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Stress Testing Credit Risk: Comparison of the Czech Republic and Germany

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Foreword

The Financial Stability Institute is pleased to present the winning FSI Award paper for 2008. This award, announced every two years at the time of the International Conference of Banking Supervisors, was established to encourage thought and research on issues relevant to banking supervisors. This year, numerous papers were received on a variety of topics written by supervisors from around the world.

A jury of highly qualified individuals chose the winning paper. The group was chaired by Mr Malcolm Knight, General Manager of the Bank for International Settlements. It also included Mrs Ruth de Krivoy, former President of the Central Bank of Venezuela; Mr Kaarlo Jännäri, former Director General of the Finnish Financial Supervision Authority; Mr Peter Praet, Director of the National Bank of Belgium and co-chair of the Basel Committee's Research Task Force; and Mr Stefan Walter, Secretary General of the Basel Committee on Banking Supervision.

The jury members and the FSI are pleased to announce that the paper authored Mr Petr Jakubík of the Czech National Bank and Mr Christian Schmieder of the European Investment Bank (and formerly of the Deutsche Bundesbank) has been selected as the winner of the 2008 FSI Award. The authors present their study of credit risk modelling and stress testing within the context of a Merton-type one-factor model. A comparison of data from the corporate and household sectors of the Czech Republic and Germany is made in an effort to model aggregate credit risk. They then establish a framework for stress testing credit risk.

Congratulations to Mr Jakubík and Mr Schmieder and the other supervisors who submitted their work for consideration. Their interest in analysing and potentially improving supervisory methods provides a true service to the supervisory community.

Josef Tošovský
Chairman
Financial Stability Institute
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1. Introduction[1]

In quantitative terms, credit risk is the most important risk in banking books. This has recently been evidently shown again in the example of the US subprime crisis. Moreover, the crisis occurred despite various improvements in credit risk management, for example the progress in the field of credit risk analysis applied by banks on the portfolio level – spurred by Basel II[2] – as well as the increasing availability of a wide range of instruments that make credit risk more liquid, for example securitization and credit derivatives. Hence, credit risk remains a major threat to financial stability in a globalised financial world, where the cross-border contagion of crises particularly threatens countries with a weak banking sector.

Credit risk analysis for the financial sector as a whole can be perceived as a crucial means to prevent the occurrence of financial crises. This can be realised by means of a regular robustness test on a country's banking sector against credit risk, for example by means of stress tests being carried out by supervisory bodies and central banks providing hints to detect financial system fragility.[3]

Within the framework of credit risk modelling and stress testing, this study investigates and compares two countries, a

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[1] The findings, interpretations and conclusions expressed in this paper are entirely those of the authors and do not represent the views of any of the authors' institutions. We thank Miroslav Singer, Vladimír Pikora, Thilo Liebig, Christoph Memmel and Pierre Tychon for their valuable comments and support. The financial support for this project by the Czech National Bank is gratefully acknowledged. Petr Jakubík: Czech National Bank and the Institute of Economic Studies of Charles University in Prague. Contact: Petr.Jakubik@cnb.cz. Christian Schmieder: Deutsche Bundesbank and European Investment Bank. Contact: C.Schmieder@eib.org.

[2] See BCBS (2006), European Commission (2006) and the respective national transposition of Basel II.

[3] The importance of this mission is underlined, for example, by the fact that the Basel Committee on Banking Supervision mandated a working group of the Research Task Force to further investigate stress testing.

new EU member state – the Czech Republic, and the largest EU economy – Germany. We seek to provide answers to various questions, notably: which macroeconomic variables are the most important to explain credit risk; whether there are country-specific differences; and what impact the occurrence of unfavourable macroeconomic circumstances can have on the macro and micro (portfolio) level, respectively.

The data employed in this study cover a period of eight years from 1998 to 2006 for the Czech Republic and twelve years from 1994 to 2006 for Germany. The investigated time horizon covers multiple periods of severe macroeconomic stress, namely the consequences of the Asian crisis in 1997 and the Russian crisis in 1998, as well as the crisis in the financial markets after 11 September 2001.

When it comes to credit risk analysis for the Central and Eastern European transitional economies (and also for many other transformation economies and developing countries), a key limitation is the availability of data as time series are usually (still) relatively short with various structural breaks. Accordingly, the data in this study have been selected very carefully to prevent misinterpretation.[4] In order to arrive at a meaningful comparison between the Czech Republic and Germany, equivalent data has been sought.

We investigate both the corporate sector and the household sector and find that credit risk for corporates can be modelled based on similar macroeconomic variables for both countries, despite fundamental differences in the default rate (time) pattern. Credit risk modelling for the household sector turns out to be more challenging for both countries as there are apparently other variables than solely economic ones to explain the default rate pattern. These findings do, in general, confirm previous studies.

[4] We draw on the findings of the analysis carried out by Jakubík (2007).

In the second step, we use the outcome of our credit risk modelling for macro stress testing purposes. In the third step we translate the outcome of the macro stress tests into a Basel II-type micro stress test of a hypothetical, but realistic, credit portfolio. We find that the impact of the macroeconomic shocks we study are substantially higher in the Czech Republic than in Germany, both on the macro and micro level. For a stress test of medium severity, we find an increase in aggregate corporate default rates of more than 100% and a rise in Basel II internal ratings-based (IRB[5]) minimum capital requirements on the credit portfolio level of up to 60% in the Czech case within one year. The figures for Germany are an increase in corporate default rates of 40% and an augmentation of IRB minimum capital requirements of roughly 30% for the same credit portfolio. For the household sectors of both countries, the impact is much less pronounced compared to corporates, with a higher impact in the German case; but the outcome is less robust and has to be therefore carefully interpreted.

Our contribution to the literature is two-fold: first, we provide a comprehensive framework for stress testing; second, we directly compare a new EU member state with one of the large "old" EU economies. While there is no directly comparable study in the literature, we confirm the finding of previous studies that stress events have a more material effect in less developed economies.

The paper is organised as follows. Section 2 provides an overview of related studies. Section 3 describes the model used to analyse credit risk for the corporate and household sectors. In Section 4, the underlying data are presented. Next, the corporate and household models are calibrated for the

[5] Credit Portfolio Risk in this study is measured based on the minimum capital requirements according to Basel II, which closely resemble Credit Value-at-Risk figures. We refer to the internal ratings-based (IRB) approach, see BCBS (2006), para. 272.

Czech Republic and Germany in Section 5 and 6, respectively. Section 7 is devoted to stress testing. Finally, section 8 provides conclusions.

2. Related Literature

A key motivation for this study originates from the Basel II framework, concluded in 2006, which seeks to provide a more meaningful framework to the solvency of financial institutions and financial stability compared to the previous Basel Accord dating back to 1988.[6] Within this framework, our focus is on the investigation of business cycle effects on the aggregate default rate of corporates and households and the link to banks' capital requirements.

Various studies have been carried out in this domain. On the micro level, a comprehensive survey of the literature dealing with cyclical effects on the major credit risk parameters, namely the probability of default (PD), loss given default (LGD) and the exposure at default (EAD) has been provided by Allen and Saunders (2004). Accordingly, all three credit risk parameters are found to be highly exposed to cyclical effects, implying a considerable impact on portfolio credit risk and capital requirements.[7] In a recent study carried out by

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[6] Another important catalyst for stress testing was the Financial Sector Assessment Program (FSAP) of the International Monetary Fund (IMF) and the World Bank initiated in the late 1990s.

[7] The study by Catarineu-Rabell et al. (2003), for example, was an early one to show that the choice of a Point-in-Time (PIT) rating system can evoke the procyclicality of capital requirements, whereas Through-the-Cycle (TTC) rating systems are more neutral. Through a comparison of PIT rating systems and TTC systems, Löffler (2004) finds that the differences are mainly driven by cyclical components of default risk. Bangia et al. (2002) and Trück and Rachev (2005), for example, have shown that there is a substantial impact of default probabilities and rating transitions on portfolio credit risk.

Düllmann et al. (2007), this has also been demonstrated to hold true for credit correlations.

While most of the numerous studies on the cyclicality of portfolio credit risk have been carried out for developed countries, the outcome that capital requirements vary strongly with the economic cycle and are driven by a complex interaction of different credit risk parameters does also hold true for developing countries, although pertinent empirical evidence remains more limited to date. Vallés (2006), for example, finds that the implementation of a through-the-cycle (TTC) rating system in emerging countries would be very difficult due to the high volatility of credit risk, indicating that portfolio credit risk might fluctuate even more strongly than in the developed world.

Despite the numerous contributions on how credit risk parameters fluctuate over the business cycle, stress testing of (real) credit portfolios remains relatively limited to date, particularly due to the limitation of publicly-available data. Among the few contributions were Peura and Jokivuolle (2004) and Rösch and Scheule (2007).

There is a wide range of research on the macroeconomic perspective of credit risk. The seminal question becomes how to model the aggregated credit risk of an economy or specific sectors such as corporates and households, respectively. Various approaches have been followed in the literature, such as applying econometric analysis on a firm-specific level including macroeconomic variables (Bunn and Redwood 2003) and using multi-factor credit portfolio models. A seminal model in the latter context has been proposed by Wilson (1997a, 1997b), known as Credit Portfolio View™, which has been used for macro stress testing by Virolainen (2004), for example. In terms of the dependent variable, macro stress tests have typically been analysed based on loan loss provisions (LLPs) or non-performing loans (NPLs). A recent review of different modelling has been provided by Sorge and Virolainen (2006).

Various macroeconomic variables have been found to be relevant as drivers of credit risk, the main drivers for corporate credit risk being GDP growth and interest rates (Virolainen 2004). Other relevant variables were corporate indebtedness, inflation, industrial production, real wages, the stock index, and the oil price (Virolainen 2004). For the household sector, empirical evidence is more limited, with unemployment and interest rates being among the most relevant variables referred to.

