

## Annex - Risk weight and asset correlation

### Scope

This memorandum is to show that the capital requirements produced by the model underlying the Benchmark Risk Weight (BRW) function are particularly sensitive to the hypothesis of asset correlation, since the corresponding loss distribution function may assume very different shapes. Chart 1 highlights such diversity by comparing the loss distribution of a portfolio composed of 1,000 exposures, all of which have the same probability of default equal to 1% obtained from Monte Carlo simulations of 100,000 scenarios. The distributions presented in Chart 1 refer to the following cases independence ( $\rho=0$ ), “low” correlation ( $\rho=2\%$ ), the correlation which is implied in the Retail function ( $\rho=8\%$ ), the correlation which is implied in the Corporate function ( $\rho=20\%$ ).

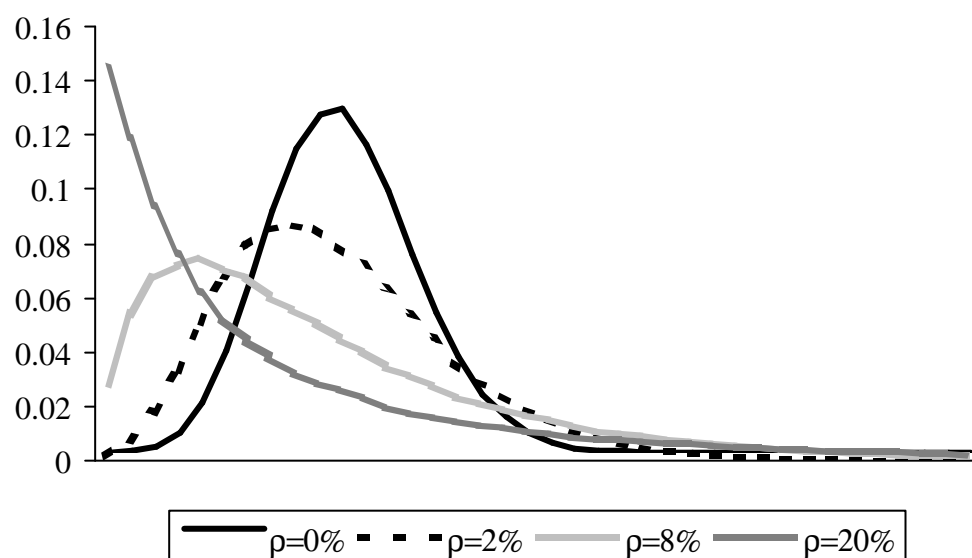


Chart 1 – The varying loss distribution according to different degrees of asset correlation

As the chart shows, the 20% correlation assumption leads to effects which can hardly be considered realistic in SME portfolios such as a probability of no loss equal to 14.6% and, symmetrically, a remarkable fatness of the tail of the loss distribution. At the same time the VaR increases from 1.8% in the case of independence to about 9.4% if  $\rho=20\%$  (see Table 1).

Table 1 shows also the capital requirements resulting from the BRW function according to various correlation hypotheses: the crucial dependence of this model results on the correlation parameter stresses the attention required in the calibration.

r	PD	BRW	CR	VaR <sub>99,5%</sub>
0.0%	1%	13	1.1%	1.8%
2.0%	1%	31	2.5%	2.6%
8.0%	1%	63	5.1%	4.9%
10.0%	1%	73	5.9%	5.7%
<b>20.0%</b>	<b>1%</b>	<b>125</b>	<b>10.0%</b>	<b>9.4%</b>
40.0%	1%	243	19.5%	18.2%

Table 1 – Capital requirement and VaR produced by the BRW according to different degrees of asset correlation

## Methodology

It is possible to calculate the asset correlation using default data<sup>1</sup> by first estimating the default correlation  $\rho$

$$\mathbf{r} = \frac{Cov(p_i, p_j)}{\sqrt{p_i(1-p_i)p_j(1-p_j)}} = \frac{E[D_i, D_j] - p_i p_j}{\sqrt{p_i(1-p_i)p_j(1-p_j)}}$$

where  $p_i, p_j$  are the default rates of grades  $i$  and  $j$  and  $Cov(p_i, p_j)$  is the historical covariance. Then, the joint default probability is:

$$E[D_i, D_j] = p_{i,j} = \mathbf{r} \sqrt{p_i(1-p_i)p_j(1-p_j)} + p_i p_j$$

Using this value it is possible to extract the implied asset correlation solving for  $\rho$  tilde the following equation:

$$p_{i,j} = \int_{-\infty}^{\Phi^{-1}(p_i)} \int_{-\infty}^{\Phi^{-1}(p_j)} \frac{1}{2p\sqrt{1-\tilde{\mathbf{r}}^2}} \exp\left\{-\frac{1}{2(1-\tilde{\mathbf{r}}^2)}(x_i^2 + x_j^2 - 2\tilde{\mathbf{r}}x_i x_j)\right\} dx_i dx_j$$

This methodology could produce outcomes which are biased if the variance estimations come from small samples. The correction proposed<sup>2</sup> is:

$$v^2 = \frac{ns^2 - \mathbf{m}(1-\mathbf{m})}{n-1}$$

where  $s^2$  is the standard variance estimator,  $\mathbf{m}$  is the weighted average default rate and  $n$  is the armonic mean of the number of issuers.

