Systemic Risk Contributions: A Credit Portfolio Approach[☆]

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Abstract

We put forward a framework for measuring systemic risk and attributing it to individual banks. Systemic risk is coherently measured as the expected loss to depositors and investors when a systemic event occurs. The risk contributions are calculated based on derivatives of the systemic risk measure, thus, ensuring a full risk allocation among institutions. Applying our methodology to a panel of 54 to 86 of the world's major commercial banks for a 13-year time span with monthly frequency, we are not only able to closely match the list of G-SIBs. We also can use individual risk contributions to compute bank-specific capital surcharges: systemic capital charges as well as countercyclical buffers. We therefore address both dimensions of systemic risk – cross-sectional and time-series – in a single, integrated approach. As the analysis of risk drivers confirms, the main focus of macroprudential supervision should be on a solid capital base throughout the cycle and de-correlation of banks' asset values.

Keywords: Systemic Risk, Systemic Risk Contributions, Systemic Capital Charge, Expected Shortfall, Importance Sampling

JEL: G21, G28, C15, C63

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1. Introduction

This paper aims to contribute to the ongoing regulatory and academic debate on systemic risk and risk contributions of systemically important banks. It puts forward a macroprudential approach that internalises negative external effects by building up a sufficient capital buffer in the financial system to bear (most of) those costs. For this purpose it relies on measures of potential costs to society associated with rare but severe systemic events. Furthermore, it is designed to mitigate cyclical effects which are typical for a risk-sensitive capital buffer.

In this paper we define systemic risk by large-scale losses, incurred by the system as a whole when low-probability systemic events occur. The notion of systemic importance is therefore directly linked to the systemic risk contribution of an individual bank given as the bank's share in the system-wide losses. Taking this definition as a basis, we put forward an approach for *measuring systemic risk* and decomposing it into the *contributions of individual institutions*. Making use of the risk contributions, bank-specific systemic capital surcharges on top of the microprudential capital requirements can be computed, as we describe in this paper. To mitigate a potentially procyclical effect of regulation, we also suggest how a countercyclical capital add-on may be calculated to be maintained during the periods of overoptimistic markets and extensive risk taking.

To assess the system-wide loss we, first, model a banking sector as a portfolio comprising banks' liabilities net of capital. Then we utilise a widely used credit risk model to assess the tail risk of that portfolio. The model inputs are the banks' individual probabilities of default, size of their liabilities net of capital and banks' sensitivity to systematic factors, which capture correlations between banks' asset returns.

We make the notion of systemic risk operational by defining it as the *expected shortfall* (ES) of the portfolio of the banks' liabilities computed at a probability level q. That is, systemic risk is measured as the expected loss to the depositors and other banks' creditors in the worst $100\cdot(1-q)$ per cent of cases. Depending on his/her tolerance towards the probability of a systemic event (given as $1-q$), the regulator can either fix q at a given level or vary $q(t)$ over time depending on the average default conditions in the banking sector. Thereby, using time-varying $q(t)$ would help to mitigate possible procyclical effects of the regulation because the regulator would then automatically take into account that the estimates of default risk based on market information may be inappropriately low (high) during a boom (bust).

In order to break down extreme portfolio losses into the contributions of individual banks, we draw on a rich literature on coherent, additive risk contributions for credit portfolios. Employing marginal risk contributions based on the partial derivatives of the portfolio ES with respect to the institutions' relative portfolio weight allows for a complete allocation of the system-wide risk to the individual banks.

In summary, we see the following aspects as the main contribution of this paper:

1. We suggest a method for measuring systemic risk as the expected extreme loss of the portfolio containing banks' liabilities. That measure of systemic risk reflects the potential losses to the economic agents (depositors and investors) and not merely banks' equity capital impairment.
2. We provide a full allocation of the systemic risk across institutions based on the Euler allocation principle for the assessment of banks' systemic importance. This method for assessing systemic risk contributions remains feasible irrespective of the size and composition of the banking system under consideration.
3. We provide an empirical example in order to investigate the main drivers of systemic risk and systemic importance.
4. Regarding possible policy implications, we address both dimensions of systemic risk in one integrated approach: the cross-sectional dimension by designing a bank-specific systemic risk surcharge above the minimum required capital as a continuous function of a bank's systemic importance. And the time dimension by imposing a capital buffer in times of overoptimistic markets in order to mitigate possible procyclical effects.

The approach suggested for measuring and attributing systemic risk has several merits. It is based on a credit portfolio model that is well understood and widely applied in the praxis of risk management. Its application is, in principle, not limited to listed financial institutions since it can also be adapted to non-listed companies as long as reliable estimates of their probability of default and of their sensitivity with regard to systematic risk factors can be obtained. Furthermore, our method can be used for projections of systemic risk or for stress testing based on predictions or on stressed values of input parameters. The model can be utilised either via simulation as in this paper or by using a fast analytical approximation as reported in Düllmann and Puzanova (2011).

Apart from its technical merits, our method has further advantages: It takes direct account of main risk drivers such as the size and individual default risk of financial institutions and correlation among interconnected entities. Because the probability of default is a function of the financial leverage, which is in turn a ratio of total assets to the weighted average of long-term and short-term liabilities, the model also takes the leverage into account. However, on the issue of interconnectedness, we have to point out that the model does not go beyond the notion of linear correlation (it does not incorporate contagion effects or tail dependence). Nonetheless, the multi-factor correlation structure suggested is rich and allows for a differentiated treatment of different groups of banks. This reflects the fact that episodes of financial distress often arise from the exposure of groups of institutions to common risk factors and that intragroup dependence is higher than intergroup dependence.

The remainder of the paper is structured as follows: Section 2 provides a brief review of selected literature. Section 3 outlines the modelling approach and the calculation method for the risk contributions of individual banks. Section 4 presents an empirical example of a system comprising world's major commercial banks and analyses the impact of different risk drivers. In section 5 we address the possible policy implications of the proposed methodology. Finally, we summarise the main results in section 6.

2. Related literature

In this section we briefly review the literature on systemic risk (contributions), which our paper is most closely related to, and point out the aspects that distinguish our approach from the others. A more comprehensive review of recent approaches for detecting the tail risk of a financial system by examining direct and indirect financial sector interlinkages can be found, for instance, in the IMF's Global Financial Stability Report (IMF, 2009, pp 73-149) and in Galati and Moessner (2011).

The approach presented in this paper relies on market information about interlinkages among banks. The study of financial sector interlinkages using market prices of financial instruments has a long tradition and a rational explanation. For instance, De Nicolo and Kwast (2002) argue that the information contained in banks' equity returns can be used to measure total (direct and indirect) dependence since stock prices reflect market participants' collective evaluation of the future prospects of the firm, including the total impact of its interactions with other institutions. The dependence structure of the banking system as given in our paper can be inferred from the empirical correlation of banks' equity returns.

Equity returns and other market data are widely used to measure the fragility of financial institutions at individual and aggregate levels. For example, Bartram et al. (2007) estimate the default probabilities for a large sample of international banks from time series of equity prices and also from equity option prices, based on the assumptions of Merton's structural model (Merton, 1974). They use this information to construct indicators for a systemic event. In our paper we use the estimates of banks' default probabilities obtained from Moody's KMV, whose model is also based on Merton's fundamental idea.

Huang et al. (2009) deduce risk-neutral default probabilities for major banks from their CDS spreads and asset return correlation from the co-movement of equity returns. Using these key parameters as input in a portfolio credit risk model, the authors suggest computing an indicator of systemic risk, namely the price of insurance against large default losses in the banking sector. As in our paper, the banking sector is represented by a hypothetical portfolio that consists of debt instruments issued by a pre-selected group of banks. The theoretical insurance premium equals the risk-neutral expectation of portfolio credit losses given that the losses exceed some minimum share of the sector's total liabilities. Our approach is different from the approach described as we use objective probabilities of default and, thus, can deduce the actual losses to depositors and investors in case of a systemic event. Furthermore, we define the systemic event not by means of a given system-wide loss threshold, but rather by setting the probability threshold for the occurrence of a systemic event.

Another application of the portfolio approach based on market data can be found in Segoviano and Goodhart (2009). The authors utilise the "nonparametric consistent information multivariate density optimising methodology" in order to obtain the joint multivariate density of the banks' asset value movements. Based on this information, several indicators of banking stability can be constructed: (i) the joint probability of distress of all banks in the portfolio; (ii) a banking stability index that reflects the number of banks expected to become distressed once at least one bank has become distressed; (iii) the conditional probabilities of distress for individual banks or specific groups of banks. The authors, however, do not consider the issue of individual risk contributions.

Also by virtue of the joint probability distribution of banks' assets Lehar (2005) specifies the following indicators of systemic risk: (i) an asset-value-related systemic risk index derived by computing the probability that a group of banks with a total amount of assets greater than a certain fraction of all banks' assets goes bankrupt within a short period of time; (ii) a number-of-defaults-related systemic risk index derived by computing the probability that a certain number of banks go bankrupt within a short period of time; (iii) the value of a hypothetical deposit insurance, its volatility as well as the individual volatility contributions.

While the methods described above mostly focus on monitoring systemic risk, Adrian and Brunnermeier (2011) suggest an approach for measuring the *contributions* that individual banks make to systemic risk. For this purpose the authors make use of the quantile regression technique and construct the so called ΔCoVaR measure of banks' risk contributions. A bank's ΔCoVaR can be described as the difference between the VaR of the system conditional on the bank being in distress and the VaR of the system conditional on the bank being in the median state. This measure of systemic risk contributions relies heavily on the observations of extremely negative stock returns of banks and is only applicable in the Gaussian setting, in which it is also additive, as shown by Jäger-Ambrozewicz (2011). Otherwise, the individual risk contributions cannot be aggregated to calculate the system-wide risk. They could even be misleading. For instance, an application of the CoVaR methodology to a non-Gaussian setting with tail dependence would result in a paradoxical outcome whereby the system with tail dependence is less risky than the Gaussian system. By contrast, we suggest a methodology that ensures additivity of the risk contributions by construction and can be extended to a non-Gaussian setting as long as feasible sampling algorithms exist for the corresponding non-

Gaussian random variables.

