The cost of a guarantee for bank liabilities: revisiting Merton

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1. Introduction

Information based on market prices can be a useful tool for assessing a bank's condition (see, eg, Flannery (1998)). Market prices contain all the information currently available to market participants and lend themselves to being combined into meaningful indicators supplying bank supervisors with additional assessment criteria. One such indicator may be the value of an explicit third-party guarantee for part or all of a bank's liabilities or the value of an implicit guarantee, eg under a de facto too-big-to-fail doctrine. A guarantee for bank liabilities transforms a formerly risky debt into a riskless obligation by shifting the risk from the creditors to the guarantor. As with every other insurance, such a transfer of risk has value for the insured party and imposes a cost on the insurer. Provided that the guarantee is politically binding, that cost is essentially the same for both explicit and implicit guarantees. The cost itself varies with the amount of risk transferred: high-risk banks impose high cost on the guarantor and vice versa. Because of the connection between the cost of the guarantee and the risk of the bank, estimates of that cost lend themselves to being used not only to fix actuarially fair insurance premia (ie compensations for third-party guarantees), but also to give bank supervisors an (additional) indication of the condition of a bank - whether that bank has a guarantee or not. The question, of course, is how to estimate that cost.

In 1977, Merton showed that a guarantee for the liabilities of a bank is equivalent to a European-style put option written on the assets of the bank. According to this, the value of the guarantee, ie the actuarial insurance premium attached to the given protection, is a decreasing function of the asset-to-liability ratio and an increasing function of the volatility of the asset returns. This insight gave rise to an extensive body of literature on the valuation of (deposit) guarantees and prepared the ground for bank-specific estimates on the basis of market data collected over quite a short time period. One of the most influential implementations of Merton's concept is the model conceived by Ronn and Verma (1986). Their relatively easy-to-use method became a kind of standard approach and was subsequently adopted by Giammarino et al (1989), Duan et al (1992), Sheldon (1996) and others.

The beauty of Merton's concept and Ronn and Verma's implementation lies in its simplicity. But as in the old saying, this beauty comes at a price. Like the Black-Scholes option pricing model on which it is based, the Merton-Ronn-Verma approach is subject to shortcomings and distortions that result in a tendency to misprice far-out-of-the-money and deep-in-the-money options (see, eg, Black (1988)). The root of the problem is that the standard Black-Scholes option pricing model assumes the price of the underlying asset to follow a geometric Brownian motion, ie it assumes normally distributed returns. Empirically, however, the returns of a bank's assets are not normally distributed. They rather show a negative skewness, ie a relatively higher density in the area of (highly) negative returns. As a result, the standard Merton-Ronn-Verma approach systematically underestimates the default probability - empirical default probabilities tend to be significantly higher than those implied by the Merton-Ronn-Verma approach - and, consequently, the cost of the guarantee.

The purpose of this paper is to present a suggestion on how to overcome that weakness while trying to retain as much of the original model's simplicity as possible. The suggestion we make is an empirical enhancement of the Ronn-Verma model that is based on a procedure employed by KMV Corporation.[2] The modification itself consists of an ad hoc adjustment of the model-implied volatility of a bank's asset returns to a level which brings the default probability consistent with the original model into line with rating agencies' empirical findings on default probabilities of corporate bonds. The adjusted volatilities are then used to generate an alternative set of estimates for the cost of the guarantee. Mainly for illustrative purposes, both the standard approach and the modified version are

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[2] KMV Corporation is a provider of credit risk measurement and management technology to financial services firms.

estimated on a monthly basis from July 1997 to June 2000 using data on selected Swiss banks with full, partial or no explicit third-party guarantee. We find large variations over time as well as across institutions and make out a substantial difference in level between the original model and the modified version.

The remainder of the paper is organised as follows: Section 2 develops the original as well as the modified model employed to estimate the cost of the guarantee for bank liabilities. Section 3 discusses the data and gives some background information on the characteristics of the banks considered; Section 4 presents the findings, and Section 5 contains a summary and conclusions.

2. Models

As shown by Merton (1977), a guarantee for a single, homogeneous term, discount debt issue can be modelled as a European put option written on the debtor's assets which in this case serves as the underlying. The exercise price of the put option is equal to the maturity value of the debt obligation, and its maturity corresponds to the maturity of the debt issue. By writing such an option, the guarantor gives the creditors the right to sell him their claims on the debtor's assets at a price equal to the outstanding liabilities. Since it is a European option, it can only be exercised on the maturity date. If at that time the value of the debtor's assets is smaller than the value of the liabilities, ie if the debtor is bankrupt, the creditors' put option will be exercised. According to the above, the value of the guarantee, G, at the maturity date is

$$G = Max[0, L - A], \tag{1}$$

where L stands for the liabilities and A for the assets. Applying this analogy to a real bank, however, raises practical difficulties. The boundary condition in equation (1) requires the same maturity date for all the debtor's obligations. A real bank doesn't meet that requirement, of course. Its liabilities rather mature over a broad spectrum of dates. Merton (1977) suggests overcoming that difficulty by reinterpreting the maturity of the liabilities as the length of time until the next bank audit. After examination by the auditors, the bank is either declared solvent or bankrupt. If the bank has a positive net worth, the put option, according to equation (1), has zero value and expires unused; it will then be replaced by a new one. If, on the other hand, the bank is bankrupt, the option has a positive value $(L - A)$ and will thus be exercised.

