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Speculative Growth, Overreaction, and the Welfare Cost of Technology-Driven Bubbles*

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Abstract

This paper develops a general equilibrium model to examine the consequences of technology-driven asset price bubbles for capital accumulation, growth, and welfare. Equity prices in the model exhibit "excess volatility" because speculative agents overreact to observed technology shocks. I show that this behavior tends to be self-confirming, particularly when temporary technology innovations are perceived to be permanent. In model simulations, speculative behavior gives rise to intermittent equity price bubbles that coincide with positive innovations in technology, investment and consumption booms, and faster trend growth, reminiscent of the U.S. economy during the late 1920s and late 1990s. The welfare cost of speculation (relative to rational behavior) depends crucially on parameter values. Speculation can improve welfare if risk aversion is low and agents underinvest relative to the socially-optimal level. But for higher levels of risk aversion, the welfare cost of speculation is large, typically exceeding one percent of per-period consumption.

Keywords: *Endogenous Growth, Business Cycles, Excess Volatility, Asset Pricing, Speculative Bubbles.*

JEL Classification: E32, E44, G12, O40.

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Bubbles are often precipitated by perceptions of real improvements in the productivity and underlying profitability of the corporate economy. But as history attests, investors then too often exaggerate the extent of the improvement in economic fundamentals.

Federal Reserve Chairman Alan Greenspan, August 30, 2002.

1 Introduction

The magnitude of short-term movements in stock prices remains a challenge to explain within a framework of rational, efficient markets. Numerous empirical studies starting with Shiller (1981) and LeRoy and Porter (1981) have shown that stock prices appear to exhibit "excess volatility" when compared to the discounted stream of ex post realized dividends.[1] Another prominent feature of stock price data is the intermittent occurrence of sustained run-ups above estimates of fundamental value, so-called speculative bubbles, that can be found throughout history in various countries and asset markets.[2] The dramatic rise in U.S. stock prices during the late 1990s, followed similarly by U.S. house prices during the mid 2000s, are episodes that have both been described as bubbles. The former episode was accompanied by a boom in business investment, while the later was accompanied by a boom in residential investment. Both booms were later followed by falling asset prices and severe retrenchments in the associated investment series, as agents sought to unwind the excess capital accumulated during the bubble periods. Coincident booms in stock prices and investment also occurred during the late 1920's—a period that shares many characteristics with the late 1990s. In particular, both periods witnessed major technological innovations that contributed to investor enthusiasm about a "new era."[3]

This paper develops a general equilibrium model to examine the consequences of technology-driven asset price bubbles for capital accumulation, growth, and welfare. I introduce excess volatility by assuming that agents engage in a form of speculative behavior that manifests itself by overreaction to observed technology shocks. In constructing their Euler equation forecasts, speculative agents behave like gamblers whose bets about the future are magnified relative to a rational player who seeks to maximize winnings (lifetime utility).[4]

The framework for the analysis is a real business cycle model with endogenous growth and capital adjustment costs, along the lines of Barlevy (2004). I allow for the possibility of an Arrow-Romer type productive externality, such that agents may underinvest relative to the socially-optimal level. The presence of the externality yields an endogenous separation

[1] Shiller (2003) provides a recent update on this literature.

[2] For an overview of historical bubble episodes, see the collection of papers in Hunter, Kaufman, and Pomerleano (2003)

[3] As described further below, the similarities between the two periods is noted by Shiller (2000), Gordon (2006), and White (2006).

[4] Harrison and Kreps (1978) employ a different characterization of speculation in a model that involves heterogenous beliefs and an exogenous constraint on short sales. Speculative agents are defined as those who are willing to pay more than their estimate of fundamental value due to the prospect of reselling later at a higher price. This form of speculation is employed in the model of Scheinkman and Xiong (2003).

between consumption and dividends, where dividend growth is more volatile than consumption growth, as in long-run U.S. data. The severity of the underinvestment problem turns out to be important for analyzing the welfare consequences of fluctuations which, in this model, can affect the economy's trend growth rate.

Under rational expectations, the technology response coefficient in the agent's Euler equation forecast rule is small in magnitude, such that the equilibrium price-dividend ratio is nearly constant for reasonable levels of risk aversion. In contrast, the price-dividend ratio in long-run U.S. stock market data is volatile and highly persistent, i.e., close to a random walk. The model result obtains because rational agents understand that technology shocks give rise to both income and substitution effects which work in opposite directions. The two effects exactly cancel when the intertemporal elasticity of substitution in consumption (the inverse of the coefficient of relative risk aversion) is unity, representing logarithmic utility. In this case, the technology response coefficient in the rational forecast rule is zero, such that the equilibrium price-dividend ratio is constant.

In the speculation model, the agent's forecast rule features a stronger technology response coefficient, generating excess volatility of equity prices. I calibrate the technology response coefficient so that the model matches the volatility of the price-dividend ratio in long-run U.S. data. The calibration procedure abstracts from the underlying source of excess volatility.[5] Nevertheless, I demonstrate that overreaction behavior tends to be self-confirming, particularly when agents misperceive the stochastic process for technology shocks. Due to the self-referential nature of the agent's decision problem, the initial use of a stronger technology response coefficient in the agent's forecast rule serves to increase the magnitude of the actual response coefficient that the agent subsequently estimates using observable data. Using a real-time learning algorithm in the non-linear version of the model, I show that overreaction behavior can persist for an extremely long time (thousands of periods) when agents mistakenly believe that innovations to technology are permanent, when in fact, innovations are only temporary. This form of misspecification is intended to capture some of the flavor of "new era enthusiasm." Even when I allow for real-time learning about the technology process itself, long-lived overreaction behavior remains a frequent outcome of the learning algorithm. In the calibrated model, I show that the forecast errors observed by the speculative agent are close to white noise, making it difficult for the agent to detect a misspecification of the forecast rule. Moreover, from the individual agent's perspective, switching to a fundamentals-based forecast rule (which involves a weaker technology response) would appear to reduce forecast accuracy, so there is no incentive to switch.

In model simulations, speculative overreaction gives rise to intermittent asset price bubbles that coincide with positive innovations in technology, investment and consumption booms, and faster trend growth, reminiscent of the U.S. economy during the late 1920s and late 1990s. The model can also generate prolonged periods where the price-dividend ratio remains in the vicinity of the fundamental value. So long as technology shocks remain small, there is no

[5] Reduced-form modeling devices such as this are often employed in macroeconomics. Examples include Calvo-type sticky price models which abstract from the underlying source of price stickiness, or money-in-the-utility-function models which abstract from the underlying role played by money in facilitating transactions.

practical distinction between a speculative agent and a rational agent. Due to the nonlinear nature of the model solution, the simulated price-dividend ratio exhibits non-Gaussian features, such as positive skewness and excess kurtosis. These features are also present in the data.

Interestingly, the speculation model outperforms the rational model in matching the relative volatilities of consumption growth, investment growth, and output growth. The presence of capital adjustment costs causes investment growth in the rational model to exhibit about the same volatility as output growth, whereas investment growth in the data is about three times as volatile as output growth. Barlevy (2004, p. 983) acknowledges the difficulty of generating sufficient investment volatility in a rational model with capital adjustment costs. However, in the speculation model, the agent's overreaction behavior significantly magnifies the volatility of investment, bringing the model much closer to the data.

Finally, I examine the welfare costs of fluctuations that can be attributed to either: (i) speculative overreaction, or (ii) business cycles. Welfare costs are measured by the percentage change in per-period consumption that makes the agent indifferent between the two economies being compared. The welfare cost of speculation (relative to rational behavior) depends crucially on parameter values. Speculation can improve welfare if risk aversion is low and agents underinvest relative to the socially-optimal level. But for higher levels of risk aversion, the welfare cost of speculation is large, typically exceeding one percent of consumption. Similarly, the welfare cost of business cycle fluctuations in the speculation model (relative to a deterministic model) increase rapidly with risk aversion.

The welfare results involve a complex interaction of several effects. Fluctuations in the model can affect both the mean and volatility of consumption growth. If fluctuations decrease mean growth, then a smaller fraction of resources will be devoted to investment. Less investment implies a higher initial level of consumption which, as noted by Barlevy (2004), can mitigate the negative effects of slower growth. But if the economy is subject to an underinvestment problem, then higher initial consumption is less desirable. Finally, as the curvature of the utility function increases, consumption growth volatility becomes more costly in terms of welfare. Which of these effects dominate depends ultimately on parameter values.

An important unsettled question in economics is whether policymakers should take deliberate steps to prevent or deflate asset price bubbles.[6] Those who advocate leaning against bubbles point out that excessive asset prices can distort economic and financial decisions, creating costly misallocations that can take years to dissipate. Others argue that policies intended to prick a suspected bubble would likely send the economy into a recession, thereby foregoing the benefits of the boom that might otherwise continue. The welfare results obtained here provide some support for both points of view.

1.1 Related Literature

The term "excess volatility" implies that asset prices move too much to be explained by changes in dividends or cash flows. The behavioral finance literature has examined a wide variety of evidence pertaining to this phenomenon. Controlled experiments on human sub-

[6] For an overview of this literature, see Lansing (2008).

jects suggest that people's decisions are influenced by various "heuristics," as documented by Tversky and Kahneman (1974). The "representativeness heuristic" is a form of non-Bayesian updating whereby subjects tend to overweight recent observations relative to the underlying laws of probability that govern the stochastic process. De Bondt and Thaler (1985) find evidence of overreaction in comparing returns of portfolios comprised of prior winning and losing stocks. Arbarbanell and Bernard (1992) and Easterwood and Nutt (1999) find evidence that security analysts' earnings forecasts tend to overreact to new information, particularly when the information is positive in nature. Daniel, et al. (1998) develop a model where investors' overconfidence about the precision of certain types of information causes them to overreact to that information. In the laboratory asset market of Caginalp et al. (2000), prices appeared to overreact to fundamentals and to be driven by previous price changes, i.e., momentum.

This paper relates to a long list of research that explores the links between non-fundamental asset price movements and investment in physical capital. Theoretical research that examines rational bubbles in overlapping generations models with productive externalities or market imperfections includes Saint Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), Oliver (2000), and Caballero et al. (2006).[7] Unlike these papers, the bubbles explored here are driven by agents' excessive bets about the future. Moreover, I use a calibrated version of the model to compute the welfare costs of the capital misallocation that results from this behavior.

The capital adjustment cost formulation in the model implies that movements in the equity price are linked directly to movements in investment, as in a standard Tobin's q framework. Along these lines, an empirical study by Barro (1990) finds that changes in real stock prices since 1891 have strong explanatory power for the growth rate of business investment. Studies by Chirinko and Schaller (2001, 2007), Gilchrist et al. (2005), and Campello and Graham (2007) all find evidence of a significant empirical link between stock price bubbles and investment decisions by firms.

Dupor (2002, 2005) examines the policy implications of non-fundamental asset price movements in monetary real business cycle model with capital adjustments costs. Non-fundamental asset price movements are driven by exogenous "expectation shocks" that a drive a wedge between the true marginal product of capital and the market return observed by firms when making their investment decisions. The volatility of these shocks is calibrated to match a return volatility statistic for the S&P 500 index, analogous to the procedure used here to calibrate the speculative agent's technology response coefficient. He finds that optimal monetary policy should lean against non-fundamental asset price movements.

Jaimovich and Rebelo (2007) develop a behavioral real business cycle model that allows for non-rational expectations. In one version, agents are overconfident about the precision of news about future technology innovations, which causes them to overreact to that news, as in the model of Daniel, et al. (1998). The model abstracts from the underlying source of the agent's overconfidence, which is analogous to the approach taken here.

In a recent paper, Hassan and Mertens (2008) consider the welfare costs of excess volatility

[7] Barlevy (2007, pp. 54-55) provides an overview of this literature.

in a capitalist-worker model where the forecasts of capital owners are perturbed away from the rational expectation by an exogenous shock, similar to the model of Dupor (2002, 2005). They find that the welfare cost of excess volatility can be quite high (on the order of 4 percent of per-period consumption) because volatility depresses the steady state capital stock and hence the wages of workers.

This paper postulates a particular form of less-than-rational behavior (overreaction) and then explores the economic consequences in a standard model. Other asset pricing research along these lines includes: Delong et al. (1990), Barsky and Delong (1993), Timmerman (1996), Barberis, Schleifer, and Vishney (1998), Brock and Hommes (1998), Cecchetti, Lam, and Mark (2000), Abel (2002), Abreu and Brunnermeier (2003), Scheinkman and Xiong (2003), Lansing (2006, 2007), and Adam, Marcet, and Nicolini (2008), among others.

