Electronic Miniaturization and Economic Growth

Pablo D. Azar[1]

Federal Reserve Bank of NY

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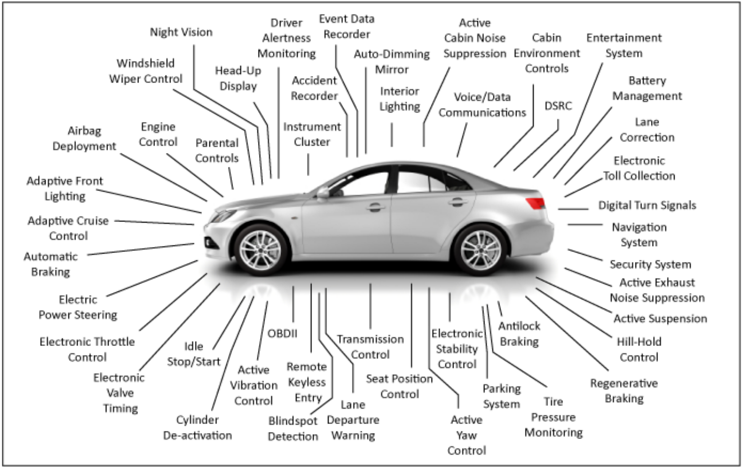
Introduction

- The supply chains of many industries have become more complex and introduced new input combinations over time.
- For example, agricultural machinery now relies on satellites, sensors, electronic components, computer hardware and software.
- Most of these changes in supply chains would not have been possible without Moore's Law.

Example I: Automobile Electronics



Source: Clemson Vehicular Electronics Laboratory

Example II: Warehouse Robots

Photo Credit: Joel Eden Photography/Kiva Systems

Questions

- How do physical constraints affect industries' supply chains ?
- What is the effect of electronic miniaturization on TFP Growth?

This Paper

- A tractable model of production with physical constraints.
- Firms can combine sets of inputs, as long as these sets satisfy some combinatorial constraint depending on inputs' physical properties.
- Under standard assumptions, output grows with the number of feasible input combinations.
- Structural Simulation: Between 13% and 22% of productivity growth in the 1960-2019 period can be attributed to physical changes in the size of electronic components. This effect is highest during the 1990s.

Related Literature

- Combinatorial Growth : Romer (1993), Weitzman (1998), Auerswald et. al. (2000), Ghiglino (2012), Acemoglu and Azar (2020), Jones (2021).
- Production Networks and Distortions Leontief (1951), Long and Plosser (1983), Ciccone (2002), Gabaix (2011), Jones (2011), Acemoglu et. al. (2012), Acemoglu et. al. (2017), Bartleme and Gorodnichenko (2015), Liu (2017), Biglio and La'o (2017), Baqaee (2017), Baqaee and Farhi (2019ab), Caliendo et. al. (2018), Oberfield (2018), Taschereau-Dumouchel (2020), Acemoglu and Tahbaz-Salehi (2021), Liu and Tsyvinski (2020)..
- Effect of Information Technology on GDP Baily and Gordon (1988), Brynjolfsson and Yang (1996), Triplett (1986,1989,1996,1999,2003), Bosworth and Triplett (2000), Baily (2002), Joregnson and Stiroh (1999,2000), Aizcorbe et. al. (2003), Jorgenson (2005), Barefoot et. al. (2018), Byrne et. al. (2017).

Roadmap

- The Model
- Equilibrium Characterization
- Structural Empirical Exercise
- Extensions
- Conclusion

The Model: Market Structure

Firms in each industry produce one good, and all markets are competitive. There are two types of industries.

1. Primary industries, indexed by $j \in \mathcal{J} = \{1, ..., J\}$. Each primary good is produced using other primary goods and labor as inputs, and has a size $\theta_j \in \mathbb{R}_{\geq 0}$.
2. A final industry. The final good is produced using a subset $S \subset \mathcal{J}$ of primary goods, and labor.