a. The standard approach

The standard approach upon which we build the proposed modifications is the Merton-Ronn-Verma approach, ie the method proposed by Merton (1977) and implemented by Ronn and Verma (1986).[3] Hence, making the standard assumptions of the Black-Scholes option pricing model, the formula for the value of the guarantee, G, can be derived analogously to the formula of a standard put option as derived by Black and Scholes (1973). It can be written as

$$G = -A \cdot N(-d_1) + L \cdot N(-d_2), \tag{2}$$

where

$$d_1 = \frac{ln(A / L) + (\sigma_A^2 / 2) \cdot T}{\sigma_A \cdot \sqrt{T}},$$

$$d_2 = d_1 - \sigma_A \cdot \sqrt{T},$$

[3] Ronn and Verma (1986) use a modified closure condition in their model, which sets the default point - the asset value at which a firm defaults - below the value of total liabilities. That feature is not adopted here. We set the default point at its theoretical level instead, ie where assets equal total liabilities, thereby taking account of the change in institutional conditions and the reduced (public) tolerance towards forbearance.

$N(\cdot)$ is the cumulative density function of a standard normal random variable, σ_A denotes the volatility of the (logarithmic) rate of return on the bank's assets, *A*, and *T* refers to the life of the option. *L* stands for the nominal value of the liabilities and can be taken from the bank's balance sheet.

In order to make the value of the guarantee comparable across banks, it has to be normalised, ie stated in terms of a certain amount, *k*, of insured liabilities.[4] The value of the guarantee per *k* Swiss francs of insured liabilities, *g*, can then be determined by multiplying both sides of equation (2) by (k / L). Thus, equation (2) transforms into

$$g = \frac{k}{L} \cdot G = -k \cdot \frac{A}{L} \cdot N(-d_1) + k \cdot N(-d_2), \tag{2a}$$

where d_1 and d_2 are the same as above.

The solution of the model presented in equations (2) and (2a) requires estimates of the market value of the bank's assets, *A*, and the volatility of their returns, σ_A. Both of these variables cannot be observed. However, they can be determined by means of two additional equations. The first equation goes back directly to Black and Scholes. In their seminal 1973 paper, they point out that it is possible to also view the equity of a firm as a European call option on the assets of that firm. In that framework, Black and Scholes consider the creditors as de facto owning the assets of the firm. By issuing a call option on these assets, they give the shareholders the right to buy the assets back. According to this, the shareholders' call option will be exercised if on the maturity date the value of the assets exceeds the value of the outstanding liabilities, ie if the firm is solvent. Hence, on the maturity date the value of that option representing the firm's equity, *E*, is equal to

$$E = Max[0, A - L],$$

where *A* and *L* are defined as above. Since the parameters of that call option are exactly the same as those of the put option written by the guarantor and held by the creditors, the shareholders' exercising behaviour must be the exact opposite of the exercising behaviour of the creditors: if the shareholders exercise, the creditors don't and vice versa. Given the standard assumption of the Black-Scholes pricing model, the formula for the equity viewed as a call option can then be written as

$$E = A \cdot N(d_1) - L \cdot N(d_2), \tag{3}$$

where d_1, d_2 and $N(\cdot)$ are the same as in equation (2).

The second equation required to determine the market value of the assets, *A*, and the standard deviation of the rate of return on *A*, σ_A, can be derived from Ito's lemma and postulates a functional relationship between the volatility of the equity returns, σ_E, and σ_A (see, for example Hull (1997)). From Ito's lemma, it follows that the standard deviation of the process *dE*(*A*, *t*) is

$$\sigma_E = \frac{A \cdot \frac{\partial E}{\partial A}}{E} \cdot \sigma_A, \tag{4}$$

where $\frac{\partial E}{\partial A}$ is the delta of the option and equal to $N(d_1)$.

By means of equations (3) and (4), the observable market value of equity, *E*, and the volatility of equity returns, σ_E, it is now possible to simultaneously estimate the two unknowns *A* and σ_A by a numerical routine. Together with the other known variables - *L* and *T* - these estimates can then be inserted into equations (2) and (2a) to solve for the respective values of the guarantee, *G* and *g*.

[4] For the calculations, *k* is set equal to CHF 1,000 ($k = 1{,}000$).

b. The modified approach

It is well known that the Black-Scholes option pricing model tends to misprice far-out-of-the-money and deep-in-the-money options (see, eg, Black (1988)). Unfortunately, that weakness has an effect on both options considered in the Merton-Ronn-Verma approach: for a solvent bank, the put option in equation (2) representing the guarantee is usually far out of the money, and the call option in equation (3) representing the equity is usually deep in the money. Hence, “the holes in Black-Scholes” may cause distortions in both cases. These distortions become apparent when empirical default probabilities are compared to corresponding default probabilities extracted from the Black-Scholes option pricing formula used in the Merton-Ronn-Verma approach. In this comparison, the latter tend to be significantly lower than the empirical data indicate. This underestimation of default probabilities can be attributed to the fact that the normal distribution of returns as assumed by the Black-Scholes model is only an imperfect description of reality. The deviations are especially strong in the area of highly negative returns, where the real distribution, which has negative skewness, shows a significantly higher density. We try to take account of this problem by combining the standard approach described above with empirical elements. Specifically, we use rating agencies’ empirical findings on default probabilities to increase the model-derived volatility of asset returns to a level which brings the corresponding default probability into line with the empirical findings. In other words, we equate the density in the relevant part of the tail of the model-implied distribution with that of the real distribution and thereby increase the volatility of asset returns. The adjusted volatilities obtained by equalising the densities in the relevant part of the tail of the model-implied and the real distribution[5] are then reinserted into equations (2) and (2a) in order to obtain alternative estimates of the fair insurance premia.

