Interbank Market and Macroprudential Tools in a DSGE Model

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Abstract

The interbank market helps to regulate liquidity in the banking sector. Banks with outstanding resources usually lend to banks that are in needs of liquidity. Regulating the interbank market may actually benefit the policy stance of monetary policy. Even more, introducing an interbank market allows better identification of the final effects of different type of shocks in the economy, specially those that are macroprudential in nature. We evaluate the introduction of an interbank market in which there are two types of banks and a central bank that has the ability to issue money (which is consistent with an interest rate pass-through). Our model is a DSGE in which the monetary policy authority has macroprudential tools (collateral hair-cut and reserve requirements) in addition to the policy interest rate. These macroprudential tools can complement the traditional interest-rate channel, amplifying its effectiveness.

Keywords: collateral constraints, banks, hair-cut, interbank market.

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1. Introduction

The interbank market plays an important role in the transmission process from monetary policy to economic activity because it helps allocate resources

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[1] We are grateful to Connel Fullenkamp, Lawrence Christiano, Phillip Bacchetta, Andreas Fisher, Romain Baeriswyl, Carlos Montoro, Daira Vega, Jorge Loyola, Paul Castillo, Nikita Cespedes, and Marco Vega, and participants at the research seminars at the Central Bank of Peru, the Swiss National Bank, and the Gerzensee Study Center for helpful comments. Special thanks to the comments made by staff members of the Swiss National Bank (SNB) and the Bank of International Settlement (BIS, at Basel). Yessenia Collahua provided excellent research assistance. All remaining errors are our own.

between financial institutions. Ignoring this market is equivalent to ignoring its effectiveness in amplifying or damping the effects of monetary policy. Furthermore, financial frictions (usually tied-up to the credit market) and regulation (hair-cuts, reserve requirements, and collateral constraints) are important components of the interbank market.

The financial accelerator of Bernanke et al. (1999) usually amplifies, spreads, and gives more persistence to different types of shocks in the economy, particularly shocks that directly affect financial intermediaries. After the financial crisis of 2007 - 2008, several economists use Bernanke et al. (1999) as a stepping stone for valid extensions of the original model. One of those extensions is the inclusion of an interbank market. As Walsh (2010) points out, imperfect credit markets make the policy interest rate insufficient to characterize the monetary policy stance. Moreover, credit effects may arise when frictions are present in these financial markets. Thus, one source of motivation for recent research is the nature of the transmission of monetary policy through more than one interest rate (interest rate pass-through) and the conditions of such transmission (the interbank lending market).

The recent literature reviews of Carrera (2012) and Roger and Vlcek (2012), highlight the lack of models in which the interbank market is modeled. In that regard, the work of Gerali et al. (2010), Curdia and Woodford (2010), Dib (2010), and Hilberg and Hollmayr (2011) are among the first on this arena.

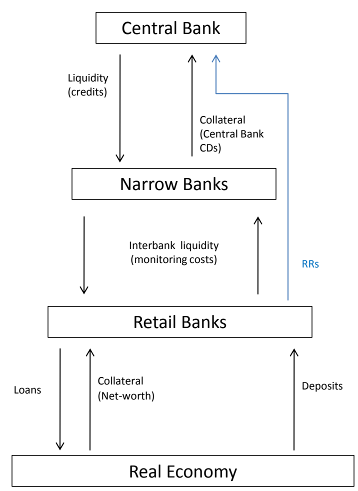
The banking sector in Gerali et al. (2010) encompasses many banks each composed of two "retail" branches and one "wholesale" unit. The first retail branch is responsible for giving out differentiated loans to households and entrepreneurs; the second for raising deposits. The wholesale unit manages the capital position of the group. In Curdia and Woodford (2010), the frictions associated with financial intermediation (intermediation requires real resources and bank lending activities create opportunities for borrowers to take out loans without being made to repay) determine both the spread between borrowing and lending rates and the resources consumed by the intermediary sector. Dib (2010) introduces the distinction between banks that only raise deposits and banks that only give out credit, and sets them up in an interbank market in which the first group of banks borrows from the second group.

Hilberg and Hollmayr (2011) take a different approach and separate the interbank market in two types of banks: commercial banks and investment banks. Hilberg and Hollmayr notice that only a few banks actually interact with the central bank, and then fund the rest of the banking system. While the capital of the banks plays an important role in Gerali et al. (2010) and Dib (2010), for Hilberg and Hollmayr (2011) it is the structure of the market and collateral that matters the most.

We partially follow on the structure of Hilberg and Hollmayr (2011) (see

Figure 1). The hierarchical interbank market is a good representation of the structure in the U.S. (only Primary Dealers deal with the central bank whereas a vast group of commercial banks is not allowed to deal directly with the monetary authority) and in Europe (only 6 out of 2500 allowed banks participate in the bidding process in main refinancing operations of the ECB and other banks rely on interbank funding).[2]

Figure 1: Interbank market structure



We depart from Hilberg and Hollmayr (2011) in two dimensions: (i) retail banks are subject to required reserves and (ii) narrow banks incur in monitor-credit costs. Dinger and Hagen (2009) point out that banks are particularly good at identifying the risk of other banks and present evidence of the importance of interbank transactions. We add monitoring costs in the same fashion as Curdia and Woodford (2010). In doing this, we find that reserve requirements actually strengthen the effects of the interest rate, a result that helps understand the importance of macroprudential tools.

The rest of the paper is organized as follows. Section 2 describes a model with an interbank market. Section 3 describes the calibration procedure. Section 4 presents our results. Finally, section 5 concludes.

[2]See Walsh (2010), chapter 11, for a description of the FED's operating procedures, and http://www.ny.frb.org/markets/primarydealers.html for more information on the FED's Primary Dealers.

2. The Model

In this paper we use a standard real sector in line with the financial accelerator of Bernanke et al. (1999) (with some additional elements of Cohen-Cole and Martínez-García (2010)) and follow on the interbank market structure of Hilberg and Hollmayr (2011), with bank monitoring costs in Curdia and Woodford (2010) fashion.

2.1. Households

There are infinite households that have an identical utility function. The utility function of each household is additively separable in consumption, (C_t), and labor, (H_t), in other words:

$$\sum_{t=0}^{\infty} \beta^t E_t \left[\frac{(C_t - bC_{t-1})^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \chi_H \frac{H_t^{1+\varphi^{-1}}}{1+\varphi^{-1}} \right] \tag{1}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, b is the habit parameter in household consumption, $\sigma > 0$ is the elasticity of intertemporal substitution, and $\varphi > 0$ is the Frisch elasticity of labor supply.

Household's income comes from renting labor to wholesale producers at competitive nominal wages, W_t. It also comes from the ownership of retailers and capital producers which rebate their total real profits, Π_t^R and Π_t^K respectively. The unanticipated profits of the banking system are also fully rebated in each period, Π_t^{NB} an Π_t^{RB}. Households' also obtain their income from interests on their one-period real deposits in the banking system, D_{t-1}. With this disposable income, households finance their aggregate consumption, C_t, open new deposits, D_t, and pay their real (lump-sum) tax bill, T_t. The households' budget constraint is defined then as:

$$C_t + T_t + D_t = \frac{W_t}{P_t} H_t + R_{t-1}^D D_{t-1} \frac{P_{t-1}}{P_t} + \Pi_t^R + \Pi_t^K + \Pi_t^{NB} + \Pi_t^{RB} \tag{2}$$

where R_t^D is the nominal short-term interest rate offered to depositors, and P_t is the consumption price index (CPI). As a convention, D_t denotes real deposits from time t to $t+1$. Therefore, the interest rate R_t^D paid at $t+1$ is known and determined at time t.

From the household's first order conditions:

$$\beta^{t+1} b E_t \left[(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} \right] = \beta^t (C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \lambda_t P_t \tag{3}$$

$$\beta^t \chi_h H_t^{\frac{1}{\varphi}} = \lambda_t W_t \tag{4}$$

$$E_t\left[\lambda_{t+1}\right]R_t = \lambda_t \tag{5}$$

We solve for the Euler equation that links consumption to the real interest rate for deposits and past consumption.

$$\begin{aligned}(C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \beta b E_t\left[(C_{t+1} - bC_t)^{\frac{-1}{\sigma}}\right] = \\ R_t^D E_t\left[\frac{P_t}{P_{t+1}}\left\{(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} - \beta b(C_{t+2} - bC_{t+1})^{\frac{-1}{\sigma}}\right\}\right]\end{aligned} \tag{6}$$

And we also solve for the labor supply.

$$\frac{W_t}{P_t} = E_t\left[\frac{\chi_H H_t^{\frac{1}{\varphi}}}{(C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \beta b(C_{t+1} - bC_t)^{\frac{-1}{\sigma}}}\right] \tag{7}$$

2.2. Wholesale Producers

There are infinite wholesale producers who employ entrepreneurial (H_t^E) and household (H_t) labor combined with rented capital goods (K_t) in order to produce wholesale goods (Y_t^W). The technology used is Cobb-Douglas:

$$Y_t^W = e^{a_t}(K_t)^{1-\psi-\varrho}(H_t)^{\psi}(H_t^E)^{\varrho} \tag{8}$$

where a_t is a productivity shock. In this constant returns-to-scale technology, the non-managerial and managerial labor shares in the production function are determined by the coefficients $0 < \varrho < 1$ and $0 < \psi < 1$. As in Bernanke et al. (1999), the managerial share is assumed to be very small. The productivity shock follows an AR(1) process of the following form:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{9}$$

where ε_t^a is normal i.i.d. with zero mean and σ_a^2 variance and ρ_a captures the degree of persistance of this shock.

Wholesale producers seek to maximize their profits:

$$P_t\Pi_t^W \equiv P_t^W Y_t^W - R_t^W K_t - W_t H_t - W_t^E H_t^E \tag{10}$$

The first order conditions for this problem result in the usual demands for labor (household and entrepreneurial) and capital,

$$R_t^W = (1-\psi-\varrho)\frac{P_t^W Y_t^W}{K_t} \tag{11}$$

$$W_t = \psi\frac{P_t^W Y_t^W}{H_t} \tag{12}$$

$$W_t^E = \varrho \frac{P_t^W Y_t^W}{H_t^E} \tag{13}$$

Wholesale producers make zero profits. Households, who own these firms, do not receive any dividends. Entrepreneurs receive income from their supply of managerial labor and rented capital to wholesalers. Wholesale producers rent capital from the entrepreneurs and return the depreciated capital after production has taken place.