Acharya et al. (2010) define systemic risk contributions as institutions' *marginal expected shortfall* which, at first glance, appears to be very similar to our approach. They define their measure by the worst 5% of equity returns measured from a historical time series at daily frequency. By contrast, we define the marginal expected shortfall following the Euler principle and incorporate all current information on portfolio risk, including, for instance, the current probabilities of default. Therefore, both implementations of the marginal expected shortfall measure differ substantially. Furthermore, the authors implicitly assume that the capital shortfall captured in their paper by decreasing market capitalisation is a reliable measure of a bank's systemic importance and a proxy for its negative externality. This definition implies that the externality of the bank's failure consists in the cost of its re-capitalisation, ie bail-out, meaning that the implicit government guarantee, taken as granted until credible resolution plans are implemented, will continue to be an acceptable option for tax payers. As opposed to than, we consider potential losses to the depositors (or the deposit guarantee schemes) and other creditors due to a bank's failure/restructuring as proxy for the effect of negative externalities imposed on society by systemically important banks. Finally, the authors embed their risk measure into an economic model to determine an optimal taxation policy for systemic risk, which is a valuable extension not addressed in our paper.

Finally, we refer to the paper by Tarashev et al. (2010). Having computed the systemic risk as the tail risk of a portfolio comprised of banks' debts, the authors use a game theoretical Shapley value concept in order to attribute the system-wide risk to the individual banks. Thereby, for each individual institution, its contribution to the risk of all possible subportfolios in which this institution is present has to be computed. Unfortunately, due to the rapidly increasing computational complexity,[1] the Shapley value methodology can only be applied to either small or homogeneous portfolios. Compared to that, the Euler allocation approach put forward in our paper remains feasible for large and heterogenous portfolios (or financial systems). Moreover, according to Denault (2001), the Euler allocation can also be motivated by game theory as the marginal contributions correspond to the Aumann-Shapley value that lies at the core of a coalitional game. Furthermore, compared to the one-factor asset return decomposition adopted by Tarashev et al. (2010), utilisation of a multi-factor model in our paper allows for a more flexible and realistic modelling of financial interlinkages.

Since the focus of this paper is on the application of the credit portfolio methodology using market and balance sheet data, we refer to the IMF's GFSR (IMF, 2009) as well as the references therein for more information on network analysis and domino effects. Moreover,

[1]There are 2^n possible subportfolios for n banks in the system

De Bandt and Hartmann (2000) provide a comprehensive survey on the theoretical and empirical literature on contagion in banking and financial markets as well as in payment and settlement systems. See also Nier et al. (2007) for further useful references. An example of an integrated systemic risk framework which combines standard techniques from market and credit risk management with a network model of a banking system is the OeNB's Systemic Risk Monitor, see Boss et al. (2006).

3. The modelling approach

In this section we describe our modelling approach for systemic risk measurement and allocation. In subsection 3.1 we make the notions of systemic risk and systemic importance operational by laying down a structural model for banks' asset returns and a credit portfolio model for the losses in a banking system. Subsection 3.2 presents a simulation solution for the risk measures proposed.

3.1. Model set-up

We model a banking system as a portfolio of n assets, the assets representing individual banks. The portfolio's loss distribution describes the risk of the entire banking system. The portfolio incurs losses only if one or more banks are in default. For the ith bank, the exposure at default, EAD_i, is defined as the book value of its liabilities after deducting capital. We normalise portfolio losses by using the institutions' relative portfolio weights given as $w_i = EAD_i / \sum_{i=1}^{n} EAD_i$. The loss given default, LGD_i, represents a fraction of the total liabilities which specifies the potential costs to the bank's creditors induced in the course of recovery/restructuring/resolution. An event of default occurs at a predefined time horizon (set to one year in this paper) with the unconditional default probability p_i. The default event is captured by the Bernoulli random variable $D_i \sim Be(p_i)$.

In the spirit of the structural credit risk framework, a bank is in default when its asset return hits or falls below its default threshold at the pre-specified time horizon. The default threshold is given as the quantile of the bank's asset return distribution at the point p_i. We assume that the standardised asset returns $\{X_i\}_{i=1,\dots,n}$ are multivariate normally distributed with a full-rank correlation matrix. To explain the origin of the linear dependence, we decompose $\{X_i\}$ into a systematic and an idiosyncratic component by means of a multi-factor model. Following Pykhtin (2004) we assume that the asset return of an institution i depends on a composite systematic risk factor Y_i, which is a convex combination of a set of independent standard normally distributed systematic risk factors $\{Z_k\}_{k=1,\dots,m}$ with $m \ll n$. The *idiosyncratic part* of the asset return variation is captured by an independent standard normally distributed shock ϵ_i.

The modelling framework for the risk drivers $\{X_i\}_{i=1,\ldots,n}$, the default indicators $\{D_i\}_{i=1,\ldots,n}$ and our target variable – the portfolio loss rate PL – can now be formally summarised as follows:

$$X_i = a_i Y_i + \sqrt{1-a_i^2}\,\epsilon_i, \qquad a_i \in (0,1) \tag{3.1}$$

$$Y_i = \sum_{k=1}^{m} \alpha_{ik} Z_k, \qquad \sum_{k=1}^{m} \alpha_{ik}^2 = 1 \tag{3.2}$$

$$Z_k, \epsilon_i \overset{iid}{\sim} N(0,1) \text{ for all } k = 1,\ldots,m \text{ and } i = 1,\ldots,n$$

$$D_i = 1 \Leftrightarrow X_i \in \left(-\infty, \Phi^{-1}(p_i)\right] \tag{3.3}$$

$$PL = \sum_{i=1}^{n} w_i \cdot LGD_i \cdot D_i. \tag{3.4}$$

In the expressions above, the factor loading a_i specifies the sensitivity of the particular institution to the systematic risk factor, and the asset correlation between distinct institutions i and j is given by $\rho_{i,j} = a_i a_j \rho_{Y_i,Y_j}$, where $\rho_{Y_i,Y_j} = \sum_{k=1}^{m} \alpha_{ik}\alpha_{jk}$ denotes the correlation between the two composite factors.

As already mentioned in section 1, we are primarily interested in the ES at the level q as a coherent measure of portfolio tail risk. But for the sake of completeness, we also report the formulae for VaR, which defines the threshold for the ES measure. Let us denote the (discrete) cumulative distribution function of the portfolio loss rate by $F_{PL}(\cdot)$ and its quantile function by $F_{PL}^{-1}(\cdot)$. Then, VaR and ES can be defined as follows:

$$VaR_q(PL) = F_{PL}^{-1}(q) = \inf\left\{x \in [0,1] \,:\, F_{PL}(x) \geqslant q\right\} \tag{3.5}$$

$$ES_q(PL) = \frac{1}{1-q}\int_q^1 VaR_t(PL)\,dt. \tag{3.6}$$

As shown in Acerbi and Tasche (2002), if the distribution of portfolio loss were continuous, (3.6) would coincide with the tail conditional expectation (TCE) defined as

$$TCE_q(PL) = E\left[PL \mid PL \geqslant VaR_q(PL)\right]. \tag{3.7}$$

For a discrete loss distribution, however, the expression above has to be augmented with a correction term which adjusts the TCE measure upwards if the probability of the portfolio losses at the point $VaR_q(PL)$ does not coincide with q. The TCE of a discrete distribution adjusted in this way, becomes the coherent ES measure:

$$
\begin{aligned}
ES_q(PL) = {} & E\big[PL \mid PL \geqslant VaR_q(PL)\big] \\
& + \frac{1}{1-q} VaR_q(PL)\big[F_{PL}\big(VaR_q(PL)\big) - q\big].
\end{aligned}
\tag{3.8}
$$

After computing the overall tail risk, we turn to the calculation of the individual risk contributions that satisfy the full allocation property, ie their sum equals the total system-wide risk. For this purpose we use the Euler allocation or the marginal risk contributions based on the derivatives of the tail risk measure with respect to the portfolio weights of individual positions. A marginal contribution measures the impact of a small change in the portfolio weight of a bank on the total tail risk of the whole portfolio. Multiplied by the exposure weight of the bank, it results in the bank's (additive) contribution to portfolio tail risk given as follows:

$$
VaR_q(w_i \mid PL) = w_i \frac{\partial}{\partial w_i} VaR_q(PL), \tag{3.9}
$$

$$
ES_q(w_i \mid PL) = w_i \frac{\partial}{\partial w_i} ES_q(PL). \tag{3.10}
$$

The Euler allocation principle has proved useful in portfolio-oriented risk management, particularly for the purpose of economic capital allocation, performance measurement, portfolio optimisation and risk-sensitive pricing. For more information on the concept of Euler contributions as well as related literature and economic motivation see Tasche (2008). For an axiomatic approach to coherent risk measures and capital allocation see Kalkbrener (2005).

In the next subsection we provide an estimation algorithm for portfolio tail risk and risk contributions based on importance sampling.

3.2. Estimation via importance sampling

In order to compute ES, VaR and the marginal risk contributions we employ the importance sampling (IS) methodology developed by Glasserman and Li (2005) for the Gaussian framework. Their two-stage IS algorithm provides an asymptotically efficient estimator for the low-probability $1-q$ of systemic events (which is equivalent to the estimation of VaR_q in terms of simulation efficiency). Moreover, Glasserman (2006) provides further results on the IS estimation of VaR, ES and corresponding tail risk contributions. For the sake of completeness we provide details on the IS algorithm adopted in this paper in the Appendix. We use that IS algorithm for simulation of the portfolio loss distribution $F_{PL}(\cdot)$.

