Basel Accord Insurance Working Group

Comments to the Consultative Document on Operational Risk

Annex: Methodology of the Capital Charge Calculation

1. Introduction

In its January 2001 Consultative Document on Operational Risk the Basel Committee outlines four different approaches for capital assessment of operational risks. These approaches differ in their degree of sophistication as well as implementation needs. In this Annex, the Working Group focuses only on the "Internal Measurement Approach" (IMA) and on the "Loss Distribution Approach" (LDA).

We first provide a brief outline of IMA and refer the interested reader for details to the Consultative Document. By IMA the required capital is given by:

$$C = \sum_{ij} \gamma_{ij}\ EI_{ij}\ PE_{ij}\ LGE_{ij}$$

where the indices i and j refer to business line and risk type, respectively. For each pair ij we denote by EI_{ij}, PE_{ij} and LGE_{ij} the exposure indicator, the probability of event and loss given event, respectively.

The expected loss of ij for an organization of exposure EI_{ij} is defined to be:

$$EL_{ij} = EI_{ij} \times PE_{ij} \times LGE_{ij}$$

In particular, $PE_{ij} \times LGE_{ij}$ corresponds to the expected loss of an organization with unit exposure. The unexpected loss of ij is obtained by multiplying EL_{ij} with a pre-specified constant factor γ_{ij}, which is derived from the industry data. In the following we seek to investigate IMA in the light of actuarial models and provide some refinements. In the following sections, for the sake of simplicity, we will consider only one line of business and one risk class and, hence, leave out the indices i and j.

2. Actuarial Models

This section introduces the notion of exposure indicator and its relation to the expected loss as well as to the unexpected loss under different assumptions of available data. The distinction between IMA and LDA arises mainly from the nature of available data. If losses are gathered on an individual basis rather than on an aggregate basis, LDA provides a more natural and accurate framework for capital calculation.

2.1. Exposure Indicator

The expected loss of a bank over a specified time horizon (period) is expressed as:

$$EL = \mathrm{E}[N]\mathrm{E}[X],$$

where N is the random variable of number of losses corresponding to that period of time (frequency) and X the one of loss amount (severity). This equation assumes independence of the two random variables of frequency and severity. In practice, the assumption of independence is proved to be valid.

An alternative way to express the expected loss is obtained by introducing the notion of exposure indicators. The expected frequency can be expressed by:

$$\mathrm{E}[N] = EI_N \times PE \ ,$$

where EI_N is defined to be *frequency exposure indicator* and PE denotes the *probability of event*. By definition PE corresponds to the expected frequency at unit exposure.

By expressing $\mathrm{E}[N]$ as in the previous equation, we made the following assumptions:

- existence of an exposure indicator that correlates with the frequency,
- linear correlation of the exposure indicator with the frequency.

The assumption of linearity may not always be valid. Nevertheless, the methodology developed below can be easily adopted to a non-linear dependency of frequency from the respective exposure indicator. Similarly, multi-exposure models may be utilized to obtain a higher degree of accuracy in estimating the frequency.

In analogy to the frequency and subject to the same assumptions, the average loss amount can be expressed as:

$$\mathrm{E}[X] = EI_X \times LGE \ ,$$

where EI_X denotes the severity exposure indicator, and LGE corresponds to the average loss at unit exposure.

By virtue of these definitions, the expected loss is expressed as:

$$EL = EI_N \times EI_X \times PE \times LGE$$

The motivation for assuming two different exposure indicators is to reflect the fact that for some risk types the underlying exposures of frequency and severity may be different. Furthermore, as shown below, the operational risk capital exhibits different scaling behavior for frequency and severity exposure indicators.

As an example of a risk type with two different exposure indicators, we consider losses in the area of trading. The number of transactions or that of traders may serve as a proxy for a frequency exposure indicator, whereas severity may rather depend on the transaction volume. An additional example is provided by employee related claims. In this case, one may consider number of employees as a frequency exposure indicator and assume losses to obey a random process that is not subject to any scaling requirement, i.e., $EI_X = 1$.

We conclude this section by defining the expected loss per unit exposure indicator of frequency and severity EL_1 by:

$$EL_1 = PE \times LGE$$

2.2. Data Format

In this section we address the issue of the required data. We assume two different scenarios of available data. Depending on the granularity of the available data the computational accuracy of the capital will be different.

