Estimating the term structure of interest rates from French data

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The data used

The data used for estimating zero coupon yield curves cover three categories of government issues:

- French franc-denominated OAT bonds (Obligations Assimilables du Trésor) with maturities at issue ranging between seven and 30 years, which have been the main instrument used for financing the government since the mid-1980s.[2]
- Treasury notes, or BTANs (Bons du Trésor à taux fixe et intérêts ANnuels), with maturities of two to five years, which are used for medium-term financing.
- Treasury bills, or BTFs (Bons du Trésor à taux Fixe et intérêts précomptés), which are issued with maturities up to one year, offering a wide choice of maturities at the time of issue. The prices quoted are those of each Friday.

OATs are issued through a process of assimilation: they are often issued with the same characteristics as existing OATs (ie the same coupon and maturity). At the first coupon date, all the new issues are pooled with the earlier releases.

OATs are no longer issued with maturities of less than a year. With the latter category, the liquidity tended to diminish, which can lead to abrupt price swings. Indeed, market operators make their decision on the basis of yield to maturity, and a slight variation in the latter has a very strong impact on the price of assets with only a short time remaining to maturity. A comparable phenomenon occurred in the case of BTANs, leading the Treasury to stop issuing them with maturities of less than one month. The prices and yield to maturity of BTFs were calculated to make them consistent with data on OATs and BTANs, whose yields are based on a 365-day year.

For all securities, coupons are paid once a year and are subject to taxation. Households are liable to a withholding tax of 18.1% on income. For the business sector, the same rate applies as with taxes on profits (34%). For non-residents, the tax rate depends on the bilateral agreement with the country concerned.

Some notes on the estimations

In selecting data for the estimations, the following rules apply.

Concerning OATs, only the most liquid of the fixed rate and French franc-denominated issues (except strips) are used. For liquidity reasons, the following issues are excluded: OATs with a maturity of less than one year, BTANs of less than one month and BTFs of less than one week. In estimating the zero coupon yield curves, tax effects are not taken into account.

The estimation goes back to January 1992. The prices or yield to maturity quoted are those of each Friday. For OAT data, the prices used correspond to the last price; for BTAN data, the price is the average between the bid and ask prices quoted; for BTF data, the yield is the average between the bid and ask yields quoted.

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[2] French franc-denominated Treasury bonds were the main instrument used for financing the government until 1985.

Two specifications are used for the interpolations: the original Nelson-Siegel function and the augmented function as proposed by Svensson. The parameters of each function are obtained for each observation date by minimising the weighted sum of the square of the errors on the prices of all of the securities, using a non-linear estimation method. The weights are the interest rate sensitivity factors of prices. In fact, this function can be seen as an approximate criterion defined on the yields to maturity. This method, strictly based on the yields to maturity, would make the estimation process longer, because a system of non-linear equations has to be solved for each iteration. Thus, a criterion obtained by taking an approximation of the estimated interest rate on the basis of a first-order Taylor approximation is substituted for this function. The function is then minimised and can be interpreted as a criterion established on the weighted prices. The latter represents the derivative of the price with regard to the yields to maturity or, in other words, the interest rate sensitivity of prices.[3]

A constraint is imposed on the parameters, so that the estimated curve goes through the shortest-term interest rate available at each observation date.

In view of the number of coefficients and the high degree of non-linearity of the function to be optimised, the parameters of the "augmented" Nelson-Siegel relationship are obtained in two stages.[4] This cuts down the estimation time and thus reduces the risk of false convergence. At first, the basic function proposed by Nelson and Siegel is estimated, using as the initial coefficients values that suit all of the possible configurations of the term structure of interest rates.[5] After convergence, the results are used as the initial values for estimating the "augmented" Nelson-Siegel function. The two parameters that are specific to the augmented part of the function, which are not available in the first step, are initialised with 0 and 1 for the extra β and τ respectively. This procedure makes it possible to start the second step with values that can be assumed to be close to the real parameters of the model. After making the estimates, the term structure of interest rates found is checked to see if it justifies the use of the "augmented" relationship rather than the basic Nelson-Siegel relationship.

The selection between the basic and the augmented Nelson-Siegel functions is based on the Fisher test (at the 5% significance level). Confidence intervals based on the data method are also estimated.

The estimated zero coupon yield curves are published in Section 4 of the Bank of France's *Bulletin Digest*.

References

Ricart, Roland and Pierre Sicsic (1995): "Estimating the term structure of interest rates from French data", Bank of France, *Bulletin Digest*, no 22, October.

3 $\underset{\tilde{\alpha}}{Min} \sum_{i=1}^{n} \left\{ \left(P_i(t,m) - P_i(t,m,\tilde{\alpha}) \right) / \Phi_i \right\}^2$, where $P(t,m)$ is the market value of a bond, m the time to maturity, n the number of issues, α the parameters vector, and $\Phi_i = \frac{\partial P_i(t,m)}{\partial r_i(t,m)} = c \sum_{i=1}^{[m]+1} \frac{-(m-[m]+i-1)}{(1+r_i(t,m))^{m-[m]+i}} - \frac{100m}{(1+r_i(t,m))^{m+1}}$, where c is the coupon of a bond, expressed as a percentage of its par value.

4 The estimates are made using Gauss software.

5 The initial coefficients, which are constant for all estimate dates, were obtained by testing various possibilities in order to come up with the smallest possible number of non-convergence points over the period 1992-94.