To this end, we first create a functional link between the respective bank’s estimated distance to default and the probability of default based on empirical data. The distance to default is a measure of soundness used by KMV Corporation. It expresses the size of a firm’s capital buffer in terms of standard deviations of its rate of return on assets according to the formula

$$DtD = \frac{A-P}{A \cdot \sigma_A}.$$

Here DtD stands for the distance to default, P denotes the point of default, defined as the asset value at which a firm defaults, and A and σ_A are the same as above. The distance to default combines asset value, business risk and leverage in a single measure of default risk. The higher the distance to default, the lower the default risk of the firm and vice versa. KMV Corporation assumes the point of default of firms to lie somewhere between total liabilities and short-term liabilities. But since banks are heavily regulated and closely supervised, we assume the point of default to be equal to total liabilities and therefore set $P=L$. With estimates of A and σ_A available from the standard approach described above and the nominal value of the liabilities, L, known, results for DtD can be obtained easily. In order to create the functional link between the distance to default and the empirical probability of default, a two-step procedure is applied. In step one, we estimate the distance to default for a sample of 40 international banks with ratings ranging from AAA to BBB- (ie investment grade ratings). The distances to default are then regressed on the corresponding ratings for long-term debt using the equation

$$DtD = u + v \cdot \log R + \varepsilon\ , \tag{5}$$

where DtD is the distance to default as defined above, R the rating category[6] and ε the error term. In this way, a functional relationship between the distance to default and the rating is created. The resulting distances to default for each rating category are shown in column 2 of Table 1 below. In step two, we replace the ratings by their corresponding historical default probabilities and, after adjustments for small-sample noise in the higher rating categories,[7] create a function approximating the

5 There is no need to know the exact shape of the real distribution, since all we need is the density below the default point, ie the empirical probability of default.

6 The ordinal ratings are replaced by cardinal numbers ranging from 1 for AAA to 10 for BBB-. The parameters u and v are: $u = 6.263118$ and $v = 0.816924$.

7 The empirical default frequencies listed in column 3 of Table 1 are the one-year cumulative default rates from Standard & Poor’s. They show that defaults of highly rated firms within one year are very rare. As a result, the data in the higher rating categories tend to be noisy. To obtain monotonically rising default rates and to take into consideration that insolvencies of

relationship between the distance to default and the adjusted empirical default probability. This function can then be used to translate a bank's distance to default into a default probability that is consistent with the empirical tendency of corporate bonds to default. The translation function itself is a composite function created by linking up a hyperbola and a parabola at $DtD = DtD_{BBB-}$; the hyperbola translates the DtD values above $DtD = DtD_{BBB-}$, the parabola those below. The fit was achieved by minimising (i) the squared differences between the hyperbola's and the parabola's respective slopes at $DtD = DtD_{BBB-}$, (ii) the divergence of the parabola's intercept with the vertical axis from the 100% mark, and (iii) the squared differences between the empirical and the approximated values.[8] The approximated default probabilities corresponding to the respective rating categories are shown in column 5 of Table 1.

Table 1

Relationship between rating, distance to default and default frequencies

Col 1		**Col 2**	**Col 3**	**Col 4**	**Col 5**
Rating		**Estimated distance to default**	**Empirical default frequency (1 year; in %)**	**Adjusted default frequency (1 year; in %)**	**Approximated default frequency (1 year; in %)**
ordinal	**cardinal**				
AAA	1	6.263	0.000	0.010	0.007
AA+	2	5.697	0.000	0.015	0.013
AA	3	5.365	0.000	0.020	0.020
AA-	4	5.130	0.000	0.025	0.030
A+	5	4.948	0.030	0.030	0.042
A	6	4.799	0.040	0.040	0.060
A-	7	4.673	0.070	0.070	0.084
BBB+	8	4.564	0.130	0.130	0.118
BBB	9	4.468	0.200	0.200	0.167
BBB-	10	4.382	0.220	0.220	0.242

From the above, it is obvious that only the upper part of the translation function - the one described by the hyperbola - is well supported by empirical data. The parabola, on the other hand, has only two anchor points: one at $DtD = DtD_{BBB-}$, the other at DtD=0, where the empirical probability of default, PD_e, is 100%. This restriction is mostly due to the fact that sub-investment grade banks are not the norm and satisfactory data are correspondingly scarce. Estimates falling into the lower part of the spectrum (where $DtD < DtD_{BBB-}$) therefore have to be treated with due care.