The Model: Primary Industries

- Each primary industry j has a strictly concave, continuous and increasing production function with constant returns to scale

$$Y_j = F_j((X_{jk})_{k=1}^{J}, L_j). \tag{1}$$

- X_{jk} is primary industry j's demand for industry k's good, and L_j is its labor demand.
- I treat the vector of physical properties θ_j as exogenous. However, there may be common factors affecting the properties of multiple primary industries, so that

$$(\theta_1, ..., \theta_J) = \Theta(\zeta) \tag{2}$$

where $\zeta \in \mathbb{R}^q$ is a vector of common factors, and $\Theta : \mathbb{R}^q \to \mathbb{R}^{J \times p}$.

Primary Industries: Leontief Example with Size Spillovers

- Primary industry j has a Leontief production function

$$Y_j = \min\{A_j L_j, \min\{\frac{X_{jk}}{\alpha_{jk}}\}\}.$$

- To produce one unit of good j, firms need to use α_{jk} units of good k.
- Size is additive, so that

$$\theta_j = \sum_{k=1}^{J} \alpha_{jk}\theta_k + \zeta_j$$

where $\zeta_j \in \mathbb{R}$ is some idiosyncratic factor affecting good j's size.

- If $(I - \alpha)$ is invertible, then

$$\theta = \Theta(\zeta) = (I - \alpha)^{-1}\zeta.$$

The Model: Final Industry Production

- The final industry has access to a menu of production functions

$$Y(S) = A(S)F(S, (X_j)_{j \in S}, L) \tag{3}$$

  where
  - $S \subset \{1, ..., J\}$ denotes industry i's set of suppliers;
  - $A(S)$ is a Hicks-Neutral productivity shifter;
  - F is increasing, strictly concave and has constant-returns to scale
  - L is the amount of labor used
  - $X = \{X_j\}_{j \in S}$ is the vector of primary good demands.
- The dependence of the productivity term $A(S)$ on S is the crucial feature of the model. By combining a richer set of inputs an intermediate industry may achieve a greater level of productivity.
- However, not all subsets S are feasible...

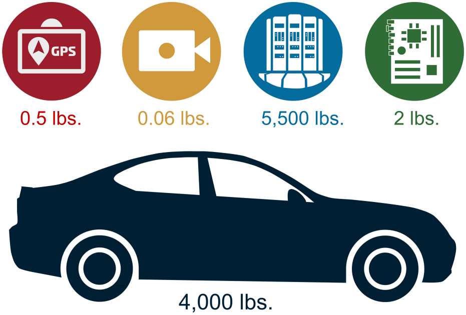
The Model: Final Industry Constraint

- The input set S of the final industry must satisfy the Knapsack constraint

$$\sum_{j \in S} \theta_j \leq \tau.$$

- I denote the collection of of all feasible subsets by

$$\mathcal{F}(\theta, \tau) = \{S \in 2^{\mathcal{J}} : \sum_{j \in S} \theta_j \leq \tau\}.$$



The Model: Households

- There is a representative household which only consumes the final good. Its utility function is $U(C_f)$.
- The representative household has one unit of labor endowment, which it supplies inelastically.
- The wage is the numeraire, so that $W = 1$.
- Denoting the price of the final good by P_f, the household's budget constraint is given by

$$P_f C_f \leq 1.$$

Equilibrium Definition

Equilibrium An equilibrium is a tuple $(P^*, S^*, C^*, L^*, X^*, Y^*)$ such that

1. Firms choose S^*, L^*, X^* to minimize unit costs, taking P^* as given
2. Prices equal firms' unit costs
3. Households choose C^* to maximize utility subject to their budget constraint, taking P^* as given
4. All markets clear

Equilibrium: Primary Firms' Problem

- A firm in primary industry j faces a standard cost-minimization problem

$$K_j(P) = \min_{X_{jk}, L_j} \sum_{k=1}^{J} P_k X_{jk} + L_j$$
$$\text{Subject to:} \quad F_j(X_j, L_j) \geq 1.$$

- Since F_j is strictly concave, this has a unique solution X_j^*, L_j^*.

Equilibrium: Assumption I

Assumption (Assumption 1)

There exists a positive vector $P^*_{pri} = (P^*_1, ..., P^*_J)$ such that $P^*_{pri} = K(P^*_{pri})$.