On the basis of the simulated distribution $\hat{F}_{PL}(\cdot)$, we estimate the tail risk measures according to the following equations, where k denotes one of s simulation runs:

$$\widehat{VaR}_q(PL) = \inf\big\{x \in [0,1] \ : \ \hat{F}_{PL}(x) \geqslant q\big\}, \tag{3.11}$$

$$\begin{aligned}\widehat{ES}_q(PL) = & \frac{\sum_{k=1}^{s} PL^k \, \mathbb{1}_{[\widehat{VaR}_q(PL),1]}\big(PL^k\big)\, l\big(PL^k\big)}{\sum_{k=1}^{s} \mathbb{1}_{[\widehat{VaR}_q(PL),1]}\big(PL^k\big)\, l\big(PL^k\big)} \\ & + \frac{1}{1-q}\widehat{VaR}_q(PL)\big[\hat{F}_{PL}\big(\widehat{VaR}_q(PL)\big) - q\big].\end{aligned} \tag{3.12}$$

As for a suitable IS estimator for the tail risk contributions, we refer to Tasche (2000) for the results on the additive contributions associated with quantile-based risk measures. The author proves that under certain continuity conditions imposed on the joint probability distribution of the individual loss variables $L_i = w_i \cdot LGD_i \cdot D_i$, the marginal contributions derived via differentiation of VaR and TCE can be represented in terms of the conditional expectation:

$$w_i \frac{\partial}{\partial w_i} VaR_q(PL) = E\big[L_i \mid PL = VaR_q(PL)\big], \tag{3.13}$$

$$w_i \frac{\partial}{\partial w_i} TCE_q(PL) = E\big[L_i \mid PL \geqslant VaR_q(PL)\big]. \tag{3.14}$$

Obviously, the risk contributions given above fulfil the full allocation condition. Additionally taking the correction of the risk measure for a discrete loss distribution into account, we obtain the following IS estimators for the additive tail risk contributions:

$$\widehat{VaR}_q(L_i \mid PL) = \frac{\sum_{k=1}^{s} w_i \cdot LGD_i \cdot D_i^k \, \mathbb{1}_{\{\widehat{VaR}_q(PL)\}}\big(PL^k\big)\, l\big(PL^k\big)}{\sum_{k=1}^{s} \mathbb{1}_{\{\widehat{VaR}_q(PL)\}}\big(PL^k\big)\, l\big(PL^k\big)}, \tag{3.15}$$

$$\begin{aligned}\widehat{ES}_q(L_i \mid PL) = & \frac{\sum_{k=1}^{s} w_i \cdot LGD_i \cdot D_i^k \, \mathbb{1}_{[\widehat{VaR}_q(PL),1]}\big(PL^k\big)\, l\big(PL^k\big)}{\sum_{k=1}^{s} \mathbb{1}_{[\widehat{VaR}_q(PL),1]}\big(PL^k\big)\, l\big(PL^k\big)} \\ & + \frac{1}{1-q}\widehat{VaR}_q(L_i \mid PL)\big[\hat{F}_{PL}\big(\widehat{VaR}_q(PL)\big) - q\big].\end{aligned} \tag{3.16}$$

Applying the IS technique outlined in the Appendix instead of a plain Monte Carlo simulation, substantially reduces variance when estimating portfolio tail risk and individual risk contributions, as shown in Düllmann and Puzanova (2011).

4. Empirical results

In this section we carry out an empirical study of systemic risk and individual risk contributions for a system containing large, internationally active commercial banks. We first describe our dataset in subsection 4.1. We then report results and investigate the impact of the main risk drivers in the two subsequent subsections: results on systemic risk in subsection 4.2 and results on the banks' risk contributions in subsection 4.3.

4.1. Dataset

The dataset used for the empirical analysis comprises a sample of the world's largest commercial banks over a time span from January 1997 to January 2010. The number of banks varies between 54 and 86 depending on IPOs, mergers and data availability. The one-year probability of default is estimated on a monthly basis by the expected default frequency (EDF) obtained from Moody's KMV CreditEdge. The EDFs range from 0.01% to 19%, with the median value 0.07% before September 2008 and 0.32% thereafter. We set the EAD equal to the book value of the bank's liabilities net of capital, also obtained from CreditEdge on a yearly basis. We transform the yearly observations into monthly data by linear interpolation. In the absence of a reliable estimate of a bank's LGD, we use the value of 100% for all banks,[2] which implies the maximum loss rate. Since the LGD is modelled as a deterministic variable, the risk contributions are linear in LGD and, therefore, its specific number does not affect our main results.

We define the systematic risk factors by the geographical regions in which the banks are headquartered. Table 1 presents summary statistics of the size distribution of the banks in the sample across 6 regions: Europe, North America, South America, Africa, Japan, Asia and Pacific excluding Japan. The banks listed in the table accounted for about two-thirds of worldwide banking industry assets in 2007/2008 as approximated by the assets of the largest 1,000 banks reported by IFSL (2010).

We set the asset return correlation within the geographical regions to the asset return correlation average of 42%, estimated for large banks on the basis of Moody's KMV GCorr module, as reported by Tarashev et al. (2010, p 21). It implies homogenous factor loadings $a_i \equiv a = \sqrt{0.42}$. The heterogeneity in the dependence structure arises from the correlation between the region-specific systematic risk factors. As a proxy for the correlation coefficients of systematic risk factors, ie $\rho_{Y_{reg(i)},Y_{reg(j)}}$, we use the Pearson's correlation estimated from the monthly returns of the Dow Jones Total Market (DJTM) total return indices for the banking sector in the respective geographical regions, obtained from Datastream for the time

[2] Tarashev et al. (2010) set the LGD rate to 55% without giving any reasons.

period from the beginning of 2005 to the end of 2009. In Table 2 we report the final asset correlation matrix used in our study. The elements on the main diagonal equal the asset correlation between two banks headquartered in the same geographic region, $a^2 = 0.42$. The off-diagonal elements specify the asset correlation between the banks headquartered in two different regions, $\rho_{i,j} = a^2 \rho_{Y_{reg_i}, Y_{reg_j}}$.

Table 1: Liability (LBS) size distribution of all banks within the sample at the beginning of 2008, aggregated by country.

Region	Country	Number of banks	Aggregate LBS	
			USD billion	% of total
EU	Austria	1	265	0.49
	Belgium	2	1,286	2.39
	Denmark	1	606	1.12
	France	3	5,571	10.33
	Germany	4	4,155	7.71
	Greece	1	111	0.21
	Iceland	1	64	0.12
	Italy	3	2,146	3.98
	Netherlands	2	3,179	5.90
	Norway	1	244	0.45
	Russia	1	146	0.27
	Spain	3	1,988	3.69
	Sweden	3	1,122	2.08
	Switzerland	2	3,079	5.71
	United Kingdom	6	8,758	16.24
AMN	Canada	5	2,093	3.88
	USA	11	7,274	13.49
AMS	Brazil	3	352	0.65
AFR	South Africa	3	322	0.60
JP	Japan	5	4,577	8.49
AS&P	Australia	5	1,589	2.95
	China	10	3,456	6.41
	Hong Kong	2	212	0.39
	India	2	305	0.57
	Singapore	3	353	0.65
	South Korea	3	654	1.21
Total		86	53,907	100

The estimates of the correlation between the *banking sector indexes*, ie $\widehat{\rho}_{Y_{reg_i}, Y_{reg_j}}$, can be figured out from Table 2. They reveal substantial differences between geographical regions, which supports our choice of a multi-factor instead of the single-factor model. For example, the correlation between the bank indexes in Africa and Japan is as small as 32%, whereas the correlation between the bank indices in Europe and North America is as large as 80%.

Table 2: The asset correlation matrix used in the empirical example.

	EU	AMN	AMS	AFR	JP	AS
EU	0.42	0.33	0.27	0.26	0.18	0.36
AMN		0.42	0.18	0.19	0.16	0.31
AMS			0.42	0.21	0.19	0.28
AFR				0.42	0.14	0.26
JP					0.42	0.19
AS						0.42

We use the dataset described to calculate the portfolio ES as well as the individual ES contributions for all banks that are in the sample at the end of each particular month and discuss the results in the following.

4.2. Drivers of systemic risk of the banking system

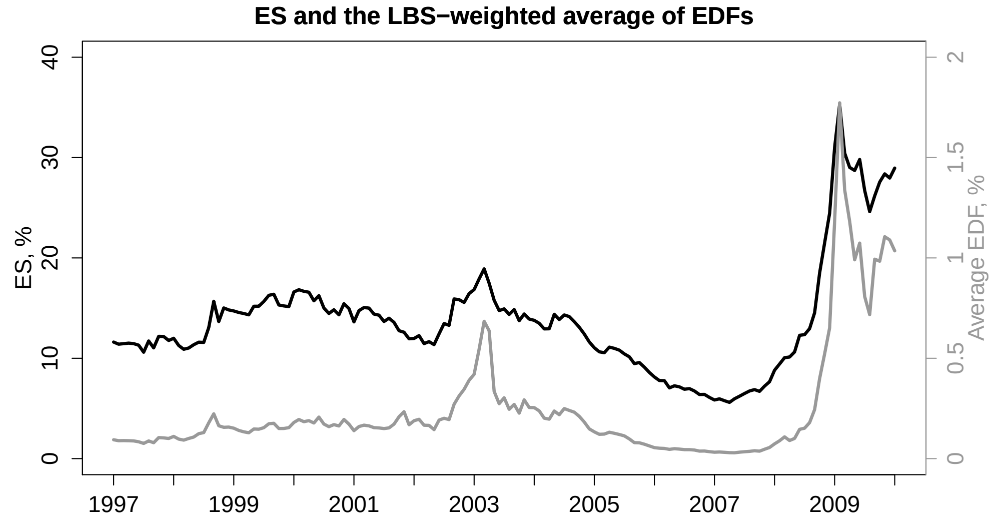
In this subsection we investigate the impact of the main risk drivers – the average default conditions in the system, the size and concentration of the system and the correlation among the banks' assets – on the evolution of systemic risk over time.

Figure 1 shows evolution of the ES over the sample period in comparison with the weighted average of the underlying EDF figures. The ES ranges from 6% to 35% of the total liabilities (LBS) of all banks that are in the sample at a given point in time. In monetary units, the ES is lowest in April 2007, amounting to $845.8bn. It peaks in February 2009 amounting to as much as $19,890bn. As can be seen from the graph, the ES very closely matches the pattern of the average estimated probabilities of default, indicating that overall default risk is the main driver of the tail risk in a banking sector. The Pearson's correlation between the two variables amounts to 92% and the estimated Spearman's correlation equals 88%.[3] For the typically small EDFs (more than 95% of EDF figures in the sample are less than 1%), the ES grows linearly with the average individual default risk. For larger EDF figures, however, the ES becomes less sensitive to the average default risk, which can also be seen in Figure 2. We will come back to that figure when discussing the impact of concentration and correlation on systemic risk.

Further analysis shows that the ES is not very sensitive to the variation in the total exposure at risk, ie total LBS, which increased more or less steadily over the sample period,

[3]Spearman's correlation is a measure of concordance. It increases in magnitude as the two variables become closer to being perfect monotone (possibly non-linear) functions of each other.

Figure 1: Evolution of the portfolio expected shortfall (ES, black line, left axis) expressed as a percentage of the total portfolio liabilities (LBS). Also plotted is the weighted average of EDFs (gray line, right axis), the weights being the shares of individual banks in the total LBS.



having started at \$7,121bn and peaking at \$57,950bn. The estimated Spearman's correlation between the ES (as a percentage of LBS) and LBS is not significantly different from zero (at the 95% confidence level).

