21%
2006	41,247	0.69%	0.26%	0.16%	0.11%	0.20%	0.00%	0.25%	0.21%	0.07%
2007	57,661	0.46%	4.67%	5.53%	0.33%	2.51%	0.17%	2.74%	2.44%	2.12%
2008	66,374	0.17%	24.93%	29.00%	8.39%	15.98%	19.40%	13.50%	18.65%	16.11%
Total	325,443	0.92%	3.74%	3.34%	0.83%	1.97%	1.74%	1.89%	1.97%	1.73%

Panel B: Securitization characteristics

Year	All	Junior	Mezzanine	Senior	Thin	Thick	OY ≤ 2004	OY2005	OY2006	OY2007
1997	10,957	0.90%	0.00%	0.00%	0.46%	0.17%	0.27%			
1998	12,839	0.52%	0.12%	0.00%	0.33%	0.12%	0.19%			
1999	13,855	0.73%	0.32%	0.02%	0.41%	0.31%	0.35%			
2000	14,941	0.96%	0.12%	0.00%	0.33%	0.30%	0.31%			
2001	16,309	1.53%	0.42%	0.00%	0.75%	0.48%	0.58%			
2002	18,814	3.40%	0.50%	0.06%	1.68%	0.65%	1.08%			
2003	21,416	1.95%	0.75%	0.02%	1.23%	0.54%	0.85%			
2004	22,728	1.60%	0.97%	0.22%	1.14%	0.76%	0.94%			
2005	28,302	0.73%	0.18%	0.01%	0.41%	0.12%	0.27%			
2006	41,247	0.61%	0.11%	0.02%	0.21%	0.18%	0.32%	0.01%		
2007	57,661	8.56%	1.09%	0.03%	3.80%	0.37%	0.83%	0.62%	5.79%	
2008	66,374	40.08%	13.55%	1.54%	22.97%	4.62%	2.92%	11.55%	26.50%	25.88%
Total	325,443	5.13%	1.51%	0.16%	2.81%	0.72%	0.74%	4.06%	16.14%	25.88%

This table shows the impairment rates for all observations, per deal and tranche characteristics. Impairment rates are high in 2002 and 2007/2008. Impairment rates per rating category fluctuate over time. Impairment rates per asset portfolio type increase in 2002 for CDOs and in 2008 especially for CDOs, HELs and MBSs. The asset classes CMBS and RMBS are aggregated to the category MBS due to the limited number of impairment events. The impairment rate has particularly increased in 2008 especially for resecuritizations, all subordination levels and tranches originated in years prior to the GFC.

Table 4 shows the impairment rates for rating categories at origination (Panel A) and at the beginning of the observation year (Panel B). In both panels, the impairment rate increases for lower rating categories (ie from Aaa-A to Caa) and fluctuates over time with a dramatic increase during the GFC for all rating classes. The relative increase decreases during the GFC with the rating quality (ie from Caa to Aaa-A). Ironically, investors were most surprised by the increase of impairment rates of highly rated securitizations.[19]

Table 5 shows the impairment rates for asset portfolio (Panel A) and securitization characteristics (Panel B). Impairment rates are high in 2002 and 2007/2008. Impairment rates per rating category fluctuate over time. Impairment rates per asset portfolio type increased in 2002 for CDOs and in 2008 especially for CDOs, MBSs and HELs. HELs include sub-prime mortgage loans and the impairment risk increased to a larger degree than the one of MBSs. It can also be seen that HELs and MBSs did not experience an economic downturn in 2002. The asset classes CMBS and RMBS are aggregated to the category MBS due to the limited number of impairment events. The impairment rate has increased in 2008 especially for resecuritizations. The levels of the impairment rates are fundamentally different between the various asset portfolio categories. Impairment rates of junior tranches increased more than impairment rates of senior tranches. Impairment rates of thin tranches increased more than impairment rates of thick tranches and the ones of more recent vintage (with regard to the GFC) more so than the ones of older vintage.

3.2 H1 – Impairment risk hypotheses

Table 6 presents two probit models linking the impairment events with CRA ratings. Model 1 takes the dummy-coded ratings (reference category: Aaa-A) into account. Model 1 shows that CRA ratings explain the credit risk. As measures for in-sample accuracy of the models the Pseudo-R^2, re-scaled R^2, and the area under the receiver operating characteristic curve (AUROC) are calculated (see Agresti (1984)).[20] The parameter estimates increase from rating Aaa-A to rating Caa and are significant. This demonstrates that the ratings imply higher impairment risk from Aaa to Caa and that ratings explain impairment risk.

