Forecasting Cross-Sections of Frailty-correlated Default

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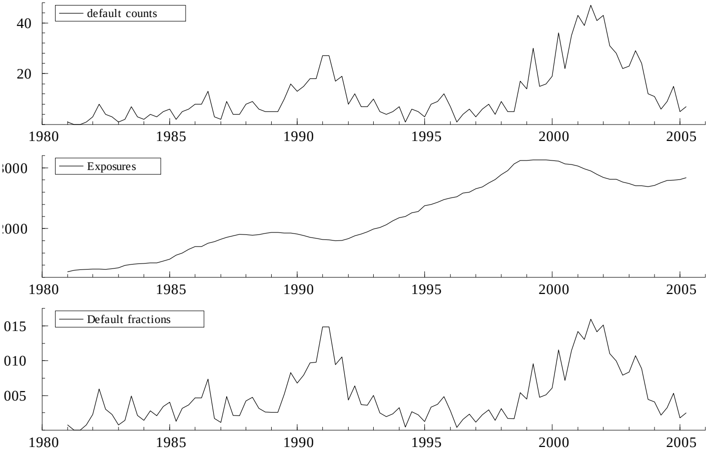
Introduction: What is done?

Know: Defaults cluster over time. Observed aggregate default fractions vary over time. A lot higher in 'bad times' than in 'good times'.



Introduction: What is done?

- We propose a novel econometric model for estimating and forecasting cross sections of time-varying conditional default probabilities. These can be used as inputs for one-year ahead VaR-levels and stress-testing loan portfolios.
- The model combines the non-Gaussian panel data model of Koopman and Lucas (2008) with the main features of Stock and Watson's (2002, 2005) approximate dynamic factor model.
- New feature: The model captures the systematic variation in corporate default counts across industry, rating, and age groups by using both unobserved components as well as dynamic factors from a large panel of selected macroeconomic and financial data.

Introduction II: unobserved risk factors

- Unobserved risk factors matter. A few observables are not enough, cf. Das, Duffie, Kapadia, Saita (2007).
- Econometric problem: No analytic expression for key distributions, such as $p(UC|\text{default counts})$ and log-likelihood.
- Simulation based techniques required. Duffie, Eckner, Guillaume, Saita (2006) use Simulated EM with Gibbs Sampling. Wendin and McNeil (2007) fully Bayesian.
- We remain in ML framework. Use simulation methods based on Importance Sampling derived for non-Gaussian models in State Space Form, see Durbin and Koopman (1997, 2001, 2002).
- Usually not many macros due to dependence on simulation methods. Include MANY macros by extracting what they have 'in common'.

What does the combined model look like?

$$
\begin{aligned}
y_{jt} | f_t^{uc}, F_t &\sim \text{Binomial}(k_{jt}, \Pi_{jt}) \\
\Pi_{jt} &= (1 + e^{-\theta_{jt}})^{-1} \\
\theta_{jt} &= \lambda_j + \beta_j f_t^{uc} + \gamma_j' F_t \\
f_t^{uc} &= \phi f_{t-1}^{uc} + \sqrt{1-\phi^2}\eta_t, \quad \eta_t \sim \text{NID}(0,1) \\
x_{it} &= \Lambda_i F_t + e_t
\end{aligned}
$$

$$
\begin{aligned}
\lambda_j &= \lambda_0 + \lambda_{1,d_j} + \lambda_{2,a_j} + \lambda_{3,s_j} \\
\beta_j &= \beta_0 + \beta_{1,d_j} + \beta_{2,s_j} \\
\gamma_{r,j} &= \gamma_{r,s_j}, \quad r = 1, 2
\end{aligned}
$$

