Collateral, Financial Intermediation, and the Distribution of Debt Capacity

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Who Has Debt Capacity?

Research question

- Which investors have debt capacity available to take advantage of investment opportunities due to temporarily low asset prices?

Main Results

Three main substantive results

- Result 1: Productive and/or poorly capitalized borrowers may exhaust their debt capacity rather than conserve it.
  - Cost of conserving debt capacity is opportunity cost of foregone investment.

- Result 2: Borrowers who exhaust debt capacity may be forced to contract when asset prices and cash flows are low.
  - Capital less productively deployed in such times.

- Result 3: Intermediary capital may be more scarce in such times forcing borrowers to contract by more.

Main Results (Cont'd)

Two main theoretical results

- Result 4: Collateral constraints due to limited enforcement
  - Link between two classes of models

- Result 5: Model of financial intermediaries as collateralization specialists
  - Role for intermediary capital

Additional Results

Additional implications of the model

- Debt capacity
  - Definition
  - Endogenous and jointly determined with investment
- Role of long term debt
- Implementation with loan commitments
- Higher collateralizability may make contraction more severe
  - "Financial innovation"
- Dynamics of minimum down payment requirements
  - "Lending standards"

Abridged Literature Review

Dynamic models of collateral

- Kiyotaki and Moore (1997)
  - ... motivated by incomplete contracting (à la Hart and Moore 1994)

- Kehoe and Levine (1993)
  - ... motivated by limited contract enforcement/limited commitment

Models of financial intermediary capital

- Holmström and Tirole (1997) and Diamond and Rajan (2000)

Model

Borrowers

- 3 dates: 0, 1, and 2. Let $\mathcal{T} \equiv \{1, 2\}$
- Continuum of borrowers with measure 1. Types $n \in \mathcal{N}$. Density of type n: $\psi(n)$. Suppress types whenever possible.
- Preferences

$$E\left[d_0 + \sum_{t \in \mathcal{T}} d_t\right]$$

- Endowment w_0 at time 0 and no other endowment.
- Return $E[Af(k)]$ in cash flow at time $t+1$ for investment of k at time t. Capital depreciates at rate δ.

Lenders

- Risk neutral and discount the future at rate $\beta < 1$.
- Large endowment of funds in all dates and states.
- Cannot operate the technology.
- ... willing to lend in state-contingent way at expected return $R \equiv 1/\beta > 1$.

Model (Cont'd)

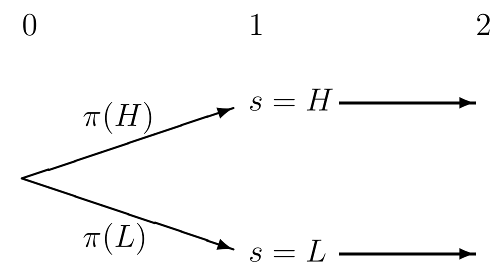
Collateral constraints due to limited enforcement

- Limited enforcement
  - ... agents can abscond with cash flows and fraction $1-\theta$ of capital (default); agents not excluded from borrowing
- ... implies collateral constraints
  - ... agents can borrow up to θ times the resale value of capital against each state.

Price of capital

- Consumption goods can be transformed into capital goods (and vice versa) at rate ϕ_0 and $\phi_t(s)$ at time 0 and t in state s, $s \in \mathcal{S} \equiv \{H, L\}$.

Figure 1: Time Line



Model with Limited Enforcement

Contracting problem with limited enforcement

- $$\max_{\{d_0, d_t(s), l_0, l_1(s), k_0, k_1(s), b_{t-1}(s)\}_{s\in\mathcal{S}, t\in\mathcal{T}}} d_0 + \sum_{s\in\mathcal{S}} \pi(s) \left\{ \sum_{t\in\mathcal{T}} d_t(s) \right\} \tag{1}$$

