Towards an Integrated Measurement of Credit and Market Risk[1]

T. Breuer[2], M. Jandačka[2], G. Krenn[3,4]

This version: July 14, 2005

Abstract

We investigate two important aspects of the integrated measurement of credit and market risk: (1) Market risk factor modelling at time horizons longer than usual in pure market risk management (we consider a three months horizon). We conclude that (a) aggregating models for high frequency data in general leads to worse results than discarding the high frequency data and estimating the models for low frequency returns only from low frequency data, (b) in a comparison of models with the same aggregation level, models which take into account GARCH effects fare better than constant volatility models.

(2) For two sample portfolios we compare profit-loss distributions derived from different perspectives towards risk: pure market risk, pure credit risk, and integrated credit and market risk perspective. Measuring market and credit risk in an integrated way spots risks that are hidden to a simple addition of pure market and credit risk numbers.

---

[1]The statements made herein are the authors' opinions, and do not necessarily reflect the views of their respective institutions. We would like to thank Burkhard Raunig and Martin Summer for useful comments.

[2]PPE Research Centre, FH Vorarlberg, Achstrasse 1, A-6850 Dornbirn

[3]Oesterreichische Nationalbank, Otto Wagner-Platz 3, A-1090 Wien

[4]Corresponding author. email: `Gerald.Krenn@oenb.at`

1 Introduction

Credit and market risk are intertwined for two reasons. First, credit risk depends on market risk factors because default probabilities, values of collateral, and values of claims may depend on interest rates, exchange rates, or other market prices. Second, market risk depends on credit risk factors because the default of a counter-party might open up a previously closed position.

Not fully capturing the inter-linkage between credit and market risk still is a source of shortcomings in the risk management of financial institutions. Determining over-all risk measures by summing up separately calculated credit and market risk measures may result in ignoring potential losses that could have been identified by using an integrated view. Integrated credit and market risk management can help individual institutions to achieve a better understanding of their overall risk position, thus contributing to improvements in the stability of the financial system as a whole.

This paper consists of two parts which investigate two important aspects of the integrated measurement of credit and market risk:

1. Risk factor modelling: Credit and market risk should be measured on the same time horizon. Usually this will be the longer time horizon of credit risk. Therefore one needs a measurement of market risk at long time horizons.

2. Portfolio valuation: The valuation of the portfolio should not add up separate losses from credit and market risk factor changes. In particular, the model should take into account that some risk factors (e.g. interest rates, exchange rates) influence both credit and market risk.

In the first part we compare different models for univariate market risk factor changes over a time horizon of 60 trading days. The models use an aggregation of 1, 5, 10, 20, 30, and 60 day changes to arrive at a distribution forecast for 60 day changes. All models are taken with and without GARCH-effects, and with residuals assumed to be distributed according to a normal distribution, a Student-t distribution, and a combination of historical body and Pareto distributed tails. Statistical out-of-sample tests lead us to the following main results: (A) Aggregating models for high frequency data in general leads to worse results than discarding the high frequency data and estimating the models for 60 day returns only from 60 day data. (B) In a comparison of models with the same aggregation level, models which take into account GARCH effects fare better than constant volatility models. This is even true when no aggregation takes place at all, i.e. when we start from 60 day returns. It has been argued that over horizons longer than usual in market risk management (10 days), GARCH effects vanish and returns tend to a normal distribution. This is in contradiction to our empirical findings for a 60 days horizon.

In a second part, we compare various ways of jointly calculating risk measures for credit and market risk. The first way (*separate view*) is to calculate separately the credit risk and the market risk profit/loss distributions, to calculate the risk measures for both profit/loss distributions; and to add up the two risk measures. The separate view corresponds to a bank with independent credit and market risk management units, each generating scenarios and calculating risk measures at its own. The total risk capital is simply the sum of credit risk capital and of market risk capital. Some might consider this procedure as conservative and argue that correlation effects will usually cause total risk to be smaller than the sum of credit risk and market risk. We argue that this view is mistaken. We show that total risk can be considerably higher than the sum of market and credit risk.

The second way (*integrated view*) is to start from a joint distribution of credit and market risk factor changes; then to calculate from it the profit/loss distribution resulting from joint moves of all risk factors. From this joint profit/loss distribution the risk measure is calculated.

For two sample portfolios, one consisting of bonds, the other of European options, we compare risk capital calculated in this integrated way to the sum of market and credit risk capital. It turns out that for both sample portfolios integrated risk capital is higher. This implies that the integration of credit and market risk reveals risks which are hidden to a separate view of credit and market risk.