By means of the functional link between the estimated distance to default and the empirical default probability, we can now estimate the alternative values of the guarantee, *G* and *g*. This too is done in two stages. In the first step, we adjust the model-implied volatility of asset returns to a level which brings the default probability generated by the standard model into line with our estimate from the translation function. The probability of default generated by the standard model described above corresponds to $N(-d_2)$ in equation (2). According to the Black-Scholes option pricing formula,

highly rated firms do indeed happen, we replaced the original default frequencies for the four highest rating categories by the (italic) figures shown in column 4 of Table 1.

8 The resulting translation function has the form

$$PD_e = \begin{cases} \dfrac{a}{(DtD - b)^c}, \text{ if } DtD \geq DtD_{BBB-} \\ d + e \cdot DtD + f \cdot DtD^2, \text{ if } DtD < DtD_{BBB-} \end{cases}.$$

The upper part represents a hyperbola with *a*=0.04594783, *b*=3.90965221, *c*=2.21549399, the lower part a parabola with *d*=100.00091548 (intercept where PD_e=100%), *e*=-44.39572746, *f*=4.9361389. PD_e denotes the empirical probability of default, and *DtD* the distance to default.

$N(-d_2)$ is the probability that a put option will be exercised. In our application of that formula, the put option represents the guarantee; it will be exercised if at the maturity date the bank is bankrupt. Hence, $N(-d_2)$ is equivalent to the bank's current probability of default (the probability that in its current condition the bank becomes insolvent within the life of the option) and we can set

$$PD_e = N(-d_2) = N\left(-\frac{ln(A/L) + (\sigma_A^2/2)\cdot T}{\sigma_A \cdot \sqrt{T}} + \sigma_A \cdot \sqrt{T}\right). \tag{5}$$

Here, PD_e is the empirical probability of default as calculated by the translation function described above and $N(-d_2)$ is defined as in equation (2). With PD_e determined, equation (5) can implicitly be solved for the corresponding volatility of asset returns, σ_A. In step two, these modified volatilities are inserted into equations (2) and (2a) in order to generate the modified values of the guarantee and the guarantee per CHF 1,000 of insured liabilities, respectively.

From a theoretical point of view, this "rough-and-ready" approach may be disputable. It is inconsistent insofar as one part of the analysis builds on the assumption that the underlying stochastic process for the evolution of the banks' asset values follows a Brownian motion, whereas the other part explicitly attempts to correct for some of the inadequacies of that very assumption. But we see this empirically enhanced version of the standard Merton-Ronn-Verma approach primarily as a practitioner's method that tries to remedy the well known empirical shortcomings of the standard model. In the absence of a coherent and consistent alternative that would both overcome the empirical shortcomings of the standard approach and be implementable at a similar cost in terms of data requirement and computational burden, we see the modified version as a viable alternative.

3. Data

The two approaches described in the preceding section - the standard Merton-Ronn-Verma approach and the modified version - require as input the equity value, *E*, the volatility of past equity returns, σ_E, the nominal value of total liabilities, *L*, and the life of the option, *T*, ie market prices as well as balance sheet data. In order to be eligible for either approach, therefore, a bank has to have actively traded shares. The quotations of these shares are taken from Datastream; the volatility of equity returns, σ_E, is calculated on the basis of 90 trading days; the balance sheet data required for the banks to which the two methods are applied are taken from the monthly banking statistics of the Swiss National Bank. As suggested by Merton (1977), we reinterpret the life of the option, *T*, as the time between audits and, following Marcus and Shaked (1984) and others, set it equal to one year.[9]

In order to derive the translation function for the empirically enhanced version, we used the historical default probabilities from Standard & Poor's. The balance sheet data of the international banks used to create the relationship between the distance to default and the rating are taken from the International DataBook of Thomson BankWatch. The relationship itself was built on a point-in-time basis, taking into consideration that some of the disturbing (business cycle) effects inherent in that approach are filtered out by diversifying the sample of banks used to create that relationship over several countries. Such business cycle effects occur, because, eg, a double-A rating does not have the same quality in an economic upswing as during a downturn. A translation function taking account of this, therefore, would tend to move left (downturn) and right (upswing). By choosing a point-in-time approach, such movements are prevented in favour of a stable relationship that tends to overestimate the cost of a guarantee during downturns and vice versa.

[9] Giammarino et al (1989) argue that since the periodicity of audits is difficult to estimate empirically, the assumption of one year is somewhat arbitrary. However, they also point out that while increasing the time to maturity will increase the estimated option values, the cross-sectional comparison of bank performance is robust to changes in the maturity, *T*. For further comments on this issue, see also Ronn and Verma (1986).

By means of these data, we generate monthly estimates for the (normalised) cost of a guarantee for bank liabilities over a three-year period from July 1997 to June 2000.[10] While doing so, we implicitly assume that every month the banks receive a new put option. This means that the liabilities are rolled over on a monthly basis, so that the life of the option at the beginning of each month is always equal to one year (see Ronn and Verma (1986)).

