# International Business Cycles and Financial Frictions[*]

Wen Yao[†]

Bank of Canada

January 2012

## Abstract

This paper builds a two-country DSGE model to study the quantitative impact of financial frictions on business cycle co-movements when investors have foreign asset exposure. The investor in each country holds capital in both countries and faces a leverage constraint on her debt. I show quantitatively that financial frictions along with foreign asset exposure give rise to a multiplier effect that amplifies the transmission of shocks between countries. The key mechanism is that a negative shock in the home country reduces the wealth of investors in both countries, which tightens their leverage constraints, leading to a fall in investment, consumption, and hours worked in the foreign country. Compared to the existing literature, which tends to produce either negative or positive but small cross-country correlations, this model produces positive and sizable correlations that are consistent with the data. The model can account for most of the investment, employment and consumption correlations and predicts more than half of the output correlation. In addition, the model shows that, consistent with empirical findings, when investors have more foreign asset exposure to the other country, the output correlation between the two countries increases.

1

# 1    Introduction

The question this paper addresses is the quantitative impact of financial frictions on the business cycle co-movements between countries when investors have foreign asset exposure. The breakout and spread of the 2007 financial crisis highlights the importance of financial frictions for international business cycle co-movements: European investors who were exposed to US mortgage-backed securities experienced a fall in their net worth when the US market collapsed. The decline in net worth tightened their leverage constraint and led to a contraction in investment activities in Europe. To analyze this mechanism, this paper embeds this type of financial friction within an international real business cycle model and concludes that the presence of leverage constraints helps the model do a better job of accounting for the correlations of output, investment and employment in the data. In addition, the model also shows that as foreign asset exposure increases, business cycles become more synchronized.

I build a two-country model where credit contracts are imperfectly enforceable and business cycles are driven by technology shocks. Each country has two types of agents: an investor and a saver. The investor holds both domestic and foreign capital. She receives risky returns by renting her capital to the market production firm. She also borrows from the domestic saver to finance her capital holdings. Because the investor cannot promise to repay her loans, she faces a leverage constraint that limits her loans to be smaller than a portion of the market value of her total capital holdings. The saver makes use of the domestic capital in home production and lends her savings to the investor. Both agents work at the market production firm. Since I am interested in evaluating business cycle implications quantitatively, I model explicitly endogenous labor supply and capital accumulation. These ingredients are important for two reasons. First, variation in hours contributes to most of the business cycle fluctuations. Second, financial frictions can generate a large amplification effect when capital is fixed. Introducing capital accumulation disciplines the exercise empirically.

The financial frictions and foreign asset exposure in this model together generate a multiplier effect that amplifies the transmission of shocks across countries. Output correlation across countries is driven up through this financial channel. When a

1

negative technology shock hits the domestic market, the demand for capital in the home country falls, which forces down the price of domestic capital. The price decline leads to a tightening of investors' leverage constraint in both countries. Borrowing is reduced globally and therefore demand for capital in the foreign country also declines. Prices of foreign assets fall, triggering another round of decline in investment and output. A multiplier effect arises since the decline in investment lowers asset prices and investors' net worth, further pushing down investment. With the presence of the financial frictions and foreign asset exposure, the shock spills over from one country to another and thus drives up the business cycle correlations.

To judge the empirical relevance of my framework, I conduct a quantitative exercise aimed at exploring whether the existence of financial frictions can improve the model's ability to account for cross-country correlations of output, employment and investment. I calibrate the model to match the data from the US and the rest of the industrial world. The model is then solved using an iterative second-order perturbation method developed by Heathcote and Perri (2009). This is because when agents have multiple assets, in the steady state where risk is absent, the returns on the assets are the same. Therefore the portfolio shares are not determinate and we need to use information from higher-order perturbation to pin down steady state portfolios.

The main findings of the paper are the following. First, the simulation result shows that the presence of financial frictions together with foreign asset exposure improves the business cycle co-movements along several dimensions: the calibrated model produces positive and sizable correlations of output, investment and employment. The model can match closely the investment and consumption correlations as in the data. The model also indicates an output correlation of 0.34, which accounts for more than half of the output correlation in the data. Moreover, the model predicts a positive employment correlation that is closer to the data than the model without financial frictions. Compared to the previous literature, which tends to predict either negative or positive but relatively small business cycle correlations, this model makes good progress by taking financial frictions into account.

Second, substantial differences exist in impulse response functions between ver-

2

sions of the model with and without financial frictions. Let me take the IRFs for hours as an example; other IRFs will be discussed later in the main text. When the leverage constraint is present, after a decline in productivity in country 1, hours fall in both countries. Hours fall in country 1 because of lower wages. Hours fall in country 2 because of the leverage constraint. Since the fall in productivity leads to a decline in the asset price in country 1, which tightens the leverage constraint of country 2's investor, capital used in country 2's production is reduced. Hence hours in country 2 also fall. However, in the case where financial frictions are absent, when productivity in country 1 falls, country 1's hours decline but country 2's hours increase because country 2 is relatively more productive.

Third, this model also predicts that when the investor increases her foreign asset exposure to the other country, the output correlation between the two countries increases. This result is consistent with the evidence documented in Imbs (2006) that output correlations rise with financial integration.

This paper is related to several strands of the literature. The first strand addresses the co-movements of international business cycles. Backus, Kehoe, and Kydland (1992) showed that in a complete market model, output, investment and labor are negatively correlated because of efficient allocation of resources across countries. Baxter and Crucini (1995), Kollmann (1996), and Heathcote and Perri (2002) introduced incomplete markets. However, they find that incomplete markets do not help much in matching the business cycle correlations in the data, because there is little need for insurance markets.

The second strand is a recent and growing literature analyzing financial frictions in an open economy context, including Gertler, Gilchrist, and Natalucci (2007), Faia (2007) and Devereux and Yetman (2010). Gertler, Gilchrist, and Natalucci (2007) build a small open economy model with credit frictions to explore the connection between the exchange rate regime and financial distress in the case of the 1997 Korea crisis. Faia (2007) studies financial frictions in a two-country DSGE model showing that business cycle synchronization increases when economies have similar financial structures, while it decreases with the degree of financial openness. However, these two papers and the previous literature did not study the impact of financial frictions

3

when the constrained agents have foreign capital exposure.

The paper by Devereux and Yetman (2010) is the closest to my work in that it studies financial frictions and capital portfolio choice in a two-country model. In contrast to my paper, their model lacks capital accumulation and endogenous labor choice, which are the key ingredients for business cycle fluctuations.

The third strand is the international portfolio choice literature, pioneered by Tille and van Wincoop (2007) and Devereux and Sutherland (2009) with a recent contribution by Heathcote and Perri (2009). This literature uses higher-order perturbation to solve optimal portfolio allocations in DSGE models.

The paper is organized as follows. In Section 2, I describe the model economy, highlight the key mechanism and show how to solve this model. In Section 3, I discuss the calibration of the model. In Section 4, I present the main results. I compare the results from a model with financial frictions and a model without financial frictions. I also provide some intuition for the results. In Section 5, I provide several robustness checks. Section 6 concludes.

## 2　Model

In this section I outline a two-country, one-good international business cycle model. The world economy consists of two countries, home (country 1) and foreign (country 2), which are the same size. Each country has three sectors: a household sector, a market production sector and a capital producer sector. The household sector is populated with two types of infinitely lived agents: an investor and a saver. The investor and saver are distinct from each other in order to motivate lending and borrowing. Adding the market production sector allows agents to derive returns from capital and labor. Moreover, I have the capital producer to facilitate the introduction of variation in capital price.

I assume that capital is mobile across the countries but labor is immobile across the countries. The following subsections detail the economic choice faced by agents in the two economies, the structure of production and the relevant market clearing conditions.

4

## 2.1 Household

There are two types of households in the model: an investor and a saver. The investors can buy the capital installed both domestically and abroad. They rent the capital to the market production firm and receive a risky return. At the same time, they can also borrow from domestic savers to finance their capital holdings. Investors account for a fraction $n$ of all households. The rest of the households participate only in the domestic bond market and I refer to them as savers. Similar to the assumption made in Bernanke, Gertler and Gilchrist (1999), I assume that investors have the ability to transform capital into a factor that can be used in the production of the market good. However, since savers do not have this ability, they will purchase capital to be used only in home production. Savers are assumed to be more patient than investors such that, in equilibrium, savers always want to lend to investors. Finally, the credit friction comes in the form of a leverage constraint: the debt that investors borrow cannot exceed a certain fraction of their total asset value.

### 2.1.1 Investor

Investors in each country $i$ choose consumption $c_{it}^I$, provide labor services $l_{it}^I$, and make a portfolio choice among domestic capital, foreign capital and domestic debt. Their utility is given by the following expression:

$$E_t \sum_{t=0}^{\infty} \left[\beta(C_{it}^I, L_{it}^I)\right]^t \frac{1}{1-\gamma} \left(c_{it}^I - \psi^I \frac{(l_{it}^I)^{1+\theta}}{1+\theta}\right)^{1-\gamma} \qquad i = 1,2 \qquad (1)$$

The investor has a Greenwood-Hercowitz-Huffman (GHH) preference, which is widely used in the open economy literature. Early work includes Correia, Neves and Rebelo (1995). GHH preference is chosen because there is no wealth effect on labor supply. As a result, only a substitution effect operates on hours and suggests that the path of hours will closely follow that of output.[1] To ensure stationary equilibrium, I follow

---

[1]GHH preferences are commonly used in the open economy literature, dating back to Mendoza (1991) and Devereux, Gregory and Smith (1992). Recent examples include Mendoza and Smith (2002) and Raffo (2009).

Mendoza (1991) to assume an endogenous discount factor.

$$\beta(C_{it}^I, L_{it}^I) = \left(1 + C_{it}^I - \psi^I \frac{(L_{it}^I)^{1+\theta}}{1+\theta}\right)^{-\omega^I}$$

The discount factor is external in the sense that a household takes $\beta(C_{it}^I, L_{it}^I)$ as exogenous. ($C_{it}^I$ and $L_{it}^I$ are the aggregate level of consumption and hours of investors.) As shown in Schmitt-Grohe and Uribe (2003), internalizing the discount factor makes negligible quantitative differences.

The period budget constraint of a representative investor is given by

$$
\begin{aligned}
c_{it}^I + q_{it}^k k_{ii,t+1}^I + q_{jt}^k k_{ij,t+1}^I &= w_{it} l_{it}^I + q_{it}^b B_{it+1}^I - B_{it}^I \\
&+ ((1-\delta)q_{it}^k + R_{it}^k)k_{ii,t}^I + ((1-\delta)q_{jt}^k + R_{jt}^k)k_{ij,t}^I
\end{aligned}
\tag{2}
$$

Here $q_{it}^k$ denotes the price of capital in country $i$, $q_{it}^b$ denotes the price of a bond in country $i$ ($q_{it}^b = \frac{1}{1+R_{it}^b}$ where $R_{it}^b$ is the risk-free rate), and $k_{ijt+1}^I$ denotes the capital in country $j$ held by an investor from country $i$. In each period, the investor receives a return $R_{it}^k$ ($R_{jt}^k$) by renting the capital to the market production firm in country $i$ ($j$). She also receives labor income by supplying labor to the market production firm. She then sells the capital after depreciation back to the capital producer at price $q_{it}^k$ ($q_{jt}^k$). By assumption, the investor is less patient than the saver; therefore, in equilibirum she will always borrow from the saver at the risk-free rate to finance the purchase of capital for the next period.