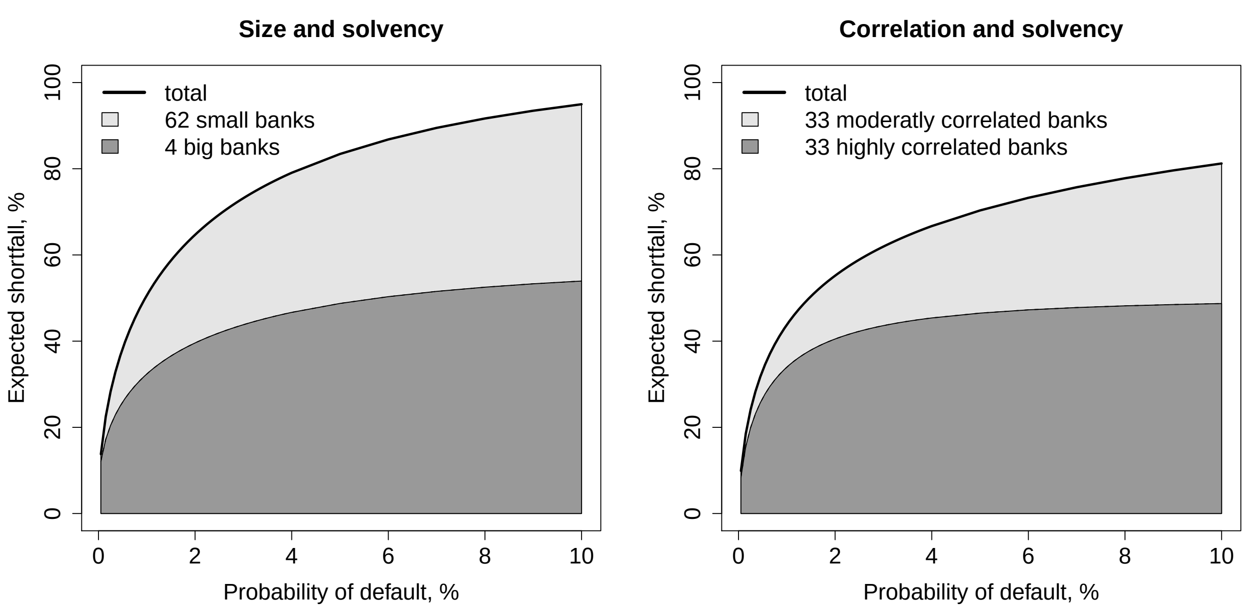
The fact that the ES is lowest at the onset of the financial crisis is due to the market-price-based point-in-time estimates of the banks' default probabilities obtained from Moody's KMV CreditEdge. Those estimates rely on the expectations of the market participants about the future performance of a bank, which is incorporated in the bank's stock returns. Because the market participants were very optimistic about the risk-taking behavior of banks precisely during the phase when risks had been accumulated in the banking sector, the EDF figures are lowest in the run-up to the sub-prime crisis. This is why the systemic risk measure in our example is also a point-in-time measure, ie a measure that captures all relevant information available today about the forthcoming one-year period. It cannot be considered as a forecast of systemic risk in later periods in the future. In order to obtain a forecast, some reliable projections for the future individual probabilities of default (and for asset correlation) should be used as model inputs. Moreover, by using stressed probabilities of default (and asset correlations), stress tests can be easily accomplished within the modelling framework suggested in this paper.[4]

In order to assess the sensitivity to the concentration of a financial system, we isolate its impact for different levels of default probabilities using a simulation exercise based on a

[4]The results of a stress-test application, now work in progress, will be reported elsewhere.

stylised portfolio. For this purpose we consider the special case of a single-factor model and define a stylised banking system populated by 66 banks which all share the same probability of default. The banks can be separated into two groups, each accounting for 50% of the overall liabilities. We define the first group as a set of 62 equally-sized small banks and the second group as a set of 4 equally-sized large banks. To keep the exposures to the single systematic risk factor constant across the system, we set the pairwise asset correlation to 42%. The results for this financial system are presented in the left-hand panel of Figure 2. Notwithstanding the fact that both groups are equally sized, the group of the big banks accounts for more than 50% of the overall ES owing to its greater concentration. This effect is even more pronounced for small probabilities of default (below 1%), which are typical for the banking sector. For instance, for the common probability of default of 0.15% the share of the four large banks in the overall ES is as high as 77%. It still amounts to 64% for a default probability of 0.5%. Hence, among relatively sound institutions, the banks with larger exposures at distress affect the overall tail risk more. Rising probabilities of default ceteris paribus initially lead to considerably higher tail risk, but then the incremental tail risk diminishes faster in PD for the concentrated banking sector than for the sector comprising many small banks.

Figure 2: Drivers of systemic risk. In each case, two groups of banks are considered. Those two groups differ from each other either in terms of the size of the banks included (left-hand plot) or the magnitude of the intragroup correlation (right-hand plot). Each group accounts for half of the total portfolio exposure. The expected shortfall is given as a percentage of the total liabilities in the system.



To investigate the ES's sensitivity with respect to the common risk factor, we isolate the impact of asset correlation by dividing our highly stylised portfolio into two homogeneous groups comprising 33 equally-sized banks each. The first group is only moderately exposed to the systematic risk with a pairwise intragroup asset correlation of 20%. By contrast, the banks assigned to the second group are highly correlated with a coefficient of 60%. The right-hand panel of Figure 2 illustrates the intuitive result that a higher sensitivity to the systematic risk factor, ie a higher asset correlation, is linked to a higher systemic risk contribution, since the probability of joint failures increases. For the common probability of default of 0.15%, the share of the 33 highly correlated banks in the overall ES is as high as 84%. It still amounts to 80% for a default probability of 0.5%.

Using the same stylised portfolio we can additionally investigate the interaction between different risk drivers and their joint influence on systemic risk. For this purpose we conduct another simulation study for different values of PDs, asset correlations and concentration levels. The results collected in Table 3 confirm the conclusions just described and deliver additional insights. Thus, if the banks are equally sized and the sectors are equally concentrated (see the last and second last panels), the ES increases as the asset correlation and/or average default risk rises. For a given correlation structure, however, a highly concentrated sector containing only a few large banks makes a disproportionately large contribution to the ES (compare the third panel with the fourth one). This effect is more pronounced for smaller PDs, indicating that even well capitalised large banks may pose a high risk for a concentrated banking system. If a banking sector is concentrated *and* the correlation among the banks is exceptionally high, the sector's ES contribution becomes very high especially for larger default probabilities; thereby the contribution of the small-bank sector declines materially (compare the second panel with the third one). This indicates that a capital deterioration for large, highly correlated banks very quickly becomes extremely dangerous. Thereby, many small banks can become systemic as a herd when the correlation among them is high. But they would still contribute less than comparably correlated big banks, at least as long as the correlation is not perfect. Comparing the first lines of the first and third panels we see that, for larger PDs, the strongly correlated small banks may produce enough joint default events to match the tail risk of failures in a highly concentrated, but less correlated large-bank sector. But when default and correlation conditions are the same for small and large banks, hypothetical mergers of small banks do not increase the tail risk in the system considerably, whereas a hypothetical splitting of large banks reduces systemic risk substantially (compare the second panel with the fourth one).

Summarising, our findings point to the following interpretation of the risk drivers' impact on systemic risk:

Table 3: ES's sensitivity to default probability, correlation and concentration

		$p = 1\%$	$p = 0.5\%$	$p = 0.1\%$
ρ_1, ρ_2	Number of banks	Expected shortfall		
42%, 42%	$n_1 = 62$	18.23	12.46	4.84
	$n_2 = 4$	32.69	26.42	14.78
	$n = 66$	50.92	38.89	19.61
20%, 60%	$n_1 = 62$	8.73	5.62	2.17
	$n_2 = 4$	42.04	33.13	17.80
	$n = 66$	50.76	38.74	19.96
	$n_1 = 4$	18.93	14.26	10.77
	$n_2 = 62$	28.90	22.62	6.36
	$n = 66$	47.83	36.88	17.13
	$n_1 = 33$	9.50	6.23	2.27
	$n_2 = 33$	32.91	25.37	11.77
	$n = 66$	42.41	31.60	14.04
10%, 30%	$n_1 = 33$	5.31	3.66	1.44
	$n_2 = 33$	14.64	11.14	4.03
	$n = 66$	19.95	14.73	5.47

Note: We consider the one-factor model and denote the intrasector correlation by ρ_j, $j = 1, 2$. The intersector correlation equals $\sqrt{\rho_1 \rho_2}$. The number of banks in sector j is given by n_j with $n = n_1 + n_2$. All banks have the same probability of default p. Expected shortfall (ES) and ES contributions are expressed as a percentage of the total liabilities in the banking system.

- Our empirical analysis has identified changes in the overall default conditions in the banking sector as the main driver of the evolution of systemic risk over time, whereas changes in the total size of the banks' liabilities are far less important in this context. (The particular impact of the asset correlations was not investigated in our analysis since they were kept constant over time.)

- Concentrated sectors containing a few large banks impose a considerable systemic risk even if these banks are well capitalised. Their impact on systemic risk increases fast when the equity capital buffer deteriorates.

- A high asset correlation makes small banks systemic as a herd although they would not contribute to systemic risk as much as comparably correlated large banks with the same aggregated amount of liabilities.

Having investigated the main drivers of systemic risk, we proceed with an analysis of the drivers of individual risk contributions, and thus the drivers of systemic importance.

4.3. Drivers of systemic importance of individual institutions

Now we turn back to the empirical dataset described in subsection 4.1 in order to analyse the risk drivers of banks' systemic importance. Thereby we first consider the evolution of systemic risk contributions over time and then the cross-sectional risk distribution.

Figure 3 shows dynamics of the ES contributions of 15 selected banks as a percentage of the overall liabilities compared with the banks' relative size (their share in the overall liabilities) and EDFs. Because in our empirical example the asset correlation is fixed over time, we focus on the impact of the banks' default risk and size and investigate these drivers of individual banks' systemic importance with regard to the *time dimension.* For most banks in the sample, the relationship between the ES contribution and the probability of default is slightly concave. The Spearman's correlation is significantly positive for all but 2 banks (for which it is not significantly different from zero) and ranges from 15% to 99% with the median observation of 76%.