Numerous papers seek to account for the behavior of the stock market, consumption, or investment using fully-rational models where agents' current decisions are affected by the anticipation of changes in the future trend growth rate or the future level/profitability of technology. Research along these lines includes Zeira (1999), Greenwood and Jovanovic (1999), Hobijn and Jovanovic (2001), Pástor and Veronesi (2006), Beaudry and Portier (2006), Jermann and Quadrini (2007), Johnson (2007), Angeletos, Lorenzoni, and Pavan (2007), and Christiano et al. (2008).

Finally, McGrattan and Prescott (2007) acknowledge that the basic neoclassical growth model with rational expectations cannot account for the boom in U.S. business investment that occurred during the late 1990s. Along the lines of Hall (2001), they argue that accounting for investment in intangible capital helps to reconcile the model with the data.

2 Historical Motivation

The basic premise of the paper is that investors overreact to technological change. A reading of stock market history lends support to this view. Shiller (2000) argues that major stock price run-ups have generally coincided with the emergence of some superficially-plausible "new era" theory that involves the introduction of new technology. Figure 1 depicts four major run-ups in real U.S. stock prices.[8] Shiller associates each run-up with the following technological advances that contributed to new era enthusiasm:

- Early 1900s: High-speed rail travel, transatlantic radio, long-line electrical transmission.
- 1920s: Mass production of automobiles, travel by highways and roads, commercial radio broadcasts, widespread electrification of manufacturing.
- 1950s and 60s: Widespread introduction of television, advent of the suburban lifestyle, space travel.
- Late 1990s: Widespread availability of the internet, innovations in computers and information technology, emergence of the web-based business model.

[8] The series for real stock prices, real dividends, and real consumption employed in the paper are derived from Robert Shiller's website (http://www.econ.yale.edu/~shiller/), updated using revised data through 2008.

In comparing the late 1920s with the late 1990s, Gordon (2006) and White (2006) both emphasize the simultaneous occurrence of major technological innovations, a productivity revival, excess capital investment, and a stock market bubble fueled by speculation. Schwert (1989, 2002) documents the pronounced increase in stock market volatility that occurred during both periods, particularly in technology-related stocks in the late 1990s. Cooper et al. (2001) document a pronounced "dotcom effect" in the late 1990s, whereby internet-related corporate name changes produced permanent abnormal returns. The authors attribute their results to a form a speculative mania among investors for "glamour" industries that are associated with new technology.

Using data on newly-issued patents, Nicholas (2008) argues that the 1920s was "a period of unprecedented technological advance." The September 7, 1929 edition of *Business Week* famously remarked "For 5 years at least, American business has been in the grip of an apocalyptic holy-rolling exaltation over the unparalleled prosperity of the 'new era' upon which we, or it, or somebody has entered." The March 8, 1999 cover story of *Business Week* proclaimed "The high-tech industry is on the cusp of a new era in computing in which digital smarts won't be tied up in a mainframe, minicomputer, or PC. Instead, computing will come in a vast array of devices aimed at practically every aspect of our daily lives." Figure 2 illustrates the similarity of the stock price movements that took place during the two periods.

From 1996 until its peak in 2000 real business investment expanded at an average compound growth rate of 10 percent per year—about 2.5 times faster than the growth rate of the U.S. economy as a whole. Much of the surge in business investment in the late 1990s was linked to computers and information technology. During these years, measured productivity growth picked up, which was often cited as evidence of a permanent structural change—one that portended faster trend growth going forward.[9] A recent analysis by Ireland and Schuh (2008) concludes that the productivity revival of the 1990s was temporary rather than permanent. But at the time, widespread belief in the so-called "new economy" caused investors to bid up stock prices to unprecedented levels relative to dividends (Figure 3). The rise and fall of potential output growth (a proxy for the new economy's speed limit) coincides roughly with cyclical movements in the stock market (Figure 4). The figure motivates consideration of a model where excess volatility and bubbles can affect the economy's trend growth rate.

The investment boom of the late 1990s now appears to have been overdone. Firms overinvested in new productive capacity in an effort to satisfy a level of demand for their products that proved to be unsustainable.[10] Caballero et al. (2006) argue that rapidly rising stock prices provided firms with a low-cost source of funds from which to finance their investment projects. The resulting surge in capital accumulation served to increase measured productivity growth which, in turn, helped to justify the enormous run-up in stock prices. Figure 5 shows

[9] For an optimistic assessment at the time, see Oliner and Sichel (2000). For a sceptical view, see Gordon (2000).

[10] Gordon (2003) documents the many transitory factors that boosted the demand for technology products during the late 1990s. These include: (1) telecom industry deregulation, (2) the one-time invention of the world-wide-web, (4) the surge in equipment and software demand from the now-defunct dotcoms, and (4) a compressed personal computer replacement cycle heading into Y2K.

that the trajectory of the S&P 500 stock index, both before and after the bubble peak, is strikingly similar to the trajectory of investment.

On January 13, 2000, near the peak of the stock market, Fed Chairman Alan Greenspan raised the possibility that investors might have overreacted to recent productivity-enhancing innovations:

> "When we look back at the 1990s, from the perspective of say 2010...[w]e may conceivably conclude from that vantage point that, at the turn of the millennium, the American economy was experiencing a once-in-a-century acceleration of innovation, which propelled forward productivity, output, corporate profits, and stock prices at a pace not seen in generations, if ever. Alternatively, that 2010 retrospective might well conclude that a good deal of what we are currently experiencing was just one of the many euphoric speculative bubbles that have dotted human history. And, of course, we cannot rule out that we may look back and conclude that elements from both scenarios have been in play in recent years."

Figure 6 shows that one can observe similar comovement between asset prices and investment in the recent U.S. housing market. Real house prices rose sharply from 2000 to 2006 while real residential investment experienced an unprecedented boom. Both series have since reversed course dramatically. An accommodative interest rate environment, combined with a proliferation of new mortgage products (loans with little or no down payment, minimal documentation of income, and payments for interest-only or less), helped fuel the run-up in house prices.

On April 8, 2005, near the peak of the housing market, Fed Chairman Alan Greenspan offered the following optimistic assessment of new technology:

> [T]he financial services sector has been dramatically transformed by technology... Information processing technology has enabled creditors to achieve significant efficiencies in collecting and assimilating the data necessary to evaluate risk and make corresponding decisions about credit pricing. With these advances in technology, lenders have taken advantage of credit-scoring models and other techniques for efficiently extending credit to a broader spectrum of consumers...Where once more-marginal applicants would simply have been denied credit, lenders are now able to quite efficiently judge the risk posed by individual applicants and to price that risk appropriately. These improvements have led to rapid growth in subprime mortgage lending.

Feldstein (2007), citing a number of studies, argues that the rapid growth in subprime lending during these years was driven in part by "the widespread use of statistical risk assessment models by lenders." The subprime lending boom was later followed by a sharp rise in delinquencies and foreclosures, massive write-downs in the value of securities backed by subprime mortgages and derivatives, the collapse of a number of large financial institutions, and, most recently, a serious financial crisis prompting unprecedented government intervention in U.S. private capital markets. In retrospect, enthusiasm for a "new era" in credit risk modeling appears to have been overdone. Persons (1930. pp. 118-119) describes the fallout from an earlier era of rapid credit expansion as follows:

> "[I]t is highly probable that a considerable volume of sales recently made were based on credit ratings only justifiable on the theory that flush times were to continue indefinitely...When the process of expanding credit ceases and we return to a normal basis of spending each year...there must ensue a painful period of readjustment."

Shiller (2008) argues that the recent U.S. housing market experience bears striking similarities to previous real estate booms and busts in U.S. history. In an exhaustive historical study of financial market bubbles in many countries, Borio and Lowe (2002) argue that episodes of sustained rapid credit expansion, booming stock or house prices, and high levels of investment, are almost always followed by periods of economic stress as bubble-induced excesses are unwound.

3 Model

The representative agent is a capitalist-entrepreneur who maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\alpha} - 1}{1-\alpha} \right], \tag{1}$$

subject to the budget constraint

$$c_t \ + \ i_t \ = \ y_t, \qquad c_t,\ i_t > 0 \tag{2}$$

where c_t is consumption, i_t is investment, y_t is output (or income), β is the subjective time discount factor, and α is the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution). When $\alpha = 1$, the within-period utility function can be written as $\log(c_t)$. The symbol E_t represents the mathematical expectation operator.

Output is produced according to the technology

$$y_t \ = \ A \exp(z_t)\, k_t^{\theta} h_t^{1-\theta}, \qquad A > 0, \quad \theta \in (0, 1], \tag{3}$$

$$z_t \ = \ \rho\, z_{t-1} \ + \ \epsilon_t, \qquad \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right), \quad z_0 \text{ given}, \tag{4}$$

where k_t is the agent's stock of physical capital and z_t represents a persistent, mean-reverting technology shock. When $\theta < 1$, output is also affected by h_t, which represents the stock of human capital or knowledge. Following Arrow (1962) and Romer (1986), I assume that h_t grows proportionally to, and as a by-product of, accumulated private investment activities. This "learning-by-doing" formulation is captured by the specification $h_t = K_t$, where K_t is the economy-wide average capital stock per person which the agent takes as given. In equilibrium, all agents are identical, so we have $k_t = K_t$ which is imposed after the investment decision is made. When $\theta < 1$, the private marginal product of capital is less than the social marginal product such that agents underinvest relative to the socially-optimal level.

Resources devoted to investment augment the stock of physical capital according to the law of motion

$$k_{t+1} \;=\; B\, k_t^{1-\lambda}\, i_t^{\lambda}, \qquad B>0, \quad \lambda \in (0,1], \quad k_0 \text{ given}, \tag{5}$$

which reflects capital adjustment costs along the lines of Lucas and Prescott (1971). Equation (5) can be interpreted as a log-linearized version of the following specification employed by Jermann (1998) and Barlevy (2004):

$$\frac{k_{t+1}}{k_t} \;=\; 1-\delta+\psi_0\left(\frac{i_t}{k_t}\right)^{\psi_1} \;\simeq\; B\left(\frac{i_t}{k_t}\right)^{\lambda}, \tag{6}$$

$$\lambda = \frac{\psi_0\psi_1\left(\widetilde{i/k}\right)^{\psi_1}}{1\,-\,\delta\,+\,\psi_0\left(\widetilde{i/k}\right)^{\psi_1}}, \qquad B = \frac{1\,-\,\delta\,+\,\psi_0\left(\widetilde{i/k}\right)^{\psi_1}}{\left(\widetilde{i/k}\right)^{\lambda}},$$

where λ and B are Taylor series coefficients and $\widetilde{i/k} = \exp\{E[\log(i_t/k_t)]\}$ is the approximation point.[11]

The agent's first-order condition with respect to k_{t+1} is given by

$$\frac{i_t\, c_t^{-\alpha}}{\lambda k_{t+1}} \;=\; E_t\, \beta\, c_{t+1}^{-\alpha}\left[\frac{\theta y_{t+1}}{k_{t+1}} \;+\; \frac{(1-\lambda)\, i_{t+1}}{\lambda k_{t+1}}\right], \tag{7}$$

where k_{t+1} is known at time t. The first-order condition can be rearranged to obtain the following standard asset pricing equation

$$\underbrace{i_t/\lambda}_{p_t} \;=\; E_t\,\beta\left[\frac{c_{t+1}}{c_t}\right]^{-\alpha}[\underbrace{\theta y_{t+1}-i_{t+1}}_{d_{t+1}} \;+\; \underbrace{i_{t+1}/\lambda}_{p_{t+1}}], \tag{8}$$

where $p_t \equiv i_t/\lambda$ is the ex-dividend price of an equity share with claim to a perpetual stream of dividends $d_t \equiv \theta y_t - i_t$. When $\theta = 1$, consumption is equal to dividends, analogous to the Lucas (1978) endowment economy. When $\theta < 1$, consumption strictly exceeds dividends, owing to the presence of the learning-by-doing externality which can be viewed as separate source of income for the agent. The term $\beta\,(c_{t+1}/c_t)^{-\alpha}$ is the stochastic discount factor.

