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High and Low Frequency Correlations in Global Equity Markets*

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Abstract

We model high and low frequency variation in global equity correlations using a sample of 43 countries, including developed and emerging markets during the period 1995-2008. Such variations are characterized by a multifactor asset pricing structure with second-moments dynamics leading to high frequency correlations that mean revert toward the smooth low frequency ones. We correct for non-synchronous biases from using international returns at high frequencies. We find that global correlations showed a remarkable increase during the recent financial turmoil, but the effect was uneven across countries. Those that experienced the largest increases in both correlation components were mainly emerging markets.

JEL classification: C32, C51, C52, G12, G15

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I. Introduction

This paper provides a framework to separate short- and long-term dynamic components of the correlation structure of international equity returns. The evolution of this structure plays an important role in understanding and evaluating the impacts of financial globalization not only on portfolio allocation and risk management practices, but also on financial sector stability. This is especially important in the context of the recent global financial crisis that will likely lead to major changes in regulation and to a reorganization of the international financial system.

While it is recognized that correlations change over time, the attention has been directed primarily to the dynamics of high frequency conditional correlations ─which better describes short-term correlation patterns─, leaving aside the slow-moving low frequency component of correlations that is most likely associated with long-term dynamics. This paper presents a new measure of low frequency global correlations based on the Factor-Spline-GARCH model of Rangel and Engle (2008), which provides a semi-parametric framework to extract smooth low frequency correlation components from high frequency financial data exploiting a factor asset pricing structure. The analysis of Rangel and Engle (2008) focuses on the US market, and it is based on a single factor CAPM asset pricing framework. The present article extends this specification by using a multifactor APT setup in a global context.[1] Specifically, the long-term correlation component is driven by the low frequency volatilities of three global regional factors (North America, Europe, and Asia) and idiosyncratic (country-specific) volatilities. The high frequency correlation component incorporates the effects of time varying loadings and unobserved latent factors within this global context.

[1] Other studies that have used multifactor models to characterize international equity returns include Brooks and Del Negro (2005), Bekaert, Hodrick, and Zhang (2008), and Pukthuanthong and Roll (2009).

This paper is similar to the recent study of Bekaert, Hodrick, and Zhang (2008), but our framework specifies parametrically the dynamic equations that describe the short-term variation in the volatility of the factors, idiosyncratic (country-specific) volatilities, and the loadings. It does not rely on multiple estimations of rolling regressions and allows us to exploit the information from daily returns to characterize term correlation dynamics. Moreover, our framework can be easily implemented in forecasting applications (see Rangel and Engle (2008)). Another recent paper related to ours is Pukthuanthong and Roll (2009). They also use daily data and an APT factor structure to model broad market index returns for a comprehensive sample of countries; however, their focus is not on the dynamics of global correlations. Instead, they suggest an alternative measure for the degree of global integration based on the explanatory power of global factors through time. To discuss the appropriateness of correlations to measure global market integration is, however, beyond the aim of the present paper. Our goal is to examine and model term correlation patterns in a global context. In this regard, our study is the first one that incorporates jointly short- and long-term correlation behavior in international markets including a large number of countries with different degrees of economic development.

Indeed, the empirical analysis sheds more light on how the correlation structure changes in developed and developing markets. Moreover, it examines the scope of this evolution in the recent period of financial distress that started with the credit-crunch of August 2007 and has developed over the whole year of 2008. Correlations in this distress period are compared with those of the pre-crisis period. Consistent with the existing evidence on the asymmetric response of international correlations to highly volatile bear markets (e.g., Longin and Solnik (1995, 2001), and Ang and Bekaert (2002)), it is found that high and low frequency correlations have increased substantially during the current financial crisis, but the effect has been unevenly distributed across countries. Emerging markets have shown higher increases in their correlation with the rest of the world due to the dominance of the elevated systematic risk over the increasing idiosyncratic volatility observed in this period. This result is relevant to evaluate changes in the scope of diversification benefits that recently have relied heavily on investments in the emerging world, as suggested by Goetzmann, Li, and Rouwenhorst (2005).

Non-synchronous trading around the world has limited the use of high frequency data in correlation analyses. In this paper, we also incorporate a synchronization step (based on the framework of Burns, Engle, and Mezrich (1998)) that allows us to exploit the information from daily data to estimate the two term correlation components. Results suggest that the suggested synchronization schemes reduce substantially the non-synchronous bias. Indeed, the average fitted correlations based on daily data are close to the weekly measures, but they retain important dynamic features (such as correlation persistence and asymmetric impacts) that are usually weakened under time aggregation.

The low frequency correlations introduced in this paper are relatively easy to estimate despite the dimensionality of the system. The smooth nature of such components simplifies the comparison between the evolution of correlations and slow-moving fundamental economic variables. For instance, the low frequency component can be associated with macroeconomic variables, which is convenient for policy analysis. Indeed, it has been found in the literature that market volatility is a big component of correlations in the U.S. market (see Engle (2007)) and country-specific long-term volatility responds to changes in local macroeconomic conditions (see Engle and Rangel (2008)). Overall, it provides a new framework for practitioners and policy makers to assess long run comovements, which is useful in international diversification and global market regulation.

In this paper, we maintain a balance between data availability and the number of countries analyzed. Low frequency correlations are estimated for 43 countries, including developed economies and emerging markets, using daily data during the period January 1995 to December

2008. The starting year was selected to include China in our analysis, since this emerging market has become largely influential in recent years and it started disclosing regular daily stock market data in 1995.

The paper is organized as follows: Section two describes the specification of the multifactor spline-GARCH model for high and low frequency correlations (FSG-DCC model hereafter) and discusses its main properties. Section three describes the data and the problem of non-synchronous trading activity in international markets. Section four presents estimation results and discusses empirical patterns of global correlations, and Section five concludes.

II. The model

The model specification follows the multivariate version of the FSG-DCC model of Rangel and Engle (2008). In this setup, equity returns are described by the arbitrage pricing theory (APT) of Ross (1976), where *K* pervasive factors characterize systematic movements of equity returns. Their residual variation is asset-specific (idiosyncratic). Hence, the return process of asset *i* is described as follows[2]:

$$r_{i,t} = \beta_i ' F_t + u_{i,t}, \tag{1.1}$$

where $F_t = (f_{1,t}, f_{2,t}, ..., f_{k,t})'$ is a vector of pervasive factors, $\beta_i = (\beta_{i,1}, \beta_{i,2}, ..., \beta_{i,k})'$ is a vector of factor loadings, and $u_{i,t}$ denotes the idiosyncratic return of asset *i*. Under a perfect unconditional factor structure, the following assumptions are satisfied:

[2] To simplify notation, returns are considered as unexpected returns. For instance, if $R_{i,t}$ denotes the observed return, then the unexpected return is defined as $r_{i,t} = R_{i,t} - E_{t-1}(R_{i,t})$.

$$\operatorname{cov}(u_{i,t},u_{j,t}) = \begin{cases} 0, & if\ i \neq j \\ \sigma^2_{u,i}, & if\ i = j \end{cases}, \quad E(u_{i,t}) = 0 \ \ \forall i \tag{1.2}$$

$$\operatorname{cov}(u_{i,t}, f_{k,t}) = 0, \quad for\ each\ i,k \tag{1.3}$$

Thus, risk is split into systematic and idiosyncratic risk. It is standard choosing zero-mean orthogonal factors, without loss of generality.[3] Therefore, we can add the assumption:

$$\operatorname{cov}(f_{i,t}, f_{j,t}) = \begin{cases} 0, & if\ i \neq j \\ \sigma^2_{f_i}, & if\ i = j \end{cases}, \quad E(f_{i,t}) = 0 \ \ \forall i \tag{1.4}$$

Hence, the standard factor model imposes the following structure in the covariance matrix of returns:

$$\operatorname{cov}(r_{i,t},r_{j,t}) = \beta_i{}'\Sigma_F\beta_j{}' + \begin{cases} 0, & if\ i \neq j \\ \sigma^2_{u,i}, & if\ i = j \end{cases}, \tag{1.5}$$

where Σ_F denotes the unconditional variance of the vector of factors, which elements are described in (1.4). From this structure, we can write the unconditional covariance matrix of a vector of returns $\mathbf{r}_t = (r_{1,t},\ldots,r_{N,t})$ as:

$$\operatorname{cov}(\mathbf{r}_t,\mathbf{r}_t{}') = B\Sigma_F B' + \Sigma_u, \tag{1.6}$$

[3] Orthogonalizing the factors only facilitates the unconditional interpretation of the factor loadings, but it does not have other effects in the model.

where $B=(\beta_1,\beta_2,...,\beta_N)'$, and Σ_u denotes the unconditional variance of the idiosyncratic terms, which elements are defined in (1.2). The previous expression summarizes the unconditional restrictions imposed by the factor structure; however, maintaining such restrictions conditionally imposes constant factor loadings and rules out the impacts of latent factors that may suddenly appear in some periods of time. As argued in Engle (2009) and Rangel and Engle (2008), allowing for temporal deviations from conditions (1.2)-(1.4) permit us to incorporate the effects of time variation in the factor loadings as well as the effects of latent unobserved factors on the dynamic behavior of equity correlations.

Following this intuition, we incorporate the mentioned effects by relaxing conditions (1.2)-(1.4) and modeling the dynamics of the conditional covariances across factors and idiosyncratic returns, including the dynamic covariation within the group of factors and within the group of idiosyncratic terms. This strategy and the factor structure in (1.1), lead to the following specification of the conditional covariance at time t (given the information set Φ_{t-1} that includes all the available information up to time t-1):

$$\mathrm{cov}_{t-1}(\mathbf{r}_t,\mathbf{r}_t')=B\Sigma_{F,t}B'+B\,\mathrm{cov}_{t-1}(F_t,\mathbf{u}_t')+\mathrm{cov}_{t-1}(\mathbf{u}_t,F_t')B'+\Sigma_{u,t}, \qquad (1.7)$$

where $\Sigma_{F,t}\equiv\mathrm{cov}_{t-1}(F_t,F_t')$ and $\Sigma_{u,t}\equiv\mathrm{cov}_{t-1}(\mathbf{u}_t,\mathbf{u}_t')$. These conditional covariance matrices do not need to be diagonal. Even if the factors are unconditionally uncorrelated, (and satisfy (1.4)), they can be conditionally correlated. This deviation from the unconditional restrictions is transitory as long as the conditional covariance mean reverts to the unconditional restriction in (1.4). Similarly, the idiosyncratic terms might satisfy (1.2), but they are allowed to be conditionally correlated and to mean revert toward their unconditional expectation. A case in which only the first and last terms of equation (1.7) appear in the covariance specification corresponds to a covariance model that captures the dynamic effect of latent unobserved factors. When we add the two terms in the middle of (1.7), which capture the dynamic covariation between factors and idiosyncrasies, we have a specification that can capture the impacts of both time variation in the factor loadings and latent unobserved factors.[4]

To specify the model, we need to parameterize all the terms in equation(1.7). We follow the FSG-DCC model of Rangel and Engle (2008) that combines the Spline-GARCH framework of Engle and Rangel (2008) and the factor structure given in (1.1) to separate a low frequency correlation component from the high frequency correlation terms. Specifically, under conditional normality, the factor model in (1.1) can be written in its conditional form as:

$$\mathbf{r_t} \mid F_t, \Phi_{t-1} \sim N(BF_t, H_{u,t}), \qquad F_t \mid \Phi_{t-1} \sim N(0, H_{F,t}), \tag{1.8}$$

where

$$\begin{aligned} &H_{u,t} = \Sigma_{u,t} R_{r,r,t} \Sigma_{u,t} \quad and \quad H_{F,t} = \Sigma_{F,t} R_{f,f,t} \Sigma_{F,t}\,, \\ &R_{r,r,t} \quad and \quad R_{f,f,t} \quad are \quad correlation\ matrices\,, \\ &\Sigma_{u,t} \quad and \quad \Sigma_{F,t} \quad \sim \quad Diagonal\ Spline\text{-}GARCH \end{aligned} \tag{1.9}$$

Under this setup, $\Sigma_{u,t} = \Gamma_{u,t} D_t^2$ and $\Sigma_{F,t} = \Gamma_{F,t} G_t^2$, where $\Gamma_{u,t} = diag\{\tau_{i,t}\}$, $D_t = diag\{g_{i,t}^{1/2}\}$, for i=1,2,…,N, $\Gamma_{F,t} = diag\{\tau_{f,j,t}\}$, and $G_t = diag\{g_{f,j,t}^{1/2}\}$, for j=1,2,…,K. Following Engle and

[4] This result follows from applying Proposition 1 of Rangel and Engle (2008) to the multifactor case. If $\mathrm{cov}_{t-1}(\mathrm{r_t}, \mathrm{r_t}') = B\Sigma_{F,t}B' + \Sigma_{u,t}$, the specification captures the temporal effect of latent factors. If this covariance includes all the terms in (1.7), the model incorporates both: time variation in the factor loadings and latent unobserved factors. As explained in this proposition, the time variation in the loadings is temporal from a constant level, which is given by the matrix *B* in the multifactor case.

Rangel (2008), the $\tau_{i,t}$'s are specified as exponential quadratic splines and the $g_{i,t}$'s are unit asymmetric GARCH processes. Element by element, we have:

$$
\begin{aligned}
u_{i,t} &= r_{i,t} - \beta_i ' F_t = \sqrt{\tau_{i,t} g_{i,t}} \varepsilon_{i,t}, \;\; i = 1,...,N \\
f_{j,t} &= \sqrt{\tau_{f,j,t} g_{f,j,t}} \varepsilon_{f,j,t}, \;\; j = 1,...,K,
\end{aligned}
\tag{1.10}
$$

where the high and low frequency variance components of the idiosyncratic terms are defined as:

$$
\begin{aligned}
g_{i,t} &= \left(1 - \theta_i - \phi_i - \frac{\gamma_i}{2}\right) + \theta_i \frac{u_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{u_{i,t-1}^2 I_{r_{i,t-1}<0}}{\tau_{i,t-1}} + \phi_i g_{i,t-1} \\
\tau_{i,t} &= c_i \exp\left(w_{i0} t + \sum_{s=1}^{k_i} w_{is} \left((t - t_{s-1})_+\right)^2 \right), \; for \;\; i = 1,...,N,
\end{aligned}
\tag{1.11}
$$

and the variance components of the factors are:

$$
\begin{aligned}
g_{f,j,t} &= \left(1 - \theta_{f,j} - \phi_{f,j} - \frac{\gamma_{f,j}}{2}\right) + \theta_{f,j} \frac{{f_{j,t-1}}^2}{\tau_{f,j,t-1}} + \gamma_{f,j} \frac{{f_{j,t-1}}^2 I_{f_{j,t-1}<0}}{\tau_{f,j,t-1}} + \phi_{f,j} g_{f,j,t-1} \\
\tau_{f,j,t} &= c_{f,j} \exp\left(w_{f,j,0} t + \sum_{s=1}^{k_{f,j}} w_{f,j,s} \left((t - t_{s-1})_+\right)^2 \right), \;\; for \; j = 1,...,K
\end{aligned}
\tag{1.12}
$$