The empirical evidence for a link between the *relative* ES contribution and a bank's share in the total liabilities, which can be considered as a proxy for the relative size of a bank, is more ambiguous in the time dimension. As Figure 4 exemplifies, banks may become more or less systemically important even if their relative size does not change. Moreover, the Spearman's correlation in the sample is not significantly different from zero for 21% of the banks. Only for some 5 banks can a linear relationship be assumed. For the other banks with significantly positive Spearman's correlation coefficients no definite functional relation can be identified. Overall, the evolution of the systemic risk contribution of a bank is mostly driven by the changes in the bank's default probability, whereas the changes in relative size of the bank are far less important. However, the *co-movement* of the bank's share in the total liabilities of the banking system with its own default risk can amplify changes in its ES contribution. That is, when a bank grows faster than the competitors without bolstering its capital base appropriately, which means ceteris paribus a rising leverage and higher probability of default, then the bank's contribution to the over-all risk of the system increases rapidly.

Turning to the distribution of systemic risk among the individual banks at a given point in time, the impact of institutions' size is more pronounced in the *cross-sectional* dimension. The cross-sectional estimates for the Spearman's correlation between the relative size and the relative ES contribution range from 79% to 94% over the sample period, with a median observation of 90%. By contrast, the cross-sectional Spearman's correlation between the individual probability of default and the ES contribution is insignificant for only 51 out of 157 months and no definite functional relation can be identified. In other words, for a given level of systemic risk at a given point in time, the risk allocation among the contributors is governed to a large extent by the banks' relative size. Moreover, as can be seen in Figure 4,

Figure 3: Dynamics of the banks' individual contributions to the portfolio expected shortfall as a percentage of the total portfolio liabilities (solid black lines) in comparison with the EDFs (solid gray lines) and individual shares in the total liabilities in the system (dashed lines). The y-axes are given in percentages.

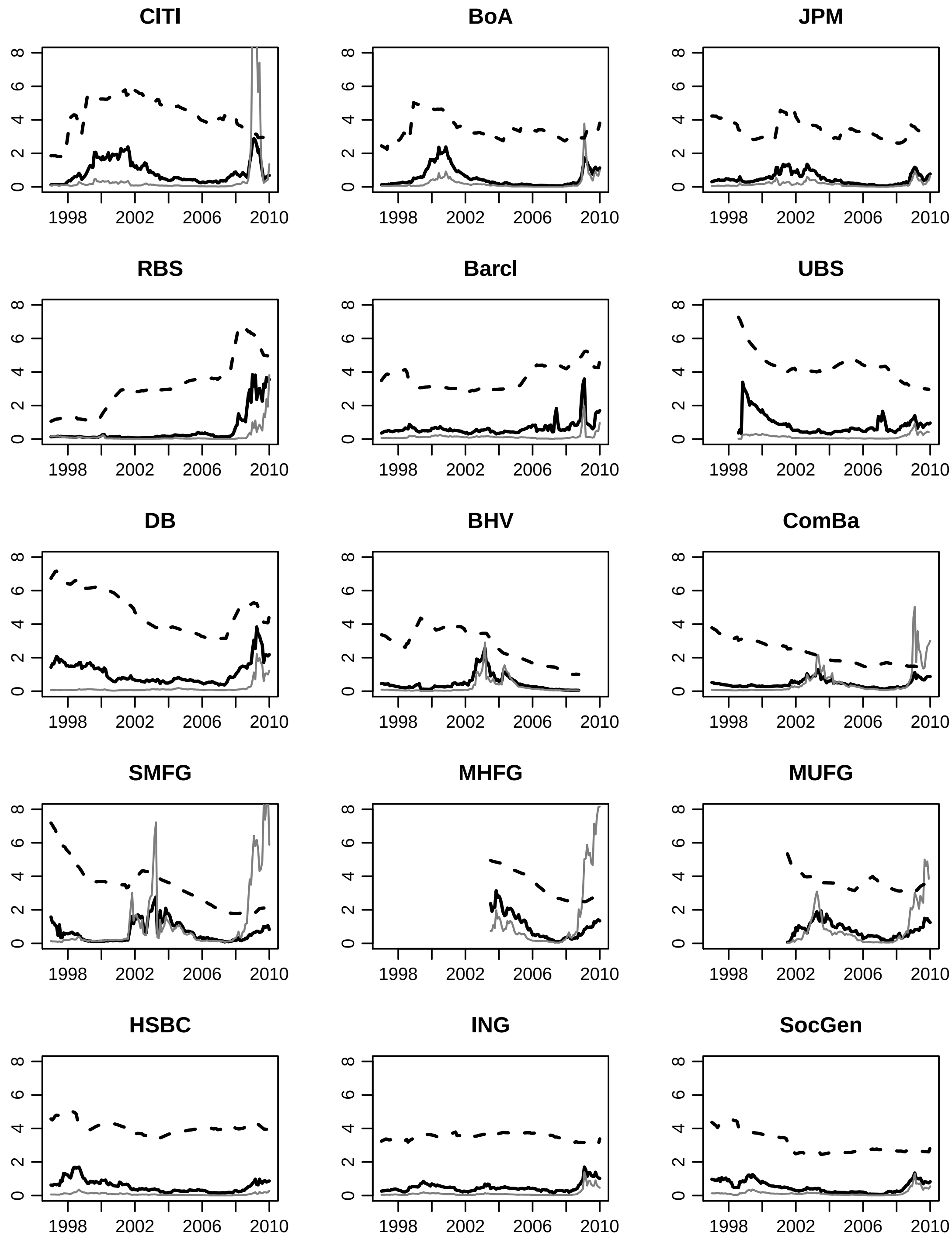
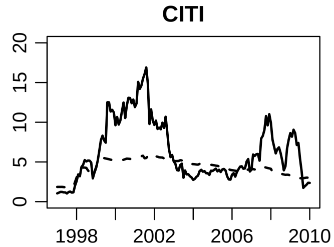


Figure 4: Dynamics of the CITIGROUP's individual share in the portfolio expected shortfall (solid line) in comparison with its share in the total liabilities in the system (dashed line). The y-axis is given in percentages.

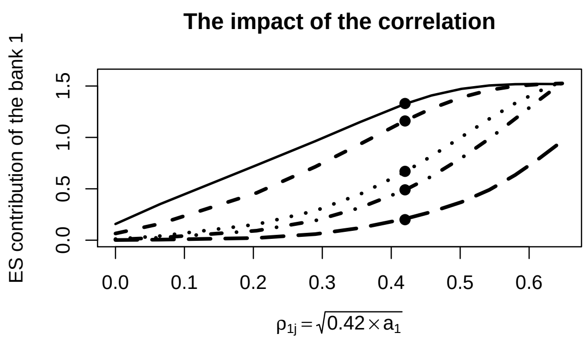


large banks may make a disproportionately large contribution to the tail risk of the system they belong to. This is the case when their share in the system-wide ES is greater than their share in the total liabilities of the system. So, there are always some 4 to 10 biggest risk contributors in our sample that account for more than 50% of the ES. Although those banks are at the same time the largest in the sample, their share in the sector-wide liabilities is well below 50% (it lies between 15% and 37%, to be precise).

Speaking about the banks' contributions to systemic risk at a particular point in time, it is important to stress that not only a bank's individual characteristics affect its systemic importance, but also the characteristics of the system which the bank is a part of. A bank's risk contribution depends on the size of other banks in the system and on the respective default probabilities as well as on asset correlations with other banks. This can be, for example, deduced from the analytical formulae for approximation of the ES contributions derived in Düllmann and Puzanova (2011).

Turning to the issue of asset correlation, which was fixed over time in the empirical exercise, we conduct a simulation study for a highly stylised bank system in order to isolate the impact of systematic risk factor sensitivity on a bank's ES contribution and, thus, on its systemic importance. For the special case of the one-factor model, in a given system comprising 66 equally sized banks we set the factor loadings of all but one bank to $a_i = \sqrt{0.42}$, for $i \in \{2, \ldots, 66\}$. For a given level of the common default probability p, those banks contribute equally to systemic risk. The first bank, however, may contribute more or less than the others, depending on its factor loading a_1. We try various values for a_1 and report the outcomes in Figure 5. The contribution of the first bank is the lowest for $a_1 = 0$, ie zero correlation with other banks. In this case the contribution to the ES at the confidence level $q = 0.999$ amounts to the stand-alone contribution, ie $1/66 \times p \times 100$ per cent of total liabilities for $p \geq 1 - q$. The contribution is the highest for $a_1 = 1$, ie maximum asset correlation in this example, given by $\sqrt{0.42} \approx 0.65$. Thereby, the maximum value of the ES contribution does not exceed $1/66 \times 100 \approx 1.515\%$ per cent of total liabilities.

Figure 5: The impact of the asset correlation on the systemic importance of a bank for five different values of the common probability of default: $p = 0.1, 0.05, 0.01, 0.005, 0.001$ (from the top to the bottom). The expected shortfall (ES) contribution of any other bank in the system is marked by a filled circle for each level of p. The values on the y-axis are given as percentages of the total liabilities in the system.



Summarising, our findings point to the following interpretation of the risk drivers' impact on the banks' systemic importance as measured by their contribution to the ES of the system:

- Evolution of the individual systemic risk contributions over time is governed to a large extent by changes in the probability of default and is less sensitive to changes in the institutions' relative size.
- The most dangerous development from the perspective of financial stability is if a bank grows faster than its competitors without strengthening its capital base, which leads to increasing leverage.
- Given a particular level of tail risk at a particular point in time, the distribution of the relative risk contributions depends strongly on the size distribution of the banks. But there are institutions whose share in the system's ES is greater than their share in the overall liabilities, indicating a disproportionately large impact on the system.
- If a bank's assets are not perfectly correlated with the composite systematic risk factor, its ES contribution is always less than the expected loss given the bank's default (here: liabilities). A bank's contribution increases along with its sensitivity to the systematic risk factors, whereby the increase is steeper for higher probabilities of default.