I assume that the investor may default on her debt; thus she always has to put down collateral against her debt. That is, the investor faces a collateral constraint (or leverage constraint) that restricts her debt to be smaller than a fraction $\kappa$ of the value of the asset offered as collateral.

$$B_{it+1}^I \le \kappa(q_{it}^k k_{ii,t+1}^I + q_{jt}^k k_{ij,t+1}^I) \quad \text{where } 0 \le \kappa \le 1 \tag{3}$$

Here $B_{it+1}^I$ denotes the amount of debt that she can borrow from the domestic saver and $\kappa$ controls the leverage ratio. This form of leverage constraint is in the style of

Kiyotaki and Moore (1997) and Mendoza and Smith (2002). Since the debt level is linked directly to the investor's total asset value, any fluctuation in either country's capital price will have an immediate impact on the borrowing capacity of the investors in both countries. Therefore, both the leverage constraint and the foreign capital exposure are the key ingredients that help to amplify the transmission of technology shocks across countries.

The FOCs for the investor are

$$q_{it}^k U_{c_{i,t}}^I = \beta_t^I E_t U_{c_{i,t+1}}^I ((1-\delta)q_{it+1}^k + R_{it+1}^k) + \kappa \mu_{it} q_{it}^k \tag{4}$$

$$q_{jt}^k U_{c_{i,t}}^I = \beta_t^I E_t U_{c_{i,t+1}}^I ((1-\delta)q_{jt+1}^k + R_{jt+1}^k) + \kappa \mu_{it} q_{jt}^k \tag{5}$$

$$q_{it}^b U_{c_{i,t}}^I = \beta_t^I E_t U_{c_{i,t+1}}^I + \mu_{it} \tag{6}$$

$$w_{it} = \psi^I (l_{it}^I)^\theta \tag{7}$$

where $\mu_{it}$ is the Lagrange multiplier on country $i$'s leverage constraint. We can see that when $\mu_{it}$ is positive, the investor wants to borrow more from the saver but is constrained by the leverage constraint.

In order to introduce home bias in the investor's capital holdings, I assume that there is some iceberg cost for investing in the foreign market. This can be interpreted as the fact that it is more difficult for a domestic investor to gather information in the foreign market. Following Devereux and Sutherland (2009) and Tille and van Wincoop (2007), the return from the foreign country that the investor receives is subject to a constant cost $\tau$. Therefore, (5) can be rewritten as

$$q_{jt}^k U_{c_{i,t}}^I = \beta_t^I E_t U_{c_{i,t+1}}^I ((1-\delta)q_{jt+1}^k + R_{jt+1}^k - \tau) + \kappa \mu_{it} q_{jt}^k \tag{8}$$

### 2.1.2   Saver

Consider a saver with GHH preferences described by

$$E_0 \sum_{t=0}^{\infty} \left[\beta(C_{it}^S, L_{it}^S)\right]^t u\left(c_{it}^{SM}, c_{it}^{SH}, l_{it}^{SM} l_{it}^{SH}\right) \tag{9}$$

7

where

$$u\left(c_{it}^{SM}, c_{it}^{SH}, l_{it}^{SM} l_{it}^{SH}\right) = \frac{1}{1-\gamma}\left(c_{it}^S - \psi^S \frac{(l_{it}^S)^{1+\theta}}{1+\theta}\right)^{1-\gamma} \tag{10}$$

and

$$c_{it}^S = \left(\lambda\left(c_{it}^{SM}\right)^e + (1-\lambda)\left(c_{it}^{SH}\right)^e\right)^{1/e}$$

$$l_{it}^S = l_{it}^{SM} + l_{it}^{SH}$$

The period utility is defined over four arguments: $c_{it}^{SM}$ is the consumption of a market good in country $i$, $c_{it}^{SH}$ is the consumption of a home good, $l_{it}^{SM}$ is labor time spent in market production and $l_{it}^{SH}$ is labor time spent in home production. The elasticity of substitution between $c_{it}^{SM}$ and $c_{it}^{SH}$ is given by $\frac{1}{1-e}$. The discount factor is defined similarly to that of an investor:

$$\beta(C_{it}^S, L_{it}^S) = \left(1 + C_{it}^S - \psi^I \frac{(L_{it}^S)^{1+\theta}}{1+\theta}\right)^{-\omega^S}$$

where $\omega^S$ represents the elasticity of the discount factor to the composite $1 + C_{it}^S - \psi^I \frac{(L_{it}^S)^{1+\theta}}{1+\theta}$.

At each date, the saver is subject to a market budget constraint that allocates total income between two uses: the purchase of the market consumption good and the purchase of household capital. Capital is sold back to the capital producer after being used in home production. I assume that capital depreciates at rate $\delta$. For simplicity, I assume it to be the same as the depreciation rate in the market production sector. The saver receives an interest payment on the bond she purchased. She also gets labor income by supplying labor to the market production firm. If $w_{it}$ is the wage rate, and $q_{it}^b$ is the price of the bond, then the budget constraint can be written as

$$c_{it}^{SM} + q_{it}^k k_{ii,t+1}^S = w_{it} l_{it}^{SM} + (1-\delta)q_{it}^k k_{ii,t}^S + q_{it}^b B_{it+1}^S - B_{it}^S \tag{11}$$

The saver is also subject to the home production constraint at each date

$$c_{it}^{SH} = G(k_{ii,t}^S, l_{it}^{SH}) \tag{12}$$

8

I assume that home production has a Cobb-Douglas production technology of the form

$$G(k_{ii,t}^S, l_{it}^{SH}) = (k_{ii,t}^S)^{\alpha_2}(l_{it}^{SH})^{1-\alpha_2} \tag{13}$$

Solving the saver's problem leads to the following FOCs:

$$q_{it}^k U_{c_m,t}^S = \beta_t^S E_t \left( U_{c_m,t+1}^S(1-\delta)q_{it+1}^k + U_{c_h,t+1}^S G_K(k_{ii,t+1}^S, l_{it+1}^{SH}) \right) \tag{14}$$

$$q_{it}^b U_{c_m,t}^S = \beta_t^S E_t U_{c_m,t+1}^S \tag{15}$$

$$w_{it} = \frac{\psi^S(l_{it}^{SM} + l_{it}^{SH})^\theta}{(c_{it}^S)^{1-e} \lambda (c_{it}^{SM})^{e-1}} \tag{16}$$

$$G_L(k_{ii,t}^S, l_{it}^{SH}) = \frac{\psi^S(l_{it}^{SM} + l_{it}^{SH})^\theta}{(c_{it}^S)^{1-e}(1-\lambda)(c_{it}^{SH})^{e-1}} \tag{17}$$

## 2.2   Capital Producer

In each country, there is one representative capital producer who operates in a perfectly competitive market. At the end of period $t$, the capital producer purchases final goods $i_{it}$ and the undepreciated physical capital $(1-\delta)k_{i,t}$ that has been used in period $t$'s production cycle. The capital producer uses these inputs to produce new installed capital $k_{it+1}$ using the following constant return to scale production technology

$$k_{i,t+1} = (1-\delta)k_{i,t} + \phi\left(\frac{i_{i,t}}{k_{i,t}}\right)k_{i,t}$$

I assume that the construction of new capital goods is subject to adjustment costs, whereas the repair of old capital goods is not. The following specification for adjustment cost is adopted

$$\phi\left(\frac{i_{i,t}}{k_{i,t}}\right) = \frac{g_1}{1-\pi}\left(\frac{i_{i,t}}{k_{i,t}}\right)^{1-\pi} + g_2$$

where $\phi(\cdot)$ is a positive, concave function. I denote the price of the new capital to be $q_{i,t}^k$, then the parameter $\pi$ controls the elasticity of $q_{i,t}^k$ with respect to the investment to capital ratio. This specification allows the shadow price of installed capital to

diverge from the price of an additional unit of capital, i.e., it permits variation in the price $q_{i,t}^k$. Similar to Kiyotaki and Moore (1997), the idea is to have asset price variability contribute to volatility in the investor's balance sheet.

Since the marginal rate of transformation from previously installed capital to new capital is unity, the price of old capital is also $q_{i,t}^k$. The firm's profit at time t is

$$\Pi_{i,t} = q_{i,t}^k k_{i,t+1} - q_{i,t}^k (1-\delta) k_{i,t} - i_{i,t}$$

The capital producer therefore solves

$$\max_{k_t, i_t} \ \Pi_{i,t} = q_{i,t}^k k_{i,t+1} - q_{i,t}^k (1-\delta) k_{i,t} - i_{i,t}$$

$$s.t. \ k_{i,t+1} = (1-\delta) k_{i,t} + \phi \left( \frac{i_{i,t}}{k_{i,t}} \right) k_{i,t}$$

Solving the maximization problem above leads to the following expression for capital price

$$q_{i,t}^k = \frac{1}{\phi' \left( \frac{i_{i,t}}{k_{i,t}} \right)} \tag{18}$$

Moreover, the new installed capital produced in each country $(k_{i,t})$ is bought by three types of agents: the domestic investor $(k_{ii,t}^I)$, the foreign investor $(k_{ji,t}^I)$ and the domestic saver $(k_{ii,t}^S)$.

$$k_{1,t} = n k_{11,t}^I + n k_{21,t}^I + (1-n) k_{11,t}^S$$

$$k_{2,t} = n k_{12,t}^I + n k_{22,t}^I + (1-n) k_{22,t}^S$$

## 2.3   Production

The structure of the market production firm is straightforward. The firm lives for only one period and has a Cobb-Douglas production function in capital and labor.

The production of the market good is subject to a stochastic technology shock $z_{it}^m$.

$$F(z_{i,t}^m, k_{i,t}^I, l_{i,t}) = e^{z_{it}^m}(k_{i,t}^I)^{\alpha_1}(l_{i,t})^{1-\alpha_1} \tag{19}$$

The firm rents capital from domestic and foreign investors

$$k_{i,t}^I = n(k_{ii,t}^I + k_{ji,t}^I) \tag{20}$$

and it also rents labor from the domestic investor and the domestic saver

$$l_{i,t} = nl_{i,t}^I + (1-n)l_{i,t}^{SM} \tag{21}$$

The optimality conditions for the firm are

$$w_{i,t} = F_L(z_{i,t}^m, k_{i,t}^I, l_{i,t}) \tag{22}$$

$$R_{i,t}^k = F_K(z_{i,t}^m, k_{i,t}^I, l_{i,t}) \tag{23}$$

I assume that the law of motion for the technology shock to market production is given by a stationary VAR of the form

$$\begin{bmatrix} z_{1t}^m \\ z_{2t}^m \end{bmatrix} = \begin{bmatrix} \rho_1^m & \rho_2^m \\ \rho_2^m & \rho_1^m \end{bmatrix} \begin{bmatrix} z_{1t-1}^m \\ z_{2t-1}^m \end{bmatrix} + \begin{bmatrix} \epsilon_{1t}^m \\ \epsilon_{2t}^m \end{bmatrix} \tag{24}$$

where $\rho_1^m$ represents the persistence of the technology shock and $\rho_2^m$ represents the spillover effect of the technology shock. The innovation follows

$$\begin{bmatrix} \epsilon_{1t}^m \\ \epsilon_{2t}^m \end{bmatrix} \sim N(0, \Sigma) \text{ with correlation matrix } \begin{bmatrix} \sigma_1^m & \\ \phi^m & \sigma_2^m \end{bmatrix} \tag{25}$$

where $\phi^m$ is the correlation between the two technology shocks.