2.3. Capital Goods Producers

There are infinite capital goods producers who at time t combine aggregate investment goods (X_t) and depreciated capital ($(1-\delta)K_t$) to manufacture new capital goods (K_{t+1}). The production of new capital is limited by technological constraints. We assume that the aggregate stock of new capital considers investment adjustment cots and evolves following the law of motion:

$$K_t = (1-\delta)K_{t-1} + \Phi(X_t, X_{t-1})X_t \tag{14}$$

where Φ is an investment adjustment cost function. We follow Christiano et al. (2005) and describe the technology available to the capital good producer as:

$$\Phi(X_t, X_{t-1}) = \left[1 - 0.5 \frac{\kappa \left(\frac{X}{X_{t-1}} - 1 \right)^2}{\frac{X_t}{X_{t-1}}} \right] \tag{15}$$

where $\frac{X_t}{X_{t-1}}$ is the investment growth rate and $\kappa > 0$ regulates the degree of concavity of the technological constraint.

A capital goods producer chooses his investment demand (X_t) and their output of new capital (K_{t+1}), to maximize the expected discounted value of their profits and solves the following problem:

$$\sum_{t=0}^{\infty} E_t \left\{ M_t^H P_t (Q_t K_{t+1} - (1-\delta)\overline{Q}_t K_t) - \Phi(X_t, X_{t-1})X_t \right\} \tag{16}$$

where M_t^H is a stochastic discount factor. Since households own the capital good producers, capital producers consider the household's stochastic discount factor defined as:

$$M_t^H = \frac{\beta^{t+\tau} \{ (C_{t+\tau} - bC_{t-1+\tau})^{\frac{-1}{\sigma}} - \beta b (C_{t+1+\tau} - bC_{t+\tau})^{\frac{-1}{\sigma}} \}}{\beta^t \{ (C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \beta b (C_{t+1} - bC_t)^{\frac{-1}{\sigma}} \}} \frac{P_t}{P_{t+\tau}} \tag{17}$$

where Q_t is the price of new capital for entrepreneurs, and determines the relative cost of investment in units of consumption (Tobin's Q), $\overline{Q}_t$ is the resale value of old capital, and $\overline{Q}_t = o_t Q_t$ is a random shock.

The first order conditions (optimization process of the capital goods producers) yield a standard link between Tobin's Q (Q_t) and investment (X_t):

$$Q_t \left[\left(1 - 0.5\kappa \frac{\left(\frac{X_t}{X_{t-1}} - 1\right)^2}{\frac{X_t}{X_{t-1}}} \right) + (-0.5)\kappa \frac{\left[\left(\frac{X_t}{X_{t-1}}\right)^2 - 1\right]}{\left(\frac{X_t}{X_{t-1}}\right)^2} \frac{X_t}{X_{t-1}} \right] =$$

$$1 + \beta E_t \{ \left[\frac{(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} - \beta b (C_{t+2} - bC_{t+1})^{\frac{-1}{\sigma}}}{(C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \beta b (C_{t+1} - bC_t)^{\frac{-1}{\sigma}}} \right]$$

$$\left[\frac{-0.5\kappa \left[\left(\frac{X_t}{X_{t-1}}\right)^2 - 1\right]}{\left(\frac{X_t}{X_{t-1}}\right)^2} \right] \left(\frac{X_t}{X_{t-1}}\right)^2 Q_{t+1} \} \tag{18}$$

This type of firm has profits because X_{t-1} and X_t are predetermined at time t and cannot be adjusted freely. Aggregate profits for the capital good producer are then defined as:

$$\Pi_t^K = Q_t K_{t+1} - (1 - \delta) Q_t K_t - X_t \tag{19}$$

2.4. Retailers

There are infinite retailers that buy a homogeneous good from wholesalers and differentiate it costlessly in order to sell it to households, entrepreneurs, and capital good producers (for consumption or investment). Variety is valued (love for variety) and so retailers gain monopolistic power to charge a retail mark-up.

Retailers set prices to maximize their profits. Their re-optimizing processes are constrained by nominal rigidities as in Calvo (1983). The retailer maintains its previous period price with a probability $0 < \alpha < 1$ which implies that with probability $(1 - \alpha)$ he is allowed to optimally reset his price. The government gives them a subsidy (τ^R) which eliminates the retail mark-up distortion.[3]

Assuming symmetric optimal price and optimal allocation of expenditures, the aggregate real profits received by the household are:

[3]In this set up, it can be shown that $\tau^R = \frac{1}{\theta}$. See Appendix A.4.1 for the optimal pricing process.

$$\Pi^R = Y_t - (1-\tau^R)\frac{P_t^W}{P_t}Y_t^W \tag{20}$$

where τ^R is the subsidy for producing at the optimum level, P_t is the retail price, and P_t^W is the wholesale price.

The aggregate CPI can be described as:

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)(P_t^z)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{21}$$

where $\theta > 1$ is the elasticity of substitution across varieties and P_t^z is the optimal price when re-optimization is possible.

Let P_t^* be another aggregate price index. This variable helps characterize the magnitude of the efficiency distortion due to sticky prices as follows:

$$P_t^* = [\alpha(P_{t-1}^*)^{-\theta} + (1-\alpha)(P_t^z)^{-\theta}]^{\frac{-1}{\theta}} \tag{22}$$

Supply equal to demand of wholesale goods implies:

$$Y_t = \left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^W \tag{23}$$

2.5. Entrepreneurs and Retail Banks

Entrepreneurs supply one unit of managerial labor ($H_t^E = 1$). They accumulate real net worth (N_t) and take real loans (L_t) in order to buy capital from capital good producers.

At the end of period t, the entrepreneur receives a nominal wage, W_t^E, and earns income from capital rented for the production of wholesale goods, $R_t^W K_t$, as well as from the resale value on the depreciated capital bought by capital goods producers, $(1-\delta)P_t Q_t K_t$.

After repaying their loans (L_t) entrepreneurs can appropriate a fraction of the aggregate capital income ($R_t^W K_t + (1-\delta)P_t Q_t K_t$).Using resources coming from managerial wages and capital rental rates, the entrepreneurs buy new capital (K_{t+1}) and decide how much to consume (C_t^E). The production of wholesale goods at time $t+1$ requires capital goods. The non-consumed portion of their income is their net worth (N_t). Entrepreneurs use N_t as well as L_t to fund the acquisition of the stock of new capital ($Q_t K_{t+1}$), thus, an entrepreneur's balance sheet can be described as:

$$Q_t K_{t+1} = L_t + N_t \tag{24}$$

where Q_t is the relative price of capital.

Then the nominal return on capital with respect to its acquisition cost (R_t^E) can be defined as the ratio between income from renting capital and selling it after depreciation over its nominal cost:

$$R_t^E = \frac{R_t^{\omega} K_t + (1-\delta) P_t Q_t K_t}{P_{t-1} Q_{t-1} K_t} \tag{25}$$

At t, entrepreneurs borrow from the banks and must agree on a contract in order to buy new capital, K_{t+1}. The debt has to be repaid at time $t+1$. In case of default at time $t+1$, banks can only appropriate the total capital returns of the entrepreneur at that time, i.e. $\omega R_{t+1}^E P_t Q_t K_{t+1}$.[4]

The cut-off $\overline{\omega}$ defines the threshold for default of entrepreneurs:

$$\overline{\omega} R_{t+1}^E P_t Q_t K_{t+1} = R_t^L P_t L_t$$

$$\overline{\omega} = \frac{R_t^L P_t L_t}{R_{t+1}^E P_t Q_t K_{t+1}} \tag{26}$$

where $\overline{\omega} R_{t+1}^E$ is the minimum return that entrepreneurs require in order to pay back to the bank, and $R_t^L P_t L_t$ is the payment amount agreed with the bank.

There is a continuum of retail banks that offer contracts with lending rate R_t^L, obtain deposits at rate $R_t^D(j)$ in a market characterized by monopolistic competition, and take the interest rate on the interbank market R_t^{IB} as given. On the liabilities side this type of bank has deposits $D_t(j)$ that are subject to reserve requirements (RR_t) and interbank funds ($IB_t(j)$) that are obtained from households and narrow banks, respectively. These funds are invested by providing loans $L_t(j)$ to entrepreneurs which, together with reserves, constitute the asset side of the retail bank's balance sheet.

Table 1: Balance Sheet of Retail Banks

Assets	Liabilities
Loans (L_t)	Deposits (D_t)
Reserves ($RR_t D_t$)	Interbank loans (IB_t)

The balance sheet identity for this bank is:

$$L_t = (1 - RR_t) D_t + IB_t \tag{27}$$

When the idiosyncratic shock is below the threshold, $\omega < \overline{\omega}$ the bank forecloses the firm and pays a monitoring cost in order to absorb it's assets. Thus,

[4] ω is the idiosyncratic shock over the returns on capital, as in Bernanke et al. (1999).

in the default case, the bank keeps $(1-\mu)\omega R^E_{t+1}P_tQ_tK_{t+1}$ and the entrepreneur walks out empty handed.