- Assumption 1 is clearly necessary for the existence of equilibrium.
- If K is strictly concave, then such P^*_{pri} is unique (Kennan, 2001).
- Even if production is Leontief and $K_j(P_{pri}) = \sum_{k=1}^{J} \alpha_{jk} P_k + L_j$ is linear, prices are unique and given by

$$P^*_{pri} = (I - \alpha)^{-1} \mathbf{1}$$

when $(I - \alpha)^{-1}$ exists and all of its entries are non-negative.

Equilibrium: Final Firms' Problem I

- We want to compute the cost function $K(P, \tau)$ of the final industry.
- We can break the cost minimization problem into two parts:
  1. For a fixed S, define a cost function $K(S, P, A, \tau)$ given by

$$K(S, P, A, \tau) = \min_{(X_j)_{j \in S}, L} \sum_{j \in S} P_j X_j + L,$$

$$\text{Subject to :}$$

$$A(S) F(S, (X_j)_{j \in S}, L) \geq 1$$

  2. The chosen set of suppliers minimizes the unit cost function:

$$S^* \in \arg\min_{S \in \mathcal{F}(\theta, \tau)} K(S, P, A(S), \tau).$$

- The cost function $K(P, \tau) = K(S^*, P, A(S^*), \tau)$.

Equilibrium : Final Firms' Problem II

- The cost function $K(P,\tau) = K(S^*, P, A(S^*), \tau)$.
- We can decompose this into a random factor, and a deterministic component (which may depend on S).
- More formally, we can write

$$K(P,\tau) = \underbrace{1/A(S)}_{\text{Random Productivity}} \times \underbrace{\overline{K}(S,P,\tau)}_{\text{Deterministic Cost}} .$$

.

Equilibrium: Assumption II

Assumption (Assumption 2)

The idiosyncratic productivity terms $(A(S))_{S \subset \mathcal{F}(\theta,\tau)}$ *are drawn identically and independently from a Frechet Distribution with CDF*

$$Pr[X \leq x] = e^{-x^{-\kappa}},$$

where $\kappa > 1$.

- Assumption 2 will allow us to tractably compute output and primary good demands.
- The Assumption can be relaxed so that $A(S) = \phi(S) \cdot \psi(S, \tau)$, where $\phi(S)$ is a Frechet random variable, and $\psi(S, \tau)$ is some deterministic productivity associated with set S and sector τ.

Equilibrium Characterization: Output

Theorem

Suppose Assumptions 1 and 2 hold. Then an equilibrium exists, and

$$Y = \phi \cdot \Big(\sum_{S \in \mathcal{F}(\theta,\tau)} \overline{K}(S,P,\tau)^{-\kappa} \Big)^{\frac{1}{\kappa}}$$

where ϕ is a Frechet Random Variable with shape parameter κ.

- Intuition: Similar to Eaton-Kortum (2002).
- Final industry firms will choose the set S which minimizes $\frac{\overline{K}(S,P,\tau)}{A(S)}$.
- This is equivalent to maximizing $\frac{A(S)}{\overline{K}(S,P,\tau)}$.
- The reciprocal cost function is Frechet, with shape κ and scale $\frac{1}{\overline{K}(S,P,\tau)}$.

Special Case: Unit Prices

Theorem

Suppose Assumptions 1 and 2 hold. Then an equilibrium exists, and

$$Y = \phi \cdot \Big(\sum_{S \in \mathcal{F}(\theta,\tau)} \overline{K}(S, P, \tau)^{-\kappa} \Big)^{\frac{1}{\kappa}}$$

where ϕ is a Frechet Random Variable with shape parameter κ.

- Suppose that the deterministic costs are $\overline{K}(S, P, \tau)$ are all equal to 1.
- Then $Y = \phi \cdot |\mathcal{F}(\theta, \tau)|^{\frac{1}{\kappa}}$.
- In this case, output is a function of the number of feasible sets satisfying the knapsack constraint.

Questions

- How do physical constraints affect industries' supply chains ?
- What is the effect of electronic miniaturization on TFP Growth?