The model is completed by adding dynamics to the covariation across factor and idiosyncratic innovations. We characterize such dynamics using the dynamic conditional correlation (DCC) framework of Engle (2002). Specifically, the vector $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, ..., \varepsilon_{N,t}, \varepsilon_{f,1,t}, ..., \varepsilon_{f,K,t})' \sim DCC$, and its correlation structure can be expressed as a partitioned correlation matrix:

$$
V_{t-1}(\varepsilon_t) = \begin{pmatrix} R_{r,r,t} & R_{r,f,t} \\ R_{f,r,t} & R_{f,f,t} \end{pmatrix},
\tag{1.13}
$$

where $R_{r,r,t}$ describes the correlations across idiosyncratic innovations, $R_{r,f,t}$ characterizes the covariances across idiosyncratic and factor innovations, and $R_{f,f,t}$ describes the correlation across factor innovations. Hence, the whole model parameterizes the conditional covariance matrix of returns in equation (1.7) as:

$$\text{cov}_{t-1}(\mathbf{r}_t, \mathbf{r}_t') = B\Gamma_{f,t}^{1/2} G_t R_{f,f,t} G_t \Gamma_{f,t}^{1/2} B' + BR_{f,r,t} D_t \Gamma_t^{1/2} + \Gamma_t^{1/2} D_t R_{r,f,t} B' + \Gamma_t^{1/2} D_t R_{r,r,t} D_t \Gamma_t^{1/2} \quad (1.14)$$

Moreover, the following expression defines the low frequency covariance:

$$\Upsilon_{t-1} = B\Gamma_{f,t} B' + \Gamma_t^{1/2} \bar{R}_{r,r} \Gamma_t^{1/2}, \quad (1.15)$$

where $\bar{R}_{r,r}$ is the unconditional correlation of idiosyncratic innovations. The assumption that factors and idiosyncrasies are unconditionally uncorrelated is maintained by imposing the restriction $\bar{R}_{r,f} = 0$. For this reason, the two terms in the middle of (1.14) vanish in the low frequency equation. The first and last terms are derived from the assumption that the factors are unconditionally uncorrelated ($\bar{R}_{f,f} = I_{k\times k}$) and the property that the unit-GARCH variances mean revert to one ($E(G_t) = I_{K\times K}$ and $E(D_t) = I_{N\times N}$). Rangel and Engle (2008) proved that the high frequency correlation component mean reverts toward the time-varying low frequency term in the one-factor case. The generalization of this result to the multifactor case is straightforward from the properties mentioned above. Hence, the high frequency correlation, defined as:

$$corr_{t-1}(\mathbf{r}_t, \mathbf{r}_t') = diag\{\text{cov}(\mathbf{r}_t, \mathbf{r}_t')\}^{-1/2} \text{cov}_{t-1}(\mathbf{r}_t, \mathbf{r}_t') diag\{\text{cov}(\mathbf{r}_t, \mathbf{r}_t')\}^{-1/2}, \quad (1.16)$$

mean reverts toward the following smooth time-varying function that is better suited to describe long-term correlation behavior:

$$LFR_t = diag\{B\Gamma_{f,t}B' + \Gamma_t^{1/2}\overline{R}_{r,r}\Gamma_t^{1/2}\}^{1/2}(B\Gamma_{f,t}B' + \Gamma_t^{1/2}\overline{R}_{r,r}\Gamma_t^{1/2})diag\{B\Gamma_{f,t}B' + \Gamma_t^{1/2}\overline{R}_{r,r}\Gamma_t^{1/2}\}^{1/2} \quad (1.17)$$

Equations (1.14)-(1.17) summarize the high and low frequency dynamics of the correlation structure of equity returns. We apply this model to describe these two types of dynamics in international markets. However, the application needs to consider the typical issue of asynchronous data in international exchanges. The following section presents two strategies to apply the multifactor Factor-Spline-GARCH framework to non-synchronized data.

III. Data and the Non-synchronous Trading Problem

Non-synchronous trading around the world is an empirical issue that arises when we use international data at high frequencies. The common practice suggests employing weekly data as the highest frequency to avoid the synchronization problem. However, given that daily data (or even higher frequency data) is available for many countries, it would be desirable to exploit all the available information when measuring, modeling, and forecasting global correlations.

a) Synchronization Methods

A few papers have addressed directly the issue of non-synchronicity. The biases that arise when closing prices are stale (as well as bias correction approaches) have been examined in Scholes and Williams (1977) and Lo and MacKinlay (1990). Burns, Engle, and Mezrich (1998) introduce a statistical model to "synchronize" returns data associated with asset prices that are not measured at the same time. Specifically, this last study suggests synchronizing the data first and then fitting the correlation model to the synchronized data.

The problem of non-synchronicity in the international context can be illustrated with an example of three stock markets whose opening and closing times are different, but they are fixed and the time difference is known (e.g., the US, Europe, and Asia). Figure 1 illustrates this problem. If we want to synchronize returns with respect to the latest market, we can subtract the unsynchronized part and add the missing synchronized component to the returns of the earlier markets. For instance, focusing on this example, the synchronized return of Europe (in terms of the US) would be:

$$s_{Et} = r_{Et} - \xi_{t-1} + \xi_t, \tag{1.18}$$

where r_{Et} is the observed return at day *t* and ξ_t denotes the return we would have observed from the closing time of market 2 (Europe on day t) to the closing time of market 3, the U.S., on the same day. The problem is that ξ_t is not observable. Burns, Engle, and Mezrich (1998) estimate this unobserved component using the linear projection of the observed unsynchronized return on all the available information up to the time of synchronization. Thus, from (1.18) the estimated synchronized return is:

$$\hat{s}_{2t} = r_{2t} - \hat{\xi}_{t-1} + \hat{\xi}_t, \ \ where\ \hat{\xi}_t = E_t(r_{2,t+1} \,|\, \{r_{1t}, r_{2t}, r_{3t}\}, \Phi_t). \tag{1.19}$$

In the case of N unsynchronized assets, these equations can be represented as a system where the vector of unsynchronized returns, $S_t = (s_{1,t}, ..., s_{N,t})'$, follows a first order vector moving average (VMA(1)) with time-varying covariance matrix:

$$S_t = v_t + Mv_{t-1}, \; V_{t-1}(v_t) = H_{v,t}, \tag{1.20}$$

and the synchronized return along with their variance are estimated as:

$$\widehat{S}_t = (I + \widehat{M})v_t, \quad V_{t-1}(\widehat{S}_t) = (I + \widehat{M})\widehat{H}_{v,t}(I + \widehat{M}), \tag{1.21}$$

where I is the NxN identity matrix and $\widehat{M}$ is the estimated coefficient of the VMA(1) model.

The factor structure presented in Section II can be combined with this synchronizing approach to estimate high and low frequency correlations from daily data. However, the estimation will require an additional synchronization step that might introduce substantial estimation errors. In this paper, we examine two strategies to implement a synchronization step in the estimation of correlations. The first strategy, labeled "Synchronization 1", is the simplest. It applies the framework of Burns, Engle, and Mezrich (1998) to the observed returns before estimating the factor model. Specifically, we synchronize first returns and factors using (1.20) and (1.21). Then, we estimate the FSG-DCC model of Section II using the two-step GMM approach described in Rangel and Engle (2008).

The second strategy, labeled "Synchronization 2", constructs consistent estimates of the factor loadings and then applies the synchronization method as an intermediate step in the FSG-DCC estimation. Specifically, the unsynchronized observed returns and factors can be modeled as a VMA (1) of the form:

$$\mathbf{y}_t = \begin{pmatrix} F_t \\ \mathbf{r}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\varepsilon}_{F,t} \\ \boldsymbol{\varepsilon}_{r,t} \end{pmatrix} + \begin{pmatrix} A_{FF} & A_{Fr} \\ A_{rF} & A_{rr} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_{F,t-1} \\ \boldsymbol{\varepsilon}_{r,t-1} \end{pmatrix}, \qquad (1.22)$$

where A_{FF}, A_{Fr}, A_{rF}, *and* A_{rr} are matrices of coefficients of dimensions (KxK), (KxN), (NxK), and (NxN), respectively. Then, as illustrated in (1.21), the synchronized vector of factors and returns is:

$$\mathbf{s}_t = \begin{pmatrix} \tilde{F}_t \\ \tilde{\mathbf{r}}_t \end{pmatrix} = \left\{ I + \begin{pmatrix} A_{FF} & A_{Fr} \\ A_{rF} & A_{rr} \end{pmatrix} \right\} \begin{pmatrix} \boldsymbol{\varepsilon}_{F,t} \\ \boldsymbol{\varepsilon}_{r,t} \end{pmatrix} \qquad (1.23)$$

Now, assuming that synchronized returns follow the factor structure in (1.1), the factor loadings are defined by $\tilde{B} = \mathrm{cov}(\tilde{\mathbf{r}}_t, \tilde{F}_t{}')\Sigma_{\tilde{F}}^{-1}$. Element by element, $\tilde{\beta}_{i,k} = \dfrac{\mathrm{cov}(\tilde{f}_{k,t}, \tilde{r}_{i,t})}{\mathrm{var}(\tilde{f}_{k,t})}$. Moreover, in the same spirit as Scholes and Williams (1977), the following proposition characterizes the synchronized factor loadings in terms of the unsynchronized coefficients.

Proposition 1: *Consider the return process in Equation* (1.1) *and assume that the vector of factors and returns follows the VMA(1) process in Equation* (1.22). *Then the synchronized factor loadings associated with the synchronized system defined by* (1.23) *and the factor structure in* (1.1), *take the following form:*

$$\tilde{\beta}_{i,k} = (\beta_{i,k} + \beta_{i,k}^{+} + \beta_{i,k}^{-}) \frac{Var(f_{k,t})}{Var(\tilde{f}_{k,t})}, \;\; k = 1,\dots,K, \;\; i = 1,\dots,N \qquad (1.24)$$

where $\beta_{i,k} = \dfrac{\operatorname{cov}(f_{k,t}, r_{it})}{Var(f_{k,t})}$, $\beta^{+}_{i,k} = \dfrac{\operatorname{cov}(f_{k,t+1}, r_{it})}{Var(f_{k,t+1})}$, *and* $\beta^{-}_{i,k} = \dfrac{\operatorname{cov}(f_{k,t-1}, r_{it})}{Var(f_{k,t-1})}$; r_{it} *and* $\{f_{k,t}\}_{k=1}^{K}$ *denote unsynchronized returns and factors.*

The proof of this proposition is presented in the Appendix and its main implication for our analysis is that the "true" factor loadings can be consistently estimated from adding leads and lags of the unsynchronized factors into the system.[5] Hence, instead of estimating (1.1), the first estimation step of "Synchronization 2" consists in estimating factor loadings, idiosyncratic innovations, and their volatilities from the following unsynchronized system of unexpected returns:

$$\mathbf{r}_t = BF_t + B^{-}F_{t-1} + B^{+}F_{t+1} + \mathbf{u}_t \tag{1.25}$$

From this step, we obtain estimates of the synchronized matrix of loadings ($\tilde{B}_s = \widehat{B} + \widehat{B}^{-} + \widehat{B}^{+}$), estimates of the unsynchronized idiosyncratic returns ($\widehat{\mathbf{u}}_t$) and of their spline-GARCH volatilities. The second step requires synchronizing the vector $\left(F_t{}', \widehat{\mathbf{u}}_t{}'\right)$ using the approach of Burns, Engle, and Mezrich (1998). From this step, we obtain an estimator of the synchronization matrix, and a vector of synchronized factors and idiosyncratic returns, $\left(\tilde{F}_t{}', \tilde{\mathbf{u}}_t{}'\right)$. In the third step, we fit a DCC model for this synchronized vector. As in (1.7), the correlation structure of the synchronized system takes the form:

$$\operatorname{cov}_{t-1}(\tilde{\mathbf{r}}_t, \tilde{\mathbf{r}}_t{}') = \tilde{B}\Sigma_{\tilde{F},t}\tilde{B}' + \tilde{B}\operatorname{cov}_{t-1}(\tilde{F}_t, \tilde{\mathbf{u}}_t{}') + \operatorname{cov}_{t-1}(\tilde{\mathbf{u}}_t, \tilde{F}_t{}')\tilde{B}' + \Sigma_{\tilde{\mathbf{u}},t} \tag{1.26}$$

[5] It is assumed that $Var(f_{k,t}) = Var(\tilde{f}_{k,t})$. This is consistent with Scholes and Williams (1977) if the factors are serially uncorrelated.

Using the estimates obtained in the previous three steps, we have all the ingredients to derive synchronized forms of equations (1.14) and (1.17), and estimate another "synchronized" version of the FSG-DCC model.

An important difference between the two synchronization methods described above is that while the first approach carries the possible estimation errors from fitting the VMA model in (1.21) since the initial step, the second method introduces such noise terms in an intermediate step that does not affect the consistent estimation of the factor loadings. If the estimation error associated with the VMA model is small, then the two approaches should deliver very similar results. We assess these synchronization methods empirically by evaluating their ability to fit benchmark correlation measures that are free of biases from asynchronous data.

Regarding the practical implementation of the methods, we use a finite order vector autoregression (VAR) approximation to estimate the VMA coefficients, as suggested by Lutkepohl and Poskitt (1991) and Galbraith, Ullah, and Zinde-Walsh (2002).[6] This approach is convenient by its simplicity and works well for processes with roots far away from the unit circle, a property that is reasonable in our case.

b) Data

We study daily returns of equity markets in 43 countries including developed and emerging economies. All returns are denominated in US dollars. Table 1 shows the countries and their information regarding development classification, exchanges, and market indices. The

[6] We examined VAR specifications of various orders, ranging from 2 to 8. Beyond three lags, we did not find important differences. Thus, VAR models of order 3 were our choice to approximate the VMA(1) system in (1.22).

composition of the sample was based on data availability. The equity data was obtained from Datastream, Global Financial Data, and Bloomberg. The FX data was obtained from Datastream and Bloomberg. The sample period starts in February 1995 and ends in December 2008. The starting year was selected to include China in our analysis since this emerging market has become largely influential in recent years and it started disclosing regular daily stock market data in 1995. As a result, we have 3612 daily observations in the sample.

We also perform analyses using weekly data to form benchmark models that are in line with other approaches, which use lower frequency data to circumvent the non-synchronicity problem (e.g., Dumas et al. (2003) and Bekaert et al. (2008)). The weekly data includes 723 observations.

Regarding the factors, we use observed economic factors that incorporate global market returns in three main regions: America, Europe, and Asia. For America (factor 1), we use the S&P500. For Europe (factor 2) and Asia (factor 3), we use the MSCI Europe Index and the MSCI Pacific Index (excluding Japan), respectively. The three indices are measured in US dollars. They represent the market risk of regions that are not synchronized in terms of their trading activities. Hence, they follow the patterns described in Figure 1 and need to be included in the synchronizing processes described above.