2 Modelling market returns at longer horizons

Pure market risk management often assumes holding periods of not more than 10 days. When credit and market risk are analyzed in an integrated way it comes natural that distributions of market risk factor returns have to be modelled at longer time horizons. In this paper we concentrate on a time horizon of 60 trading days, corresponding to three calendar months. When choosing a model for the longer run returns, several choices can be made. Since market data are available at higher frequencies it is possible to use the high frequency data to estimate a reliable model of high frequency changes, and then aggregate the high frequency changes to the longer time horizon. In doing so, different basic periods can be chosen. By aggregation one can possibly exploit the availability of data at higher frequencies in order to get more reliable estimates for the lower frequency. On the other hand, estimation and/or modelling errors are likely to be magnified by the aggregation procedure. Next, we can include possible GARCH effects into the model or take a constant volatility model. Furthermore, we can choose among several possible distributions for the residuals, e.g. normal, Student-t or a Pareto fitting of the tails. The goal of this part of the paper is to examine whether aggregating high frequency changes, providing for GARCH effects and considering alternative residual distributions has an effect on the reliability of forecasts of 60 days return distributions.[5]

For choosing a model for the distributions of the various time series we use statistical *out-of-sample* tests of the 60 days *density forecasts* produced by the 36 different models under consideration.[6] We apply these models to 19 different market risk factors, including equity indices, interest rates, and exchange rates.[7] Based on the test results we try to identify a *small* set of models able to cover *all* the time series fairly well, rather than picking for each time series the optimal model. The motivation for this strategy is robustness under the inclusion of new time series.

The first part of this paper is structured as follows: We describe the aggregation of short term changes to long term changes in Section 2.1. In Section 2.2 we discuss the aggregation of short term GARCH (1,1) models. The models are described in Section 2.3 and the test procedure in Section 2.4. Section 2.5 presents and discusses of the test results.

2.1 Aggregating one-period to multi-period return distributions

We concentrate on a time horizon of 60 trading days. Market data are available at a higher frequency, usually daily or even intra-day. This opens

[5]Models with autoregressive terms are not considered. For market risk factors, arguments related to arbitrage rule out the existence of significant autoregressive terms when transaction costs are low.

[6]The models are listed in Section 2.3.

[7]The market risk factors are listed in Section 2.5.

the possibility to model the distributions of returns[8] over shorter periods (1, 5, 10, 20, 30, or 60 days), and then aggregate these distributions in order to arrive at forecasts of 60 days returns. In this way one can possibly exploit the availability of higher frequency data in order to get more reliable estimates. On the other hand, estimation and/or modelling errors might be magnified by the aggregation.

When we confine ourselves to the standard deviation of the return distribution, a simple aggregation method is given by multiplying by the square root of time. This works correctly if we have i.i.d. returns. Volatility clustering observed in the markets implies that returns are not i.i.d. This finding questions the appropriateness of the square root of time-method.

Let us denote an m-period return during time t and $t+m$ by ${}_m r_t$.[9] When the returns r_t are discrete, the probability that ${}_m r_t$ is equal to y is the probability that $\sum_{i=0}^{m-1} r_{t+i}$ is equal to y. In this case one has to sum the probabilities of all possible paths of $\{r_{t+i}\}_{i=0}^{m-1}$ which sum up to y. In analogy, for continuous one-period returns the density function of ${}_m r_t$ is

$$f_{{}_m r_t}(y) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{r_{t+m-1}}\left(y - \sum_{i=1}^{m-1} x_i\right) \prod_{i=1}^{m-1} f_{r_{t+i-1}}(x_i) dx_1 \dots dx_{m-1}, \qquad (1)$$

where f_{r_t} denotes the density function of the one-period returns r_t, conditional on the previous realizations of r_t.

In general, this multi-period density function can be evaluated only numerically. Therefore, we use a Monte Carlo simulation in order to approximate the aggregated distribution function of the density (1). We simulate 10.000 paths of m steps by drawing for each step from the distribution given by the density f_{r_t}. Each path yields a value after step m, which is a draw from the aggregated distribution. Simulating enough paths gives a sufficient approximation of the aggregated distribution.

2.2 Aggregated GARCH processes

A number of stylized facts about the volatility of financial asset prices have emerged over the years, and have been confirmed in numerous studies (see [11]). A good model must be able to capture and reflect these stylized facts.

[8] By return we mean the logarithmic change of a risk factor.

[9] We abbreviate the 1-period return ${}_1 r_t$ during time t and $t+1$ by r_t. As we consider log-returns, we have ${}_m r_t = \sum_{i=0}^{m-1} r_{t+i}$.

Volatility Clustering. Many studies (see [11]) report evidence that large changes (in either direction) in the prices of an asset are often followed by large changes (in either direction), and small changes are often followed by small changes. The implication of such volatility clustering is that volatility shocks today will influence volatility over some time in the future.

Heavy tails. It is well established that the unconditional distribution of asset returns has heavy tails. Extreme moves occur more frequently than could be expected if returns were normally distributed.

GARCH processes are a popular tool for the description of financial time series because they are known to describe volatility clustering. Aggregated GARCH processes can also capture the heavy tails even if the one-period distribution is normal.

Definition 1 *The sequence $\{r_t, t \in \mathbb{Z}\}$ is defined to be generated by a* strong GARCH(1,1) process *if*

$$\begin{aligned} \epsilon_t &:= r_t/\sigma_t \sim D(0,1) \qquad i.i.d., \\ \sigma_t^2 &= a + b r_{t-1}^2 + c\sigma_{t-1}^2, \end{aligned} \tag{2}$$

where $D(0,1)$ specifies a distribution of errors with mean zero and unit variance.

The class of strong GARCH processes is somewhat restricted because it assumes that the errors are identically distributed and independent.[10] In the sequel, we consider strong GARCH processes.

For the estimation of the GARCH parameters we use a Quasi Maximum Likelihood method: The GARCH parameters are estimated under the assumption that the residual distribution $D(0,1)$ is standard normal. But our models of the marginals, as described in Section 2.3, also use other residual distributions, namely the Student-t and distributions with "historical body and Pareto fittted tails". For the consistency and asymptotic normality of Quasi Maximum Likelihood Estimations of GARCH models we refer to [1] and [4].