Stress testing is commonly used by central banks and regulatory bodies to identify vulnerabilities of the financial sector overall or to determine specific risks for individual institutions.[8] This can generally be realised in a top-down manner or by referring to a bottom-up framework.[9] Sorge and Virolainen (2006) provide a literature overview on analytical approaches to macro stress tests and apply these models to the case of Finland (see also Jones et al. (2004) and Sorge (2004)). Boss et al. (2006) give an overview on the stress testing framework followed by the Austrian central bank. The rationale is to explain industry-specific default rates by means

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[8] Macro stress tests are also carried out by the regulatory bodies of the two countries analysed in this study, the Czech National Bank and the Deutsche Bundesbank. In the Czech case, the stress testing framework was gradually elaborated since 2004 by Čihák (2004) and Čihák et al. (2007). Based on a forecast of the macroeconomic environment in terms of historical worst-case scenarios, credit risk models as developed by Jakubík (2007) are applied. The Deutsche Bundesbank takes a two-step approach: besides using aggregate data available in the Central Bank to analyse financial stability (top-down approach), it also uses information collected from single institutions via a survey (bottom-up approach) (Deutsche Bundesbank 2007). In the first case that is similar to the current study, the Bundesbank uses a macro module to define macroeconomic stress events via macroeconomic variables, which are then fed into a microeconomic module (the so-called banking module) to assess which banks would run into difficulties (Deutsche Bundesbank 2007, p. 106f.).

[9] For an overview of stress tests applied by German banks see Deutsche Bundesbank (2004) and Deutsche Bundesbank (2007).

of macroeconomic risk factor changes in order to be able to apply industry sector-specific stress tests. Credit risk models with macroeconomic variables are also employed in stress test exercises carried out by the Bank of England. Within this framework, Pain (2003) found an empirical relationship between banks' loan loss provisions and macroeconomic indicators such as GDP growth, real interest rates, credit growth and the concentration of the domestic loan portfolio. For the new EU member states, studies remain more limited. Overall, previous studies find that a relatively limited number of macroeconomic factors permit credit risk for the corporate sector to be explained in a meaningful way, including in times of financial turbulence.[10]

3. Credit Risk Modelling with Macroeconomic Variables

Within this study, we refer to the loaded one-factor credit risk model, being also the basis for the calculation of Basel II capital requirements for credit risk (Gordy (2003)). To explain credit risk, the idea is thereby to model the aggregated credit risk conditional on the macroeconomic environment, whereby the model does become a quasi multi-factor model while keeping its conceptual simplicity. The one-factor model has been employed for credit risk modelling by several authors, for example by Hamerle et al. (2004), Rösch (2005) and Jakubík (2007). Unlike linear models, this relatively simple non-linear model better captures the highly complex relationships of economies which are hardly linear in the real world. Subsequently, the basic cornerstones of the model and the ways it will be used for credit risk modelling are outlined.

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[10] See Borio et al. (2001), for example.

A random variable with a standard normal distribution is assumed for the standardised logarithmic asset returns of firm *i* at time *t*:

$$R_{it} = \sqrt{\rho}F_t + \sqrt{1-\rho}U_{it} \qquad (1)$$

where R_{it} denotes the logarithmic asset return for each firm *i* of an economy at time *t* and F_t corresponds to the logarithmic asset return of an economy at time *t*, which is assumed to be a random variable with a standard normal distribution. This variable represents the part of the asset return which is not firm-specific and can thus denote the general economic conditions for the profitability and creditworthiness of firms in an economy. U_{it} denotes the firm-specific asset return, which is again assumed to be random with a standard normal distribution. The two random variables are assumed to be serially independent. The portion of risk that is systematic is defined by ρ_i, the correlation of the firm's asset return with the systematic factor F_t.

Given these assumptions, the logarithmic asset return of firm *i* at time *t* is also standard normally distributed. The model is based on the Merton model, according to which a default occurs if the return on a firm's assets falls below a certain barrier *T*, the default threshold. Formally,

$$P(Y_{it} = 1) = P(R_{it} < T), \qquad (2)$$

where *Y* denotes a binary random variable with two potential states, borrower *i* defaults (1) and does not default (0), respectively, at time *t* and *T* is the default threshold.

In order to model aggregate macroeconomic credit risk by means of different macroeconomic indicators, it is – unlike in the case of Gordy's Basel II one-factor-model (Gordy 2003) – further assumed that the value of the default threshold *T* depends on the economic cycle. This is modelled by taking a linear combination of macroeconomic variables (x_{jt}) to represent the value of the default threshold *T*.

The final form of the macroeconomic one-factor credit risk model used in this study is shown in equation (3), where Ψ denotes the distribution function of the standard normal distribution that represents the impact of changes in the macroeconomic indicators, β_0 is a constant and β_j are the coefficients of the macroeconomic variables, x_{ij}:

$$p_{it} = P(R_{it} < T) = P(\sqrt{\rho}F_t + \sqrt{1-\rho}U_{it} < \beta_0 + \sum_{j=1}^{K}\beta_j x_{jt}) = \Psi(\beta_0 + \sum_{j=1}^{K}\beta_j x_{jt}) \qquad (3)$$

The default probability conditional on the realisation F_t of a random unobservable factor representing the state of the economy at time t corresponding to the default probability (3) is given by formula (4).

$$p_i(f_t) = P(U_{it} < \frac{\beta_0 + \sum_{j=1}^{K}\beta_j x_{jt} - \sqrt{\rho}f_t}{\sqrt{1-\rho}}) = \Psi\left(\frac{\beta_0 + \sum_{j=1}^{K}\beta_j x_{jt} - \sqrt{\rho}f_t}{\sqrt{1-\rho}}\right) \qquad (4)$$

If we furthermore assume a homogenous portfolio of firms in the economy whose asset returns follow process (1), the default rate in the economy is – based on the law of large numbers – equivalent to the firm's default probabilities. Accordingly, the model may then be applied to homogenous sub-sectors of the economy such as the corporate sector and the household sector, for example.

Accordingly, the specification of the model resulting from (3) is as follows:

$$df_t = \psi(c + \sum_i \beta_i x_i) \qquad (5)$$

where df_t denotes the dependent variable of the model (ie the respective default rate based on the NPL inflows), β_i is the coefficient vector and x is the vector of the macroeconomic variables and c is a constant.

Due to recalculation by the cumulative distribution function of a normal distribution, the coefficients of equation (5) can not be interpreted as the commonly used elasticities of the impacts of the relevant macroeconomic factors on credit risk. Rather, the joint effect of a change in different dimension in measured.[11]

In order to estimate model (3), a relationship with a conditional number of defaults of firms depending on the realisation of random variable *F, f_t,* the latent factor, is used. This number is, under the given assumptions, again random and has a binomial distribution with the conditional probability $p_i(f_t)$ given by equation (4) and the number of firms N_t.

$$D(f_t) \approx Bi(N_t, p(f_t)) \qquad (6)$$

The model is then calibrated by maximising a likelihood function. In order to ensure its robustness, notably to avoid for calibration bias, the obtained residuals are tested for autocorrelation and heteroscedasticity.

4. Data

Next, the composition of the dependent variable, the inflow of non-performing loans[12] (NPLs) and the macroeconomic

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[11] For this reason, we cannot calculate elasticities of the macroeconomic factors employed by the final specification of the model. The impact of these indicators on the sectoral default probability can only be evaluated conditional upon the others factors. By consequence, the effect of the change of the selected factor is different according to the level of the other macroeconomic indicators incorporated into the model.

[12] In fact, we first calculated the inflow of NPLs on a quarterly basis. Second, we derived the annual default rate by dividing the sum of the quarterly NPL inflows by the average volume of outstanding loans during the respective one-year sliding window. As already outlined in the literature review, also bankruptcy data-based insolvency rates could be used as the dependent variable. This approach was used, for example, by Virolainen (2004) or Bos et al. (2006).

indicators will be outlined both for the Czech Republic and Germany. The data are generally annual observations derived from one-year sliding windows on quarterly data.

4.1 Data for the Czech Republic

In the case of the Czech Republic, the inflow of the NPLs was not separately available for the household and corporate sector. Moreover, the original raw data taken from the Czech National Bank (ČNB (2007)) were quarterly sectoral NPL stocks. In order to obtain sectoral NPL flows, the NPL stocks have been adjusted for write-offs, sales and the enforcement of such exposure in the banks' books. Accordingly, the total annual NPL flows have been split into corporate, household and other credit. This has been done based on expert judgment, facilitated by robustness checks based on credit register data, where the actual monthly data have been available since November 2002. Finally, the annual NPL flows (henceforth also default rates) being used as the dependent variable of this study were calculated.

The relatively volatile time pattern of the corporate and household default rates for the Czech Republic are shown in Figure 1 and Figure 3, respectively.

The macroeconomic data for the Czech Republic have been taken from the time series archives (ARAD) of the Czech National Bank. Similar to the existing literature (see section 2), we considered the growth rate of real GDP in the Czech economy and in its most important trade partner economies, the level of real exchange rates, nominal and real interest rates, inflation, the unemployment rate and the ratio of loans-to-GDP for the corporate model. For the household model, the unemployment rate, nominal and real interest rates, inflation, the interest rate gap, the real GDP growth rate, the output gap, the ratio of loans-to-GDP, and the ratio of interest paid to income or disposable income have been considered for the specification of the model.[13]

The macroeconomic variables for the corporate model were available from 1998 Q3 and thus shorter than the dependent variable, so the model was estimated for the time period from 1998 Q3 to 2006 Q4 (see Table 1). For the household sector, the dataset used for the calibration of the model spans the period from 1997 Q3 to 2006 Q4 (see Table 1).