IV. Empirical Results

Our empirical analysis explores high and low frequency patterns of correlations in international capital markets. We proceed by estimating three variations of the FSG-DCC model described in Section II. The first one is the FSG-DCC model based on weekly data; the second one is the FSG-DCC model based on daily data and “Synchronization 1”; and the last one is the FSG-DCC model based on daily data and “Synchronization 2”.[7] In addition, we compute a model free benchmark correlation measure. Following Bekaert, Hodrick, and Zhang (2008), we estimate the sample correlation matrix for every half year in the sample using weekly data.[8] If correlations have low frequency variation, it should appear in the sequence of 6-month sample correlations. Hence, we compare its temporal patterns with the low frequency component of the FSG-DCC correlations. We also use this benchmark to evaluate the synchronization methods using a mean absolute error (MAE) metric.

a) Evidence from weekly data

We first analyze global correlation behavior from weekly equity returns. Estimation results of the FSG-DCC model are shown in Table 2. The first three rows correspond to the factors. Their volatility show significant persistence that fluctuates between 0.7 and 0.78, the ARCH effects are not statistically significant, and the asymmetric leverage effects are strong and significant at the 5% level. Regarding the country estimates, the last three columns within the section of mean parameters show the estimated factor loadings (betas). With exception of China and Venezuela, at least one of the loadings is statistically significant for every country. Regarding the section of volatility parameters, the persistence coefficient is significant for all the countries and its average is 0.725; the ARCH effect is significant at the 10% level for 23 cases and its average is 0.083; and the asymmetric effect is significant at this level for 25 cases and its average is 0.058. The number of knots fluctuates between 1 and 5, and its average is 2.2. The bottom section of Table 2 presents the estimates of the DCC parameters associated with the second step of the FSG-DCC estimation. The two parameters are estimated using the Composite-Likelihood approach of Engle, Shephard and Sheppard (2008). They are both significant at the 5% level.

[7] “Synchronization 1” and “Synchronization 2” are described in Section III.

[8] Bekaert, Hodrick, and Zhang (2008) use this 6-month sample correlation measure to evaluate a number of factor specifications.

Figure 2 illustrates dynamic features of high and low frequency correlation components using the case of Germany and Japan as an example. The dashed line represents the high frequency component (HFC) and the smooth line describes the low frequency one (LFC). The dimensionality of the problem complicates a display of all the correlations in the system. Instead, we illustrate the main patters of correlations using cross-sectional averages. For instance, suppose that $\widehat{\rho}_{i,j,t}^{(m)}$ is the estimated equity correlation from model *m*, associated with markets *i* and *j*, at time *t*. Then, the average correlation of market *i* is defined as $\overline{\rho}_{i,t}^{(m)} = \sum_{j,j\neq i}^{N} \widehat{\rho}_{i,j,t}^{(m)}$, and the global average correlation at time *t* is $\overline{\rho}_{t}^{(m)} = \sum_{i=1}^{N} \overline{\rho}_{i,t}^{(m)}$.

Figure 3 presents global average correlations of the weekly FSG-DCC (HFC and LFC), and those of the 6-month sample correlations. It shows how the 6-month sample correlations lie around the smoother LFC process. The global HFC describes shorter term correlation behavior in global markets. Overall, this figure illustrates that, at different degrees of smoothness, the three series show evidence of time-varying patterns in global correlations that vary from 0.25 to 0.48, in the LFC case, from 0.17 to 0.69, in the 6-month sample case, and from 0.22 to 0.75, in the HFC case. Their main features suggest that equity correlations increased slowly during the period 1995-2001; then they showed a slight decline from 2001-2004; and finally, they showed a substantial increase during the last two years of the sample (2006-2008). This is consistent with the existing empirical evidence about asymmetric response of correlations to highly volatile bear markets (e.g., Ang and Bekaert (2002), Ang and Chen (2002) and Longin and Solnik (1995, 2001)). To explain this behavior in terms of the factor structure of our model, Figures 4 and 5 illustrate dynamic patterns of average idiosyncratic and systematic volatilities, respectively.[9] The factor structure suggests that increases in idiosyncratic volatilities are associated with declines in correlations (everything else equal), and increases in the volatilities of the factors lead to increases in correlations (everything else equal). Figures 4 and 5 suggest that idiosyncratic volatilities, measured from either the spline-GARCH model or the 6-month samples, have moved in line with the factor volatilities, which is also consistent with recent empirical studies that have found a strong comovement between these volatilities (e.g., Campbell et al. (2001) and Guo and Savickas (2006, 2008)). Based on these dynamic patterns and the factor model implications, we can argue that the two volatility effects have had opposite impacts on global correlations. For example, during the last two years of the sample, average idiosyncratic volatilities are increasing, but not as much as the factor volatilities. Overall, the rise in factor volatilities appears to dominate the idiosyncratic effect causing the observed increase in global equity correlations.

b) Evidence from daily data

An additional goal of this study is to explore the empirical performance of different strategies that allow the use of information at higher frequencies to estimate dynamic correlations in an international context. In this subsection, we use daily data and apply the synchronization strategies described in section III. Table 3 shows the estimation results of the FSG-DCC model based on daily data and "Synchronization 1". Regarding the factors, the GARCH effects are all significant and bigger than those in Table 2. This is consistent with previous empirical evidence that shows that volatility persistence tends to decrease with time aggregation in ARCH type models (e.g., Chou (1988)). The ARCH and asymmetric effects tend to be smaller, which is also

[9] The average of idiosyncratic volatilities is defined as: $\bar{\sigma}_{u,t} = \sum_{i=1}^{N} \sigma_{u,i,t}$, where $\sigma_{u,i,t}$ is the conditional volatility of the idiosyncratic innovation in (1.1).

consistent with this empirical evidence on ARCH effects and time aggregation. The asymmetric effects are all significant and dominate the ARCH terms. Regarding the country mean parameters, at least one of the estimates of the factor loadings is statistically significant for every country. In terms of the idiosyncratic variance, the persistence parameter is highly significant for all the cases and its mean is 0.80, which is substantially bigger than that of the weekly case. The asymmetric volatility coefficients are all positive and significant, except those of Italy, Poland, and Venezuela, and the corresponding average is 0.07. The ARCH terms are significant for almost all cases (exceptions are Ireland and Spain), and their average is 0.056. These averages are slightly smaller that those based on weekly data. The average number of knots associated with the spline functions is 4.82, more than twice the average value from the weekly estimation. The bottom part of Table 3 presents the DCC estimates that drive the dynamic behavior of correlations across idiosyncratic and factor innovations. Both DCC coefficients are statistically significant. The persistent effect is bigger than that of the weekly case and the updating effect is slightly smaller.

Table 4 shows the estimation results for the FSG-DCC model based on daily data and "Synchronization 2". The results associated with the factors are very similar to those in Table 3. Although this is also the case for the country estimates, there are noticeable differences in terms of their statistical significance. In this case, among the mean coefficients, at least two factor loadings are significant for every case.[10] The GARCH and ARCH effects are always highly significant. The asymmetric effect is statistically insignificant only for four cases. Regarding the average values of these effects, the major difference with respect to Table 3 is that the average ARCH effect is slightly bigger and the average number of knots is slightly smaller. In terms of the second-stage DCC parameters, the persistence coefficient tends to be bigger and the updating coefficient smaller.

[10] We mark a synchronized loading as statistically significant if at least one of the unsynchronized coefficients (associated with the contemporaneous, the leaded, and lagged factors) is statistically significant (see Equation (1.24)).

Besides the differences in terms of magnitude and statistical significance, we evaluate the empirical fit of these two synchronized FSG-DCC specifications following the MAE approach of Bekaert, Hodrick, and Zhang (2008). Specifically, we average the fitted daily values of (low frequency) correlations over five-day periods and form weekly correlations. Then, we compare these estimated weekly low frequency correlations with those of a benchmark correlation model based on weekly data using the following MAE metric:

$$MAE_{CORR} = \frac{1}{\tilde{T}} \sum_{\tilde{t}=1}^{\tilde{T}} \left(\sum_{i=1}^{N} \sum_{j>i}^{N} | \widehat{\rho}_{i,j,\tilde{t}}^{(benchmark)} - \widehat{\rho}_{i,j,\tilde{t}}^{(FSG-DCC)} | \right), \tag{1.27}$$

where, $\tilde{t}$ refers to the new aggregated time unit in which the benchmark is varying and $\tilde{T}$ denotes the total number of periods.[11] If a synchronized factor model has the appropriate structure, it should capture as much as possible of the low frequency variation in the global correlations presented in Figure 3. We use the two weekly models shown in this figure to compare the synchronized FSG-DCC model. Moreover, in order to make clearer the benefits of using a synchronization scheme, we add a FSG-DCC specification that ignores the non-synchronous data issue. We label this model as the non-synchronized FSG-DCC. Table 5 presents the MAE statistics. With regard of the 6-month sample benchmark, the two synchronized models show a similar performance, they are close to the weekly FSG-DCC, which is not affected by the non-synchronous bias. In contrast, the MAE associated with the non-synchronized FSG-DCC is far from these values (it is almost 12% bigger). The bottom panel shows the results using the weekly FSG-DCC benchmark. In this case, the daily FSG-DCC based on "Synchronization 2" dominates. Moreover, its MAE is 45% smaller than that of the non-synchronized model.

[11] Equation (1.27) is the equally weighted version of the $ABSE_{CORR}$ statistic used by Bekaert, Hodrick, and Zhang (2008).

These in-sample fit statistics appear to moderately favor the daily FSG-DCC model based on "Synchronization 2".[12] Figures 6 and 7 present an example of the correlation and volatility components of this model for a group of four countries in different time regions that include one emerging market (Brazil) and three developed markets (France, Japan, and the UK). Figure 6 shows the corresponding high and low frequency correlations along with rolling correlations (based on a 100-day window). Figure 7 illustrates high and low frequency patterns in the idiosyncratic volatilities of these countries. There is a clear upward trend in the low frequency correlations within the group of developed economies. In contrast, the correlation trend with respect to Brazil shows a declining pattern during 2008. These effects can be explained by looking at the behavior of idiosyncratic volatilities in Figure 7. While for the developed countries low frequency components of idiosyncratic volatilities show non-increasing behavior, Brazil shows a remarkable upward trend in its long-term idiosyncratic volatility during the last two years.

The aggregated behavior of the synchronized daily model is further illustrated in Figure 8, which summarizes the estimation results of Table 4. They include the factor loadings of each country, and time aggregates of low frequency correlations and idiosyncratic volatilities. The figure also shows aggregates of rolling correlations (based on weekly data and a window of one year) to illustrate again the good fit of the daily model. The most correlated countries are developed economies and the less correlated group is formed by emerging markets. Consistent with a factor structure, it is also clear that countries with higher levels of idiosyncratic volatilities tend to have lower correlations with the rest of the world. This is by construction when we look at the FSG-

[12] However, additional out-of sample forecasting tests might be important to further address the issue of model selection. We leave these exercises to future research and, focusing on the in-sample results, we illustrate the performance of the synchronized models using the specification based on "Synchronization 2".

DCC low frequency correlation aggregates. However, the model-free average rolling correlations lie very closely to the model based correlations. This indicates that a factor structure provides a good framework to explain global correlation behavior.

c) Market Distress and Global Correlations

The financial markets distress we have observed since August 2007 has produced important increases in systematic volatilities as well as in global correlations (see figures 3-5). Idiosyncratic volatility has also increased ─offsetting part of the systematic impacts ─, but its levels are still below those observed in 1998 (see Figure 4). As mentioned earlier, most of the recent increase in correlations can be attributed to the dominance of the systematic component. Figure 9 illustrates this rise in correlations around two points that can be seen as inflection points in the current financial crisis. One corresponds to the beginning of the credit-crunch on August 6, 2007 and the other to the bankruptcy of the investment bank Lehman Brothers on September 15, 2008. The figure shows that both average high and low frequency correlations (from the synchronized FSG-DCC model) have increased around 20% from the beginning of the credit-crunch to a week before the bankruptcy of Lehman. However, while the average low frequency correlation showed a rise of only 5% from the bankruptcy of Lehman to December 2008, the high frequency component increased 29% during this period. This highlights an interesting feature of the model related to the smoothness of the low frequency component that is less responsive to some shocks that may be of temporary nature, or that are close to the boundary and not so informative about longer term effects.

A natural question is whether such increases in global correlations are equally distributed across countries. This can be explored by looking at the changes in the correlation of each country with the rest of the world, from the pre credit-crunch period to December 2008.[13] Figure 10 shows such distribution of changes in average low frequency correlations. Even though the average correlation increased for all countries (except Canada) the distribution is far from even. About 13 out of 43 countries experienced a rise in correlations above 30%. With the exception of New Zealand, Japan, and Austria, all of them are emerging markets. In contrast, a group of 17 countries experienced moderate increases in their correlations with the rest of the world in percentages below 20%. Among these countries, only three of them are emerging markets (Argentina, Brazil, and Russia). Regarding the changes on average high frequency correlations, Figure 11 presents their distribution. In this case, nineteen countries show average correlation increases above 50%. As before, most of them are emerging markets (15 out of 19). It is remarkable the case of Venezuela that experienced an increase of almost 200% in its average high frequency correlation with the world during the fall of 2008.

To further understand the results in Figure 10, it is useful to look at the changes in low frequency idiosyncratic volatilities during the recent crisis period (see Figure 12). Among the group of 13 countries that experienced increases in their average low frequency correlation with the rest of the world, nine of them had only moderate increases (below 30%) in their long-term idiosyncratic volatilities. Regarding the group of 17 countries with moderate correlation changes, ten of them experienced high increases (above 60%) in their long-term idiosyncratic volatilities. These results suggest that systematic effects have dominated the low frequency reactions in most of the emerging world during the current financial crisis. Of course, the results in Figure 11 involve more complex interactions between time varying systematic and idiosyncratic volatilities, time varying betas, latent unobserved factors, and dynamic correlations across factors.

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[13] The pre credit-crunch correlations are obtained from correlation estimates of the FSG-DCC model based daily data, “Synchronization 2”, and a sample period from February 1995 to August 3 2007. We focus on the last year of this sample and compute the average correlation of each country with the rest of the world.

V Concluding Remarks

This study models high and low frequency variation in global equity correlations using a comprehensive sample of 43 countries, including developed and emerging markets, during the period 1995-2008. The modeling approach modifies and extends the Factor-Spline-GARCH (FSG-DCC) model of Rangel and Engle (2008) by allowing for dynamic interaction of multiple factors, and by explicitly introducing a synchronization step in the estimation process to correct for biases from non-synchronous trading activity in international markets.

This multifactor version has the same properties as the single-factor model of Rangel and Engle (2008). Specifically, the high frequency correlation component exploits the factor pricing structure and the dynamic interactions among factors and idiosyncratic terms to incorporate the effect of time varying betas and latent unobserved factors on the short-term correlation behavior. The model handles conditional correlation across the factors that may be non-trivial, even for factors that are unconditionally uncorrelated. The low frequency component also exploits the functional form imposed by both the factor structure and the long-term behavior of systematic and idiosyncratic volatilities. These features characterize the long-term correlation behavior and determine the level to which high frequency correlations mean revert.

The international context in which this model is implemented brings to the discussion the issue of non-synchronous trading activity in international markets. The paper explores different alternatives to address this problem, including the standard approach of using lower frequency data, and two other strategies that explicitly synchronize the components of the factor model. These strategies adapt the framework of Burns, Engle, and Mezrich (1998) in order to be applied in the factor setup and to be implemented within the estimation steps of the FSG-DCC model.

Regarding the empirical fit, the results show that the weekly FSG-DCC describes well the empirical dynamic features of global correlations that are obtained from a model-free benchmark. In addition, the daily synchronized FSG-DCC models are evaluated with respect to their ability to fit low frequency correlation behavior, which is proxied by the weekly unbiased correlation measures. Results suggest that the two synchronization schemes reduce substantially the non-synchronous bias. Moreover, while their fitted values are close to the weekly measures, they retain important dynamic features (such as persistence and asymmetric impacts) that are usually weakened under time aggregation.