The class of strong GARCH processes is not closed under temporal aggregation. The density function (1) of the aggregated returns does not necessarily have the same form as the density of one-period returns. For example,

[10]Strong, semi-strong, and weak GARCH processes are outlined e.g. in [9]

even if all one-period returns are distributed normally, multi-period aggregated returns need not be normal or even elliptic. This is a key phenomenon of aggregating time dependent return distributions.

Figure 1 compares the tails of the one-period and the aggregated two-period distribution. The aggregated returns are fat tailed although one-period returns are normal. An aggregation of normally distributed returns can therefore explain the empirical fact that financial time series have fat tails.

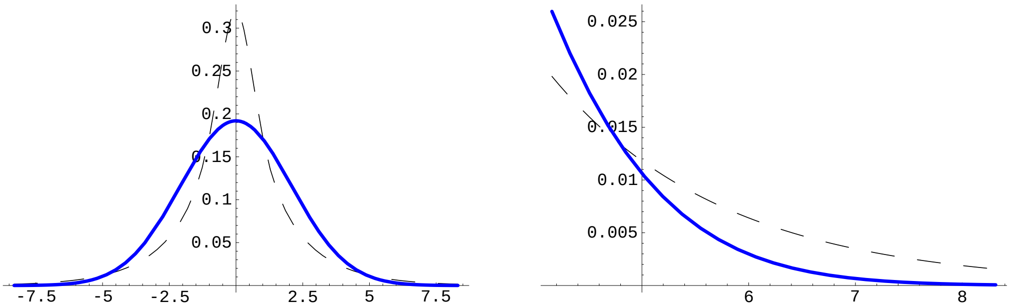

Figure 1: *Example of a 2-period aggregated density function (dashed line), compared with a normal density function with the same mean and volatility (solid line). The aggregated distribution has fat tails.*

Figure 2 shows how the (excess) kurtosis increases with the aggregation level. This picture points to a possible difficulty of the aggregation method. Whereas the kurtosis increases with the aggregation level, we know from empirical investigations that high frequency data tend to be more fat tailed than low frequency data.

2.3 Models for 60 days return distributions

In order to account for possible excess kurtosis, either of the one-period return distributions or of the residuals $D(0, 1)$ of the GARCH models, we consider several possible distributions: normal, Student, and EVT. The EVT-distribution results from modelling the body of the distribution by historic simulation and the left and right tails by a Generalised Pareto distribution. For the left tails we took the lowest 10%, for the right tail the highest 10%. The details of the procedure are described in [13].

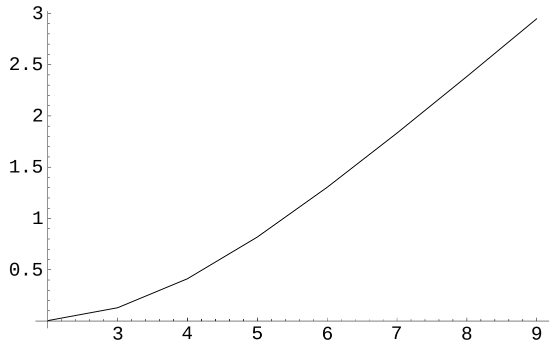

Figure 2: *Excess kurtosis of m-period returns where the one-period returns r_t follow a GARCH(1,1) process with $a = b = c = \sigma_t = 0.2$.*

Combining the various possibilities for aggregation levels, residual distributions, and GARCH or constant volatility, we get various models for the market risk factors. The terminology of model names uses

- xxd means that the model aggregates distributions for xx day returns to arrive at the 60 days return distribution, as described in Section 2.1.
- G in the model name means that the one-period distributions are modelled with GARCH(1,1) as described in Section 2.2.
- The last part of the model name represents the distributions of errors: "norm" for the normal distribution, "t" for the Student-t distribution, and "EVT" for the distribution with the body modelled by historic simulation and the tails by a Pareto distribution, as described in Section 2.3.

In this terminology the models for the marginals of market risk factors are:

1d_norm	1d_t	1d_EVT	1d_G_norm	1d_G_t	1d_G_EVT
5d_norm	5d_t	5d_EVT	5d_G_norm	5d_G_t	5d_G_EVT
10d_norm	10d_t	10d_EVT	10d_G_norm	10d_G_t	10d_G_EVT
20d_norm	20d_t	20d_EVT	20d_G_norm	20d_G_t	20d_G_EVT
30d_norm	30d_t	30d_EVT	30d_G_norm	30d_G_t	30d_G_EVT
60d_norm	60d_t	60d_EVT	60d_G_norm	60d_G_t	60d_G_EVT

2.4 Testing procedures

In order to test the 60 days distribution forecasts produced by the various models, it is not enough to assess whether the means, variances, or some quantiles of the distributions were correctly predicted.[11] For many applications the overall distributional properties are important, not just the means or variances. Therefore, based on [14], we test for the adequacy of the density forecasts of the entire distribution.