In equilibrium, the present discounted value of the retail banks' profits must be driven down to zero by competition. Next period profits are:

$$E_t\left[\Pi^{RB}_{t+1}\right] = \left[\int_{\overline{\omega}}^{\infty} R^L_t L_t dF(\omega) + (1-\mu)\int_0^{\overline{\omega}} \omega R^E_{t+1}Q_tK_{t+1}dF(\omega) + R^{RR}_t RR_t D_t(i) - R^D_t(i)D_t(i) - \frac{\kappa^D}{2}\left(\frac{R^D_t(i)}{R^D_{t-1}(i)}-1\right)^2 R^D_t D_t - R^{IB}_t IB_t\right]\frac{P_t}{P_{t+1}} \tag{28}$$

where $F(\omega)$ is the cumulative density of the idiosyncratic shock. This expression can be simplified using the entrepreneur's balance sheet and the threshold definition in order to substitute away L_t and R^L_t which yields:

$$E_t\left[\Pi^{RB}_{t+1}\right] = \left[g(\overline{\omega})R^E_{t+1}p_t - \left(\frac{(R^D_t(i) - R^{RR}_t RR_t)}{1-RR_t} + \left(1 - \frac{(1-RR_t)}{p_t-1}\left(\frac{R^D_t(i)}{R^D_t}\right)^{\epsilon}\frac{D_t}{N_t}\right)\left(R^{IB}_t - \frac{(R^D_t(i) - R^{RR}_t RR_t)}{1-RR_t}\right)\right)(p_t-1) - \frac{\kappa^d}{2}\left(\frac{R^D_t(i)}{R^D_{t-1}(i)}-1\right)^2 R^D_t\frac{D_t}{N_t}\right]N_t\left(\frac{P_t}{P_{t+1}}\right) \tag{29}$$

where $p_t \equiv \frac{Q_tK_{t+1}}{N_t}$ and $g\left(\overline{\omega}\right) \equiv \left[\overline{\omega}\Pr\left(\omega > \overline{\omega}\right) + (1-\mu)\, E\left(\omega \mid \omega < \overline{\omega}\right)\Pr\left(\omega < \overline{\omega}\right)\right]$, which is increasing in $\overline{\omega}$ as long as $\overline{\omega} < \omega^*$. This will be the case given some restrictions on the parameters imposed by Bernanke et al. (1999).

On the other hand, entrepreneur's benefit would be:

$$\left(\int_{\overline{\omega}}^{\infty}\omega R^E_{t+1}Q_tK_{t+1}dF(\omega) - R^L_t L_t\right)\left(\frac{P_t}{P_{t+1}}\right) \tag{30}$$

Similar substitutions will yield:

$$[E(\omega \mid \omega > \overline{\omega})\Pr(\omega > \overline{\omega}) - \overline{\omega}\Pr(\omega > \overline{\omega})]R^E_{t+1}p_tN_t\left(\frac{P_t}{P_{t+1}}\right) \tag{31}$$

The optimal contract is determined choosing $\overline{\omega}$, p_t and $R^D_t(i)$. Given that N_t is a positive exogenous variable at this point it can be ignored from the objective function. Given a well behaved distribution of ω (e.g.: log-normal), it can be demonstrated that $f(\overline{\omega}) \equiv E(\omega \mid \omega > \overline{\omega}) \Pr(\omega > \overline{\omega}) - \overline{\omega} \Pr(\omega > \overline{\omega})$ is decreasing in $\overline{\omega}$.

The Lagrangian of the problem would be:

$$
\begin{aligned}
\max_{\overline{\omega}, p_t, R_t^D(i)} \mathcal{L} = & \sum_{t=0}^{\infty} \left(M_t^H f(\overline{\omega}) R_{t+1}^E p_t N_t \frac{P_t}{P_{t+1}} \right) \\
& + \lambda \sum_{t=0}^{\infty} \left[M_t^H \left(g(\overline{\omega}) R_{t+1}^E p_t - R_t^{IB}(p_t - 1) \right. \right. \\
& + \left(\frac{R_t^D(i)}{R_t^D} \right)^{\epsilon} \frac{D_t}{N_t} (R_t^{IB}(1 - RR_t) - R_t^D(i) + R_t^{RR} RR_t) \\
& \left. \left. - \frac{\kappa^d}{2} \left(\frac{R_t^D(i)}{R_{t-1}^D(i)} - 1 \right)^2 R_t^D \frac{D_t}{N_t} \right) \right] N_t \frac{P_t}{P_{t+1}}
\end{aligned}
\tag{32}
$$

where M_t^H is the households' stochastic discount factor, previously defined.

The solution to this problem yields the financial accelerator equation of Bernanke et al. (1999), linking the external finance premium to the ratio between value of the assets and the net worth of a firm:

$$
\frac{R_{t+1}^E}{R_t^{IB}} = \left[\frac{P_t Q_t K_t}{N_t} \right]^{\upsilon} \tag{33}
$$

The external finance premium comes from the framework of Bernanke et al. (1999). This financial accelerator links the spread on capital returns and the leverage of the entrepreneurs-borrowers. Moreover, the costly-state verification theory implies that external funding to the entrepreneur is more expensive than internal funding.

Given our assumption of monopolistic competition in the market for deposits (a la Gerali et al. (2010)), the retail bank does not find it optimal to perfectly arbitrage between its sources of funding. Adjustment costs and monopolistic competition imply the following relationship between the deposit rate (R_t^D) and the net cost of funding obtained from the interbank market.

$$
\begin{aligned}
\kappa^d \left(\frac{R_t^D}{R_{t-1}^D} - 1 \right) \frac{R_t^D}{R_{t-1}^D} \frac{P_t}{P_{t+1}} = & \\
& \left(-1 - \epsilon + \epsilon \frac{R_t^{IB}(1 - RR_t) + R_t^{RR} RR_t}{R_t^D} \right) \frac{P_t}{P_{t+1}} + \\
& SDF \left(\frac{D_{t+1}}{D_t} \right) \kappa^d \left(\frac{R_{t+1}^D}{R_t^D} - 1 \right) \left(\frac{R_{t+1}^D}{R_t^D} \right)^2 \frac{P_{t+1}}{P_{t+2}}
\end{aligned}
\tag{34}
$$

Once the contract has been defined, we can characterize the evolution of entrepreneurial net worth:

$$N_t = \gamma f(\overline{\omega_t}) R_t^e Q_{t-1} K_t \left(\frac{P_{t-1}}{P_t}\right) + \frac{W_t^E}{P_t}$$

where γ is the probability that an entrepreneur will stay in the market next period. Entrepreneurial consumption is constituted by the assets of defaulting entrepreneurs that exit the market:

$$C_t^E = (1-\gamma) f(\overline{\omega}_t) R_t^E Q_{t-1} K_t \left(\frac{P_{t-1}}{P_t}\right)$$

2.6. Narrow banks

There is a continuum of narrow banks as well. Each narrow bank acts as a friction on the interbank market and behaves as an agent on its own.[5]

Table 2: Balance Sheet of Narrow Banks

Asset	Liabilities
Central Bank CDs (CD_t)	Central bank credit (L_t^{CB})
Interbank Loans (IB_t)	

This type of bank maximizes with respect to interbank lending (IB_t). The interest rate on the interbank market (R_t^{IB}) is the outcome of the profit-maximizing behavior of both the retail bank and the narrow bank. Even more, this type of bank takes the policy rate (R_t^P) set by the central bank as given. The liability side consists of central bank credit (L_t^{CB}) obtained via Open Market Operations and the assets side is composed of loans to retail banks and required reserves. The balance sheet of each narrow bank is as follows:

$$CD_t + IB_t = L_t^{CB} \tag{35}$$

The narrow bank faces collateral constraints. The liquidity obtainable by each individual narrow bank is L_t^{CB}. The left hand side shows the Central Bank CDs (CD_t) that are accepted as collateral by the central bank.

[5]Hilberg and Hollmayr (2011) argues that a hierarchical interbank market follows on the structure found in the U.S. where only Primary Dealers deal with the central bank whereas a vast group of commercial banks is not allowed to directly deal with the monetary authority and in Europe only 6 out of 2500 allowed banks participate in the bidding process in main refinancing operations of the ECB and other banks rely on interbank funding.

Narrow bank's profits are defined as:

$$\Pi_t^{NB} = [R_t^{IB} IB_t + R_t^{CD} CD_t - R_t^{REPO} L_t^{CB} - \Xi(IB_t)] \frac{P_t}{P_{t-1}} \tag{36}$$

where R_t^{IB} is interest rate on the interbank market, R_t^{CD} is interest rate on the Central Bank CDs, R_t^{REPO} is the interest rate charged on report operations, and $\Xi(IB_t)$ is the monitoring cost.[6]

For simplicity, R_t^{CD} is tied up to the policy rate such that $R_t^{CD} = \theta^{CD} R_t^P$ with $0 < \theta^{CD} < 1$ and $R_t^{REPO} = \theta^{REPO} R_t^P$ with $1 < \theta^{REPO} < 2$.

The constraint can be described as the Hair-cut: Narrow bank uses Central Bank CDs as collateral in order to borrow from the central bank: $CD_t = (1 - HC_t) L_t^{CB}$, where HC_t is the hair-cut.

A narrow bank maximizes its profits which implies:

$$\left(R^{IB} - \frac{1}{HC} \left(\theta^{REPO} - (1 - HC)\theta^{CD} \right) R^P \right) = \Xi' (IB) \tag{37}$$

Thus, the interbank interest rate depends positively on the policy interest rate and the volume of interbank lending.

2.7. Central Bank

The balance sheet of the central bank has, on the liability side, central bank's CDs (that are eligible assets for open market operations, CD_t) and excess reserves ER_t that the central bank receives from retail banks. The central bank is able to choose the fraction of credit that has to be covered by collateral (hair-cut) in the form of certificates of deposits. On the asset side the central bank has credit.

Table 3: Balance Sheet of the Central Bank

Asset	Liabilities
Central bank credit (L_t^{CB})	Excess reserves (ER_t)
	Central Bank CDs (CD_t)

Therefore, the balance sheet of the central bank is as follows:

[6] See Curdia and Woodford (2010) for more details on this set up of monitoring costs

$$L_t^{CB} = ER_t + CD_t \tag{38}$$

The central bank receive new reserve deposits from retail banks, issue CDs, and earns interests on its report operations. These funds are used to pay a (small) interest rate on reserves, CDs' interest, and issue new credit (repos). Any exceeding funds are transfered to the central government (ST_t).

$$RR*D+CD+R_{-1}^{REPO}L_{-1}^{CB}(\frac{P_{-1}}{P})+T = R_{-1}^{RR}RR_{-1}D_{-1}(\frac{P_{-1}}{P})+R_{-1}^{CD}CD_{-1}(\frac{P_{-1}}{P})+\tau^r\frac{P^w}{P}Y^w+L^{CB}+ST \tag{39}$$

The central bank in this model controls the liquidity situation on the interbank market but also responds to distortions of macroeconomic variables by an interest rate rule. If contemporaneous inflation is above its target, the central bank reacts by increasing the short term interest rate. In addition the central bank also reacts to deviations of output from its long run trend. The Taylor rule is then defined as:

$$\left(\frac{R_t}{\bar{R}}\right) = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{P_{t+1}}{P_t}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{1-\rho_R} e^{\varepsilon_t^R} \tag{40}$$

where ρ_R is interest rate rigidity, ϕ_π is weight of inflation in the Taylor rule, ϕ_y is weight of output-gap in the Taylor rule.