The model's adjustment cost specification (5) implies a direct link between the equity price p_t and investment in physical capital i_t, consistent with a standard Tobin's q framework. This feature is also consistent with the observed comovement between U.S. asset prices and the corresponding investment series shown earlier in Figures 5 and 6. Although the model implies perfect comovement between p_t and i_t, this prediction could be relaxed by introducing stochastic variation in the adjustment cost parameter λ.

[11] Since the functional form of the constraint affects the agent's intertemporal optimality condition, the economic environment considered here is not isomorphic to that of Jermann (1998) and Barlevy (2004).

The gross return from holding the equity share from period t to $t+1$ is given by

$$R_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t} = \frac{i_{t+1}}{i_t}\left[\frac{\lambda\theta y_{t+1}}{i_{t+1}}+1-\lambda\right], \tag{9}$$

which shows that return volatility is driven by the volatility of investment growth and by shifts in the output-investment ratio.

To facilitate a solution to the agent's problem, the first-order condition (8) must be rewritten in terms of stationary variables. If we define the price-consumption ratio as $x_t \equiv p_t/c_t = (i_t/\lambda)/c_t$, then the budget constraint (2) can be used to derive the following expressions for the equilibrium allocations:

$$c_t = \left[\frac{1}{1+\lambda\, x_t}\right] y_t, \tag{10}$$

$$i_t = \left[\frac{\lambda\, x_t}{1+\lambda\, x_t}\right] y_t, \tag{11}$$

$$d_t = \left[\frac{\theta-(1-\theta)\,\lambda\, x_t}{1+\lambda\, x_t}\right] y_t, \tag{12}$$

where $y_t = A\exp(z_t)\, k_t$ in equilibrium. The price-dividend ratio can be written as

$$\frac{p_t}{d_t} = \frac{i_t/\lambda}{d_t} = \frac{x_t}{\theta-(1-\theta)\,\lambda\, x_t}. \tag{13}$$

which is a non-linear function of the price-consumption ratio x_t. When there is no productive externality, we have $\theta = 1$ such that $p_t/d_t = x_t$.

An expression for equilibrium consumption growth can be obtained by combining (10), (11), and (5) to yield

$$\begin{aligned}\frac{c_{t+1}}{c_t} &= \left[\frac{1+\lambda\, x_t}{1+\lambda\, x_{t+1}}\right]\frac{y_{t+1}}{y_t} = \left[\frac{1+\lambda\, x_t}{1+\lambda\, x_{t+1}}\right]\exp(z_{t+1}-z_t)\,\frac{k_{t+1}}{k_t},\\ &= BA^{\lambda}\left[\frac{(1+\lambda\, x_t)^{1-\lambda}\,(\lambda\, x_t)^{\lambda}}{1+\lambda\, x_{t+1}}\right]\exp\left[z_{t+1}-(1-\lambda)\; z_t\right].\end{aligned} \tag{14}$$

Substituting the above expression into equation (8) together with $y_{t+1} = c_{t+1}+i_{t+1}$ yields the following transformed version of the first-order condition in terms of stationary variables:

$$\frac{x_t^{1-\lambda\phi}\exp\left[\phi\,(1-\lambda)\, z_t\right]}{(1+\lambda\, x_t)^{(1-\lambda)\phi}} = E_t\,\underbrace{\widetilde{\beta}\left[\frac{\theta+x_{t+1}\,(1-\lambda+\lambda\theta)}{(1+\lambda\, x_{t+1})^{\phi}}\right]\exp(\phi\, z_{t+1})}_{w_{t+1}}, \tag{15}$$

$$\phi \equiv 1-\alpha, \qquad \widetilde{\beta} \equiv \beta\left[B\,(A\lambda)^{\lambda}\right]^{\phi},$$

where w_{t+1} represents the non-linear combination of variables which the agent must forecast.

The observed technology shock z_t and the existing capital stock k_t uniquely determine the amount of per capita output according to (3). Each period, the agent must only decide the fraction of available output to be devoted to investment, with the remainder devoted to consumption. Since $i_t/c_t = \lambda x_t$, the agent's decision problem can be formulated equivalently in terms of the price-consumption ratio x_t, which is stationary.

3.1 Rational Expectations

The transformed first-order condition (15) is a non-linear stochastic difference equation. Except for the special case of log utility $(\phi = 0)$, an exact analytical solution cannot be obtained. To facilitate an analytical solution, both sides of equation (15) are approximated as power functions around the points $\widetilde{x} = \exp\{E[\log(x_t)]\}$ and $\widetilde{z} = 0$ to obtain:

$$a_0 \left[\frac{x_t}{\widetilde{x}}\right]^{a_1} \exp\left[\phi\,(1-\lambda)\,z_t\right] \quad = \quad E_t \underbrace{b_0 \left[\frac{x_{t+1}}{\widetilde{x}}\right]^{b_1} \exp\left(\phi\, z_{t+1}\right)}_{w_{t+1}} \tag{16}$$

where a_0, a_1, b_0, and b_1 are Taylor series coefficients that depend on $\widetilde{x}$, as defined in Appendix A. The right-side of equation (16) is a log-linear approximation of the non-linear composite variable w_{t+1}. In equilibrium, movements in $\log(w_{t+1})$ and $\log(x_{t+1})$ are both driven solely by movements in the technology shock z_{t+1}.

Equation (16) requires the agent to construct a joint conditional forecast of x_{t+1} and z_{t+1}. Anticipating the possibility (explored in later sections) that the agent may not be endowed with knowledge of the stochastic process for z_{t+1}, it is convenient to recast the first-order condition (16) in terms of the composite variable w_t as follows

$$a_0 \left[\frac{w_t}{\widetilde{w}}\right]^{\frac{a_1}{b_1}} \exp\left[\phi\left(1-\lambda-\frac{a_1}{b_1}\right) z_t\right] \;=\; E_t w_{t+1} \tag{17}$$

$$\text{where} \qquad \frac{w_t}{\widetilde{w}} = \left[\frac{x_t}{\widetilde{x}}\right]^{b_1} \exp\left(\phi\, z_t\right), \qquad \widetilde{w} = b_0 = \exp\left\{E\left[\log\left(w_t\right)\right]\right\}.$$

The recast first-order condition highlights the self-referential nature of the agent's decision problem, whereby the realized value of w_t depends on the agent's conditional forecast of that same variable.

The rational solution to the recast first-order condition (17) is given by the following proposition.

Proposition 1. *The approximate rational law of motion for the composite variable w_t is given by*

$$w_t \;=\; \widetilde{w}\exp\left(m\, z_t\right),$$

where $\widetilde{w} = \exp\{E[\log(w_t)]\}$ is the approximation point and the technology response coefficient is given by

$$m \;=\; \frac{\phi\,\left[a_1 - (1-\lambda)\, b_1\right]}{a_1 - \rho b_1}.$$

Proof: See Appendix A.

Given the rational law of motion for w_t, the corresponding rational forecast rule is

$$E_t w_{t+1} \;=\; \widetilde{w}\,\exp\left[m\rho\, z_t \,+\, \tfrac{1}{2}m^2\sigma_\varepsilon^2\right]\,, \tag{18}$$

which assumes that the agent has knowledge of the technology shock parameters ρ and σ_ε^2. This assumption will be relaxed in later sections.

Making use of the definitional relationship between w_t and x_t shown in equation (17), it is straightforward to recover the following law of motion for the rational price-consumption ratio:

$$x_t \;=\; \widetilde{x}\exp\left(\gamma\, z_t\right), \tag{19}$$

$$\text{where} \qquad \gamma \;=\; \frac{m-\phi}{b_1} \;=\; \frac{\phi\left[\rho-(1-\lambda)\right]}{a_1-\rho b_1}$$

In the special case of logarithmic utility, we have $\phi = 0$ such that $m = \gamma = 0$, resulting in $w_t = \widetilde{w}$ and $x_t = \widetilde{x}$ for all t. From equation (13), the price-dividend ratio p_t/d_t is also constant in the logarithmic case. When $\phi \neq 0$, the variables w_t, x_t, and p_t/d_t all respond to technology shocks. The direction of movement depends on the relative magnitudes of the income and substitution effects of the shock, which in turn are governed by the intertemporal elasticity of substitution (IES), as given by $1/(1-\phi)$. When the risk coefficient $\alpha > 1$, we have $\phi < 0$, such that IES < 1. The sign of γ depends not only on the sign of ϕ, but also on the sign of $\rho - (1-\lambda)$.[12] In the baseline model calibration, the result is $\gamma > 0$. In this case, the substitution effect dominates the income effect and the agent's investment increases relative to consumption in response to a positive tenchnology shock. For moderate levels of risk aversion, the variables w_t, x_t, and p_t/d_t exhibit very little volatility because the income and substitution effects largely offset one another.

3.2 Speculative Overreaction

I introduce excess volatility by assuming that agents overreact to observed technology shocks when making forecasts about the future. Later, in Section 5, I demonstrate that this behavior tends to be self-confirming, particularly when agents misperceive the stochastic process driving technology shocks. The form of misperception that I consider is intended to capture some of the flavor of "new era enthusiasm" that has characterized historical bubble episodes.

Suppose that agents mistakenly believe that technology shocks are governed by the following driftless random walk

$$z_t = z_{t-1} + u_t \qquad\qquad u_t \sim N\left(0, \sigma_u^2\right), \tag{20}$$

which implies that agents perceive technology innovations to be permanent, when in fact, innovations are temporary. In the numerical simulations, I set $\rho = 0.9$, a typical value for

[12] For all calibrations examined, $a_1 - \rho b_1 > 0$.

annual data. When $\rho = 0.9$, the true stochastic process (4) implies that z_t is stationary but highly persistent. A large amount of data would be required before agents could reject the hypothesis of $\rho = 1$. Given a series of observations of z_t, agents could compute the perceived permanent innovation variance from the relationship $\sigma_u^2 = Var\left(\Delta z_t\right)$. The true stochastic process for z_t implies $Var\left(\Delta z_t\right) = 2\sigma_\epsilon^2 / \left(1+\rho\right)$. Hence, in the calibration of the speculation model, I set the perceived permanent innovation variance equal to $\sigma_u^2 = 2\sigma_\varepsilon^2 / \left(1+\rho\right)$.

To derive an approximate solution to the speculation model, I again make use of a recast first-order condition which is formulated in terms of the composite variable $w_{s,t}$. The subscript "s" denotes values associated with the speculation model which differ from those in the rational model. The recast first-order condition is

$$a_{0s} \left[\frac{w_{s,t}}{\widetilde{w}_s}\right]^{\frac{a_{1s}}{b_{1s}}} \exp\left[\phi\left(1-\lambda-\frac{a_{1s}}{b_{1s}}\right) z_t\right] \;=\; \widehat{E}_t w_{s,t+1} \tag{21}$$

$$\text{where} \qquad w_{s,t} = \widetilde{w}_s \left[\frac{x_{s,t}}{\widetilde{x}_s}\right]^{b_{1s}} \exp\left(\phi\, z_t\right).$$

The symbol $\widehat{E}_t$ represents the speculative agent's forecast conditioned on beliefs about the law of motion of $w_{s,t}$ and the stochastic process governing z_t. The Taylor series coefficients for the speculation model are denoted by a_{0s}, a_{1s}, b_{0s}, and b_{1s}, which have the same functional form as their rational counterparts. The approximation point for the actual law of motion of $w_{s,t}$ is given by $\widetilde{w}_s = b_{0s} = \exp\left\{E \log\left(w_{s,t}\right)\right\}$.

Analogous to the rational model, the speculative agent's perceived law of motion (PLM) for $w_{s,t}$ takes the following form

$$w_{s,t} \;=\; \widetilde{w}_s \exp\left(m_s\, z_t\right), \tag{22}$$

where $\widetilde{w}_s$ and m_s can be interpreted as parameters to be estimated from observable data. Making use of the above PLM and the perceived technology shock process (20), the corresponding speculative forecast rule is

$$\widehat{E}_t w_{s,t+1} \;=\; \widetilde{w}_s \exp\left[m_s\, z_t \,+\, \tfrac{1}{2} m_s^2 \sigma_u^2\right]. \tag{23}$$

Substituting the above forecast into the approximate first-order condition (21) yields the following actual law of motion (ALM) for $w_{s,t}$:

$$w_{s,t} \;=\; \widetilde{w}_s \exp\left\{\left[\frac{m_s\, b_{1s}}{a_{1s}} + \frac{\phi\left[a_{1s} - \left(1-\lambda\right) b_{1s}\right]}{a_{1s}}\right] z_t\right\}, \tag{24}$$

which takes the same form as the PLM. The important point to note is that the actual response coefficient (given by the term in square brackets) is linearly increasing in the perceived response coefficient m_s. Thus, if the agent initially adopts a value of m_s that is larger in magnitude than the rational expectations value m, the resulting overreaction and excess volatility will tend to be self confirming. Moreover, when b_{1s}/a_{1s} is close to 1, as it is in the calibrated version of the model, a real-time learning algorithm can deliver extremely long-lived overreaction behavior.