Consider a time series of returns r_t $(t = 1, \ldots, n)$ generated from some true conditional densities $f_t(.)$ $(t = 1, \ldots, n)$. Now some model produces a series of 60 days conditional density forecasts $p_t(.)$ $(t = 1, \ldots, n)$. The task is to evaluate whether the true conditional densities $f_t(.)$ agree with the predicted conditional densities $p_t(.)$. Applying the Rosenblatt transformation (see [15]) to the observed returns r_t,

$$r_t \mapsto z_t := \int_{-\infty}^{r_t} p_t(u)du \tag{3}$$

we get a transformed series z_t which should be i.i.d. U(0,1) if the predicted conditional densities $p_t(.)$ agree with the true conditional densities $f_t(.)$. Applying the inverse of the normal distribution function

$$z_t \mapsto n_t := \Phi^{-1}(z_t), \tag{4}$$

produces a series n_t which is standard normally i.i.d. if the original returns r_t are distributed according to the predicted densities p_t (see [2]).

Berkowitz [2] applied a likelihood-ratio test to the z_t against the first order autoregressive alternative $n_t - \mu = \rho_1(n_{t-1} - \mu) + \epsilon_t$ to test for i.i.d. N(0,1). Instead, we can perform a Kolmogorov-Smirnov test for the simple hypothesis that the n_t are sampled from a standard normal distribution. This is our Test 1. A model is accepted if the p-value is higher than 5%.

In order to test additionally whether the *conditional* variance of the n_t is constant and equal to one, de Raaij and Raunig [14] consider the regressions

$$\begin{aligned} n_t &= \beta_0 + \beta_1 n_{t-1} + u_t & (5)\\ n_t^2 &= \gamma_0 + \gamma_1 n_{t-1}^2 + v_t & (6) \end{aligned}$$

where u_t and v_t are non-autocorrelated with zero expectation conditional on their own past values. In case the n_t have zero mean and are uncorrelated

[11] Backtesting for example amounts to a test of a quantile of the predicted distributions.

we have $\beta_0 = 0$ and $\beta_1 = 0$. In case the n_t have constant conditional unit variance we have $\gamma_0 = 1$ and $\gamma_1 = 0$. To test whether these restrictions are satisfied, de Raaij and Raunig [14] propose a joint Wald test of the four equalities $\beta_0 = 0$, $\beta_1 = 0$, $\gamma_0 = 1$, and $\gamma_1 = 0$. Additionally, they use the Jarque-Bera test to see whether the n_t have skewness zero and kurtosis equal to three. The Jarque-Bera test without the Wald test would not be very powerful since it does not test for mean and variance.

According, we perform the following Test 2. A model is accepted if the p-value of the Jarque-Bera test is higher than 5% *and* the p-value of the joint Wald test for $\beta_0 = \beta_1 = \gamma_1 = 0$ and $\gamma_0 = 1$ is higher than 5%.

2.5 Test results

The two tests outlined in Section 2.4 were applied to 19 market data time series. For these series we used daily data from Bloomberg starting at the dates indicated below and ending 28 February, 2004.

Equity Indices	
Dow Jones Industrial Average	03-Jan-1970
DAX	03-Jan-1970
Nikkei 225	06-Jan-1970
Austrian Traded Index	09-Jan-1986
FTSE 100	04-Jan-1984
Swiss Market Index	02-Jul-1988
Interest Rates	
US Govt. 3 months	02-Jun-1983
US Govt. 6 months	02-Jun-1983
US Govt. 2 years	01-Feb-1977
US Govt. 5 years	03-Jan-1970
US Govt. 10 years	03-Jan-1970
US Govt. 30 years	02-Dec-1980
Germany Euro-deposits 6 months*	03-Jan-1975
Germany Govt. 10 years	04-Jan-1989
Japan Govt. 10 years	23-Oct-1987
Exchange Rates	
EUR/USD	05-Jan-1971
EUR/GBP	05-Jan-1971
EUR/CHF	05-Jan-1971
EUR/JPY	05-Jan-1971

* Source: Datastream

Table 1 summarises the test results. For each model, this table shows for how many of the 19 time series the model was accepted in Test 1 and Test 2. Additionally, the average of p-values of the rejected time series is given. A low value of this average indicates that rejections on average were unambiguous.

According to Test 2, which is more selective, the 60d_G_EVT model is best. It is acceptable for 15 out of 19 time series. The second best model, 60d_EVT, is acceptable for 13 out of 19 time series. According the more liberal Test 1, the situation is less clear. 60d_G_norm is acceptable for 17 time series, 60d_norm and 30d_t are acceptable for 16 time series; 60d_G_EVT, 60d_G_t, and 60d_EVT are acceptable for 15 time series.

Furthermore we see that results improve as the length of the basic period increases and the number of aggregation steps decreases. Aggregating models for high frequency data in general leads to worse results than discarding the high frequency data and estimating the models for 60 days returns only from 60 days data.

We also see that in a comparison of models with the same aggregation level, models which take into account GARCH effects fare better than constant volatility models. For most models the GARCH versions are better than the constant volatility versions. The two only exceptions are 20d_G_t, which according to Test 2 is worse than the 20d_t model, and 30d_G_t, which according to Test 1 is worse than the 30d_t model. For all other models, in both tests the GARCH versions are better or equally good as the constant volatility versions.

It might be interesting to note that Test 2 is more selective than Test 1. For all models, Test 2 accepts this model for fewer or equally many time series as Test 1.