2.8. Government

The government intertemporal budget constraint is:

$$GS_t = GS_{t-1} + ST_t - G_t \tag{41}$$

where GS_t are government savings, ST_t are transfers from the central bank, and G_t is government expenditure.

The government's utility function takes into account a preference for smooth expenditure. Henceforth, it will assume the form:

$$U(G_t) = \left(G_t - \frac{aG_t^2}{2}\right) \tag{42}$$

The problem of the government is:

$$\max_{G_t, ST_t} \mathcal{L} = E_t \sum_{t=0}^{\infty} \left[\left(G_t - \frac{aG_t^2}{2}\right) + \lambda_t(-GS_t + GS_{t-1} + ST_t - G_t)\right] \tag{43}$$

First order conditions will yield the following fiscal policy Euler equation:

$$G_t = E_t[G_{t+1}] \tag{44}$$

2.9. Resource Constraint

All that is left to tie up our model is to define the resource constraint. Production of the final good is allocated to private consumption (by households and entrepreneurs), to investment (by capital goods producers), government spending, and to cover costs from monitoring (required to enforce loan contracts). The resource constraint takes the following form:

$$\begin{aligned} P_tY_t = &P_tC_t + P_tC_t^E + P_tX_t + P_tG_t - \\ &(1 - f(\omega) - g(\omega))R_t^E P_{t-1}Q_{t-1}K_t + \Xi(IB_t) \end{aligned} \tag{45}$$

where the last two terms are the loss due to Costly State Verification (CSV) on defaulting entrepreneurs and resources used up to monitor the activities of banks.

3. Calibration

Our calibration of the model's parameters captures the key features of the U.S. economy. In Table 4 and 5 we report the calibration values and steady state values and ratios.

Regarding the households, the steady-state gross domestic inflation rate (P_t/P_{t-1}) is set equal to 1.00. The discount factor, (β) is set to 0.99 to match the historical averages of nominal deposit and risk-free interest rates, R_t^D and R_t. The risk-aversion parameters in workers' utility functions (σ) is set to 1. Assuming that workers allocate one third of their time to market activities, we set the parameter determining the weight of leisure in utility (χ_H) and the inverse of the elasticity of intertemporal substitution of labor (φ) to 1.0 and 0.33, respectively. The habit formation parameter, (b), is set to 0.75, as estimated in Christiano et al. (2010).

The capital share in aggregate output production $(1-\psi-\varrho)$ and the capital depreciation rate (δ) are set to 0.33 and 0.025, respectively. The parameter measuring the degree of monopoly power in the retail-goods market (θ) is set to 6, which would have implied a 20 per cent markup.

The nominal price rigidity parameter (α) in the Calvo price is set to 0.75, implying that the average price remains unchanged for four quarters.

Monetary policy parameters ϕ_π and ϕ_Y are set to values of 1.5 and 0.005, respectively, and these values satisfy the Taylor principle (see Taylor (1993)).

Following Bernanke et al. (1999), the steady-state leverage ratio of entrepreneurs $(1-N/K)$, is set to 0.5, matching the historical average. The steady-state elasticity of the external finance premium (υ) is set at 0.05, the value that is used by Bernanke et al. (1999).

Table 4: Parameter Calibration

Preferences			
$\beta = 0.99$	$\sigma = 1$	$\varphi = 0.333$	$\chi_h = 1$
$b = 0.75$	$\eta = 0.9779$	$\theta = 6$	
Monetary policy			
$\rho_R = 0.7$	$\phi_\pi = 1.5$	$\phi_y = 0.005$	
Technologies			
$\delta = 0.025$	$\psi = 0.66$	$\varrho = 0.01$	$\kappa = 8$
Government			
$\tau^R = 0.166$			
Nominal rigidities			
$\alpha = 0.75$			
Financial sector			
$\upsilon = 0.0506$	$rrss = 0.06$	$hcss = 0.8$	
Exogeneous processes			
$\rho_a = 0.95$	$\rho_{hc} = 0.9$	$\rho_{RR} = 0.9$	

Table 5: Steady-State Values and Ratios

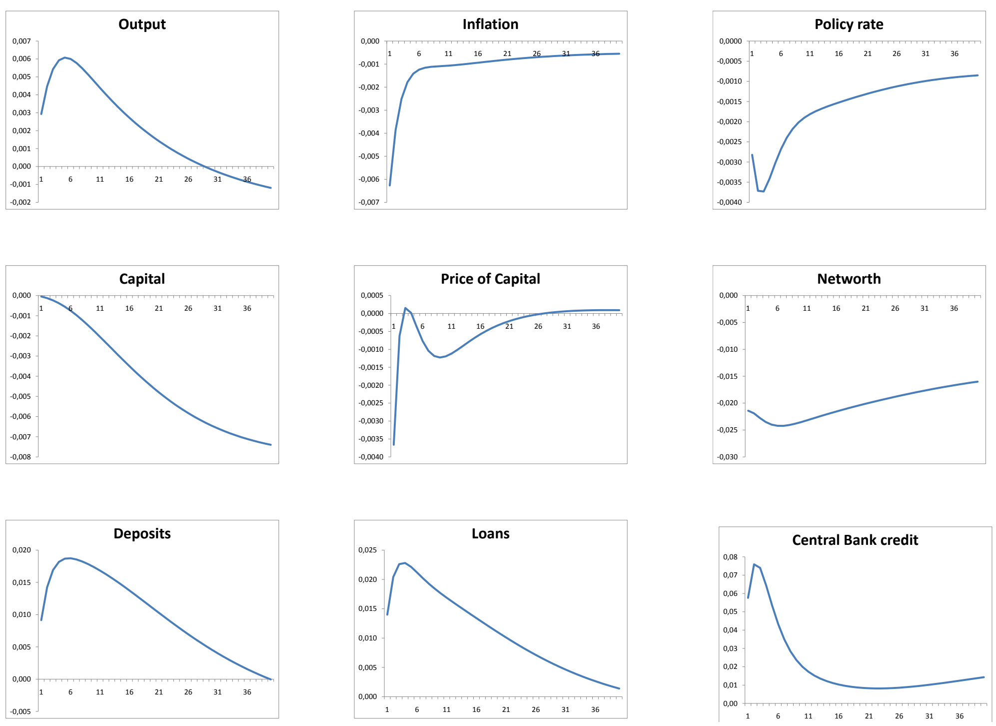
Variables	Definitions	Values
π	inflation	1.0000
R	policy rate	1.0141
R^D	deposit rate	1.0097
C/Y	consumption to output	0.606
I/Y	investment to output	0.18
K/Y	capital stock to output	6.753

4. Results

Figure 2 shows the model's impulse responses to a productivity shock. Most variables exhibit fairly standard behaviour. Output returns slowly to steady state thanks to habit formation, adjustment costs associated to investment and the shock's own persistance. In addition, inflation and the policy rate decrease.

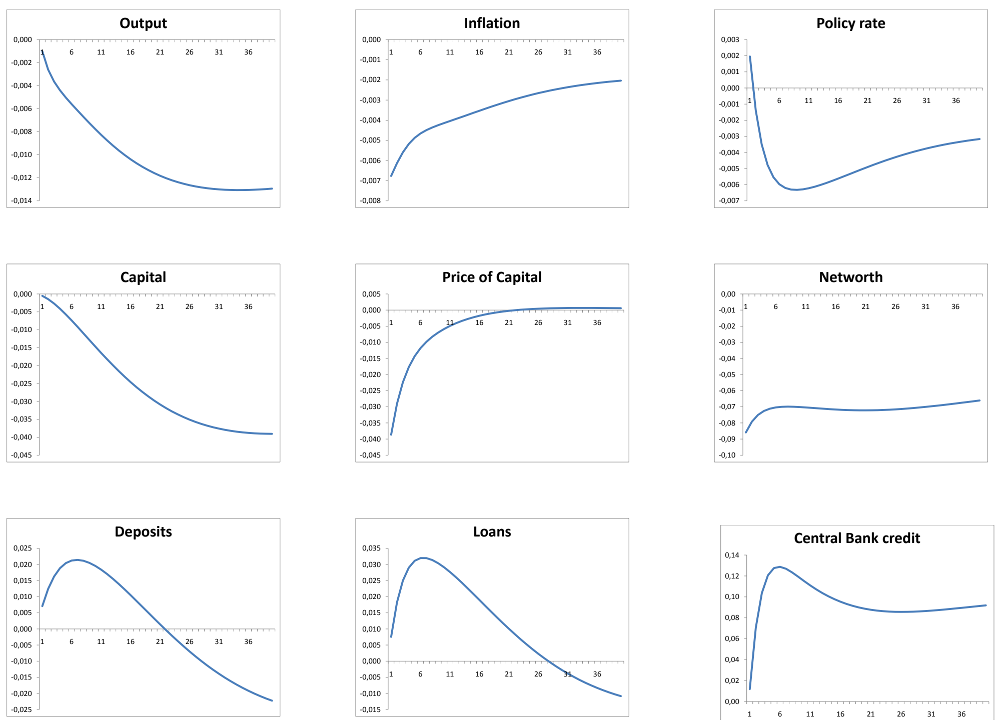
Debt contracts are signed in nominal terms and with rate-setting banks. In fact, deflation increases the real value of debt obligations through asset-price effects on the collateral value. Similar to Gerali et al. (2010) the banking sector is imperfectly competitive, markups applied on loan rates raise the cost of debt servicing. A given deflation leaves debtors with a higher burden of real debt obligations which weigh more on their resources and on their spending, dampening the supply shock. Given that capital moves very slowly, the demand for loans decreases. Consistent with this, capital's relative price recovers quickly after the shock hits and then slowly goes back to steady state. The decrease in the policy rate is transmitted to the deposit rate, however, the greater incomes overcome that effect. As a result of the greater demand for loans (given the greater economic activity and decrease in the net worth), retail bank's funding requirements from narrow banks increases, prompting the still positive but decreasing central bank funding observed in the last panel of Figure 2.