Making use of the definitional relationship between $w_{s,t}$ and $x_{s,t}$ shown in equation (21), we can recover the following actual law of motion for the speculative price-consumption ratio:

$$x_{s,t} \;=\; \widetilde{x}_s \exp\left(\gamma_s \, z_t\right), \tag{25}$$

$$\text{where} \qquad \gamma_s \;=\; \frac{m_s \,-\, \phi\,(1-\lambda)}{a_{1s}},$$

which takes the same form as the rational price-consumption ratio (19), but has a different technology response coefficient. Given $x_{s,t}$, the speculative price-dividend ratio is computed from

$$\frac{p_{s,t}}{d_{s,t}} = \frac{x_{s,t}}{\theta - (1-\theta)\,\lambda\, x_{s.t}}. \tag{26}$$

Due to the non-linear nature of the above equation, the presence of excess volatility in $x_{s,t}$ can generate sharp run-ups and crashes in the price-dividend ratio which resemble patterns observed in long-run U.S. data.

4 Model Calibration

A time period in the model is taken to be one year. The speculative agent's perceived coefficient m_s is calibrated so that the model matches the volatility of the price-dividend ratio in long-run annual U.S. data. The remaining parameters of the speculation model are chosen simultaneously to match various empirical targets, as summarized in Table 1. For example, the volatility of the technology shock innovation σ_ε is chosen so that the model matches the standard deviation of real per capita consumption growth of nondurables and services in long-run annual U.S. data. Appendix B contains the approximate analytical moments that are used to calibrate the speculation model.

The rational model employs the same deep parameter values as the speculation model. The rational agent's technology response coefficient m is endogenous. For the quantitative analysis, I examine a range of values for the externality parameter θ and the risk coefficient α. The baseline calibration is $\theta = 0.4$ and $\alpha = 1.5$. When either θ or α is changed, the remaining parameters are adjusted to maintain the same targets shown in Table 1.

Given the calibrated values of λ and B shown in Table 1, equation (6) can be used to recover the implied curvature parameter ψ_1 for comparison with Barlevy (2004). Assuming an annual depreciation rate of $\delta = 0.1$, equation (6) yields $\psi_1 = 0.59$ when $\theta = 0.4$, and yields $\psi_1 = 0.12$ when $\theta = 1.0$. Barlevy (2004) considers values in the range $0.12 \leq \psi_1 \leq 0.26$ for an endogenous growth model that corresponds to the $\theta = 1.0$ case. As $\psi_1 \to 1.0$, the implied adjustment costs approach zero. Hence, the calibration methodology used here delivers lower implied adjustment costs when $\theta < 1$.

Table 1: Example Calibrations for the Speculation Model

Parameter	Value		Description/Empirical Target
θ	0.4	1.0	Capital share of income.
α	1.5	1.5	Coefficient of relative risk aversion.
A	0.333	0.333	Mean capital-output ratio = 3.
λ	0.070	0.014	Mean investment-output ratio = 0.25.
B	1.216	1.056	Mean consumption growth = 1.98 %.
σ_ε	0.059	0.091	Volatility of consumption growth = 3.99 %..
ρ	0.9	0.9	Technology shock persistence in annual data.
σ_u	0.060	0.093	Perceived innovation variance $\sigma_u^2 = 2\sigma_\varepsilon^2/(1+\rho)$.
β	0.967	0.949	Mean price-dividend ratio = 26.6.
m_s	1.165	2.257	Volatility of price-dividend ratio = 13.8.
m	-0.427	-0.234	Rational model value.
γ_s	1.416	2.434	Speculation model value.
γ	0.070	0.257	Rational model value.

When $\theta = 0.4$, the parameter values in Table 1 yield $\gamma_s = 1.416$ from equation (25) and $\gamma = 0.070$ from equation (19). Consequently, the speculative price-consumption ratio $x_{s,t}$ will exhibit much more volatility than the rational price-consumption ratio x_t. The positive sign of the rational coefficient γ reflects the dominance of the substitution effect over the income effect for these parameter values.

When $\theta = 0.4$, the parameter values in Table 1 yield $b_{1s}/a_{1s} = 0.907$, which appears in the actual law of motion (24). When $\theta = 1$, the result is $b_{1s}/a_{1s} = 0.960$. Consequently, the actual response coefficient in the actual law of motion (24) will increase nearly one-for-one with the perceived response coefficient m_s.

5 Self-Confirming Nature of Overreaction

5.1 Forecast Errors

Figure 7 illustrates the self-confirming nature of overreaction. The left panel plots the actual technology response coefficient in the actual law of motion (24) as a function of the perceived response coefficient m_s. The plotted line has a slope of 0.907 and a vertical intercept of -0.078. In contrast, the 45-degree line has a slope of 1.0 and a vertical intercept of 0.0. At the baseline calibration, the perceived coefficient of $m_s = 1.165$ generates an actual response coefficient of 0.978. Thus, a large amount of data would be required before the agent could statistically reject the hypothesis that the perceived coefficient is the same as the actual coefficient.[13]

The right panel of Figure 7 figure plots the autocorrelation of percentage forecast errors observed by the agent for the baseline calibration of $\alpha = 1.5$ and $\theta = 0.4$. The percentage

[13] Because the agent mispercieves the technology process, the fixed point of the map shown in the left of Figure 7 does not coincide with the rational reponse coefficient m. The fixed point of the map occurs at $m_s = -0.842$, whereas $m = -0.427$ from Table 1.

forecast error is defined as

$$err_{s,t+1} = \log\left(\frac{w_{s,t+1}}{\widehat{E}_t w_{s,t+1}}\right), \tag{27}$$

where $w_{s,t+1}$ is derived from the actual law of motion (24) and $\widehat{E}_t w_{s,t+1}$ is derived from the perceived law of motion, as shown in equation (23). For the rational model, the percentage forecast error is defined similarly, but in this case w_{t+1} and $E_t w_{t+1}$ are both derived from the actual law of motion shown in Proposition 1. The figure shows that large values of m_s yield forecast errors which exhibit near-zero autocorrelation, making it difficult for the agent to detect a misspecification of the forecast rule.

Table 2 provides some additional properties of the forecast errors computed from a long simulation of the model. The root-mean squared percentage error is given by $RMSPE = 100\left[Var\left(err_{s,t+1}\right) + E\left(err_{s,t+1}\right)^2\right]^{0.5}$ The simulated time series for $w_{s,t}$ and w_t are computed by solving the original nonlinear first-order condition (15) at each time step of the simulation, as described in Appendix C.

Table 2: Properties of Percentage Forecast Errors

	$\theta = 0.4$		$\theta = 1.0$	
	Rational Model	Speculation Model	Rational Model	Speculation Model
Mean	0.01 %	0.00 %	0.00 %	1.53 %
RMSPE	2.52 %	6.96 %	2.14 %	20.8 %
Corr. Lag 1	−0.01	0.07	−0.01	−0.04
Corr. Lag 2	0.00	0.07	0.00	−0.03
Corr. Lag 3	0.00	0.06	0.00	−0.02

Note: Statistics are from 15,000 period simulation with $\alpha = 1.5$.

The table shows that forecast errors observed by the speculative agent remain close to white noise even for higher order lags. Experiments with the model show that the forecast errors become more persistent at higher levels of risk aversion. Intuitively, the vertical intercept of the solid blue line in the left panel of Figure 7 becomes more negative as the risk coefficient α increases, thus producing a wider gap between the actual and perceived values of the technology response coefficient. At the baseline calibration with $\theta = 0.4$, we have $RMSPE \simeq 7\%$ in the speculation model versus $RMSPE \simeq 2.5\%$ in the rational model.

Although not shown in the table, one can also compute the forecast errors that arise when the rational fundamentals-based forecast rule (18) is used to predict the realized value of $w_{s,t+1}$ in the speculation model. These errors would be of interest to a speculative agent who is contemplating a switch to a fundamentals-based forecast. In deciding whether to switch forecasts, the agent could keep track of the forecast errors associated with each method. Before any switch occurs, the actual law of motion for $w_{s,t}$ would still be governed by (24). In simulations, the speculative forecast rule (23) significantly outperforms the rational fundamentals-based forecast when predicting the realized value of $w_{s,t+1}$. For example, when $\theta = 0.4$, we have $RMSPE \simeq 7\%$ in the speculation model versus $RMSPE \simeq 18\%$ when the fundamentals-based forecast is used to predict the realized value of $w_{s,t+1}$. From the perspective of an individual agent, switching to the fundamentals-based forecast would appear to reduce forecast accuracy, so there is no incentive to switch. In other words, an individual agent would become "locked-in" to the speculative forecast so long as other agents are following the same approach.[14]

5.2 Real-Time Learning

Figure 8 plots sample real-time learning paths for the baseline calibration $\alpha = 1.5$ and $\theta = 0.4$. Details of the learning algorithm are contained in Appendix C. For each simulation, the starting value for the technology response coefficient in the agent's forecast rule is either above or below the rational fixed point value of $m = -0.427$. The starting values for the forecast rule parameters are maintained for the first ten periods of the simulation. Each period thereafter, the agent computes a new set of parameters from the sample moments of the observed time series, using all past data. The figure plots the first 3,000 periods of a 50,000 period simulation.

In the left panel of Figure 8, the agent's forecast is constructed in the form of (23) using the misperceived technology process (20). In the right panel, the agent's starts with the misperceived technology process, and then constructs a forecast in the form of (18) where estimates for ρ, and σ_ε are computed each period from the sample moments of the observed technology shocks.

The simulations show that the estimated technology response coefficient is path-dependent, exhibiting sensitivity not only to the starting value, but also to the particular sequence of random shocks. If the agent's initial forecast is characterized by overreaction, then overreaction can persist for thousands of periods, particularly in the left panel where temporary technology innovations are perceived to be permanent. The estimated response coefficients fluctuate rapidly at first, but then settle into nearly-flat trajectories thereafter.

In the left panel, when the starting value of the technology response coefficient is 3.0, the ten simulated learning paths yield an average response coefficient of 1.27 after 3,000 periods. After 50,000 periods, the response coefficients range from a low of 0.173 to a high of 1.49, with an average value of 0.805. In the right panel, the corresponding ten simulated learning paths yield an average response coefficient of 0.893 after 3,000 periods. After 50,000 periods, the response coefficients range from a low of -0.144 to a high of 1.27, with an average value of 0.361. While movement in the direction of the rational reponse coefficient ($m = -0.427$) does occur on average, the learning process is extremely slow. Moreover, individual learning paths can imply significant overreaction behavior even after 50,000 periods.

The intuition for the slow learning process is straightforward. The map that relates the perceived response coefficient to the actual response coefficient (Figure 7) lies very close to the 45-degree line. Consequently, small amounts of sampling variation in the moments of the observed time series can often overcome the map's weak convergence properties toward a fixed point. The map shown in Figure 7 is based on a log-linear approximation of the model. In contrast, the real-time learning algorithm is defined by a system of nonlinear stochastic difference equations. The nonlinear features of the algorithm affect the sampling variation

[14] Lansing (2006) examines the concept of forecast lock-in using a standard Lucas-type asset pricing model.

in the observed time series, thereby influencing the speed of convergence and the end-of-simulation values. Overall, the simulations suggest that long-lived overreaction behavior can be justified as a possible outcome of a self-referential learning process where the agent acts like an econometrician.

6 Model Simulations

This section examines the ability of the speculation model to match various features of U.S. data.