4.2 Data for Germany

For the German economy, the availability of data is, in general, more favourable than for the Czech Republic, although the time series have been impacted by German re-unification and by the introduction of the Euro. However, data for the dependent variable, NPLs, were available only fairly recently for regulatory purposes, spurred by the stress testing framework of the IMF and the World Bank, and thus not accessible to the public. Overall, the rationale for the German case was to attempt at referring to analogous variables as in the Czech case, in order to enable a meaningful comparison.

The NPL inflows for the German economy used in this study are based on the data reported to the Deutsche Bundesbank in the framework of regulatory reporting since 1994.[14] In order to arrive at quarterly data, the annual amounts have been interpolated and then divided by the sum of the corporate and

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[13] Disposable income was modeled by using average wages and household consumption. Interest paid was modeled as the product of the credit volume and the annual PRIBOR plus a specific interest rate spread paid by the households according to CNB data.

[14] These data have been compiled from the figures reported by the German banks in the framework of banking supervision and are not publicly available.

household credit volume for the given quarter.[15] The NPLs were again not available separately for corporates and households. The disaggregation has been realised as follows: first, the portions of the household and corporate credit on the total non-bank credit volume in Germany for the respective quarter have been taken.[16] Next, these portions have been combined with the insolvency rate[17] to disentangle the ratio into corporate and household NPLs. In this way, the quarterly NPLs for the German corporates and households were available for the period from 1994 to 2006.[18]

The NPLs for German corporate loans do approximately remain on the same level during the 1990s before there is a steady increase until 2002, and a substantial and continuous decrease afterwards (Figure 2). For the NPLs of household credit, we observe a steady increase over the whole observation period.[19] This is reflected in the rapid and

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15 This has been done by taking the reported NPLs for the respective period as the reference value for the second quarter. Subsequently, the figures for the other periods have been interpolated and extrapolated for the data in the first and last year, respectively.

16 Corporate debt is thereby referred to both as corporate debt and the debt of self-employed persons, amounting to 37% and 17%, respectively, of the total debt to non-banks in 2007.

17 Given that the insolvency rate for the household sector is only available since its introduction in 1999, the insolvency rate of 1999 has also been used for 1994 to 1998. The generally very low level of household insolvencies at this time can justify this approach. For the period after 1999, the annual insolvencies recorded by CreditReform have been referred to, which have subsequently been interpolated to arrive at quarterly data. For the corporate sector, annualized quarterly insolvency rates recorded by the German statistical office have been used.

18 In the same way as for the Czech Republic, the NPL inflows have been adjusted for the public sector and the entrepreneurs.

19 A private insolvency code was only introduced in 1999. One of the reasons for the increase in the private default rate may also have been that this opportunity was only gradually „discovered“ by the population.

continuous increase in the insolvency rates for households, although the default level remains relatively low.[20]

The macroeconomic data for Germany have been taken from the time series archives of the Deutsche Bundesbank and Eurostat.[21] For the corporates, various indicators have been taken into account, namely the real GDP growth, the change in the real effective exchange rate[22], the inflation rate, the corporate credit to GDP ratio, the unemployment rate, the development of industry production and the inflow of order bookings in the industry. For the household sector, five indicators have been particularly considered, namely the unemployment rate, the nominal and real interest rate (EURIBOR), the household credit to GDP ratio, the net household income in nominal terms and the household savings rate.

The table below provides an overview of the time span of data available for the two models and two country specifications. As shown in the table, the time series for Germany date back three to four years longer than in the Czech case.

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20 The relatively low level of household default rates for Germany reflects the very benign economic development of the German economy after the Second World War, at least until German re-unification as well as the economic model followed, namely the social market economy („Soziale Marktwirtschaft“). This economic model has brought forward a very solid middle-class population participating in growth and a comparably minor low social class, further facilitated by a low level of unemployment and a strong role of social systems. Given that the market for securitisation in Germany is relatively limited, it can also be excluded that household defaults were „hidden“ in securitisation data and thereby not captured in credit register data.

21 We took the GDP values from the Eurostat archive and the remaining time series from the Deutsche Bundesbank.

22 We used the real effective exchange rate with the EER-44 group.

Table 1

Overview of the time series available to calibrate the model per sector and country

Model	Czech Republic		Germany	
	House-holds	**Corporate**	**House-holds**	**Corporate**
Timeframe	1997 Q3–2006 Q4	1998 Q3–2006 Q4	1994 Q1–2006 Q4	1994 Q1–2006 Q4

5. Credit Risk Model for the Corporate Sector

Key macroeconomic determinants used to explain the development of the corporate default rate in an economy are, among others, (real) interest rates (1), exchange rates (2), inflation (3), the state of the economy and that of key business partners, denoted by the growth rate (4) as well as the level of corporate indebtedness (5).[23] For both countries, at least one variable from each "domain" has been considered.

After testing various specifications based on expert judgment, namely by seeking to include at least one variable from all five domains discussed above and by considering meaningful time lags, we arrived at country-specific models being most useful to depict the historic default rates. The inclusion of as few

[23] Other relevant factors include the legal environment of a country, but particularly also forward-looking factors such as the slope of the yield curve and stock indices, for example. Key macroeconomic determinants of the profitability of non-financial firms are discussed in CNB (2007), for example.

variables as possible facilitates stress testing, which will be performed in the next step. For both countries, the model contains four variables, two of which are the same for both countries with the same time lags as shown in Table 4.

For the Czech Republic, the model comprises the real exchange rate (*e*), inflation (π), the GDP (*gdp*), and the indebtedness of the corporate sector as a portion of the gdp (*debt*) with the time lags as shown below:

$$df_t = \psi(c + \beta_2 e_{t-2} + \beta_3 \pi_{t-1} + \beta_{4,1} gdp_t + \beta_5 debt_{t-4}) \quad (7)$$

In order to take into account that the considered time period constitutes a transition period for the Czech economy, we also explicitly controlled for potential structural breaks. This has been done by recursively running the model and consecutively including longer time periods in the estimation. Accordingly, we analysed whether the coefficients were stable. The outcome is shown in the Appendix, including two standard error bands around the estimated coefficients. As shown in the graph, the coefficients exhibit a relatively high instability by the end of 2003 and beginning of 2004, which points to the presence of a structural break at that time.[24] To further trace this observation, we added a dummy variable to the model, defined as 1 for the first period and 0 otherwise (see formula 8 below). It could be shown that the dummy variable was highly significant and the performance of the model increased when we chose the first period until the third quarter of 2003.[25]

As further elaborated below, in terms of the impact of these findings on the results, we found that the sensitivity of the

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[24] This was also confirmed by Chow tests, whereby the hypothesis of no structural break was rejected at a confidence level of less than 1%.

[25] No further structural break was identified, which was also confirmed by additional Chow tests.

corporate credit model for the Czech Republic to macroeconomic changes is lower since 2003 as the Czech economy moved towards a low risk post-transition economy by that time. One reason for that is that inefficient firms increasingly did not survive as the economy matured. Besides, the structural break was driven by bank privatisations that resulted in a restructuring of their internal processes, especially credit granting.

$$df_t = \psi(c + \beta_2 e_{t-2} + \beta_3 \pi_{t-1} + \beta_{4,1} gdp_t + \beta_5 debt_{t-4} + \beta_6 dum_t) \qquad (8)$$

Descriptive statistics on the macroeconomic variables included for the Czech Republic are shown below.

Table 2

Descriptive statistics for the macroeconomic variables for the Czech Republic

Variable	Notation	Mean	StD (%)	Min	Max
Real Exchange Rate	e	0.934	0.07	0.805	1.046
Inflation (%)	π	2.70	2.44	–0.40	12.00
Real GDP Growth Czech Republic (%)	*gdp*	3.59	1.89	–0.38	6.66
Corporate Credit to GDP (%)	*debt*	23.82	11.99	16.51	50.24

For Germany, the final model looks as follows:

$$df_t = \psi(c + \beta_1 ir_{t-1} + \beta_{4,1} gdp_t + \beta_{4,2} indprod_{t-3} + \beta_5 debt_{t-4}) \qquad (9)$$

The German model comprises the nominal interest rate (*ir*), the GDP (*gdp*), industrial production (*indprod*) and the debt to GDP ratio (*debt*). It is striking that the lag for the two variables included in both models, the GDP and the debt ratio, turned out to be the same. Descriptive statistics on the macroeconomic variables for Germany are shown below.

Table 3

Descriptive statistics for the macroeconomic variables for Germany

Variable	Notation	Mean	StD (%)	Min	Max
Nominal Interest Rate (%)	*ir*	4.49	2.14	2.08	9.70
Real GDP Growth (%)	*gdp*	1.72	1.65	–1.96	7.69
Industry Production (%)	*indprod*	1.30	3.54	–8.89	6.95
Corporate Credit to GDP	*debt*	56.64	3.19	51.49	62.01

The table below summarises the variables included with the coefficients, the time lag, the significance level and the standard error.

Table 4

Macroeconomic credit risk model for the Czech and German corporates

Variable	Notation	Czech case			German case		
		Lag	Coefficient	Std. error	Lag	Coefficient	Std. error
Constant			–3.060***	0.358		–2.6997***	0.07141
Nominal Interest Rate (β_1)	*ir*		NA		–1	2.2194***	0.4919
Real Exchange Rate (β_2)	*e*	–2	1.062***	0.323		NA	
Inflation (β_3)	π	–1	–4.850***	0.636		NA	
GDP ($\beta_{4,1}$)	*gdp*	0	–4.609***	1.079	0	–3.3677***	0.3280
Industry production ($\beta_{4,2}$)	*indprod*		NA		–3	–0.8215***	0.1464
Credit-to-GDP ratio (β_5)	*debt*	–4	3.006***	0.246	–4	1.0871***	0.1213
Dummy variable (β_6)	*dum*	0	0.238***	0.043	0	0.0400***	0.0125

Significance level: **: Significant at 5% level; ***: Significant at 1% level.