Model 2 includes the ratings as well as the dummy-coded rating years (reference category: 1997). The rating years are significant which implies that the realized impairment rates differ between the years. This has been pointed out by previous studies on corporate ratings (compare eg Loeffler (2004) which conclude that ratings average the risk over the business cycle.[21] In other words, Model 2 shows that CRA ratings do not explain the increased level of impairment risk especially during economic downturns. We include rating year dummies in all subsequent models to control for this and further analyze the prediction quality of ratings in hypothesis H3.

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[19] Please note that inconsistencies may reflect the accuracy as well as the stochastic nature of impairment events. The latter is particularly relevant if the number of observations is low for a given category. One example is the impairment rates for the rating classes Ba (16.49%) and B (4.68%) in 2007 in Panel B of Table 4. These inconsistencies are in line with reports by the data-providing CRA (compare Moody's Investors Service (2008)).

[20] All measures are bounded between zero (lowest fit) and one (highest fit).

[21] Such models are also known as through-the-cycle models.

Table 6

The link between impairment risk, CRA ratings and time

Variable	Model 1	Model 2
Intercept	-2.1517***	-3.2346***
	0.0062	0.0741
Baa	0.8351***	1.0397***
	0.0107	0.0133
Ba	1.1900***	1.4301***
	0.0133	0.0163
B	1.3276***	1.5209***
	0.0167	0.0202
Caa	2.0038***	2.2803***
	0.0287	0.0344
1998		-0.1159
		0.1051
1999		0.0142
		0.0933
2000		-0.1526
		0.0955
2001		0.1083
		0.0855
2002		0.3217***
		0.0804
2003		0.1596**
		0.0807
2004		0.1622**
		0.0796
2005		-0.4408***
		0.087
2006		-0.5317***
		0.0859
2007		0.6662***
		0.0749
2008		1.7862***
		0.0741
Pseudo R-square	0.0520	0.1220
R-square rescaled	0.1818	0.4265
AUROC	0.7688	0.9231

This table shows parameter estimates for the probit models Model 1 to Model 2. The model specification is $P(D_{ijt} = 1) = \Phi(\beta' \chi_{ijt})$. Standard errors are in parentheses. The significance is indicated as follows: ***: significant at 1%, **: significant at 5%, *: significant at 10%. AUROC is the area under the receiver operating characteristic curve (see Agresti (1984)).

Model 1 shows that CRA ratings explain the credit risk over time. Model 2 shows that CRA ratings are unable to explain changes in the increased level of impairment risk over time.

Table 7

The link between impairment risk, CRA ratings, asset portfolio and securitization characteristics, with rating year dummies

Variable	Model 3	Model 4	Model 5	Model 6 (prior GFC)	Model 7 (GFC)
Intercept	–5.6417***	–2.8000***	–4.5874***	0.2176***	–7.0547***
	0.1575	0.0750	0.1694	0.3047	0.2006
Baa	0.9849***	0.6949***	0.5668***	0.8263***	0.5472***
	0.0138	0.0143	0.0152	0.0481	0.0169
Ba	1.4267***	1.0748***	0.9934***	1.4125***	0.9244***
	0.0170	0.0172	0.0183	0.0510	0.0208
B	1.6326***	1.1510***	1.2224***	1.8561***	1.0900***
	0.0216	0.0212	0.0228	0.0558	0.0268
Caa	2.3478***	1.9833***	1.9779***	2.5822***	1.7801***
	0.0365	0.0356	0.0382	0.0665	0.0495
CDO	0.5059***		0.5925***	–0.3066***	2.1625***
	0.0263		0.0274	0.0428	0.0801
HEL	0.5885***		0.4660***	–0.4728***	1.9970***
	0.0245		0.0252	0.0419	0.0789
MBS	–0.2606***		–0.4380***	–1.1824***	1.0394***
	0.0253		0.0262	0.0475	0.0791
Resecuritisation	0.2355***		0.3450***	–0.0909	0.3954***
	0.0528		0.0561	0.1530	0.0634
Deal size	0.1220***		0.0994***	–0.1383***	0.1657***
	0.0071		0.0077	0.0151	0.0090
Subordination		–2.6234***	–3.4892***	–1.4095***	–4.0653***
		0.0602	0.0792	0.1708	0.0935
Thickness		–0.5138***	–0.6260***	–0.5851***	–0.5317***
		0.0388	0.0454	0.0893	0.0538
Year Dummies	Yes	Yes	Yes	Yes	Yes
Pseudo R-square	0.1355	0.1328	0.1476	0.0246	0.2231
R-square rescaled	0.4735	0.4643	0.5159	0.4048	0.4729
AUROC	0.9427	0.9416	0.9540	0.9507	0.9171