  subject to the budget constraints at date 0, 1, and 2,

  $$\begin{aligned} w_0 + l_0 &\geq d_0 + \phi_0 k_0 && (2) \\ A_1(s) f(k_0) + \phi_1(s) k_0 (1-\delta) + l_1(s) &\geq d_1(s) + \phi_1(s) k_1(s) + R b_0(s), \ \forall s, && (3) \\ A_2(s) f(k_1(s)) + \phi_2(s) k_1(s)(1-\delta) &\geq d_2(s) + R b_1(s), \quad \forall s \in \mathcal{S}, && (4) \end{aligned}$$

  lender's ex ante participation constraint at date 0,

  $$\sum_{s\in\mathcal{S}} \pi(s) \left\{ \sum_{t\in\mathcal{T}} R^{-(t-1)} b_{t-1}(s) \right\} \geq l_0 + \sum_{s\in\mathcal{S}} \pi(s) R^{-1} l_1(s), \tag{5}$$

  enforcement constraints at date 1 and 2,

  $$\begin{aligned} d_1(s) + d_2(s) &\geq \hat{d}_1(s) + \hat{d}_2(s), \qquad \forall s \in \mathcal{S}, && (6) \\ d_2(s) &\geq A_2(s) f(k_1(s)) + \phi_2(s) k_1(s)(1-\theta)(1-\delta), \ \forall s \in \mathcal{S}, && (7) \end{aligned}$$

  and non-negativity constraints on dividends and capital,

  $$d_0 \geq 0, \quad d_t(s) \geq 0, \quad k_0 \geq 0, \quad k_1(s) \geq 0, \qquad \forall s \in \mathcal{S} \text{ and } t \in \mathcal{T}, \tag{8}$$

  where $\{\hat{d}_t(s)\}$ dividends that the borrower can achieve after default.

Model with Limited Enforcement (Cont'd)

Outside option: No exclusion after default

- $\{\hat{d}_t(s), \hat{k}_1(s), \hat{b}_1(s)\}_{t\in\mathcal{T}}$ maximize

$$\sum_{t\in\mathcal{T}} d_t(s) \tag{9}$$

subject to

$$\underbrace{A_1(s)f(k_0) + \phi_1(s)k_0(1-\theta)(1-\delta)}_{\text{borrower's net worth after default}} + b_1(s) \ \geq\ d_1(s) + \phi_1(s)k_1(s), \tag{10}$$

(4), (7), and (8).

- Difference to Kehoe and Levine (1993): production and **outside option**
  - ... they assume exclusion from intertemporal trade after default.

- **Recursive structure:** agent's problem after default is identical to continuation problem when agent keeps promises, except net worth is different.

- Similar outside option in Lustig (2007) (endowment economy) and Lorenzoni and Walentin (2007) (constant returns to scale).

Role for Long Term Debt?

Irrelevance

- **Lemma 1** *Considering state-contingent one period debt is sufficient.*
- No gains from long term contracting.
  - If the borrower promises to pay $Rb_0(s)$ in state s at time 1, he receives an amount of funds $\pi(s)b_0(s)$ at time 0.
- Intuition:
  - Enforcement constraints restrict credible promises to payments with present value less than or equal to the value of capital that borrower cannot abscond with.
  - Any contract satisfying this restriction can be implemented with one period debt contracts.
- In contrast:
  - Long term contracts not irrelevant with outside option as in Kehoe and Levine (1993).
  - No borrowing at all with outside option as in Bulow and Rogoff (1989).

Collateral Constraints due to Limited Enforcement

Equivalence

- **Lemma 2** *Enforcement constraints (6) and (7) are equivalent to collateral constraints*

$$\begin{aligned} \phi_1(s)\theta k_0(1-\delta) &\geq Rb_0(s), \qquad \forall s \in \mathcal{S}, \qquad (11) \\ \phi_2(s)\theta k_1(s)(1-\delta) &\geq Rb_1(s), \qquad \forall s \in \mathcal{S}. \qquad (12) \end{aligned}$$

- Advantages:
  - Simple decentralization of optimal dynamic lending contract
    - ... implementation with state contingent secured loans
    - ... equilibrium with collateral constraints (similar to constraints in Kiyotaki and Moore (1997), but state contingent)
      - related: equilibrium with solvency constraints (Alvarez and Jermann (2000))
  - Constraint set in problem with collateral constraints is convex.
- Notion of (state contingent) **debt capacity**: $R^{-1}\phi_1(s)\theta(1-\delta)$

Role for Loan Commitments?

