Risk and Liquidity in a System Context[*]

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Abstract

This paper explores the pricing of debt in a financial system where the assets that borrowers hold to meet their obligations include claims against other borrowers. Assessing financial claims in a system context captures features that are missing in a partial equilibrium setting. It is possible for spreads to *fall* as debts *rise*, as debt-fuelled increases in asset prices and stronger balance sheets reinforce each other. Conversely, it is possible that *de-leveraging* leads to *increases* in spreads, as is often observed during crises.

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1 Introduction

Many assets in the financial system are claims against other borrowers. As such, the value of such assets fluctuate with the strength of the borrowers' balance sheets. When the web of claims and obligations link financial entities together into a tightly knit system, the relative values of liabilities and assets (and hence net worth), the availability of credit, and asset prices are interrelated and fluctuate together. We may also expect feedback elements that serve to magnify the responses to shocks. Balance sheet changes will affect asset prices and asset price changes will affect balance sheets. The loop thus created may generate amplified responses to any shocks to the financial system.

The role of easy credit conditions in setting the general tenor of financial market conditions has returned as a topical issue in the wake of the unprecedented monetary easing in the United States in recent years. Indeed, recent developments in financial markets have posed a challenge to central bankers and other public officials. For many observers, the signals emanating from the financial markets - in the form of low long-term interest rates, compressed yield spreads and low implied volatility - have painted a benign economic picture that gives little weight to the potential down-side risks.[1]

In a financial system with interlinked claims and obligations, one party's obligation is another party's asset. When calculating the equity value of the financial system as a whole, such claims and obligations cancel out. What remains as the equity value of the financial system as a whole is the marked-to-market value of the "fundamental" assets - assets that are not the obligation of some other party. The larger is the value of fundamental assets, the

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[1] Official publications express these worries in more guarded terms. See Bank of England (2004a, 2004b) and IMF (2005a, 2005b).

larger is the equity value of the financial system as a whole, and the stronger is the average balance sheet in the system. In this sense, an increase in the value of fundamental assets is a rising tide that lifts all boats.

As a case in point, consider the following passage from a recent commentary in the *Wall Street Journal*.[2]

> While many believe that irresponsible borrowing is creating a bubble in housing, this is not necessarily true. At the end of 2004, U.S. households owned $17.2 trillion in housing assets, an increase of 18.1% (or $2.6 trillion) from the third quarter of 2003. Over the same five quarters, mortgage debt (including home equity lines) rose $1.1 trillion to $7.5 trillion. The result: a $1.5 trillion increase in net housing equity over the past 15 months.

The author minimizes the dangers from the $1.1 trillion increase in debt by appealing to the marked-to-market value of housing equity. By valuing the entire housing stock at the current marginal transaction price, the marked-to-market value of housing may be a poor indicator of the potential liquidation value when a substantial chunk of the housing stock is put up for sale. However, the marked-to-market value is the relevant measure when assessing the market price of claims (such as mortgages) backed by the housing stock. The effects are then transmitted further up the chain to mortgage-backed securities, and then to collateralized debt obligations (CDOs), and then to claims against financial institutions that hold such CDOs. At each step in the chain, obligations are backed by claims further down the chain.

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[2] "Mr. Greenspan's Cappuccino" Commentary by Brian S. Wesbury, *Wall Street Journal,* May 31, 2005. The title makes reference to Alan Greenspan's comments on the "froth" in the U.S. housing market.

In a tightly-knit financial system, externalities transmitted through balance sheets are unavoidable and have far-reaching consequences. A transaction in the market affects more than the parties directly involved in the transaction itself, since the price determined in the transaction is used to price other assets and obligations. As such, the transaction has a spillover effect on the balance sheets of other entities in the financial system.

The externalities cut both ways, however. Just as house buyers paying exorbitant prices exert positive externalities that buoy up others' balance sheets, the distressed selling by defaulting home owners exert negative externalities that undermine others' balance sheets. In this sense, asset price booms fuelled by lax credit conditions and slumps fuelled by financial distress can be seen as mirror images of each other. The common thread is the feedback from asset price changes to changes in the strength of balance sheets, and the spillover effects across market participants. One of the main tasks of the paper is provide a unifying framework that can accommodate both types of phenomena, and to chart the propagation mechanisms both "on the way up" and "on the way down".

I begin by outlining a framework that can be used to assess the value of claims and obligations in a system of interlocking balance sheets. The basic problem can be posed in the following way. The marked-to-market value of my claim against A depends on A's creditworthiness, and so depends on the value of A's claims against B, C, etc. However, B or C may have a claim against me, and so we are back full circle. The task of valuing claims in a financial system thus entails solving for a consistent set of prices - that is, solving a fixed point problem.

I show that such a problem has a well-defined solution, and the value of all claims can be priced uniquely as a function of the underlying parameters

of the financial system - the level and seniority profile of debt, the structure of balance sheet interconnections, and (crucially) the current marginal transaction price of fundamental assets. When the level of debt is treated as a choice variable, it is possible to undertake comparative statics exercises on the externalities that are transmitted through the financial system. I do this both "on the way up" and "on the way down" by showing how the framework can be used to study lending booms and market crashes.

I draw on several strands in the literature. The formal apparatus owes much to the literature on lattice-theoretic ideas made popular in economics by Topkis (1978) and Milgrom and Roberts (1990, 1994). A recent paper by Eisenberg and Noe (2001) showed how tools from lattice theory can be applied to solve an allocation problem in bankruptcy, and the framework reported here builds on their insights.

To the extent that balance sheets serve as the conduit for the transmission of shocks, my framework is related to the large literature on the collateral role of assets in amplifying shocks to the financial system. The research to date has distinguished two distinct channels. The first is the increased credit that operates through the *borrower's* balance sheet, where increased lending comes from the greater creditworthiness of the borrower (Bernanke and Gertler (1989), Kiyotaki and Moore (1998, 2001)). The second is the channel that operates through the *banks'* balance sheets, either through the liquidity structure of the banks' balance sheets (Bernanke and Blinder (1988), Kashyap and Stein (2000)), or the cushioning effect of the banks' capital (Van den Heuvel (2002)). All these features make an appearance in my framework.

The results reported here are also related to the developing theoretical literature on the role of liquidity in asset pricing (Acharya and Pedersen (2005), Brunnermeier and Pedersen (2005a, 2005b), Morris and Shin (2004)).

Although my framework has little to contribute to the study of market microstructure issues, the common thread is the relationship between funding conditions and the resulting market prices of assets.

The theme of financial distress examined here is also closely related to the literature on liquidity drains that deal with events such as the stock market crash of 1987 and the LTCM crisis in the summer of 1998. Gennotte and Leland (1990) and Geanakoplos (2003) provide analyses that are based on competitive equilibrium. The framework to be presented below could be seen as a reduced-form representation of a fully-fledged competitive analysis. It provides a simplified representation that draws attention to the key balance sheet spillovers without having to flesh out the full competitive economy. Provided that the shortcomings of such a reduced-form representations are borne in mind, the simplicity of the framework can aid our understanding of the balance sheet propagation effects. I begin by outlining the formal apparatus. Applications to liquidity drains and lending booms then follow.