This table shows parameter estimates for the probit model Model 3 to Model 7. The model specification is $P(D_{ijt} = 1) = \Phi(\beta' \chi_{ijt})$. Standard errors are in parentheses. The significance is indicated as follows: ***: significant at 1%, **: significant at 5%, *: significant at 10%. AUROC is the area under the receiver operating characteristic curve (see Agresti (1984)).

The inclusion of asset portfolio (Model 3 and 5) and securitization (Model 4 and 5) characteristics after controlling for credit rating and rating year explains impairment risk. The ramifications are that CRA ratings do not sufficiently account for the impairment risk stipulated by asset portfolio and securitization characteristics for given rating years. The division of the data into pre-GFC and GFC years shows that the asset portfolio characteristics (asset portfolio category, resecuritization status and deal size) are cyclical as the parameter sign changes while the securitization characteristics are not cyclical. CRAs are unable to measure both relationships.

Table 7 confirms that the inclusion of asset portfolio (Model 3 and 5, Model 6 and 7) and securitization (Model 4 and 5, Model 6 and 7) characteristics after controlling for credit ratings add to the explanation of impairment risk. The ramifications are that CRA ratings do not sufficiently account for the average impairment risk stipulated by asset portfolio and securitization characteristics over time.

The split of the data into pre-GFC and GFC years shows that the asset portfolio characteristics (asset portfolio category, resecuritization status and deal size) are cyclical as the parameter sign changes while the securitization characteristics are not cyclical. Impairment risk is significantly lower (higher) for CDO, HEL, MBS, resecuritization and big deals before (during) the GFC than during (before) the GFC. Likewise, subordination and tranche thickness are negatively related to impairment risk and ratings are not able to explain this.

In summary, we reject the hypothesis H1a that ratings contain all information about the average asset quality of the asset portfolio relevant for impairment risk. In addition, we reject hypothesis H1b that ratings contain all information about the characteristics of securitizations relevant for impairment risk. CRAs do not take into account all the available asset portfolio and securitization information that is relevant to explaining impairment risk. The important ramifications are that i) CRAs may have to include such characteristics into the rating models or ii) users such as investors or prudential regulators should apply asset portfolio specific impairment rates to ratings when interpreting CRA ratings.[22]

3.3 H2 – Agency incentive hypotheses

Commercial CRAs may have a monetary incentive to bias the measures of impairment risk. The analyzed incentive hypotheses relate to the origination process during which a CRA may underestimate the risk in general (as fee revenue is high at origination) or during economic booms (as origination volumes and therefore fee revenue is high during economic booms).[23]

Model 8 in Table 8 shows that different origination years (also known as vintages) differ in risk. Models 9 and 10[24] show that ratings are unable to explain the risk of the different vintages.

Even more interestingly, Models 11 and 12 show that the vintage risk differs between the years prior to the GFC and during the GFC. During the GFC, the risk which is not reflected in ratings, increases for more recent origination and is highest for securitizations, which were originated immediately before the GFC. Vice versa, during years before the GFC, the risk which is not reflected in ratings decreases for more recent originations.

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[22] Despite the common use of ratings as metric risk measures, CRAs often claim to assess the relative risk, which essentially implies that a rating of a higher alphabetic order involves a lower level of financial risk. In an extension, all models were estimated controlling for the annual average impairment rate to ascertain that the findings relate to the absolute (calibration) as well as relative (discrimination) level of risk. The results are comparable to the ones reported in Tables 6 and 7.

[23] In addition, Bolton et al (2009) argue that investors are naive and reputational risk is low.

[24] Model 10 controls for the rating year. Please note that the panel data set looks at origination and monitoring years, ie years between origination and maturity of securitizations.