Definition

- Binding agreement to provide loan at specific date for fee paid up front.

Why take out a loan commitment?

- So far all loans have zero NPV when extended, that is $l_1(s) = b_1(s)$ and

$$NPV_1(s) = -l_1(s) + R^{-1}Rb_1(s) = 0.$$

  Then loan commitments are unnecessary (and fees are zero).

- Suppose for fee $c_0(s) > 0$ at time 0, the lender agrees to provide loan $l_1(s) > b_1(s)$ in state s at time 1 such that (due to competitive pricing)

$$c_0(s) + \pi(s)R^{-1}\{-l_1(s) + R^{-1}Rb_1(s)\} = 0.$$

- Suppose borrower conserves debt capacity $b_0(s) < R^{-1}\phi_1(s)\theta k_0(1-\delta)$.
  - Alternative and equivalent implementation: loan commitment for $l_1(s) \equiv b_1(s) + R(\hat{b}_0(s) - b_0(s))$ where $\hat{b}_0(s) \equiv R^{-1}\phi_1(s)\theta k_0(1-\delta)$.

  Loan commitments are **equivalent** to saving contingent debt capacity.

- Borrowers, who choose to exhaust debt capacity, choose not to arrange loan commitments!

Dynamics of Minimum Down Payments

Minimum down payment requirements

- Define minimum down payments $\wp_0$ (and similarly $\wp_1(s)$) as

$$\wp_0 \equiv \phi_0 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s) \phi_1(s) \theta(1-\delta)$$

Expected capital appreciation affects minimum down payment

- Minimum down payment as fraction of the price of capital is low when the price of capital is expected to rise, for example, when

$$\sum_{s \in \mathcal{S}} \pi(s) \phi_1(s) / \phi_0$$

  is high.

- Consistent with anecdotal evidence on minimum down payment requirements (or **lending standards**).

Distribution of Debt Capacity

Who conserves debt capacity?

- Simplifying assumptions
  - Define return $R_1(k_0, s)$ as $R_1(k_0, s) \equiv \frac{A_1(s)f'(k_0)+\phi_1(s)(1-\theta)(1-\delta)}{\wp_0}$ (and similarly $R_2(k_1(s), s)$).
  - With constant returns to scale, $f(k) = k$, $f'(k) = 1$, $R_1(s) \equiv R_1(k_0, s)$ (and similarly $R_2(s)$).
  - **Assumption 1** $R_2(s) > R$, $\forall s \in \mathcal{S}$.
- **Proposition 1** ***Productive borrowers exhaust their debt capacity***, *that is, if* $\sum_{s\in\mathcal{S}} \pi(s)R_1(s)R_2(s) > \max_s\{RR_2(s)\}$, *then* $k_0 = \frac{1}{\wp_0}w_0$ *and* $V_0(w_0) = \sum_{s\in\mathcal{S}} \pi(s)R_1(s)R_2(s)w_0$. *Less productive borrowers conserve their net worth, that is, if the condition is not met,* $k_0 = 0$, $k_1(s') = \frac{R}{\pi(s')}w_0$, *and* $V_0(w_0) = RR_2(s')w_0$, *where* s' *such that* $R_2(s') = \max_s\{R_2(s)\}$.
- With constant returns, either exhaust debt capacity or conserve it all.

Relative Contraction of Productive Firms

Can firms contract?