3 The Importance of Modelling and Evaluating Joint Moves of Credit and Market Risk Factors

In this section we investigate the importance of integrated credit and market risk measurement as compared to summing up separate risk numbers for credit and for market risk. In standard credit risk models market risk factors are usually assumed to be deterministic [6]. This is a major obstacle to an integration of market and credit risk.

An integrated view of credit and market risk might be necessary in sit-

Model	Test 1 (KS) # accept.	av. P for non-acc.	Test 2 (JB+W) # accept.	av. P for non-acc.
1d_norm	0	0.0003	0	0
1d_t	0	0.0063	0	0.0003
1d_EVT	0	0.0004	0	0
1d_G_norm	0	0.0022	0	0.0001
1d_G_t	2	0.0068	0	0.0004
1d_G_EVT	0	0.0002	0	0
5d_norm	0	0	0	0
5d_t	1	0.0005	0	0
5d_EVT	0	0.0001	0	0
5d_G_norm	0	0.0005	0	0
5d_G_t	1	0.0012	0	0.0001
5d_G_EVT	0	0.0002	0	0
10d_norm	0	0.0013	0	0.0001
10d_t	2	0.0057	0	0.0003
10d_EVT	1	0.0021	0	0.0001
10d_G_norm	0	0.0051	0	0.0003
10d_G_t	3	0.0027	0	0.0001
10d_G_EVT	2	0.0018	0	0.0001
20d_norm	2	0.0016	0	0.0001
20d_t	6	0.0147	1	0.0008
20d_EVT	1	0.0078	0	0.0004
20d_G_norm	2	0.0064	0	0.0003
20d_G_t	7	0.0143	0	0.0008
20d_G_EVT	4	0.0051	0	0.0003
30d_norm	5	0.0147	0	0.0008
30d_t	16	0.01	8	0.0009
30d_EVT	8	0.0208	1	0.0012
30d_G_norm	5	0.0186	0	0.001
30d_G_t	14	0.021	11	0.0026
30d_G_EVT	10	0.0172	2	0.001
60d_norm	16	0.0201	9	0.002
60d_t	13	0.0118	9	0.0012
60d_EVT	15	0.0207	13	0.0034
60d_G_norm	17	0.0016	10	0.0002
60d_G_t	15	0.0068	11	0.0008
60d_G_EVT	15	0.0201	15	0.005

Table 1: *Summary of test results. For each model, this table shows for how many of the 19 time series the model was accepted in Test 1 and Test 2. Additionally, the average of p-values of the rejected time series is given.*

uations, in which credit risk depends on market risk factors, or market risk exposure depends on credit risk. A historical example of such a situation are the portfolios held by some banks during the Russian crisis of 1998. Those banks had dollar/rubel forwards with Russian banks and matching rubel/dollar forwards with US banks. These positions were fully hedged against moves in the dollar/rubel exchange rate. Furthermore, for these positions default risk was irrelevant as long as the exchange rate did not move. If one counter-party defaulted, it was always possible to get the currency deliverable to the other counter-party on the market at no loss if the exchange rate did not move. And a move of exchange rates was very improbable in those times of the managed rubel exchange rate regime. So from a pure market risk and a pure credit risk point of view the risk of the portfolio was zero. However, during the Russian crisis in August 1998, adverse credit events and market moves occured *simultaneously.* The Russian counterparties defaulted and at the same time the value of the rubel dropped dramatically. The USD deliverable to the US banks had to be purchased on the market and the rubel they got in return were not much worth. This led to considerable losses for the banks involved.

In the present paper we consider two sample portfolios with similar characteristics. The bond portfolio we consider consists of a long and a short zero coupon bond due in 11 years with a face value of 1,000 EUR. The short bond position is assumed to be default risk-free, while the long position is with an obligor of average Austrian default risk. The value of the portfolio (to a risk neutral investor) at a time horizon of one year is thus given by

$$1_{\{\text{no default in year 1}\}} 1000(1+r_{10})^{-10} \prod_{t=1}^{10} (1-p_t) - 1000(1+r_{10})^{-10}, \qquad (7)$$

where r_{10} is the risk free 10 year EUR interest rate, and p_t are the probabilities of default in year t starting at the time horzizon in one year. The recovery rate is assumed to be zero.

The default probabilities p_t are assumed to be determined from some linear combination of macroeconomic variables, as in Wilson [16] or Boss [3]:

$$p_t = \frac{1}{1+e^{-y_t}}, \qquad (8)$$

where, for the sake of simplicity, y_t is assumed to depend only on the one most relevant macroeconomic risk factor, namely the Austrian industry production

IP_t in year t via the equation $\Delta y_t = \beta_1 + \beta_2 \Delta \mathrm{IP}_t$. The constants β_1, β_2 are estimated from default data.

The portfolio pricing formula (7) is valid only for a risk neutral investor. The reason for this is that via (8) the default probabilities depend on industry production, which is not a traded asset. Therefore there is no unique risk neutral probability measure, the market is incomplete. However, any individual investor with some given utility function can evaluate the portfolio by requiring that the expected utility of the portfolio payoff equals the utility of the portfolio value. For risk neutral investors, i.e. if the utility function is linear, the portfolio value determined in this way is the expected payoff of the portfolio. This is given by the portfolio value function (7).