## 2.4   Market Clearing

There are two sets of market clearing conditions: the bond market clearing and the good market clearing. Since the bond market is assumed to be domestic, the total bond within a country is zero, which gives the following conditions.

$$nB^I_{1t+1} + (1-n)B^S_{1t+1} = 0 \tag{26}$$

$$nB^I_{2t+1} + (1-n)B^S_{2t+1} = 0 \tag{27}$$

Now I develop the aggregate resource constraint for this economy.

$$nc^I_{1t} + (1-n)c^{SM}_{1t} + nc^I_{2t} + (1-n)c^{SM}_{2t} + i_{1t} + i_{2t} = F(z^m_{1t}, k^I_{1t}, l_{1t}) + F(z^m_{2t}, k^I_{2t}, l_{2t}) \tag{28}$$

The good market clearing gives the result that total market output is used in total market consumption and total investment.

## 2.5   Model Mechanism

This section reviews the main mechanism of the model and highlights important parameters. I show that both financial frictions and foreign asset exposure are important in leading to the increase in business cycle co-movements. When a negative technology shock hits the domestic market, the demand for capital in the home country falls, which forces down investment and the price of domestic capital. The degree to which the price of capital falls depends on the parameter $\pi$, which controls the elasticity of price with respect to the investment to capital ratio. As $\pi$ becomes larger, the capital price is more variable in response to a change in investment.

$$q^k_{i,t} = \frac{1}{\phi'\left(\frac{i_{i,t}}{k_{i,t}}\right)} = \frac{1}{g_1}\left(\frac{i_{i,t}}{k_{i,t}}\right)^\pi$$

From the investor's leverage constraint below, we see that the fall in the domestic asset price leads to a tightening of investors' leverage constraint in both countries. Borrowing is thus reduced globally. Since the leverage ratio is $\frac{1}{1-\kappa}$ for a given $\kappa$, as

$\kappa$ becomes bigger, the leverage ratio is higher. Hence the decline in global borrowing is steeper.

$$B_{it+1}^I \leq \kappa(q_{it}^k k_{ii,t+1}^I + q_{jt}^k k_{ij,t+1}^I) \quad \text{where } 0 \leq \kappa \leq 1$$

As borrowing falls globally, demand for capital in the foreign country also declines, which pushes down the price of the foreign asset, leading to another round of credit tightening. A multiplier effect arises, since the decline in investment lowers asset prices and investors' net worth, further pushing down investment.

From the equation above, we can see that by considering foreign exposure, the foreign asset price has an immediate effect on the balance sheet of domestic investors. Along with the presence of the financial frictions, the technology shock spills over from one country to another and thus drives up the business cycle correlations.

## 2.6    Solution Method

This model is solved using an iterative second-order perturbation method adopted from Heathcote and Perri (2009). The standard method for analyzing a DSGE model is to take a linear approximation around a deterministic steady state. However, this method cannot be used to solve the current model because when we have more than one asset, in the steady state the returns are the same across assets. Hence the portfolio shares are indeterminate: any share of domestic and foreign capital holdings will be consistent with the steady state. The way to find a steady state portfolio share is to use information from the higher-order approximation. The detailed algorithm is documented in the Appendix.

# 3    Calibration

I now proceed to choose parameter values, setting some numbers on the basis of a priori information and setting others according to the steady state conditions. A period in the model corresponds to one quarter. The sample period in the data is from 1972:1 to 2008:4. Table 1 gives a summary of the calibration.

## 3.1  Preference and Production Parameters

The intertemporal elasticity of substitution (IES) is set to 0.5, which is standard in the literature. The parameter $\omega^S$, which controls the saver's discount factor, is set to 0.039 to match an annual interest rate of 4%. Following Bernanke, Gertler and Gilchrist (1999), I use the investor's discount factor to match an interest premium on borrowed funds of 2%, approximately the historical average spread between the prime lending rate and the six-month Treasury bill rate. This gives $\omega^I$ the value of 0.112. The implied steady state discount factor for the saver is 0.99 and the implied steady state discount factor for the investor is 0.97. For the elasticity of labor supply, in line with Greenwood, Hercowitz and Huffman (1988), I calibrate it to be 1.7, which corresponds to $\theta = 0.6$.

The depreciation rate $\delta$ is set to 0.025, corresponding to an annual depreciation rate of 10%. I now use $\alpha_1$(capital share of market production), $\alpha_2$(capital share of home production), $\psi^I$(investor's labor supply level), $\psi^S$(saver's labor supply level) and $\lambda$(share of market consumption good) to match the following five observations: the market capital-to-output ratio, the home capital-to-output ratio, the market hours for the investor, the market hours for the saver and the home hours for the saver. According to Greenwood, Rogerson and Wright (1993), the home capital to output ratio is 5, where home capital is defined as consumer durables plus residential structures. Since the total capital to output ratio is around 12, as given by Cooley and Precsott (1995), the market capital to output ratio is set to 7. I choose the hours worked for market production to be 0.33 and the hours spent on home production to be 0.25. This calibration gives a capital share of market production ($\alpha_1$) of 0.29 and a capital share of home production ($\alpha_2$) of 0.40.

The only preference parameter that is left unspecified is $e$, the elasticity of substitution between the market and home consumption good. A higher value of $e$ means that the saver is more willing to substitute consumption of one sector's output with consumption of the other sector's output. The empirical evidence on $e$ is controversial. Eichenbaum and Hansen (1990) suggest that the two goods are very close to perfect substitutes. Benhabib, Rogerson and Wright (1991) use PSID data to estim-

ate this elasticity, which results in a value of 0.8 for $e$. For the benchmark model, I use an intermediate value among existing estimates, $e = 0.9$. In what follows I will also consider several alternative values of $e$ as a robustness test.

Following Bernanke, Gertler and Gilchrist (1999), the elasticity of the capital price with respect to the investment to capital ratio, $\pi$, is taken to be 0.25. This is one of the key parameters in the model since capital price is crucial for determining the level of loans for the investor and hence the global investment level. However, there is no firm consensus in the literature about what this parameter value should be. A reasonable assumption about the adjustment cost suggests that the value should lie within a range of 0. to 0.5.[2] The parameter $\tau$ controls the degree of home bias. When this cost is absent, only 14% of the investor's capital holdings are domestic, exhibiting a substantial bias against home capital. This observation is consistent with the theory, since when an agent's labor income is correlated with her home capital return, to diversify this risk the agent will take a larger position in the foreign country. I set $\tau$ to be 0.091 such that 75% of the investor's assets are domestic.

When the leverage constraint is binding, the leverage ratio is $\frac{1}{1-\kappa}$ for a given $\kappa$. In this model, I calibrate the leverage ratio to be 3, according to Dedola and Lombardo (2010). This number is higher than the leverage ratio used in Bernanke, Gertler and Gilchrist (1999), since I consider not only non-financial firms but also financial firms.

In this model, savers do not have access to the equity market; therefore, I calibrate the share of savers to match the fraction of the population who do not participate in the stock market. According to the Survey of Consumer Finances (2007), about half of US households have become stock owners. Therefore, I set the share of savers to be 0.5.

---

[2]Jermann (1998) uses a value of 0.23. Christensen and Dib (2008) give an estimate of 0.59 by Bayesian estimation. Meier and Muller (2006) report an estimate of 0.65, close to that of Christensen and Dib (2008).

## 3.2 Technology Parameters

For the benchmark calibration of technology, I follow the estimates from Heathcote and Perri (2004). They estimate the parameters of the bivariate shock process using estimates of Solow residuals. They subtract a common deterministic growth trend from Solow residuals and then estimate by least squares. In this case, the productivity shocks still display high persistence and positively correlated innovations, but they no longer find evidence of spillovers. This gives the following estimates

$$\begin{bmatrix} z_{1t}^m \\ z_{2t}^m \end{bmatrix} = \begin{bmatrix} 0.91 & 0. \\ 0. & 0.91 \end{bmatrix} \begin{bmatrix} z_{1t-1}^m \\ z_{2t-1}^m \end{bmatrix} + \begin{bmatrix} \epsilon_{1t}^m \\ \epsilon_{2t}^m \end{bmatrix}$$

where

$$\begin{bmatrix} \epsilon_{1t}^m \\ \epsilon_{2t}^m \end{bmatrix} \sim N(0, \Sigma) \text{ with correlation matrix } \begin{bmatrix} 0.006 \\ 0.25 & 0.006 \end{bmatrix}$$

In the sensitivity analysis, I also use the productivity estimates from Backus, Kehoe and Kydland (1992), where there is some evidence of spillover. The estimates are

$$\begin{bmatrix} z_{1t}^m \\ z_{2t}^m \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} z_{1t-1}^m \\ z_{2t-1}^m \end{bmatrix} + \begin{bmatrix} \epsilon_{1t}^m \\ \epsilon_{2t}^m \end{bmatrix}$$

and I maintain the same covariance matrix as in Heathcote and Perri (2004).

# 4 Results

In this section, I analyze the quantitative implications of my model. First, I report the moments generated by the model and compare them with the data. Second, I look at the impulse response functions (IRFs) of the technology shock to analyze the model mechanism.

## 4.1    Moments

The results of the simulation under the benchmark calibration are summarized in Table 2. The first column of Table 2 shows the statistics calculated from the data. Panels (A), (B) and (C) are calculated from US time series for the period 1972:1 to 2008:4. The statistics from panel (D) represent the correlation of US series with series from the rest of the industrial world (which is an aggregate of Europe, Japan and Canada). The details of the aggregation of the rest of the world data are shown in the Appendix. Except for net exports, all series are logged and filtered by the Hodrick-Prescott filter with a smoothing parameter of 1600. In the table, output and consumption refer to market output and consumption while investment refers to total investment.[3]

The third column of Table 2, "Model 2: Constrained with 25% Foreign Exposure," is our benchmark model with calibrations documented in Section 3. The second column of Table 2, "Model 1: Unconstrained," is the same as Model 2 except that the investor does not face a leverage constraint. The last column, "Model 3: Constrained with 86% Foreign Exposure," is the model where instead of imposing a 75% home bias, I let the investor fully diversify her portfolio such that, as shown in the calibration, she holds 86% of the capital in the foreign market.

We first compare the data with the results from the constrained economy (Model 2). As we see from the cross-country correlations in panel (D) of Table 2, the constrained economy matches the investment and consumption correlations well. The model produces more than half of the output correlation in the data and matches most of the employment correlation. When we compare the constrained economy (Model 2) with the unconstrained economy (Model 1), we can see that having the leverage constraint in the model gives an overall improvement in the cross-country correlations. The unconstrained economy predicts a consumption correlation and an output correlation that are too low, relative to the data. The constrained economy does better, predicting a higher level of consumption and output correlations. In terms of investment and employment, both models predict positive correlations,

---

[3]The definition of investment is consistent with that in the data.

while the constrained economy is closer to the moments in the data.

Overall, the model with constraint performs better in terms of the cross-country correlations. The presence of the leverage constraint increases the correlation of consumption, employment and output, while it decreases the correlation of investment. As will be shown in the IRF analysis in Section 4.2, those improvements are introduced by the financial frictions.