The sample of banks used to illustrate the modified Merton-Ronn-Verma approach consists of eight Swiss banks - seven cantonal banks and one big bank. Cantonal banks have a special status within the Swiss banking system. The main feature distinguishing them from other banks is their establishment on the basis of cantonal legislation.[11] Among their other specific characteristics, the two most prominent are state control and state guarantees. The 24 cantonal banks in existence are fully controlled by the respective canton, although some of them have issued non-voting shares to the general public in recent years. In all but two cases, state ownership is accompanied by a state guarantee that covers all the liabilities of the banks.[12] Thus, the cantons are not only the owners, but also the guarantors of the cantonal banks. This is about to change though: since 1999, a state guarantee is no longer a necessary characteristic of a cantonal bank, and it is now sufficient for the canton to hold more than one third of the bank's equity and voting rights. Several cantons want to make use of that possibility and reduce their controlling interest, although most of them have no plans to change the terms of the guarantee. Despite the many common attributes, there are also substantial differences among the cantonal banks, particularly in terms of size and business activity. Most cantonal banks only operate in their respective canton, and so their size and business activities strongly depend on the canton's population and economic strength. Measured by total assets, six cantonal banks rank among the top 10 Swiss banks. Many of them are involved in business activities similar to those of the big banks, except that they are barely involved in foreign markets. Most of the seven cantonal banks that met our requirements are medium-sized. They represent varying economic areas and, to some extent, pursue different corporate policies. Unfortunately, the market for their shares is not very deep, and thus a caveat regarding the quality of the data seems appropriate.[13] There is no such problem for the big bank in our sample; its shares are among the most traded in the market. Like the other non-cantonal banks, the big bank has no third-party guarantee for its liabilities. But together with most other Swiss banks, it is a signatory of the Swiss Bankers Association's "Agreement on Depositor Protection in case of Compulsory Bank Liquidation". In the event of a bankruptcy moratorium or bankruptcy proceedings against a signatory bank, this agreement provides limited advance payment, supplied by other signatory banks, for legally privileged depositors.

4. Findings

In a nutshell, running the bank data through the two models described in Section 2 produces three major findings:

- the results are highly sensitive towards the method applied
- they show significant variations over time
- they reveal large cross-sectional differences

[10] The transition from daily to monthly data for both the equitiy price and the volatility of equity returns was made by taking the average over the respective month.

[11] Switzerland operates a three-tier system of government, with the 26 cantons placed between the Confederation and the municipalities.

[12] The two exceptions are the Banque Cantonale de Genève and the Banque Cantonale Vaudoise, both part of our sample. The former has only a limited guarantee, the latter no guarantee at all.

[13] Despite this qualification, equity prices are still the best market data available. For mid-sized firms, bond markets are generally less liquid than stock markets and indicators based on bond prices are accordingly less sound.

The results are presented below, both graphically and in tabulated form. For each of the eight banks examined, we show the estimates of the normalised cost of the guarantee, *g*, for both the standard model (see Table 2 and Figure 1) and the modified version (see Table 3 and Figure 2).[14]

The sensitivity of the results to the choice of method is quite astonishing. Compared with the standard Merton-Ronn-Verma approach, the modified version shows significantly higher estimates. For our sample, the numbers are at least five times higher when the empirically enhanced model is applied. Moreover, that factor becomes much larger when the estimates get smaller, ie the farther out of the money the option is (see Figures). The finding that smaller values for the cost of the guarantee are raised relatively more than larger values can be seen as an indication that the tendency of the Black-Scholes option pricing model to misprice far-out-of-the-money options is indeed alleviated by the modified approach.

It is obvious that the results depend a lot on the form of the function that translates the distance to default extracted from the standard model into default probabilities that are in line with empirical findings. The composite translation function described in Section 2b is just one of several possibilities and open to challenge. Still, the results obtained suggest that the method matters a lot and that the recalibration based on empirical data brings measures into dimensions which seem closer to what one would expect. The problem, of course, is that an empirical verification of both the Merton-Ronn-Verma approach and the modified version is hardly possible, since the central variables - the market value of assets, *A*, and the volatility of their returns, σ_A - cannot be observed. In order to understand the results, especially those that seem incredibly high, it is important to know exactly what they mean. The normalised cost of a guarantee for a bank's liabilities can also be seen as a rough approximation of the default probability: if the value of the guarantee is, eg, CHF 50 per CHF 1,000 of liabilities, then the probability of default is roughly 5%. Those 5% mean that, given the current values for volatilities, leverage etc the bank will default within one year with a probability of 5%. In reality, however, the parameters change continuously, basically with every transaction made. High values for the cost of the guarantee, therefore, can be the result of a short-lived overreaction of the market upon some bad news and thus need not remain.

Our results also reveal large variations over time for both methods. These variations reflect macroeconomic as well as bank-specific events. They are also in line with the anecdotal evidence. The main underlying driver of the variation over time is the share price, ie the assessment by market participants. Both models are highly sensitive towards such changes. Price variations not only affect the leverage, they also have quite a significant impact on volatilities. The sensitivity to price changes is exacerbated by a highly non-linear relationship between the volatilities and the cost of the guarantee. That market participants are the driving force behind the results generated by the two approaches is indicative of the limitations of market-based methods in general: these methods express the expectations going into the share price, but not necessarily the default risk. As a consequence of this, perverse expectations generate perverse results.