Figure 2: Responses to a Productivity Shock



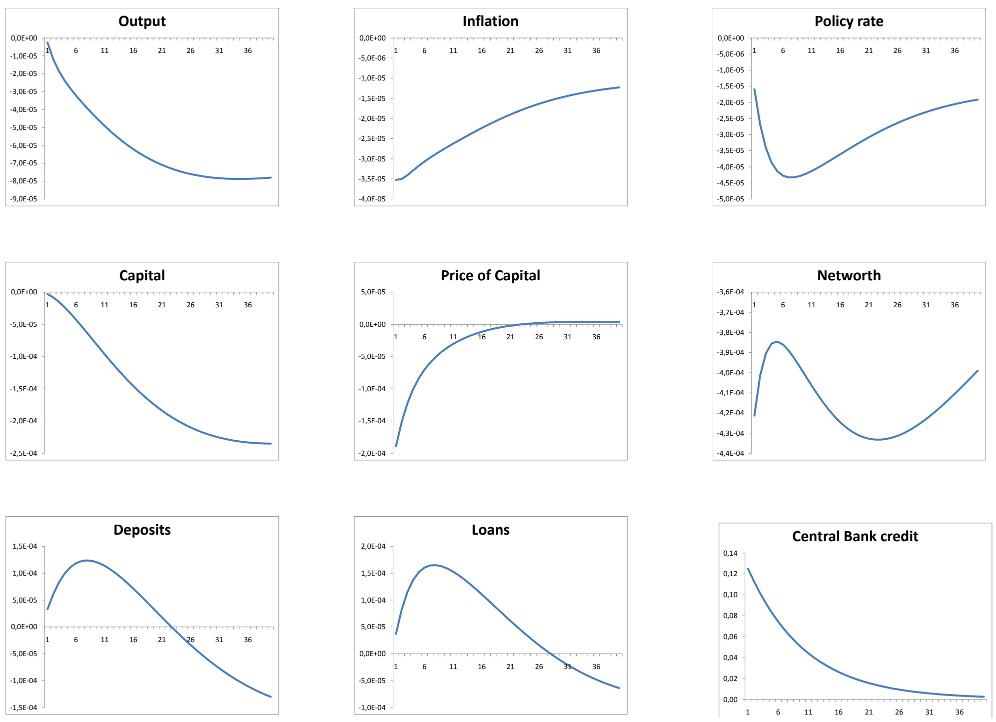
The model's impulse responses to a (negative) monetary policy shock are shown in Figure 3. Output decreases and returns slowly to steady state. The demand contraction has a negative impact on inflation, which will lead the monetary authority to decrease the policy rate gradually. The increase in interest rates punishes entrepreneurial net worth (and the relative price of capital), resulting in an increase in funding requirements from the financial sector. Loans increase given that capital is fixed in the short run. There is an increase in deposits given the initial higher rates paid on them but eventually the wealth effect prevails and deposits fall, prompting an increase in retail banks' demand for interbank funds which translates in more credit being required from the narrow bank, and then from the central bank.

Figure 3: Responses to Monetary Policy Shock



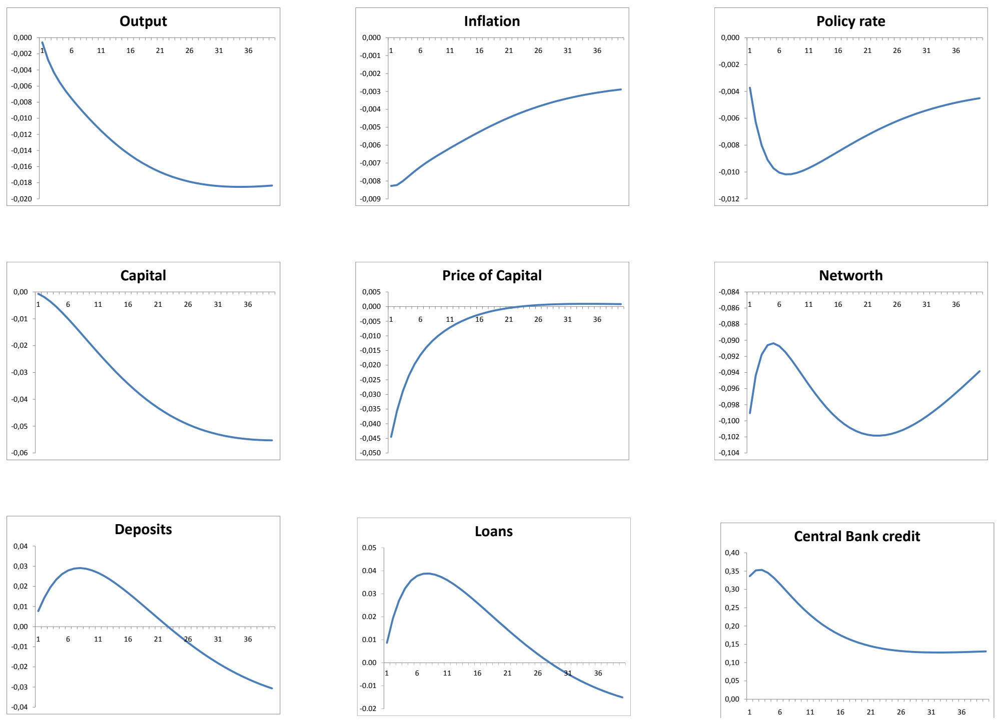
The hair-cut shock corresponds to an increase in the collateral requirements of the central bank in order to offer credit to narrow banks. In steady state, narrow banks are required to maintain central bank's CDs to back up at least 20% of the credit received from the central bank. Figure 4 shows the effects of an increase in this requirement from 20% to 30%. Output decreases and prices fall given the decrease in aggregate demand. Interbank interest rate increases slightly given the higher funding costs of investment banks (the policy rate is only one component of average funding costs, the hair-cut has a positive effect on costs as well). The fall in output drives down investment and capital. This results are relatively small compared to the ones of reserve requirements.

Figure 4: Responses to a Hair-cut Shock



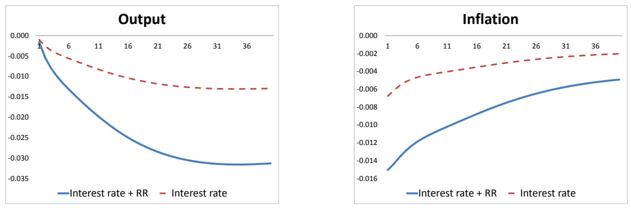
Reserve requirement shock depicted in Figure 5 corresponds to an increase in reserve requirements from 6% to 9%. The resulting decrease in aggregate demand pushes down output and inflation. This time, the policy response of the central bank is much stronger, pushing down the interbank rate. The negative impact of this shock on the relative price of capital results in the gradual decrease of capital itself. Thus, loans eventually decrease too. The long run impact on deposits is a consequence of the negative income effect on consumer's budget. Central bank credit expands in order to finance the higher loans required to finance capital acquisition, it decreases as capital falls. In the long run, central bank expands credit again in order to cover the funding requirements of retail banks whose deposits are falling steadily.

Figure 5: Responses to a Reserve Requirement Shock



In Figure 6 we present the results of the combined effects of contractionary interest rate and a reserve requirement policy shocks. Reserve requirements strenghten the reaction of output and inflation to an interest rate policy shock. In this particular case, reserve requirements act as a tax on the intermediation process, making more expensive the use of deposits as a source for funding the lending process. In addition, the narrow bank faces also a higher cost for getting credits from the central bank. The costs of both sources of funding get up, which strenghtens the reaction-force to the interest rate increase. This result also suggest that lower movements in interest rate can achieve the same desired inflation and output, if used together with a consistent reserve requirement policy.

Figure 6: Responses to a combined policy shocks of Interest rate and Reserve Requirements



5. Conclusions

When the central bank regulates the interbank market, by means of reserve requirements or collateral requirements, the monetary authority also affects the liquidity in the banking sector, first, and the economy, later. This way of affecting bank funding, without any use of the policy interest rate, is a Macroprudential tool.

In terms of modeling, the introduction of an interbank market allows a better identification of the final effects of different type of shocks in the economy. Important conclusions such as the complementarity of a central bank's tools can be potentially answered in a model with this additional feature. This is not an easy task because we need to endogenously introduce money in the model. In doing so, we then can evaluate the impact of a particular interbank structure in the economy.

The properties of Macroprudential tools, developed in this model, are combined with the traditional effect of an interest rate policy shock. The complementarity of these two tools is one of our results. The role of reserve requirements as a tax to the financial intermediation, increases the cost of funding economic activity through deposits and strenghtens the power of the interest rate shock over output and inflation. In other words, a central bank can achieve similar reaction on inflation and output with a lower increase of the policy interest rate if reserve requirements are increased at the same time.

Our results are in line with those of Carrera (2012) and Whitesell (2006). In his review of the relevant literature, Carrera (2012) finds that complementarity of these policy tools are normally achieved on different modeling strategies, however there is room for more research that highlights the role of collaterals and the mechanism by which these tools operate. In the same line, Whitesell (2006) shows that combined policies of interest rate and reserve requirements results in lower volatility of the policy interest rate.

Regarding the hair-cut as a Macroprudential tool, we do not find significant results. As Carrera (2012) argues, this mechanism is more effective when certain conditions are met (as in the presence of a financial crisis or a zero lower-bound environment). The research question remains in our agenda, how would hair-cuts matter if those conditions are met?

While the research conclusion for this paper is clear enough, this model can be extended to consider the possibility of collateral from retail banks to either narrow banks or a shadow banking system. The flexibility of our model allows for questions that are directly related with the liquidity of the financial system, and that is part of our research agenda.