In terms of the within-country moments, in general the model with constraint gives moments that are closer to the moments in the data. The constrained economy replicates the level of output volatility in the data; however, output in the unconstrained economy (2.52%) is more volatile than in the constrained economy (1.84%). The high volatility of the unconstrained economy is introduced by the frequent substitution between market and home consumption. In terms of relative volatility, both models over-predict the volatility of consumption while they under-predict the volatility of investment in the data. For the within-country correlations, both models give positive correlations of net exports with output while we see negative correlations in the data.[4]

We then compare the difference induced by financial exposure. The investor in Model 2 holds 25% of capital in the foreign market, while the investor in Model 3 holds 86% of capital in the foreign market. The impact of this foreign asset exposure on the business cycle co-movements is immediate. If we look at the cross-country correlations, output correlation increases from 0.34 to 0.52. Consumption and labor also rise because of the increased synchronization of output. Investment correlation, on the other hand, falls. As foreign capital exposure increases, foreign asset prices will have a more profound impact on the debt level of the investor, which in turn influences domestic investment and output. Hence, the output correlation is driven up by increased foreign asset exposure.

---

[4]This result is also found in several other papers studying international business cycle correlations, such as Backus, Kehoe and Kydland (1992) and Heathcote and Perri (2002).

## 4.2 Impulse Responses

In this section, I explain why the behavior of the three models differs. I analyze the response of the two-country economy to a one-standard-deviation negative shock in country 1. As in all the subsequent figures, the time units on the graphs are to be interpreted as quarters.

Figures 1-4 show the impact of a one-standard-deviation decline in country 1's technology shock. The upper panel shows country 1's response and the lower panel shows country 2's response. In each plot, the dashed line corresponds to the impulse response in the unconstrained economy (Model 1) and the solid line corresponds to the impulse response in the constrained economy (Model 2). In the figure, output and consumption refer to market output and consumption, while investment refers to total investment.

### 4.2.1 Model with Leverage Constraint

We first analyze the response of the constrained economy (Model 2). When a negative shock hits, the demand for capital in country 1 immediately falls, leading to a 0.9% decline in investment in country 1. Following the weak demand for capital, the price of capital in country 1 falls 0.22%. Since investors hold leveraged portfolios across countries, the decline in asset prices in country 1 leads to a reduction in total wealth for investors in both countries. Therefore, the leverage constraints are tightened globally and the debts that investors are eligible to lend are reduced. We observe a 0.6% decline of debt in country 1 and a 0.2% decline of debt in country 2. After the global decline of debt, not only do investors in country 1 have a weak demand for capital, but so do investors in country 2. Hence, investment and capital price fall in country 2 as well. The decline in capital price thus triggers another round of declines in investment and output.

Since the decline in the demand for capital reduces the income of the investor, the investor's consumption falls. As the savers suffer from a decline in their wage income, savers' consumption is also reduced. Overall, total market consumption in country 1 falls around 1.4% and that of country 2 falls around 0.2%.

19

Upon the negative shock to productivity, wages fall in country 1; hence, the investor's and saver's labor supplies are reduced immediately in country 1. We observe a 0.6% decline in total market labor for country 1. For the market labor in country 2, since there is no wealth effect on investor's labor for GHH preferences, the investor's market labor supply in country 2 does not move. However, since the saver faces substitution between home and market consumption, the wealth effect and substitution effect both affect her labor supply.[5] When the shock hits, the wealth effect dominates the substitution effect and the saver in country 2 increases her market labor supply. However, this effect is minimal as it only leads to a one-basis-point movement. Next period, the substitution effect becomes larger and the saver's market labor supply falls.

The output of market production in country 1 falls by 1.4% in the next period, and through the transmission mechanism introduced by the leverage constraint, the output of market production in country 2 falls by around 0.1%.

In country 2, we also see an increase in the capital used in home production. Because the domestic saver holds a portfolio of domestic capital and a bond, a decline in the demand for the bond makes the saver shift her assets to domestic capital; therefore, the capital used in home production increases.

### 4.2.2 Model with No Leverage Constraint

We then analyze the response of the unconstrained economy (Model 1). When the investor is not facing a leverage constraint, an unexpected one-standard-deviation decline in country 1's productivity leads to a fall in the return on market capital in country 1. We observe a fall in the debt level in country 1, because home production becomes more productive than market production, making the saver shift her holdings from a bond to home capital. From Figure 1 we also observe an increase in the debt level in country 2. This is because the investor from country 2 suffers from her investment loss in country 1 caused by the low return on market capital and she does not face any form of collateral constraint; therefore she increases her debt to

---

[5]From Equations (16) and (17) we see that consumption shows up in the FOC for saver's labor choice, therefore affecting the saver's labor decision.

compensate for her investment loss. The changes in the debt level in country 2 also lead to a decline in the purchase of home capital in country 2, because the increase in the debt level indicates an increase in the bond holdings of the saver. As the saver holds more bonds, she rebalances her portfolio by reducing her exposure to home capital.

Because of the decline in productivity in country 1's market sector, country 2 now looks more productive. Market capital flows from country 1 to country 2; thus, Figure 3 shows an increase in market capital in country 2. Market output in country 2 follows a pattern similar to that of market capital: market output in country 2 increases after the shock. Investment and capital prices in both countries fall. In country 1, the decline in investment is mainly driven by market capital: investment in market capital falls because of a lower return. In country 2, the decline in investment is mainly driven by home capital: the investment in home capital falls because of the saver's portfolio balancing.

### 4.2.3   Comparison

After examining the two scenarios separately, we now put them together for comparison. There are several points to note. First, upon a negative technology shock to country 1, market output in country 2 declines in the constrained economy, whereas it increases in the unconstrained economy. The response of the unconstrained economy is similar to the situation in a standard model with complete markets: capital flows into the more productive country, leading to negative responses of the production factors. The effect of the financial frictions becomes apparent when we look at the response of the constrained economy. The presence of the leverage constraint limits investors' ability to invest in both countries. Since they are constrained from getting more loans, they do not have many resources to invest; therefore, although country 2's investment opportunity is better, market capital in country 2 still declines.

Second, the decline of consumption in country 2 in the constrained economy is nearly three times as much as in the unconstrained economy. For the unconstrained economy, country 2's consumption declines only 0.06%. Since the investor is not

21

constrained, she can borrow from the saver to cushion her investment loss; therefore, her consumption is barely affected. However, for the constrained economy, the investor cannot borrow as much as she wants; hence, consumption is affected to a greater degree, leading to a 0.18% decline.

Third, investment in country 2 falls less in the constrained economy than in the unconstrained economy. The reason is the following: investment is defined as total investment in the country, which means that it is the sum of investment in market production and investment in home production. In the constrained economy, investment in the market sector falls because of the tightened leverage constraint for the investor. However, investment in the home sector rises because the saver shifts her portfolio from bonds to home capital. These two forces work against each other and the fall in market investment outweighs the increase in home investment, leading to an overall decline in investment. In the unconstrained economy, market investment in country 2 rises because relative productivity in country 1 is now higher. At the same time, investment in the home sector declines because now the saver shifts her portfolio from home capital to bonds. These two forces result in a decline in the total investment level in country 2 and the magnitude is larger than in the constrained economy.

Fourth, the debt levels in the two countries move in the same direction in the constrained economy, whereas the debt levels move in different directions in the unconstrained economy. For the constrained economy, debt falls in both countries because the leverage constraints in both countries are tightened. For the unconstrained economy, country 2's investor increases her debt to offset the loss in investment in country 1.

To briefly sum up, the differences between the two models discussed above are exactly introduced by the financial frictions. The financial frictions drive up output, consumption and employment correlations and drive down the investment correlation.

### 4.2.4 Degree of Foreign Exposure

In this section, I look at different degrees of foreign asset exposure. I compare two cases: Model 2, in which investors are holding 25% of their capital in the foreign market, and Model 3, in which investors are holding 86% of their capital in the foreign market. Figures 5-8 show the impact of a one-standard-deviation decline in country 1's technology level. The upper panel shows country 1's response to the shock and the lower panel shows country 2's response. In each plot, the solid line corresponds to the impulse response for the 25% foreign exposure economy and the dashed line corresponds to the 86% foreign exposure economy.

Given the same level of decline in capital prices in country 1 for both economies (Figure 5), it is straightforward to see that the more foreign capital the investor holds, the more she suffers from tightening of the leverage constraint. This idea is confirmed in Figure 5, which shows the response of the debt level in country 2. We notice that when the investor holds 86% of foreign capital, her debt level falls three times more than in the case where she holds only 25% of foreign capital. The debt level further influences other economic activities; hence output and consumption decrease. Therefore, a larger balance-sheet exposure to risky foreign capital results in business cycles that are more synchronized between the two countries.

## 5    Sensitivity Analysis

In this section, we report the results of the sensitivity analysis with respect to some key parameters in the model. Specifically, I explore some alternative values for the investment adjustment cost, leverage ratio, shock process and elasticity of substitution between home and market goods.

### 5.1    Adjustment Cost

The parameter $\pi$ controls the elasticity of the capital price with respect to the investment to capital ratio. As discussed in the calibration, the estimate of this elasticity varies in the literature. A recent paper by Christensen and Dib (2008) estimates $\pi$

to be 0.59 using data on investment. Other papers, such as Meier and Muller (2006), give an even higher value of 0.65. Therefore, as a robustness check, we set $\pi$ to 0.5, implying a larger investment adjustment cost and a slower response of investment than the benchmark model. We also set $\pi$ to 100; in this case the adjustment cost is so large that investment does not move at all. The model is then reduced to a version where capital is fixed in each country, which is similar to the setup of Devereux and Yetman (2010).[6] I argue that a significant difference exists between the model with and without capital accumulation. When capital cannot move across countries, the business cycle synchronization becomes stronger.

Table 3 shows the simulation results when $\pi$ is 0.25, 0.5 and 100, respectively. As $\pi$ increases and as we move from left to right of the table, we see an increase in the cross-country correlations in all macro variables. The important role that $\pi$ plays in the propagation mechanism is obvious. When the investment adjustment cost becomes higher, the capital price responds more to a technology shock. Since the capital price has an immediate impact on the investor's balance sheet, it influences the level of loans and the investor's future investment decisions. Therefore, when the investment adjustment cost increases, business cycles are more synchronized. A higher adjustment cost, on the other hand, also implies that investment is less responsive to shocks. Therefore, we see a decline in the investment volatility.

## 5.2   Leverage Ratio

Now I experiment with a higher leverage ratio for the investor's leverage constraint. As shown in the previous section, the leverage constraint serves as an important channel for the propagation of technology shocks. From the leverage constraint below, we see that as $\kappa$ becomes bigger, the bigger the impact the investor's asset value has on the eligible loans.

$$B_{it+1}^I \leq \kappa(q_{it}^k k_{ii,t+1}^I + q_{jt}^k k_{ij,t+1}^I)$$

---

[6]However, the two models are still not the same since this one has endogenous labor. Capital in this case can be interpreted as land, which is not mobile across countries but nevertheless can be owned by different investors.

Since many financial firms have higher leverage ratios, we set $\kappa$ to 0.8, corresponding to a leverage ratio of 5 as a robust check. As seen from Table 4, output volatility increases compared to an economy with lower leverage. There is also 0.07 increase in the cross-country correlations of consumption, output and labor.

## 5.3  Different Shock Process

In the benchmark calibration, there is no spillover between the two technology shocks. Therefore I conduct a sensitivity analysis regarding the spillovers. The calibration for the technology shock is taken from Backus, Kehoe and Kydland (1992), where the persistence of the shock is 0.906 and the spillover is 0.088. The covariance matrix for the innovation remains the same.