2   Framework

There are n agents in the financial system, whom I shall simply call "investors" for ease of reference. The only qualification to be a member of the financial system is to have a balance sheet, and so these agents could be banks, households or some other type of financial intermediary.

All investors are assumed to be risk-netrual. By assuming risk-netrality, I abstract from discussions of shifts in risk-aversion or shifts in "risk appetite" that often crops up in commentaries of asset price booms and market distress. One of the main objectives of the analysis is to show how such phenomena can be explained without reference to shifts in attitudes to risk.

Investors may issue debt to fund their activities, and if they do so, they

issue zero coupon debt that matures at date T. Investor i may issue several seniority classes of debt. Denote by $\bar{x}_{ij}$ the face value of the debt issued by investor i in the seniority class j. Thus, $\bar{x}_{i1}$ is the face value of the most seniority tranche of debt issued by investor i, and $\bar{x}_{i2}$ is the next most senior, and so on. Denote by π_{ijk} the proportion of investor i's jth senior debt held by investor k. Since all debt is held by some investor in the financial system, we have for any i and j,

$$\sum_{k=1}^{n} \pi_{ijk} = 1 \tag{1}$$

We impose a mild regularity condition for later use. Assume that there are unleveraged investors in the financial system who have no debt outstanding. We make the assumption that for all debt instruments, some of it is held by an unleveraged investor. The condition can be weakened somewhat without affecting the main conclusions, but the stronger version simplifies the arguments. Formally, the regularity condition states that for any i and j, there exists an investor k with zero debt such that $\pi_{ijk} > 0$.

We now turn to the asset side of investor i's balance sheet. Investor i is endowed with a project that yields random but positive payoff at date T, with mean w_i. If investor i is a household, then w_i may be interpreted as the endowment of wage income. The realizations across investors' random endownments is governed by some joint density g. There is also a "fundamental" asset that is not the obligation of any other investor, and we denote the price of the fundamental asset at date 0 as v. Investor i holds y_i units of the fundamental asset.

The only condition imposed on the joint density of payoffs is that the terminal cash flows for any investor is higher conditional on a high value of v. Formally, if $G_i\left(.|v\right)$ is the cumulative distribution over investor i's payoff

at date T conditional on the date 0 asset price v, then $v' \geq v$ implies

$$G_i\left(.|v\right) \geq G_i\left(.|v'\right) \tag{2}$$

so that we have an ordering based on first degree stochastic dominance. We do not need to specify the density over the realizations over v itself. In fact, it will be important for later applications that we allow for heterogenous private valuations of the fundamental asset at date N. However, any holding of the asset is constantly marked to market at the prevailing transaction price.

Denote by x_{ij} the market value of investor i's debt in seniority class j. The market value of i's debt is less than its face value, but is increasing in the value of i's assets that back the debt. Assume that the risk-free interest rate is zero. Then, x_{ij} approaches its face value $\bar{x}_{ij}$ as the value of i's assets that back the obligation becomes large. Since investor i holds proportion π_{jki} of investor j's debt of seniority class k, the market value of investor i's assets, denoted by a_i, is given by

$$a_i = w_i + vy_i + \sum_j \sum_k \pi_{jki} x_{jk} \tag{3}$$

The market value of debt will depend on the valuation of assets, which in turn depends on the market value of the debt issued by other borrowers. The pricing of debt must also be consistent with the uncertainty in endowments at the maturity date T. Thus, the market value of debt must be solved in the context of the financial system as a whole.

Merton (1974) noted that the market value of i's debt is the price of a portfolio that consists of a short position in a put option on i's assets with strike price equal to the face value of the debt together with a cash holding of the face value of the debt itself. Following Merton's approach, we may conjecture that the market value of investor i's debt is a well-defined

function of the market value of i's assets at the time. Figure 1 depicts such a relationship, where $x_i = \sum_j x_{ij}$ denotes the market value of the total debt of investor i, and θ denotes the set of parameters that underpin the pricing relationship. The parameter θ is defined as

$$\theta = (\bar{x}, v, w) \tag{4}$$

where $\bar{x}$ is the profile of face values of debt $\bar{x}_{ij}$, v is the price of the fundamental asset and w is the profile of the expected endowments w_i. We have
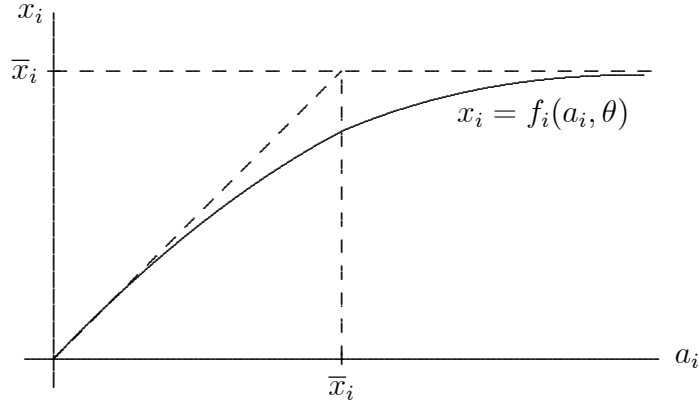

Figure 1: Market value of total debt x_i of investor i

the following pair of preliminary results.

Lemma 1 *There exist functions $\{f_{ij}\}$ such that*

$$x_{ij} = f_{ij}(a_i, \theta) \tag{5}$$

where each f_{ij} is non-decreasing in a_i, and is bounded above by $\bar{x}_{ij}$.

Lemma 2 *The market value of equity is non-decreasing in a_i. That is, the function e_i defined as*

$$e_i \equiv a_i - \sum_j f_{ij}(a_i, \theta) \tag{6}$$

is non-decreasing in a_i.

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The proofs of lemmas 1 and 2 are given in the appendix. Lemma 2 is the intuitive result that the market value of equity cannot decrease as one's asset value increases. Figure 1 conveys the intuition. The market value of equity is the gap between the 45 degree line and the curve x_i. As a_i becomes larger, the gap becomes larger. Lemma 1 verifies that Merton's (1974) perspective on the pricing of defaultable claims is valid in a system context, also.

From (5), we have the following system for the determination of the value of financial system claims.

$$
\begin{aligned}
x_{11} &= f_{11}\left(a_1\left(x\right),\theta\right) \\
x_{12} &= f_{12}\left(a_1\left(x\right),\theta\right) \\
&\vdots \\
x_{nk} &= f_{nk}\left(a_n\left(x\right),\theta\right)
\end{aligned}
$$

The notation $a_i\left(x\right)$ makes explicit the fact that asset values depend on the vector x of all claims in the financial system, where

$$
x = \left(x_{11}, x_{12}, \cdots, x_{21}, x_{22}, \cdots, x_{nk}\right)
$$

We can write this system as

$$
x = F\left(x,\theta\right) \tag{7}
$$

A consistent set of valuations is a fixed point x of the mapping F. It turns out that there is a unique fixed point x, so that the consistent set of debt prices are well-defined as a function of the parameters $\theta = \left(\bar{x}, v, w\right)$. This result is proved formally in the next section.