Table 8

The link between impairment risk, CRA ratings and incentive characteristic

Variable	Model 8	Model 9	Model 10	Model 11 (prior GFC)	Model 12 (GFC)
Intercept	–2.6520***	–3.2727***	–3.3453***	–3.2831***	–4.7431***
	0.0193	0.0264	0.0770	0.0789	0.1628
Baa		1.0302***	1.1717***	1.0443***	1.1979***
		0.0134	0.0146	0.0423	0.0157
Ba		1.4544***	1.5794***	1.5101***	1.5950***
		0.0164	0.0178	0.0433	0.0201
B		1.7405***	1.7628***	1.6905***	1.7774***
		0.0208	0.0224	0.0464	0.0270
Caa		2.5912***	2.7181***	2.4704***	2.9414***
		0.0344	0.0394	0.0616	0.0604
OY1998	0.3606***	0.1266***	0.1171**	0.1030**	0.5618***
	0.0350	0.0446	0.0477	0.0486	0.2010
OY1999	0.4210***	0.1307***	0.1160***	0.1294***	0.1775***
	0.0335	0.0423	0.0469	0.0473	0.2043
OY2000	0.4817***	0.1095**	0.0866	0.0745	0.3565***
	0.0339	0.0426	0.0474	0.0486	0.1893
OY2001	0.3010***	–0.0353	–0.0878***	–0.1836***	0.7052***
	0.0341	0.0428	0.0488	0.0526	0.1750
OY2002	0.2784***	0.0618	0.0282***	–0.2806***	1.1220***
	0.0324	0.0400	0.0490	0.0596	0.1679
OY2003	0.1400***	0.0613	–0.0233***	–0.8856***	1.1371***
	0.0329	0.0404	0.0521	0.1153	0.1660
OY2004	0.2993***	0.2212***	0.1029***	–0.8876***	1.1386***
	0.0281	0.0346	0.0497	0.1611	0.1640
OY2005	0.8911***	1.0017***	0.8465***	–1.0269***	1.8801***
	0.0219	0.0279	0.0445	0.2151	0.1623
OY2006	1.6489***	1.7959***	1.5317***		2.5416***
	0.0207	0.0267	0.0435		0.1620
OY2007	2.0051***	2.2405***	1.5700***		2.5816***
	0.0226	0.0286	0.0447		0.1623
Year Dummies	No	No	Yes	Yes	Yes
Pseudo R-square	0.0744	0.1246	0.1440	0.0205	0.2094
R-square rescaled	0.2602	0.4356	0.5035	0.3377	0.4439
AUROC	0.8533	0.9266	0.9479	0.9285	0.8995

This table shows parameter estimates for the probit model Model 8 to Model 12. The model specification is $P(D_{ijt} = 1) = \Phi(\beta' \chi_{ijt})$. Standard errors are in parentheses. The significance is indicated as follows: ***: significant at 1%, **: significant at 5%, *: significant at 10%. AUROC is the area under the receiver operating characteristic curve (see Agresti (1984)).

The risk of securitization differs for each origination year (OY) and CRAs are unable to measure this element. In addition, Model 11 and Model 12 show that the risk of recent origination years is high for the GFC and low for years prior to the GFC.

In order to test the hypotheses H2a and H2b, we replace the origination year dummies by the time since origination (TSO) and the securitization volume at origination (SVO). TSO is equal to one in the origination year and greater than one in monitoring years.[25]

Table 9

The link between impairment risk,CRA ratings and incentive characteristics (cont.)