From the last column of Table 4, we observe an increase in the consumption correlation and a decline in output, investment and labor correlations. When there is spillover between technology shocks, the consumption correlation increases from 0.45 to 0.62. This is because a negative shock to one country signals that the other country's output will also decline in the future. Consumers in that country take this into account and lower their current consumption. Therefore, the consumption correlation goes up when a technology shock spills over from one country to the other. Kehoe and Perri (2002) find a similar effect in their paper.

## 5.4  Elasticity of Substitution between Goods

Since the estimates of the elasticity of substitution between home and market goods span a wide range, we experiment with different values of $e$: 0.9, 0.5 and 0.1. As $e$ gets smaller, it is more difficult to substitute between the two goods. Table 5 shows the simulation results for different values of $e$. As $e$ gets smaller, the correlations of output, labor and consumption become smaller, while the investment correlation becomes larger.

The decline in the correlations of consumption, output and labor becomes apparent when we look at the saver's FOC for labor choice. Because the saver substitutes between home and market goods, consumption starts to have an impact on the labor

choice. In other words, the wealth effect starts to show up in the saver's labor choice. Therefore, as $e$ gets smaller, the saver starts to supply more labor because she feels poorer. This then reduces the correlation of labor. Thus the correlation of output is also reduced. The increase in the investment correlation occurs because when $e$ is close to zero, it becomes very difficult to substitute between market and home consumption; therefore, the role of home production is reduced to a minimum. Thus, we see an increase in the investment correlation.

# 6    Conclusions

This paper argues that financial frictions are important for international business cycles because they magnify the propagation of technology shocks across countries through the balance sheet of leveraged investors.

I have shown that incorporating financial frictions and exposure to foreign assets, which seems to be an important aspect of the recent financial crisis, helps us do a better job of accounting for business cycle correlations across countries. The calibrated model can match most of the investment, employment and consumption correlations as in the data. The model indicates an output correlation that accounts for more than half of the output correlation in the data. Moreover, the model also shows that, consistent with the data, when investors have more foreign asset exposure to the other country, the output correlation between the two countries increases.

My study reaffirms the growing attention in the open economy literature to integrating financial market frictions into otherwise standard two-country models. I document the importance of including financial frictions and foreign asset exposure in the analysis. Since this model is able to replicate some key facts of international business cycles, I believe that this framework is a promising one for conducting further research, particularly on welfare analysis and the design of monetary and fiscal policies.

# References

[1] Backus, D., P.J. Kehoe and F. Kydland (1992) "International Real Business Cycles," Journal of Political Economy, vol.100 no.4

[2] Baxter, M. and M.J. Crucini (1995) "Business Cycles and the Asset Structure of Foreign Trade," International Economic Review, vol. 36 no. 4

[3] Benhabib, J., R. Rogerson and R. Wright (1991) "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," Journal of Political Economy 99, 1166-1187

[4] Bernanke, B., M. Gertler and S. Gilchrist (1999) "The Financial Accelerator in a Quantitative Business Cycle Framework," Handbook of Macroeconomics, vol 1

[5] Christensen, I. and A. Dib (2008) "The Financial Accelerator in an Estimated New Keynesian Model," Review of Economic Dynamics 11 155-178

[6] Cooley, T.F. and E.C. Prescott (1995) "Economic Growth and Business Cycles," Frontiers of Business Cycle Research, Princeton University Press.

[7] Correia, I, J.C. Neves and S. Rebelo (1995) "Business Cycles in a Small Open Economy," European Economic Review 39 1089-1113

[8] Dedola, L. and G. Lombardo (2010) "Financial Frictions, Financial Integration and the International Propagation of Shocks," Working Paper

[9] Devereux, M, A.W. Gregory and G.W. Smith (1992) "Realistic Cross-Country Consumption Correlations in Two-Country, Equilibrium, Business Cycle Model," Journal of International Money and Finance 11 3-16

[10] Devereux , M. Sutherland (2009) "Country Portfolios in Open Economy Macro Models," Working Paper

[11] Devereux , M. and J. Yetman (2010) "Leverage Constraints and the International Transmission of Shocks," NBER Working Paper 16226

[12] Eichenbaum, M. and L.P. Hansen (1990) "Estimating Models with Intertemporal Substitution using Aggregate Time Series Data," Journal of Business and Economic Statistics 8, 44-66

[13] Faia, E (2007) "Finance and International Business Cycles," Journal of Monetary Economics 54 1018-1034

[14] Gertler, M., S. Gilchrist and F.M. Natalucci (2007) "External Constraints on Monetary Policy and the Financial Accelerator," Journal of Money, Credit and Banking, vol. 39, no.2-3

[15] Greenwood, J., Z. Hercowitz, and G. Huffman (1988) "Investment, Capacity Utilization, and the Real Business Cycle," American Economic Review vol. 78 no. 3

[16] Greenwood, J., R. Rogerson, and R. Wright (1993) "Putting Home Economies into Macroeconomics," Federal Reserve Bank of Minneapolis Quarterly Review, Summer, 1-17

[17] Heathcote, J and F. Perri (2002) "Financial Autarky and International Business Cycles," Journal of Monetary Economics, 49 601-627

[18] Heathcote, J and F. Perri (2004) "Financial Globalization and Real Regionalization," Journal of Economic Theory 119 207-243

[19] Heathcote, J and F. Perri (2009) "The International Diversification Puzzle Is not as Bad as You Think," Working Paper

[20] Iacoviello, M (2005) " House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," America Economic Review, vol 95 no. 3

[21] Imbs, J (2006) "The Real Effects of Financial Integration," Journal of International Economics 68 (2006) 296-324

28

[22] Jermann, U. (1998) "Asset Pricing in Production Economies," Journal of Monetary Economics 41, 257-275

[23] Kehoe, P. and F. Perri (2002) "International Business Cycles with Endogenous Incomplete Markets," Econometrica, vol 70 no.3 907-928

[24] Kiyotaki and Moore (1997) "Credit Cycles," Journal of Political Economy, vol 105, no. 2

[25] Kollmann, R. (1996) "Incomplete Asset Markets and the Cross-Country Consumption Correlation Puzzle," Journal of Economic Dynamics and Control 20 945-961

[26] Meier, A. and G Muller (2006) "Fleshing out the Monetary Transmission Mechanism: Output Composition and the Role of Financial Frictions," Journal of Money, Credit, Banking 38, 1999-2133

[27] Mendoza, E.G. (1991) "Real Business Cycles in a Small Open Economy," American Economic Review, vol 81 no.4

[28] Mendoza, E.G. and K.A. Smith (2002) "Margin Calls, Trading Costs, and Asset Prices in Emerging Markets: The Financial Mechanics of the 'Sudden Stop' Phenomenon," NBER Working Paper 9286

[29] Raffo, A (2009) "Technology Shocks: Novel Implications for International Business Cycles," Working Paper, Board of Governors of the Federal Reserve System

[30] Schmitt Grohe, S. and M. Uribe (2003) "Closing Small Open Economy Models," Journal of International Economics, 61 163-185

[31] Tille, C. and E.van Wincoop (2007) "International Capital Flows," Staff Report no. 280 Federal Reserve Bank of New York

# A    Computation

This appendix describes an algorithm for computing the equilibrium portfolios in open economy DSGE models. To a large extent, existing open economy models ignore portfolio composition, analyzing financial linkage between countries in terms of net foreign assets, with no distinction made between assets and liabilities. There is a growing literature that tries to develop methods to solve portfolio problems in these models. This work has been pioneered by Devereux and Sutherland (2009) and Tille and van Wincoop (2007) with a recent contribution by Heathcote and Perri (2009). The idea of these three methods is essentially the same: If we have more than one asset, then the asset returns in the steady state are the same. Therefore, the portfolios are indeterminate in the steady state. In order to use the perturbation method to solve the model, we need steady state portfolio shares to perturb around. In general, we use information from second-order perturbations to determine the steady state portfolios.

To be specific, in my model the steady state returns to capital in market production are the same across countries. Therefore, although the total amount of capital used in market production is known, the distribution is indeterminate: home and foreign investors can hold an arbitrary portion of the total market capital. I use the algorithm developed by Heathcote and Perri (2009) in solving this model.

Step 1 : Calculate the non-stochastic symmetric steady state equilibrium. We denote the steady state as $[\lambda_{11}, \lambda_{22}, X, Y]$, where $\lambda_{11}$ is the market capital in country 1 held by country 1 investors, and $\lambda_{22}$ is the market capital in country 2 held by country 2 investors. $X$ is the steady state of non-portfolio state variables and $Y$ is the steady state of non-portfolio control variables. The first-order conditions pin down the value of $X$ and $Y$, while any value of $\lambda_0 = \lambda_{11} = \lambda_{22}$ is consistent with the equilibrium.

Step 2: Compute the decision rules $\lambda_{11,t+1} = g_1(\lambda_{11,t}, \lambda_{22,t}, X_t)$, $\lambda_{22,t+1} = g_2(\lambda_{11,t}, \lambda_{22,t}, X_t)$, $X_{t+1} = g_3(\lambda_{11,t}, \lambda_{22,t}, X_t, \epsilon_{t+1})$, $Y_t = g_4(\lambda_{11,t}, \lambda_{22,t}, X_t)$ up to second order around the steady state. The decision rules are computed using methods from Schmitt-Grohe and Uribe (2004). In order to apply their methods,

I add a small quadratic adjustment cost for changing the portfolio from its steady state. However, we do not know whether the steady state portfolio $\lambda_0$ we guessed is the same as the average equilibrium portfolio in the true stochastic economy.

Step 3: Simulate the model for a large number of periods using the computed decision rules from Step 2. Compare the average portfolio shares with the steady state portfolio. If they are different, then we update the steady state portfolio with the average portfolio and return to Step 2. If the difference between them is within a certain tolerance level, then that means the initial steady state $\lambda_0$ is a good approximation of the long-run portfolio holdings and we take it as the solution to our model.

This algorithm is tested in Heathcote and Perri (2009) by comparing it to the model solution where the analytical form of the portfolio is known. The comparison shows that this algorithm gives a good approximation to the model and enjoys a rapid convergence.

# B   Data

The data series come from the OECD Quarterly National Accounts (QNA). For the US, GDP, consumption and investment correspond to Gross Domestic Product, Private plus Government Final Consumption Expenditure and Gross Fixed Capital Formation (all at constant prices). The employment data, which come from OECD Main Economic Indicators, use the (deseasonalized) civilian employment index series. The import and export series at constant prices are from OECD Quarterly National Accounts.

For the data for the rest of the world, we construct an aggregate of Canada, Japan and 19 European countries. The 19 European countries include Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Norway, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey and the United Kingdom. For GDP, consumption and investment, I aggregate all the countries to create a single fictional non-US country by first rebasing each series in 2005 national currency constant prices and then expressing everything in 2005 US dollars using PPP exchange rates.

Employment for the rest of the world is aggregated using constant weights that are porportional to the number of employed persons in each area in 2005. An employment series for the 19 European countries is not available before 2001; therefore I use employment for Austria, Finland, France, Germany, Italy, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom between 1984:1 and 2000:4. For the period 1972:1 to 1983:4, I use aggregated employment data from the same set of countries between 1984:1 and 2000:4 except Portugal. For the period 1962:1 to 1971:4, I use aggregated data from Finland, Germany, Italy, Sweden and the United Kingdom. These were the only European countries for which I could find consistent and comparable employment series.