2.2.1. Aggregate Loss Data

Assumes following data to be given for I banks and T periods:

a) number of losses $N_{i,t}$,

b) aggregate period loss $S_{i,t}$, and

c) exposure indicators $EI_{N;i,t}$ and $EI_{X;i,t}$.

2.2.2. Individual Loss Data

Assumes following data to be given for I banks and T periods:

a) number of losses $N_{i,t}$,

b) individual period losses $X_{i,t,n}$ with $n = 1, \cdots, N_{i,t}$, and

c) exposure indicators $EI_{N;i,t}$ and $EI_{X;i,t}$.

2.3. Parameter Estimation

To estimate the parameters PE and LGE, we first assume the data to be given in the aggregate form. By means of this data, PE is calculated to be[1]:

$$PE = \frac{\sum_{i=1}^{I}\sum_{t=1}^{T} N_{i,t}}{\sum_{i=1}^{I}\sum_{t=1}^{T} EI_{N;i,t}}$$

For the sake of notation and as the data of a bank for different periods can be considered as the data of different banks of a given period, we drop, in the following, the period index and represent a banks data of different periods by assuming different banks for the data of each period. Hence, the last equation can be rewritten as:

[1] To obtain this, one assumes a Poisson random variable N for frequency and applies the maximum likelihood method to obtain the Poisson parameter PE.

$$PE = \frac{\sum_{k=1}^{K} N_k}{\sum_{k=1}^{K} EI_{N;k}},$$

where $K = I \times T$. Similarly, we define:

$$LGE = \frac{\sum_{k=1}^{K} \frac{S_k}{EI_{X;k}}}{\sum_{k=1}^{K} N_k}.$$

Estimation of PE and LGE for individual loss data can be easily reduced to the aggregate loss data case by applying the identity $S_k = \sum_{n=1}^{N_k} X_{k,n}$. Alternatively, LGE can be computed as the expected value of the distribution function corresponding to samples $\left\{\frac{X_{k,n_k}}{EI_{X;k}}\right\}$.

2.4. Expected Loss

From the previous consideration, we obtain:

$$EL_1 = PE \times LGE = \frac{\sum_{k=1}^{K} \frac{S_k}{EI_{X;k}}}{\sum_{k=1}^{K} EI_{N;k}}.$$

For a bank of arbitrary exposure the expected loss can now be expressed as $EL = EI_N \times EI_X \times EL_1$. This equation, clearly, estimates a bank's expected loss level by combining its *exposure data* with the industry's *loss experience*, expressed in EL_1. However, in the presence of sufficient loss data, one may alternatively rely only on a bank's own loss data to estimate PE and LGE, and, hence, EL_1. Such an approach,

clearly, takes into account many qualitative parameters, such as risk management quality, that are best reflected in a bank's internal data. However, insufficiency of internal data, be it due to good luck or good management in the past, may lead to an underestimation of the future loss burden.

2.5. Capital Calculation

In this section we calculate the unexpected loss as a proxy for capital charge. Unexpected loss may be defined as standard deviation or as a percentile (less expected value) of some target distribution.

In the following, we first demonstrate the scaling behavior of the unexpected loss with regard to the exposure indicators of frequency and severity. We then proceed by computing the unexpected loss for the two different data scenarios introduced in section 2.2.

2.5.1. Scaling Behavior of Unexpected Loss

Let us define as the unexpected loss the standard deviation of the aggregate loss S. Assuming a Poisson random variable for frequency, by virtue of the formula