The dynamics of industry production is determined from a vector autoregressive modell from a set of five macroeconomic variables. The first macroeconomic variable is industry production itself, the four others are the three month interest rate, the EUR/USD exchange rate, the inflation rate, and the slope of the yield curve. The dynamics of the macroeconomic variables is determined by

$$\boldsymbol{m}_t = \boldsymbol{c} + A\boldsymbol{m}_{t-1} + \boldsymbol{\epsilon}_t, \tag{9}$$

where A is a matrix of autoregressive terms and the components of the error vector $\boldsymbol{\epsilon}_t$ are assumed to be normally distributed with $(\mathbf{0}, \Sigma)$ and independent of $\boldsymbol{\epsilon}_{t-1}$. The parameter vector $\boldsymbol{c}$ and the matrix A are estimated from the historical data of the five macroeconomic variables. By writing this model 10 time steps into the future we get forecasts of $m_{1,t} = \mathrm{IP}_t$ (and if desired also of the other macroeconomic variables $m_2, \ldots, m_5$). In a simplifying low-dimensional picture one can consider the extected value of the portfolio at the time horizon of one year to be a function only of future values of the market risk factor r_{10} and of the credit risk factor IP_1, with the expectation taken over the variables $\mathrm{IP}_2, \ldots, \mathrm{IP}_{10}$. Figure 3 plots the expected portfolio value as a function of these two risk factors.

To estimate the model we used quarterly data from 1969 to 2003 for r_{10} and the macroeconomic risk factors. In order to avoid seasonal effects the quarterly data were aggregated to a yearly basis. The default probabilities p_t are determined as the yearly actual default rates among all Austrian companies. For the log-returns of r_{10} we assumed by a GARCH(1,1)-model with normal residuals. To get a joint distribution of the market and the credit risk factor we coupled the marginal distributions of the risk factors by a Gaussian copula.

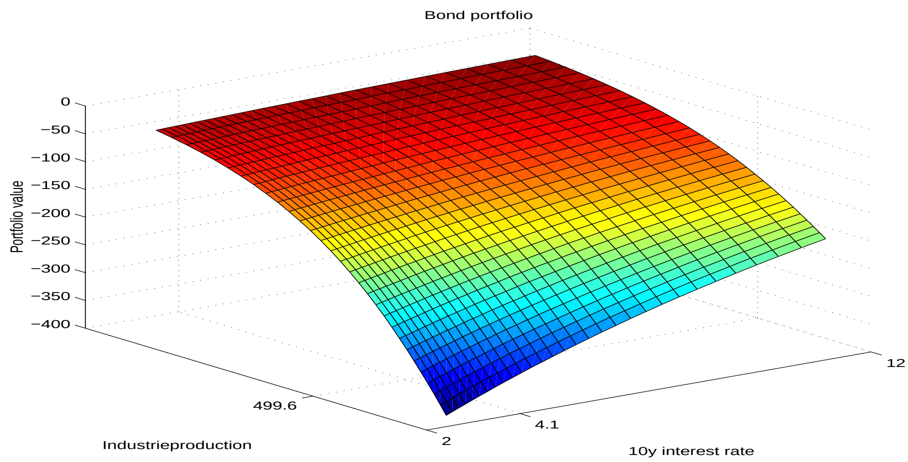

Figure 3: *Expected present value of the bond portfolio as a function of the 10 yr interest rate and industry production at a time horizon of one year, with the expectation taken over the variables $IP_2, \ldots, IP_{10}$. The portfolio value does not change too much if only the interest rate, or only the industry production changes from its current level of 4.1% resp. 499.6. However, if both change simultaneously, the losses are considerable. This gives a first intuition why an integrated view on market and credit risk is necessary.*

How does the risk profile of this portfolio look like from an integrated market and credit risk perspective, as compared to a mere sum of market and credit risk? To investigate this question we approximated four different profit/loss functions of the portfolio from 1.000.000 Monte Carlo paths of the market risk factor r_{10} and the macro-model (9) including $\mathrm{IP}_1, \ldots, \mathrm{IP}_{10}$.

The four profit/loss functions are the following. (0) Pure market risk, without taking into account the possibility of default is zero, since the portfolio is perfectly hedged to moves of the market risk factor r_{10}. This is displayed in Column (0) of Table 2.

(1) Credit risk by itself, without taking into account the possibility of interest rate changes, is considerable. Column (1) of Table 2 gives Value at Risk (VaR) and Expected Shortfall (ES) numbers of the profit/loss simulated from scenarios with the interest rate fixed at its current value and with varying values of industry production. The left tail of this profit/loss distribution is the dashed function in Figure 4 with a step at 696.5 EUR. The worst thing that can happen in this pure credit risk perspective is that the counter-party

of the long position defaults in the first year. This occurs with a probability of roughly 2.2% and leads to a loss of 696.5 EUR.

(2) A first crude integration of market and credit risk is achieved by considering the market risk at a constant probability of default. Column (2) of Table 2 gives VaR and ES numbers of the profit/loss simulated from scenarios with varying levels of the interest rate and with industry production fixed at its current value. The left tail of this profit/loss distribution is the dashed-dotted line in Figure 4. In this perspective a default of the counterparty of the long position in the first year does not lead to one certain loss number. Rather, the effect of the default depends on the level of the interest rate. In case of adverse moves of the interest rate the loss can be considerably larger than 696.5 EUR.