As shown in the table, the outcome of our empirical analysis demonstrates a substantial influence of the real *exchange rate* on corporate credit risk in the Czech Republic. This effect is in line with our expectations, as it particularly applies to export-oriented countries.[26] We used the real effective exchange rates of the Czech koruna deflated by the consumer price index (CPI) lagged by two quarters. The sign of the coefficients indicates, as expected, that a stronger real exchange rate of the domestic currency affects corporate credit risk positively, ie the default rate increases with an appreciation of the koruna.

The negative impact of *inflation*[27] on firms' default rate, being less evident than the effect of other variables, was also confirmed. The reasoning for this is as follows: While companies can also be negatively affected by the increase in the prices of their inputs, an increase in the market prices of firms' outputs will, by contrast, affect firms in the opposite way. From a debtor point of view, the increase in the price level in the economy corresponds to a decrease of the real value of a credit obligation. Although permanent inflation leads to additional costs and harms the economy, inflation tends to improve the financial situation of debtors in the short run and *ceteris paribus* leads to a decrease of companies' default rates.

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[26] This reason is as follows: an appreciated exchange rate raises the prices of domestic goods in foreign currency so that firms become less competitive outside the country of origin. Given that, in general, the price of goods in foreign currency at the world market is highly determined by the cost/profit ratio of firms, appreciated exchange rates ultimately tend to result in higher default rates in the corporate sector. This effect can be assumed to be substantially higher for Czech firms than for German firms, the main reason being that German firms benefit from the "domestic" Euro area in this context, where most of the German exports are located. The negative correlation for the Czech Republic has been empirically found by CNB (2007), for example.

[27] As given by the 4-quarter change in the consumer price index (CPI).

An economic boom will definitely influence default rates in the corporate sector as the demand for goods and services produced by non-financial firms increases. Consequently, the profit of companies increases and corporate default rates decrease. Perhaps not surprisingly, therefore, the outcome of the regression shows that the corporate default rate in the Czech economy depends negatively on the growth rate of real *GDP*[28] over the previous year.

The vulnerability of the corporate sector would be expected to depend on the extent of its indebtedness. The size of the effect would be expected to depend on how much a higher financial leverage increases default probabilities, which will particularly occur in case of an unexpected macroeconomic shock, denoted by the corporate debt to GDP ratio (*credit-to-GDP*). As to this indicator, we found that a lag of four quarters turned out to be the best specification, with a positive effect on the corporate default rate.[29]

Moreover, it turned out that neither real and nominal interest rates, as well as the real GDP growth rate in the EU-15, EU-25, EA-12 and Germany, nor the unemployment rate, were among the most important explanatory variables. Although some of them had significant univariate prediction power for the corporate default rate, they did not contribute to the prediction power of the multivariate model in a meaningful way as they were correlated with other variables included in the model specification. This does particularly apply to nominal interest rates, which were not included in the Czech model as expected due to the correlation with the exchange rates.

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[28] We consider the growth of GDP compared to the same quarter in the prior year.

[29] Debt is defined as total outstanding bank loans to the non-financial corporate sector.

For Germany, the nominal *interest rate*[30] has been found to be a key macroeconomic variable.[31] There are several reasons for that, one notably being that the traditional long-term financing via bank debt plays an important role for the German economy. This particularly applies to the German SMEs (the "Mittelstand"), which often have a close relationship with one or a small number of house bank(s). An increase in interest rates result in a prompt increase in the funding rates of the corporates and thus to a worsening of firms' ability to meet their financial obligations due to more expensive financial resources. Unlike the case of the Czech Republic, neither the real exchange rate nor the inflation rate was among the most significant variables. In the first case, the introduction of the Euro seems to have had an important contribution in reducing foreign exchange rate dependency for the German firms. Similarly, given that economic growth in Germany has been relatively sparse during the period considered, inflation was generally relatively low and thus not significant.

In the same way as for the Czech Republic, the unlagged real *GDP* growth rate significantly contributed to the explanation of the German corporate default rate. In addition, the *industry production* variable contributed as a cyclical indicator to the explanation of the default rate.[32] Unlike all other macroeconomic variables considered in this study, the latter indicator has the specific characteristic of being forward looking. However, it turned out that this variable resulted in

[30] Until 1999, we used one-month annual German interbank rates and afterwards we used one-month annual EURIBOR rates.

[31] The level of interest rates has a substantial effect on corporate credit financing, notably for credit with short-term duration, constituting a substantial portion of newly granted credit particularly in the Czech Republic, but also in Germany.

[32] We used annual changes of the index value relative to the quarter in the year before.

autocorrelation, so that it has not been considered for the final specification.

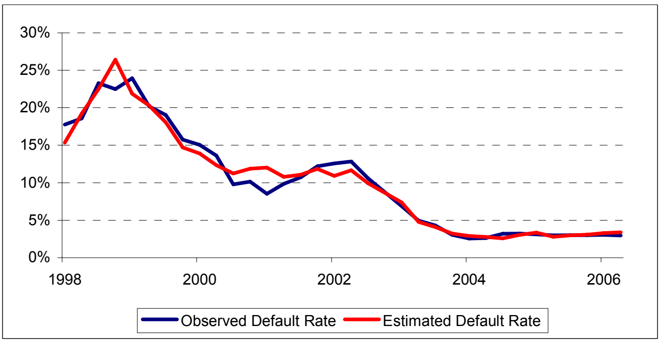
Also, the corporate *credit-to-GDP ratio* with a lag of four quarters was included in the final specification for Germany. This clearly indicates that the aggregated level of the corporate indebtedness plays a crucial role for the prediction of corporate default rates.

When estimating both models, the latent factor was significant. This implies that the corporate default rate in the economy is affected by additional factors other than those included in the model. Nevertheless, the graphs below clearly show that the calibration based on a limited number of variables was relatively successful.

Figure 1 demonstrates the performance of the estimated model for the Czech corporate sector. The figure clearly shows the decrease of the corporate NPL-based default rate from a very high level of around 20% at the end of the 1990s to a level of 10% from 2000 to 2002, before the NPL rate decreased to an intensity of 3% from 2004 on, the time pattern being captured by the model in a meaningful way.

Figure 1

Performance of the model for the Czech corporate sector[33]

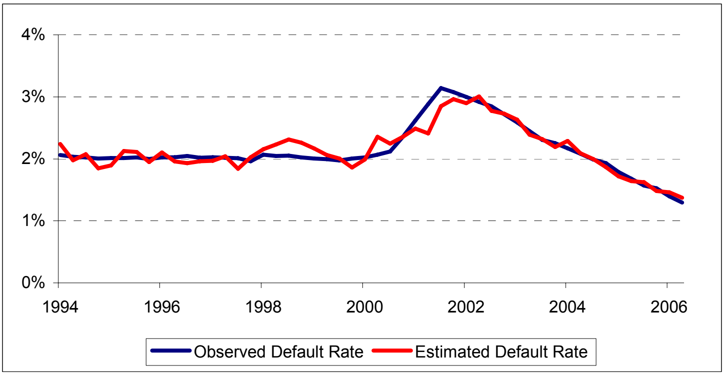


[33] The default rate denotes the non-performing loan inflow rate.

Figure 2 shows the equivalent outcome of the model calibration for Germany. The NPL inflow rate for the German corporates fluctuates around 2% until 2000, reaches a peak of around 3% in 2002 and decreases again to the previous level and then further to 1.5% afterwards. As shown in the graph, the model is able to explain the increase in the default rate and the subsequent decrease. In addition, the model predicts some slight increases, particularly around the time of the Asian and Russian crises in 1998/1999, which, however, can not be perceived in the corporate default rate in Germany.

Figure 2

Performance of the model for the German corporate sector



Due to the non-linearities of the models, the estimated β coefficients cannot be interpreted as the elasticity of the default rates with respect to the specific macroeconomic indicator change. However, we can calculate mean elasticities that represent the effect of changes of macroeconomic indicators (x_i) on the aggregated default rate (df). In a general way, the formula for the elasticities can be expressed by the following equation:

$$E_i = \frac{\frac{\partial df}{\partial x_i}}{\frac{df}{x_i}} = \frac{\frac{\partial \psi(c + \sum_{i=1}^{k} \beta_1 x_i)}{\partial x_i}}{\frac{\psi(c + \sum_{i=1}^{k} \beta_1 x_i)}{x_i}} = \beta_i \phi(c + \sum_{i=1}^{k} \beta_1 x_i) \frac{x_i}{\psi(c + \sum_{i=1}^{k} \beta_1 x_i)} \tag{10}$$

Table 5 shows the elasticities of the macroeconomic indicators (x_i) at their mean values. The signs of the elasticities are all in line with the signs of the coefficients (β_i). The default rate for the Czech economy is much more sensitive to changes in the GDP than in the German case. Furthermore, contrary to Germany, the corporate default rate in the Czech economy is very sensitive to exchange rate changes.