# C   Table and Figures

Table 1: Benchmark Calibration

| Parameter | Description | Value |
|:---------:|:------------|:------|
| $\gamma$ | inverse of IES | 2 |
| $\omega^I$ | controls investor's discount factor | 0.112 |
| $\omega^S$ | controls saver's discount factor | 0.039 |
| $\theta$ | controls elasticity of labor supply | 0.6 |
| $\alpha_1$ | capital share of market production | 0.29 |
| $\alpha_2$ | capital share of home production | 0.40 |
| $\psi^I$ | controls level of investor's labor | 3.08 |
| $\psi^S$ | controls level of saver's labor | 1.32 |
| $\lambda$ | share of market good consumption | 0.57 |
| $e$ | controls ES between home and market good | 0.9 |
| $\delta$ | depreciation | 0.025 |
| $\pi$ | invesment adjustment cost | 0.25 |
| $\tau$ | iceberg cost | 0.091 |
| $n$ | measure of investors | 0.5 |
| $\kappa$ | controls leverage ratio | 2/3 |

*Note*: The first column shows the parameters that need to be calibrated. The second column describes the parameters and the last column shows the calibrated values for the parameters.

## Table 2: Model Moments - Benchmark Model

|  | Data | Model 1 Unconstrained | Model 2 Constrained 25% Foreign Exposure | Model 3 Constrained 86% Foreign Exposure |
|---|---|---|---|---|
| **(A) Standard Deviation in %** | | | | |
| Output | 2.06 | 2.52 | 1.84 | 1.78 |
| Net Export | 0.39 | 0.28 | 0.21 | 0.16 |
| | | | | |
| **(B) Standard Deviation relative to Ouput** | | | | |
| Consumption | 0.63 | 1.07 | 1.01 | 0.99 |
| Investment | 2.82 | 0.55 | 0.67 | 0.77 |
| Labor | 0.67 | 0.73 | 0.71 | 0.71 |
| | | | | |
| **(C) Cross-Correlation with Output** | | | | |
| Consumption | 0.82 | 0.99 | 0.99 | 0.98 |
| Labor | 0.86 | 1 | 1 | 1 |
| Investment | 0.95 | 0.91 | 0.94 | 0.96 |
| Net Export | -0.45 | 0.54 | 0.53 | 0.46 |
| | | | | |
| **(D) Cross-Country Correlations** | | | | |
| Consumption | 0.44 | 0.28 | 0.45 | 0.75 |
| Output | 0.61 | 0.23 | 0.34 | 0.52 |
| Investment | 0.46 | 0.76 | 0.46 | 0.29 |
| Labor | 0.43 | 0.23 | 0.34 | 0.54 |

*Note*: The first column shows the statistics calculated from the data. Panels (A), (B) and (C) are calculated from US time series for the period 1972:1 to 2008:4. The statistics from panel (D) represent the correlation of US series with series from the rest of the industrial world. The third column, "Model 2," is the benchmark model. The second column, "Model 1," is the same as Model 2 except that the investor does not face the leverage constraint. The last column, "Model 3," is the same as Model 2 except that the investors have more exposure to foreign capital.

Table 3: Sensitivity Analysis - Adjustment Cost

| | Data | Benchmark Model $\pi = 0.25$ | Sensitivity Test $\pi = 0.5$ | $\pi = 100$ |
|---|---|---|---|---|
| **(A) Standard Deviation in %** | | | | |
| Output | 2.06 | 1.84 | 1.94 | 2.22 |
| Net Export | 0.39 | 0.21 | 0.26 | 0.34 |
| | | | | |
| **(B) Standard Deviation relative to Ouput** | | | | |
| Consumption | 0.63 | 1.01 | 1.06 | 1.18 |
| Investment | 2.82 | 0.67 | 0.45 | 0.00 |
| Labor | 0.67 | 0.71 | 0.72 | 0.72 |
| | | | | |
| **(C) Cross-Correlation with Output** | | | | |
| Consumption | 0.82 | 0.99 | 0.99 | 0.99 |
| Labor | 0.86 | 1 | 1 | 1 |
| Investment | 0.95 | 0.94 | 0.95 | 0.93 |
| Net Export | -0.45 | 0.53 | 0.55 | 0.48 |
| | | | | |
| **(D) Cross-Country Correlations** | | | | |
| Consumption | 0.44 | 0.45 | 0.56 | 0.70 |
| Output | 0.61 | 0.34 | 0.41 | 0.53 |
| Investment | 0.46 | 0.46 | 0.70 | 0.91 |
| Labor | 0.43 | 0.34 | 0.42 | 0.54 |

*Note*: The first column shows the statistics calculated from the data. The second column is the benchmark model. The last two columns are for different values of the investment adjustment cost.

Table 4: Sensitivity Analysis - Leverage and Shocks

| | Data | Benchmark Model | Sensitivity Test | |
| --- | --- | --- | --- | --- |
| | | | High Leverage | BKK |
| **(A) Standard Deviation in %** | | | | |
| Output | 2.06 | 1.84 | 2.18 | 1.64 |
| Net Export | 0.39 | 0.21 | 0.24 | 0.31 |
| **(B) Standard Deviation relative to Ouput** | | | | |
| Consumption | 0.63 | 1.01 | 1.09 | 1.12 |
| Investment | 2.82 | 0.67 | 0.53 | 0.38 |
| Labor | 0.67 | 0.71 | 0.72 | 0.70 |
| **(C) Cross-Correlation with Output** | | | | |
| Consumption | 0.82 | 0.99 | 0.99 | 0.98 |
| Labor | 0.86 | 1 | 1 | 1 |
| Investment | 0.95 | 0.94 | 0.92 | 0.89 |
| Net Export | -0.45 | 0.53 | 0.54 | 0.57 |
| **(D) Cross-Country Correlations** | | | | |
| Consumption | 0.44 | 0.45 | 0.52 | 0.62 |
| Output | 0.61 | 0.34 | 0.41 | 0.31 |
| Investment | 0.46 | 0.46 | 0.61 | 0.34 |
| Labor | 0.43 | 0.34 | 0.41 | 0.29 |

*Note*: The first column shows the statistics calculated from the data. The second column is the benchmark model. The third column is the model with a leverage ratio of 5. The last column is the model with the technology process from Backus, Kehoe and Kydland (1992).

Table 5: Sensitivity Analysis - Elasticity of Substitution between Goods

| | Data | Benchmark Model $e = 0.9$ | Sensitivity Test $e = 0.5$ | $e = 0.1$ |
|---|---|---|---|---|
| **(A) Standard Deviation in %** | | | | |
| Output | 2.06 | 1.84 | 1.51 | 1.42 |
| Net Export | 0.39 | 0.21 | 0.22 | 0.22 |
| | | | | |
| **(B) Standard Deviation relative to Ouput** | | | | |
| Consumption | 0.63 | 1.01 | 0.86 | 0.81 |
| Investment | 2.82 | 0.67 | 0.86 | 0.93 |
| Labor | 0.67 | 0.71 | 0.57 | 0.53 |
| | | | | |
| **(C) Cross-Correlation with Output** | | | | |
| Consumption | 0.82 | 0.99 | 0.99 | 0.99 |
| Labor | 0.86 | 1 | 1 | 1 |
| Investment | 0.95 | 0.94 | 0.97 | 0.97 |
| Net Export | -0.45 | 0.53 | 0.59 | 0.60 |
| | | | | |
| **(D) Cross-Country Correlations** | | | | |
| Consumption | 0.44 | 0.45 | 0.46 | 0.45 |
| Output | 0.61 | 0.34 | 0.31 | 0.30 |
| Investment | 0.46 | 0.46 | 0.62 | 0.64 |
| Labor | 0.43 | 0.34 | 0.27 | 0.23 |

*Note*: The first column shows the statistics calculated from the data. The second column is the benchmark model. The last two columns are for different values of elasticity of substitution between home and market goods.

Figure 1: IRF Comparison: Unconstrained vs Constrained Economy
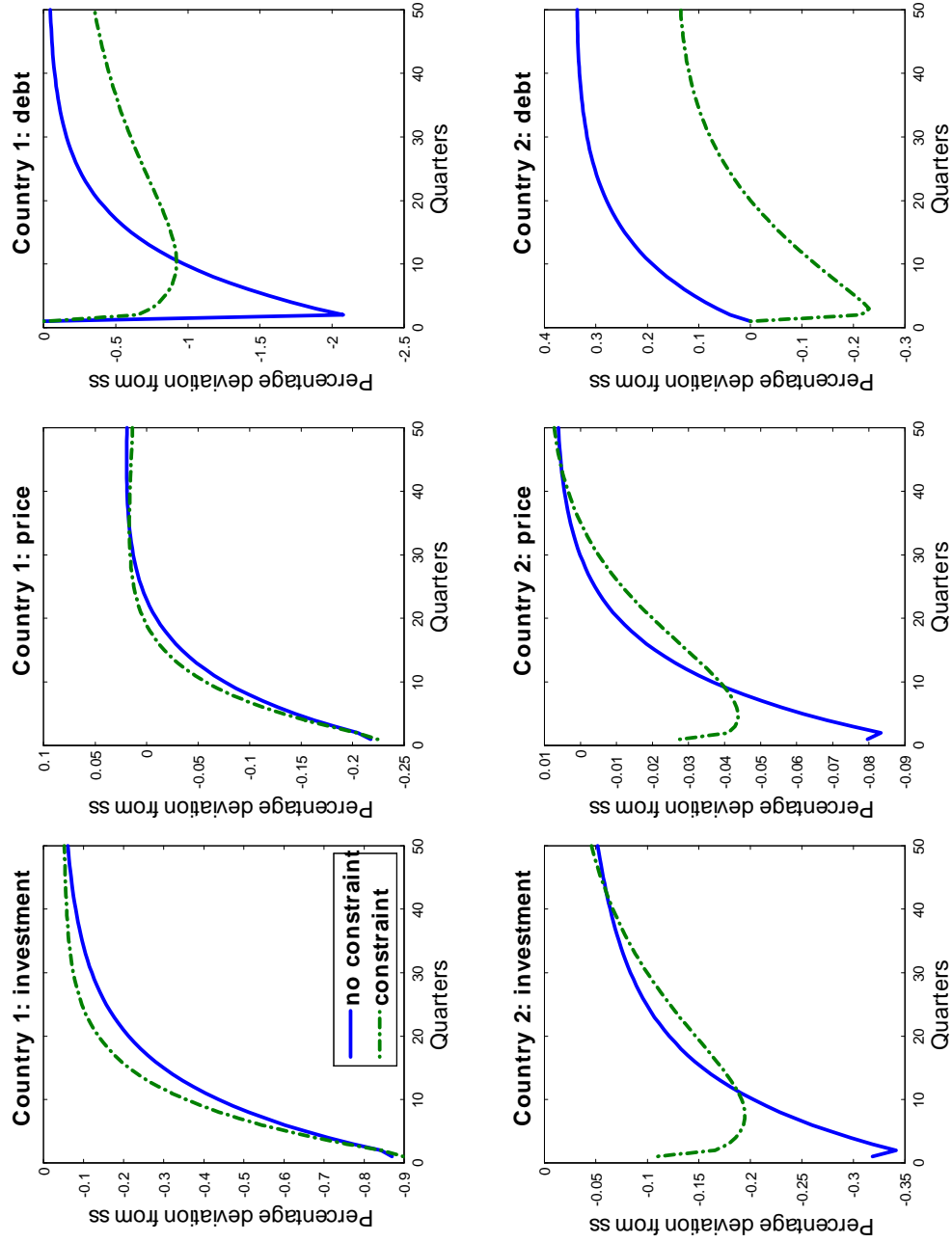
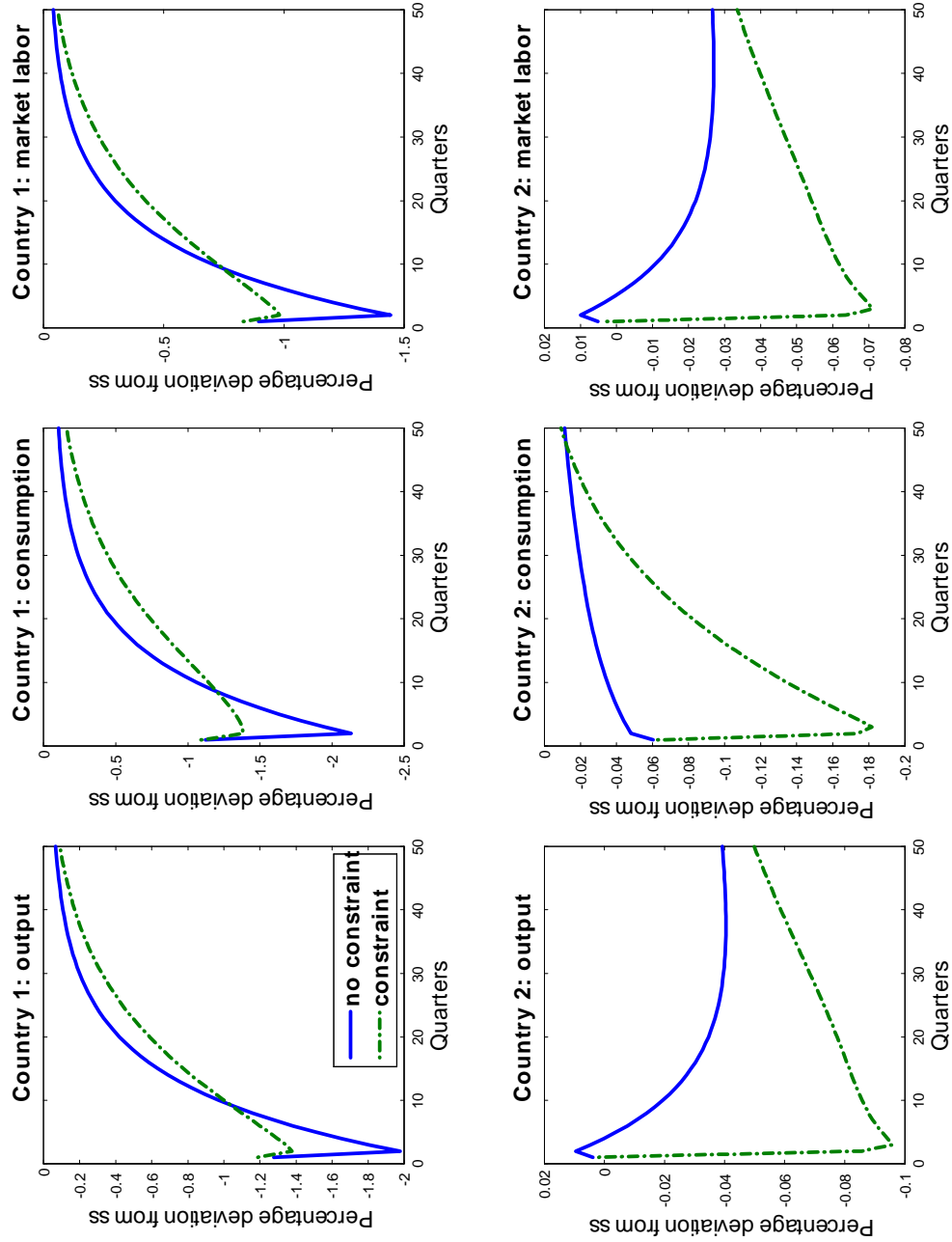Figure 2: IRF Comparison: Unconstrained vs Constrained Economy