Of course the correction can be applied only if the number of issuers and of defaults are known (not only the time series of default rates); so, given the data available, we can correct the asset correlation estimates just for S&P's and BIP.

## Results

Applying this method on real data we got the following results:

1. Italian official data<sup>3</sup> ("Decay and mortality of cash financing", Base Informativa Pubblica, Banca d'Italia) leads to correlation coefficients which are sensibly lower than the one implied by the current proposal. The 20% assumption is, thus, not supported by the empirical evidence in the Italian case.
2. Even the estimate of the implied asset correlation using S&P's data on default rates<sup>4</sup> leads to results not consistent with the 20% (the average corrected asset correlation is around 7-8%
3. Moody's<sup>5</sup> default experience is more consistent with the Basle hypothesis since the "within" grade asset correlation is around 19.9%. However it should be considered that the higher asset correlation could

<sup>1</sup> See for example JP Morgan, 1997, *CreditMetrics*.

<sup>2</sup> See Gordy M., A Comparative Anatomy of Credit Risk Models, Appendix B

<sup>3</sup> See for example Sironi A., C. Zazzara, *Bancaria*, 2001, forthcoming publication.

<sup>4</sup> Standard & Poor's. Ratings Performances 1999. We drop firms rated AAA because of the absence of defaults (this, of course, raises the average asset correlation estimate).

easily be the consequence of small samples bias. Chart 2 shows the particular dynamic behavior of the Aa and A default rates that, increasing the default variance estimates for those grades, raises the average asset correlation. As mentioned before, the data at our disposal does not allow us to apply the variance correction in this case.

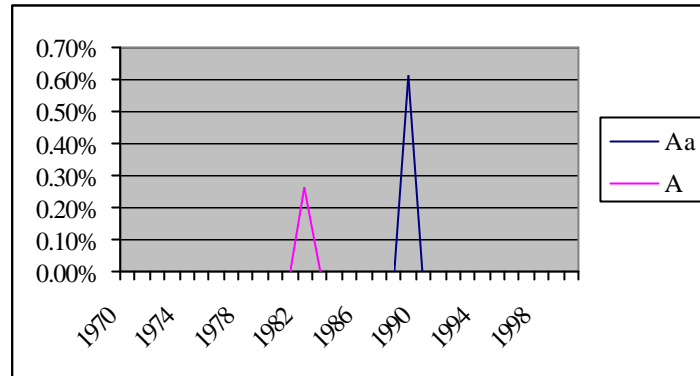


Chart 2 – Moody's Aa, A default rates over time

Statistics	Standard & Poor's							Base Inf. Pubh.		Moody's					
	AAA	AA	A	BBB	BB	B	CCC	x<5000	x>5000	Aaa	Aa	A	Baa	Ba	B
average # of Issuers	156.0	425.1	735.3	476.6	328.4	343.8	37.5	848829	129750	//	//	//	//	//	//
average # of Defaults	0.0	0.1	0.3	1.0	3.0	17.7	7.8	2187.7	348.1	//	//	//	//	//	//
average default rate (r)	0.00%	0.01%	0.04%	0.22%	1.03%	4.98%	18.37%	2.59%	2.58%	0.00%	0.02%	0.01%	0.14%	1.20%	6.50%
Uncorrected Asset Correlation	//	1734%	2016%	877%	1125%	611%	1437%	077%	219%	//	3063%	2688%	1659%	1308%	1229%
Corrected Asset Correlation	//	//	1424%	052%	788%	464%	825%	076%	213%	//	//	//	//	//	//