Data

- To answer these questions, I combined data from three sources:
  1. Product-level Quantity, Price, and Weight (IHS Markit, Census Import Data). 2019 snapshot.
  2. Industry productivity (NBER-CES Manufacturing Data). Every year between 1958-2011.
  3. Service Productivity Data from BLS (since 1987).
- I will focus on heavy manufacturing industries (SIC codes 3401-3999) as opposed to light manufacturing industries (food,ceramic,glass,apparel,etc.), which are less likely to have electronic components.

Data Summary: Weight

Panel A: Manufacturing Industries Ranked by Volume (TEUs)			
Top 5 Industries		Bottom 5 Industries	
Truck and bus bodies	12.000	Other transportation equipment, nspf, and parts, nspf	0.001
Truck trailers	1.000	Dental equipment, supplies, and parts, nspf	0.001
Travel trailers and campers	0.670	Small arms ammunition, nspf	0.001
Machine tools, metal-forming, and parts, nspf	0.220	Electric lamps	0.002
Conveyors and conveying equipment, and parts, nspf	0.208	Telephone and telegraph apparatus, and parts, nspf	0.002

Data Summary: Volume

Panel B: Manufacturing Industries Ranked by Weight (Lbs)			
Top 5 Industries		Bottom 5 Industries	
Truck and bus bodies	31,378.010	Guided missiles and space vehicles, and parts, nspf	9.645
Truck trailers	7,607.040	Telephone and telegraph apparatus, and parts, nspf	10.907
Travel trailers and campers	3,179.060	Dolls and stuffed toy animals	11.352
Machine tools, metal-forming, and parts, nspf	2,601.450	Electric lamps	12.138
Rolling mill machinery, and parts, nspf	2,416.508	Dental equipment, supplies, and parts, nspf	13.553

Data Summary: Density

Panel C: Manufacturing Industries Ranked by Density (Lbs/TEUs)			
Top 5 Industries		Bottom 5 Industries	
Other transportation equipment, nspf, and parts, nspf	36,788.140	Guided missiles and space vehicles, and parts, nspf	857.333
Fabricated plate work	28,377.130	Truck and bus bodies	2,614.834
Structural metal parts, nspf	28,208.570	Aircraft	3,306.925
Fabricated structural metal products, nspf	25,487.860	Aircraft equipment, nspf	3,748.971
Small arms ammunition, nspf	23,185.870	Travel trailers and campers	4,744.866

Data Summary: Price-Per-Lb

Panel D: Manufacturing Industries Ranked by Price ($/Lbs)			
Top 5 Industries		Bottom 5 Industries	
Guided missiles and space vehicles, and parts, nspf	259.448	Structural metal parts, nspf	0.617
Aircraft	117.643	Architectural and ornamental metal work, nspf	0.797
Missile and rocket engines	90.623	Fabricated structural metal products, nspf	1.008
Aircraft equipment, nspf	68.309	Truck trailers	1.102
X-ray apparatus and tubes and related irradiation apparatus	38.773	Bolts, nuts, screws, rivets, and washers	1.367

Structural Simulation: Extrapolating Electronic Weights

- Take the weight $\theta_s(2019)$ of semiconductors in 2019 as exogenous.
- Use Moore's Law to extrapolate the weight $\theta_s(t) = 2^{\frac{1}{1.5}\cdot(2019-t)}\theta_s(2019)$ of semiconductors at time t. Let

$$\Delta\theta_s(t) = \theta_s(t) - \theta_s(2019).$$

- Compute the effect the spillover effect of semiconductor miniaturization on the weights of computers and electronics as

$$\Delta\theta_E(t) = (I - \alpha_E)^{-1}\Delta\theta_s(t)$$

where α_E is the input-output matrix restricted to computers and electronics industries.