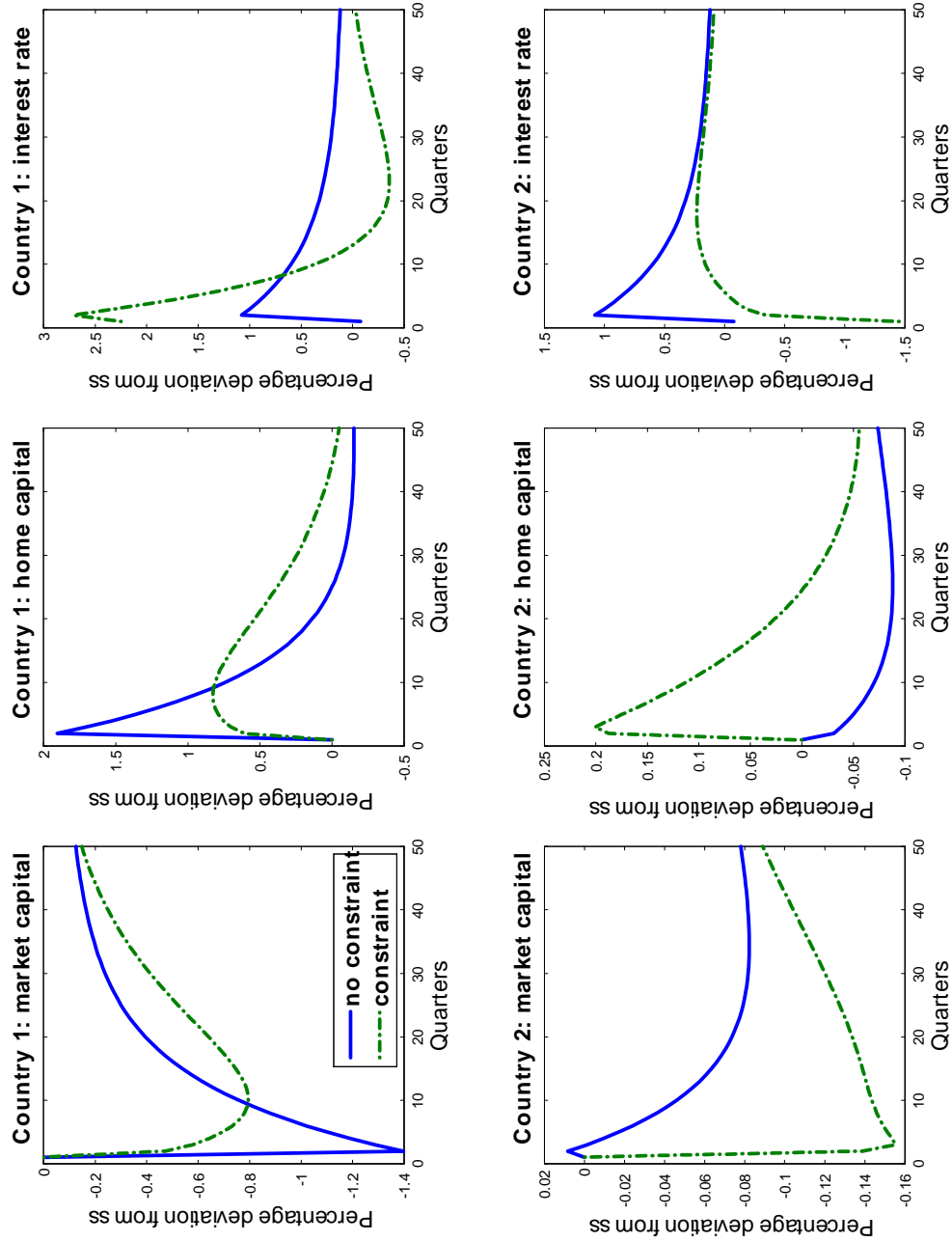Figure 3: IRF Comparison: Unconstrained vs Constrained Economy

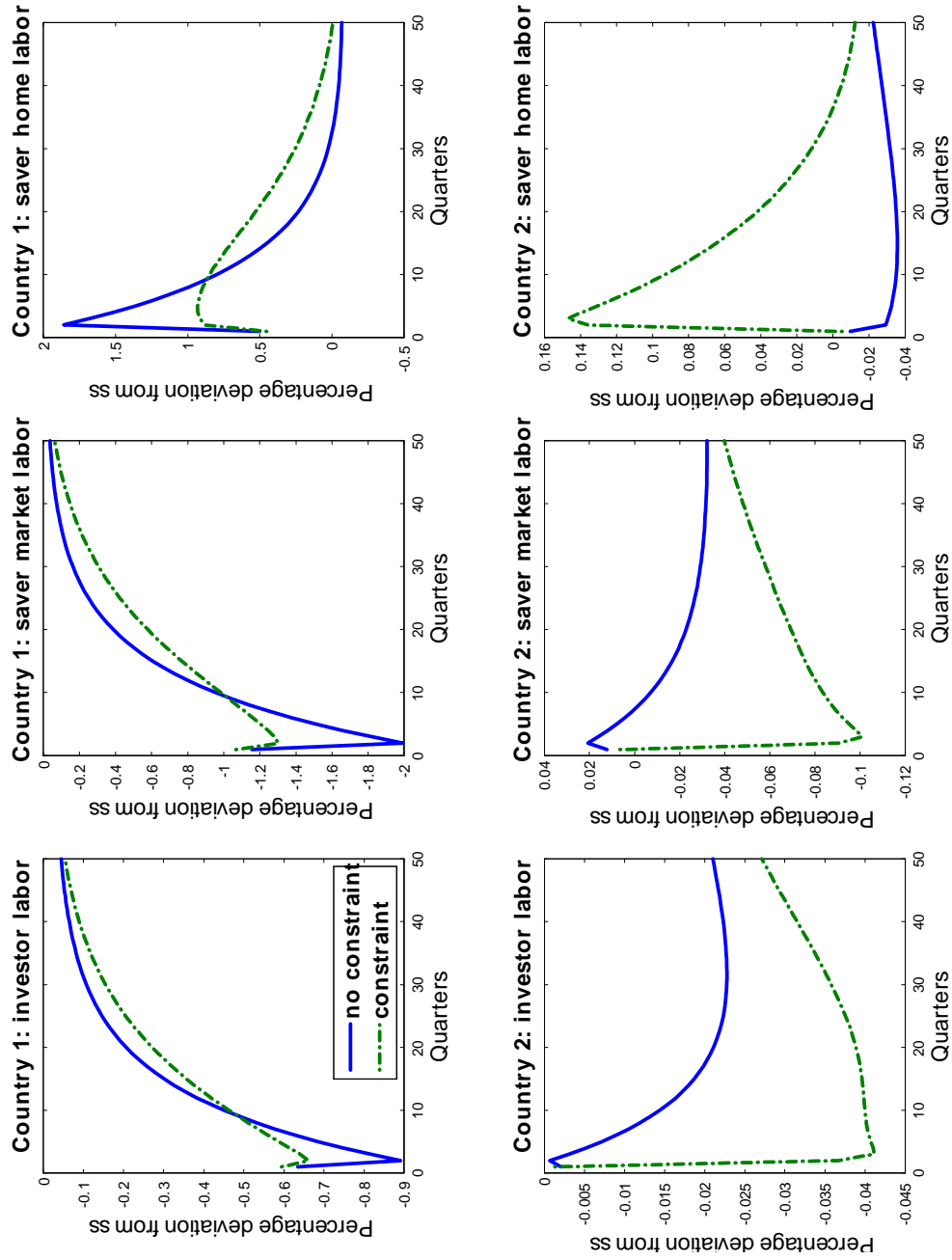Figure 4: IRF Comparison: Unconstrained vs Constrained Economy