Table 5

Mean elasticities for the included variables for the Czech and German corporate model

Czech Republic

Variables	*gdp*	*e*	π	*Debt*
Elasticities	–0.29	1.80	–0.28	1.58

Germany

Variables	*gdp*	*indprod*	*ir*	*Debt*
Elasticities	–0.14	–0.03	0.28	1.7

The robustness of the specifications has been ensured by means of three tests: first, a likelihood test (chi-square test); second, a test on autocorrelation in the residuals; third, a test on heteroscedasticity in the residuals. Accordingly, it has been found that the corporate models are relatively robust for both countries. More specifically, the likelihood test was performed to evaluate the quality of the model: For both countries, the test rejected the null hypothesis at a confidence level of less than 1%. In addition, the non-linearity of the model was taken into account by investigating the pseudo-coefficients suggested by Estrella (1998), Cragg-Uhler (1970) and Veall-Zimmermann (1992), yielding values of close to one and thus supporting the quality of the model. A test on autocorrelation of the model was done by using the Q-statistics. Autocorrelation in the residuals is absent at the 5% confidence level. Moreover, potential heteroscedasticity was investigated by means of the Breusch-Pagan test. For both countries, the presence of heteroscedasticity was not indicated, implying that both models can be assumed to be unbiased.

In the last step, out-of-sample tests were performed. This was realised by using data until 2004 to calibrate the models and more recent data to test the forecasting ability of the models. More precisely, the corporate credit risk model was calibrated for the period 1998 Q3 to 2004 Q4 in the case of the Czech economy and period 1994 Q1 to 2004 Q4 in the German case. Then we generated forecasts for the period 2004 Q1 to 2006 Q4 for both models and calculated the forecast bias, standard forecast error and mean square forecast error. Next, these evaluation statistics were compared with a random walk model. The outcome shows that the models also perform relatively well in an out-of-sample context as shown in Table 6 and Figure 13 in the Appendix. The better performance of the random walk model in the Czech case was driven by the low volatility of the corporate default rate in the period considered (2004–2006).

Table 6

Out-of-sample tests for the corporate sector

2004 1Q– 2006 4Q	Czech case		German case	
	Forecast	Random walk	Forecast	Random walk
Forecast bias	0.000439	–0.000441	0.000033	0.000986
Standard forecast error	0.003441	0.002113	0.000588	0.000298
Mean square forecast error	0.000012	0.000005	0.000000	0.000001

6. Credit Risk Model for the Household Sector

For the analysis of credit risk for households, we refer to the same aggregate top-down model as in the case of the corporates.

Overall, there are at least three different economic factors that are particularly relevant for the prediction and explanation of household default rates, namely household income (1), the level of indebtedness (2) and the macroeconomic environment in a broader sense (3).

In general, a financial crisis for households usually occurs as follows: should the disposable household income decrease and fall under a certain threshold, households have to sell their assets. If there are no assets left to be sold, a default event occurs.

From the set of variables considered for the households (see data section), two macroeconomic variables have been included in the models for the Czech Republic and for

Germany. However, the included variables were from different domains: while the included variables in the Czech case are both "purely" macroeconomic variables (ie from the third domain), the relevant variables in the German case refer to household income and level of indebtedness as shown in Table 7.

For the Czech model calibrated for the time period from 1997 Q3 to 2006 Q3[34], it was found that both the unemployment rate (*u*) and the real interest rate (*r*) have a positive effect on household default rates. For Germany (1994 Q1 – 2006 Q4), the two most significant variables were household income (*inc*) and the household debt to GDP ratio (*debthouse*) with a negative and positive effect on the private default rate, respectively.

The most important macroeconomic driver for household default is the *unemployment rate* as it significantly affects the households' income. In the case that the key breadwinner of a heavily indebted household loses his/her job, for example, the household is usually not able to compensate for this deficit, so default becomes highly likely. In the Czech case, the *unemployment rate* was found to be relevant with a lag of four quarters, which corresponds to the lagged impact on payment discipline in the event of loss of employment.[35] In the case of the real interest rate, the statistically best results were achieved for a lag of three quarters – apparently the duration

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[34] The quarterly time series of the annual default rate was generated from the monthly series of the annual default rate calculated using relationship (2) by averaging the three monthly figures corresponding to the relevant quarter. Although the default rate obtained using equation (2) was available from 1994, the time series on which the model was estimated had to be shortened as a result of some lags in the model and due to the shorter series of the other macroeconomic indicators included in the model.

[35] The loan is initially repaid from savings or redundancy payment; payment discipline is affected only after that.

of an altered interest rate fixing on household defaults. The real *interest rate* was calculated by deflating the annual PRIBOR by annual changes in the CPI. The relevance of real interest rates is highly driven by the fact that a substantial portion of household debt is based on floating interest rates. In addition, the use of this variable allows capturing the impact of interest rates and inflation simultaneously. It is worth noting that the National Bank of Denmark uses the same variables as found for the Czech Republic (Danmarks Nationalbank 2007). The other variables considered such as nominal interest rates, inflation, the interest rate gap, the real GDP growth rate, the output gap and the ratio of interest paid to income or disposable income were not significant.

The development of the Czech household default rate is graphically shown below, exhibiting a much less volatile time pattern than the corporate default rate for the country.

Figure 3

Performance of the model for the Czech household sector

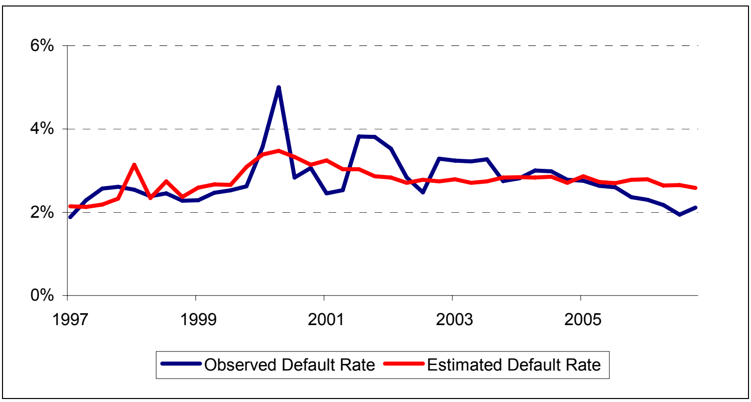

Table 7

Macroeconomic credit risk model for the Czech and German households

Variable	Notation	Czech case			German case		
		Lag	Coefficient	Std. error	Lag	Coefficient	Std. error
Constant			–2.224***	0.071		–5.5656***	0.1072
Household Income (β_1)	*inc*	NA	NA	NA	0	–5.7912***	0.9244
Credit-to-GDP ratio (β_2)	*debthouse*	NA	NA	NA	–4	5.7186***	0.2788
Unemployment Rate ($\beta_{3,1}$)	*u*	–4	3.695***	0.846	NA	NA	NA
Real Interest Rate ($\beta_{3,2}$)	*r*	–3	1.808**	0.596	NA	NA	NA

Significance level: **: Significant at 5% level; ***: Significant at 1% level.

Also for Germany, the calibration of the model turned out to be less straightforward as for the corporate case. In the German case, it was by far the *credit-to-GDP ratio,* ie the aggregate level of household debt relative to GDP that had the strongest effect on the household default rate, being the only ratio that reflects the immense and continuous increase in the household default rate in Germany during the last ten years. It is important to highlight at this point that an official household default code has only existed since 1999, and that its existence has become known only fairly recently, so the increase in the default rate can be partially driven by more and more people becoming aware of this opportunity. Hence, the results for the German household sector have to be interpreted with caution.

Moreover, the decent economic development in Germany during the last decade was also partially reflected in *household income*. The household income constitutes the primary source whereby credit is financed, so it plays a crucial role for the household default.[36]

The other structural economic indicators that were considered, for example the savings rate and interest rate levels, did not reflect the general development and therefore were not included in the model.

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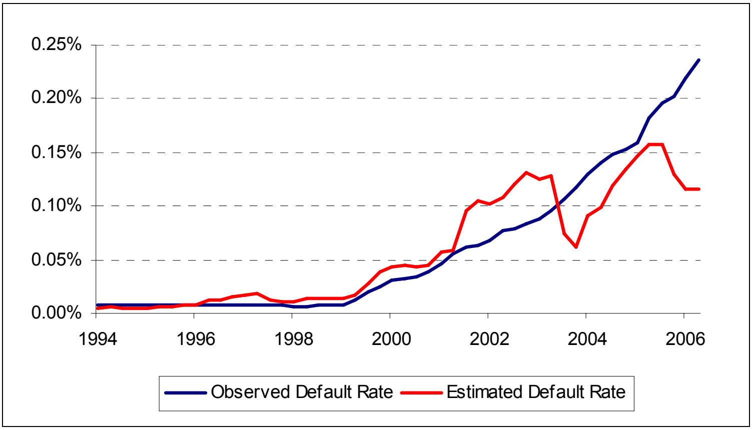
[36] More specifically, the income to installment ratio is relevant, given that this ratio is usually relatively stable over time as households typically have a regular income in form of a salary or pension. In addition, households often possess financial assets, for example real estate or personal assets, which can be liquidated in case of a financial shortcoming.

Figure 4

Performance of the model for the German household sector



In sum, the outcome for both countries reveals that the aggregate household models were less successful in explaining household default compared to the corporate case, and there were apparently less similarities across countries. An important reason could be that there were apparently developments beyond the economic sphere that explain the evolution of the household default rate, being of a socio-economic nature. In the German case, for example, there was, similar to other western European countries, a tendency toward a decline in the welfare of many private households, particularly in case of unemployment, notably also due to a reduction of social contributions. Combined with a change in family structures, namely an increasing divorce rate, this led to an increasing level of single-person households and migration into larger cities where rents were increasing etc.

Subsequently, mean elasticities were calculated for both models. We found very low elasticities of the Czech household default rate particularly with respect to the real interest rates.

In the German case, the elasticities were much higher. Nevertheless, further tests outlined below point to a possible bias of the estimates of the German model and a poor performance of the models for the households overall.