Finally, similar to Dib (2010), we plan to compare our results againts the results of a model with only a financial accelerator (as in Bernanke et al. (1999)). Our prior is that taking into account the interbank market dampens the Bernanke et al. (1999) financial accelerator effect, a result that is similar to the model with capital requirements in Dib (2010).

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AppendixA. The Model - Levels

AppendixA.1. Household

The first order conditions for the household problem are:

$$\frac{\partial L}{\partial C_t} = \beta^t (C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \lambda_t P_t + \beta^{t+1} E_t \left[(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} \right] (-b) = 0 \quad \text{(A.1)}$$

$$\frac{\partial L}{\partial H_t} = -\beta^t \chi_h H_t^{\frac{1}{\varphi}} + \lambda_t W_t = 0 \quad \text{(A.2)}$$

$$\frac{\partial L}{\partial D_{t+1}} = -\lambda_t + E_t[\lambda_{t+1}] R_t^D = 0 \quad \text{(A.3)}$$

From A.1:

$$\beta^{t+1} b E_t \left[(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} \right] = \beta^t (C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \lambda_t P_t \quad \text{(A.4)}$$

From A.2:

$$\beta^t \chi_h H_t^{\frac{1}{\varphi}} = \lambda_t W_t \quad \text{(A.5)}$$

From A.3:

$$E_t[\lambda_{t+1}] R_t = \lambda_t \quad \text{(A.6)}$$

From A.5 and A.6 we get the supply of labor:

$$\beta b E_t \left[(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} \right] = (C_{t+1} - bC_t)^{\frac{-1}{\sigma}} - \frac{\chi_h}{W_t} H_t^{\frac{1}{\varphi}} P_t \quad \text{(A.7)}$$

$$\frac{P_t}{W_t} = E_t \left[\frac{(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} - \beta b (C_{t+1} - bC_t)^{\frac{-1}{\sigma}}}{\chi_h H_t^{\frac{1}{\varphi}}} \right] \quad \text{(A.8)}$$

$$\frac{W_t}{P_t} = E_t \left[\frac{\chi_h H_t^{\frac{1}{\varphi}}}{(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} - \beta b (C_{t+1} - bC_t)^{\frac{-1}{\sigma}}} \right] \quad \text{(A.9)}$$

From A.4 we get the following expression:

$$\lambda = -\frac{\beta^{t+1}}{P_t} b E_t \left[(C_{t+1} - bC_t)^{\frac{-1}{\sigma}} \right] + \frac{\beta^t}{P_t} (C_t - bC_{t-1})^{\frac{-1}{\sigma}}$$

Replacing in A.6:

$$E_t\left[\frac{\beta^t(C_t-bC_{t-1})^{\frac{-1}{\sigma}}-\beta^{t+1}b(C_{t+1}-bC_t)^{\frac{-1}{\sigma}}}{P_t}\right]=R_t^D E_t\left[\frac{\beta^t(C_{t+1}-bC_t)^{\frac{-1}{\sigma}}-\beta^{t+1}b(C_{t+2}-bC_{t+1})^{\frac{-1}{\sigma}}}{P_{t+1}}\right]$$

$$\frac{\beta^t}{P_t}\left[(C_t-bC_{t-1})^{\frac{-1}{\sigma}}-\beta bE_t\left((C_{t+1}-bC_t)^{\frac{-1}{\sigma}}\right)\right]=R_t^D E_t\left[\frac{\beta^t}{P_{t+1}}\{(C_{t+1}-bC_t)^{\frac{-1}{\sigma}}-\beta b(C_{t+2}-bC_{t+1})^{\frac{-1}{\sigma}}\right]$$

Re-written, we get the Euler equation:

$$\begin{aligned}\{(C_t-bC_{t-1})^{\frac{-1}{\sigma}}-\beta(C_{t+1}-bC_t)^{\frac{-1}{\sigma}}=\\ R_t^D E_t\left[\frac{P_t}{P_{t+1}}\{(C_{t+1}-bC_t)^{\frac{-1}{\sigma}}-\beta b(C_{t+2}-bC_{t+1})^{\frac{-1}{\sigma}}\}\right]\end{aligned}\tag{A.10}$$

AppendixA.2. Entrepreneurs: Return of the Entrepreneurs

The nominal return for an entrepreneur is given by:

$$R^E=\frac{R^\omega K_{t-1}}{P_{t-1}Q_{t-1}K_{t-1}}+\frac{(1-\delta)P_tQ_tK_{t-1}}{P_{t-1}Q_{t-1}K_{t-1}}$$

$$R^E=\left[\frac{R^\omega}{P_tQ_t}+(1-\delta)\right]\frac{P_tQ_t}{P_{t-1}Q_{t-1}}\tag{A.11}$$

Re-arrenging, we obtain the real return of entrepreneur:

$$\frac{R_t^E}{(1+\pi^p)(1+\pi^q)}=\left[\frac{R^\omega}{P_tQ_t}+(1-\delta)\right]\tag{A.12}$$

where R_t^ω is the payment for unit of K (real terms net of depreciation).

AppendixA.3. Wholesale Producers

The production function of the Wholesaler is:

$$Y_t^\omega=exp(a_t)K_{-1}^{1-\psi-\zeta}H^\psi He^\zeta\tag{A.13}$$

The wholesale producer has to pick up K_{t-1}, H_t, and H_t^E to maximize the following profit function:

$$P_t^W exp(a_t) K_{-1}^{1-\psi-\zeta} H_t^{\psi} H_t^{E\zeta} - R_t^{\omega} K_{t-1} - W_t H_t - W_t^E H_t^E \tag{A.14}$$

The first order conditions for the wholesalers are:

$$\frac{\partial \pi^{\omega p}}{\partial K_{-1}} = P^{\omega} exp(a_t)(1-\psi-\zeta) K_{-1}^{1-\psi-\zeta} h^{\psi} he^{\zeta} - R^{\omega} = 0$$

$$\frac{\partial \pi^{\omega p}}{\partial h} = P^{\omega} exp(a_t) \psi K_{-1}^{1-\psi-\zeta} h^{\psi-1} he^{\zeta} - W = 0$$

$$\frac{\partial \pi^{\omega p}}{\partial he} = P^{\omega} exp(a_t) \zeta K_{-1}^{1-\psi-\zeta} h^{\psi} he^{\zeta-1} - W^e = 0$$

From these conditions we obtain the demand for production factors:

$$R^{\omega} k_{-1} = (1-\psi-\varrho) P_t^W Y_t^W \tag{A.15}$$

$$W_t H_t = \psi P_t^W Y_t^W \tag{A.16}$$

$$W_t^E H_t = \varrho P_t^W Y_t^W \tag{A.17}$$

AppendixA.4. Retailer

AppendixA.4.1. Optimal pricing

There are differentiated varieties of an homogeneous good, defined as $Y_z(z)$, for $z \in [0,1]$ retailer. Final goods are bundles of these differentiated varieties, aggregated by:

$$Y_t = \left[\int_1^0 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta-1}{\theta}}$$

where $\theta > 1$ is the elasticity of substitution across varieties.

Then, the price index is given by:

$$P_t = \left[\int_1^0 P_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$$

where $P_t(z)$ is the price of retailer z.

The optimal allocation of expentidute to each variety is given by:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta} Y_t$$

A retailer z chooses $\tilde{P}_t(z)$ to maximize:

$$\sum_{\tau=0}^{\infty} E_t \left[\alpha^\tau M_{t,t+\tau} \tilde{Y}_{t,t+\tau}(z)(\tilde{P}_t(z) - (1-\tau^R)P^W_{t+\tau})\right]$$

where

$$M_{t,t+\tau} = \beta^\tau \left[\frac{(C_{t+\tau} - bC_{t-1+\tau})^{\frac{-1}{\sigma}} - \beta b(C_{t+1+\tau} - bC_{t+\tau})^{\frac{-1}{\sigma}}}{(C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \beta b(C_{t+1} - bC_t)^{\frac{-1}{\sigma}}}\right] \frac{P_t}{P_{t+\tau}}$$

is the household's stochastic discount factor for $\tau-$ periods ahead (same as the one of capital producers), P_t^w is the nominal price for the wholesale goods, $\tilde{Y}_{t,t+\tau}(z) = \left(\frac{\tilde{P}_t(z)}{P_{t+\tau}}\right)^{-\theta} Y_{t+\tau}$ is the demand given taht prices remain fixed at $\tilde{P}_t(z)$, $\tau^R = \frac{1}{\theta}$ is a subsidy that eliminate the retail mark-up distinction, and $\alpha(0 < \alpha < 1)$ is the probability that retailers maintain its previous period price.

The first order condition for this problem is:

$$\Sigma_{t=0}^{\infty} E_t \left[(\alpha\beta)^t \frac{\{(C_{t+\tau} - bC_{t-1+\tau})^{\frac{-1}{\sigma}} - \beta b(C_{t+1+\tau} - bC_{t+\tau})^{\frac{-1}{\sigma}}\}}{\{(C_t - bC_{t-1})^{\frac{-1}{\sigma}} - \beta b(C_{t+1} - bC_t)^{\frac{-1}{\sigma}}\}} \tilde{Y_z}(z) \left(\frac{\tilde{P}_t(z)}{P_t} - \frac{\theta}{\theta-1}(1-\tau^R)\frac{P^W_{t+1}}{P_{t+1}}\right)\right] = 0$$

where: $\frac{\theta}{\theta-1}$ is the retail mark-up, and $\frac{P_t^W}{P_t}$ is the price of wholesale output in units of consumption.

All re-optimizing retailers face a symmetric problem, then, the aggregate CPI can be written as:

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)\tilde{P}_t(z)^{1-\theta}]^{\frac{1}{1-\theta}}$$

where $\tilde{P}_t(z)$ is the symmetric (optimal) price.