2.1 Nature of the Problem

To gain some intuition for the result, it is useful to address the issue from the point of view of someone attempting to solve the problem through an iterative

process. One approach would be to start from a "conservative" viewpoint, where the claims against other investors are given zero value initially. Thus, investor i's assets would be calculated by putting a pessimistic value on the claims against other investors. Then we can obtain the initial estimate x^1 for debt values based on this conservative assumption (note the superscript notation):

$$x^1 = F(0, \theta)$$

Iterating this relationship, we have the sequence $x^1, x^2, x^3, \cdots$ defined by

$$x^{t+1} = F\left(x^t, \theta\right)$$

Provided that the endowments and fundamental asset values are positive, we would have

$$0 \lneqq x^1 \tag{8}$$

where the notation $0 \lneqq x^1$ indicates that $0 \neq x^1$ and $0 \leq x^1$. Furthermore, since F is a non-decreasing function of x, applying F to both sides of (8) preserves the inequality. If $x^1 = x^2$, then we have found a fixed point. Otherwise, we have $0 \lneqq x^1 \lneqq x^2$. By iteration, we either have a fixed point x^k for finite k, or we have the inequalities

$$0 \lneqq x^1 \lneqq x^2 \lneqq x^3 \lneqq \cdots$$

Since each component x_{ij} of x lies in the closed and bounded interval $[0, \bar{x}_{ij}]$, the sequence converges from below to some well-defined limit. If F were continuous at the limit, such a limit would be a fixed point of the mapping F, and so we will have found one set of consistent debt valuations.

There is, however, an alternative iterative approach, which is to choose an "optimistic" starting point, where all the claims are evaluated at their face

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values $\bar{x}$. The initial estimate x^1 for debt values based on this "optimistic" assumption would be

$$x^1 = F(\bar{x}, \theta)$$

and the sequence $x^1, x^2, x^3, \cdots$ is defined as before by $x^{t+1} = F(x^t, \theta)$. Following an analogous line of reasoning as above, we obtain the decreasing sequence

$$\bar{x} \gneqq x^1 \gneqq x^2 \gneqq x^3 \gneqq \cdots$$

that converges from above to some well-defined limit. Such a limit would be potentially another, distinct fixed point of the mapping F. Without additional argument, we could not guarantee that the limit from above would coincide with the limit from below, nor indeed, that there were not yet more fixed points that we are missing from the iterative procedure. However, if we could show that there is precisely one fixed point of the mapping F, then it would be easy to compute this fixed point using one of the iterative procedures we have sketched here.

2.2   Unique System of Debt Prices

It turns out that there is a unique fixed point of the mapping F in the debt pricing problem. Following Eisenberg and Noe (2001), the argument makes use of tools from lattice theory, as popularized in economics by Topkis (1978) and Milgrom and Roberts (1990, 1994). For completeness of the discussion, I begin by covering some preliminary ground.

A *complete lattice* is a partially ordered set $(X, \leq)$ with the property that every non-empty subset $S \subseteq X$ has both a greatest lower bound $\inf(S)$ and a least upper bound $\sup(S)$ that belong to the set X. In our context, we can define a complete lattice with the set X given by

$$X \equiv [0, \bar{x}_{11}] \times [0, \bar{x}_{12}] \times \cdots \times [0, \bar{x}_{nk}]$$

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and the ordering $\leq$ is given by the usual component-wise order so that $x \leq x'$ when $x_{ij} \leq x'_{ij}$ for all components i, j. Tarski (1955) showed that monotonic functions on complete lattices have a highest and lowest fixed point.

Lemma 3 *(Tarski's Fixed Point Theorem) Let $(X, \leq)$ be a complete lattice and F be a non-decreasing function on X. Then there are x^* and x_* such that $F(x^*) = x^*$, $F(x_*) = x_*$, and for any fixed point x, we have $x_* \leq x \leq x^*$.*

For the sake of self-contained discussion, as well as for later adaptation for use in my argument, I reproduce the proof of Tarski's fixed point theorem here. Define the set S as

$$S = \{x | x \leq F(x)\} \tag{9}$$

and define x^* as $x^* \equiv \sup S$. For any $x \in S$, $x \leq x^*$. Since F is non-decreasing, $x \leq F(x) \leq F(x^*)$. Thus, $F(x^*)$ is also an upper bound for S. But x^* is defined as the *least* upper bound of S. Thus

$$x^* \leq F(x^*) \tag{10}$$

Applying F to both sides of (10), we have $F(x^*) \leq F(F(x^*))$. But this implies that $F(x^*) \in S$, so that $F(x^*)$ is bounded by x^*. That is, $F(x^*) \leq x^*$. Taken together with (10), this means that $F(x^*) = x^*$. Any other fixed point of F must belong to S, and so x^* is the largest fixed point. The smallest fixed point x_* is defined as $\inf \{x | x \geq F(x)\}$, and the argument is exactly analogous.

In our debt pricing problem, we can use the particular features of our problem to show that the smallest and largest fixed points coincide, so that we have a unique fixed point.

Theorem 4 *There is a unique profile of debt prices x that solves $x = F(x, \theta)$.*

The argument for theorem 4 is as follows. From Tarski's fixed point theorem (lemma 3) there are solutions x and x' such that $x_{ij} \leq x'_{ij}$ for all i, j. Suppose, contrary to the theorem, that there is more than one solution. Then for some i, j, we have

$$x'_{ij} > x_{ij} \tag{11}$$

From our regularity assumption, there is an unleveraged investor h that holds some fraction of this debt. Denote by e'_h investor h's equity value under x', and denote by e_h her equity value under x. For an unleveraged investor, the value of equity is just the value of assets. Thus,

$$
\begin{aligned}
e'_h &= w_h + vy_h + \sum_j \sum_k \pi_{jkh} x'_{jk} \\
&> w_h + vy_h + \sum_j \sum_k \pi_{jkh} x_{jk} \\
&= e_h
\end{aligned}
\tag{12}
$$

In other words, investor h's equity value under x' is strictly larger than her equity value under x.

Meanwhile, the asset value of investor i under x' is at least as large as her asset value under x. This is true for all investors in the financial system. From lemma 2, the equity value of an investor under x' is at least as large as that under x. Thus, for all investors i, $e'_i \geq e_i$. Denote $e \equiv \sum_i e_i$ and $e' \equiv \sum_i e'_i$. From (12) and the fact that $e'_i \geq e_i$ for all i, we have

$$e' > e \tag{13}$$

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Meanwhile, we can calculate aggregate equity by adding up across investors.

$$
\begin{aligned}
e &= \sum_i a_i - \sum_i \sum_j x_{ij} \\
&= \sum_i \left(w_i + v y_i + \sum_j \sum_k \pi_{jki} x_{jk} \right) - \sum_i \sum_j x_{ij} \\
&= \sum_i (w_i + v y_i) \qquad\qquad\qquad\qquad\qquad (14) \\
&= \sum_i \left(w_i + v y_i + \sum_j \sum_k \pi_{jki} x'_{jk} \right) - \sum_i \sum_j x'_{ij} \\
&= e'
\end{aligned}
$$

where (14) follows from fact that $\sum_i \pi_{jki} = 1$. Hence,

$$
e = e' \qquad\qquad\qquad\qquad\qquad (15)
$$

But then we have a contradiction, since (15) is incompatible with (13). Thus, we conclude that $x' = x$, and so theorem 4 holds.