Overall, the findings discussed in this section confirm the need to study systemic risk in a portfolio context instead of on a single entity basis. It shows that high concentration increases the fragility of a banking sector substantially, all other things being equal. With regard to policy implications, our results emphasise that tailoring macroprudential instruments simply to the size of a financial institution would not adequately address the issue of negative externalities: For the major systemic risk contributors, the share in the ES may be much higher than the relative size would suggest. Our analysis of systemic risk drivers indicates that the main focus of macroprudential banking supervision should be on a solid capital base throughout the cycle and de-correlation of banks' asset values.[5]

5. Policy tools – A capital charge for systemic risk and a mitigant of procyclical effects

Macroprudential regulation should address both dimensions of systemic risk, as underlined by Borio (2009) among others: The *cross-sectional dimension*, considered in subsection 5.1, relates to the distribution of aggregate risk in a financial system at a given point in time. The corresponding policy issue consists in the calibration of prudential instruments according to the banks' contributions to systemic risk. The *time dimension*, addressed in subsection 5.2, covers the evolution of aggregate risk over time. The corresponding policy issue is to find a way to reduce potentially procyclical effects of regulatory tools based on a measure of system-wide financial risk.

5.1. Cross-sectional implementation and systemic capital charge

Our findings confirm that strengthening the capital base in the banking sector (which entails smaller default probabilities and leverage) would, in fact, significantly reduce systemic risk. This result is in line with the ongoing regulatory debate. In particular, the policy setters – FSB, IMF and BIS – have been jointly working on macroprudential frameworks (FSB et al., 2011) whose broad aim is to reduce the probability of failure of global systemically important banks (G-SIBs) by increasing their going-concern loss absorbency and to reduce the extent or impact of G-SIBs' failure by improving global recovery and resolution frameworks.[6] In order to achieve the first goal, the Basel Committee on Banking Supervision issued a consultative document BCBS (2011) in which it elaborates an assessment methodology for systemic importance of G-SIBs using a set of balance-sheet and other indicators complemented with supervisory judgement. The selected indicators reflect five categories of systemic importance – the size of banks, their interconnectedness and global activity, their substitutability and complexity –, each of these categories being equally weighted to form a score value. Based on that score, the BCBS groups the G-SIBs identified into four buckets of increasing systemic importance and proposes an additional loss absorbency requirement of 1 to 2.5 per cent of

[5] For example, the ring-fencing of retail banking and prohibition of activities which would lead to a market risk exposure or an exposure against other financial institutions, as suggested in ICB (2011), aims in this direction.

[6] In terms of our portfolio approach, the intended policy measures aim at reducing default probabilities and LGDs of G-SIBs.

risk-weighted assets. An extra bucket with a 3.5 per cent capital surcharge is envisaged to penalise banks that keep becoming more systemically important.

Our model-based approach could also be used to identify, say, the top 30 systemically important banks according to banks' relative contributions to systemic risk:[7] Table 4 shows which banks were in the top 30 throughout their respective sample period. There is a strong overlap between banks in the first column in Table 4, which were permanently observed among the top 30 contributors to systemic risk in our empirical study, and the set of 29 G-SIBs who have been identified and publicly announced by the FSB on 4th November 2011 (FSB, 2011). ABN AMRO, which was in the top 30 during its sample period according to our model, dropped out of the sample at mid-2008 due to acquisition. It is understandably not on the list of G-SIBs based on data for end-2009 (BCBS, 2011, p 10). Thus, INDUSTRIAL AND COMMERCIAL BANK OF CHINA (ICBC) is the only bank from the first column of the Table not on the G-SIB-list of the FSB. Five more banks from the list of G-SIBs can be found in the second column of the Table: RBS, which was not among top 30 only in 2002-2003, DEXIA and UNICREDIT, which were among top 30 from 2003 on and BANK OF CHINA and NORDEA, which were among top 30 from 2008 on. Another bank from the list, WELLS FARGO, can be found in the third column. It was among the top 30 from 1999 to 2003 as well as from 2009 on. As to the remaining five banks which are on the list of G-SIBs but not in the Table: (i) BANK OF NEW YORK MELLON and STATE STREET (two rather small but important US banks, each having about $20,000bn in assets under custody) could not be identified as top-30 contributors to systemic risk by means of our model and (ii) BANQUE POPULAIRE CdE (formed by the 2009 merger of CNCE and BFBP, (semi-) cooperative banks) as well as GOLDMAN SACHS and MORGAN STANLEY (two security brokers and dealers) are not contained in our sample focused on the world's largest *commercial* banks.

Overall, our model-based approach matches very closely the results presented by FSB.[8] However, to come up with the list of G-SIBs, not only data from many sources had to be collated together. The information on some of the indicators even was directly collected from banks, see BCBS (2011). By contrast, in our empirical analysis we only use balance-sheet and market information available to the public. We are able to produce results for an extended time span and to update them frequently without any additional effort on collecting the data.

Of course, the relative bank ranking based on marginal risk contributions can be used in order to build discrete buckets of systemically important banks in the sense of the BCBS'

[7] In fact, the BCBS admits that a model-based approach for estimating of individual banks' contributions to systemic risk would be another option, but also points out that such models are still "at a very early stage of development" to be implemented in regulatory practice (BCBS, 2011, p 3).

[8] The discrepancies might be attributable to the supervisory judgment option, see BCBS (2011, p 3).

Table 4: Commercial banks ranking among top 30 contributors

Permanently	Frequently	Occasionally
ABN AMRO	**BANK OF CHINA**	BANK OF NOVA SCOTIA
BANK OF AMERICA	BBVA	DANSKE BANK
BARCLAYS	BHV	KBC GROUP
BNP PARIBAS	CANADIAN IMPERIAL BANK	SUMITOMO
CITIGROUP	CHINA CONSTRUCTION BANK	TORONTO DOMINION BANK
COMMERZBANK	**DEXIA**	**WELLS FARGO**
CRÉDIT AGRICOLE	INTESA SANPAOLO	WOORI
CREDIT SUISSE	**NORDEA**	
DEUTSCHE BANK	RBC	
HSBC	**RBS**	
ICBC	RESONA HOLDINGS	
ING	**UNICREDIT**	
JPMORGAN CHASE	WACHOVIA	
LLOYDS		
MITSUBISHI UFJ		
MIZUHO		
SANTANDER		
SOCIÉTÉ GÉNÉRALE		
SUMITOMO MITSUI		
UBS		

Note: The banks in the first column were permanently observed among the top 30 contributors to systemic risk throughout their respective sample periods. The banks in the second column were in the top 30 in $\geq 50\%$ of their respective observations. The banks in the third column were in the top 30 in $\geq 30\%$ of their respective observations.

approach outlined at the beginning of this subsection. Such a bucketing approach with a penalty bucket may, to some extent, provide incentives for banks become less systemically important by actions that reduce the value of the indicator variables. This approach does not establish a direct link between the bank's risk contribution and the capital charge. Therefore, it is not designed to internalise the losses or costs associated with a systemic event. By contrast, our model-based marginal ES approach would directly address the issue of banks' systemic importance and their impact on the system by tailoring the capital requirements to the system-dependent risk contributions. Therefore, from a conceptual point of view and disregarding model risk, our approach may be better suited to internalise the negative externalities and incentivise the banks to shrink their ES contributions by reducing (risk-weighted) leverage and/or the correlation with other banks.[9]

[9]Like other approaches known to us, the model presented in this paper only relies on the correlation

The basic idea is that there should be enough capital in the system to withstand the losses which may materialise during a systemic event. This could be achieved by setting the banks' total minimum capital requirements in accordance with their systemic risk contributions. However, it may be the case that due, for instance, to low correlation with other banks in the system, a bank's ES contribution falls short of microprudential capital requirements (which are stand-alone, not system-dependent and subject to a different assessment methodology). Therefore, we suggest a continuous, bank-specific *systemic capital charge* (SCC) be determined on top of the minimum required capital (MRC). We consider the ith institution at time t that is subject to MRC. The key idea is to charge the difference between the systemic risk contribution and the original regulatory minimum capital requirement. If the MRC exceeds the systemic risk contribution of a bank, then no add-on for systemic risk is charged. The following equation summarises this definition of an ES-based SCC:

$$SCC_i(PL,t) = \max\left\{ EAD_i(t)\frac{\partial}{\partial w_i(t)} ES_q(PL,t) - MRC_i(t), 0 \right\}. \tag{5.1}$$

According to the banks' figures on their total regulatory capital holdings, in 2006/2007 54 out of 63 banks, for which the data could be obtained from Bankscope, were well capitalised in the sense that their reported capital exceeded $MRC_i(t) + SCC_i(PL,t)$ as defined in (5.1). In 2008/2009 the same was true for only 11 out of 72 banks.

Further refinements could be contemplated when computing ES contributions, which are inputs to equation (5.1). For example, where a credible recovery and resolution plan shows that the losses associated with a bank's failure can with certainty be reduced to a fraction of the bank's liabilities, we can incorporate this information into the model by adjusting LGD_i.

It should be pointed out that computing macroprudential capital requirements may be more complex than computing risk contributions themselves. The simple formula (5.1) suggests setting the capital surcharges according to the currently observed risk and capital levels and does not take into account subsequent changes in the overall systemic risk landscape. But, once new capital requirements are implemented, the banks' probabilities of default (and potentially also the asset correlations) decline, resulting in lower tail risk and changing absolute and relative risk contributions. For this reason Gauthier et al. (2010) suggest an iterative procedure to solve for the fixed point at which the capital allocation in the system is consistent with the banks' risk contributions. Such reallocation of capital not only means that undercapitalised banks raise capital or de-leverage, but also that overcapitalised banks increase their

between financial institutions. We are not aware of any approaches in this field of research that would allow an investigation of the causality links between the behavior of a single bank and the response of the system, or the other way around.

leverage. A superior approach, that is not simply based on the *re*allocation of the given total capital, would require knowledge of the optimal total level of capital in the banking system. This optimal amount of capital is not necessarily to cover systemic risk completely, since the tail risk in the system can be far too high to be fully backed with capital.[10] Therefore, the level of the total capital requirements could be on average lower than the amount of the ES. The remaining systemic risk should be borne by uninsured debt holders (not by the public).

To sum up, a capital surcharge based on banks' individual contributions to system-wide risk would reduce the competitive advantages of being systemically important. This is a highly relevant consequence especially until credible resolution regimes are in place. Moreover, the SCC would strengthen the capital base of those institutions which contribute to the fragility of the banking sector more than their stand-alone risk profile suggests.