	All years			prior GFC			GFC		
Variable	**Model 13**	**Model 14**	**Model 15**	**Model 16**	**Model 17**	**Model 18**	**Model 19**	**Model 20**	**Model 21**
Intercept	–2.6666***	–20.5275***	–14.5172***	–3.3839***	–8.3009***	–10.6136***	–1.9064***	–25.7527***	–15.6255***
	0.0781	0.2852	0.3355	0.0824	0.5122	0.5702	0.0176	0.3568	0.4297
Baa	1.0849***	1.1121***	1.1182***	1.0418***	1.0367***	1.1168***	1.1585***	1.1516***	1.1845***
	0.0139	0.0138	0.0140	0.0421	0.0417	0.0432	0.0154	0.0151	0.0155
Ba	1.5241***	1.5944***	1.5976***	1.5260***	1.5595***	1.6511***	1.5786***	1.6101***	1.6337***
	0.0170	0.0173	0.0175	0.0430	0.0432	0.0454	0.0197	0.0197	0.0202
B	1.7323***	1.8604***	1.8897***	1.7317***	1.8248***	1.9094***	1.7911***	1.8360***	1.9216***
	0.0215	0.0225	0.0228	0.0458	0.0474	0.0491	0.0264	0.0266	0.0279
Caa	3.0060***	2.8240***	3.1527***	2.6315***	2.8019***	2.7880***	3.1612***	2.7189***	3.3976***
	0.0417	0.0397	0.0437	0.0604	0.0628	0.0629	0.0612	0.0518	0.0688
TSO	–0.2554***		–0.1692***	0.0274***		0.0644***	–0.3807***		–0.2996***
	0.0042		0.0049	0.0057		0.0062	0.0055		0.0063
SVO		0.7006***	0.4759***		0.2062***	0.2901***		0.8718***	0.5094***
		0.0109	0.0129		0.0206	0.0224		0.0133	0.0159
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Pseudo R-square	0.1360	0.1364	0.1400	0.0195	0.0200	0.0204	0.2031	0.1933	0.2103
R-square rescaled	0.4755	0.4767	0.4895	0.3213	0.3285	0.3362	0.4305	0.4098	0.4458
AUROC	0.9399	0.9376	0.9424	0.9184	0.9181	0.9187	0.8953	0.8790	0.9008

This table shows parameter estimates for the probit model Model 13 to Model 21. The model specification is $P(D_{ijt} = 1) = \Phi(\beta' \chi_{ijt})$. Standard errors are in parentheses. The significance is indicated as follows: ***: significant at 1%, **: significant at 5%, *: significant at 10%. AUROC is the area under the receiver operating characteristic curve (see Agresti (1984)).

The first panel (all years) shows that the impairment risk given ratings (ie which is not explained by ratings) decreases with time since origination. This confirms that CRAs may have an incentive to assign i) too low risk ratings in origination years to increase fee revenue and ii) too high risk ratings in monitoring years to maintain stable default and rating migration performance measures. The second and third panel show that this effect is mainly driven by occurrence of the GFC. In addition, impairment risk given ratings increases with the securitization activity at origination. This result holds for all years, the years before and during the GFC.

Table 9 shows that the negative parameter estimate (panel for all years) for the time since origination (TSO) implies that the level of impairment risk (given the rating) decreases over

[25] High SVO indicates that a tranche was originated in a high securitization volume year (ie especially 2002 and later). Low SVO indicates that a tranche was originated in a low securitization volume year (ie especially before 2002).

time. The relative fee revenue is high at origination and low thereafter. The implication is that the impairment risk given ratings (ie which is not explained by ratings) decreases over time. This confirms that CRAs may have an incentive to assign i) too low risk ratings in origination years to increase fee revenue and ii) too high risk ratings in monitoring years to maintain stable default and rating migration performance measures. The second and third panel show that this effect is mainly driven by the occurrence of the GFC. Thus we reject the hypothesis H2a that rating-implied impairment risk and time since origination are positively correlated.

In addition, a high securitization volume at origination (when absolute fee revenue is high) implies high impairment risk after controlling for rating. This result holds for the years before and during the GFC. Thus we reject the hypothesis H2b that rating-implied impairment risk and rating intensity at origination are negatively correlated.

Both hypothesis tests suggest that impairment risk is under-represented by ratings when fee revenue is high, which is the case at origination and during an economic boom when origination volume is high.

3.4 H3 – Prediction hypothesis

Ratings are generally applied as proxies for future impairment risk. The information content of corporate bond issue ratings has been analysed (compare, eg, Blume et al (1998)). However, no evidence for CRA ratings on securitizations has been presented. Our previous results show that credit ratings do not include all relevant risk factors and are overoptimistic when fee revenue is high. Therefore we now check how this affects the ability for predicting future impairment risk.

The forecasting power of credit ratings is tested by an approach related to (Rajan et al (2008)) which directly links ratings to future impairment risk. The approach proceeds in three steps.