We also found large cross-sectional differences among the eight banks considered. This is not surprising, since banks differ in a number of ways (activities, geographical focus and reach, degree of diversification, leverage, quality of management, etc) and have different risk appetites. But behind the cross-sectional differences, there are also similarities - parallel movements among several banks - to be made out. These are indicative of macro shocks affecting most banks in a similar way, or even of contagion or information effects that can occur when a bank-specific shock undermines confidence in the whole banking system (the latter might indeed have been the case in late 1998).

[14] In the Figures, we set lower bounds for the value of the normalised guarantee at CHF=0.0001 for the standard approach and CHF=0.01 for the modified version.

Table 2
Normalised cost of a guarantee according to standard approach (in CHF)

	Bank A	Bank B	Bank C	Bank D	Bank E	Bank F	Bank G	Bank H
Jul 97	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Aug 97	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0009
Sep 97	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0063
Oct 97	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000	0.0000	0.0089
Nov 97	0.0000	0.0000	0.0000	0.0000	0.0003	0.0001	0.0000	0.0693
Dec 97	0.0000	0.0000	0.0000	0.0000	0.0006	0.0025	0.0000	0.3240
Jan 98	0.0000	0.0006	0.0000	0.0000	0.0011	0.0032	0.0000	0.2927
Feb 98	0.0000	0.0017	0.0000	0.0000	0.0004	0.0038	0.0000	0.2555
Mar 98	0.0000	0.0007	0.0000	0.0000	0.0075	0.0006	0.0000	0.0365
Apr 98	0.0000	0.0000	0.0328	0.0000	0.1390	0.0137	0.0001	0.0034
May 98	0.0000	0.0000	0.0746	0.0000	0.4391	0.0254	0.0002	0.0074
Jun 98	0.0000	0.0001	0.2729	0.0000	0.4661	0.0148	0.0002	0.0097
Jul 98	0.0000	0.0022	0.2127	0.0000	0.4403	0.0078	0.0000	0.0970
Aug 98	0.0000	0.0328	0.0054	0.0000	0.0883	0.0001	0.0000	0.6936
Sep 98	0.0000	0.1039	0.1373	0.0000	0.0169	0.0008	0.0000	12.4808
Oct 98	0.0000	0.2577	14.5261	0.0000	0.2851	0.1790	0.0000	76.4331
Nov 98	0.0000	0.1261	20.8101	0.0000	0.2197	2.0673	0.0000	86.8817
Dec 98	0.0000	0.0449	24.7906	0.0000	0.1437	1.7521	0.0000	88.4928
Jan 99	0.0000	0.0072	29.8607	0.0000	0.0408	0.9403	0.0000	59.9656
Feb 99	0.0000	0.0000	6.7599	0.0000	0.0002	0.0926	0.0000	6.3781
Mar 99	0.0000	0.0000	2.6153	0.0000	0.0000	0.0000	0.0000	1.2769
Apr 99	0.0000	0.0000	4.0059	0.0000	0.0000	0.0000	0.0000	0.2198
May 99	0.0000	0.0000	1.1688	0.0000	0.0000	0.0000	0.0000	0.0620
Jun 99	0.0000	0.0000	0.0493	0.0000	0.0000	0.0000	0.0000	0.0149
Jul 99	0.0000	0.0000	0.0039	0.0000	0.0000	0.0000	0.0000	0.0179
Aug 99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0929
Sep 99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0693
Oct 99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0538
Nov 99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0947
Dec 99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0272
Jan 00	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0340
Feb 00	0.0000	0.0000	0.0064	0.0000	0.0001	0.0000	0.0000	0.1531
Mar 00	0.0000	0.0000	0.0452	0.0000	0.0124	0.0000	0.0001	0.0377
Apr 00	0.0000	0.0000	0.1049	0.0000	0.0362	0.0000	0.0001	0.0096
May 00	0.0000	0.0000	0.0406	0.0000	0.0276	0.0000	0.0007	0.0025
Jun 00	0.0000	0.0000	0.0117	0.0000	0.0134	0.0000	0.0002	0.0002

Table 3
Normalised cost of a guarantee according to modified approach (in CHF)