(3) Full integration of market and credit risk is achieved by varying the market and the credit risk factors simultaneously according to their joint distribution. Column (3) of Table 2 gives VaR and ES numbers of the profit/loss simulated from scenarios with varying levels of the interest rate and of industry production. The left tail of this profit/loss distribution is the solid line in Figure 4. As for the crude integration (2), with full integration (3) the effect of default in the first year depends on the interest rate level and can therfore be considerably higher than 696.5 EUR. Additionally, the default probability of the counterparty during years 2 through 11 can vary due to changes in industry production. This results in higher loss potentials than for crude integration (2).

Measuring market and credit risk in an integrated way spots risks that are hidden to a simple addition of market and credit risk numbers. The size of the newly detected risks are displayed in the last two columns of Table 2. The second last column gives the integration effect achieved by the crude integration: risk numbers for market risk with constant default probability minus the sum of risk numbers for pure market risk and pure credit risk. The last column shows the effects of full integration: risk numbers for integrated market and credit risk minus the sum of risk numbers for pure market risk and pure credit risk. Positive numbers in these columns indicate that the risk measured in an integrated view is higher than the sum of pure market and pure credit risk. At all quantiles, the integrated point of view gives consistently higher VaR and ES numbers.

We also observe that the difference between full integration and crude integration (market risk plus constant PD) is not always small. In particular for the 5% and the 2.5% confidence levels, crude integration conveys the false

VaR α	(0) MR no CR	(1) CR no MR	(2) MR unchgd CR	(3) Intd MR&CR	(2)-(1)-(0) integr. effect	(3)-(1)-(0) integr. effect
0.001	0	696.5	766.8	768.1	70.3	71.6
0.0025	0	696.5	750.0	747.2	53.5	50.7
0.005	0	696.5	730.9	727.6	34.4	31.0
0.01	0	696.5	701.8	700.9	5.2	4.3
0.025	0	226.6	142.6	228.9	-84.0	2.3
0.05	0	198.3	137.8	199.4	-60.5	1.1

ES α	(0) MR no CR	(1) CR no MR	(2) MR unchgd CR	(3) Intd MR&CR	(2)-(1)-(0) integr. effect	(3)-(1)-(0) integr. effect
0.001	0	696.5	782.3	782.7	85.8	86.2
0.0025	0	696.5	767.3	766.8	70.8	70.3
0.005	0	696.5	753.7	751.6	57.1	55.1
0.01	0	696.5	734.8	732.8	38.3	36.3
0.025	0	623.5	612.9	623.7	-10.7	0.1
0.05	0	416.0	376.2	416.8	-39.8	0.9

Table 2: *Value at Risk and Expected Shortfall numbers for the bond portfolio at various confidence levels and with various risk measurement techniques. Column (0) displays risk numbers from a pure market risk point of view, and column (1) from a pure credit risk point of view. Column (2) shows risk numbers from a market risk point of view which takes into account the possibility of default, but at a constant default probability. Column (3) gives risk numbers from a fully integrated market and credit risk perspective. The size of the integration effect is given in the last two columns. The positive numbers indicate higher risk in the integrated perspective.*

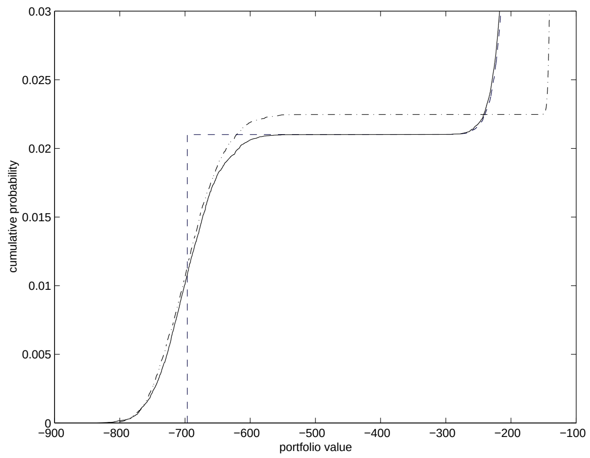

Figure 4: *The cumulative distribution functions of the profit/loss of the bond portfolio for various risk measurement techniques. The profit/loss distribution from a pure market risk point of view (Method (0)) is not visible on the plot, since it is peaked at zero. Visible are the profit/loss distributions from a pure credit risk point of view (Method (1), dashed line), from a market risk point of view incorporating credit risk at a constant default probability (Method (2), dashed-dotted line), and from a fully integrated market and credit risk perspective (Method (3), solid line).*

illusion that credit and market risk set off each other. Full integration reveals that this is not the case. Crude integration is not a reliable substitute for full integration.

Finally consider a portfolio of matching long and short European calls on the Dow Jones Industrial index with a strike price of 8500 USD maturing exactly at the time horizon of one year. (Since the options mature exactly at the time of evaluation, the risk-free interest rate and the volatility of the underlying are irrelevant for the option value, which is simply the difference of the value of the underlying and the strike price.) The long position is exposed to default risk, whereas for the short position default of the counterparty is irrelevant. Table 3 compares the VaR and ES numbers for the option portfolio under the four risk measurement perspectives. As the last two columns show, risk is consistently higher in the integrated perspective for this portfolio as well.