5.2. A mitigant of procyclical effects

Within the presented framework the evolution of systemic risk over time is mainly driven by the co-movement of the probabilities of default in the banking sector. In Figure 1 we saw how the use of point-in-time estimates of the default probability based on market prices can induce procyclicality in the tail risk measure. Market-based measures suggest that the system is strongest in times when market volatility is below average and market participants accumulate large amounts of risk. During a downturn or turbulent markets characterised by rising volatility/uncertainty, probabilities of default increase and the tail risk measure soars. Therefore, when the SCCs are introduced such as to incentivise the banks to reduce their risk contributions, one question remains: how to mitigate a possible destabilising effect of cyclical fluctuations in the over-all level of systemic risk especially during boom phases when risks are estimated to be very low? A possible solution is described below.

Consider the situation during a boom, when the market volatility is exceptionally low so that default probabilities based on market information may underestimate the actually excessive risk taking in the system. This may lead to an adverse effect that the ES-based SCCs are the lowest just in the run-up to a bust. To counteract such an effect we suggest that the regulators adjust their tail-risk tolerance level over time using $q(t)$ instead of a fixed q. A possible way of doing so, would be to link $q(t)$ to the cross-sectional exposure-weighted

[10]During the time period under consideration the system-wide exposure (ie $\sum_{i=1}^{n} LBS_i(t)$, with n rising over time) increased from 23 to 100 per cent of the global GDP whereas the amount of the tail risk varied between 6.8 and 29 per cent of the global GDP according to $ES_{q=0.999}$. The IMF's figures on the world GDP were taken.

average of estimated default probabilities in the banking sector:

$$q(t) = 1 - \sum_{i=1}^{n} w_i(t) \cdot p_i(t). \tag{5.2}$$

This implies higher $q(t)$ (or lower tail-risk tolerance $1 - q(t)$) during the times, when the market-information-based probabilities of default are very low. Since the ES is the higher the further we go into the tail of the probability distribution of losses, $q(t) > 99.9\%$ would result in $ES_{q(t)} > ES_{q=0.999}$ during a boom phase. Therefore, the difference between $ES_{q(t)}$ and ES_q enables us to identify the periods of an overoptimistic behavior in the financial sector, when the risk of default is perceived to be exceptionally low.

As long as such overoptimistic market conditions last, it may be advisable to put a *countercyclical capital buffer* (CCB) on top of the SCC. The CCB for an individual bank can be calculated as the amount of $ES_{q(t)}$ not covered by the sum of the bank's MCR and SCC:

$$CCB_i(PL,t) = \max\left\{EAD_i(t)\frac{\partial}{\partial w_i(t)}ES_{q(t)}(PL,t) - \big(MRC_i(t) + SCC_i(PL,t)\big), 0\right\}. \tag{5.3}$$

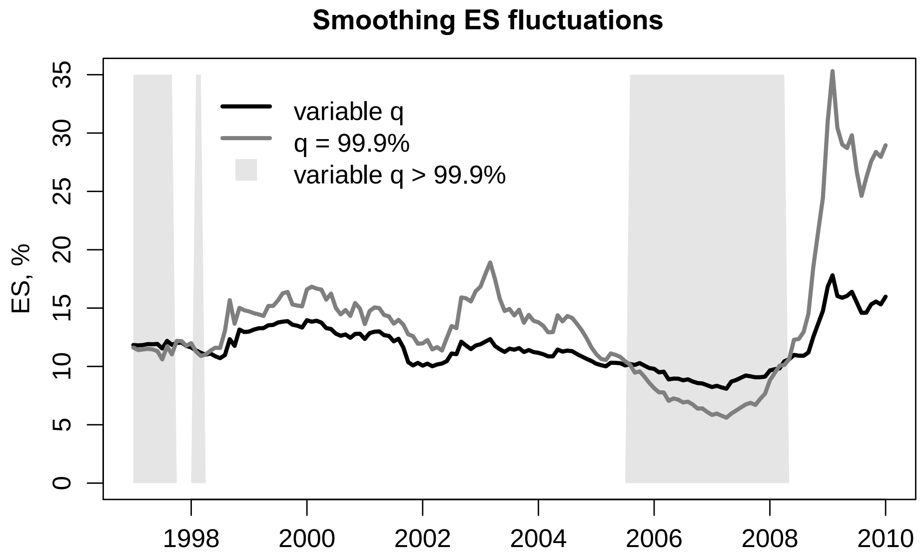
The CCB will be likely introduced only on an infrequent basis and released as soon as market conditions worsen (ie when $ES_{q(t)}$ approaches ES_q). As in the case of the SCC, however, a deeper understanding of the possible reaction of the banking sector to the additional capital requirements remains a topic for further research.[11]

The evolution of $ES_{q(t)}$ for the portfolio under consideration as compared to $ES_{q=0.999}$ is shown in Figure 6. Remarkably, the range of variation of the ES based on this time-varying probability level is considerably smaller than for $ES_{0.999}$ (8.09% to 17.81% versus 5.61% to 35.31% of total liabilities). The probability level $q(t)$ ranges from 98.23% to 99.97% with the median value of 99.85%. It is the highest from the second half of 2005 until the second half of 2008 (a shaded area in the figure) – the period of a risk build-up before the outbreak of the global financial crisis. In this period of low-volatility markets and excessive risk taking CCBs should rightly have been introduced.

To sum up, the presented method for measuring systemic risk and individual risk contributions based on an adoption of the credit portfolio approach to a banking system has an appealing feature that not only it can be applied for the identification of the systemically important institutions. More than that, it can be used as an integrated approach for calculation of both the systemic capital charges and countercyclical capital buffers, therefore addressing

[11] As well as the question, whether or not the CCBs based on $ES_{q(t)}$ as suggested in this paper are in line with the Basel III provisions on CCBs, which are supposed to counteract an excessive credit growth.

Figure 6: Evolution of the portfolio ES calculated according to the time-varying tolerance levels $q(t)$ (black line) versus ES at the constant tolerance level $q = 99.9\%$ (gray line).



both cross-sectional and time-series dimensions of systemic risk.

6. Conclusions

Addressing the system-wide risk of a banking system through macroprudential regulation requires an approach that internalises the potential costs of a systemic failure. We develop such an approach by, first, assessing systemic risk and, second, attributing this risk to individual banks, while the emphasis is on the attribution method. We employ for this purpose the Euler allocation principle, which is widely used in the risk management of financial institutions.

In this paper a banking system is modeled as a portfolio consisting of those banks in the global financial system which may be deemed systemically important. From a public purse perspective, we model systemic risk in terms of the expected shortfall (ES) of this portfolio. The expected losses conditional on exceeding a given level of regulatory tolerance reflect the potential costs incurred by banks' depositors and investors in a low-probability event such as a systemic crisis.

The portfolio approach used has the additional advantage that the modelling requirements are based on standard risk management techniques. The method provides a tool to assess the systemic importance of major financial institutions based on publicly available information including market prices. Moreover, the model can, in principle, be applied to smaller, not publicly traded institutions as well, provided that their probabilities of default and their exposures to common risk factors can be estimated based on available information.

After the tail risk of the whole financial system has been quantified by means of the system-wide ES, it is allocated to the individual banks based on their marginal risk contributions. An important advantage of this method is the full allocation property, which means that the sum of systemic risk contributions attributed to individual institutions equals the system-wide risk in the aggregate. For the purpose of simulating the portfolio loss function, upon which the calculation of the portfolio ES and the risk contributions is based, we adopt a two-stage importance sampling method.

We also apply the approach proposed to a sample of the world's major commercial banks and calculate the ES and ES contributions on a monthly basis for a time period of 13 years. Based on the results of the empirical study, we, firstly, were able to match very closely the list of G-SIBs revealed by FSB. Thereby, advantage of our approach is that it can be utilised only using publicly available information, whereas to come up with the list of G-SIBs information on some of the indicators had to be directly collected from banks. Secondly, we could analyse the impact of systemic risk drivers both in the time-series and cross-sectional dimension.

The main findings of the empirical study with regard to the risk drivers and corresponding policy implications may be summarised as follows:

- On the one hand, the possibility of joint defaults in a banking sector (made up of individual probabilities of default and correlations among the banks) governs to a large extent the evolution of systemic risk over time, whereas changes in the total size of the banks' liabilities are far less important in this context. Therefore, a solid capital base throughout the cycle and the de-correlation of banks' asset values should be the main focus of macroprudential banking supervision.

- On the other hand, given a particular level of systemic risk at a particular point in time, the distribution of the relative risk contributions depends strongly on the size distribution of the banks. But still there are institutions whose share in the system's ES is greater than their share in the overall liabilities, indicating a disproportionately large impact on the system. Therefore, although the large banks should always be subject to more intensive supervision, tying the bank-specific systemic capital surcharges solely to the institution's size would not address the issue of negative externality. The surcharges should be directly linked to the banks' systemic risk contributions, which are calculated not on a stand-alone basis but in interaction with other banks.

- Finally, concentrated sectors containing a few big banks impose a considerable systemic risk even if these banks are well capitalised. Thus, in a system comprised of only a few large banks it may be sensible to split them up.

Having computed banks' individual contributions to overall systemic risk, we can depart from the binary approach, whereby some firms would be considered of systemic importance and others would not, which would leave room for regulatory arbitrage. Using individual tail risk contributions, the binary concept can be refined to the desired degree either by introducing several systemic rating categories or by utilising a direct functional link between an institution's marginal contribution to systemic risk and its degree of systemic importance.

Relying on the marginal ES contributions as a measure of the institutions' systemic importance, policy tools can be adjusted accordingly. One policy option would be to impose a systemic capital charge as the amount of the systemic risk contribution not covered by minimum capital requirements. Increasing overall risk-based capital requirements would reduce the probability of systemically important banks becoming distressed. Moreover, since additional capital is costly, this would address the negative externality posed by a possible failure of a systemically relevant bank. Regarding the time dimension of systemic risk, we suggest a countercyclical capital add-on to be maintained when the markets are exceptionally overoptimistic in their perception of the risk of default, such as to mitigate excessive risk taking. This capital add-on may be calculated as the positive difference between the ES, as measured using a time-varying level of the regulator's tolerance toward the probability of a systemic event, and the ES, as measured using a fixed tolerance level of, say, 0.1%.