Firstly, a probit regression is estimated for each year

$$P\left(D_{ijt} = 1\right) = \Phi\left(\beta' \chi_{ijt}\right) \tag{4}$$

where χ_{jit} are dummy variables for the ratings, which are observed at the beginning of the observation period. Next, the linear predictor for the subsequent year is calculated:

$$\hat{\eta}_{ijt+1} = \hat{\beta}' \chi_{ijt+1} \tag{5}$$

and the impairment probability prediction for the subsequent year

$$\hat{p}_{ijt+1} = \Phi\left(\hat{\beta}' \chi_{ijt+1}\right) \tag{6}$$

using the estimated coefficients $\hat{\beta}$ from Equation (4). Finally, the forecasting power is assessed by running a probit regression (Model 22).

$$P\left(D_{ijt+1} = 1\right) = \Phi\left(\gamma_0 + \gamma_1 \hat{\eta}_{ijt+1}\right) \tag{7}$$

We test for $\gamma_0 = 0$ and $\gamma_1 = 1$, i.e., whether ratings provide perfect forecasts. As a robustness check a linear regression is estimated (Model 23):

$$D_{ijt+1} = \delta_0 + \delta_1 \cdot \hat{p}_{ijt+1} + \varepsilon_{ijt+1} \tag{8}$$

so that $E\left(D_{ijt+1}\right) = P\left(D_{ijt+1}\right) = \delta_0 + \delta_1 \cdot \hat{p}_{ijt+1}$ where $\delta_0 = 0$ and $\delta_1 = 1$.

Again, we test for $\delta_0 = 0$ and $\delta_1 = 1$. All steps are repeated for each year from 1999 to 2008 where in the probit regression (4) all data up to year t are used. Table 10 shows the parameter estimates from each regression Model 22 (Equation 7). Table 11 contains the estimation results from each regression Model 23 (Equation 8).

Table 10

The link between realized and predicted impairment risk (probit regression)

	(1)	(2)	(3)	(4)	(5)
Prediction year	γ_0	γ_1	**Pseudo R^2**	**R^2 Rescaled**	**AUROC**
1999	–0.7917***	0.6206***	0.0079	0.741	0.851
	(0.1668)	(0.0587)			
2000	0.1750	1.1776	0.0158	0.3852	0.949
	(0.2309)	(0.1210)			
2001	–0.1547	0.8558***	0.0180	0.2607	0.905
	(0.1321)	(0.0540)			
2002	0.5501***	1.1008*	0.0375	0.3328	0.926
	(0.1160)	(0.0529)			
2003	–0.1045	0.9276	0.0271	0.2896	0.913
	(0.0995)	(0.0482)			
2004	–0.6379***	0.6700***	0.0193	0.1916	0.821
	(0.0820)	(0.0351)			
2005	–0.3331**	1.1792**	0.0131	0.3553	0.958
	(0.1376)	(0.0854)			
2006	0.2745*	1.5383***	0.0121	0.4276	0.941
	(0.1596)	(0.1008)			
2007	0.6017***	0.9468***	0.0442	0.2127	0.839
	(0.0493)	(0.0192)			
2008	1.4974***	0.9788**	0.1453	0.2482	0.750
	(0.0252)	(0.0098)			

This table shows the results of out-of-sample prediction probit regression Model 22. The model specification is $P(D_{ijt+1}=1)=\Phi(\gamma_0+\gamma_1\hat{\eta}_{ijt+1})$. Standard errors are in parentheses. The significance is indicated as follows: ***: significant at 1%, **: significant at 5%, *: significant at 10%. The tested hypotheses are that $\gamma_0=0$ and $\gamma_1=1$.

The estimated parameters γ_0 and γ_1 are statistically different from $\gamma_0=0$ and $\gamma_1=1$. The ramification is that CRA ratings do not predict impairment risk.