Among the empirical results, we find substantial variation in the short and long-term components of global correlations during the sample period. Their average level roughly doubled during these 23 years. Moreover, about 50% of such increases in magnitude occurred during the period 2006-2008, which incorporates part of the recent financial turmoil. Indeed, consistent with the existing evidence on the asymmetric response of international correlations to highly volatile bear markets, the two aggregated global correlation components showed a remarkable rise during these last two years as a result of a dominant effect of systematic volatility; however, the long-term correlation component showed a more moderated response. Interestingly, the changes in global correlations during this period have not been evenly distributed across countries. Some countries, mainly emerging markets, have experienced higher increases in their comovements with the rest of the world. This is partially explained by the behavior of their idiosyncratic volatility in relation with the volatilities of the systematic factors. These countries showed relatively low increases in their idiosyncratic volatilities that nonetheless were not able to offset the rise in the systematic global volatility. Overall, these results have relevant implications for assessing changes in the benefits from international diversification that have largely relied on emerging markets investments.

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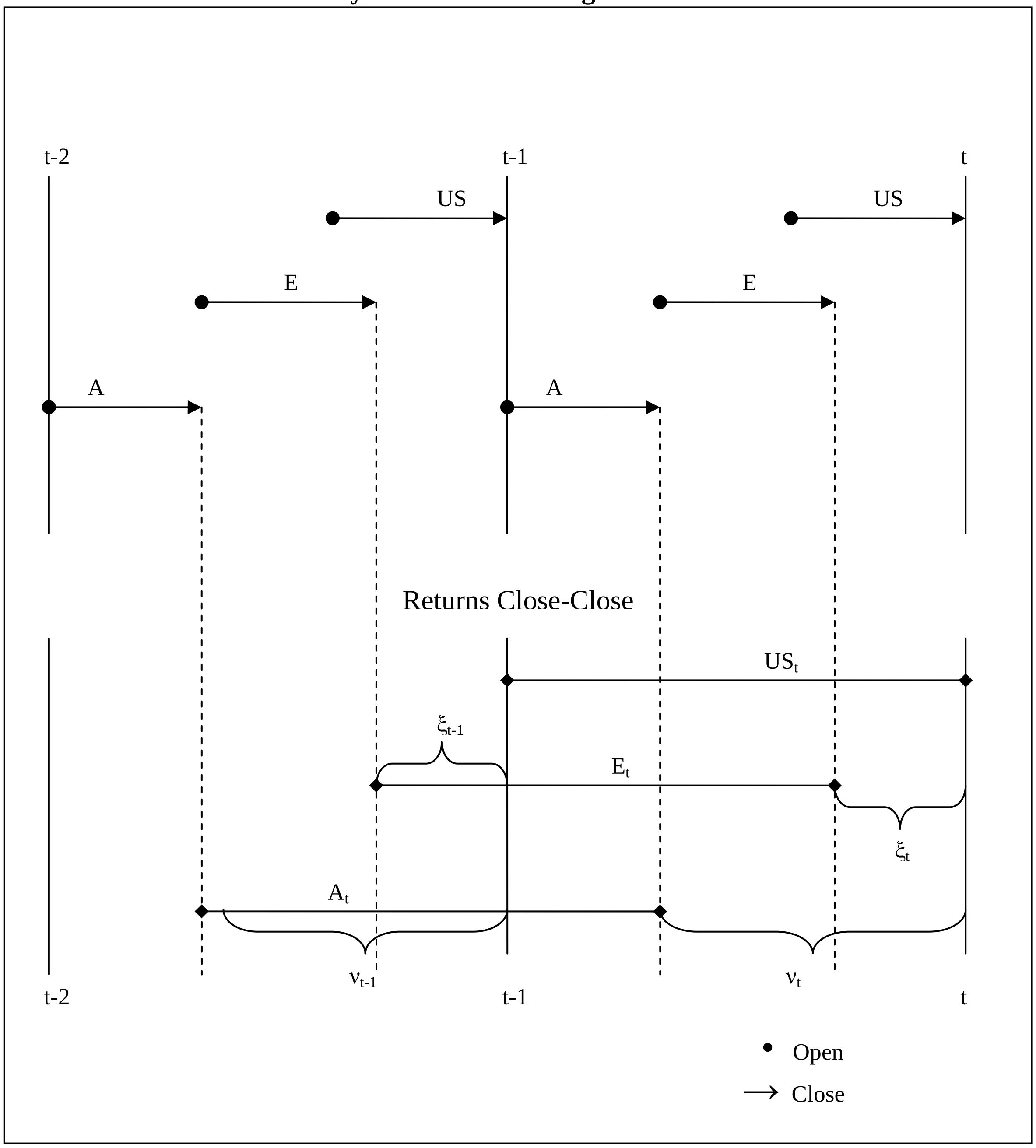
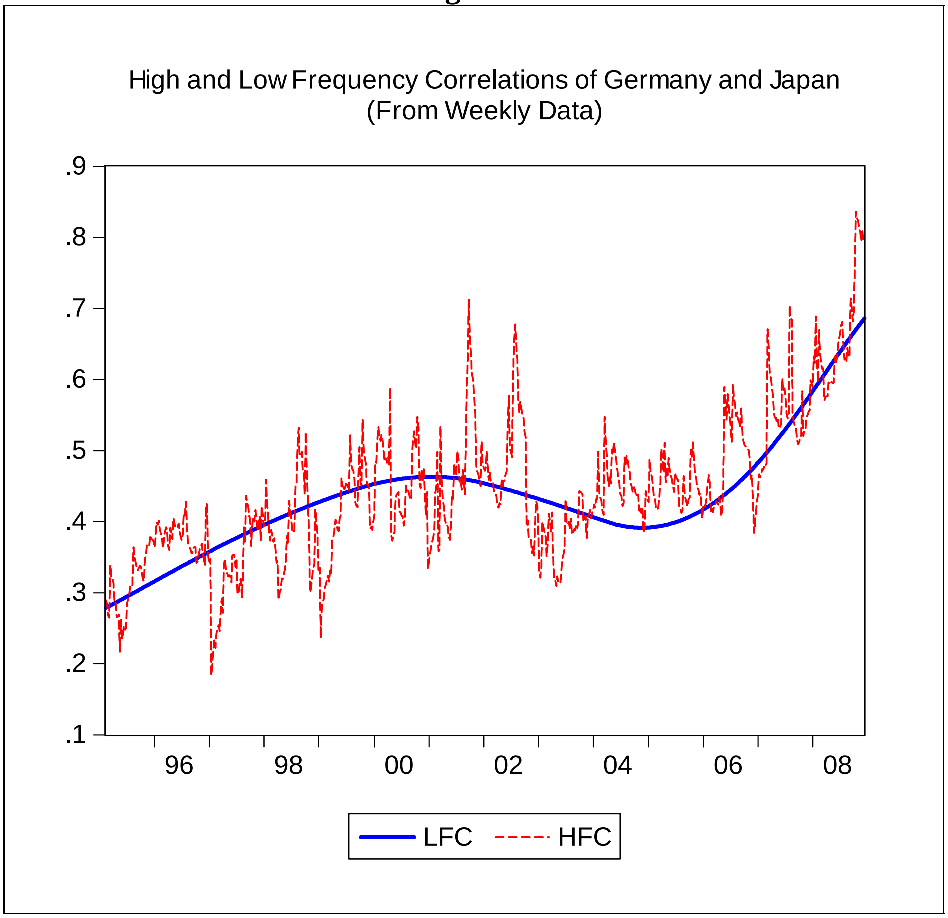
Figures and Tables

Figure 1
Asynchronous Trading Periods



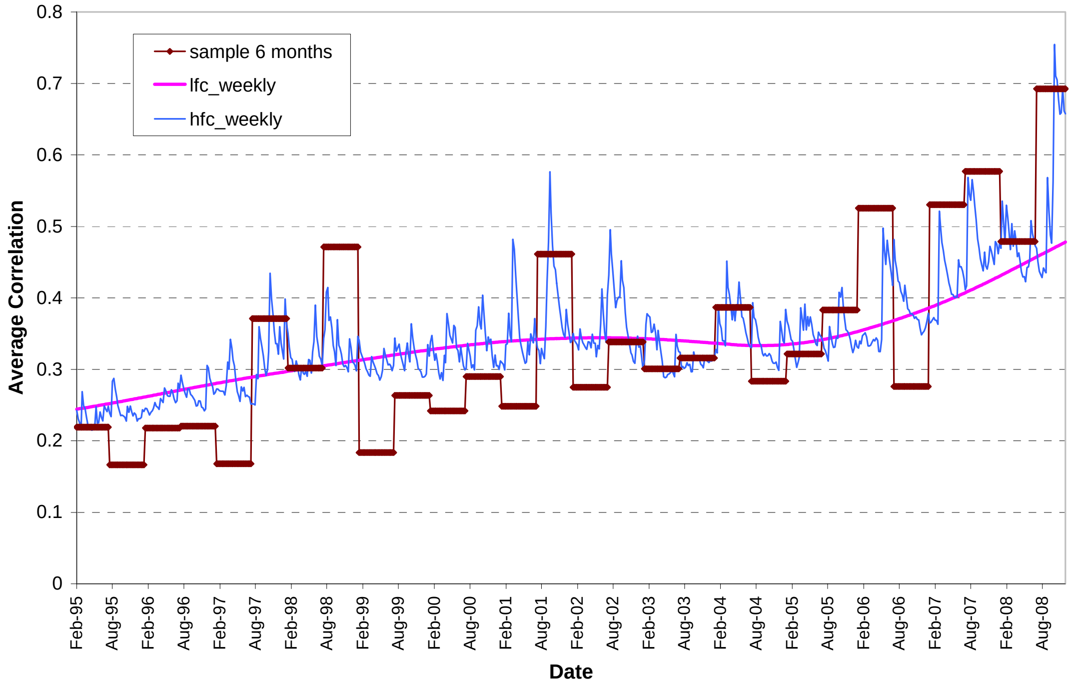
Notes: This figure illustrates the problem of non-synchronous trading in three markets, denoted by US, E, and A. The top panel shows the non-synchronous opening and closing times of each market. The bottom panel describes the unsynchronized observed returns and the unobserved missing fractions associated with the non-matching trading times.

Figure 2



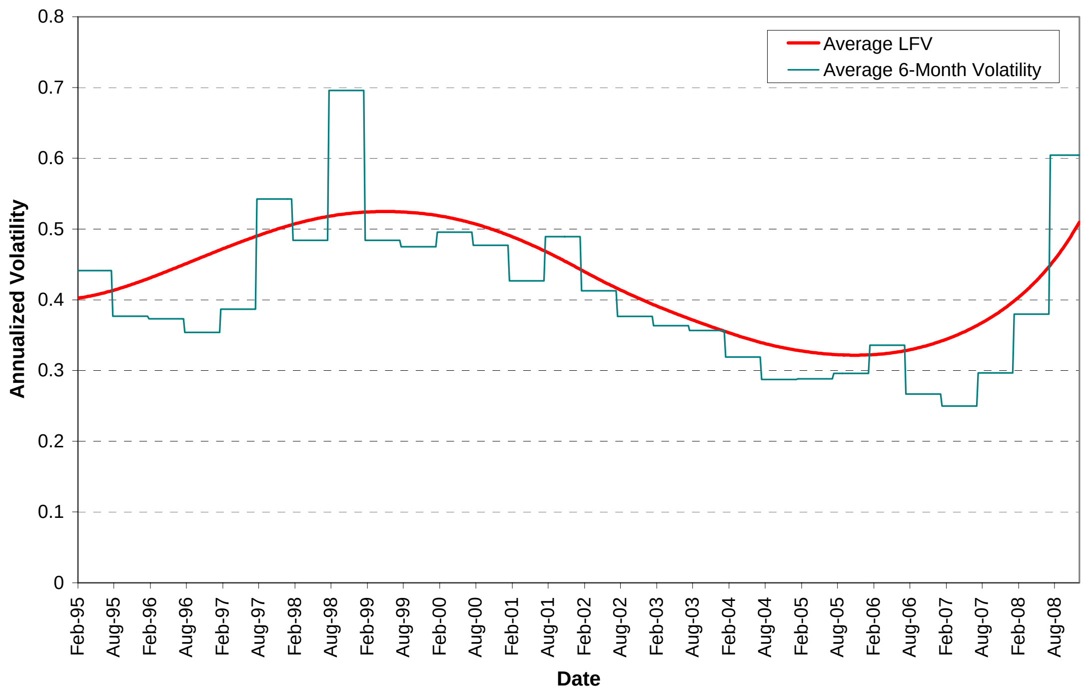
Notes: This figure presents the fitted FSG-DCC correlations between Germany and Japan. The estimation is based on weekly data. HFV stands for "High frequency correlation" and LFV refers to "Low frequency correlation".

Figure 3
Global Average Correlations over Time (From Weekly Data)



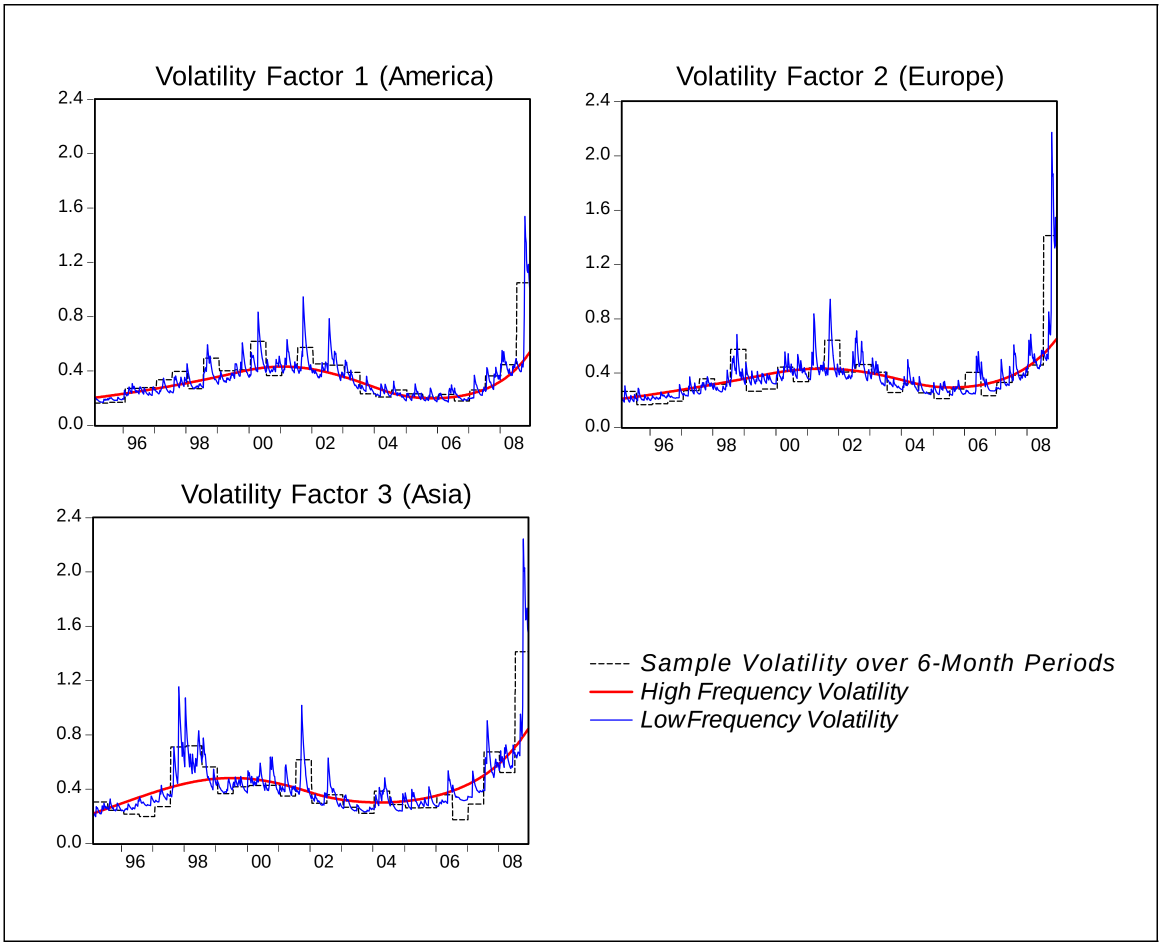
Notes: This figure presents the cross sectional average of equity correlations from the FSG-DCC Model and the 6-month sample correlation model. This average is defined as $\overline{\rho}_t^{(m)} = \sum_{i=1}^{N} \overline{\rho}_{i,t}^{(m)}$, where $\overline{\rho}_{i,t}^{(m)}$ is the average correlation of country i (at time t) with the rest of the world, according to model m. Specifically, $\overline{\rho}_{i,t}^{(m)} = \sum_{j,j \neq i}^{N} \widehat{\rho}_{i,j,t}^{(m)}$, where $\widehat{\rho}_{i,j,t}^{(m)}$ is the correlation between country i and country j at time t, estimated from model m. The estimations are based on weekly data. The sample period goes from February 1995 to December 2008.