Summarising, the portfolio approach, which we put forward for modelling a banking system, can help us to understand the complex nature of systemic risk in terms of its cross-sectional dimension as well as its evolution over time. The suggested integrated approach not only delivers a continuous measure of banks' systemic importance, which we use for calculation of the bank-specific systemic capital surcharges. This approach can also be used for calculation of countercyclical capital buffers to be maintained during the booms in order to mitigate possible procyclical effects of regulatory tools based on a tail-risk measure. Further theoretical and empirical research, however, is required to ensure that model-based policy tools for the treatment of systemic risk are viable and robust before they are put into practice.

AppendixA. Importance sampling algorithm for the portfolio loss distribution

In this appendix we briefly describe the two-stage IS algorithm adopted for simulation of the portfolio loss distribution and refer to Glasserman and Li (2005) for further details.

Default events in a portfolio are typically positively correlated. They tend to occur simultaneously, driven by systematic risk factors. Thus, the first essential step in the variance reduction of the IS estimates is to modify the joint Gaussian distribution of systematic risk factors $\mathbf{Y} = (Y_1 \ldots Y_n)'$ in order to produce more "bad" realisations (negative values, in our case), so that joint defaults in the portfolio occur more frequently, leading to large portfolio losses.

The transformation could be accomplished by shifting the mean of $\mathbf{Y}$ from $\mathbf{0}$ to $\boldsymbol{\mu}$, leaving the initial correlation matrix (denoted by Σ) unchanged. Depending on x_q, which is the quantile of the portfolio loss distribution associated with the chosen confidence level q, the new mean vector can be found according to the solution of the following maximisation problem:

$$\boldsymbol{\mu}_{x_q} = \arg\max_{\mathbf{y}} \left\{ -\theta x_q + C_{PL|\mathbf{Y}}(\theta) - \frac{1}{2}\mathbf{y}'\Sigma^{-1}\mathbf{y} \right\}, \tag{A.1}$$

with $C_{PL|\mathbf{Y}}(\theta) = \sum_{i=1}^{n} \ln\left(1 - p_i(y_i) + e^{w_i \cdot LGD_i \cdot \theta} p_i(y_i)\right)$ being the cumulant generating function of the conditional portfolio loss distribution. Here $\mathbf{y} = (y_1, \ldots, y_n)'$ denotes a realisation of the random vector $\mathbf{Y}$.

Now, when we have a realisation of the systematic risk factors, the second step in the variance reduction is to modify the banks' conditional probabilities of default so as to increase the likelihood of individual banks' failure. To do so, we shift the mean of the conditional loss distribution into the region $[x_q, 1)$ by increasing conditional default probabilities.

To make the conditional expected loss equal the threshold x_q, we set the conditional default probabilities, initially given by

$$p_i(Y_i) = \Phi\left(\frac{\Phi^{-1}(p_i) + a_i Y_i}{\sqrt{1 - a_i}}\right), \tag{A.2}$$

to their exponentially tilted values $p_i(Y_i, \theta)$, which depend on the tilting parameter θ:

$$p_i(Y_i; \theta) = \frac{e^{\theta \cdot w_i \cdot LGD_i} p_i(Y_i)}{1 - p_i(Y_i) + e^{w_i \cdot LGD_i \cdot \theta} p_i(Y_i)}. \tag{A.3}$$

The optimal tilting parameter $\theta_{x_q}(\mathbf{y})$ can be found by solving:

$$\theta_{x_q}(\mathbf{y}) = \left\{ \theta \ : \ \sum_{i=1}^{n} w_i \cdot LGD_i \cdot p_i(y_i; \theta) = x_q \right\}. \tag{A.4}$$

If $x_q > E[PL \mid \mathbf{y}]$, then $\theta_{x_q}(\mathbf{y})$ is positive and the tilted default probabilities $p_i(y_i; \theta_{x_q}(\mathbf{y}))$ are greater than the original ones, leading to larger portfolio losses. Otherwise, $\theta_{x_q}(\mathbf{y})$ is negative and should be set to zero in order to estimate the tail risk, because there is no advantage in reducing $p_i(y_i)$. In other words, the appropriate choice of tilting parameter in our setting is:

$$\theta^{+}_{x_q}(\mathbf{y}) = \max\{0, \theta_{x_q}(\mathbf{y})\}. \tag{A.5}$$

It is important to accentuate the fact that there is no need for a repetitive computation of shifting and tilting parameters for different loss levels x_q. Although the parameters μ_{x_q} and $\theta^+_{x_q}(\mathbf{y})$ depend on a particular loss quantile, it is sufficient for a practical implementation to choose only one value of x_q. This loss level should be located in the tail, close to $VaR_q(PL)$ and can be chosen on the basis of a brief preliminary MC simulation run. The exact position of the loss threshold is not critical. For the chosen value of x_q, equation (A.1) needs to be solved numerically only once before starting the first simulation run. $\theta^+_{x_q}(\mathbf{y})$ has to be determined once for each realisation $\mathbf{y}$.

Taking this information into account, we suggest the following IS simulation algorithm:

- Choose an appropriate loss level x_q.
- Find $\boldsymbol{\mu}_{x_q}$ by solving (A.1).
- For each replication $k = 1, \dots, s$:
  - sample a vector $\mathbf{y}$ from $N(\boldsymbol{\mu}_{x_q}, \Sigma)$;
  - calculate $p_i(y_i)$ according to (A.2);
  - set $\theta^+_{x_q}(\mathbf{y})$ as in (A.5) by solving (A.4);
  - calculate $p_i\big(y_i; \theta^+_{x_q}(\mathbf{y})\big)$ according to (A.3);
  - generate default indicators $D_i(y_i)$ from $Be\Big(p_i\big(y_i; \theta^+_{x_q}(\mathbf{y})\big)\Big)$;
  - calculate portfolio loss PL^k in each kth simulation run as in (3.4);
  - calculate the corresponding likelihood ratio:

$$l\big(PL^k\big) = \exp\left[-\theta^+_{x_q}(\mathbf{y})PL^k + C_{PL|\mathbf{Y}}\big(\theta^+_{x_q}(\mathbf{y})\big) + \frac{1}{2}\boldsymbol{\mu}'_{x_q}\Sigma^{-1}\boldsymbol{\mu}_{x_q} - \boldsymbol{\mu}'_{x_q}\Sigma^{-1}\mathbf{y}\right].$$

- Calculate the empirical cumulative distribution function of the portfolio loss rate:

$$\hat{F}_{PL}(x) = 1 - \frac{1}{s}\sum\nolimits_{k=1}^{s} \mathbb{1}_{(x,1]}\big(PL^k\big)\, l\big(PL^k\big), \quad x \in [0,1].$$

References

Acerbi, C. and Tasche, D. 2002. On the Coherence of Expected Shortfal. *Journal of Banking and Finance*, 26(7):1487–1503.

Acharya, V., Pedersen, L., Philippon, T., and Richardson, M. 2010. Measuring Systemic Risk. Working Paper.

Adrian, T. and Brunnermeier, M. 2011. CoVaR. Working Paper.

Bartram, S., Brown, G., and Hund, J. 2007. Estimating Systemic Risk in the International Financial System. *Journal of Financial Economics*, 86(3):835–869.

BCBS 2011. Global Systemically Important Banks: Assessment Methodology and the Additional Loss Absorbency Requirement. BIS, Consultative Document.

Borio, C. 2009. Implementing the Macroprudential Approach to Financial Regulation and Supervision. Financial Stability Review 13, Banque de France.

Boss, M., Krenn, G., Puhr, C., and Summer, M. 2006. Systemic Risk Monitor: A Model for Systemic Risk Analysis and Stress Testing of Banking Systems. In *Financial Stability Report*, pages 83–95. OeNB.

De Bandt, O. and Hartmann, P. 2000. Systemic Risk: A Survey. Working Paper 35, ECB.

De Nicolo, G. and Kwast, M. 2002. Systemic Risk and Financial Consolidation: Are They Related? *Journal of Banking and Finance*, 26(5):861–880.

Denault, M. 2001. Coherent Allocation of Risk Capital. *Journal of Risk*, 4(1):1–34.

Düllmann, K. and Puzanova, N. 2011. Systemic Risk Contributions: A Credit Portfolio Approach. Deutsche Bundesbank, Discussion Paper Series 2: Banking and Financial Studies, No 08/2011.

FSB 2011. Policy Measures to Address Systemically Important Financial Institutions. Announcement.

FSB, IMF, and BIS 2011. Macroprudential Policy Tools and Frameworks. Update to G20 Finance Ministers and Central Bank Governors.

Galati, G. and Moessner, R. 2011. Macroprudential Policy – A Literature Review. Working Paper 337, BIS.

Gauthier, C., Lehar, A., and Souissi, M. 2010. Macroprudential Capital Requirements and Systemic Risk. Working Paper.

Glasserman, P. 2006. Measuring Marginal Risk Contributions in Credit Portfolios. *Journal of Computational Finance*, 9(2):1–41.

Glasserman, P. and Li, J. 2005. Importance Sampling for Portfolio Credit Risk. *Management Science*, 51(11):1643–1656.

Huang, X., Zhou, H., and Zhu, H. 2009. A Framework for Assessing the Systemic Risk of Major Financial Institutions. Working Paper 281, BIS.

ICB 2011. Final report. Recommendations of the Independent Commission on Banking.

IFSL 2010. Banking 2010. Research report, International Financial Services London.

IMF 2009. Global Financial Stability Report. April.

Jäger-Ambrozewicz, M. 2011. Closed Form Solutions of Measures of Systemic Risk. Working Paper.

Kalkbrener, M. 2005. An Axiomatic Approach to Capital Allocation. *Mathematical Finance*, 15(3):425–437.

Lehar, A. 2005. Measuring Systemic Risk: A Risk Management Approach. *Journal of Banking and Finance*, 29(10):2577–2603.

Merton, R. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance*, 29(2):449–470.

Nier, E., Yang, J., Yorulmazer, T., and Alentorn, A. 2007. Network Models and Financial Stability. *Journal of Economic Dynamics and Control*, 31(6):2033–2060.

Pykhtin, M. 2004. Multi-Factor Adjustment. *Risk Magazine*, 17(3):85–90.

Segoviano, M. and Goodhart, C. 2009. Banking Stability Measures. IMF Working Paper.

Tarashev, N., Borio, C., and Tsatsaronis, K. 2010. Attributing Systemic Risk to Individual Institutions. BIS Working Papers No 308.

Tasche, D. 2000. Risk Contributions and Performance Measurement. Working Paper.

Tasche, D. 2008. Capital Allocation to Business Units and Sub-Portfolios: The Euler Principle. Working Paper.