- **Proposition 2** *For open set of parameters, borrowers are "forced to" contract, that is,* $\exists s \in \mathcal{S}$ *such that* $k_1(s) < k_0$*.*
- When $k_0 > 0$, then

$$k_1(s) = \left(\frac{A_1(s) + \phi_1(s)(1-\theta)(1-\delta)}{\phi_1(s) - R^{-1}\phi_2(s)\theta(1-\delta)} \right) k_0$$

- Thus, **productive borrowers contract**
  - ... when cash flows ($A_1(s)$) are sufficiently low.

Effect on average productivity

- More productive firms may contract, while less productive firms expand.
  - Productive firms exhaust debt capacity & have low cash flow/net worth.
  - Less productive firms conserve debt capacity and expand.
- Lower average productivity in such times.

Effect of Collateralizability on Contraction

Leverage and severity of contraction

- **Proposition 3** *With higher collateralizability, borrowers, who exhaust debt capacity, may be forced to contract by more. Suppose the parameters are as in Proposition 2; then* $\frac{\partial}{\partial\theta}\left(\frac{k_1(s)}{k_0}\right) < 0$ *as long as* $\frac{\phi_1(s)}{\phi_2(s)} > \frac{1}{R}\frac{k_1(s)}{k_0}$.

Two effects of leverage (higher θ)

- Less "free net worth" since able to pledge larger fraction of funds at time 0.
- Lower minimum down payment requirement due to greater ability to borrow going forward.
- Opposite direction, but as long as price of capital not too much higher at time 2, first effect dominates.
  - Higher leverage due to **higher pledgeability leads to more severe contraction** in capital.
  - "Financial innovation."

Role of Borrower Net Worth

Effect of borrower net worth on debt capacity

- Simplifying assumptions
  - **Assumption 2** $R_2(k_1(s), s) > R$, $\forall s \in \mathcal{S}$.
  - **Assumption 3 (i)** $R_2(k, H) < R_2(k, L)$, *for* k *in the relevant range; and* **(ii)** $k_1(H) > k_1(L)$, *where* $k_1(s) \equiv (A_s(s) f(w_0/\wp_0) + \phi_1(s) w_0/\wp_0(1-\theta)(1-\delta))/\wp_1(s)$ *for* w_0 *in the relevant range.*

- **Borrowers conserve some debt capacity** for the low state as long as they are not too constrained.

- **Proposition 5** *Suppose Assumption 3 holds. Then there exist* $\underline{w}_0 < \bar{w}_0$ *such that* **(i)** *for* $w_0 \le \underline{w}_0$, $\lambda_0(s) > 0$, $\forall s \in \mathcal{S}$, $k_0 = \frac{1}{\wp_0} w_0$, *and* $k_1(s) = \frac{1}{\wp_1(s)}(A_1(s) f(k_0) + \phi_1(s) k_0 (1-\theta)(1-\delta))$*;* **(ii)** *for* $\underline{w}_0 < w_0 < \bar{w}_0$, $\lambda_0(H) > 0$ *and* $\lambda_0(L) = 0$*; and* **(iii)** *for* $\bar{w}_0 \le w_0$, $\lambda_0(s) = 0, \forall s \in \mathcal{S}$, $R_2(k_1(H), H) = R_2(k_1(L), L)$, *and* $R = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s)$.

Financial Intermediation

Financial intermediaries as "collateralization specialists"

- **Financial intermediaries** ...
  - ... are lenders with particular ability to collateralize claims, in particular, ability to reduce amount of capital borrowers can abscond with to $1-\theta^i$ ($\theta^i > \theta$) (similar to monitoring in Diamond (2007))
  - ... have limited capital w_0^i,
  - ... and are themselves subject to the same limited enforcement constraints.

- **Role for intermediary capital**
  - Intermediary capital required to finance extra $\theta^i - \theta$ since cannot in turn borrow against that amount due to limited enforcement constraints.
  - Dynamic model of intermediary capital; net worth of intermediaries is a state variable.

Direct vs. Intermediated Finance

Borrower's problem

- For exposition, one period problem here; borrower can borrow in state contingent way from direct lenders and financial intermediaries.

- Direct lenders lend to intermediaries, but, to simplify, notation as if providing direct finance.

