# SOME COMMENTS ON
# "FUNDAMENTAL REVIEW OF THE TRADING BOOK"

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With the document "Fundamental review of the trading book" the Basel Committee on Banking Supervision provides an intensive examination for the supervision of the market risks of financial institutions. According to the invitation for comments on p.7 of this document [6] we are happy to point out a number of topics which seem important to us for a consistent framework of regulation.

Essentially we restrict our comments to the following five points:

- the advantages of the Expected Shortfall ($ES$),
- the integration of stress scenarios into risk measurement,
- the possibility of supervisory arbitrage,
- the definition of time consistency of multiperiod risk assessment, and
- the problem of the boundary between banking book and trading book.

(A) **$ES$ versus $VaR$:**

We underpin the appraisal of ES and insist on the following well-known issues:

$ES$, as a (one period) coherent risk measure takes into account diversifications of risks via subadditivity. As noted in [1], Section 2.3., (a) to (d), this has consequences for the definition of the financial institution subject to external supervision, as well as for the management of internal supervision.

Contrary to $VaR$, $ES$ always deals with the question: "How bad is bad?" While this difference may be irrelevant when treating normal or exponentially decreasing distributions, it becomes important when it comes to heavy tailed distributions, as e.g. for catastrophic or systemic events.

As shown in [14] and [9], $ES$ allows for a fair internal allocation of capital for different lines of business.

It should however be mentioned that the dependence of $ES$ on the mere distribution of the loss $X$ (law invariance) may prove a serious drawback when there is need of dealing with change of numéraire (or "risk-free" benchmark investment device) or change of currency.

(B) **Integrating stress scenarios into the risk measurement:**

Any (one period) coherent risk measure, in particular $ES$, carries intrinsically a set of "test probabilities" also known as "stress scenarios". This should allow for building relatively realistic models, by examining how the test probabilities relate to actual uncertainty.

In the case of $ES$ the test probabilities are characterized by their density with respect to the historical or to a risk neutral base probability. Indeed it has been shown in [9], Chapter 4, Example 4

and also in [10], that a "base" probability $\mathbb{P}$ together with a "confidence level" $\alpha$ defines expected shortfall for a random variable $X$ mathematically as:

$$\begin{aligned} ES(X) \quad = \quad & \text{supremum of expectations of } X \text{ over the probabilities} \\ & \text{having a density } f \text{ w.r.t. } \mathbb{P} \text{ satisfying } f \leq 1/(1-\alpha). \end{aligned} \tag{1}$$

These probabilities represent scenarios under which a given loss $X$ is tested. Moreover, if $\mathbb{P}$ is a pricing probability of traded securities one can consider that $ES$ indeed "stress tests the loss $X$ for market risk". In addition to the 10-day $VaR$, the "Basel 2.5" amendment introduced a "stressed $VaR$" which mimics the standard $VaR$ under a stress scenario ([6] p. 60, see also [7]). A coherent risk measure can capture a unified treatment of standard and stress scenarios.

If $\mathbb{P}$ incorporates some type of macroeconomic uncertainty, as for example good or bad state of the "economy", the use of a coherent risk measure can reveal dependencies (risk compounding effects) between losses of many exposures . This may encourage to look for systemic risk measurement as well as for model risks in an $ES$ framework.

(C) **Supervisory Arbitrage**[1]**:**

There may be a possibility of supervisory arbitrage when $ES$ is defined out of the historical probability rather than out of a pricing measure of traded securities:

If no probability $f(\omega)\mathbb{P}(d\omega)$ with a density $f$ as in (1) happens to be a pricing measure, it can be proved that there exists a tradeable and zero-cost portfolio $D$ with negative $ES$. By subadditivity of $ES$ it will result that, for any loss $X$, adding a sufficiently large multiple $\lambda$ of $D$ to $X$ provides a new loss $X + \lambda D$ with negative $ES$: every loss is thus transformed at zero cost into an acceptable one! Such a warning can be traced back to Section 4.3 in [1] and is also illustrated in [3] and [4].

The possibility of supervisory arbitrage may be considered an extreme case of the broader problem of over-estimation of diversification benefits, the Committee has expressed its concern about at various places in [6].

(D) **Time-consistency:**

A reasonable system of multiperiod risk assessments should have the following property of time-consistency, as defined in [13] and [12]: If a date-2 exposure $X_2$ satisfies, at date 1, in all cases the supervisory requirement and if there are no cash flows at date 1, it should also meet the supervisory requirement at date 0. A family of risk assessments with this property of time-consistency allows a unified treatment of risky positions with different time horizons (see also **(E)**).

The "square-root-of-time" rule which fabricates out of a one-day (one period) risk measure a ten-day (one period) risk measure, stems from a purely Gaussian (or log-normal) and stationary setting. Its general application is more than questionable, as mentioned in [6]. In any case it does not address the question of consistency of supervisory requirements.

[1]We present a notion of supervisory arbitrage which however should not be confounded with the issue of the boundary between banking book and trading book (see [6] Annex 1).

When measuring risk over more than one period, $ES$ should be used not as a longer period measure but — with adapted parameters and in a conditional way — as a device of a backward induction in time, in order to avoid inconsistencies over time of risk assessments and solvability decisions[2].

It is the concept of generators in multiperiod risk measurement, as introduced in [8] and developed in [12] which leads to a date-0 assessment of a date-2 risk $X_2$, as $ES_0(\alpha_1 \cdot ES_1(X_2))$ (here $X_1 = 0$), where each indexed $ES$ is a (conditional) expected shortfall for the period from "index" to "index+1", $\alpha_1 \in (0, 1)$.

Time-consistency of the risk measurement implies the Bellman principle (see [2] and [5]) which guaranties that optimal risk-avoiding portfolios are identical whether calculated periodically or over a global time horizon. Without this property imposed capital requirements may lead to different results according to the way of calculation. (Unfortunately, the Solvency Capital Requirement ($SCR$) in Solvency II is not defined in a time-consistent way and juridical altercations may result; similar difficulty happens with the Swiss Solvency Test (see [5])).

(E) **Towards removing the boundary between banking and trading books:**

Finally it might be tempting to guess that a good time-consistent risk assessment may help to overcome the deficiencies stemming from the present differences in capital requirements for similar risk exposures depending whether they are hold in the trading or in the banking book. Also the Committee's proposed approach to factoring in market liquidity (Annex 4 of [6]) should be accessed via an appropriate time-consistent assessment.

At least conceptually, a unified method should be sketched as a leitmotiv – especially for the models-based approach – leading to a convincing supervisory accounting principle (see also [11]). We believe that these questions are important fields of further research.

## References


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[2] The Section 5.3. of [2] may lead the unsuspicious reader to the impression that when information, portfolio changes, or supervision are available or compulsory at several future dates, ES can be used at each of them, with different time horizons. Yet the paper includes a warning concerning the non consistency of $ES$ analysis at different dates for a fixed time horizon.

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