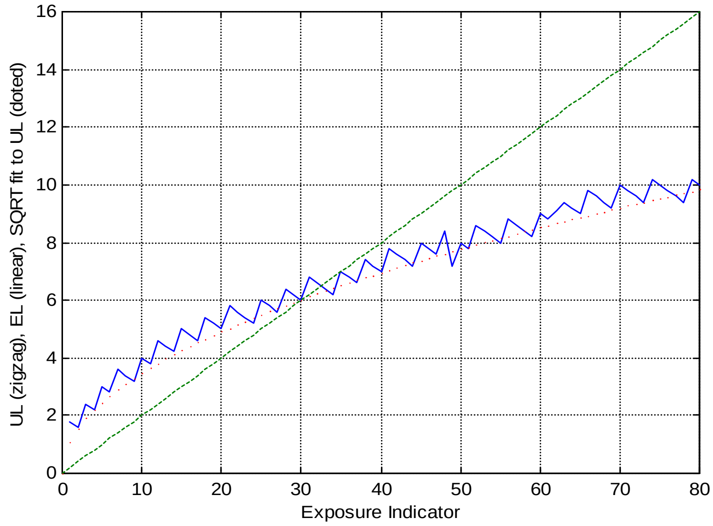
$$\mathrm{Var}[S] = \mathrm{E}\left[X^2\right]\mathrm{E}[N],$$

we obtain for the transformation $X \to EI_X\ X$ and $N \to EI_N\ N$:

$$\sigma_S \to EI_X \sqrt{EI_N}\ \sigma_S .$$

The scaling property suggested by the last equation is by no means restricted to standard deviation. In fact, a similar scaling behavior can be proved for the unexpected loss as defined by $UL = VAR - EL$, where $VAR = \mathrm{F}_S^{-1}(0.99)$ is assumed.

To illustrate this, we performed a simulation assuming $PE = 0.2$, $EI_N = [1, 2, 3, \cdots, 79, 80]$, $LGE = 1$ and $EI_X = 1$. Following figure depicts the scaling behavior of the UL as a function of the frequency exposure EI_N.



2.5.2. Unexpected Loss in Case of Aggregate Loss Data (IMA)

Henceforth we assume the unexpected loss to be defined as $UL = VAR - EL$ with VAR being the value at risk at some high tolerance level $(\geq 99^{th})$.

Let UL_1 denote the unexpected loss per unit exposure indicator for frequency and severity. By imposing the scaling property, for an arbitrary exposure we assume the following approximation:

$$UL = EI_X \sqrt{EI_N}\, UL_1$$

Furthermore, by defining γ to satisfy $UL_1 = \gamma\, EL_1$, we can write:

$$UL = \gamma \, EI_X \sqrt{EI_N} \, EL_1 \, .$$

This equation can also be expressed in terms of expected loss for arbitrary exposure indicators:

$$UL = \frac{\gamma}{\sqrt{EI_N}} \, EL \, .$$

By definition, γ depends on the distribution of the aggregate losses and the definition of unexpected loss. In general, calculation of γ is an involved task, as it requires an estimation of the aggregate loss distribution. Here, we only consider the following two cases.

2.5.2.1. Pure Frequency Exposure ($EI_N > 1, \; EI_X = 1$)

In this case we seek to estimate the distribution function of the aggregate losses of the banks such that the following two properties are satisfied:

- Expected value to scale as $\mu \rightarrow EI_N \, \mu$,
- Standard deviation to scale as $\sigma \rightarrow \sqrt{EI_N} \, \sigma$.

As shown in section 2.4.1., higher percentiles of the aggregate distribution function will also exhibit a similar scaling behavior as standard deviation. Since we deal with aggregate losses of banks, by virtue of the central limiting theorem, we impose the following normal distribution:

$$N(x; E_N \mu, \sqrt{E_N} \sigma) = \frac{1}{\sqrt{2\pi} \sqrt{EI_N} \, \sigma} \, e^{-\frac{(x - EI_N \, \mu)^2}{2\left(\sqrt{EI_N} \, \sigma\right)^2}} \, .$$

It is easily shown, that the maximum likelihood estimation of the parameters μ and σ results in:

$$\mu^* = \frac{\sum_{t=1}^{T} S_t}{\sum_{t=1}^{T} EI_{N;t}}, \quad \sigma^* = \frac{1}{T}\sum \frac{\left(S_t - EI_{N;t}\mu\right)^2}{EI_{N;t}}.$$

Assuming, in what follows, the unexpected loss to be defined at a 99^{th} tolerance level, γ is calculated to be:

$$\gamma = \frac{N^{-1}\left(0.99;\mu^*,\sigma^*\right) - \mu^*}{\mu^*}.$$

where $N^{-1}(\cdot;\mu^*,\sigma^*)$ is the inverse normal distribution function with parameters μ^* and σ^*.