Table 3 presents unconditional moments of asset pricing variables computed from a long simulation of the model, where $\mu^d_{t+1} \equiv \log(d_{t+1}/d_t)$ and $\mu^c_{t+1} \equiv \log(c_{t+1}/c_t)$ are the growth rates of dividends and consumption, respectively. The table also reports the corresponding statistics from long-run U.S. data.[15]

Table 3: Unconditional Asset Pricing Moments

Statistic	U.S. Data	Rational Model	Speculation Model
Mean p_t/d_t	26.6	22.8	26.6
Std. Dev.	13.8	0.42	13.7
Skew.	2.20	0.04	4.12
Kurt.	8.21	2.94	42.1
Corr. Lag 1	0.93	0.90	0.84
Mean R_{t+1}	7.84 %	6.64 %	7.26 %
Std. Dev.	17.8 %	6.63 %	12.6 %
Corr. Lag 1	0.04	−0.04	−0.06
Mean μ^d_{t+1}	1.36 %	1.94 %	1.99 %
Std. Dev.	12.2 %	5.42 %	7.80 %
Corr. Lag 1	0.11	−0.03	−0.02
Mean μ^c_{t+1}	1.98 %	1.94 %	1.99 %
Std. Dev.	3.99 %	5.82 %	3.98 %
Corr. Lag 1	0.25	−0.03	0.17

Note: Model statistics are from 15,000 period simulation with $\theta = 0.4$, $\alpha = 1.5$.

Recall that the speculation model is calibrated to match the mean and volatility of the price-dividend ratio in the data. But the model also does a good job of matching other moments. In particular, the U.S. price-dividend ratio exhibits positive skewness and excess kurtosis, which suggest the presence of nonlinearities in the data. The speculation model is able to capture these features because excess volatility in $x_{s,t}$, together with the non-linear form of equation (26), produces intermittent run-ups and crashes in the model price-dividend ratio. In contrast, the rational model delivers very low volatility, near-zero skewness, and no

[15] The sample periods for the U.S. data shown in Table 3 are as follows: price-dividend ratio 1871-2008, real equity return 1871-2008, real dividend growth 1872-2008, real consumption growth 1890-2008. The price-dividend ratio in year t is defined as the value of the S&P 500 stock index at the beginning of year $t+1$, divided by the accumulated dividend over year t.

excess kurtosis. The persistence of the price-dividend ratio in both models is inherited from persistent technology shock process with $\rho = 0.9$.

The mean equity return for both models is just slightly below the long-run U.S. average of 7.84%. The volatility of returns in the speculation model is about twice that of the rational model (12.6% versus 6.63%), but somewhat below the return volatility of 17.8% in the data. The reason the speculation model underpredicts the U.S. return volatility is because it underpredicts the volatility of dividend growth, which is one component of the return. The volatility of dividend growth in the speculation model is 7.8% whereas the corresponding figure in U.S. data is 12.2%. From equation (12), the volatility of dividend growth in either model could be increased by introducing stochastic variation in the production function parameter θ.

The speculation model is calibrated to match the first and second moments of consumption growth in the data. Consumption growth in the speculation model exhibits small positive serial correlation, with a coefficient of 0.17, which is close to the correlation coefficient of 0.25 in the data.

Figure 9 plots simulations from both models for the baseline calibration with $\theta = 0.4$ and $\alpha = 1.5$. In the top left panel, the highly persistent and volatile nature of the speculative price-dividend ratio gives rise to intermittent excursions away from the rational (or fundamental) value. At the baseline calibration, we have $\gamma_s = 1.416$, which implies that the speculative valuation ratios increase in response to a positive technology shock. The technology-driven bubble episodes in the model coincide with economic booms and excess capital formation, as shown in the lower panels of Figure 9. These episodes are reminiscent of the U.S. economy during the late 1920s and late 1990s. Interestingly, the speculation model can also generate prolonged periods where the price-dividend ratio remains in close proximity to the rational value. This is because a speculative agent behaves much like a rational agent so long as technology shocks remain small. Consequently, only a fraction of the cyclical fluctuations in the model are due to bubble-like episodes.

Figure 10 plots annual growth rates of macroeconomic variables from model simulations. Table 4 compares the volatilities of the model growth rates to those in the data.[16] In the rational model, the presence of capital adjustment costs makes the volatility of investment growth about the same as the volatility of output growth, which is counterfactual. In long-run U.S. data, investment growth is about three times more volatile than output growth. By construction, the speculation model magnifies equity price volatility which is linked directly to the volatility of investment growth. Given that output growth volatility in the two models is about the same, the excess volatility of investment growth in the speculation model results in a lower volatility of consumption growth relative to the rational benchmark. This result has important implications for the welfare analysis, which is discussed in the next section.

[16] Data on per capita real GDP from 1870-2008 was obtained from globalfinancialdata.com. Data on real business fixed investment from 1929-2008 was obtained from the U.S. Bureau of Economic Analysis/Haver Analytics.

Table 4: Volatility of Real Growth Rates

Variable	Dates	U.S. Economy	Rational Model	Speculation Model
$\Delta \log(y_t)$	1871-2008	5.28	5.93	5.94
$\Delta \log(c_t)$	1890-2008	3.99	5.82	3.98
$\Delta \log(i_t)$	1930-2008	16.2	6.24	12.2
$\Delta \log(d_t)$	1872-2008	12.2	5.42	7.80
$\Delta \log(p_t)$	1872-2008	17.9	6.24	12.2

Note: In percent, from 15,000 period simulation with $\theta = 0.4$, $\alpha = 1.5$.

6.1 Welfare Cost of Speculation and Business Cycles

This section examines the welfare costs of fluctuations that can be attributed to either: (i) speculative overreaction, or (ii) business cycles. Welfare costs are measured by the percentage change in per-period consumption that makes the agent indifferent between the two economies being compared. The details of the welfare computations are contained in Appendix D.

The basic intuition underlying the welfare results is as follows:

- Fluctuations that are driven by speculation or business cycles can affect both the mean and volatility of consumption growth.
- A decrease in mean consumption growth is associated with a smaller fraction of resources devoted to investment, and hence a higher initial level of consumption. Higher initial consumption can mitigate the welfare costs of slower growth.
- Higher initial consumption is less desirable from a welfare standpoint when agents underinvest, i.e., when $\theta < 1$.
- As risk aversion increases, consumption growth volatility becomes more costly in terms of welfare.

Which of these various effects dominate depends crucially on parameter values. Table 5 summarizes the moments of $\log(c_{t+1}/c_t)$ for three different versions of the model. For each value of the risk coefficient α, the speculation model is calibrated to match the mean and volatility of consumption growth in long-run U.S. data. The deterministic model sets $z_t = 0$ for all t such that consumption growth is constant at the deterministic steady-state value. As described in Appendix D, the initial level of consumption in the deterministic model differs from the average initial consumption levels in the fluctuating models.

Fluctuations in the price-consumption ratio affect the mean and volatility of consumption growth via equation (14), which is nonlinear. Depending on the degree of risk aversion, speculation may increase or decrease mean consumption growth relative to the rational or deterministic models. Table 5 shows that when risk aversion is very low, speculation increases mean consumption growth relative to both the rational and deterministic models, while the reverse holds true for higher risk aversion. But for any degree of risk aversion, speculation *reduces* the volatility of consumption growth relative to the rational model, consistent with the earlier discussion of Figure 10 and Table 4.

Table 5: Mean and Volatility of Consumption Growth

		$\theta = 0.4$		
α	Statistic	Deterministic Model	Rational Model	Speculation Model
0.5	Mean	1.62	1.61	2.00
	Std. Dev.	0	6.09	3.97
1.5	Mean	1.96	1.94	1.99
	Std. Dev.	0	5.82	3.98
2.5	Mean	2.06	2.12	1.98
	Std. Dev.	0	5.69	3.98

Note: In percent. Statistics are averages from a 15,000 period simulation.

Some insight into the effect of fluctuations on investment and growth can be obtained from the equation that defines the approximation point for the rational price-consumption ratio. As shown in appendix A, the approximation point is given by

$$\widetilde{x} = \exp\left\{E\left[\log\left(x_t\right)\right]\right\} = \frac{\theta\beta\exp\left[\phi\widetilde{\mu} + m^2\sigma_\varepsilon^2/2\right]}{1 - \beta\left(1 - \lambda + \lambda\theta\right)\exp\left[\phi\widetilde{\mu} + m^2\sigma_\varepsilon^2/2\right]}, \tag{28}$$

where $\widetilde{\mu}$ is the endogenous trend growth rate of consumption that depends on $\widetilde{x}$. The above equation shows that an increase in the magnitude of the technology response coefficient m that multiplies the innovation variance σ_ε^2 serves to increase the magnitude of $\widetilde{x}$. An increase in $\widetilde{x}$ implies an increase in the mean investment-consumption ratio relative to the deterministic model. This effect works through the channel of increased precautionary saving. However, there is another effect that works in the opposite direction. From equation (11). we see that i_t/y_t is an increasing concave function of the price-consumption ratio x_t. Hence, fluctuations which increase the volatility of x_t will serve to push down the average value of i_t/y_t via Jensen's inequality. Which of these two effects dominates depends on parameter values.

Table 6 summarizes the welfare cost of speculation relative to a rational model with identical parameter values. The results are also plotted in the left panel of Figure 11. Interestingly, speculation can improve welfare if risk aversion is low and agents underinvest relative to the socially-optimal level ($\theta < 1$). Higher levels of risk aversion cause the welfare cost of speculation to increase rapidly when $\theta < 1$, but the welfare costs decline a bit with risk aversion when $\theta = 1$. At low levels of risk aversion, speculation increases mean consumption growth by boosting the average investment-output ratio. Devoting more resources to investment yields a large welfare pay-off when agents underinvest. However, at higher levels of risk aversion, speculation reduces mean consumption growth by lowering the average investment-output ratio—an effect that is particularly costly when the economy already suffers from an underinvestment problem. This intuition accounts for the steeper slope of the welfare cost plot when $\theta = 0.4$. As θ increases toward 1.0, the plot rotates clockwise to become flatter, thus shifting the cross-over point where welfare costs equal zero.

When $\theta = 1$, there is no underinvestment problem. In this case, the speculation model's inefficient response to technology shocks must be weighed against the rational model's higher consumption growth volatility, with the latter taking on greater significance for welfare at higher degrees of risk aversion. Consequently, as risk aversion rises with $\theta = 1$, the welfare cost of speculation relative to the rational model exhibits a declining tendency over the range $0.5 \leq \alpha \leq 2.5$.

Table 6: Welfare Cost of Speculation

α	$\theta = 0.4$	$\theta = 0.6$	$\theta = 1.0$
0.5	-7.90	-3.93	6.20
1.0	-3.21	-2.56	4.72
1.5	0.74	-1.11	3.55
2.0	4.76	0.48	2.67
2.5	9.56	2.28	2.05

Note: In percent of per-period consumption.

Table 7 summarizes the welfare cost of business cycles in the speculation model relative to a deterministic model with identical parameter values. The results are also plotted in the right panel of Figure 11. I focus on the welfare cost of business cycles in the speculation model (as opposed to the rational model) because, by construction, the speculation model matches the empirical targets listed in Table 1, and hence is a more realistic representation of the U.S. economy for the chosen parameter values.

The general pattern of welfare costs shown in Table 7 is similar to that of Table 6. The welfare comparison between the speculation model and the deterministic model shown in Table 7 can be interpreted as a more extreme experiment in the effects removing fluctuations relative to that shown in Table 6. As before, the welfare cost plot in the right panel of Figure 11 exhibits a steeper slope when $\theta = 0.4$. Once again, this pattern reflects the magnified benefits and costs of changing the average investment-output ratio (and average initial consumption) when the economy suffers from an underinvestment problem.

Referring back to Table 5, when $\alpha = 0.5$ and $\theta = 0.4$, the deterministic model exhibits lower mean consumption growth than the speculation model: 1.62% versus 2.00%. Lower mean consumption growth in the deterministic model implies a lower investment-output ratio—a feature that is particularly costly when $\theta = 0.4$. For this parameterization, business cycles serves to raise the average investment-output ratio and thereby help to address the underinvestment problem. For this reason, and because risk aversion is low, business cycle fluctuations in the speculation model serve to *increase* welfare by 9.30%. But as risk aversion increases, the beneficial effects of fluctuations are reversed; business cycles now lower the average investment-output ratio and thereby exacerbate the underinvestment problem, producing large welfare losses.

Table 7: Welfare Cost of Business Cycles In Speculation Model

α	$\theta = 0.4$	$\theta = 0.6$	$\theta = 1.0$
0.5	-9.30	-4.87	5.16
1.0	-3.20	-2.55	4.74
1.5	1.87	-0.13	4.63
2.0	6.88	2.44	4.78
2.5	12.6	5.26	5.21

Note: In percent of per-period consumption.

Overall, the main message from Tables 6 and 7 is that technology-driven bubbles and the associated business cycle fluctuations can be very costly as risk aversion increases. For the baseline parametrization with a risk coefficient of $\alpha = 1.5$, the welfare costs in Tables 6 and 7 range from a low of -1.11% to a high 4.63%.