Table 8

Mean elasticities for the considered variables for the Czech and German household model

Czech case

Variables	*u*	*r*
Elasticities	0.12	0.01

German case

Variables	*inc*	*debt*
Elasticities	–0.23	8.13

The previous considerations were further investigated by means of robustness tests. The outcome of these tests shows that the household models were less robust, which has to be taken into account when it comes to stress testing. More specifically, the absence of autocorrelation can be confirmed for the Czech case at the 5% significance level, while this is not possible for the German case. In the latter case, this means that the model can be biased and that the dependent variable may not be properly explained by the considered macroeconomic indicators only. In addition, a test on the presence of heteroscedasticity in the residuals shows that the estimated standard errors of the coefficients can be biased.

7. Stress Testing

Next, the credit risk models are used for stress testing, which is an important means both as a tool to ensure financial stability and for Basel II purposes on a micro level. This has also become evident in terms of the US subprime crisis. Its effects are likely to remain a threat to financial stability, although the countries considered in this study, notably Germany, seem to be only moderately affected through banks' investments in US real estate securities (with the exception of specific financial institutions). The major effect resulting from the crisis can be a worsening worldwide economic environment due to a potential recession in the United States. Stress testing allows simulating various scenarios to help banks and regulators identify and prevent potential risks.

The key challenge for stress testing is the choice of scenarios. Ideally, a stress testing exercise begins with an assumed adverse macroeconomic event that is then endogenously translated into a credit risk scenario in terms of different risk factors. However, such comprehensive and complex risk modelling frameworks hardly exist in banks, so banks (but also regulatory bodies) are likely to apply other methods, namely regression analysis, historical simulation and sensitivity analysis.[37]

For the purpose of stress testing the financial system as a whole, so-called macro stress testing, we will refer to historical simulation type stress testing, as we use adverse events during the observed period to predict stress scenarios in the future as further elaborated below.

In the next step, the outcome of our macro stress tests is being used for Basel II-type stress testing at the firm level,

[37] For an overview on stress tests applied by German banks see Bundesbank (2004).

which we will refer to as micro stress testing. The stress test type applied in the case of the IRB is sensitivity analysis and historical simulation of the IRB credit parameters rather than the single risk factor which has been pre-calibrated in terms of stress.

7.1 Macro Stress Tests for the Czech and German Corporates

For both the corporate and household sector, we deal with the challenge of selecting appropriate scenarios. We will use both historic simulation-based scenarios and expert judgement-based scenarios to arrive at appropriate macro stress tests.[38]

In the underlying case with a stress test forecast period of one year, some of the variables would not have to be forecasted due to their lag of up to four quarters. However, this is not the usual case, particularly if one seeks to arrive at a longer prediction horizon. In that case, forecast models such as panel regression models or the vector error correction model (VEC) can be used to either directly model credit growth, for example, or by estimating it through macroeconomic variables.[39]

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[38] In the latter case, one could use the macroeconomic forecast of the respective central banks, for example, in the Czech case the CNB's quarterly macroeconomic forecast (CNB 2003) and in the German case the macroeconomic forecast of the Deutsche Bundesbank and the European Central Bank, respectively, and then add some additional stress according to one's considerations.

[39] Alternatively, the most important parameter used in this study, (sector-specific) credit growth, can be forecasted by means of panel regression models (for the CEECs see Cottarelli et al. (2003) and Duenwald et al. (2005)). Besides, VEC models have been used to arrive at forecasts for single countries as well as for aggregate data for several countries, for example by Hofman (2001) and Schadler et al. (2005).

For all four models, we simulate a stress test based on end-2006 data (2006 Q4) for the end 2007 situation. Accordingly, the inflation rate of 2.9%, a real effective exchange rate of 1.046, a real GDP growth rate of 5.81% and a corporate credit-to-GDP ratio of 17.69% yield a model-inferred default rate of 3.5% for Czech corporates at the end of 2006, very close to the actual value.

Next, we define two quantile-based stress scenarios of a moderate and more severe intensity, which are used for all four models and thereby allow for meaningful comparisons. We assume that the quantiles of all macroeconomic variables change by 10 percentage points (moderate stress scenario, HS 10%) and 20 percentage points (severe stress scenario, HS 20%), respectively, in the unfavourable direction. In two cases where the highest historical levels applied, we did not apply a full adjustment of the parameter (ie a 10% and 20% increase or decrease), but used the 100% quantile value for the HS 10% scenario and rounded the parameter up for the HS 20% scenario. Descriptive statistics of the historical values for all variables included in the Czech corporate model are shown in Table 2. In the moderate stress scenario (HS 10%), for example, the real effective exchange rate remains at 1.046 given that this is the maximum observed historical value, the inflation decreases to 2.5%, GDP growth decreases to 4.27% and the credit-to-GDP ratio increases to 19.67%. In addition, we add an expert based scenario which takes into account the recent development of the variables. The stress parameter values used for the three scenarios are displayed below.

Table 9

Macro stress tests applied to the Czech corporates

	Real effective FX rate	**Inflation (%)**	**GDP growth (%)**	**Credit-to-GDP (%)**	**Corporate Default rate**	**Relative to 2006 Q4**
2006 Q4 value (Quantile, %)	1.046 (100.0)	2.90% (54.5)	5.81% (81.8)	17.69% (33.3)	**3.5%**	**NA**
HS 10% scenario	1.046[1]	2.50%	4.27%	19.67%	**5.5%**	**+57%**
HS 20% scenario	1.1[2]	2.30%	3.88%	28.46%	**10.6%**	**+204%[3]**
Expert based	1.1	2.50%	4.00%	23.00%	**8%**	**+128%**

Note: HS 10% (20%) represents a moderate (severe) stress test scenario in which the quantiles of all macroeconomic variables change by 10 (20) percentage points in the unfavourable direction.

[1] This value was used because the quantile value for the current scenario is already at 100%. [2] For the 20% scenario, we assumed that the level of the real effective exchange rate would increase to 1.1, taking into account the very high level of the variable and did thereby not strictly stick to the quantile rule. [3] If the quantile rule had been strictly applied, the default rate would be 11.6% and the respective increase 233%.

As shown in Table 9, the Czech corporate default rates do substantially increase for all three stress scenarios, by 57% in the HS 10% case (to 5.5%), 128% in the expert based scenario and by 204% in the HS 20% scenario. For the historical quantile-based scenarios, it is particularly the credit-to-GDP ratio that accounts for a very high increase in the default rate. Given that the level of this variable has constantly been below 20% since the second quarter of 2003, the corresponding historical based value of the HS 20% scenario (28.46%) seems to be relatively high, so the expert based value has been chosen in a less conservative way. While the HS 20% scenario may be perceived as a severe scenario rather unlikely to happen, it shows the volatility of the corporate default rate for the Czech corporates in an illustrative way.

The outcome of the corresponding stress tests for Germany is shown below.

Table 10

Macro stress tests applied to the German corporates

	Nominal interest rate (%)	**GDP growth (%)**	**Industry produc-tion change (%)**	**Credit-to-GDP (%)**	**Corpo-rate default rate**	**Relative to 2006 Q4**
2006 Q4 value (Quantile, %)	3.54% (41.5)	3.95% (95.3)	3.92% (79.6)	53.45% (9.6)	**1.4%**	**NA**
HS 10% scenario	3.83%	3.34%	2.96%	54.18%	**1.6%**	**+13%**
HS 20% scenario	4.25%	2.59%	2.05%	54.46%	**1.9%**	**+29%**
Expert based	4.50%	2.00%	3.00%	55.00%	**2.0%**	**+38%**

Note: HS 10% (20%) represents a moderate (severe) stress test scenario in which the quantiles of all macroeconomic variables change by 10 (20) percentage points.

As clearly visible from the table, the impact of the scenarios on the default rate of German corporates is much lower than in the Czech case, the increase remaining at below 30% for the historical scenarios (even for the HS 20% scenario), while the expert based scenario, which has been chosen in a slightly more conservative way (notably in case of GDP growth), yields an increase by 38%.

The outcome that the multivariate impact of the stress test is much more severe for the Czech Republic may also be perceived at the resulting stress values of each of the macroeconomic variables of the two respective corporate models: in the German case, they are not diverging away too much from the actual value when being stressed by 10% and 20%, respectively, while the stress level in the Czech case results in more substantial differences. Nevertheless, the full range of the historical values for Germany does also show that a more severe stress event could potentially yield a more substantial increase in the corporate default rate. If one applies a stress level of 30% (HS 30%) and 40% (HS 40%), respectively, to further investigate whether the stress effect tends to be non-linear, the resulting increase in the default rate by 44% and 66%, respectively, shows that the impact remains relatively moderate compared to the Czech case, so this is not the case.

7.2 Macro Stress Tests for the Czech and German Households

For the stress tests of the household sector, it has first to be taken into account that the performance of the model was considerably weaker than for the corporate sector, so the stress testing exercise will be less precise and one is more likely to over- or under-estimate the effect of a stress event. Taking a conservative stance, one may add some cushion on the actual outcome in both directions. Hence, we will only apply the same historical, quantile-based scenarios for the households without additionally using an expert-based scenario, also in order not to stretch the use of household models too much.

The outcome of the stress tests applied to Czech households is displayed below. The result of an increase in the default rate of only 4% for the HS 10% scenario and 6% for the HS 20% scenario outlines a relatively low sensitivity of the household default rate to changes in the economic variables. As pointed out before, not all effects shown in the empirical default rates are fully captured in the model, so the result has to be perceived in a careful way.

Table 11

Macro stress tests applied to the Czech households

	Unem-ployment Rate (%)	Real Interest Rate (%)	House-hold Default Rate	Relative to 2006 Q4
2006 Q4 value (Quantile, %)	7.77% (43.2)	–0.47% (0.0)	**2.59%**	**NA**
HS 10% scenario	8.02%	0.01%	**2.69%**	**+4%**
HS 20% scenario	8.19%	0.15%	**2.75%**	**+6%**

Note: HS 10% (20%) represents a moderate (severe) stress test scenario in which the quantiles of all macroeconomic variables change by 10 (20) percentage points.