The intuition behind theorem 4 comes out clearly from the argument above. The result rests on the principle that the equity value of the *whole financial system* depends only the assets that are not the obligation of any other investor. This is natural, since when all claims and obligations are summed across investors, the total value of claims against others must match exactly the total value of obligations to others. Thus, as long as the value of the fundamentals are unchanged, the equity value of the whole system is conserved. By Tarski's fixed point theorem, if there is more than one solution, then one must be strictly larger than the other in at least one component. However, this turns out to imply that the total equity values can be strictly ordered - a contradiction.

While the uniqueness result sets an important benchmark, the most important result for applications is the comparative statics result for the unique

solution. The following result is the key to the later sections of the paper, and is a straightforward consequence of the comparative statics results due to Milgrom and Roberts (1990, 1994).

Theorem 5 *Let $x(\theta)$ be the unique solution to $x = F(x, \theta)$. Then $x(\theta)$ is increasing in the partial ordering over θ in the sense that whenever $\theta' \geq \theta$, we have $x(\theta') \geq x(\theta)$.*

The proof follows from the definition of the set S given by (9). Noting the dependence of F on the parameters θ, let us denote

$$S(\theta) \equiv \{x | x \leq F(x, \theta)\} \tag{16}$$

As θ becomes larger, the condition for inclusion becomes more accommodative. Formally, if $\theta' \geq \theta$, then $S(\theta') \supseteq S(\theta)$. Since $x(\theta') = \sup S(\theta')$ and $x(\theta) = \sup S(\theta)$, we have $x(\theta') \geq x(\theta)$. This proves the result.

Having outlined the formal apparatus, we now turn to the application of our framework to specific examples. The first example is that of liquidity drain and financial distress.

3 Solvency Constraint

Leveraged investors are subject to solvency requirements. Consider a generalized solvency requirement of the form:

$$\frac{a_i - x_i}{x_i} \geq r^* \tag{17}$$

where $x_i = \sum_j x_{ij}$ is the market value of total debt outstanding for investor i. The numerator in (17) is the market value of equity for investor i, while the denominator is the market value of i's debt. Thus, (17) stipulates that the equity to debt ratio must not fall below some minimum value r^*.
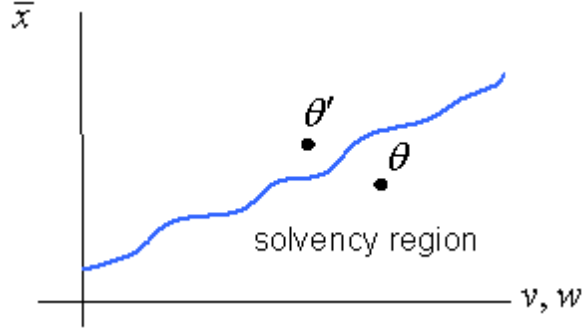
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Figure 2: Solvency region for θ

From our theorem on uniqueness (theorem 4), we know that the parameters $\theta = (\bar{x}, v, w)$ determine uniquely the value of all assets and liabilities in the financial system. So, for any given θ, either the solvency constraint (17) is satisfied for all investors, or there are some investors for which the constraint binds. We can thus partition the set of parameters θ into two mutually exclusive subsets. The *solvency region* is the set of θ for which (17) is satisfied for all investors.

The solvency region can be illustrated schematically as in figure 2. The parameter $\theta = (\bar{x}, v, w)$ has many dimensions, but figure 2 shows schematically that increases in the v or w components of θ promote solvency, as does a fall in the $\bar{x}$ components. To see this, consider increasing v or w while keeping $\bar{x}$ fixed. Since the market value of debt is bounded above by $\bar{x}$ while the assets are increasing without bound, the solvency constraint (17) must be satisfied for all i for sufficiently high values of v and w. Next, consider a fall in $\bar{x}$ while holding v and w fixed. As $\bar{x}$ approaches 0, the market values of debt also approaches 0, while the asset values remain positive, given by the value of the holding of the fundamental asset and the endowment. Thus, (17) will be satisfied for low enough $\bar{x}$. In figure 2, the boundary of the

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solvency subset can thus be represented schematically as an upward-sloping line, with the solvency region lying below the line.

When θ lies outside the solvency region, then the constraint (17) binds for at least one investor, and some remedial action must be taken by that investor. If investor i finds that the constraint is binding, then the possible remedial actions fall under two categories.

- Investor i raises additional equity. This can be achieved by selling equity claims to other investors and purchasing the fundamental asset or otherwise increasing the value of her assets a_i.

- Investor i pays down her debt. This can be done by selling some portion of her holding of the fundamental asset or liquidating some portion her endowment w_i to raise cash in order to buy back her debt, and thereby reduce the face value of her debt $\bar{x}_i$.

However, we may expect spillover effects across investors, especially if an investor attempts to regain solvency by paying down debt by selling her assets. If the market price is sensitive to the increased supply, then the fall in price will affect all other investors' balance sheets, through marking to market of the balance sheets of all investors. The effects are illustrated in figure 3. Consider a shock to the endowments w that shifts the parameters to the left to θ, which lies outside the solvency set. The vector of face values of debt $\bar{x}$ must fall in order to return to the solvency subset. If investors attempt to sell the fundamental asset or liquidate their endowments w_i inefficiently, v and w may fall. Thus, rather than returning to solvency by a direct vertical route, the arrow points south-west to θ', as illustrated in figure 3. The more sensitive is the price of the fundamental asset to increased selling pressure,
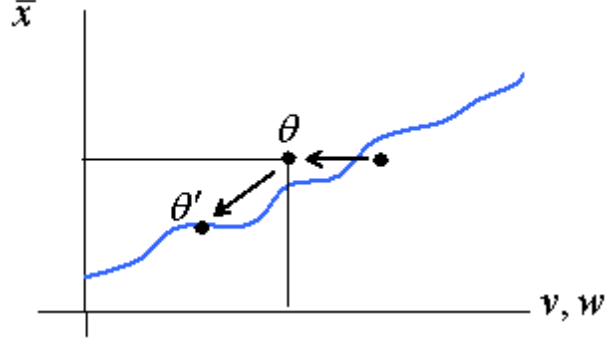
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Figure 3: Restoration of solvency

the greater are the spillover effects across investors as selling by one investor triggers the solvency constraint for other investors.

The *spread* on debt (i, j) is the discount on its market value x_{ij} relative to its face value $\bar{x}_{ij}$, and we can define the spread as:

$$1 - \frac{x_{ij}}{\bar{x}_{ij}} \tag{18}$$

The spread goes to zero as investor i's balance sheet becomes stronger, or if investor i's pays down the face value $\bar{x}_{ij}$ for fixed value of assets. However, remedial action to pay down debt may result in falls in asset prices. Such falls in asset prices may undermine the market value x_{ij}. The overall effect on spreads when there are concerted attempts to restore solvency may go either way. Figure 4 illustrates. Starting from θ, consider a shift to θ', representing a fall in face values $\bar{x}$ for fixed values of v and w. Asset prices are higher at the new point θ' so that market values of debt $\{x_{ij}\}$ are higher (or no lower) for all debt (i, j). Spreads are thus lower at θ' than at θ.