	Bank A	Bank B	Bank C	Bank D	Bank E	Bank F	Bank G	Bank H
Jul 97	0.0330	0.0123	0.0046	0.0222	0.1755	0.0178	0.0084	0.4690
Aug 97	0.0349	0.0108	0.0051	0.0281	0.1939	0.0226	0.0078	1.3249
Sep 97	0.0323	0.0118	0.0046	0.0293	0.3220	0.0311	0.0092	6.9068
Oct 97	0.0336	0.0151	0.0039	0.0226	0.6647	0.0822	0.0084	10.1136
Nov 97	0.0583	0.0265	0.0039	0.0028	0.6191	0.3203	0.0081	38.9290
Dec 97	0.0647	0.1267	0.0033	0.0028	0.8423	1.7340	0.0157	81.4728
Jan 98	0.0932	1.0228	0.0028	0.0028	1.2702	2.1901	0.0217	78.9215
Feb 98	0.0837	2.5882	0.0213	0.0051	0.7113	2.7746	0.0623	76.0511
Mar 98	0.0437	1.3499	0.0849	0.0092	8.2729	0.7412	0.1731	31.0161
Apr 98	0.0226	0.3570	20.7442	0.0287	54.4188	11.0070	0.3055	6.1351
May 98	0.0108	0.0932	34.4609	0.0408	87.9485	17.7560	0.4190	12.1686
Jun 98	0.0084	0.5544	66.0294	0.0281	89.8761	11.5534	0.4784	14.5864
Jul 98	0.0071	5.0890	58.6940	0.0164	88.1011	6.3400	0.2000	48.3826
Aug 98	0.0084	32.2470	4.1435	0.0081	43.1953	0.3429	0.1296	105.8234
Sep 98	0.0078	55.9567	45.9094	0.0060	15.5697	0.7761	0.1046	269.0045
Oct 98	0.0108	76.7553	273.1770	0.0084	72.2478	50.9360	0.0647	463.7953
Nov 98	0.0104	58.5349	307.0138	0.0084	66.3206	143.1664	0.0803	483.4771
Dec 98	0.0084	37.9735	323.6397	0.0039	55.0194	138.6418	0.0594	487.1592
Jan 99	0.0071	12.3888	342.6690	0.0019	27.9699	109.7282	0.0623	433.8823
Feb 99	0.0046	0.2414	211.3593	0.0039	0.5080	38.9401	0.0745	219.9938
Mar 99	0.0036	0.0515	151.8519	0.0189	0.1573	0.0881	0.0795	128.4346
Apr 99	0.0028	0.0572	177.0362	0.0192	0.0898	0.0293	0.0865	65.9489
May 99	0.0023	0.1003	112.8751	0.0174	0.1457	0.1559	0.0691	35.3717
Jun 99	0.0028	0.1216	23.7870	0.0100	0.1973	0.2099	0.0425	14.4969
Jul 99	0.0023	0.1036	2.2898	0.0000	0.1790	0.2022	0.0200	16.7759
Aug 99	0.0019	0.0556	0.0299	0.0000	0.1431	0.1380	0.0092	42.8121
Sep 99	0.0019	0.0113	0.0213	0.0000	0.1759	0.0178	0.0104	35.8600
Oct 99	0.0009	0.0055	0.0459	0.0000	0.1674	0.0100	0.0108	31.1975
Nov 99	0.0004	0.0084	0.0718	0.0000	0.0588	0.0133	0.0145	42.3644
Dec 99	0.0009	0.0092	0.1402	0.0000	0.0500	0.0213	0.0363	20.5136
Jan 00	0.0009	0.0145	0.3414	0.0000	0.0889	0.0226	0.0588	23.0867
Feb 00	0.0019	0.0151	3.7158	0.0009	0.4315	0.0349	0.1399	51.8753
Mar 00	0.0028	0.0123	21.9682	0.0013	14.4075	0.0311	0.3286	23.6964
Apr 00	0.0023	0.0118	36.6798	0.0013	28.1772	0.0235	0.4225	8.4864
May 00	0.0023	0.0066	20.0251	0.0019	24.1911	0.0260	0.8912	2.2371
Jun 00	0.0013	0.0028	8.0022	0.0009	15.0309	0.0204	0.04536	0.5516

Figure 1

Normalised cost of a guarantee according to standard approach (logarithmic scale; in CHF)