Figure 4
Average Low Frequency Idiosyncratic Volatility (From Weekly Data)



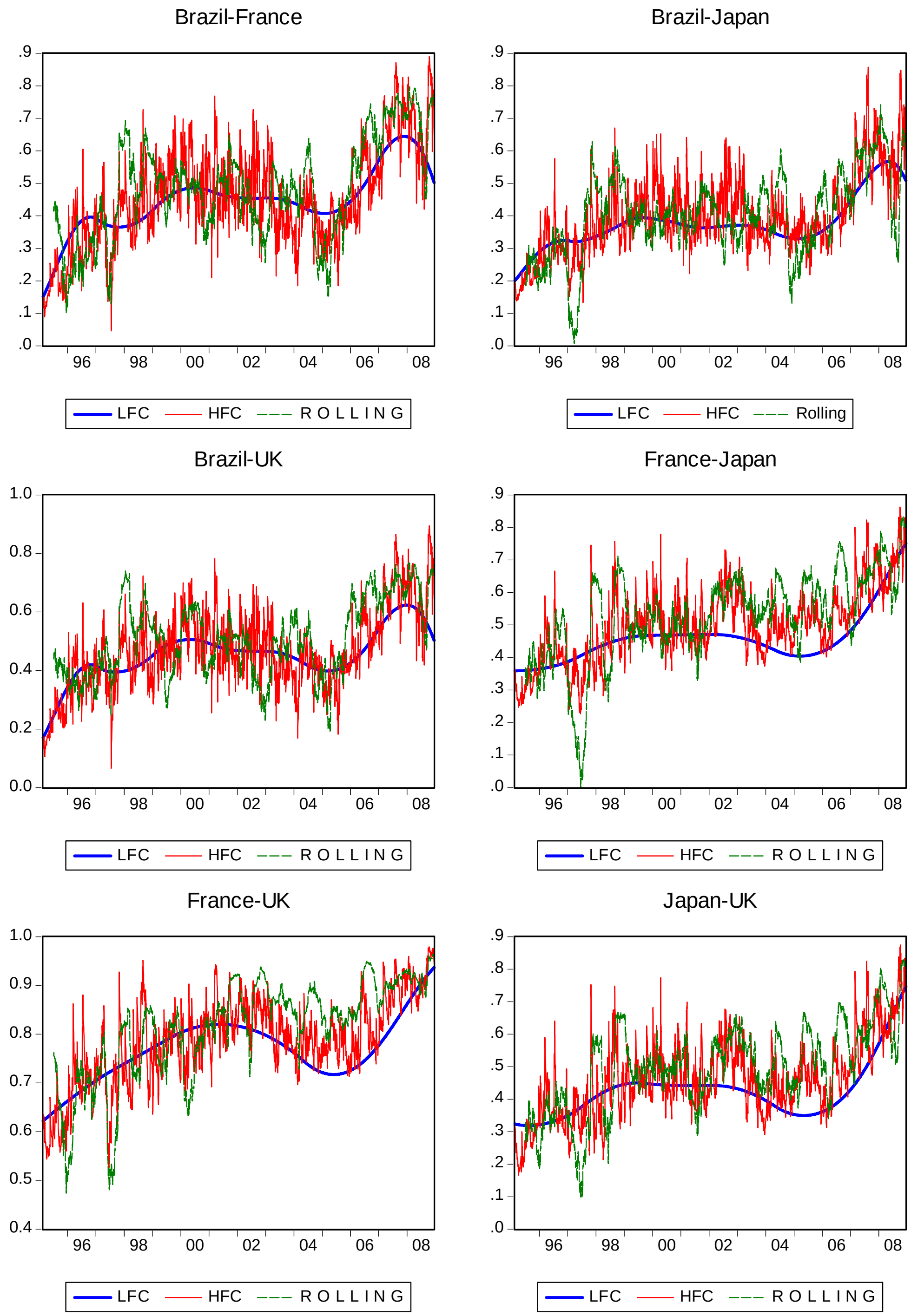
Notes: This figure presents the cross-sectional average of idiosyncratic volatilities from the FSG-Model and the 6-month sample volatility model. This average is defined as $\overline{\sigma}_{u,t} = \sum_{i=1}^{N} \sigma_{u,i,t}^{(m)}$, where $\sigma_{u,i,t}^{(m)}$ is the idiosyncratic volatility of country i at time t computed from model m. The estimations are based on weekly data. The sample period goes from February 1995 to December 2008.

Figure 5
Volatility of the Factors



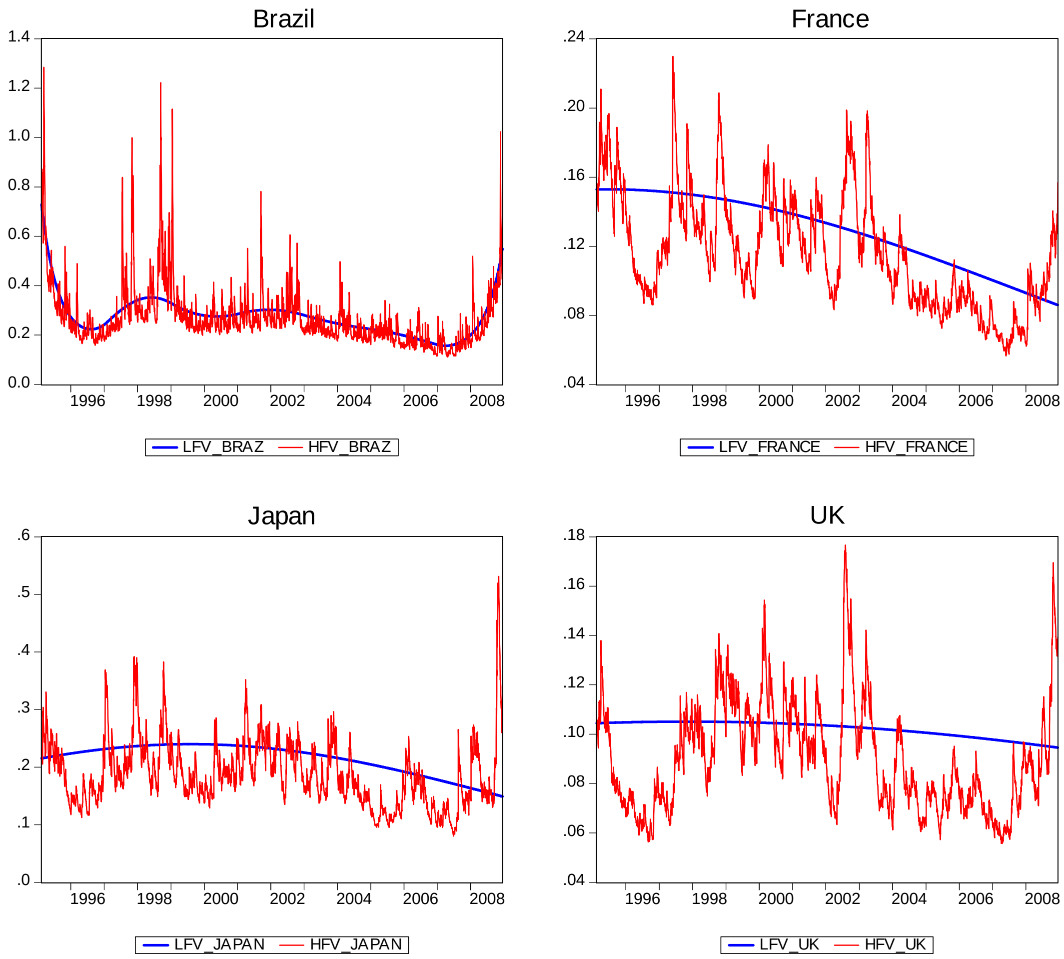
Notes: This figure presents the high and low frequency volatilities of the factors used in the FSG-DCC estimation. The sample period goes from February 2005 to December 2008. The estimates are based on weekly observations.

Figure 6
Rolling and Synchronized Equity Correlations of the FSG-DCC Model
(From Daily Data)



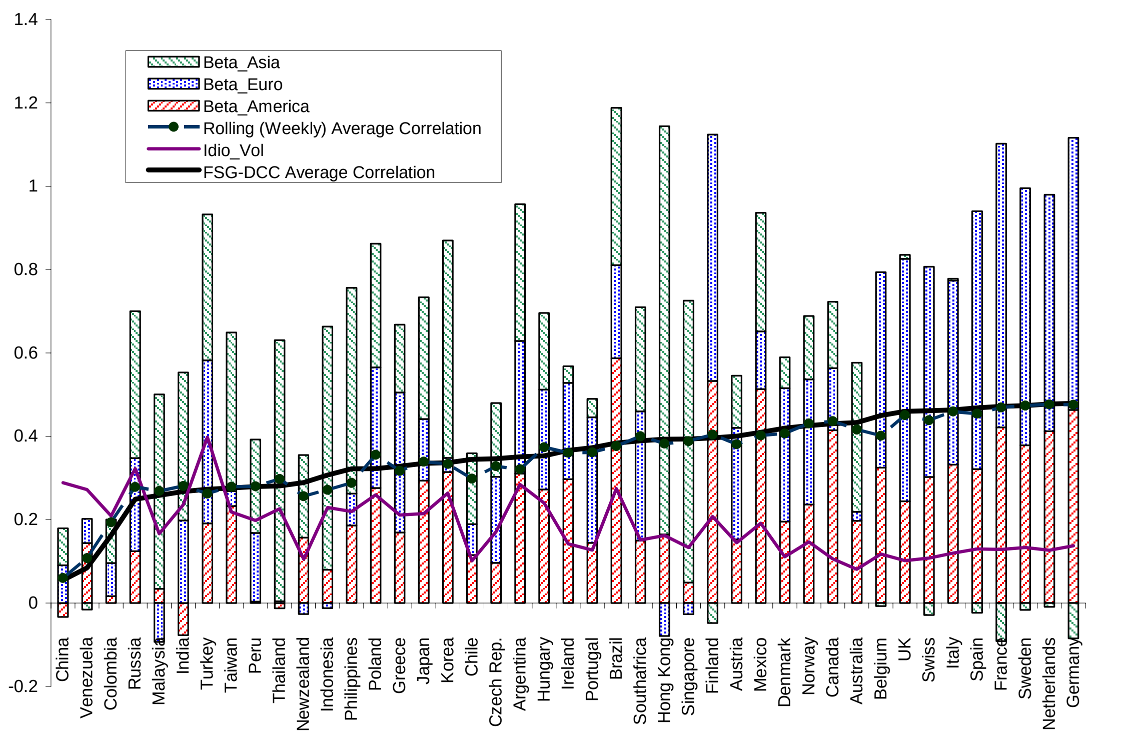
Notes: This figure shows FSG-DCC correlation estimates. The estimation uses daily returns on the stock market indices described in Table 1. The sample period goes from February 1995 to December 2008. The data is obtained from Datastream, Global Financial Data, and Bloomberg. *HFC* denotes "High frequency correlation", *LFC* refers to "Low frequency correlation", and *Rolling* corresponds to the rolling correlations using a 100-day window.

Figure 7
High and Low Frequency Idiosyncratic Volatilities



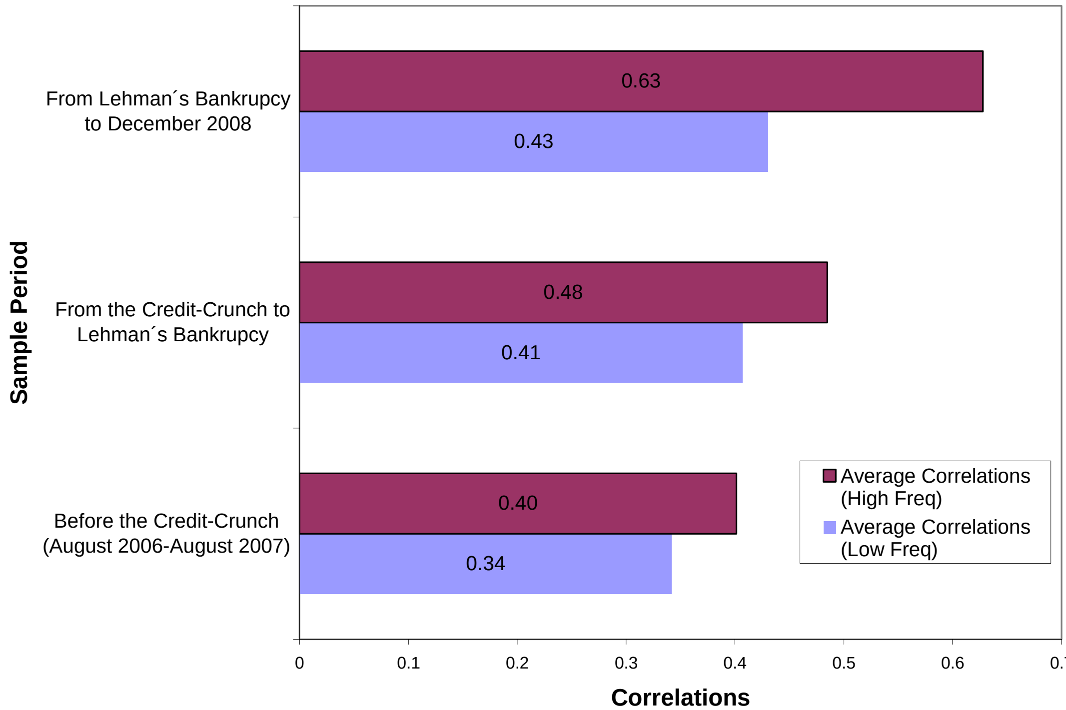
Notes: This figure shows Spline-GARCH volatility estimates. The estimation uses daily returns on the stock market indices described in Table 1. The sample period goes from February 1995 to December 2008. The data is obtained from Datastream, Global Financial Data, and Bloomberg. *HFV* denotes "High frequency volatility" and *LFV* refers to "Low frequency volatility".