In contrast, consider a leftward shift from θ to θ'', representing a fall in v or w given fixed face values $\bar{x}$. Such a shift results in a fall in market values of debt x_{ij} while the face value $\bar{x}_{ij}$ remains fixed. Hence the spreads
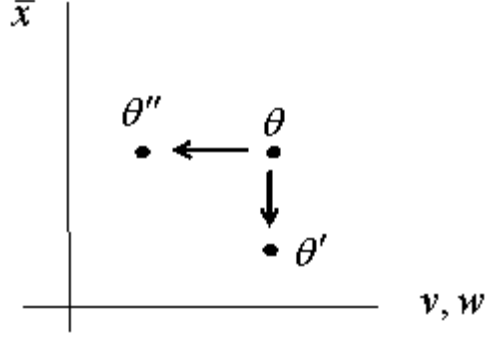
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Figure 4: Two scenarios for spreads

at θ'' will be larger than at the starting point θ. Since remedial actions will generally result in a shift in which both face values and fundamental asset values fall, the prediction on spreads are generally ambiguous. Qualitatively, the greater is the fall in v and w as a result of the concerted selling pressure, the more likely it is that de-leveraging results in an *increase* in spreads. Let us examine an example of liquidity drain and financial distress illustrating the effects outlined so far.

3.1 Liquidity Drain and Financial Distress

For the purpose of illustration, let us rule out the possibility that the investor can raise new equity, and focus attention on the possibilities of reducing debt by selling the holding of the fundamental asset. Let us also make the simplifying assumption that the liquidation value of the endowment is zero, so that an investor gains nothing by liquidating her endowments. The only way for an investor to regain solvency is to sell her holding of the fundamental asset.

Denote by s_i the amount of the fundamental asset offered for sale by investor i. Through this sale, the investor obtains cash of vs_i that can be

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used to pay down her debt. The market value of assets after the sale is

$$w_i + v\left(y_i - s_i\right) + b_i \tag{19}$$

where b_i denotes the market value of claims against other investors, given by $\sum_j \sum_k \pi_{jki} x_{jk}$. When the proceeds of the sale are used to pay down debt, the new level of debt is given by

$$x_i^0 - v s_i \tag{20}$$

where $x_i^0 = \sum_j x_{ij}^0$, the initial market value of investor i's debt. Thus, if the trader finds that her solvency constraint is binding, then the sale must be large enough so that

$$\frac{w_i + v\left(y_i - s_i\right) + b_i - x_i^0 + v s_i}{x_i^0 - v s_i} \geq r^* \tag{21}$$

If s_i satisfies the constraint with equality, we have

$$s_i = \frac{\left(1 + r^*\right) x_i^0 - w_i - v y_i - b_i}{r^* v} \tag{22}$$

But s_i must lie between zero and the total holding y_i. We can thus write the minimum sale by trader i that is compatible with the solvency constraint as:

$$s_i = \min\left\{y_i, \max\left\{0, \frac{\left(1 + r^*\right) x_i^0 - w_i - v y_i - b_i}{r^* v}\right\}\right\} \tag{23}$$

Figure 5 illustrates. We know from our comparative statics result (theorem 5) that b_i is a non-decreasing function of v. Hence, the s_i curve is strictly downward-sloping when it lies between 0 and y_i. The required sale s_i is larger when price v falls more. Summing over all traders i, we can derive the aggregate sale function

$$s\left(v\right) \equiv \sum_i s_i\left(v\right) \tag{24}$$
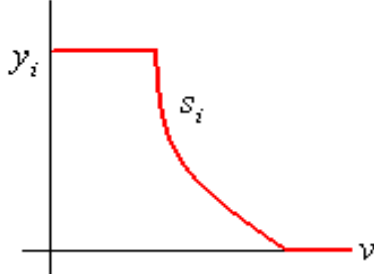
21

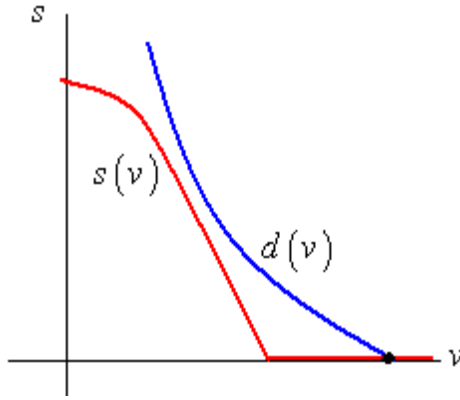Figure 5: Minimum sale to ensure solvency

Figure 6: Initial value of v

Since each s_i is decreasing in v, the aggregate required sale is also decreasing in v. Two different $s(v)$ functions are depicted in figures 6 and 7. In figure 6, the value of endowments w_i and claims b_i are high enough, so that no sales are required. Suppose that the demand curve for the fundamental asset generated by the heterogenous private valuations of the unleveraged investors are given by $d(v)$. An equilibrium price of the commodity is a price v for which $s(v) = d(v)$. In figure 6, the $s(v)$ and $d(v)$ curves intersect at a point on the horizontal axis, indicating that no forced sales are necessary to meet solvency constraints of any investor. Figure 7 paints a contrasting
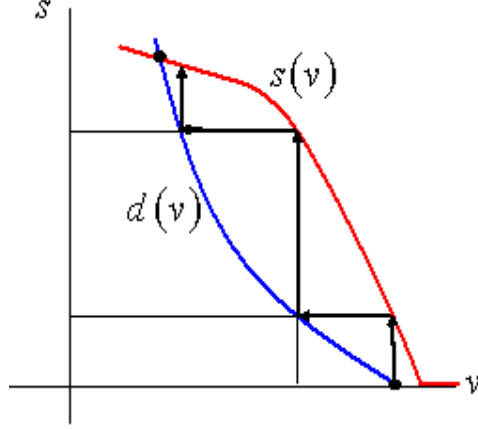
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Figure 7: Distressed value of v

picture in which the $s(v)$ curve has shifted higher (say, due to a downward shock to the endowments w) with the result that at the previous price of the fundamental asset, a strictly positive amount of sales must take place. The amount of the forced sale is indicated by the size of the right-most arrow. The forced sale introduces an endogenous downward response to the price of the fundamental asset, since the market clears at a lower price given the additional supply. When the investors' balance sheets are evaluated at this lower price, the solvency constraint binds again, as evidenced by the fact that the implied sales are now even higher, forcing yet more sales.

The price adjustment process can be depicted as a step adjustment process in the wedge below the $s(v)$ curve, but above the $d(v)$ curve. The second round sale of the commodity is implied by the $s(v)$ curve at the lower price. Given this increased supply, the price falls further, and so on. The price falls until we get to the nearest intersection point where the $d(v)$ curve and $s(v)$ curve cross. Equivalently, we may define the function Φ as

$$\Phi(v) = d^{-1}(s(v))$$

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and an equilibrium price of the commodity is a fixed point of the mapping $\Phi(.)$. The function $\Phi(.)$ has the following interpretation. For any given commodity price v, the value $\Phi(v)$ is the market-clearing price of the commodity that results when the price of the commodity on the traders' balance sheets is evaluated at price v. Thus, when $\Phi(v) < v$, we have the precondition for a downward spiral in the price, since the price that results from the sale of the commodity is lower than the price at which the balance sheets are evaluated.