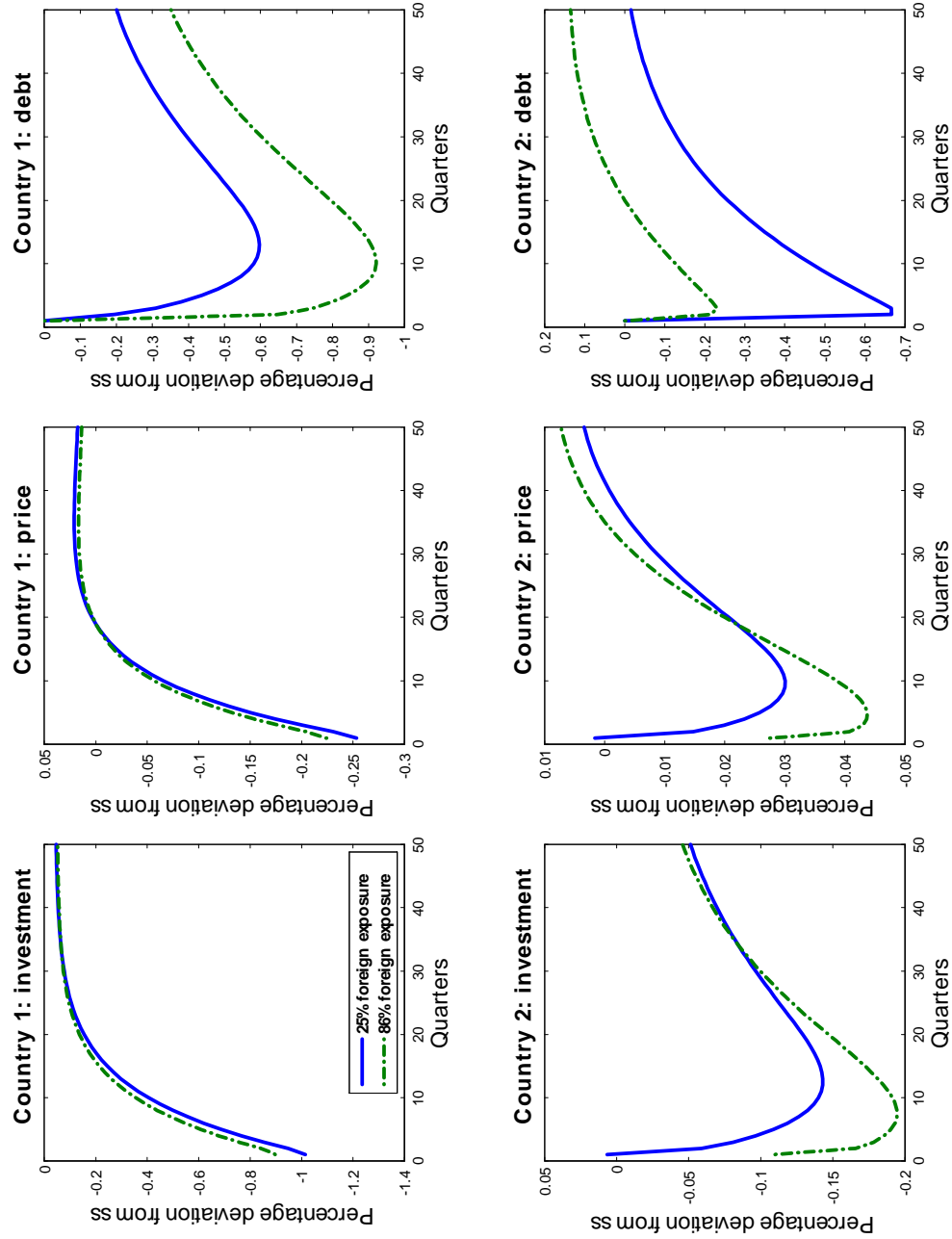Figure 5: IRF for different degrees of foreign exposure

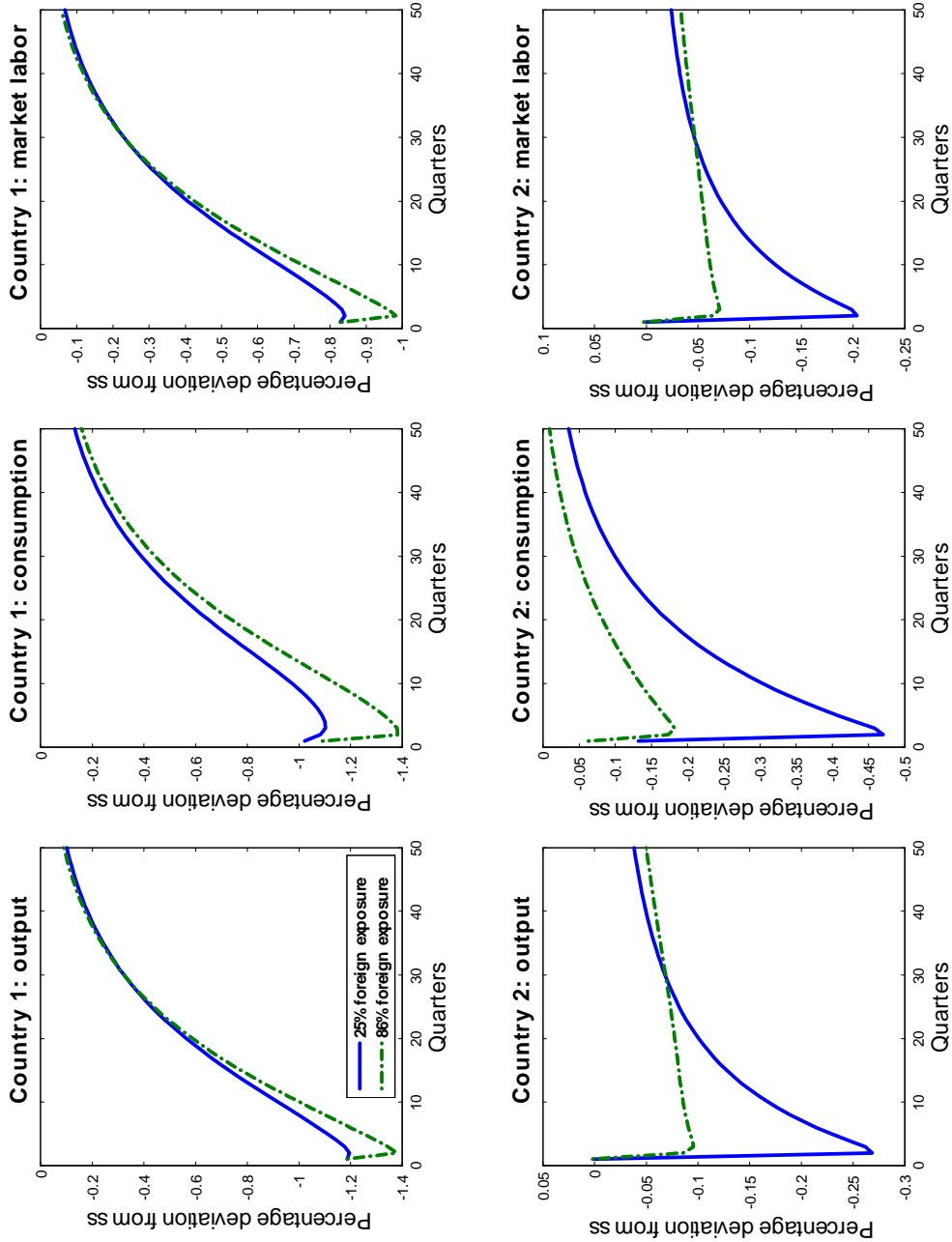Figure 6: IRF for different degrees of foreign exposure
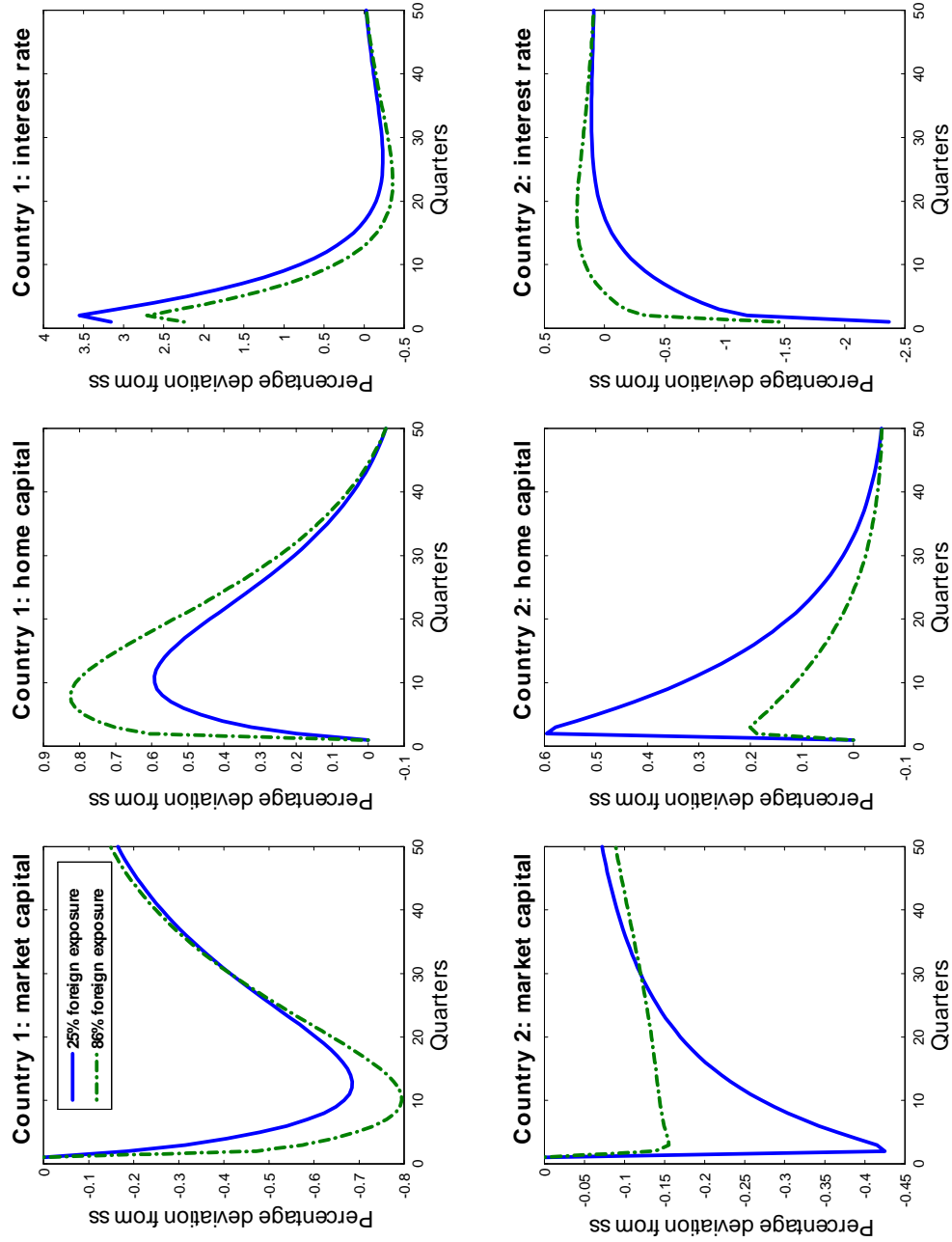
Figure 7: IRF for different degrees of foreign exposure

44

Figure 8: IRF for different degrees of foreign exposure