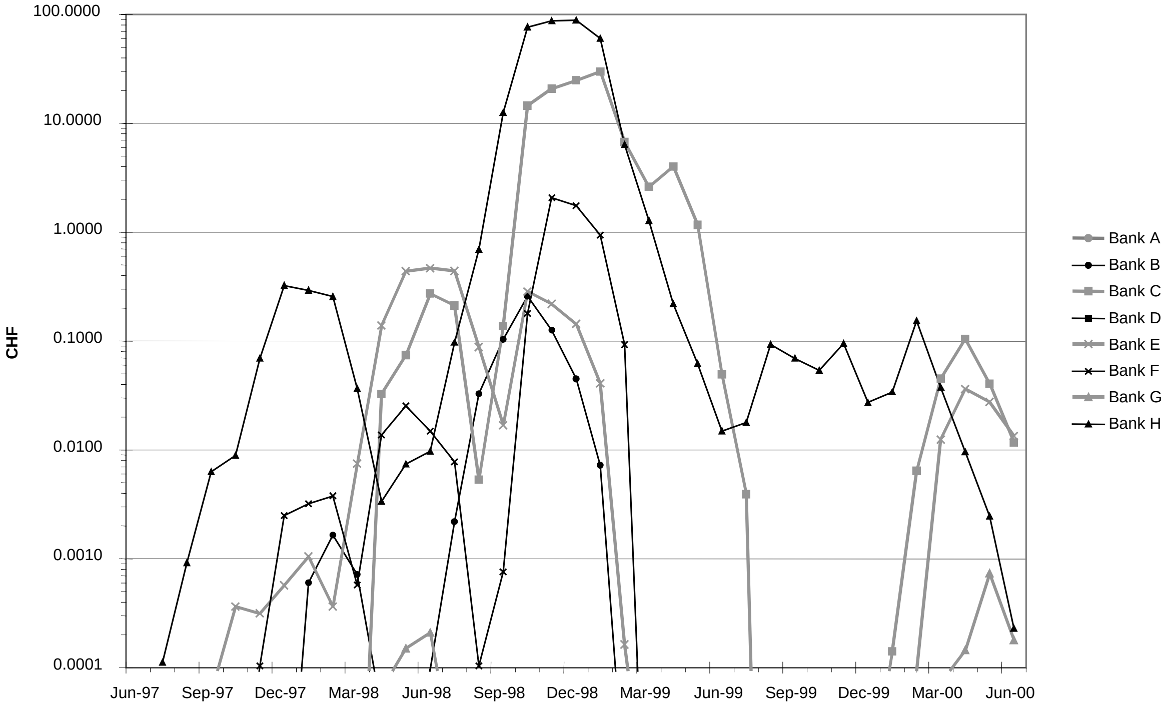
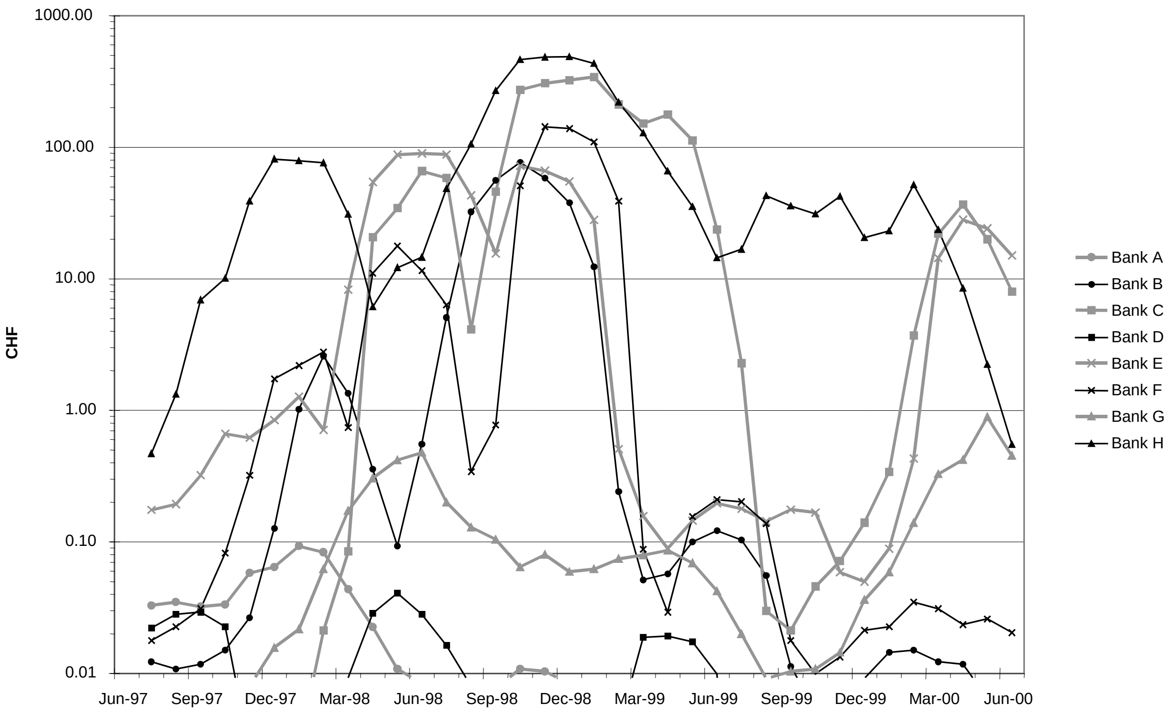


Figure 2

Normalised cost of a guarantee according to modified approach (logarithmic scale; in CHF)



5. Summary and conclusions

This paper makes use of the analogous relationship between a put option and the guarantee for a bank's liabilities in order to estimate the cost of such a guarantee for eight Swiss banks. Because of the direct connection between the cost of such a guarantee and the default risk of the bank, estimates of that cost can be used to both fix actuarially fair insurance premia and assess a bank's condition. Starting with the standard Merton-Ronn-Verma approach, we develop a modified version of the original model that incorporates empirical findings on default probabilities and thereby attempts to remedy an empirical deficiency - the "holes in Black-Scholes" - of the standard approach. Comparing the results generated by the two models shows that the (normalised) cost of the guarantee is at least five times higher when the modified version is employed. In line with the rationale of the modification, these differences become bigger the farther out of the money the option is. Apart from the significant difference in level, both methods show similar patterns of variation over time. These variations are mainly driven by the share price and can be quite large. Not surprisingly, the variations across banks are large too; they reflect different orientations and different risk appetites. But behind these variations there are also similarities, indicating macro shocks hitting many banks in a similar way or even contagion and information effects. Which of the two methods has better explanatory power for real-world data is ultimately an empirical question. Unfortunately, that question is difficult to resolve since the central variables cannot be observed. However, the deficiencies of the standard approach are known, and the modifications we suggest try to overcome them. At the end of the day, this should result in an improvement.

The findings of this paper may be of interest for any supervisor, deposit insurer or central banker who would like to use market information to quantify the value of any explicit or implicit exposure towards bank creditors or the financial condition of a bank in general. A major impediment to the implementation of contingent claims models like the ones presented is the fact that most (Swiss) banks, especially the smaller ones, do not have publicly traded shares. This, however, is a necessary condition for these models to be applicable in a useful way. But since those banks that do have publicly traded equity are usually the bigger ones and are therefore more important from both a systemic and a depositor protection point of view, the option-based approach we suggest can still be a helpful tool.

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