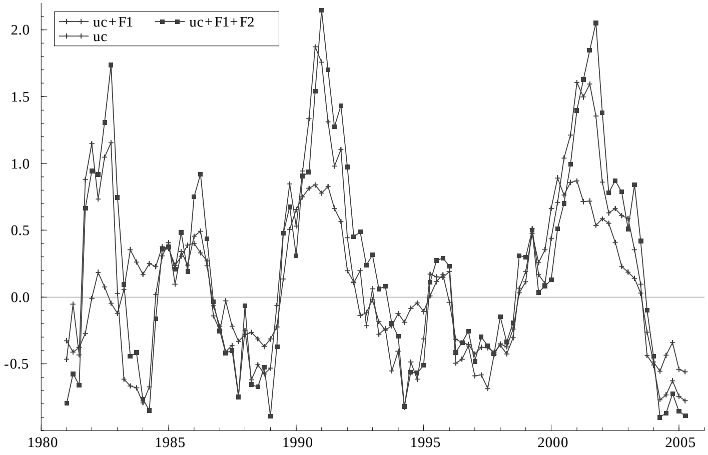
Model specification with both f_t^{uc}, F_t

par	val	t-val	par	val	t-val	par	val	t-val
λ_0	-1.50	8.91	ϕ	0.85	10.80	γ_1^{IG}	0.57	3.10
$\lambda_{1,fin}$	-0.39	2.84	β_0	0.63	3.81	γ_1^{BB}	0.38	2.59
$\lambda_{1,tra}$	-0.12	1.39	$\beta_{1,fin}$	-0.14	0.78	γ_1^{B}	0.07	0.56
$\lambda_{1,lei}$	-0.67	3.98	$\beta_{1,tra}$	0.01	0.05	γ_1^{CCC}	0.24	3.30
$\lambda_{1,utl}$	-0.43	4.34	$\beta_{1,lei}$	0.24	1.43			
$\lambda_{1,hte}$	−0.34	2.75	$\beta_{1,utl}$	0.09	0.61	γ_2^{IG}	0.36	2.18
$\lambda_{1,hea}$	-0.55	3.28	$\beta_{1,hte}$	0.18	1.12	γ_2^{BB}	0.13	0.89
$\lambda_{2,0-3}$	-0.68	5.34	$\beta_{1,hea}$	0.27	1.41	γ_2^{B}	0.40	4.28
$\lambda_{2,4-5}$	-0.38	2.94				γ_2^{CCC}	0.05	0.78
$\lambda_{2,6-12}$	-0.39	3.13	$\beta_{2,IG}$	-0.14	0.55			
$\lambda_{3,IG}$	-6.40	24.80	$\beta_{2,BB}$	-0.01	0.02			
$\lambda_{3,BB}$	-4.21	19.88	$\beta_{2,B}$	0.00	-			
$\lambda_{3,B}$	−2.63	14.72	$\beta_{2,CCC}$	-0.42	2.94			

LogLik: -3017.75

Estimation results

Smoothed Signal/Default Intensity, Investment Grade, All Firms



How well does the model forecast?

Out of sample forecasting, average 1997 - 2004.

Reduction in MAE, %		Improvement over "no factors"	Improvement over "only observables"
only $\hat{F}_t$	IG	-1.8%	-0.7%
	SG	-6.4%	-4.6%
only $\hat{f}_t^{uc}$	IG	-9.9%	-8.9%
	SG	-14.4%	-12.8%
$\hat{F}_t$ and $\hat{f}_t^{uc}$	IG	-11.1%	-10.1%
	SG	-17.2%	-15.6%
F_t and f_t^{uc}	IG	-26.0%	-25.1%
	SG	-26.8%	-25.4%

How well does the model forecast?

Out of sample forecasting, year 2001.

Reduction in MAE, %		Improvement over "no factors"	Improvement over "only observables"
only $\hat{F}_t$	IG	-5.1%	-2.8%
	SG	-6.0%	-4.0%
only $\hat{f}_t^{uc}$	IG	-14.9%	-12.8%
	SG	-20.0%	-18.3%
$\hat{F}_t$ and $\hat{f}_t^{uc}$	IG	-38.2%	-36.7%
	SG	-27.1%	-25.6%
F_t and f_t^{uc}	IG	-68.0%	-67.2%
	SG	-49.4%	-48.3%

Conclusion: What to take home from this?

- Which macro variables are the right ones to model pd's? Take very many and use what they have 'in common'.
- The presence of serially correlated unobserved factors is 'a blessing'. Smoothed risk factors today contain information about default conditions in the future.

Thanks! // Questions?