More generally, the importance placed on asset prices follows the recent theoretical literature on banking and financial crises that has emphasised the limited capacity of the financial markets to absorb sales of assets (see Allen and Gale (2004), Cifuentes, et al. (2005), Gorton and Huang (2003) and Schnabel and Shin (2004)), where the price repercussions of asset sales have important adverse welfare consequences. Similarly, the inefficient liquidation of long assets in Diamond and Rajan (2005) has an analogous effect. The shortage of aggregate liquidity that such liquidations bring about can generate contagious failures in the banking system.

4   Lending Booms

Lending booms can be understood as a mirror image of liquidity drains and financial distress. In the previous section, we considered solvency constraints that required the equity to debt ratio to be *above* some given threshold value r^*. In this section, we consider constraints that require the equity to debt ratio to be *below* some given threshold value. That is, we consider constraints of the form:
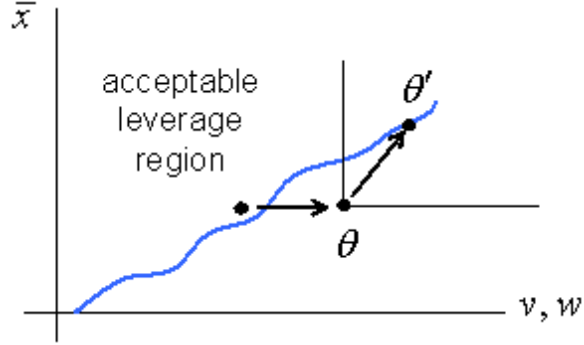
$$\frac{a_i - x_i}{x_i} \leq r^{**} \tag{25}$$

Figure 8: Adjustment to restore minimum leverage

Leveraged financial institutions such as hedge funds that rely on the magnification of returns on equity through leverage will aim to maintain some *minimum* level of leverage. More generally, when a financial institution is conscious of meeting a minimum level of return on equity, leverage must be sufficiently large to allow it to meet its target.

The comparative statics of a lending boom can be illustrated in the schematic two-dimensional diagrams for shifts in θ, as in the previous section. Figure 8 illustrates. The adjustment depicted in figure 8 is the mirror image of the adjustment required to restore solvency. Suppose a positive shock to endowments w pushes the system to θ. At this point, the leverage is too low for some investors. Minimum leverage can be restored by increasing the face value of debt $\bar{x}$. The upward sloping line in figure 8 represents the lower boundary of the minimum leverage set. If there are spillover effects in which increases in debt result in driving up the price of v of the fundamental asset, then increases in v raise the equity values of other investors, and may push their leverage down too low. These other investors will then attempt to restore leverage by increasing their face values of debt $\bar{x}_{ij}$, causing spillover effects on others.
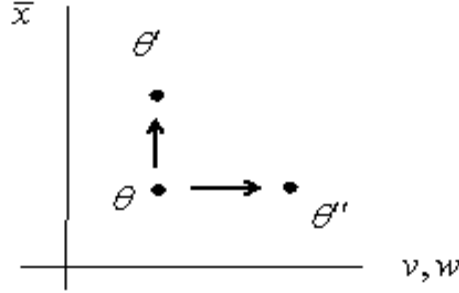
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Figure 9: Two scenarios for spreads

The effect on spreads depends on the strength of the spillover effects across investors. In figure 9, a shift upward from θ to θ' increases spreads unambiguously. This is because the equity value of the whole financial system is unchanged while the face value has increased strictly. Spreads must rise. The shift from θ to θ'' reduces spreads unambiguously, since asset values rise without a change in face values of debt. Since the adjustment to the minimum leverage set takes place in the "northeast" direction, the overall effect on spreads is ambiguous. If fundamental asset prices react sufficiently to increases in lending, then overall spreads could fall.

4.1 Example of Housing Boom

Let us illustrate the arguments by considering an example of a housing boom. The fundamental asset is residential property, and its price is denote by v. There are three groups of investors in the financial system - young households, old households and the banking sector. The three groups have the following characteristics.

- The young households are endowed with their random endowment that arrives at date T, which can be interpreted as their wage income. They

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start with no housing assets. The young households have a high private valuation for residential property, but are credit constrained, and cannot purchase property without taking on debt.

- The old households are unleveraged investors. They start out by holding all of the housing stock in the economy, even though their private valuation of property is lower than that of the young households. The old households are also the residual equity holders of the banks.

- Banks fund their activities by incurring liabilities to the old households but then lending out the proceeds to the young households.

Denote by Y the set of young households in the system and let $\bar{D}$ be the total face value of the loans granted by the banks to the young households, so that

$$\bar{D} \equiv \sum_{i \in Y} \sum_j \bar{x}_{ij} \tag{26}$$

The debt incurred by the young households are used to purchase property from the old households. The old households have a lower valuation for housing than the young households, and there is a mutually acceptable transfer of housing from the old households to the young households.

Abstracting away from the detailed matching and bargaining procedures that lead to the transaction, assume that the marginal transaction price v is an increasing function of the stock of housing that ends up in the hands of the young households, as illustrated in figure 10. Since the young must finance their purchase of housing by borrowing from the financial intermediaries, assume that the stock of housing held by the young is an increasing function of $\bar{D}$. An increase in $\bar{D}$ has two related effects. One the one hand, there is an increase in the corresponding market value of debt $D \equiv \sum_{i \in Y} \sum_j x_{ij}$. In
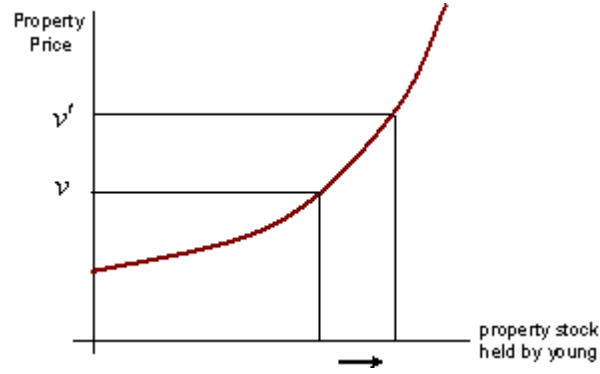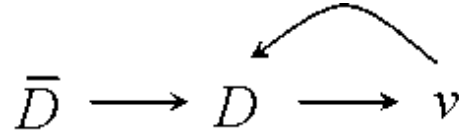
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Figure 10: Price of housing

Figure 11: Effect of increased debt

addition, there is a second-round effect of an increase in $\bar{D}$, since an increase in the price of housing increases the asset value of the young households, which raises the market value of claims against them. Figure 11 illustrates the direct and indirect effect of an increase in lending. An increase in $\bar{D}$ has a direct effect on D, but there is also an indirect effect that raises D further resulting from the boom in house prices. The informal description given above can be made more precise by appealing to our comparative statics result (theorem 5), and the adjustment depicted in figure 8. In our example, price of housing v is an increasing function of $\bar{D}$, so that an increase in debt is associated with an increase in v. In terms of figure 8, the adjustment implies a shift in the "northeast" direction.