- $$\max_{\{d_0,d_1(s),k_0,b_0(s),b_0^i(s)\}_{s\in\mathcal{S}}} d_0 + \sum_{s\in\mathcal{S}} \pi(s)d_1(s)$$

  subject to budget constraints,

  $$\begin{aligned} w_0 + \sum_{s\in\mathcal{S}} \pi(s)\{b_0(s) + b_0^i(s)\} \ &\geq \ d_0 + \phi_0 k_0 \\ A_1(s)f(k_0) + \phi_1(s)k_0(1-\delta) \ &\geq \ d_1(s) + Rb_0(s) + R_0^i b_0^i(s), \qquad \forall s \in \mathcal{S}, \end{aligned}$$

  two sets of collateral constraints,

  $$\begin{aligned} \phi_1(s)\theta k_0(1-\delta) \ &\geq \ Rb_0(s), \qquad \forall s \in \mathcal{S}, \\ \phi_1(s)\theta^i k_0(1-\delta) \ &\geq \ Rb_0(s) + R_0^i b_0^i(s), \qquad \forall s \in \mathcal{S}, \end{aligned}$$

  and $d_0 \geq 0$, $d_1(s) \geq 0$, $k_0 \geq 0$, $b_0^i(s) \geq 0$ $\forall s \in \mathcal{S}$ and $t \in \mathcal{T}$.

Dynamics with Limited Intermediary Capital

Limited intermediary capital affects spreads

- Assumption
  - **Assumption 4** $\underline{nl}_1^i(L) > 0 > \underline{nl}_1^i(H)$.
  - Loan demand from borrowers who conserve debt capacity potentially important.

- Highest spread between intermediated and direct finance in state L:

  Proposition 8 *Suppose Assumption 4 holds. Then* $\exists \varepsilon > 0$ *such that* $\forall w_0^i < \underline{w}_0^i$ *and* $\varepsilon > \underline{w}_0^i - w_0^i$, $R^i \equiv R_0^i(H) = R_1^i(L) > R$, *and* $R_0^i(L) = R_1^i(H) = R$.

- Define time 0 spread by $\varsigma_0 \equiv \sum_{s \in \mathcal{S}} \pi(s) R_0^i(s) - R$; time 1 spread in state s by $\varsigma_1(s) \equiv R_1^i(s) - R$.

  Corollary 3 *Under the conditions of Proposition 8,* $\varsigma_1(L) > \varsigma_0 > \varsigma_1(H) = 0$.

Impact of Limited Intermediary Capital on Borrowers

Effect on severity of contraction

- The scarcer intermediary capital, the more borrowers will contract in the state in which intermediary capital is scarce.

- **Proposition 9** *Suppose* w_0^i *is as in Proposition 8. If* s *such that* $\underline{nl}_1^i(s) > 0 > \underline{nl}_1^i(s')$, $s' \neq s$, *then* $\frac{d}{dw_0^i}\frac{k_1^g(s)}{k_0^g} > 0$.

Two reasons why productive borrowers contract

- First: low cash flow and low net worth in state L.
- Second: cost of intermediated funds increases in state L.

Conclusion

Distribution of debt capacity

- Productive/less well capitalized borrowers likely exhaust debt capacity.
- Borrowers who exhaust debt capacity may be forced to contract.
- Scarce intermediary capital may force borrowers to contract by even more.

Outline

Literature

- Models of collateral and debt capacity

Model

- Collateral constraints due to limited enforcement

Results

- Role for long term debt?
- Loan commitments and contingent financing
- Determinants of minimum down payment requirements
- Productivity and distribution of debt capacity
- Implications for firm investment
  - ... effect of collateralizability
- The role of borrower net worth
- Financial intermediation