Figure 8
Correlation Structure from FSG-DCC (and Synchronization 2): Average Low Frequency and Rolling Correlations, Idiosyncratic Volatilities, and Factor Loadings



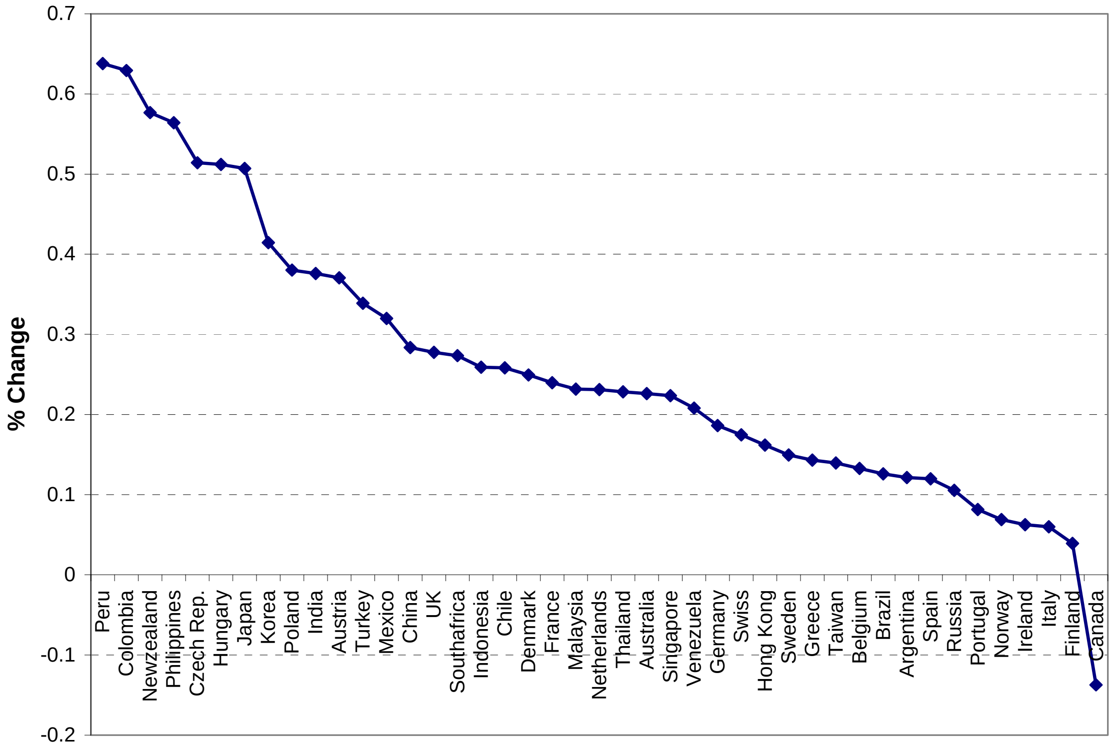
Notes: This figure summarizes the estimation results of the FSG-DCC model based on daily data and "Synchronization 2" (see Table 4). It shows average low frequency correlations of each country with respect to the rest of the world over the whole sample period (February 1995-December 2008). It also shows model-free average rolling correlations based on a 100-day window. The figure presents average idiosyncratic volatilities for each country over the whole sample period. The vertical bars correspond to the estimated synchronized factor loadings associated with the three global market factors (America, Europe, and Asia).

Figure 9
Global Average High and Low Frequency Correlations



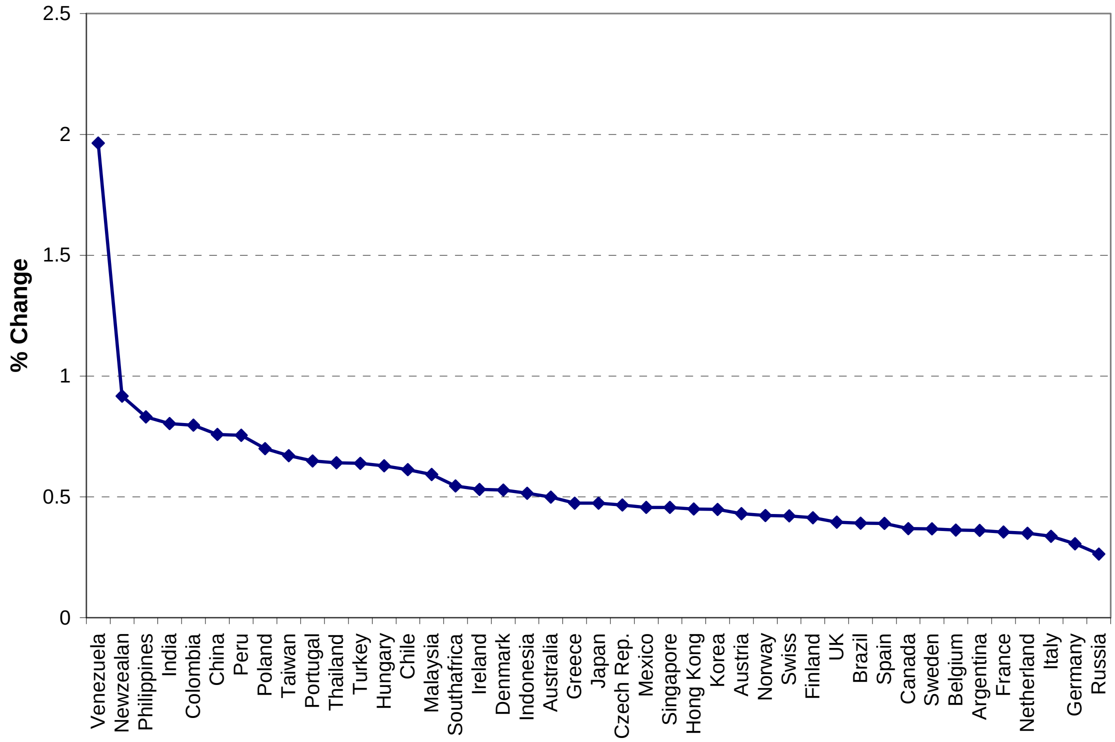
Notes: This figure shows average low frequency equity correlations for three different periods. These correlations are computed from the FSG-DCC model (based on daily synchronized data). For the first period (before the credit-crunch), the average is computed over low frequency correlations between August 3, 2006 and August 3, 2007. For the second period (from the credit-crunch to Lehman's bankruptcy), the average is computed over low frequency correlations between August 6, 2007 and September 12, 2008. The last period considers correlations between September 15 and December 15, 2008.

Figure 10
Distribution of Changes on Average Low Frequency Correlations between August 2007 and December 2008



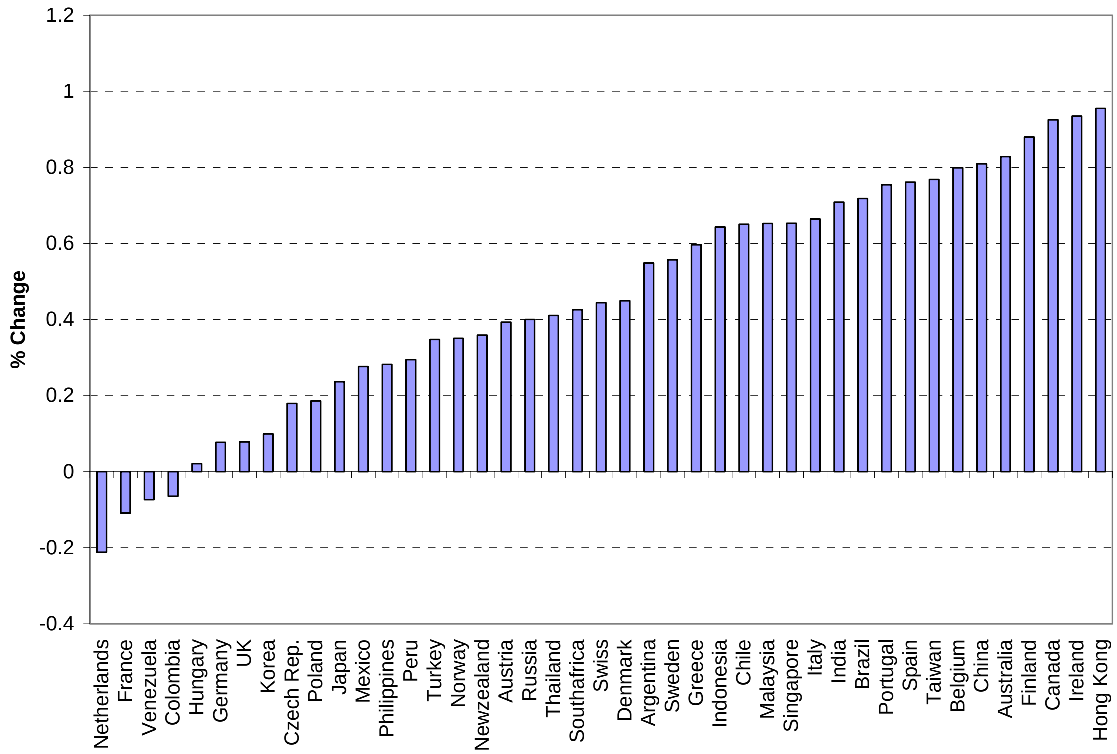
Notes: This figure shows percentage changes on average low frequency equity correlations. These correlations are computed from the FSG-DCC model (based on daily synchronized data). The changes are associated with average correlations in two periods. The first period goes from August 3, 2006 to August 3, 2007. The second period goes from September 15 to December 15, 2008.

Figure 11
Distribution of Changes on Average High Frequency Correlations between August 2007 and December 2008



Notes: This figure shows percentage changes on average high frequency equity correlations. These correlations are computed from the FSG-DCC model (based on daily synchronized data). The changes are associated with average correlations in two periods. The first period goes from August 3, 2006 to August 3, 2007. The second period goes from September 15 to December 15, 2008.

Figure 12
Change in Idiosyncratic Volatility from August 2007 to December 2008



Notes: This figure shows percentage changes on average low frequency idiosyncratic volatilities. These volatilities are computed from the FSG-DCC model (based on daily observations). The changes are associated with average idiosyncratic volatilities in two periods. The first period goes from August 3, 2006 to August 3, 2007. The second period goes from September 15 to December 15, 2008.

Table 1

Countries and Stock Exchanges

Country	Market Classification	Exchange	Name of the Market Index
Argentina	emerging	Buenos Aires	MERVAL
Australia	developed	Australian	ASX
Austria	developed	Wiener Börse	ATX
Belgium	developed	Euronext	CBB
Brazil	emerging	Sao Paulo	BOVESPA
Canada	developed	TSX Group	S&P/TXS 300
Chile	emerging	Santiago	IGPAD
China	emerging	Shanghai Stock Exchange	SSE-180
Colombia	emerging	Bogota	IGBC
Czech Republic	emerging	PSE	SE PX-50 Index
Denmark	developed	Copenhagen	KAX All-Share Index
Finland	developed	Helsinki	HEX
France	developed	Euronext	CAC-40
Germany	developed	Deutsche Börse	DAX-30
Greece	developed	Athens	Athens SE General Index
Hong Kong	developed	Hong Kong	Hang Seng Composite Index
Hungary	emerging	Budapest	Budapest SE Index
India	emerging	Mumbai	Mumbay SE-200 Index
Indonesia	emerging	Jakarta	Jakarta SE Composite Index
Ireland	developed	Irish	ISEQ Overall Price Index
Italy	developed	Borsa Italiana	Milan MIB General Index
Japan	developed	Tokyo	Nikkei 225
Korea	emerging	Korea	KOSPI
Malaysia	emerging	Bursa Malaysia	KLSE Composite
Mexico	emerging	Mexico	IPC
Netherlands	developed	Euronext	AEX
New Zealand	developed	New Zealand	New Zealand SE All-Share Capital Index
Norway	developed	Oslo	Oslo SE All-Share Index
Peru	emerging	Lima	Lima SE General Index
Philippines	emerging	Philippines	Manila SE Composite Index
Poland	emerging	Warsaw	Poland SE Index (Zloty)
Portugal	developed	Euronext	Portugal PSI General Index
Russia	emerging	Russian Exchange	Russia AKM Composite
Singapore	developed	Singapore	SES All-Share Index
South Africa	emerging	JSE South Africa	FTSE/JSE All-Share Index
Spain	developed	Spanish Exchanges (BME)	IBEX
Sweden	developed	Stockholmsbörsen	SAX All-Share index
Switzerland	developed	Swiss Exchange	Switzerland Price Index
Taiwan	emerging	Taiwan	Taiwan SE Capitalization Weighted Index
Thailand	emerging	Thailand	SET General Index
Turkey	emerging	Istanbul	Istanbul SE IMKB-100 Price Index
United Kingdom	developed	London	FTSE-250
Venezuela	emerging	Caracas	Caracas SE General Index