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The market value of debt of the young households increase both due to the upward shift in $\bar{D}$ itself, but also due to the increase in v. If the increase in v is large enough, then the increase in the market value of debt may outweigh the increase in the face value of debt, leading to a narrowing of the spread on the loans. Thus, it is possible to have a narrowing of spreads even as debt increases.

Figure 10 illustrates an important feature of property wealth in our example. The marked-to-market value of the housing stock increases in proportion to the price of the marginal traded property. However, the marked-to-market value of the housing stock is a poor guide to the liquidation value of the housing stock it a substantial chunk of the housing stock is put up for sale, as in a macroeconomic downturn.

4.2 Bank Equity

We can also examine separately the impact on the balance sheets of the banks themselves. From our comparative statics result (theorem 5), an increase in v raises the *market value D* of the loans to the young households. Since D represents the assets of the banking sector as a whole, we know from lemma 2 that the equity value of the financial sector as a whole increases. Denote by E the market value of the equity of the banking sector. Hence, we can plot E as an upward-sloping function of D as in figure 12. If the increase in E erodes the banks' leverage to take θ outside the acceptable leverage set, then those banks that have seen their leverage erode too much will attempt to restore their leverage by increasing their lending to the household sector. Banks attempt to maintain a constant leverage as measured by the E/D ratio, by raising the face value of loans sufficiently. Such a policy by the banks defines D as a function of E. The inverse function generated by this
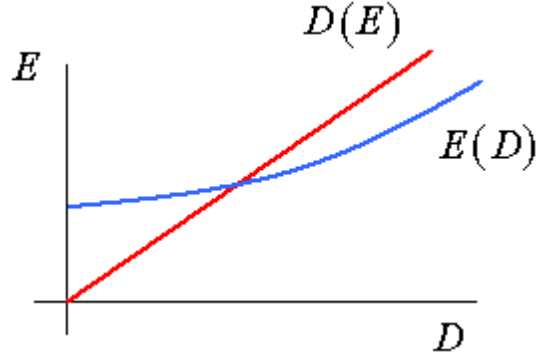
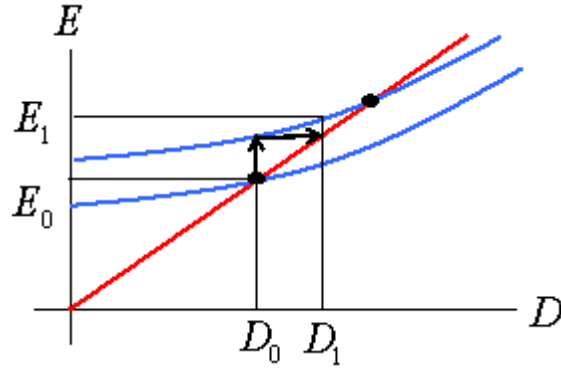Figure 12: Joint determination of leverage and bank assets

Figure 13: Comparative statics of a lending boom

increasing relationship can then be plotted in (D, E) space, as shown by line in figure 12 labelled as $D(E)$. The intersection of the two curves gives the combination of D and E that is consistent both with consistent pricing of debt within the system and constant leverage of the banks.

Figure 13 illustrates the comparative statics of an upward shift of the $E(D)$ curve that results from banks raising additional equity. The new intersection point is associated with higher debt D as well as higher bank equity E. Since our model is static, the shift should be interpreted as a

one-step jump. However, it is instructive to decompose the comparative statics analysis into a stepwise adjustment. The initial shift in the $E(D)$ curve leads to the vertical movement from (D_0, E_0) to (D_0, E_1), as indicated by the upward-pointing arrow. But the upward adjustment in bank equity leads to greater lending, and so there is an adjustment to the right, to the point (D_1, E_1). In turn, the greater lending leads to a further rise in property price and higher bank equity, and so on.

5 Avenues for Future Research

The focus of the paper has been on the liquidity of the financial system as a whole, where "liquidity" refers to the funding conditions for current and potential borrowers. For existing borrowers, rising asset prices strengthen their balance sheets making them more creditworthy. For potential borrowers, the stronger balance sheets of financial intermediaries play to their advantage, since these financial intermediaries are more willing to extend new credit, and extend credit on easier terms. The simplicity of the framework holds some promise for several lines of research.

- The framework is well-suited to pricing complex debt instruments such as collateralized debt obligations (CDOs), since CDOs are obligations that are backed by claims on others.

- Our framework is well-suited to quantitative analysis of risks (such as value at risk calculations) that take account of endogenous changes in asset prices and the feedback effects that result.

- The seniority structure can be examined separately in the pricing of securitized credit products. The system perspective will give some

assurance that the correlation structure arising from payoff interactions will be captured fully.

There are also broader conceptual issues. A number of trends have served to sharpen the effects outlined in the paper. Rajan (2005) notes how financial development has been made possible by greater use of "arms length" relationships in financial contracts, but such arms length relationships also imply greater delegation. The shortened decision horizons entailed by delegation increase the potency of market prices in causing feedback (Shin (2005)).

The key effect is the feedback between strength of balance sheets (as implied by x) and the level of debt (as given by $\bar{x}$). Strong balance sheets induce banks to increase their lending. In turn, increased lending raises property prices, leading to stronger balance sheets.

The reason why banks would increase their lending in the face of stronger balance sheets would be intimately tied to the short-term incentives facing the banks' management. Stronger balance sheets imply a larger marked-to-market equity for the bank. Suppose for the moment that shareholder value is measured in terms of return on marked-to-market equity (I return to this below). The more conscious is a bank's management to shareholder returns, the greater will be the incentive to react to the erosion of leverage by trying to restore leverage to some extent. The trend in recent years towards improved corporate governance through greater transparency, greater accountability to shareholders and greater use of incentive schemes tied to the share price will all strengthen the motives of the management to restore leverage.

The feedback from increased debt (given by $\bar{x}$) to stronger balance sheets (given by x) has to do with how quickly the increased debt translates into higher asset prices and how quickly the increase in asset prices is reflected in visibly stronger balance sheets. Here, marking to market is key. For

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the United States, the prevalence of mortgage-backed instruments as the prime source of finance for the property sector means that this pre-condition is already in place. For those economies that rely on bank lending, the accounting regime will be important. When assets and liabilities are marked to market continuously, the accounting numbers mirror the underlying market prices immediately.

Accounting numbers serve an important certification role in financial markets. They are audited numbers that carry quasi-legal connotations in bringing the management to account. As such, accounting numbers serve as a justification for actions. If decisions are made not only because you believe that the underlying fundamentals are right, but because the accounts give you the external validation to take such decisions, then the accounting numbers take on great significance. Plantin, Sapra and Shin (2004, 2005) discuss these and other dimensions of the tradeoff between mark-to-market accounting and historical cost accounting.

