Technical note on the estimation procedure for the Belgian yield curve

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The purpose of this note is to document the methodology and data used for the construction of the zero coupon yield curve that is daily estimated by the National Bank of Belgium. The yield curve is based on the functional form proposed by Nelson and Siegel (1985) and extended by Svensson (1994).

Theoretical model

The following functional form is used to represent the zero coupon yield curve:

$$r(m) = \beta_0 + \beta_1 * \left[\frac{1-\exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} \right] + \beta_2 * \left[\frac{1-\exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - \exp\left(-\frac{m}{\tau_1}\right) \right] + \beta_3 * \left[\frac{1-\exp\left(-\frac{m}{\tau_2}\right)}{\frac{m}{\tau_2}} - \exp\left(-\frac{m}{\tau_2}\right) \right] \quad (1)$$

The zero coupon yield r depends on the maturity of the bond (m) and the parameters β_0, β_1, β_2, β_3, τ_1 and τ_2. This function is used to define the discount factor $d(m)$:

$$d(m) = \exp\left(-\frac{r(m,\beta,\tau)}{100} m\right) \quad (2)$$

Each bond price can then be approximated by the discounted sum of the coupon payments and final capital:

$$P^e(m) = \sum_{i=1}^{m} \exp\left(-\frac{r(m,\beta,\tau)}{100} i\right) * Coupon + \exp\left(-\frac{r(m,\beta,\tau)}{100} m\right) * 100 \quad (3)$$

The parameters β_0, β_1, β_2, β_3, τ_1 and τ_2 are estimated by minimising the sum of squared bond price errors weighted by $(1/\Phi)$:

$$\underset{\beta_0,\beta_1,\beta_2,\tau_1,\tau_2}{Min} \sum_{j=1}^{n} \left\{ \left[P_j - P_j^e(\beta_0,\beta_1,\beta_2,\tau_1,\tau_2) \right] / \Phi_j \right\}^2 \quad (4)$$

where Φ equals the duration $*$ price$/(1+$yield to maturity) of the bond.

Application and data

The daily estimation is based on the market price of Treasury certificates and linear bonds: all outstanding Treasury certificates (with a maturity between days and one year) and all linear bonds or OLO's in Belgian francs with a maturity longer than one year (excluding line 239) are included in the sample. This means that some 45 prices are considered, of which 18 bond prices and 27 Treasury

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certificates. This sample is adjusted over time according to the information from specialists in the bond market.

The market prices of the bonds are corrected for the accrued interest calculated as a proportion of the coupon payment. There is no correction for the deviation between the day of trade (t) and the day of settlement ($t + 3$ for bonds and $t + 2$ for the Treasury certificates).

The estimation programme starts by estimating the parameters β_0, β_1, β_2 and τ_1 with fixed $\beta_3 = 0$ and $\tau_2 = 1$. Then the programme checks whether the estimation result improves by adding β_3 and τ_2. If these coefficients are not significant, the simple Nelson-Siegel formula is retained; otherwise, the extended Svensson formula is used.

References

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