Structural Simulation: Computing Changes in Productivity

- As we go back in time and semiconductors grow exponentially, some machines which are feasible to build in 2019—such as cars with assisted navigation—become infeasible.
- At the industry level, this means that machine-producing industries become less productive because they have fewer feasible input combinations that they can turn into products.
- The expected change in productivity when product weights shift from θ to θ' is

$$\Delta \log A_i = \frac{1}{\kappa}(\log |\mathcal{F}(\theta', \tau_i)| - \log |\mathcal{F}(\theta, \tau_i)|).$$

- Domar Aggregation Yields

$$\Delta \log GDP = \sum_{i=1}^{N} D_i \Delta \log A_i$$

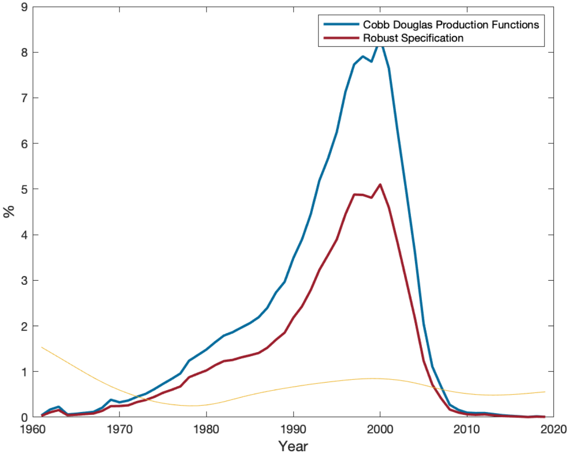
where $D_i = \frac{P_i Y_i}{GDP}$ is industry i's Domar weight.

Estimating κ^{-1}

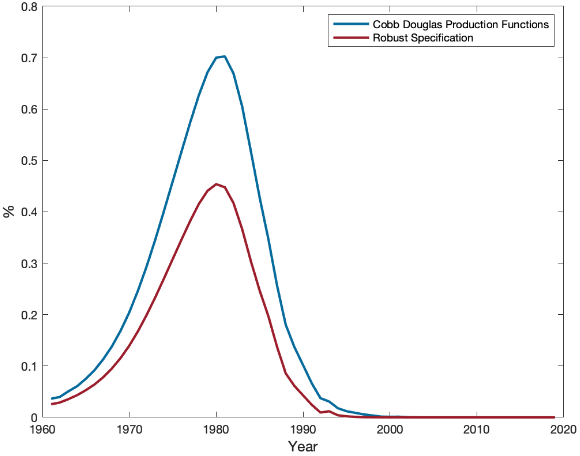
Table 5: Regression Estimates for κ^{-1}

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: All Industries						
κ^{-1}	0.004***	0.004**	0.006**	0.004*	0.004*	0.004
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)
Observations	6162	6162	1928	1928	4234	4234
Panel B: Non-Manufacturing Industries						
κ^{-1}	0.011**	0.011**	0.026**	0.006	0.010*	0.010*
	(0.005)	(0.005)	(0.011)	(0.037)	(0.005)	(0.006)
Observations	2914	2914	188	188	2726	2726
Panel C: Manufacturing Industries						
κ^{-1}	0.003	0.003	0.004*	0.004*	-0.017	-0.017
	(0.002)	(0.002)	(0.002)	(0.002)	(0.011)	(0.011)
Observations	3248	3248	1740	1740	1508	1508
Period	1960-2019	1960-2019	1960-1989	1960-1989	1990-2019	1990-2019
Growth Fixed Effects	No	Yes	No	Yes	No	Yes

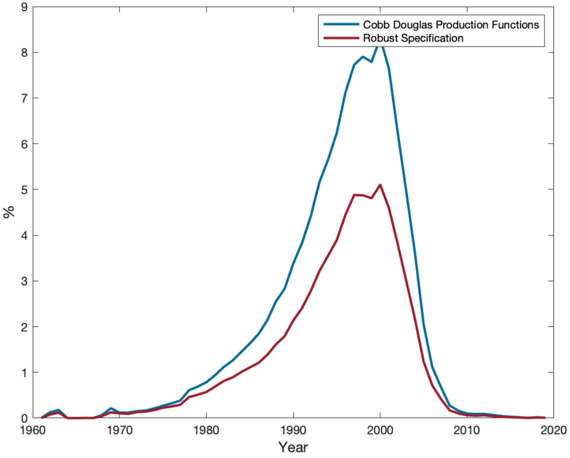
Structural Simulation: Electronic Miniaturization and Productivity Growth



Structural Simulation: Electronic Miniaturization and Productivity Growth: Manufacturing



Structural Simulation: Electronic Miniaturization and Productivity Growth: Services