To date, a thorough-going approach to marking to market has affected only a small segment of the financial sector - notably, hedge funds and other hedge fund-like institutions that deal mainly with marketable claims. Marking to market has been limited by the lack of reliable prices in deep and liquid markets for many assets. Loans, for instance, have not been traded in large enough quantities to mark the loan book to market in a reliable way. However, all this is about to change. The advent of deep markets in credit derivatives has removed the practical barriers to marking loans to market. The price of a credit default swap can be used to price a "notional" loan corresponding to the standardised characteristics of such a loan, much like the price of a futures contract on a bond, which indicates the price of a notional bond. Feasibility is no longer a hurdle to a thorough-going application of

marking to market (or will not remain a hurdle for long).

It can be argued that mark-to-market accounting has already had a far-reaching impact on the conduct of market participants through those institutions that deal mainly with tradeable securities, such as hedge funds and the proprietary trading desks of investment banks. However, even these developments will pale into insignificance to the potential impact of the marking to market of loans and other previously illiquid assets.

Accounting numbers, such as Return on Equity (ROE) have traditionally made reference to book equity (accumulated value of past profit) rather than the market price of equity claims. However, this distinction is becoming increasingly less relevant. The recent trend (as prescribed by the new accounting standards) is to feed any capital gains to the profit and loss account (the income statement) so that capital gains and losses will be reflected immediately on the balance sheet. The potential for feedback is thus enhanced, and the effects outlined in the paper take on greater significance.

Appendix

This appendix gives proofs of lemma 1 and lemma 2. Let us employ the "hat" notation "^" to denote the realized value of a random variable at the maturity date T. Thus, $\hat{x}_{ij}$ is the realized payout at date T of i's debt of seniority j, and $\hat{a}_i$ is the realized value of i's assets at date T where all claims are valued at their cash payoffs. At the maturity date, the problem is one of finding the bankruptcy solution that respects seniority. This problem was solved by Eisenberg and Noe (2001) in a similar context, and a modified version of their argument gives us a well-defined bankruptcy solution here, also. Begin by noting that the seniority structure of i's debt implies that the realized value of debt $\hat{x}_{jk}$ is a non-decreasing function of $\hat{a}_j$ given by

$$\min\left\{\bar{x}_{jk}, \max\left\{0, \hat{a}_j - \sum_{h=1}^{k-1} \bar{x}_{jh}\right\}\right\} \tag{27}$$

Denote by $\hat{x}$ the profile of realized payoffs from all debt instruments in the system. Realized asset values are functions of $\hat{x}$, as well as the realized values of endowments and the fundamental asset. There is thus a function G that is non-decreasing in $\hat{x}$ such that

$$\hat{x} = G\left(\hat{x}, \bar{x}, \hat{w}, \hat{v}\right) \tag{28}$$

By Tarski's fixed point theorem (lemma 3), there are fixed points $\hat{x}$ and $\hat{x}'$ such that $\hat{x} \leq \hat{x}'$, and any fixed point is bounded below and above by $\hat{x}$ and $\hat{x}'$ respectively.

The claim is that $\hat{x} = \hat{x}'$, so that there is a unique solution to (28). To see this, suppose that $\hat{x} \neq \hat{x}'$. Then, each component of $\hat{x}'$ is at least as large as the respective component in $\hat{x}$, and there is at least one component that is *strictly* larger in $\hat{x}'$. Denote by $\hat{e}_i$ the realized value of i's equity under

$\hat{x}$, and denote by $\hat{e}'_i$ the realized value of i's equity under $\hat{x}'$. Then for all investors i,

$$\hat{e}_i = \max\left\{\hat{a}_i - \sum_j \bar{x}_{ij}\right\} \leq \max\left\{\hat{a}'_i - \sum_j \bar{x}_{ij}\right\} = \hat{e}'_i \qquad (29)$$

Summing across investors, and making use of our assumption that some portion of each debt instrument is held by at least one unleveraged investor, we can conclude that there is at least one unleveraged investor that has a strictly larger equity value under $\hat{x}'$ than under $\hat{x}$. Hence,

$$\hat{e} = \sum_i \hat{e}_i < \sum_i \hat{e}'_i = \hat{e}' \qquad (30)$$

However, from the composition of assets, we know that

$$\begin{aligned}
\sum_i \hat{e}_i &= \sum_i \hat{a}_i - \sum_i \sum_j \bar{x}_{ij} \\
&= \sum_i (\hat{w}_i + \hat{v}y_i) \\
&= \sum_i \hat{a}'_i - \sum_i \sum_j \bar{x}_{ij} \\
&= \sum_i \hat{e}'_i \qquad (31)
\end{aligned}$$

which contradicts (30). Hence, contrary to the supposition, we must have $\hat{x} = \hat{x}'$. Thus, there is a unique solution to the bankruptcy problem at the ex post stage.

Next, denote by $E(.|v)$ the expectation at date 0 with respect to the density over realized outcomes at the maturity date T when the price of the fundamental asset is v at date 0. The joint density over realized outcomes is defined by the joint density over the random endowments $\{\hat{w}_i\}$ and the realized value of the fundamental asset $\hat{v}$ all conditional on v. Since all investors are risk-neutral,

$$x_{ij} = E\left(\hat{x}_{ij}|v\right) \qquad (32)$$

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Meanwhile, each $\hat{x}_{ij}$ is a function of $\hat{a}_i$, and the density over all asset values $\{\hat{a}_i\}$ conditional on v is well-defined. Given risk-neutrality,

$$a_i = E\left(\hat{a}_i|v\right) \equiv z_i\left(v\right) \tag{33}$$

which defines a one-to-one correspondence z_i between a_i and v. Using this one-to-one correspondence, we can define f_{ij} such that

$$
\begin{aligned}
f_{ij}\left(a_i, \theta\right) &\equiv E\left(\hat{x}_{ij}|z_i^{-1}\left(a_i\right)\right) \\
&= E\left(\hat{x}_{ij}|v\right) \\
&= x_{ij}
\end{aligned}
\tag{34}
$$

where z_i^{-1} is the inverse of z_i. For $v' \geq v$, the conditional distribution $G_i\left(.|v'\right)$ over realizations $\hat{a}_i$ dominates the conditional density $G_i\left(.|v\right)$ in the sense of first degree stochastic dominance. Since $\hat{x}_{ij}$ is non-decreasing in $\hat{a}_i$, f_{ij} is non-decreasing in $z_i^{-1}\left(a_i\right)$, and hence non-decreasing in a_i. We thus have lemma 1.

Finally, lemma 2 follows from a very similar argument. Since investors are risk-neutral,

$$
\begin{aligned}
e_i &= E\left(\hat{e}_i|v\right) \\
&= E\left(\hat{e}_i|z_i^{-1}\left(a_i\right)\right)
\end{aligned}
\tag{35}
$$

As noted above, if $v' \geq v$, then the conditional distribution $G\left(.|v'\right)$ over realizations $\hat{a}_i$ dominates the conditional density $G\left(.|v\right)$ in the sense of first degree stochastic dominance. Since $\hat{e}_i$ is a non-decreasing function of $\hat{a}_i$, (35) is non-decreasing in a_i. In other words, market equity e_i is non-decreasing in the asset value a_i, as claimed in lemma 2.

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