VaR α	(0) MR no CR	(1) CR no MR	(2) MR unchgd CR	(3) Intd MR&CR	(2)-(1)-(0) integr. effect	(3)-(1)-(0) integr. effect
0.001	0	223.2	3583.0	3714.7	3359.8	3491.5
0.0025	0	223.2	2748.2	2715.8	2525.0	2492.6
0.005	0	223.2	1956.0	1941.3	1732.9	1718.1
0.01	0	223.2	1017.3	936.2	794.1	713.1
0.025	0	0	0	0	0	0
0.05	0	0	0	0	0	0

ES α	(0) MR no CR	(1) CR no MR	(2) MR unchgd CR	(3) Intd MR&CR	(2)-(1)-(0) integr. effect	(3)-(1)-(0) integr. effect
0.001	0	223.2	4522.3	4622.6	4299.2	4399.5
0.0025	0	223.2	3661.9	3729.4	3438.8	3506.3
0.005	0	223.2	2990.6	3012.8	2767.4	2789.7
0.01	0	223.2	2224.1	2218.7	2000.9	1995.5
0.025	0	196.0	1013.2	986.4	817.2	790.4
0.05	0	98.0	506.6	493.2	408.6	395.2

Table 3: *Value at Risk and Expected Shortfall numbers for the option portfolio at various confidence levels and with various risk measurement techniques. Column (0) displays risk numbers from a pure market risk point of view, and column (1) from a pure credit risk point of view. Column (2) shows risk numbers from a market risk point of view which takes into account the possibility of default, but at a constant default probability. Column (3) gives risk numbers from a fully integrated market and credit risk perspective. The size of the integration effect is given in the last two columns. The positive numbers indicate higher risk in the integrated perspective.*

4 Conclusions

In this paper we investigated two important aspects of the integrated measurement of credit and market risk: market risk factor modelling at long time horizons and the importance considering simultaneous moves of market and credit risk factors. Measuring market and credit risk in an integrated way spots risks that are hidden to a simple addition of pure market and credit risk numbers.

Other important aspects of integration have not been addressed: (1) Measuring credit and market risk on the same time horizon requires not only a model of market risk factors at the longer time horizon of credit risk data, but also a treatment of the possible rebalancing actions of portfolio managers over the longer time horizon. (2) An integrated model should correctly model the market price dependence not just of the liabilities but also of the collaterals. In particular, it should correctly model the correlation between values of the various collaterals. This requires a multivariate model of the values of collaterals.

References

[1] Berkes I., L. Horvath, P. Kokoszka (2003): "GARCH processes: Structure and estimation", *Bernoulli* **9**, 201-227

[2] Berkwoitz J. (2001): "Testing Density Forecasts, with Applications to Risk Management", *Journal of Business and Economic Statistics* **19**, 465–475

[3] Boss M. (2002): "A Macroeconomic Credit Risk Model for Stress Testing the Austrian Credit Portfolio", *Financial Stability Report* **4**, Oesterreichische Nationalbank
http://www.oenb.at/en/img/fsr_04_tcm16-8061.pdf

[4] Busch T. (2004): "Finite Sample Properties of GARCH Quasi-Maximum Likelihood Estimators and Related Test Statistics" Working Paper, University of Aarhus, preliminary version of October 11, 2003, http://www.econ.au.dk/fag/2785/e03/garch_paper.pdf

[5] Chen X., Y. Fan, A. Patton (2004): "Simple Tests for Models of Dependence Between Multiple Financial Time Series, with Applications

to U.S. Equity Returns and Exchange Rates", London Economics Financial Markets Group Working Paper No. 483. http://ssrn.com/ abstract=513024

[6] Crouhy A., D. Galai, R. Mark (2000): "A comparative analysis of current credit risk models", *Journal of Banking and Finance* **24**, 59-117

[7] Daul St., E. De Giorgi, F. Lindskog, A. McNeil (2003): "The Grouped t-Copula with an Application to Credit Risk", *Risk* **16**, 63–76

[8] Dias A., P. Embrechts (2003): "Dynamic copula models for multivariate high-frequency data in finance", Working Paper, available at http://www.math.ethz.ch/ baltes/ftp/papers.html

[9] Drost F. C., T. E. Nijman (1993): "Temporal Aggregation of GARCH Processes", *Econometrica* **61**, 909-927

[10] Embrechts P., A. McNeil, D. Straumann (2002): "Correlation and Dependence in Risk Management: Properties and Pitfalls", pp. 176–223 in M. Dempster (ed.): *Risk Management: Value at Risk and Beyond*, Cambridge University Press

[11] Engle R. F., A. J. Patton (2001): "What good is a volatility model?", *Quantitative Finance* **1**, 237-245

[12] Jandačka M., T. Breuer (2004): "Market Risk on Long Time Horizons with Multi-Step GARCH Models", Working Paper, PPE Research Center, FH Vorarlberg

[13] McNeil A., R. Frey (2000), "Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: an Extreme Value Approach", *Journal of Empirical Finance* **7**, 271-300

[14] de Raaij G., B. Raunig (2002): "Evaluating Density Forecasts from Models of Stock Market Returns", Working Paper 59, Oesterreichische Nationalbank

[15] Rosenblatt M. (1952): "Remarks on a Multivariate Transformation", *Annals of Mathematical Statistics* **23**, 470–472

[16] Wilson T. (1997): "Portfolio Credit Risk (I)" and "Portfolio Credit Risk (II)", *Risk* **10** (9) and (10)