Barlevy (2004) estimates that eliminating business cycles can yield welfare gains of around 7 percent of per-period consumption when holding initial consumption fixed in an endogenous growth model with logarithmic utility ($\alpha = 1$) and no productive externality ($\theta = 1$) . Barlevy's rational model is calibrated to match post-World War II data, whereas the speculation model considered here is calibrated to match long-run data prior to the year 1900. Interestingly, the welfare costs of business cycles in the speculation model with $\theta = 1$ are not too far from Barlevy's results, despite differences in the capital adjustment cost formulation and the calibration methodology. Qualitatively, the results presented in Table 7 are consistent with Barlevy's finding that the welfare cost of business cycles can be large when long-run growth is endogenous.

7 Concluding Remarks

"Nowhere does history indulge in repetitions so often or so uniformly as in Wall Street," observed legendary speculator Jesse Livermore.[17] History tells us that periods of major technological innovation are typically accompanied by speculative bubbles as agents overreact to genuine advancements in productivity. Excessive run-ups in asset prices can have important consequences for the economy because mispriced assets imply some form of capital misallocation. Innovations to technology are also considered by many economists to be an important driving force for business cycles.

This paper developed a behavioral real business cycle model in which speculative agents overreact to observed technology shocks. Overreaction tends to be self-confirming; the forecast errors observed by the agent are close to white noise for moderate levels of risk aversion. Long-lived overreaction behavior is a frequent outcome of a self-referential learning process where the agent acts like an econometrician. The speculation model outperformed the rational model in capturing several features of long-run U.S. data, including the higher moments of asset pricing variables and the relative volatilities of output, investment, and consumption growth rates.

[17] From Livermore's thinly-disguised biography by E. Lefevére (1923, p. 180).

Interestingly, even from the narrow perspective of this simple theoretical model, it remains an open question whether the costs of speculative behavior outweigh the possible benefits to society. Speculation can affect the mean and volatility of consumption growth, as well as the agent's average initial consumption level. Which of these various effects dominate in terms of welfare depends crucially on the degree of risk aversion and the severity of the economy's underinvestment problem.

It should be noted, of course, that the model abstracts from numerous real-world issues that would affect investors' welfare. One noteworthy example is financial fraud. Throughout history, speculative bubbles have usually coincided with outbreaks of fraud and scandal, followed by calls for more government regulation once the bubble has burst. Indeed, the term "bubble" was coined in England in 1720 following the famous price run-up and crash of shares in the South Sea Company. The run-up led to widespread public enthusiasm for the stock market and an explosion of highly suspect companies attempting to sell shares to investors. One such venture notoriously advertised itself as "a company for carrying out an undertaking of great advantage, but nobody to know what it is." The proliferation of fraudulent stock-offering schemes led the British government to pass the so-called "Bubble Act" in 1720.[18]

The idea that speculation may yield benefits to society has a long history. Regarding the merits of speculation, J. Edward Meeker (1922, p. 419), the economist of the New York Stock Exchange, wrote:

> "Of all the peoples in history, the American people can least afford to condemn speculation...The discovery of America was made possible by a loan based on the collateral of Queen Isabella's crown jewels, and at interest, beside which even the call rates of 1919-1920 look coy and bashful. Financing an unknown foreigner to sail the unknown deep in three cockleshell boats in the hope of discovering a mythical Zipangu [land of gold] cannot, by the wildest exercise of language, be called a 'conservative investment.' "

[18] The law was officially named "An Act to Restrain the Extravagant and Unwarrantable Practice of Raising Money by Voluntary Subscription for Carrying on Projects Dangerous to the Trade and Subjects of the United Kingdom." See Gerding (2006).

A Appendix: Approximate Rational Solution (Proposition 1)

Taking logarithms of both sides of the transformed first-order condition (15) and then applying a first-order Taylor series approximation to each side yields equation (16). The Taylor-series coefficients are given by

$$a_0 = \frac{\widetilde{x}^{1-\lambda\phi}}{(1+\lambda\,\widetilde{x})^{(1-\lambda)\phi}}, \tag{A.1}$$

$$a_1 = 1 - \frac{\phi\lambda\,(1+\widetilde{x})}{1+\lambda\,\widetilde{x}}, \tag{A.2}$$

$$b_0 = \widetilde{\beta}\left[\frac{\theta+\widetilde{x}\,(1-\lambda+\lambda\theta)}{(1+\lambda\,\widetilde{x})^{\phi}}\right], \tag{A.3}$$

$$b_1 = \frac{\widetilde{x}\,(1-\lambda+\lambda\theta)}{\theta+\widetilde{x}\,(1-\lambda+\lambda\theta)} - \frac{\phi\lambda\,\widetilde{x}}{1+\lambda\,\widetilde{x}}, \tag{A.4}$$

where $\widetilde{x} = \exp\{E[\log(x_t)]\}$ is the approximation point and $\widetilde{\beta} \equiv \beta\left[B\,(A\lambda)^{\lambda}\right]^{\phi}$.

The conjectured law of motion for the composite variable is $w_{t+1} = \widetilde{w}\exp(m\,z_{t+1})$. Substituting the conjectured form into the right-side of (17), evaluating the conditional expectation, and then collecting terms, yields:

$$\frac{w_t}{\widetilde{w}} = \underbrace{\left[\frac{b_0}{a_0}\right]^{\frac{b_1}{a_1}} \exp\left[\frac{m^2\sigma_\epsilon^2 b_1}{2\,a_1}\right]}_{=1} \exp\left[\underbrace{\frac{m\rho b_1 - \phi\,[a_1-(1-\lambda)\,b_1]}{a_1}}_{=m}\, z_t\right], \tag{A.5}$$

which shows that the conjecture is correct. Solving for the undetermined coefficient m yields

$$m = \frac{\phi\,[a_1-(1-\lambda)\,b_1]}{a_1-\rho b_1}, \tag{A.6}$$

where a_1 and b_1 both depend on $\widetilde{x}$ from (A.2) and (A.4).

Setting the undetermined constant term on the right-side of (A.5) to unity yields the following nonlinear equation for the approximation point $\widetilde{x}$

$$\widetilde{x} = \frac{\theta\beta\exp\left[\phi\widetilde{\mu}+m^2\sigma_\epsilon^2/2\right]}{1-\beta\,(1-\lambda+\lambda\theta)\exp\left[\phi\widetilde{\mu}+m^2\sigma_\epsilon^2/2\right]}, \tag{A.7}$$

where $\widetilde{\mu}$ depends on $\widetilde{x}$ as shown below:

$$\exp(\widetilde{\mu}) = BA^{\lambda}\left[\frac{\lambda\,\widetilde{x}}{1+\lambda\,\widetilde{x}}\right]^{\lambda}. \tag{A.8}$$

Comparing (A.8) to equation (14) shows that $\widetilde{\mu}$ represents the endogenous trend growth rate of consumption in the rational model. Given a set of parameter values, equations (A.7) and (A.8) are solved simultaneously for $\widetilde{x}$ and $\widetilde{\mu}$. Equation (A.3) is then used to compute $\widetilde{w} = b_0$. Equation (A.6) is used to compute m. The technology response coefficient for the rational price-consumption ratio (19) is given by $\gamma = (m-\phi)/b_1$. ■

B Appendix: Approximate Moments for Calibration

The Taylor series coefficients for the speculation model are denoted by a_{0s}, a_{1s}, b_{0s}, and b_{1s}. These coefficients take the same form as equations (A.1) through (A.4), but $\widetilde{x}$ is now replaced by $\widetilde{x}_s$. Analogous to the rational solution, we have $\widetilde{w}_s = b_{0s} = \exp\{E[\log(w_{s,t})]\}$.

The approximation point $\widetilde{x}_s = \exp\{E[\log(x_{s,t})]\}$ is the solution to the following nonlinear equation

$$\widetilde{x}_s = \frac{\theta\beta\exp\left[\phi\widetilde{\mu}_s + m_s^2\sigma_u^2/2\right]}{1-\beta(1-\lambda+\lambda\theta)\exp\left[\phi\widetilde{\mu}_s + m_s^2\sigma_u^2/2\right]}, \tag{B.1}$$

where $\widetilde{\mu}_s$ depends on $\widetilde{x}_s$, as shown below:

$$\exp(\widetilde{\mu}_s) = BA^{\lambda}\left[\frac{\lambda\,\widetilde{x}_s}{1+\lambda\,\widetilde{x}_s}\right]^{\lambda}. \tag{B.2}$$

Comparing (B.2) to equation (14) shows that $\widetilde{\mu}_s$ represents the endogenous trend growth rate of consumption in the speculation model. Given a set of parameter values and the calibrated technology response coefficient m_s, equations (B.1) and (B.2) are solved simultaneously for $\widetilde{x}_s$ and $\widetilde{\mu}_s$. The response coefficient that appears in the approximate law of motion (25) for $x_{s,t}$ is then given by $\gamma_s = [m_s - \phi(1-\lambda)]/a_{1s}$.

Starting from equation (26), a Taylor series approximation for the speculative price-dividend ratio is given by

$$\frac{p_{s,t}}{d_{s,t}} = \left[\frac{\widetilde{x}_s}{\theta-(1-\theta)\lambda\,\widetilde{x}_s}\right]\left[\frac{x_{s,t}}{\widetilde{x}_s}\right]^{n_s}, \tag{B.3}$$

$$\text{where} \qquad n_s = 1 + \left[\frac{(1-\theta)\lambda\,\widetilde{x}_s}{\theta-(1-\theta)\lambda\,\widetilde{x}_s}\right].$$

The above expression implies the following unconditional moments:

$$E[\log(p_{s,t}/d_{s,t})] = \log\left[\frac{\widetilde{x}_s}{\theta-(1-\theta)\lambda\,\widetilde{x}_s}\right], \tag{B.4}$$

$$\begin{aligned} Var[\log(p_{s,t}/d_{s,t})] &= n_s^2\,Var[\log(x_{s,t})], \\ &= n_s^2\gamma_s^2\,Var(z_t), \end{aligned} \tag{B.5}$$

$$\begin{aligned} Corr[\log(p_{s,t}/d_{s,t}),\,\log(p_{s,t-1}/d_{s,t-1})] &= Corr[\log(x_{s,t}),\,\log(x_{s,t-1})], \\ &= Corr[z_t,\,z_{t-1}], \\ &= \rho. \end{aligned} \tag{B.6}$$

Given equations (B.4) and (B.5), the unconditional mean and variance of $p_{s,t}/d_{s,t}$ can be computed by making use of the properties of the log-normal distribution.[19]

[19] If a random variable v_t is log-normally distributed, then $E(v_t) = \exp\left\{E[\log(v_t)] + \frac{1}{2}Var[\log(v_t)]\right\}$ and $Var(v_t) = E(v_t)^2\{\exp(Var[\log(v_t)]) - 1\}$.