By market clearing, total demand of retailers must be equal to the total production of wholesale producers:

$$\int_0^1 Y_t(z)dz = Y_t^W$$

By optimal allocation expenditure, then:

$$Y_t = \left(\frac{P_t(z)}{P_t}\right)^{\theta} Y_t^W$$

and

$$P_t = \left[\int_1^0 P_t(z)^{1-\theta} dz\right]^{-\frac{1}{\theta}} = [\alpha(P_{t-1}^*)^{-\theta} + (1-\alpha)\tilde{P}_t(z)^{-\theta}]^{-\frac{1}{\theta}}$$

The aggegate nominal profits received by the households is:

$$\Pi_t^R = \int_0^1 [Y_t(z)P_t(z) - (1-\tau^R)P_t^W]dz$$

By optimal allocation of expenditure in each variety, then:

$$\Pi_t^R = P_t \left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^W - (1-\tau^R)P_t^W Y_t^W \tag{A.18}$$

AppendixA.5. Narrow banks

A narrow bank maximizes:

$$\max \Pi^{NB} = R^{IB} IB \left(\frac{P}{P_{+1}}\right) + R^{CD} CD \left(\frac{P}{P_{+1}}\right) - R^{REPO} L^{CB} \left(\frac{P}{P_{+1}}\right) - \Xi\left(IB\right)\left(\frac{P}{P_{+1}}\right) \tag{A.19}$$

but

$$R^{REPO} = \theta^{REPO} R^P \tag{A.20}$$

with $\theta^{REPO} > 1$ and

$$R^{CD} = \theta^{CD} R^P \tag{A.21}$$

with $\theta^{CD} < 1$, then,

$$\max \Pi^{NB} = \left(R^{IB} - \frac{1}{HC}\left(\theta^{REPO} R^P - \theta^{CD} R^P (1-HC)\right)\right)\left(\frac{P}{P_{+1}}\right) IB - \Xi\left(IB\right)\left(\frac{P}{P_{+1}}\right) \tag{A.22}$$

$$\max \Pi^{NB} = \left(R^{IB} - \frac{1}{HC}\left(\theta^{REPO} - (1-HC)\theta^{CD}\right) R^P\right)\left(\frac{P}{P_{+1}}\right) IB - \Xi\left(IB\right)\left(\frac{P}{P_{+1}}\right) \tag{A.23}$$

which leads to the following FOC

$$\left(R^{IB} - \frac{1}{HC}\left(\theta^{REPO} - (1-HC)\theta^{CD}\right) R^P\right) = \Xi'\left(IB\right) \tag{A.24}$$

AppendixA.6. Government

Government "savings" (GS) is zero in SS (thus we will approximate around $1+GS$ in the following section). The intertemporal budget constraint is defined as:

$$GS = GS_{-1} + ST - G \tag{A.25}$$

and in steady state: $ST = G$.

$$1 + GS = 1 + GS_{-1} + ST - G \tag{A.26}$$

–

From the government's FOCs:

$$(1 - aG_t) - \lambda_t = 0 \tag{A.27}$$

$$E_t \left[-\lambda_t + \lambda_{t+1}\right] = 0 \tag{A.28}$$

Combining these results we obtain the Euler equation for government expenditure:

$$E_t \left[-(1 - aG_t) + (1 - aG_{t+1})\right] = 0$$

$$(1 - aG_t) = E_t \left[(1 - aG_{t+1})\right]$$

$$1 - aG_t = 1 - aE_t \left[G_{t+1}\right]$$

$$G_t = E_t \left[G_{t+1}\right] \tag{A.29}$$

AppendixB. The Model - Log lineal

AppendixB.1. Households

All uppercase variables (except first line of each mini section) represent steady state values. Lowercase variables are deviations from steady state. No subscript implies variable is in current period.

The first derivative of the instantaneous utility of consumption
$UC * (C - hab * C_{-1})^{(\frac{1}{\sigma})} = 1$
$uc + \left(\frac{1}{\sigma}\right)\left(\frac{1}{1-hab}c - \frac{hab}{1-hab}c_{-1}\right) = 0$
-
Marginal utility of consumption
$MUC = UC - \beta * hab * UC_{+1}$

$muc = \left(\frac{1}{\sigma}\right)\left(\frac{\beta * hab}{1-\beta * hab}\left(\frac{1}{1-hab}c_{+1} - \frac{hab}{1-hab}c\right) - \frac{1}{1-\beta * hab}\left(\frac{1}{1-hab}c - \frac{hab}{1-hab}c_{-1}\right)\right)$

-

Household budget constraint

$C + T + D = \frac{W}{P}H + R^d_{-1}D_{-1}(\frac{P_{-1}}{P}) + \Pi^R + \Pi^K + \Pi^{NB} + \Pi^{RB}$

$\frac{C}{Y}c + \frac{D}{Y}d = \psi y - (1-\psi)(p^w - p) + \delta\frac{K}{Y}q - \left((\eta-1)\frac{\Xi(IB)}{Y} + g(\overline{\omega})R^e\frac{K}{Y} + R^{RR}RR\frac{D}{Y} - R^{ib}\frac{IB}{Y}\right)\pi + (\eta-1)\eta\frac{\Xi(IB)}{Y}ib_{-1} + g(\overline{\omega})R^e\frac{K}{Y}\left(\frac{dg(\overline{\omega})}{g(\overline{\omega})} + r^e + q_{-1} + k\right) + R^{RR}RR\frac{D}{Y}\left(r^{RR}_{-1} + rr_{-1} + d_{-1}\right) - R^{IB}\frac{IB}{Y}\left(r^{IB}_{-1} + ib_{-1}\right)$

-

household´s Euler equation

$\frac{MUC}{P} = \beta * R^d * \frac{MUC_{+1}}{P_{+1}}$

$muc = r^d + muc_{+1} - \pi_{+1}$

-

labor supply

$W * MUCP = \chi_h H$

$p^w - p + y + muc = (\frac{\phi+1}{\phi})h$

but $y^w = a + (1-\psi-\varphi)k_{-1} + \psi h$ then

$p^w - p + muc = \frac{\phi(1-\psi)+1}{\phi\psi}y - \frac{\phi+1}{\phi\psi}((1-\psi-\varphi)k_{-1} + a)$

-

AppendixB.2. Retailer supply and aggregation

Retailer profit

$\Pi^K = Y - (1-\tau^r) * \frac{P^w}{P} * Y^w$

$\pi^k = y - \frac{(1-\tau^r)}{\tau^r}(p^w - p)$

-

retailer demand = wholesaler supply

$(P^\epsilon) * Y = (P^*)^\epsilon * Y^w$

$y = y^w$

-

domestic price evolution, alternative cpi weights

$P^* = (\alpha * P^{*}_{-1}{}^{(-\epsilon)} + (1-\alpha) * P^{z(-\epsilon)})^{(-1/\epsilon)}$

$p^* = \alpha p^*_{-1} + (1-\alpha)p^z$

-

additional variables required to characterize price setting

$MUCPVN = MUCP * VN$

$mucpvn = mucp + vn$

-

ídem

$MUCPVD = MUCP * VD$

$mucpvd = mucp + vd$

-

domestic price evolution

$P = (\alpha * P_{-1}{}^{(1-\epsilon)} + (1-\alpha) * P^{z(1-\epsilon)})^{(\frac{1}{1-\epsilon})}$

$p = \alpha p_{-1} + (1-\alpha)p^z$

thus, $p = p*$

-

optimal retail price derivation

$P^z * (\epsilon - 1) * VD = \epsilon * (1 - \tau^r) * VD$

$p^z + vd = vn$ //

-

$VN * MUCP = Y * (P)^\epsilon * P^w * MUCP + \alpha * \beta * MUCPVN_{+1}$; //

$vn = \frac{Y}{Y+\alpha\beta VN}(y + \epsilon p + p^w) + \frac{\alpha\beta VN}{Y+\alpha\beta VN}(vn_{+1} - r^d)$

but $VN = Y/(1 - \alpha\beta)$ then

$vn = (1 - \alpha\beta)(y + \epsilon p + p^w) + \alpha\beta(vn_{+1} - r^d)$

-

$VD * MUCP = Y * (P^\epsilon) * MUCP + \alpha * \beta * MUCPVD_{+1}$

$vd = \frac{Y}{Y+\alpha\beta VD}(y + \epsilon p) + \frac{\alpha\beta VD}{Y+\alpha\beta VD}(vd_{+1} - r^d)$

but $VD = Y/(1 - \alpha\beta)$ then

$vd = (1 - \alpha\beta)(y + \epsilon p) + \alpha\beta(vd_{+1} - r^d)$

then, given that $p^z = vn - vd$

$vn - vd = (1 - \alpha\beta)p^w + \alpha\beta(vn_{+1} - vd_{+1})$

$p^z = (1 - \alpha\beta)p^w + \alpha\beta p^z_{+1}$

but $p^z = \frac{1}{1-\alpha}p - \frac{\alpha}{1-\alpha}p_{-1}$

$\frac{1}{1-\alpha}p - \frac{\alpha}{1-\alpha}p_{-1} = (1 - \alpha\beta)p^w + \alpha\beta(\frac{1}{1-\alpha}p_{+1} - \frac{\alpha}{1-\alpha}p)$

$-\alpha\beta p_{+1} + (1 + \alpha^2\beta)p - \alpha p_{-1} = (1 - \alpha)(1 - \alpha\beta)p^w$

going for Phillips curve:

$-\alpha\beta p_{+1} + (1 + \alpha^2\beta)p - \alpha p_{-1} - (1 - \alpha)(1 - \alpha\beta)p = (1 - \alpha)(1 - \alpha\beta)p^w - (1 - \alpha)(1 - \alpha\beta)p$

$-\alpha\beta(p_{+1} - p) + \alpha(p - p_{-1}) = (1 - \alpha)(1 - \alpha\beta)(p^w - p)$

$-\alpha\beta\pi_{+1} + \alpha\pi = (1 - \alpha)(1 - \alpha\beta)(p^w - p)$

Phillips curve:

$\pi = \beta\pi_{+1} + \frac{(1-\alpha)}{\alpha}(1 - \alpha\beta)(p^w - p)$

-

AppendixB.3. Capital Goods Producers

capital accumulation

$K = (1 - \delta) * K_{-1} + CPHI * X$

$k = (1 - \delta)k_{-1} + \delta x$

-

Tobin´s Q

$Q(1 + \kappa - \kappa(\frac{X}{X_{-1}})) = 1 - \frac{0.5\kappa\beta}{MUC}(\frac{MUC_{+1}Q_{+1}X^2_{+1}}{X^2} - MUC_{+1}Q_{+1})$