Conclusion

Literature

Models of collateral

- ... motivated by incomplete contracting (à la Hart and Moore 1994)
  - Kiyotaki and Moore (1997)
  - Krishnamurthy (2003), Iacoviello (2005), Eisfeldt and Rampini (2007, 2008), Lorenzoni and Walentin (2007)
- ... motivated by limited contract enforcement/limited commitment
  - Kehoe and Levine (1993, 2001, 2006)
  - Macroeconomic applications: Kocherlakota (1996), Ligon, Thomas, and Worrall (1997), Kehoe and Perri (2002, 2004), Krueger and Uhlig (2006)
  - Asset pricing applications: Alvarez and Jermann (2000, 2001), Lustig (2007), Lustig and van Nieuwerburgh (2007)
  - Corporate finance applications: Albuquerque and Hopenhayn (2004), Cooley, Marimon, and Quadrini (2004)
- ... motivated by private information of cash flows
  - Diamond (1984), Lacker (2001), Rampini (2005).

Literature (Cont'd)

Other models of collateral

- Barro (1976) ... affects borrowing rate
- Bester (1985) ... eliminates credit rationing
- Stulz and Johnson (1992) ... reduces underinvestment problem
- Rajan and Winton (1995) ... incentives to monitor
- Dubey, Geanakoplos, and Shubik (2005) and Geanakoplos (1997) ... renders market more market

Models of debt capacity

- Shleifer and Vishny (1992)
- Financial intermediation: Holmström and Tirole (1997), Bolton and Freixas (2000), Cantillo (2004), and Diamond and Rajan (2000)
- Effects on asset prices: Allen and Gale (1998, 2004), Gorton and Huang (2004), and Acharya, Shin, and Yorulmazer (2007)
- Agency models of dynamic firm financing: Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007a, 2007b), DeMarzo, Fishman, He, and Wang (2007), Atkeson and Cole (2008)

Effect of Asset Prices on Contraction

How do asset prices affect contraction?

- **Proposition 4** $\frac{\partial}{\partial \phi_1(s)} \left(\frac{k_1(s)}{k_0} \right) < 0.$

Two effects of higher price of capital at time 1 in state s

- Higher "free net worth"
- Higher minimum down payment requirement
- Second effect dominates first:
  - The higher price of capital, the more capital contracts.
- Key: Higher net worth requirements!

Financial Intermediary's Problem

Model with representative financial intermediary

- Given $R_0^i(s)$, $\forall s \in \mathcal{S}$, the intermediary solves

$$\max_{\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}} d_0^i + \sum_{s \in \mathcal{S}} \pi(s) R^{-1} d_1^i(s)$$

subject to

$$w_0^i \geq d_0^i + \sum_{s \in \mathcal{S}} \pi(s) l_0^i(s)$$

and

$$R_0^i(s) l_0^i(s) \geq d_1^i(s), \qquad \forall s \in \mathcal{S},$$

as well as $d_0^i \geq 0$, $d_1^i(s) \geq 0$, $l_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$, where $l_0^i(s)$ is the amount that the intermediary lends against state s.

Comments

- Simplified 1-period problem; clearly $R_0^i(s) \geq R$, $\forall s \in \mathcal{S}$.
- **Lemma 5** $R_0^i(H) = R_0^i(L) \equiv R_0^i$ *without loss of generality.*

Capital Structure: Intermediated vs. Direct Finance

Cross section of capital structure

- Most productive/most constrained firms borrow from financial intermediaries.

- **Proposition 6** *Suppose* $R_0^i > R$*. If* $R \geq \sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1-\delta))/\phi_0$*, then* $k_0 = 0$ *and* $V(w_0) = Rw_0$*; otherwise, if* $R_0^i \geq \mu_0^* \equiv \sum_{s \in \mathcal{S}} \pi(s) R_1(s)$*, then* $k_0 = (1/\wp_0)w_0$ *and* $V(w_0) = \mu_0^* w_0$*, and if* $R_0^i < \mu_0^*$*, then* $k_0 = (1/\bar{\wp}_0)w_0$ *and* $V(w_0) = \bar{\mu}_0^* w_0$ *where* $\bar{\wp}_0$ *and* $\bar{\mu}_0^*$ *are defined in the proof.*