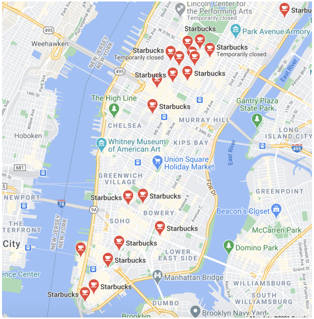


Extension I: Multiple Industries

- In practice, there is more than one industry which uses primary materials.
- We can think of different industries Y_τ whose products have different sizes τ.
- For example, biotechnology and nanotechnology industries would have $\tau \approx 10^{-9}m$. Industrial machinery would have $\tau \approx 10m$. Aviation would have $\tau \approx 100m$.
- We can define final output as a standard CES aggregator

$$Y = \left(\int_0^\infty Y_\tau^{\frac{\beta-1}{\beta}} d\tau\right)^{\frac{\beta}{\beta-1}}$$

Extension II: Other Combinatorial Constraints



(a) Facility Location

DANGEROUS GOODS & COMBUSTIBLE LIQUIDS STORAGE COMPATIBILITY CHART

Class or Subsidiary Risk	2	2	TOXIC GAS 2	2	3	4	4	4	5.1	5.2	TOXIC 6	CORROSIVE 8
FLAMMABLE GASES (2)	OK TO STORE TOGETHER	OK TO STORE TOGETHER	SEGREGATE At least 3m	SEGREGATE At least 3m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 3m	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 5m
NON TOXIC NON FLAMMABLE GASES (2)	OK TO STORE TOGETHER	OK TO STORE TOGETHER	OK TO STORE TOGETHER	OK TO STORE TOGETHER	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 3m	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 5m
TOXIC GAS (TOXIC GAS 2)	SEGREGATE At least 3m	OK TO STORE TOGETHER	MAY NOT BE COMPATIBLE CHECK MSDS AND NOTES	SEGREGATE At least 3m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 3m	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 5m
OXIDIZING GAS (2)	SEGREGATE At least 3m	OK TO STORE TOGETHER	SEGREGATE At least 3m	OK TO STORE TOGETHER	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 3m	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 5m
FLAMMABLE LIQUIDS + COMBUSTIBLE LIQUIDS (3)	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	OK TO STORE TOGETHER	SEGREGATE At least 3m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	ISOLATE	SEGREGATE At least 5m	SEGREGATE At least 3m
FLAMMABLE SOLID (4)	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 3m	OK TO STORE TOGETHER	SEGREGATE At least 3m	SEGREGATE At least 5m	SEGREGATE At least 3m	ISOLATE	SEGREGATE At least 3m	MAY NOT BE COMPATIBLE CHECK MSDS AND NOTES
SPONTANEOUSLY COMBUSTIBLE (4)	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 3m	OK TO STORE TOGETHER	SEGREGATE At least 5m	SEGREGATE At least 5m	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 3m
DANGEROUS WHEN WET (4)	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	SEGREGATE At least 5m	OK TO STORE TOGETHER	SEGREGATE At least 5m	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 5m
OXIDIZING AGENT (5.1)	SEGREGATE At least 3m	SEGREGATE At least 3m	SEGREGATE At least 3m	SEGREGATE At least 3m	SEGREGATE At least 5m	KEEP APART	SEGREGATE At least 5m	SEGREGATE At least 5m	MAY NOT BE COMPATIBLE CHECK MSDS AND NOTES	ISOLATE	SEGREGATE At least 3m	SEGREGATE At least 3m
ORGANIC PEROXIDE (5.2)	ISOLATE	ISOLATE	ISOLATE	ISOLATE	ISOLATE	ISOLATE	ISOLATE	ISOLATE	ISOLATE	OK TO STORE TOGETHER	ISOLATE	SEGREGATE At least 3m

(b) Matching Constraints

Further Results

- More general constraints and physical properties
- Reduced Form Evidence
- Growth Model

Future Work

- Use microdata on semiconductor competition to estimate oligopoly model with endogenous growth.
- Explore other combinatorial constraints (Bank Facility Location, Effect of Containerization on Trade)