The outcome of a macro stress test for the German households is displayed below. The increase in the default rate is higher than for the Czech case, with 11% for the HS 10% scenario and 28%[40] for the HS 20% scenario. It is worth highlighting that the level of default-rate increase for the two stress scenarios is very similar to the case of the German corporates. While this outcome may be striking at first glance, it again reflects the steady and rapid increase in German household default rates during recent years. Again, it has to be taken into account that the model specification is less robust than for the corporate case.

[40] If the 100% quantile had been applied for the credit-to-GDP ratio (45.20% as for the HS 10% scenario), the default rate would increase only by 21%.

Table 12

Macro stress tests applied to the German households

	House-hold Income Change (%)	Credit-to-GDP (%)	House-hold Default Rate	Relative to 2006 Q4
2006 Q4 value (Quantile)	1.09% (50.8)	45.13% (98.3)	**0.115%**	**NA**
HS 10% scenario	0.62%	45.20%[1]	**0.128%**	**+11%**
HS 20% scenario	0.15%	45.50%[2]	**0.148%**	**+28%**

Note: HS 10% (20%) represents a moderate (severe) stress test scenario in which the quantiles of all macroeconomic variables change by 10 (20) percentage points.

[1] Given that the quantile value for the current scenario is already at 98.3%, we applied the 100% quantile. [2] In the same way as for the Czech corporates, a higher value has been applied for the 20% scenario than foreseen by the formal treatment.

7.3 Stress test applied to a hypothetical bank portfolio

In the Basel II framework (BCBS 2006), stress testing is part of Pillar I and Pillar II whereby banks are asked to analyse possible future scenarios that may threaten their solvency. In the case of credit risk, this notably includes an assessment of economic or industry sector-specific downturn events, which must be chosen in a "meaningful" and "reasonably conservative" way and thereby represent at least "mild recession scenarios", but not necessarily a "worst-case scenario" (BCBS 2006, para. 435). In order to keep stress tests flexible and thereby match the specific requirements of each bank, the challenging task of the choice of stress scenarios is at the discretion of banks and has to be justified vis-à-vis the supervisory body.

Within the Basel II framework, a stress test on credit risk would comprise all IRB credit risk parameters, namely the Probability of Default (PD) (both those of borrowers and guarantors), the Loss Given Default (LGD) and the Exposure At Default (EAD)[41] If a bank applying either the Foundation IRB (FIRB)[42] or the Advanced IRB (AIRB) approach additionally uses an economic capital model, then stress testing would typically additionally comprise an assessment of credit correlations and portfolio industry sector concentration or geographic concentrations.[43]

Next, we use the outcome of the macro stress testing exercise in a top-down manner for Basel II-type micro stress testing based on a hypothetical credit portfolio.[44] The portfolio

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[41] In addition, also the Maturity (M) could be subject to stress testing.

[42] IRB stands for internal ratings-based approach.

[43] Basel II-type microeconomic stress testing is also being applied in supervisory bodies. In Deutsche Bundesbank, for example, Basel II-type microeconomic stress tests are carried out in two different forms: first, in terms of sensitivity analysis to simulate macroeconomic effects on specific credit risk parameters which have been carried out in the framework of the Quantitative Impact Studies (QIS); second, in terms of scenario analysis, based on a stochastic multi-factor model by using data from the German credit register, for example to arrive at further insights into sectoral concentrations. For further information see Deutsche Bundesbank (2007, p. 102f.).

[44] We sought to make the portfolio as realistic as possible, which has been supported by expert judgment in various dimensions, so that the values are meaningful for both countries. It is important to highlight that a real portfolio would be non-homogenous in terms of PD and EAD (but also in terms of LGD and, if the data allow for that, in credit correlations), which would alter the results. The direction of the changes depends on the actual portfolio, namely whether it exhibits mostly SMEs with similar risk characteristics or a mixture of large, medium and small corporates, the industry-sector distribution etc.

comprises 6,000 loans with 50% of the credit exposure being granted to corporates and 50% to households. This decomposes into 462 loans made to corporates and 5,538 to households with a relative exposure size of the corporate exposures to the retail (household) exposures of 12:1.[45]

In terms of the credit portfolio risk measure, we refer to the Basel II minimum capital requirements under the IRB approach. In this way, the regulatory capital requirements – in its role as a Value-at-Risk ratio for a one-year horizon and a confidence level of 99.9% – are used as an indicator of the inherent credit risk in the portfolio.[46] For the corporate loans, we apply the Basel II IRB formula for corporates (BCBS (2006), para. 272) based on four inputs, namely the PD, the LGD, the EAD and the Maturity (M). For the loans to households, the formula for other retail exposure (BCBS (2006), para. 330) is applied, whereby the maturity is not needed.

In order to allow for a meaningful comparison between the two countries, we use the two historical, quantile-based macroeconomic stress scenarios elaborated in the last section (HS 10%, HS 20%) as the basis for portfolio stress testing (ie for a stress on the PD) and measure the relative effect for both countries for the same credit portfolio.

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[45] This size ratio has been revealed as follows. We calculated the average corporate credit by dividing the total corporate credit volume in Germany by the number of corporate firms in 2007 and then assumed that the credit of the firms is equally spread over two banks. For the household sector, we similarly took the total household credit volume in Germany and divided it by the number of German households. The resulting absolute numbers were 209,000 EUR for the corporate sector and 17,000 EUR for the household sector. In terms of size ratio, we thereby assume that 12/13 (n=5538.5) of the loans are granted to households and 1/13 (n=461.5) to corporates. See Deutsche Bundesbank (2007), p. 54/55.

[46] Further information on Basel II can be found in BCBS (2006) and the respective national regulations, in the European Union coming from EC (2006).

We further assume that the portfolio is homogenous in terms of PDs and exposures within the corporate and household sector, respectively. In the first case (1), the LGD is fixed to 45% and the maturity to 2.5 years as foreseen under the Foundation IRB approach for senior unsecured debt.[47] Second (2), the LGD will be increased by 20% in relative terms (from 45% to 54%) in order to take into account a potential positive correlation between the PD and the LGD.

The resulting Herfindahl-Hirshmann-Index (HHI) of 0.00059 indicates that the credit portfolio referred to is granular in terms of EAD[48], implying that there is only minor name concentration. Furthermore, it is also assumed that the corporate portfolio is also diversified across industries and geographical sectors, so the basic assumptions for the use of the Basel II one-factor model are fulfilled. While these assumptions facilitate the use of the Basel II IRB model, the resulting credit risk requirements thus tend to be a lower bound for actual portfolio credit risk.

The regulatory IRB capital requirements for this hypothetical credit portfolio are displayed below, listing the outcome for the two stress tests without and with LGD stress for the two countries, respectively.[49] The capital requirements are measured relative to the portfolio exposure and are highlighted in bold.

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[47] In some jurisdictions, the effective maturity is used in the same way as in case of the AIRB and is thereby not fixed to 2.5 years.

[48] In the study of Gordy and Lütkebomert (2007), an average HHI of 0.001 has been determined for credit portfolios of large German banks based on data from the German credit register.

[49] It is worth highlighting that we assume that the same levels of stress (ie HS 10% and HS 20%) occur simultaneously for the households and the corporates in each case.

Table 13

Impact of macro stress tests on IRB minimum capital requirements (CR) for a hypothetical portfolio: comparison for the Czech Republic and Germany (CR are measured in % of exposure)

	Stress Scenario	Parameter	End 2006 portfolio (unstressed)	Forecasted 2007 stress portfolio	
				PD stress only (case 1)	PD and LGD stress (case 2)
Czech Republic	HS 10%	Corporate-PD (%)	3.50	5.5	
		Household-PD (%)	2.59	2.69	
		LGD (%)	45	45	54
		CR (%)	**7.82**	**8.66 (+10.7%)**	**10.39 (+32.9%)**
	HS 20%	Corporate-PD (%)	3.50	10.60	
		Household-PD (%)	2.59	2.75	
		LGD (%)	45	45	54
		CR (%)	**7.82**	**10.37 (32.6%)**	**12.45 (59.2%)**
Germany	HS 10%	Corporate-PD (%)	1.43	1.63	
		Household-PD (%)	0.115	0.128	
		LGD (%)	45	45	54
		CR (%)	**4.66**	**4.87 (+4.5%)**	**5.84 (25.3%)**
	HS 20%	Corporate-PD (%)	1.43	1.85	
		Household-PD (%)	0.115	0.148	
		LGD (%)	45	45	54
		CR (%)	**4.66**	**5.09 (+9.2%)**	**6.11 (31.1%)**

The table shows various results. First, the unstressed minimum capital requirements are 7.82% of the exposure in the Czech case, compared to 4.66% for the German case, implying that the portfolio risk in the Czech Republic exceeds the portfolio risk in Germany by a considerable portion of 68% due to higher default rates.[50] Second, the table also shows that the impact of a univariate macro stress on default probabilities (when keeping recovery rates at the same level, ie case 1) is much higher for the Czech portfolio, yielding an increase in the capital requirements of 10.7% compared to 4.5% for Germany in case of stress scenario HS 10% as well as 32.6% vis-à-vis 9.2%, respectively, for the more severe stress scenario HS 20%. This outcome shows that a macro stress event for default rates translates into a 2.4 times (HS 10%) and 3.5 times (HS 20%), respectively, higher impact on capital requirements in the Czech case, vastly driven by the higher volatility of default rates in the corporate sector.

