Notes on the estimation procedure for the Spanish term structure

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Data

At present, about 34 securities are used in the estimation. These securities are distributed as follows:

- 22-26 coupon bonds with a residual maturity between one and 30 years,
- four Treasury bills with a residual maturity of three, six, nine and 12 months,
- four bond repos with a maturity of one, seven, 15 and 30 days.

Bond repos (called *operaciones simultáneas* in Spain) are used for the very short-run maturity observations because the Treasury bill secondary market is rather illiquid for maturities less than three months, while the repo market is very active for such maturities. These repos are "American-style", that is, the buyer (the party who acquires the bond) is considered the owner of it so that he/she is free to sell it or make a reverse repo with the corresponding bond. Thus repos may be considered as Treasury bill equivalents (assuming no credit risk). The observed repo rate for maturity $m, r(m)$ is converted into a price as follows:

$$P = 100\,(1 - r(m)\,m / 360)$$

Estimation methods

The estimation goes back to January 1991. However, the estimation method used for the period January 1991-December 1994 is different from the one used since January 1995.

For the period January 1991-December 1994, the estimation method used is from Nelson and Siegel (1987), that is, the following equation is assumed for the instantaneous forward rate:

$$f_0(h) = \beta_0 + \beta_1 \exp\left(-\frac{h}{\tau_1}\right) + \beta_2 \frac{h}{\tau_1} \exp\left(-\frac{h}{\tau_1}\right) \tag{1}$$

where h is the horizon and $b = (\beta_0, \beta_1, \beta_2, \tau_1)$ is the parameter vector to be estimated. β_0 represents the instantaneous asymptotic rate and $(\beta_0 + \beta_1)$ the instantaneous spot rate. This equation is consistent with a forward rate process fulfilling a second-order differential equation with two identical roots. This functional form allows for only one local maximum or minimum along the maturity profile, according to the sign of β_2.

The spot rate function for maturity $m, r(m)$ and the forward rate function, $f_0(m)$, are related by:

$$r(m) = \int_0^m \frac{f(s)}{m}\,ds \tag{2}$$

Hence:

$$r(m) = \beta_0 + (\beta_0 + \beta_1)\,\frac{\tau_1}{m}\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) - \beta_2 \exp\left(-\frac{m}{\tau_1}\right) \tag{3}$$

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Replacing $r(m)$ in the discount function, $d(m)$:

$$d(m) = \exp(-mr(m)) \tag{4}$$

one obtains $d(m)$ as a function of $b = (\beta_0, \beta_1, \beta_2, \tau_1)$.

Therefore the price of the i-th bond is expressed as:

$$P_t^i(b) + cc_t^i = C^i \sum_{j=1}^{j=v^i} d_t\left(m_j^i; b\right) + 100 d_t\left(m_{v^i}; b\right) \tag{5}$$

$i = 1,\ldots,n$

where t is the day of observation, n is the number of observations, v^i is the number of payments for the i-th bond, and cc_t^i is the accrued interest, calculated as:

$$cc = coupon \; \frac{actual\ date - last\ payment\ date}{next\ payment\ date - last\ payment\ date}$$

C^i = coupon

$b = (\beta_0, \beta_1, \beta_2, \tau_1)$ is the parameter vector to be estimated.

The estimation is made by using a non-linear least squares procedure. More precisely, the algorithm used is from Marquardt and runs with SAS.

Once the parameter vector, $b = (\beta_0, \beta_1, \beta_2, \tau_1)$, is estimated, instantaneous forward rates are obtained from equation (1), and zero coupon rates from equation (3). The estimation criterion is to minimise the squared price error. Observed prices used in the estimation correspond to the median of prices traded during the day of observation. Tax effects are not taken into account in the estimation.

Since January 1995, the estimation method used has been from Svensson (1994). Svensson's method is identical to Nelson and Siegel's, but adding to the instantaneous forward rate function the term $\beta_3 (h/\tau_2) \exp(-h/\tau_2)$, where now the parameter vector to be estimated is $b = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$. Here, β_0 also represents the instantaneous asymptotic rate and $(\beta_0 + \beta_1)$ the instantaneous spot rate. This functional form allows for more than one local maximum or minimum along the maturity profile. Thus, Svensson's method is more flexible than Nelson and Siegel's.

Again, the estimation is made by non-linear least squares using the Marquardt algorithm. The estimation criterion is to minimise the squared price error weighted by the inverse of duration, that is:

$$\underset{b}{Min} \sum_{i=1}^{n} \left\{ \left(P_t^i - P_t^i\left(\hat{b}\right)\right) \left(\frac{1}{D_t^i}\right) \right\}^2 \tag{6}$$

where: $D_t^i = \dfrac{\partial P_t^i}{\partial y_t^i} \dfrac{(1 + y_t^i)}{P_t^i}$

y_t^i = yield to maturity

P_t^i = observed price

$P_t^i\left(\hat{b}\right)$ = estimated price

The prices used in the estimation correspond to the average between the bid and ask prices quoted at 4 pm. These prices correspond to a settlement date of $t + j$ days, where t is the trading date.[2] For the estimation, prices are valued at t and obtained from the following expression:

$$P_{t+j}^i + cc_{t+j} = \left(P_t^i + cc_t\right) \left(1 + r_t(j) \; \frac{j}{360}\right) - C^i \left(1 + r_t\left(t + j - t_1\right) \; \frac{t + j - t_1}{360}\right) \tag{7}$$

[2] Up to 30 November 1997, j was seven days. Since then, j has been three days.

where: t = observation day

P_{t+j} = average of bid-ask quoted price

cc_t = accrued interest at t

$r(j)$ = simple interest rate for j days

t_1 = date of next coupon payment

C^i = coupon if $t < t_1 < t + j$, 0 otherwise

Tax effects are not taken into account in the estimation. Since July 1997, an additional parameter (*Y*), which measures the differences in prices between non-stripped and stripped bonds (the last are traded at a lower yield), is estimated. In order to take this feature into account in the estimation, it is assumed that for the stripped bonds:

$$r^s(m) = r(m) + \gamma \tag{8}$$

where $r(m)$ is the spot rate of equation (3) and γ is the new parameter to be estimated. Therefore, the price of the i-th bond is expressed as:

$$P_t^i(b) + cc_t^i = C^i \sum_{j=1}^{v^i} d_t\left(m_j^i; b\right) + 100 d_t\left(m_{v^i} : b\right) \tag{9}$$

$i = 1, \ldots, n$

where $\begin{cases} (\beta_1, \beta_2, \beta_3, \tau_1, \tau_2) \text{ if } i \text{ is stripped} \\ (\beta_1, \beta_2, \beta_3, \tau_1, \tau_2, \gamma) \text{ if } i \text{ is not stripped} \end{cases}$

Summary of characteristics of estimation

	January 1991-December 1994	**January 1995 onwards**
Frequency of estimation	Daily	Daily
Prices used in the estimation	Median of traded prices	Average of bid-ask quoted prices
Short-run securities	Bond repos for 1, 7, 15 and 30 days Treasury bills for 3, 6, 9 and 12 months	Bond repos for 1, 7, 15 and 30 days Treasury bills for 3, 6, 9 and 12 months
Long-run securities	Treasury bonds	Treasury bonds
Estimation method	Nelson and Siegel	Svensson
Minimisation criterion	Price error	Price error weighted by the inverse of duration
Regression procedure	Non-linear least squares	Non-linear least squares
Algorithm used	Marquardt	Marquardt
Econometric package	SAS	SAS

References

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