Table 2

Estimation Results: FSG-DCC based on Weekly Data

	Mean Parameters				Variance Parameters			
Country	μ_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	θ_i	γ_i	ϕ_i	**knots**
Factor 1	0.0019 **				1.4E-05	0.170 **	0.782 **	3
Factor 2	0.0019 **				1.7E-07	0.225 **	0.698 **	3
Factor 3	0.0017 **				1.4E-02	0.182 **	0.771 **	2
Argentina	-0.0001	0.172 *	0.541 **	0.381 **	3.6E-02 *	0.034	0.887 **	5
Australia	0.0008 **	0.124 **	0.022	0.429 **	3.4E-02	-0.016	0.929 **	2
Austria	0.0011	0.052	0.358 **	0.230 **	4.9E-03	0.200 **	0.771 **	1
Belgium	0.0004	0.214 **	0.605 **	-0.026	8.9E-02 *	0.067	0.669 **	3
Brazil	0.0026 **	0.500 **	0.377 **	0.357 **	1.1E-02	0.120 **	0.873 **	1
Canada	0.0007	0.433 **	0.169 **	0.180 **	1.1E-01 **	-0.054	0.904 **	1
Chile	0.0012 **	0.132 **	0.090 **	0.188 **	1.7E-01 **	-0.032	0.815 **	1
China	0.0000	-0.137	0.132	0.073	4.1E-01 **	-0.298	0.275 **	2
Colombia	0.0033 **	0.153 **	0.150 *	0.153 **	1.7E-01 **	0.128 *	0.428 **	1
Czech Rep.	0.0004	0.103 *	0.299 **	0.279 **	5.3E-02 **	0.088 **	0.844 **	1
Denmark	0.0013 **	0.121 **	0.440 **	0.115 **	3.9E-02 **	-0.044	0.964 **	3
Finland	0.0011	0.424 **	0.624 **	-0.036	1.2E-06	0.056	0.810 **	3
France	-0.0004	0.416 **	0.685 **	-0.094	6.0E-06	0.155 **	0.600 **	4
Germany	0.0005	0.395 **	0.756 **	-0.094	2.9E-02	0.044	0.838 **	3
Greece	0.0005	0.103	0.458 **	0.141 **	4.1E-02	0.031	0.675 **	2
Hong Kong	0.0001	0.124 **	-0.072	1.007 **	3.9E-02	0.095 *	0.773 **	2
Hungary	0.0016	0.167 **	0.243 **	0.270 **	1.4E-01 **	0.085	0.449 **	3
India	0.0019	-0.093	0.362 **	0.334 **	4.3E-02	0.061	0.767 **	2
Indonesia	0.0020 *	-0.104	0.041	0.702 **	2.9E-02	0.174 **	0.694 **	2
Ireland	0.0009	0.292 **	0.298 **	0.063 *	2.0E-01 **	0.152 **	0.610 **	1
Italy	-0.0009	0.220 **	0.471 **	0.107 **	1.9E-01 **	0.021	0.548 **	2
Japan	-0.0016	0.222 **	0.160 **	0.446 **	5.1E-02 **	0.074 **	0.820 **	1
Korea	0.0001	0.288 **	-0.038	0.688 **	1.9E-01 **	-0.049	0.601 **	2
Malaysia	-0.0003	-0.040	-0.073	0.544 **	4.1E-02 **	0.094 **	0.885 **	2
Mexico	0.0028 **	0.592 **	0.154 **	0.258 **	5.4E-02 **	0.043 *	0.914 **	1
Netherlands	-0.0002	0.365 **	0.632 **	-0.004	5.8E-02 **	0.084 **	0.747 **	3
New Zealand	0.0002	0.086 **	-0.007	0.231 **	1.3E-07	0.067 **	0.927 **	2
Norway	0.0017 **	0.147 **	0.454 **	0.188 **	6.4E-02	0.060	0.775 **	2
Peru	0.0020 **	0.073	0.137 **	0.283 **	4.6E-01 **	-0.004	0.232 **	2
Philippines	-0.0005	0.051	0.008	0.589 **	1.7E-05	0.103 **	0.792 **	2
Poland	0.0005	0.298 **	0.279 **	0.366 **	9.2E-02 **	-0.005	0.803 **	2
Portugal	0.0004	0.077 **	0.397 **	0.080 **	9.5E-06	0.170 **	0.619 **	2
Russia	0.0055 **	-0.059	0.493 **	0.341 **	1.8E-01 **	0.097 *	0.597 **	2
Singapore	-0.0006	0.075 *	-0.028	0.719 **	2.2E-07	0.053 **	0.957 **	2
South Africa	0.0016 **	0.120 **	0.371 **	0.267 **	3.6E-02	0.142 **	0.733 **	2
Spain	0.0008	0.294 **	0.644 **	-0.029	2.8E-05	0.067 *	0.760 **	3
Sweden	0.0006	0.336 **	0.612 **	0.016	1.5E-02	0.107 *	0.709 **	3
Swiss	0.0005	0.297 **	0.511 **	-0.021	6.9E-02 *	0.144 **	0.632 **	3
Taiwan	-0.0002	0.004	0.114 *	0.444 **	9.6E-06	0.081 **	0.874 **	3
Thailand	-0.0013	-0.123	0.071	0.648 **	1.6E-01 **	0.002	0.580 **	2
Turkey	0.0039 **	0.056	0.586 **	0.225 **	4.3E-02 *	-0.034	0.902 **	2
UK	-0.0004	0.253 **	0.596 **	-0.014	8.2E-02 **	0.007	0.822 **	1
Venezuela	0.0031 **	-0.001	0.050	0.073	3.6E-01 **	-0.278	0.299 **	3

Correlation (DCC) Parameters

a	0.011 **
b	0.942 **

Notes on Table 2: This table shows parameter estimates of the FSG-DCC model using weekly data. The sample period is January 1995 to December 2008. All the returns are denominated in U.S. dollars. The three global factors are associated with stock market returns in America, Europe, and Asia, respectively.

The model specification is:

$$\mathbf{r}_t \mid F_t, \Phi_{t-1} \sim N(BF_t, \Sigma_{u,t}), \quad F_t \mid \Phi_{t-1} \sim N(0, \Sigma_{F,t}), \; u_{i,t} = r_{i,t} - \mu_i - \beta_i ' F_t = \sqrt{\tau_{i,t} g_{i,t}} \varepsilon_{i,t}, \; i = 1, ..., 43$$

$$F_t = \left(f_{1,t}, f_{2,t}, f_{3,t} \right)', \quad f_{j,t} = \sqrt{\tau_{f,j,t} g_{f,j,t}} \varepsilon_{f,j,t}, \; j = 1, 2, 3, \quad \beta_i = \left(\beta_{1,i}, \beta_{2,i}, \beta_{3,i} \right)'$$

$$g_{i,t} = \left(1 - \theta_i - \phi_i - \frac{\gamma_i}{2} \right) + \theta_i \frac{u_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{u_{i,t-1}^2 I_{r_{i,t-1} < 0}}{\tau_{i,t-1}} + \phi_i g_{i,t-1}$$

$$\tau_{i,t} = c_i \exp\left(w_{i0} t + \sum_{s=1}^{k_i} w_{is} \left((t - t_{s-1})_+ \right)^2 \right), \; for \; i = 1, ..., 43,$$

$$g_{f,j,t} = \left(1 - \theta_{f,j} - \phi_{f,j} - \frac{\gamma_{f,j}}{2} \right) + \theta_{f,j} \frac{f_{j,t-1}{}^2}{\tau_{f,j,t-1}} + \gamma_{f,j} \frac{f_{j,t-1}{}^2 I_{f_{j,t-1} < 0}}{\tau_{f,j,t-1}} + \phi_{f,j} g_{f,j,t-1}$$

$$\tau_{f,j,t} = c_{f,j} \exp\left(w_{f,j,0} t + \sum_{s=1}^{k_{f,j}} w_{f,j,s} \left((t - t_{s-1})_+ \right)^2 \right), \quad for \; j = 1, 2, 3,$$

$$\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, ..., \varepsilon_{43,t}, \varepsilon_{f,1,t}, \varepsilon_{f,2,t}, \varepsilon_{f,3,t})' \sim \text{DCC process with parameters (a,b).}$$

(**) denotes statistical significance at the 5% level.
(*) denotes statistical significance at the 10% level.

Table 3

Estimation Results: FSG-DCC based on Daily Data and Synchronization 1

	Mean Parameters				Variance Parameters			
Country	μ_i	$\beta_{1,i}$	$\beta_{2,i}$	$\beta_{3,i}$	θ_i	γ_i	ϕ_i	knots
Factor 1	1.87E-04				6.1E-07	0.148 **	0.901 **	3
Factor 2	3.59E-04 **				1.6E-02 *	0.114 **	0.883 **	3
Factor 3	2.07E-04				5.1E-03	0.146 **	0.853 **	6
Argentina	-2.18E-04	0.308 **	0.343 **	0.175 **	3.7E-02 **	0.145 **	0.754 **	9
Australia	3.71E-06	0.016	0.156 **	0.322 **	2.5E-02 **	0.036 **	0.913 **	2
Austria	1.90E-04	-0.016	0.406 **	0.096 **	2.7E-02 **	0.087 **	0.868 **	2
Belgium	1.28E-04	0.023	0.720 **	-0.096	4.9E-02 **	0.046 **	0.871 **	3
Brazil	-1.97E-04	0.584 **	0.274 **	0.195 **	4.3E-02 **	0.169 **	0.737 **	7
Canada	3.93E-05	0.340 **	0.221 **	0.085 **	6.0E-02 **	0.090 **	0.663 **	11
Chile	4.89E-05	0.098 **	0.089 **	0.076 **	8.1E-02 **	0.079 **	0.787 **	5
China	-3.87E-04	-0.128	0.067	0.157 **	1.1E-01 **	0.055 **	0.712 **	10
Colombia	-1.22E-04	0.080 **	0.044	0.063 **	1.9E-01 **	0.132 **	0.527 **	11
Czech Rep.	1.25E-04	0.016	0.340 **	0.117 **	4.7E-02 **	0.086 **	0.832 **	2
Denmark	9.82E-05	0.011	0.505 **	0.005	5.2E-02 **	0.043 **	0.829 **	3
Finland	6.87E-05	0.219 **	0.859 **	-0.165	3.3E-02 **	0.038 **	0.716 **	6
France	-2.14E-05	0.020	0.955 **	-0.144	3.1E-02 **	0.024 **	0.929 **	3
Germany	8.43E-05	0.034 *	0.916 **	-0.128	5.0E-02 **	0.040 **	0.894 **	3
Greece	-9.07E-05	0.011	0.498 **	0.068 **	9.7E-02 **	0.106 **	0.699 **	6
Hong Kong	-1.49E-04	-0.067	0.098 **	0.809 **	1.5E-02 **	0.062 **	0.912 **	3
Hungary	-6.76E-05	0.166 **	0.422 **	0.077 **	8.2E-02 **	0.056 **	0.809 **	2
India	2.65E-04	-0.118	0.269 **	0.240 **	6.9E-02 **	0.149 **	0.748 **	2
Indonesia	7.96E-05	-0.091	0.128 **	0.437 **	7.4E-02 **	0.162 **	0.657 **	6
Ireland	1.84E-04	0.071 **	0.420 **	0.001	9.8E-03	0.088 **	0.788 **	4
Italy	-2.94E-05	-0.022	0.809 **	-0.098	5.7E-02 **	0.004	0.898 **	3
Japan	4.06E-05	-0.015	0.366 **	0.292 **	3.0E-02 **	0.042 **	0.916 **	3
Korea	1.38E-05	-0.006	0.293 **	0.415 **	3.2E-02 **	0.045 **	0.919 **	2
Malaysia	-2.47E-05	-0.043	0.005	0.361 **	8.8E-02 **	0.073 **	0.613 **	11
Mexico	-6.39E-05	0.509 **	0.202 **	0.081 **	3.8E-02 **	0.143 **	0.780 **	4
Netherlands	-3.99E-06	0.044 **	0.890 **	-0.127	1.7E-02 **	0.076 **	0.925 **	1
New Zealand	7.30E-05	0.039 **	0.080 **	0.155 **	8.3E-02 **	0.087 **	0.592 **	6
Norway	1.51E-04	0.046 **	0.510 **	0.051 **	3.7E-02 **	0.067 **	0.879 **	2
Peru	-3.96E-06	0.017	0.216 **	0.144 **	1.8E-01 **	0.068 **	0.680 **	2
Philippines	-1.11E-04	-0.029	0.113 **	0.404 **	6.3E-02 **	0.112 **	0.826 **	2
Poland	6.57E-05	0.107 **	0.409 **	0.196 **	4.8E-02 **	0.008	0.914 **	2
Portugal	7.55E-05	0.023	0.460 **	-0.020	3.0E-02 **	0.133 **	0.719 **	7
Russia	-6.24E-05	0.147 **	0.305 **	0.162 **	6.6E-02 **	0.092 **	0.766 **	12
Singapore	-5.53E-05	-0.046	0.072 **	0.539 **	3.3E-02 **	0.081 **	0.810 **	6
South Africa	-3.69E-05	0.061 **	0.392 **	0.156 **	6.9E-02 **	0.057 **	0.859 **	1
Spain	-7.44E-06	0.000	0.838 **	-0.088	1.8E-02	0.049 **	0.816 **	7
Sweden	1.91E-05	0.046 **	0.846 **	-0.105	2.9E-02 **	0.066 **	0.833 **	4
Swiss	1.01E-05	-0.017	0.719 **	-0.082	3.0E-02 **	0.074 **	0.875 **	3
Taiwan	-2.97E-05	-0.002	0.076 **	0.360 **	2.0E-02 **	0.090 **	0.884 **	3
Thailand	2.61E-05	-0.097	0.131 **	0.434 **	8.4E-02 **	0.054 **	0.644 **	10
Turkey	-3.71E-04	0.013	0.438 **	0.290 **	6.5E-02 **	0.054 **	0.811 **	3
UK	-8.38E-05	-0.042	0.797 **	-0.077	4.3E-02 **	0.057 **	0.817 **	6
Venezuela	-3.21E-04	0.119 **	-0.058	0.047 **	2.5E-01 **	-0.103	0.663 **	10

Correlation (DCC) Parameters

a	0.007 **
b	0.985 **

Notes on Table 3: This table shows parameter estimates of the FSG-DCC model using daily data.
The sample period is January 1995 to December 2008. All the returns are denominated in U.S. dollars.
The three global factors are associated with stock market returns in America, Europe, and Asia, respectively.
The model specification is:

$$\mathbf{r}_t \mid F_t, \Phi_{t-1} \sim N(BF_t, \Sigma_{u,t}), \quad F_t \mid \Phi_{t-1} \sim N(0, \Sigma_{F,t}), \; u_{i,t} = r_{i,t} - \mu_i - \beta_i ' F_t = \sqrt{\tau_{i,t} g_{i,t}}\, \varepsilon_{i,t}, \; i = 1, ..., 43$$

$$F_t = \left(f_{1,t}, f_{2,t}, f_{3,t} \right)', \quad f_{j,t} = \sqrt{\tau_{f,j,t} g_{f,j,t}}\, \varepsilon_{f,j,t}, \quad j = 1,2,3, \quad \beta_i = \left(\beta_{1,i}, \beta_{2,i}, \beta_{3,i} \right)'$$

$$g_{i,t} = \left(1 - \theta_i - \phi_i - \frac{\gamma_i}{2} \right) + \theta_i \frac{u_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{u_{i,t-1}^2 I_{r_{i,t-1} < 0}}{\tau_{i,t-1}} + \phi_i g_{i,t-1}$$

$$\tau_{i,t} = c_i \exp\left(w_{i0} t + \sum_{s=1}^{k_i} w_{is} \left((t - t_{s-1})_+ \right)^2 \right), \; for \; i = 1, ..., 43,$$

$$g_{f,j,t} = \left(1 - \theta_{f,j} - \phi_{f,j} - \frac{\gamma_{f,j}}{2} \right) + \theta_{f,j} \frac{f_{j,t-1}{}^2}{\tau_{f,j,t-1}} + \gamma_{f,j} \frac{f_{j,t-1}{}^2 I_{f_{j,t-1} < 0}}{\tau_{f,j,t-1}} + \phi_{f,j} g_{f,j,t-1}$$

$$\tau_{f,j,t} = c_{f,j} \exp\left(w_{f,j,0} t + \sum_{s=1}^{k_{f,j}} w_{f,j,s} \left((t - t_{s-1})_+ \right)^2 \right), \quad for \; j = 1,2,3,$$

$$\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, ..., \varepsilon_{43,t}, \varepsilon_{f,1,t}, \varepsilon_{f,2,t}, \varepsilon_{f,3,t})' \sim \text{DCC process with parameters (a,b).}$$

(**) denotes statistical significance at the 5% level.
(*) denotes statistical significance at the 10% level.