Table 11

The link between realized and predicted impairment risk (linear regression)

	(1)	(2)	(3)
Prediction year	δ_0	δ_1	Adj. R^2
1999	0.0014***	0.6513***	0.0178
	(0.0005)	(0.0410)	
2000	–0.0018***	1.1613***	0.1009
	(0.0004)	(0.0284)	
2001	0.0029***	0.6721***	0.0265
	(0.0006)	(0.0319)	
2002	0.0024***	1.6082***	0.0678
	(0.0008)	(0.0435)	
2003	0.0007	0.9589***	0.0587
	(0.0006)	(0.0262)	
2004	0.0001	0.9407***	0.0683
	(0.0007)	(0.0230)	
2005	–0.0017***	0.4375**	0.0567
	(0.0003)	(0.0106)	
2006	–0.0024***	0.6031***	0.0768
	(0.0002)	(0.0103)	
2007	0.0155***	1.7140***	0.0322
	(0.0007)	(0.0391)	
2008	0.0925***	5.1955***	0.1573
	(0.0014)	(0.0467)	

This table shows the results of out-of-sample prediction linear regression Model 23. The model specification is $D_{ijt+1} = \delta_0 + \delta_1 \cdot \hat{p}_{ijt+1} + \varepsilon_{ijt+1}$. Standard errors are in parentheses. The significance is indicated as follows: ***: significant at 1%, **: significant at 5%, *: significant at 10%. The tested hypotheses are that $\delta_0 = 0$ and $\delta_1 = 1$.

The estimated parameters δ_0 and δ_1 are statistically different from $\delta_0 = 0$ and $\delta_1 = 1$. The ramification is that CRA ratings do not predict impairment risk.

It can be seen that in most years, both coefficients of either regression are statistically significant and thus different from their ideal values (Columns 1 and 2). Moreover, the respective R^2s neither increase nor decrease throughout. This implies that the ratings quality has neither consistently declined nor improved.[26] While for most years, the evidence of underprediction or overprediction is mixed, particularly the downturn years 2002, 2007 and 2008 exhibit a significant underestimation of risk by the ratings. If ratings predict impairment risk accurately, they should have anticipated the downturns and should have downgraded the transactions accordingly. However, the observation that the estimates of γ_0 and δ_0 are greater than zero indicates that impairment risk has been under-predicted by the ratings in these years. In summary, the analysis shows that the rating quality has neither consistently declined nor improved through time. In other words, there has been a mix of years of overprediction and years of underprediction of impairment risk. This indicates that CRA ratings have a limited ability to predict impairment risk.

In summary, we reject the hypothesis H3 that ratings predict impairment risk. The ramifications are that CRAs are poor predictors for impairment risk and that investors relying on predictions of future levels of impairment risk may have to build private models.[27] Alternatively, CRAs may adjust their ratings by a projection of the future state of the economy. This may be accomplished by including time-lagged variables of the level and change of the total impairment rate.

4. Discussion and outlook

To date, empirical evidence on the accuracy of ratings and risk models for securitizations is limited. The article's main objective is to analyze the impact of idiosyncratic and systematic risk characteristics on impairment risk of securitizations.

The most substantial findings are that CRA ratings for securitizations

- Do not fully account for the average credit quality in asset portfolios;
- Do not fully account for the structure of asset securitizations;
- Measure a too low impairment risk level at origination when fee revenue is high;
- Measure a too low impairment risk level if a securitization was originated in a high securitization activity year;
- Are unable to predict impairment risk.

CRA ratings (like many other commercial vendor solutions) may have to be interpreted in relation to the invested resources. Please note that the major CRAs cover a large number of rated debt issuers and issues per year[28] with a limited number of financial analysts.[29] This paper has also shown that ratings are informative with regard to the average idiosyncratic impairment risk over the business cycle.

There may be various ways to address the findings of this paper, which may include the knowledge transfer to the financial system (ie to CRAs and CRA rating users), independence between CRA fee revenue and origination process, cap for CRA fee revenues or introduction

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[26] A comparison of R^2 should be carefully interpreted as each year has a different number of observations. Please also note that our definition of rating quality differs from the definition of rating standard by Blume et al (1998), compare Footnote 10.

[27] The results confirm the findings by Loeffler (2004) for corporate ratings.

[28] For instance, in 2007, Moody's Investors Service rated 100 sovereigns; 12,000 corporate issuers; 29,000 public finance issues; and 96,000 structured finance obligations.

[29] For instance, in 2007, Moody's Investors Service employed approximately 1,000 analysts.

of minimum standards on resources spent on ratings. A public discussion is needed to transfer the findings into regulatory policy.

To date, CRAs have usually made available to the general public histories of their financial risk measures as well as the respective realizations. Little is known of the quality of models of other vendors as well as financial institution internal models as the respective information is kept private. However, recent negative earnings announcements from financial institutions suggest that other models applied in industry may share similar properties. Therefore, a formal validation of such models is important.

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