$q - \kappa(x - x_{-1}) = -\kappa\beta(x_{+1} - x)$

-

AppendixB.4. Wholesale Producer

production function
$Y^w = \exp(a) * K_{-1}{}^{(1-\psi-\varphi)} * H^\psi * (H^e)^\varphi$
$y^w = a + (1-\psi-\varphi)k_{-1} + \psi h$
-
productivity shock
$a = \rho^a * a_{-1} + \varepsilon^a$
-
capital demand
$R^w * K_{-1} = (1-\psi-\varphi) * P^w * Y^w$
$r^w + k_{-1} = p^w + y^w$
-
household labour demand
$W * H = \psi * P^w * Y^w$
$w + h = p^w + y^w$
-
entrepreneurial labour demand
$W^e * H^e = \varphi * P^w * Y^w$
$w^e = p^w + y^w$
-

AppendixB.5. Entrepreneurs

net return on capital (definition)
$R^e * P_{-1} * Q_{-1} = R^w + (1-\delta) * P * Q$
$r^e - \pi + q_{-1} = \frac{R^e-(1-\delta)}{R^e}(p^w - p + y - k_{-1}) + \frac{(1-\delta)}{R^e} q$
-
entrepreneur´s balance sheet
$Q * K = B + N$
$q + k = \left(\frac{B}{K}\right) b + \left(\frac{N}{K}\right) n$
-
entrepreneur's net worth evolution
$N_t = \gamma f\left(\overline{\omega_t}\right) R_t^e Q_{t-1} K_{t-1}\left(\frac{P_{t-1}}{P_t}\right) + \frac{W_t^e}{P_t}$
$n_t = \gamma R^{ib} \frac{K}{N}\left(r_t^e - r_{t-1}^{ib}\right) + \gamma R^{ib}\left(r_{t-1}^{ib} - \pi_t + n_{t-1}\right) + \gamma R^{ib}\left(\frac{R^e}{R^{ib}} - 1\right)\frac{K}{N}\left(r_t^e - \pi_t + q_{t-1} + k_{t-1}\right) -$
$\gamma R^{ib} loss \frac{R^e}{R^{ib}} \frac{K}{N}\left(r_t^e - \pi_t + q_{t-1} + k_{t-1}\right) + \varrho \frac{Y}{N}\left(p_t^w - p_t + y_t\right)$
$n_t \approx \gamma R^{ib} \frac{K}{N}\left(r_t^e - r_{t-1}^{ib}\right) + \left(r_{t-1}^{ib} - \pi_t + n_{t-1}\right)$
-
entrepreneur's consumption
$C_t^e = (1-\gamma)\left(R_t^e Q_{t-1} K_{t-1}\left(\frac{P_{t-1}}{P_t}\right) - \left(R_{t-1}^{ib} + \frac{loss R_t^e Q_{t-1} K_{t-1}\left(\frac{P_{t-1}}{P_t}\right)}{(Q_{t-1}K_{t-1} - N_{t-1})\left(\frac{P_{t-1}}{P_t}\right)}\right)(Q_{t-1}K_{t-1} - N_{t-1})\left(\frac{P_{t-1}}{P_t}\right)\right)$
$c_t^e = \frac{1-\gamma}{\gamma}\frac{Y}{C^e}\frac{N}{Y} n_t - \varrho \frac{1-\gamma}{\gamma}\frac{Y}{C^e}\left(p_t^w - p_t + y_t\right)$
$c_t^e \approx n_t$
-
financial accelerator
$R_{+1}^e = (Q * \frac{K}{N})^{fap} * R^{ib}$

$r^e_{+1} = fap(q+k-n) + r^{ib}$
-

AppendixB.6. Retail Bank

Retail bank balance sheet
$B = D + IB$
$b = \frac{D}{B}d + \frac{IB}{B}ib$
-
Retail bank profit
$\Pi^{RB}_{t+1} = \Bigg(g\left(\overline{\omega}\right) R^e_{t+1} p_{t+1} - \Bigg(\frac{\left(R^D_{t+1}(i) - R^{RR}_{t+1} RR_{t+1}\right)}{1-RR_{t+1}} +$
$\left(1 - \frac{(1-RR_{t+1})}{p_{t+1}-1} \left(\frac{R^D_{t+1}(i)}{R^D_{t+1}}\right)^{\epsilon} \frac{D_{t+1}}{N_{t+1}}\right) \left(R^{ib}_{t+1} - \frac{\left(R^D_{t+1}(i) - R^{RR}_{t+1} RR_{t+1}\right)}{1-RR_{t+1}} \right)$
$(p_{t+1} - 1) - \frac{\kappa^d}{2} \left(\frac{R^D_{t+1}(i)}{R^D_t(i)} - 1 \right)^2 R^D_{t+1} \frac{D_{t+1}}{N_{t+1}} N_{t+1} \left(\frac{P_t}{P_{t+1}}\right)$
- Determination of deposit rates (as in Gerali et al. (2010))
$\left(-1 - \epsilon + \epsilon \frac{R^{ib}_{t+1}(1-RR_{t+1}) + R^{RR}_{t+1} RR_{t+1}}{R^D_{t+1}}\right) \frac{P_t}{P_{t+1}} - \kappa^d \left(\frac{R^D_{t+1}}{R^D_t} - 1\right)$
$\frac{R^D_{t+1}}{R^D_t} \frac{P_t}{P_{t+1}} + SDF \frac{D_{t+2}}{D_{t+1}} \kappa^d \left(\frac{R^D_{t+2}}{R^D_{t+1}} - 1\right) \left(\frac{R^D_{t+2}}{R^D_{t+1}}\right)^2 \frac{P_{t+1}}{P_{t+2}} = 0$
-

AppendixB.7. Narrow Bank

offers funding to retail banks and takes credit from central bank
$CD + IB = L^{CB}$
$\ln(CD + IB) = \ln(L^{CB})$
$\frac{CD}{L^{CB}} cd + \frac{IB}{L^{CB}} ib = l^{CB}$
-
$CD = (1 - HC) L^{CB}$
$cd = d(\ln(1 - HC) + \ln L^{CB})$
$cd = \frac{-dHC}{1-HC} + l^{CB}$
$cd = -\frac{HC}{1-HC} hc + l^{CB}$
-
$IB + (1 - HC) L^{CB} = L^{CB}$
$IB = HC * L^{CB}$
$ib = hc + l^{CB}$
-
FOC
$\left(R^{IB} - \frac{1}{HC}\left(\theta^{REPO} - (1-HC)\theta^{CD}\right) R^P\right) = \Xi'\left(IB\right)$
$r^{IB} - \frac{R^P}{R^{IB}} \left(\left(\frac{1}{HC}\theta^{REPO} - \frac{(1-HC)}{HC}\theta^{CD} \right) \left(r^P - hc\right) - \theta^{CD} hc \right) = \frac{(\eta-1)}{\tilde{\Xi}} \frac{\Xi(IB)}{R^{IB} IB} ib$
-
-

AppendixB.8. Central Bank & Government

government spending

$ST = RR*D+\left(R_{-1}^{IB}IB_{-1}\left(\frac{P_{-1}}{P}\right) - \Xi'\left(IB_{-1}\right)IB_{-1}\left(\frac{P_{-1}}{P}\right) - IB\right) - R_{-1}^{RR}RR_{-1}D_{-1}(\frac{P_{-1}}{P})$

$\frac{ST}{Y}st = RR\frac{D}{B}\frac{B}{K}\frac{K}{Y}(rr+d)+R^{IB}\frac{IB}{B}\frac{B}{K}\frac{K}{Y}\left(r_{-1}^{IB} - \pi + ib_{-1}\right) - \eta\frac{\Xi(IB)}{Y}(\eta ib_{-1} - \pi) - \frac{IB}{B}\frac{B}{K}\frac{K}{Y}ib - R^{RR}RR\frac{D}{B}\frac{B}{K}\frac{K}{Y}\left(r_{-1}^{RR} - \pi - rr_{-1} - d_{-1}\right)$

-

budget constraint

$RR*D+CD+R_{-1}^{REPO}L_{-1}^{CB}(\frac{P_{-1}}{P})+T = R_{-1}^{RR}RR_{-1}D_{-1}(\frac{P_{-1}}{P})+R_{-1}^{CD}CD_{-1}(\frac{P_{-1}}{P})+\tau^r\frac{P^w}{P}Y^w + L^{CB} + ST$

$T = \tau^r\frac{P^w}{P}Y^w$

-

taylor rule

$\frac{R}{R^{SS}} = (\frac{R_{-1}}{R^{SS}})^{\rho^r} * (\Pi^{\phi_\pi} * (\frac{Y}{Y^{SS}})^{\phi_y})^{(1-\rho_r)} * \exp(\varepsilon^r)$

$r = \rho_r r_{-1} + (1-\rho_r)(\phi_\pi\pi + \phi_y y) + \varepsilon^r$

-

haircut shock

$hc = \rho^{HC}hc_{-1} + \epsilon^{HC}$

-

reserve requirement shock

$rr = \rho^{RR}rr_{-1} + \epsilon^{RR}$

-

AppendixB.9. Resource Constraint

$Y = C + C^e + X + G + (1 - f(\overline{\omega}) - g(\overline{\omega}))R^eQ_{-1}K(\frac{P_{-1}}{P}) + \Xi(IB_{-1}) + \frac{\kappa^d}{2}\left(\frac{R^D}{R_{-1}^D} - 1\right)^2 R_{-1}^D D_{-1}(\frac{P_{-1}}{P})$

$y = \frac{C}{Y}c+\frac{C^e}{Y}c^e+\frac{X}{Y}x+\frac{G}{Y}g-R^e\frac{K}{Y}(f'(\overline{\omega})+g'(\overline{\omega}))d\overline{\omega}+(1-f(\overline{\omega})-g(\overline{\omega}))R^e\frac{K}{Y}(r^e - \pi + q_{-1} + k)+\eta\frac{\Xi(IB)}{Y}ib_{-1}$

-

inflation definition

$\Pi = \frac{P}{P_{-1}};$

$\pi = p - p_{-1}$

-