Starting from (14), a Taylor series approximation for consumption growth in the speculation model is given by

$$\frac{c_{s,t+1}}{c_{s,t}} = \exp\left(\widetilde{\mu}_s\right) \left[\frac{x_{s,t}}{\widetilde{x}_s}\right]^{a_{2s}} \left[\frac{x_{s,t+1}}{\widetilde{x}_s}\right]^{b_{2s}} \exp\left[z_{t+1} - (1-\lambda)\ z_t\right], \tag{B.7}$$

$$\text{where} \qquad a_{2s} = \frac{\lambda\left(1+\widetilde{x}_s\right)}{1+\lambda\,\widetilde{x}_s}, \qquad b_{2s} = \frac{-\lambda\,\widetilde{x}_s}{1+\lambda\,\widetilde{x}_s},$$

and $\exp\left(\widetilde{\mu}_s\right)$ is given by equation (B.2). Given the approximate law of motion (25) for $x_{s,t}$, the above expression implies the following unconditional moments

$$E\left[\log\left(c_{s,t+1}/c_{s,t}\right)\right] = \widetilde{\mu}_s, \tag{B.8}$$

$$\begin{aligned} Var\left[\log\left(c_{s,t+1}/c_{s,t}\right)\right] = & \Big\{\left(\gamma_s a_{2s} - 1 + \lambda\right)^2 + \left(\gamma_s b_{2s} + 1\right)^2 \\ & + 2\rho\left(\gamma_s a_{2s} - 1 + \lambda\right)\left(\gamma_s b_{2s} + 1\right)\Big\}\ Var\left(z_t\right). \end{aligned} \tag{B.9}$$

C Appendix: Learning and Nonlinear Model Simulations

Real-time learning is discussed in Section 5.2 of the text. The learning algorithm is described by the following system of nonlinear stochastic difference equations:

$$\frac{x_t^{1-\lambda\phi}\exp\left[(1-\lambda)\,\phi\, z_t\right]}{\left(1+\lambda\, x_t\right)^{(1-\lambda)\phi}} = \begin{cases} \widetilde{w}_{t-1}\ \exp\left[m_{t-1}\, z_t\ +\ \frac{1}{2}m_{t-1}^2\sigma_u^2\right], \\ \text{or} \\ \widetilde{w}_{t-1}\ \exp\left[m_{t-1}\,\rho_{t-1}\, z_t\ +\ \frac{1}{2}m_{t-1}^2\sigma_{\varepsilon,t-1}^2\right], \end{cases} \tag{C.1}$$

$$w_t \equiv \widetilde{\beta}\left[\frac{\theta + x_t\left(1-\lambda+\lambda\theta\right)}{\left(1+\lambda\, x_t\right)^{\phi}}\right]\exp\left(\phi\, z_t\right), \tag{C.2}$$

$$m_t = \frac{Cov\left[\,\Delta\log\left(w_t\right),\ \Delta z_t\,\right]}{Var\left(\Delta z_t\right)}, \tag{C.3}$$

$$\widetilde{w}_t = \exp\left\{E\left[\log\left(w_t\right) - m_t\, z_t\right]\right\}, \tag{C.4}$$

$$\rho_t = 2\,Corr\left(\,\Delta z_t,\ \Delta z_{t-1}\,\right) + 1, \tag{C.5}$$

$$\sigma_{\varepsilon,t}^2 = \frac{1}{2}\left(1+\rho_t\right)Var\left(\Delta z_t\right). \tag{C.6}$$

where z_t is governed by the true stochastic process (4). Equation (C.1) is the nonlinear first-order condition (15), where the right side defines the agent's conditional forecast under learning using the most recently estimated parameters. The top forecast rule applies to the simulation where the agent misperceives the technology process, where $\sigma_u^2 = 2\sigma_\varepsilon^2/\left(1+\rho\right)$ is the calibrated variance of the perceived permanent innovation. The bottom forecast rule applies to the simulation where the agent is learning about the technology process. Given the conditional forecast and the current observed value of z_t, the left side of (C.1) is solved for x_t each period using a nonlinear equation solver. Given x_t and z_t, the nonlinear definitional relationship (C.2) is used to compute the current realization of the composite variable w_t. Given all past data on w_t and z_t, the agent computes the sample moments shown in (C.3) through (C.6), as applicable, to obtain a new set of forecast rule parameters. The sample moments in (C.3) through (C.6) are computed using first differences of the relevant variables, rather than levels, because the agent perceives that z_t either exhibits a unit root (top forecast rule) or may exhibit a unit root (bottom forecast rule).

The model simulations described in Section 5.1 and 5.2 employ an algorithm that is similar to (C.1) and (C.2), except that the forecast rule parameters are held constant throughout the simulation at the calibrated values. In the speculation model, the forecast rule parameters are $\widetilde{w}_s$, m_s, and σ_u^2. In the rational model, the forecast rule parameters are $\widetilde{w}$, m, ρ, and σ_ε^2.

For both the learning algorithm and the nonlinear model simulations, the initial condition for the price-consumption ratio and the composite variable is the deterministic steady state, which does not depend on the technology response coefficient. Hence, the speculation model and the rational model have the same steady state. The steady-state price-consumption ratio is denoted by $\overline{x}$. Steady-state consumption growth is denoted by $\overline{\mu}$. The values of $\overline{x}$ and $\overline{\mu}$ solve the following system of nonlinear equations

$$\overline{x} = \frac{\theta\beta \exp\left(\phi\overline{\mu}\right)}{1-\beta\left(1-\lambda+\lambda\theta\right)\exp\left(\phi\overline{\mu}\right)}, \tag{C.7}$$

$$\exp\left(\overline{\mu}\right) = BA^{\lambda}\left[\frac{\lambda\,\overline{x}}{1+\lambda\,\overline{x}}\right]^{\lambda}. \tag{C.8}$$

Given $\overline{x}$, the steady-state value of the composite variable is computed from:

$$\overline{w} = \beta\left[B\,\left(A\lambda\right)^{\lambda}\right]^{\phi}\left[\frac{\theta+\overline{x}\,\left(1-\lambda+\lambda\theta\right)}{\left(1+\lambda\,\overline{x}\right)^{\phi}}\right]. \tag{C.9}$$

D Appendix: Welfare Cost Computation

This appendix describes the procedure for computing the welfare costs presented in Tables 6 and 7.

D.1 Welfare Cost of Speculation

Average lifetime utility in the rational model is represented by V. Average lifetime utility in the speculation model is represented V_s. These welfare measures can be written as

$$V = \frac{-1}{\phi\left(1-\beta\right)} + E\sum_{t=0}^{\infty}\beta^t\frac{\left(c_t\right)^{\phi}}{\phi}, \qquad \phi \equiv 1-\alpha, \tag{D.1}$$

$$V_s = \frac{-1}{\phi\left(1-\beta\right)} + E\sum_{t=0}^{\infty}\beta^t\frac{\left(c_{s,t}\right)^{\phi}}{\phi}, \tag{D.2}$$

where $c_t = y_t/\left(1+\lambda\,x_t\right)$ and $c_{s,t} = y_{s,t}/\left(1+\lambda\,x_{s,t}\right)$ are the nonlinear allocation rules that govern the consumption streams. During a simulation, x_t and $x_{s,t}$ are computed using the nonlinear algorithm described in Appendix C. The unconditional mean E is approximated by the average over 5000 simulations, each 2000 periods in length, after which the results are not changed. The initial consumption levels at $t = 0$ are stochastic variables. Each simulation starts at $t = -1$ with $y_t = y_{s,t} = 1$, such that $c_t = c_{s,t} = 1/(1 + \lambda\,\overline{x})$, where $\overline{x}$ is the steady-state price-consumption ratio from equation (C.7).

The welfare cost of speculation is the constant percentage amount by which $c_{s,t}$ must be increased in the speculation model in order to make average lifetime utility equal to that in the rational model. Specifically, I solve for τ such that

$$\begin{aligned} V &= \frac{-1}{\phi(1-\beta)} + E\sum_{t=0}^{\infty}\beta^t \frac{\left[c_{s,t}(1+\tau)\right]^{\phi}}{\phi}. \\ &= \frac{-1}{\phi(1-\beta)} + (1+\tau)^{\phi}\left[V_s + \frac{1}{\phi(1-\beta)}\right], \end{aligned} \tag{D.3}$$

which yields the result

$$\tau = \left[\frac{\phi(1-\beta)V + 1}{\phi(1-\beta)V_s + 1}\right]^{\frac{1}{\phi}} - 1. \tag{D.4}$$

In the case of log utility $(\phi = 0)$, equation (D.4) becomes $\tau = \exp\left[(V - V_s)(1-\beta)\right] - 1$.

D.2 Welfare Cost of Business Cycles

The welfare cost of business cycles in the calibrated speculation model is the constant percentage amount by which $c_{s,t}$ must be increased in order to make average lifetime utility equal to that of a deterministic model with $z_t = 0$ for all t. Lifetime utility in the deterministic model V_d can be written as

$$V_d = \frac{-1}{\phi(1-\beta)} + \sum_{t=0}^{\infty}\beta^t \frac{(c_{d,t})^{\phi}}{\phi}. \tag{D.5}$$

The deterministic simulation starts at $t = -1$ with $y_{d,t} = 1$, such that $c_{d,t} = 1/(1 + \lambda\,\overline{x})$, where $\overline{x}$ is given by equation (C.7). Deterministic consumption evolves according to the law of motion $c_{d,t} = c_{d,t-1}\exp(\overline{\mu})$, where $\overline{\mu}$ is given by equation (C.8). Deterministic consumption at $t = 0$ will thus differ from average consumption at $t = 0$ in the fluctuating model.

Analogous to equation (D.4), the welfare cost of business cycles in the speculation model is given by

$$\tau = \left[\frac{\phi(1-\beta)V_d + 1}{\phi(1-\beta)V_s + 1}\right]^{\frac{1}{\phi}} - 1. \tag{D.6}$$

In the case of log utility, $(\phi = 0)$, equation (D.6) becomes $\tau = \exp\left[(V_d - V_s)(1-\beta)\right] - 1$.

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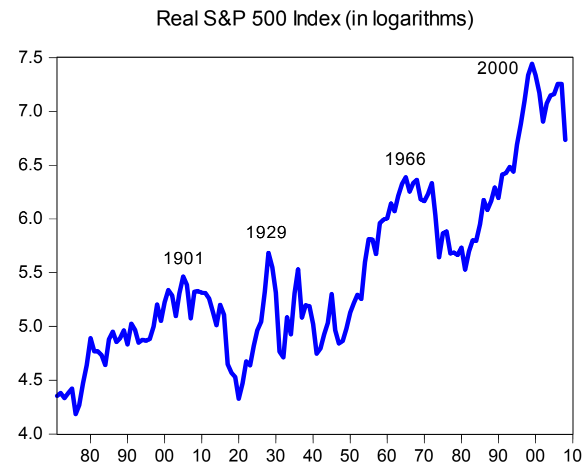

Figure 1: Four major run-ups in U.S. stock prices.

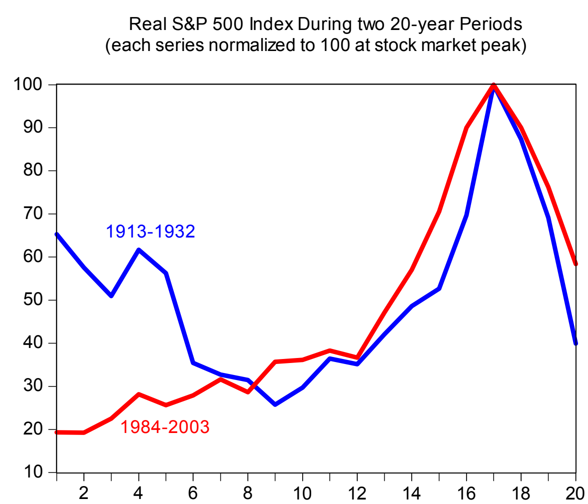

Figure 2: Comparing two bubble episodes.

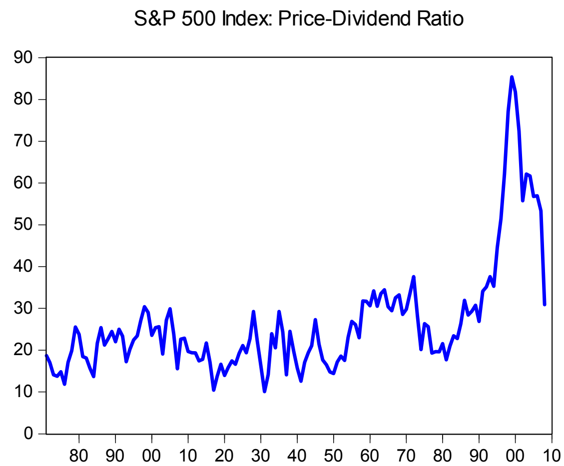

Figure 3: The price-dividend ratio reached unprecedented levels around the year 2000.

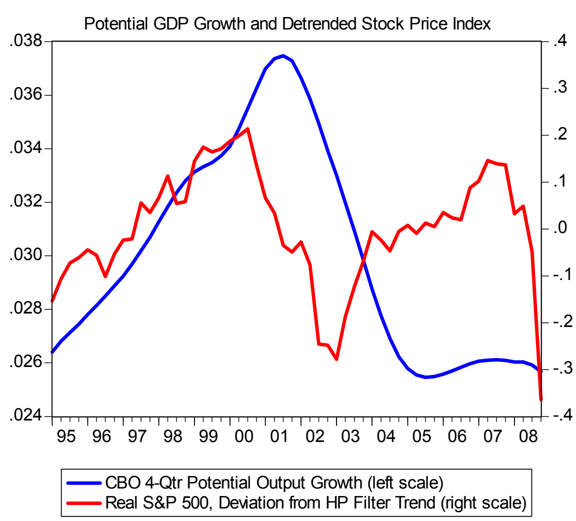

Figure 4: Rise and fall of the "new economy."