Table 4

Estimation Results: FSG-DCC based on Daily Data and Synchronization 2

	Mean Parameters								Variance Parameters						
Country	μ_i		$\beta_{1,i}$		$\beta_{2,i}$		$\beta_{3,i}$		θ_i		γ_i		ϕ_i		knots
Factor 1	1.46E-04								6.2E-08		0.157	**	0.894	**	3
Factor 2	3.59E-04	**							1.6E-02	*	0.114	**	0.883	**	3
Factor 3	2.07E-04								5.1E-03		0.146	**	0.853	**	6
Argentina	3.23E-04		0.311	**	0.318	**	0.328	**	6.0E-02	**	0.125	**	0.757	**	9
Australia	1.47E-04	*	0.198	**	0.021	**	0.358	**	2.7E-02	**	0.031	**	0.912	**	2
Austria	3.18E-04	**	0.153	**	0.268	**	0.125	**	4.6E-02	**	0.089	**	0.847	**	2
Belgium	1.57E-04		0.325	**	0.469	**	-0.007	**	5.2E-02	**	0.031	**	0.890	**	3
Brazil	3.28E-04		0.587	**	0.223	**	0.377	**	3.6E-02	**	0.186	**	0.758	**	7
Canada	1.24E-04		0.415	**	0.148	**	0.159	**	5.3E-02	**	0.081	**	0.795	**	9
Chile	2.47E-04	**	0.115	**	0.074	**	0.170	**	1.4E-01	**	0.055	**	0.721	**	6
China	-9.06E-05		-0.033		0.091	*	0.089	**	1.1E-01	**	0.056	**	0.725	**	10
Colombia	4.11E-04	**	0.017		0.080	**	0.104	**	2.3E-01	**	0.102	**	0.529	**	9
Czech Rep.	2.83E-04	*	0.097	**	0.206	**	0.177	**	4.2E-02	**	0.114	**	0.812	**	2
Denmark	2.59E-04	**	0.195	**	0.320	**	0.074	**	6.4E-02	**	0.038	**	0.840	**	3
Finland	2.15E-04		0.533	**	0.591	**	-0.048	**	9.1E-02	**	0.083	**	0.590	**	4
France	-6.57E-05		0.421	**	0.680	**	-0.091	**	2.9E-02	**	0.023	**	0.953	**	1
Germany	1.67E-04		0.463	**	0.653	**	-0.085	**	1.2E-01	**	0.012		0.671	**	8
Greece	7.37E-05		0.169	**	0.336	**	0.162	**	1.2E-01	**	0.111	**	0.663	**	6
Hong Kong	9.42E-06		0.165	**	-0.079	**	0.979	**	2.6E-02	**	0.054	**	0.931	**	2
Hungary	4.73E-04	**	0.273	**	0.239	**	0.184	**	1.1E-01	**	0.041	**	0.786	**	3
India	5.87E-04	**	-0.077		0.198	**	0.355	**	8.1E-02	**	0.125	**	0.765	**	2
Indonesia	5.68E-04	**	0.080	**	-0.012		0.584	**	1.0E-01	**	0.213	**	0.572	**	6
Ireland	3.07E-04	**	0.297	**	0.231	**	0.040	**	2.3E-02	**	0.090	**	0.732	**	4
Italy	-6.45E-05		0.332	**	0.441	**	0.004	**	1.1E-01	**	0.020		0.750	**	7
Japan	-2.80E-04		0.294	**	0.148	**	0.292	**	3.2E-02	**	0.076	**	0.918	**	1
Korea	-1.81E-05		0.315	**	0.034	**	0.522	**	3.7E-02	**	0.062	**	0.904	**	2
Malaysia	2.71E-05		0.034	**	-0.093	**	0.466	**	6.2E-02	**	0.106	**	0.792	**	6
Mexico	5.14E-04	**	0.513	**	0.139	**	0.285	**	3.3E-02	**	0.134	**	0.794	**	6
Netherlands	-5.26E-05		0.412	**	0.567	**	-0.009	**	2.5E-02	**	0.063	**	0.926	**	1
New Zealand	4.13E-05		0.157	**	-0.026	**	0.198	**	8.7E-02	**	0.042	**	0.790	**	2
Norway	4.11E-04	**	0.237	**	0.300	**	0.152	**	5.3E-02	**	0.058	**	0.805	**	8
Peru	4.89E-04	**	0.003	**	0.165	**	0.224	**	2.3E-01	**	0.076	**	0.651	**	1
Philippines	-1.58E-04		0.186	**	0.076	**	0.494	**	7.8E-02	**	0.121	**	0.787	**	2
Poland	2.66E-04		0.276	**	0.289	**	0.296	**	6.9E-02	**	0.006		0.881	**	2
Portugal	1.89E-04	*	0.144	**	0.302	**	0.044	**	9.4E-02	**	0.141	**	0.635	**	7
Russia	1.35E-03	**	0.124	**	0.223	**	0.352	**	8.2E-02	**	0.059	**	0.778	**	8
Singapore	-9.65E-05		0.049	*	-0.027		0.676	**	2.6E-02	**	0.072	**	0.850	**	6
South Africa	2.82E-04	**	0.150	**	0.311	**	0.250	**	6.6E-02	**	0.064	**	0.841	**	3
Spain	1.40E-04		0.321	**	0.618	**	-0.023	**	5.2E-02	**	0.030	**	0.864	**	3
Sweden	5.80E-05		0.379	**	0.617	**	-0.017	**	4.9E-02	**	0.089	**	0.774	**	4
Swiss	7.18E-05		0.302	**	0.505	**	-0.029	**	3.4E-02	**	0.064	**	0.882	**	3
Taiwan	-1.00E-04		0.233	**	0.036		0.381	**	2.6E-02	**	0.082	**	0.878	**	3
Thailand	-1.99E-04		-0.013		0.004		0.626	**	7.3E-02	**	0.064	**	0.712	**	10
Turkey	7.36E-04	**	0.191	**	0.391	**	0.350	**	7.3E-02	**	0.060	**	0.789	**	3
UK	-1.26E-04		0.245	**	0.581	**	0.010	**	2.9E-02	**	0.018	**	0.957	**	1
Venezuela	4.62E-04	**	0.144	**	0.058	*	-0.016	*	2.2E-01	**	-0.061		0.675	**	10

Correlation (DCC) Parameters

a	0.005	**
b	0.989	**

Notes on Table 4: This table shows parameter estimates of the FSG-DCC model using daily data.
The sample period is January 1995 to December 2008. All the returns are denominated in U.S. dollars.
The three global factors are associated with stock market returns in America, Europe, and Asia, respectively.
The model specification is:

$$\mathbf{r}_t \mid F_t, \Phi_{t-1} \sim N(BF_t, \Sigma_{u,t}), \quad F_t \mid \Phi_{t-1} \sim N(0, \Sigma_{F,t}), \; u_{i,t} = r_{i,t} - \mu_i - \beta_i' F_t = \sqrt{\tau_{i,t} g_{i,t}}\,\varepsilon_{i,t}, \; i = 1, \ldots, 43$$

$$F_t = \left(f_{1,t}, f_{2,t}, f_{3,t} \right)', \quad f_{j,t} = \sqrt{\tau_{f,j,t} g_{f,j,t}}\,\varepsilon_{f,j,t}, \; j = 1, 2, 3, \quad \beta_i = \left(\beta_{1,i}, \beta_{2,i}, \beta_{3,i} \right)'$$

$$g_{i,t} = \left(1 - \theta_i - \phi_i - \frac{\gamma_i}{2} \right) + \theta_i \frac{u_{i,t-1}^2}{\tau_{i,t-1}} + \gamma_i \frac{u_{i,t-1}^2 I_{r_{i,t-1}<0}}{\tau_{i,t-1}} + \phi_i g_{i,t-1}$$

$$\tau_{i,t} = c_i \exp\left(w_{i0} t + \sum_{s=1}^{k_i} w_{is} \left((t - t_{s-1})_+ \right)^2 \right), \; for \; i = 1, \ldots, 43,$$

$$g_{f,j,t} = \left(1 - \theta_{f,j} - \phi_{f,j} - \frac{\gamma_{f,j}}{2} \right) + \theta_{f,j} \frac{{f_{j,t-1}}^2}{\tau_{f,j,t-1}} + \gamma_{f,j} \frac{{f_{j,t-1}}^2 I_{f_{j,t-1}<0}}{\tau_{f,j,t-1}} + \phi_{f,j} g_{f,j,t-1}$$

$$\tau_{f,j,t} = c_{f,j} \exp\left(w_{f,j,0} t + \sum_{s=1}^{k_{f,j}} w_{f,j,s} \left((t - t_{s-1})_+ \right)^2 \right), \quad for \; j = 1, 2, 3,$$

$$\boldsymbol{\varepsilon}_{\mathrm{t}} = (\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{43,t}, \varepsilon_{f,1,t}, \varepsilon_{f,2,t}, \varepsilon_{f,3,t})' \sim \text{DCC process with parameters (a,b).}$$

(**) denotes statistical significance at the 5% level.
(*) denotes statistical significance at the 10% level.

Table 5

Comparison of Correlation Models with Respect to Weekly Low Frequency Correlations

Model	MAE (6-Month Sample Benchmark)	MAE (FSG-DCC Weekly Benchmark)
FSG-DCC (Daily with Synchronization 1)	158.47	44.01
FSG-DCC (Daily with Synchronization 2)	158.48	42.85
FSG-DCC Ignoring Non-Synchronicity	177.16	78.34
FSG-DCC (Weekly)	154.40	

Notes on Table 5: This table shows the Mean Absolute Error (MAE) statistics defined in (1.27) as:

$$MAE_{CORR} = \frac{1}{\tilde{T}} \sum_{\tilde{t}=1}^{\tilde{T}} \left(\sum_{i=1}^{N} \sum_{j>i}^{N} | \widehat{\rho}_{i,j,\tilde{t}}^{(benchmark)} - \widehat{\rho}_{i,j,\tilde{t}}^{(FSG-DCC)} | \right),$$

where the benchmark models are the "row" models, and the FSG-DCC models are the specifications in the first column.

Appendix 1

Proof of Proposition 1:

Consider the return process in (1.1) and the system for the joint vector of synchronized factors and returns in (1.23). Then the variance covariance matrix of this vector takes the following form:

$$Var(\mathbf{s_t}) = \left\{ I_{k+N} + \begin{pmatrix} A_{FF} & A_{Fr} \\ A_{rF} & A_{rr} \end{pmatrix} \right\} \begin{pmatrix} \sum_F & \sum_{Fr} \\ \sum_{rF} & \sum_r \end{pmatrix} \left\{ I_{k+N} + \begin{pmatrix} A'_{FF} & A'_{rF} \\ A'_{Fr} & A'_{rr} \end{pmatrix} \right\}$$

$$= \begin{pmatrix} I_K + A_{FF} & A_{Fr} \\ A_{rF} & I_r + A_{rr} \end{pmatrix} \begin{pmatrix} \sum_F & \sum_{Fr} \\ \sum_{rF} & \sum_r \end{pmatrix} \begin{pmatrix} I_K + A'_{FF} & A'_{rF} \\ A'_{Fr} & I_r + A'_{rr} \end{pmatrix} \tag{1.28}$$

$$= \begin{pmatrix} \sum_F + A_{FF} \sum_F + A_{Fr} \sum_{rF} & \sum_{Fr} + A_{FF} \sum_{Fr} + A_{Fr} \sum_r \\ A_{rF} \sum_F + \sum_{rF} + A_{rr} \sum_{rF} & A_{rF} \sum_{Fr} + \sum_{rr} + A_{rr} \sum_{rr} \end{pmatrix} \begin{pmatrix} I_K + A'_{FF} & A'_{rF} \\ A'_{Fr} & I_r + A'_{rr} \end{pmatrix},$$

where A_{FF}, A_{Fr}, A_{rF}, *and* A_{rr} are matrices of coefficients of dimensions (KxK), (KxN), (NxK), and (NxN), respectively. Hence, the synchronized covariance term, $\mathrm{cov}(\tilde{\mathbf{r}}_t, \tilde{F}_t')$, is given by the NxK bottom-left submatrix in (1.28):

$$\begin{aligned} \mathrm{cov}(\tilde{\mathbf{r}}_t, \tilde{F}_t') = {} & A_{rF} \sum_F + A_{rF} \sum_F A'_{FF} + \sum_{rF} + \sum_{rF} A'_{FF} + A_{rr} \sum_{rF} \\ & + A_{rr} \sum_{rF} A'_{FF} + A_{rF} \sum_{Fr} A'_{Fr} + \sum_{rr} A'_{Fr} + A_{rr} \sum_{rr} A'_{Fr}. \end{aligned} \tag{1.29}$$

Now, from the unsynchronized system in (1.22):

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\varepsilon}_{r,t} + A_{rF} \boldsymbol{\varepsilon}_{F,t-1} + A_{rr} \boldsymbol{\varepsilon}_{r,t-1} \\ F_t &= \boldsymbol{\varepsilon}_{F,t} + A_{FF} \boldsymbol{\varepsilon}_{F,t-1} + A_{Fr} \boldsymbol{\varepsilon}_{r,t-1}, \end{aligned} \tag{1.30}$$

and the one-period lead and lag factor terms are:

$$\begin{aligned} F_{t+1} &= \boldsymbol{\varepsilon}_{F,t+1} + A_{FF} \boldsymbol{\varepsilon}_{F,t} + A_{Fr} \boldsymbol{\varepsilon}_{r,t} \\ F_{t-1} &= \boldsymbol{\varepsilon}_{F,t-1} + A_{FF} \boldsymbol{\varepsilon}_{F,t-2} + A_{Fr} \boldsymbol{\varepsilon}_{r,t-2}. \end{aligned} \tag{1.31}$$

Thus, from (1.30) and (1.31), we can derive:

$$\begin{aligned} \mathrm{cov}(\mathbf{r}_t, F_t') &= \sum_{rF} + A_{rF} \sum_F A'_{FF} + A_{rF} \sum_{Fr} A'_{Fr} + A_{rr} \sum_{rF} A'_{FF} + A_{rr} \sum_{rr} A'_{Fr}, \\ \mathrm{cov}(\mathbf{r}_t, F_{t-1}') &= A_{rF} \sum_F + A_{rr} \sum_{rF}, \\ \mathrm{cov}(\mathbf{r}_t, F_{t+1}') &= \sum_{rF} A'_{FF} + \sum_{rr} A'_{Fr}. \end{aligned} \tag{1.32}$$

Then, it is straightforward to obtain that,

$$\operatorname{cov}(\tilde{\mathbf{r}}_{\mathrm{t}}, \tilde{F}_t') = \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_t') + \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_{t-1}') + \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_{t+1}'). \tag{1.33}$$

This expression can be rewritten as:

$$\begin{aligned} \operatorname{cov}(\tilde{\mathbf{r}}_{\mathrm{t}}, \tilde{F}_t') = {} & \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_t') Var(F_t)^{-1} Var(F_t) + \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_{t-1}') Var(F_{t-1})^{-1} Var(F_{t-1}) \\ & + \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_{t+1}') Var(F_{t+1})^{-1} Var(F_{t+1}) \end{aligned} \tag{1.34}$$

and, under the assumption that $Var(F_t) = Var(F_{t-1}) = Var(F_{t+1})$,

$$\tilde{B} \equiv \operatorname{cov}(\tilde{\mathbf{r}}_{\mathrm{t}}, \tilde{F}_t') Var(\tilde{F}_t)^{-1} = \left\{ B + B^- + B^+ \right\} Var(F_t) Var(\tilde{F}_t)^{-1}, \tag{1.35}$$

where:

$$B \equiv \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_t') Var(F_t)^{-1}, \;\; B^- \equiv \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_{t-1}') Var(F_{t-1})^{-1}, \;\; and \;\; B^+ \equiv \operatorname{cov}(\mathbf{r}_{\mathrm{t}}, F_{t+1}') Var(F_{t+1})^{-1}$$

Note that even when the factor unconditional variances show smooth variation, (1.35) holds as a very precise approximation, since in such a case $Var(F_{t-1}) \approx Var(F_t) \approx Var(F_{t+1})$.

Now, taking the *(i,k)* typical element of (1.35) we obtain (1.24). Q.E.D.