2.5.2.2. Pure Severity Exposure ($EI_N = 1,\ EI_X > 1$)

In this case, the aggregate distribution function is assumed to satisfy the following two properties:

- Expected value to scale as $\mu \to EI_X\ \mu$
- Standard deviation to scale as $\sigma \to EI_X\ \sigma$.

Due to the scaling property, γ is calculated to be:

$$\gamma = \frac{F^{-1}(0.99) - \mu}{\mu},$$

where $F_S(s)$ is the distribution function of the observations $\left\{\frac{S_k}{EI_{X;k}}\right\}$ and μ its mean.

Up to this point, the required capital charge is calculated by:

$$UL = \gamma\, EI_X \sqrt{EI_N}\; EL_1 .$$

In particular, γ and EL_1 are obtained from the industry data. To emphasize a bank's loss experience in its capital calculation and to provide incentive for better risk management, EL_1 can be replaced by the respective quantity obtained from a bank's internal loss data.

2.5.3. Unexpected Loss in Case of Individual Loss Data (LDA)

In the presence of the individual losses the calculation of the unexpected loss becomes more straightforward and accurate. First we find the loss severity of a bank with exposure EI_X. In order to achieve this we fit a distribution $F_X(x)$ to scaled individual losses $\left\{\frac{X_{k,n_k}}{EI_{X;k}} EI_X\right\}$. The frequency distribution is assumed to be Poisson with mean $\mathrm{E}[N] = EI_N \times PE$.

To compute the unexpected loss, we aggregate the loss distribution $F_X(x)$ with frequency distribution resulting in $F_S(s)$. The unexpected loss can be calculated according to $UL = F_S^{-1}(0.99) - \mu$, where μ is the expected value of $F_S(s)$.

So far the only relevant parameters of a bank in the capital calculation have been its exposure indicators. In analogy to IMA, to incorporate expected loss of a bank into its capital calculation, we define $f(\cdot)$ to by

$$f(EI_N) = \frac{UL}{\mu}.$$

From this we can compute the unexpected loss of a bank utilizing its exposure indicators EI_X and EI_N, as well as its expected loss per unit exposure EL_1:

$$UL = f(EI_N)\, EL,$$

where $EL = EI_X\ EI_N\ EL_1$.

Note that contrary to IMA, the previous equation makes no approximations for the capital computation. Computation of $f(\cdot)$ can be easily achieved via Monte Carlo simulations. Alternatively, a look up table can be set up assuming discrete values of frequency exposure.

2.6. Diversification Benefit

We consider the effect of having various different business units and risk categories on the overall capital charge. Clearly, as a result of diversification between business units and/or risk categories a simple addition of corresponding capital charges should be avoided.

In analogy to the behavior of standard deviation and under assumption of independence, we suggest the following procedure for capital calculation in the presence of various business units and risk categories[1]:

[1] In presence of correlation we apply $UL = \sqrt{\sum_{ij} UL_{ij}^2 + 2\sum_{ij<mn} UL_{ij} UL_{mn} \rho_{ij,mn}}$

$$UL = \sqrt{\sum_{ij} UL_{ij}^2} \ .$$

3. Inclusion of Insurance

In this section we seek to demonstrate the impact of an insurance cover on the reduction of required capital charge by considering two types of coverage. Thereby we assume insurance to apply to all losses and neglect issues related to the quality of the coverage, such as policy extent, credit worthiness of the insurer.

Before proceeding we point out that an inclusion of insurance by simply applying the γ factor to the expected loss of a bank after insurance leads to erroneous results. This is due to the fact that the computation of γ assumes no insurance to apply to the losses.

3.1. Aggregate Excess Loss

Assume an aggregate excess coverage attaching at D_{agg} with a coverage limit of L_{agg}. Let UL_{post} denote the unexpected loss after the insurance. Assuming P to be the insurance premium, we obtain:

$$UL_{post} = UL - L_{agg} + P \qquad \text{if } D_{agg} + L_{agg} \le UL$$

To show this, note that $UL_{post} = F_{S,post}^{-1}(0.99) - EL_{post}$. From the assumption that the coverage applies within the unexpected loss, i.e., $D_{agg} + L_{agg} \le UL$, we obtain $F_{S,post}^{-1}(0.99) = F_S^{-1}(0.99) - L_{agg}$. By assuming the retained expected loss by the bank EL_{post} to be given by $EL_{post} = EL - P$, we get $UL_{post} = UL - L_{agg} + P$.