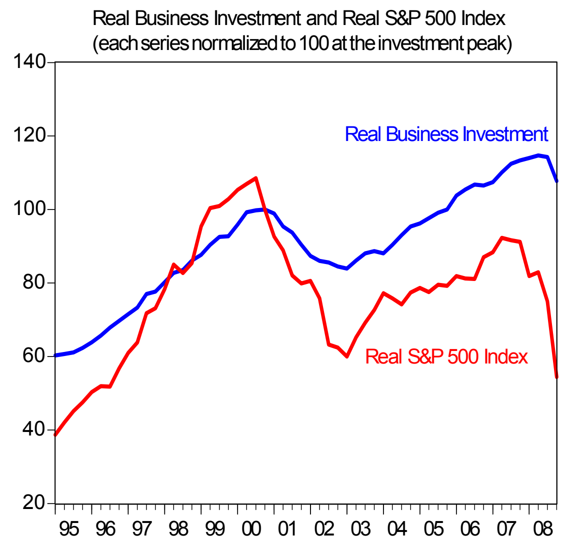

Figure 5: Comovement of business investment and stock prices.

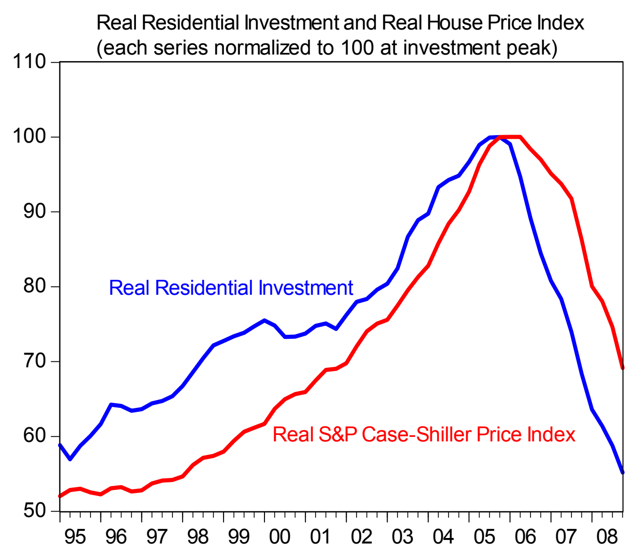

Figure 6: Comovement of residential investment and house prices.

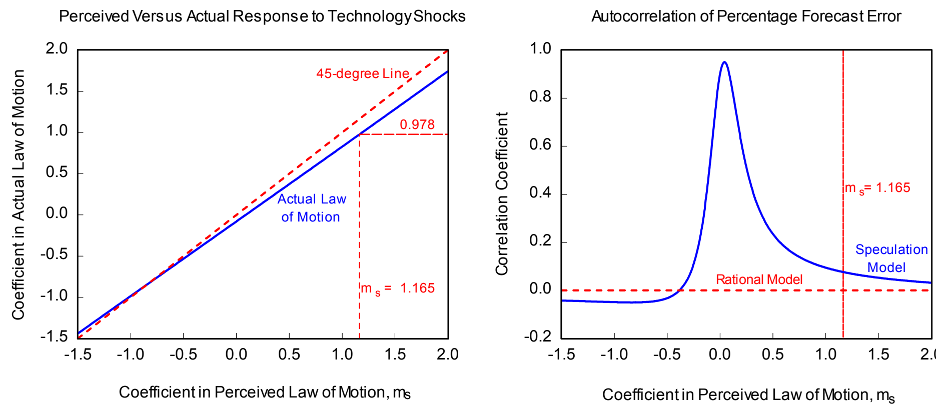

Figure 7: Overreaction behavior tends to be self–confirming.

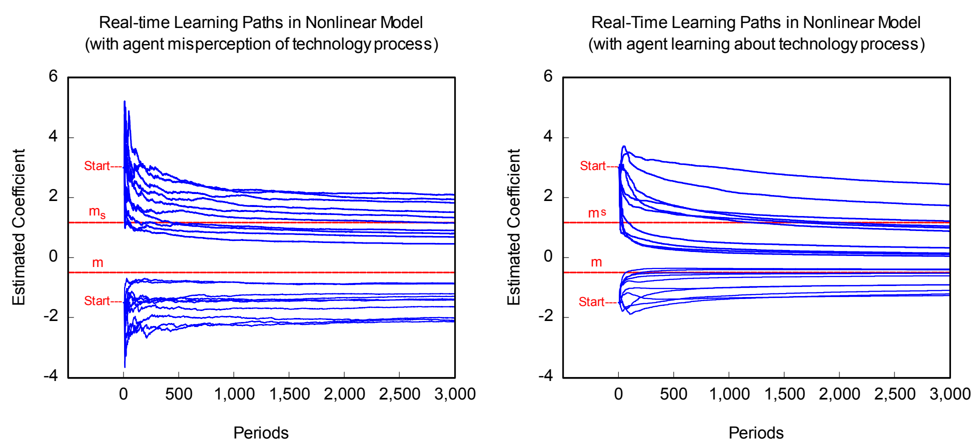

Figure 8: Long-lived overeaction is a frequent outcome of a real-time learning algorithm.

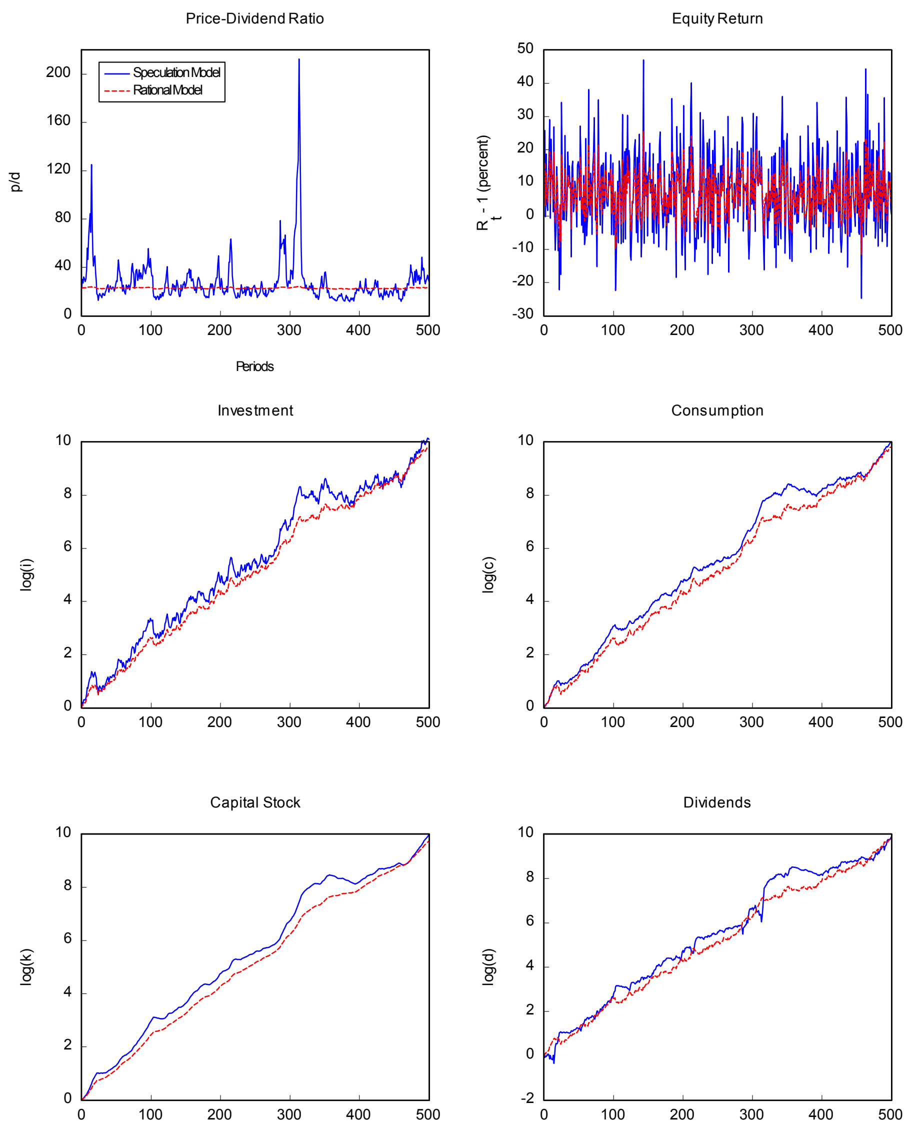

Figure 9: Bubbles coincide with economic booms and excess capital formation.

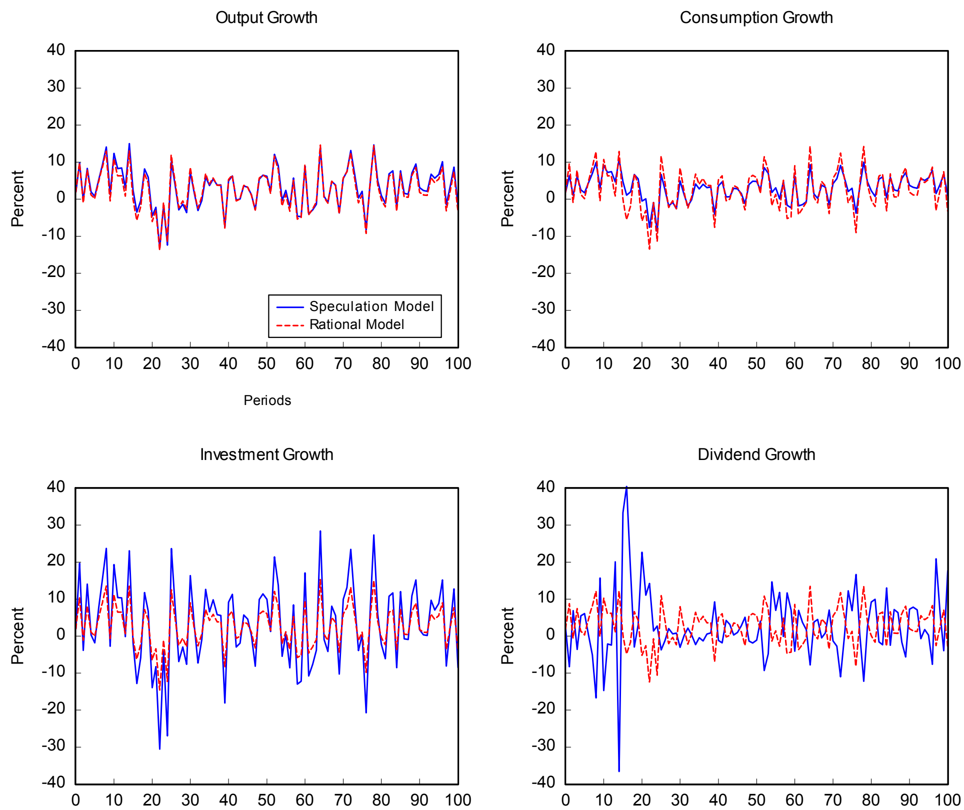

Figure 10: Speculation magnifies investment volatility but reduces consumption volatility.

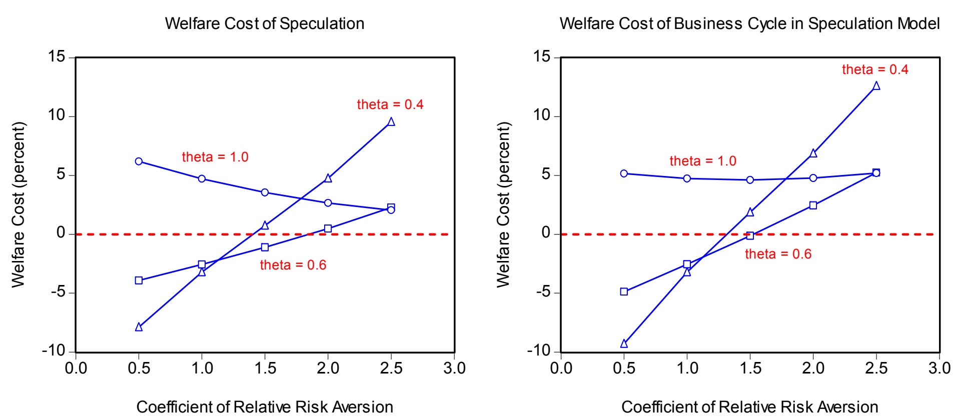

Figure 11: Welfare costs increase rapidly with risk aversion when $\theta < 1$.