Table 2 – Asset correlation estimates

## Data

Year	Moody's						Standard & Poor's							BIP	
	Aaa	Aa	A	Baa	Ba	B	AAA	AA	A	BBB	BB	B	CCC	500-cx<5,000	x>5,000
1970	0.00%	0.00%	0.00%	0.27%	4.12%	22.78%	//	//	//	//	//	//	//	//	//
1971	0.00%	0.00%	0.00%	0.00%	0.42%	3.85%	//	//	//	//	//	//	//	//	//
1972	0.00%	0.00%	0.00%	0.00%	0.00%	7.14%	//	//	//	//	//	//	//	//	//
1973	0.00%	0.00%	0.00%	0.45%	0.00%	3.77%	//	//	//	//	//	//	//	//	//
1974	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%	//	//	//	//	//	//	//	//	//
1975	0.00%	0.00%	0.00%	0.00%	1.02%	5.97%	//	//	//	//	//	//	//	//	//
1976	0.00%	0.00%	0.00%	0.00%	1.01%	0.00%	//	//	//	//	//	//	//	//	//
1977	0.00%	0.00%	0.00%	0.28%	0.52%	3.28%	//	//	//	//	//	//	//	//	//
1978	0.00%	0.00%	0.00%	0.00%	1.08%	5.41%	//	//	//	//	//	//	//	//	//
1979	0.00%	0.00%	0.00%	0.00%	0.49%	0.00%	//	//	//	//	//	//	//	//	//
1980	0.00%	0.00%	0.00%	0.00%	0.00%	4.94%	//	//	//	//	//	//	//	//	//
1981	0.00%	0.00%	0.00%	0.00%	0.00%	4.49%	0.00%	0.00%	0.00%	0.00%	0.00%	2.35%	0.00%	//	//
1982	0.00%	0.00%	0.26%	0.31%	2.72%	2.41%	0.00%	0.00%	0.43%	0.35%	3.18%	3.23%	23.53%	//	//
1983	0.00%	0.00%	0.00%	0.00%	0.91%	6.31%	0.00%	0.00%	0.00%	0.34%	1.20%	4.67%	0.00%	//	//
1984	0.00%	0.00%	0.00%	0.36%	0.83%	6.72%	0.00%	0.00%	0.00%	0.68%	0.59%	3.37%	14.29%	//	//
1985	0.00%	0.00%	0.00%	0.00%	1.75%	8.22%	0.00%	0.00%	0.00%	0.00%	1.55%	6.28%	9.52%	2.73%	2.45%
1986	0.00%	0.00%	0.00%	1.33%	2.04%	11.80%	0.00%	0.00%	0.17%	0.32%	1.35%	8.36%	17.65%	2.77%	2.31%
1987	0.00%	0.00%	0.00%	0.00%	2.71%	6.25%	0.00%	0.00%	0.00%	0.00%	0.37%	3.68%	7.81%	2.55%	2.12%
1988	0.00%	0.00%	0.00%	0.00%	1.24%	6.04%	0.00%	0.00%	0.00%	0.00%	1.05%	4.17%	19.30%	2.14%	1.62%
1989	0.00%	0.61%	0.00%	0.60%	2.98%	9.12%	0.00%	0.00%	0.00%	0.57%	0.71%	3.18%	28.85%	1.83%	1.80%
1990	0.00%	0.00%	0.00%	0.00%	3.34%	16.16%	0.00%	0.00%	0.00%	0.56%	3.17%	8.33%	31.91%	2.11%	1.73%
1991	0.00%	0.00%	0.00%	0.28%	5.30%	14.71%	0.00%	0.00%	0.00%	0.51%	2.50%	12.54%	31.15%	2.19%	1.90%
1992	0.00%	0.00%	0.00%	0.00%	0.30%	9.03%	0.00%	0.00%	0.00%	0.00%	0.00%	7.46%	27.78%	2.63%	2.43%
1993	0.00%	0.00%	0.00%	0.00%	0.55%	5.79%	0.00%	0.00%	0.00%	0.00%	0.35%	2.10%	12.50%	3.70%	4.82%
1994	0.00%	0.00%	0.00%	0.00%	0.24%	3.82%	0.00%	0.00%	0.11%	0.00%	0.27%	2.98%	19.23%	3.40%	4.08%
1995	0.00%	0.00%	0.00%	0.00%	0.67%	4.80%	0.00%	0.00%	0.00%	0.32%	0.71%	4.38%	25.00%	3.03%	3.31%
1996	0.00%	0.00%	0.00%	0.00%	0.00%	1.44%	0.00%	0.00%	0.00%	0.00%	0.65%	2.59%	3.57%	2.80%	3.34%
1997	0.00%	0.00%	0.00%	0.00%	0.19%	2.11%	0.00%	0.00%	0.00%	0.12%	0.18%	3.24%	11.11%	2.79%	2.75%
1998	0.00%	0.00%	0.00%	0.12%	0.61%	4.26%	0.00%	0.00%	0.00%	0.31%	0.76%	4.39%	35.48%	2.19%	2.19%
1999	0.00%	0.00%	0.00%	0.10%	1.14%	5.88%	0.00%	0.16%	0.08%	0.19%	1.03%	7.33%	30.38%	1.97%	1.79%
2000	0.00%	0.00%	0.00%	0.38%	1.05%	5.14%	//	//	//	//	//	//	//	//	//
p	0.00%	0.02%	0.01%	0.14%	1.20%	6.50%	0.00%	0.01%	0.04%	0.22%	1.03%	4.98%	18.37%	2.59%	2.58%
Var(p)	0.00%	0.00%	0.00%	0.00%	0.02%	0.22%	0.00%	0.00%	0.00%	0.00%	0.01%	0.07%	1.18%	0.00%	0.01%
Def. Corr.	//	0.59%	0.25%	0.52%	1.44%	3.61%	//	0.15%	0.25%	0.23%	0.86%	1.48%	7.87%	0.10%	0.33%

<sup>5</sup> Moody's Investors Service, Feb. 2001, Default and recovery rates of Corporate Bond Issuers: 2000. Again, we drop firms rated Aaa.