3.2. Excess Loss

Let assume all losses be subject to one each and every deductible d_{ee} and one each and every limit l_{ee}. In this case, the relief in capital can be best shown within the framework of LDA.

To proceed, we assume the unexpected loss to be given by $UL_S = g(N)UL_X$, where $g(\cdot)$ is a function that, for a given frequency, transforms the unexpected loss of the severity to the one of the aggregate loss. By the same definition, after insurance we obtain $UL_{S_{post}} = g(N)UL_{X_{post}}$, where S_{post} and X_{post} are the random variables of aggregate loss and severity, respectively, after the application of insurance coverage.

By assuming $d_{ee} + l_{ee} \leq UL_X$ we can insert $F_{X_{post}}^{-1}(0.99) = F_X^{-1}(0.99) - l_{ee}$ in the previous equation, obtaining:

$$UL_{S_{post}} = g(N)\left(F_X^{-1}(0.99) - l_{ee} - EL_{X_{post}}\right).$$

Moreover, by assuming that retained expected loss of a bank can be expressed as:

$$EL_{X_{post}} = EL_X - \frac{P}{N},$$

we get for the unexpected loss after insurance:

$$UL_{S_{post}} = g(N)\left(F_X^{-1}(0.99) - l_{ee} - EL_X + \frac{P}{N}\right).$$

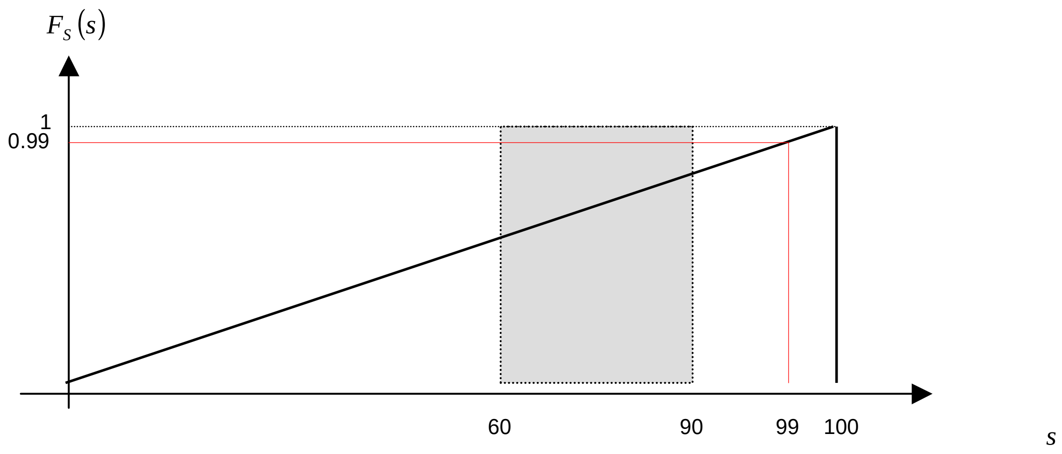
This equation can be simplified to obtain:

$$UL_{S_{post}} = UL_S - g(N)l_{ee} + \frac{g(N)}{N}P.$$

4. Working Example

In this section we illustrate the capital relief resulting from inclusion of an insurance by means of an example.

For simplicity, let assume the aggregate losses to be a uniformly distributed on $[0,100]$. Furthermore, assume an aggregate excess coverage of 30 in excess of 60.



The unexpected loss before insurance is calculated to be:

$$UL = F^{-1}(0.99) - EL = 99 - 50 = 49\,.$$

By the same definition, after insurance, the unexpected loss becomes

$$UL_{post} = F_{post}^{-1}(0.99) - EL_{post} = F^{-1}(0.99) - 30 - EL_{post}\,,$$

where EL_{post} is the expected loss retained by the bank. From $EL_{post} = 42.5$, we obtain $UL_{post} = 26.5$.

On the other hand, the premium for the layer 30 in excess of 60 is calculated to be $P = 7.5$. By applying the formula for UL_{post} introduced in 3.1., we obtain $UL_{post} = 49 - 30 + 7.5 = 26.5$.