Third, if one adds a potential stress level of the LGD of 20%[51] in order to arrive at a more comprehensive impact of capital requirements for the credit portfolio (case 2), then the increase in capital requirements is around 33% (HS 10%) and 59% (HS 20%) for the two stress cases on the Czech portfolio and 25% vis-à-vis 31% for the German portfolio. In a study based on a large European corporate credit portfolio for a similar time period, Düllmann et al. (2008) find a maximum increase in the IRB minimum capital requirements of 34% on an annual basis, which is in the same range as for the German case. However, for an economic capital model additionally taking into account

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[50] Although we focus on the relative impact of stress tests, it is worth noting that the ratio is below the loaded 8% level in both cases, reflecting that the portfolio quality is above average overall based on the end 2006 parameters.

[51] From a methodological perspective, an increase in the LGD has a linear effect on Basel II capital requirements', implying that the impact is higher than for the PD.

the variation of credit correlations, the authors find that the economic capital increase may be more than twice as high as for the IRB capital. If this was translated into the outcome of the current study, then a credit portfolio in the Czech Republic may be exposed to an increase in (economic) capital requirements by more than 100%, despite the “smoothing” effect of the Czech households.

In sum, macro stress scenarios translated into micro stress tests may lead to a considerable increase in capital requirements even for a homogenous and well-diversified portfolio that reflects the average macroeconomic conditions of the respective economy as in the underlying case. Hence, the effect can be assumed to multiply for concentrated, heterogeneous portfolio with a higher portion of corporate credit as in the underlying case, so that the increase in default rates and recovery rates are higher than for an average bank portfolio.

8. Conclusion

In this study, macroeconomic credit risk modelling for the corporate sector and the household sector has been applied to both the Czech and the German economy. In the first step, reliable and comparable data have been generated, yielding a time frame of eight years from 1998 to 2006 for the Czech Republic and 12 years from 1994 to 2006 for Germany based on annualized quarterly data. Furthermore, it has been ensured throughout the study that the results for the two countries were comparable in order to work out and explain potential differences.

Next, a Merton-type one-factor credit risk model has been used to estimate aggregate corporate and household default rates based on non-performing loans. We found that modelling was meaningful for the corporate sector, while this was not entirely the case for the household sector. Moreover, the outcome that similar variables can explain the default rates for two economies with a different default rate and volatility pattern outlines that corporate credit risk can obviously be explained by a limited set of similar variables for different economies. By contrast, further research seems to be necessary in order to better explain the time pattern of household default rates. The use of bottom-up models as applied by some regulatory bodies is promising provided that data are available.

Subsequently, macro stress testing has been applied, indicating that there can be a substantial increase in the corporate default rate (more than 100% for the Czech Republic; up to 40% for Germany) also in case of a relatively minor change of the macroeconomic environment in historic terms, particularly for the Czech corporate sector. For Germany, the impact of macroeconomic shocks is apparently much less pronounced, at least for the period considered. However, the sensitivity of the corporate sector default rates for the Czech Republic was driven by the high volatility of the macroeconomic indicators during the transition period of the late 1990s, so the results have to be carefully interpreted. For the households, the sensitivity to the macroeconomic environment is less pronounced, unlike what the recent US subprime crisis might suggest. This outcome, which also holds true for the credit portfolio level, has to be interpreted with caution, as the robustness of the household model is substantially lower than the one for the corporate sector, but it might also be seen as an indication that both economies are more robust against shocks in the household sector. The latter assumption seems to be reflected in the fact that the financial crisis has not had a considerable effect on the Czech and German household sector.

In the third step, the outcome of macro stress testing has been used for top-down Basel II-type micro stress testing of the same hypothetical credit loan portfolio for both countries. It turned out that the increase in portfolio risk induced by a macroeconomic shock is more than twice as high for the Czech Republic as for Germany. Based on regulatory IRB capital requirements, portfolio credit risk of an average bank would increase up to 60% in the Czech case and roughly 30% for Germany. In the case of portfolios concentrated in names or sectors and/or when using an economic capital model that additionally takes into account potential variations of credit correlations, the increase in capital requirements could be much higher.

In sum, it has been clearly shown how important a comprehensive analysis of credit risk is in order for regulatory bodies to be able to take measures to safeguard financial stability. Besides the numerical results of this study for the two underlying countries, we provided a template to investigate the credit risk environment of an economy. In this way, the underlying study can serve as a starting point for other countries for the monitoring of credit risk in their financial systems and thereby contribute to the detection of potential fragility of the banking sector as a means to prevent financial crises. Given that central banks and supervisory bodies usually have data on the credit portfolios of domestic banks (for example via a credit register), macroeconomic stress scenarios can be readily translated in a top-down manner for micro stress testing, in particular for those banks that are relevant from a systemic perspective. For the industry, the elaborated framework may contribute to the introduction of Basel II-type micro stress testing.

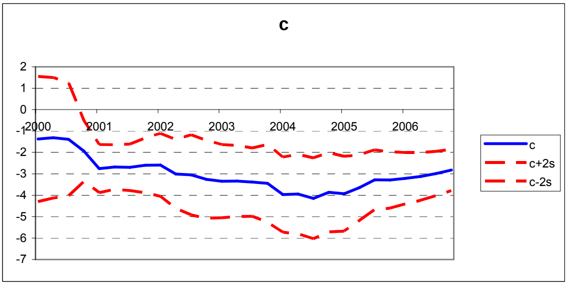
In any case, future research is needed on the highly complex issues discussed within this paper. There are various approaches by different regulatory bodies pointing in the same direction from a methodological viewpoint. However, the most fundamental issue is to ensure reliable databases with a long-term horizon, which are not available in most countries. Improvements are also needed particularly for the household sector, where the modelling turned out to go beyond the economic dimension.

Appendix

Figure 5

Analysis of coefficient stability for the Czech corporate credit risk model

(model without dummy variable for a structural break)



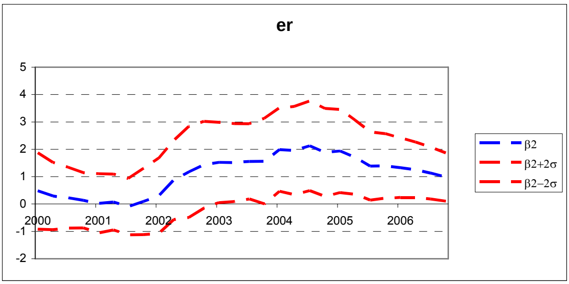
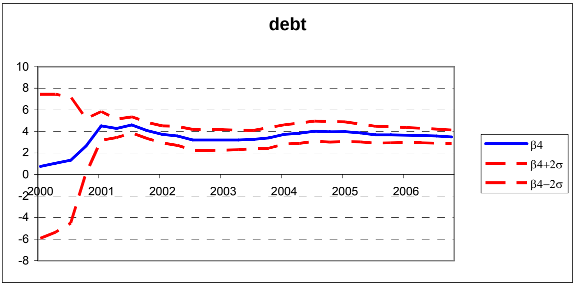
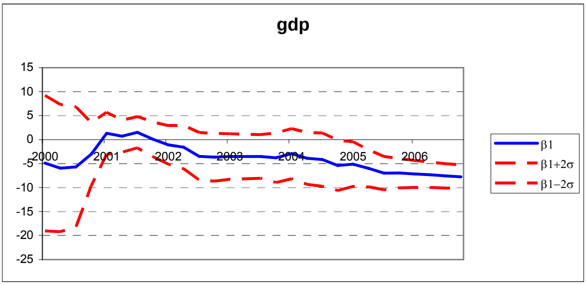


Figure 5 (cont)

Analysis of coefficient stability for the Czech corporate credit risk model

(model without dummy variable for a structural break)





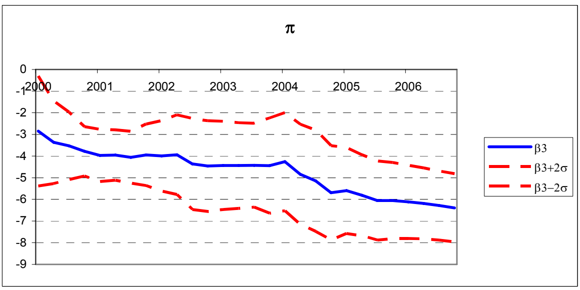
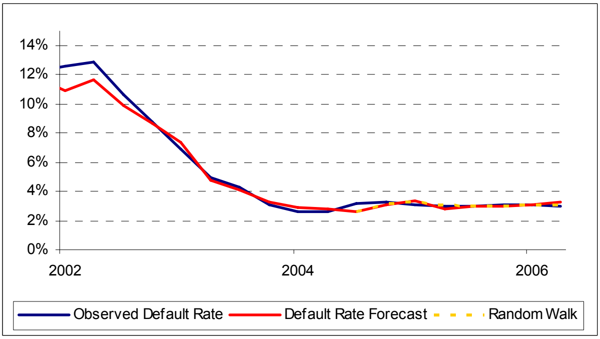


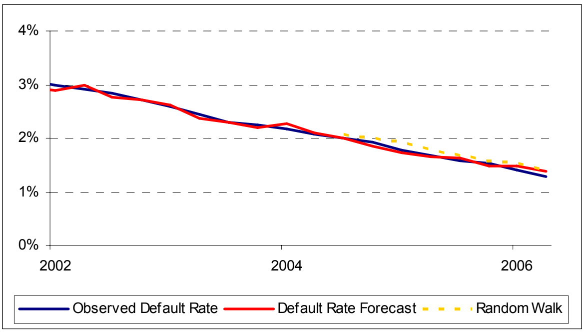
Figure 6

Out-of-sample test for the Czech and German corporate credit risk model

Czech case



German case



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